

Fluid Flow Operations
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Module - 08
Lecture - 20
Drag, Lift, Cavitation – Part 1: Drag

Welcome to massive open online course on fluid flow operations, in this lecture we will discuss regarding Drag, Lift, Cavitation as a part 1 for the module 8.

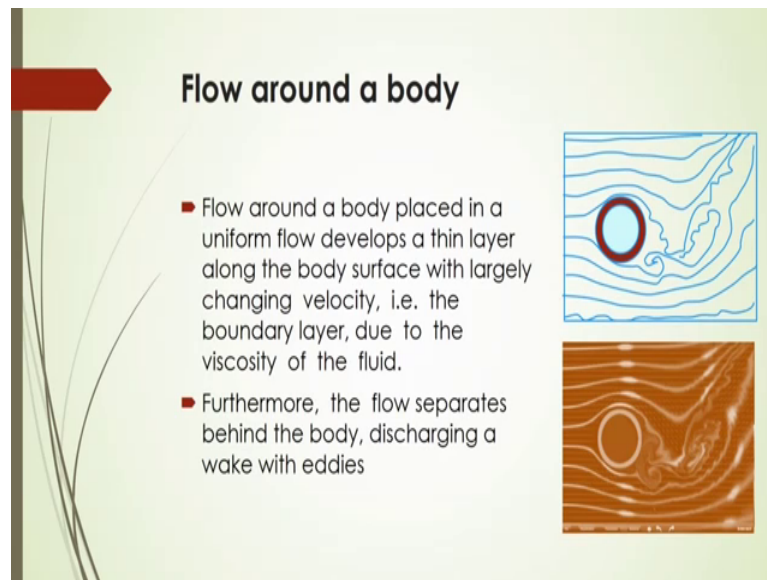
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And this lecture will include drag when flow around a body and what should be the components of the drag whenever fluid will be flowing around that body and also what should be the various cases drag phenomena for the different shapes of the body.

So, we have already discussed different issues regarding the flow channel, even turbulent flow laminar flow in the channel and how a channel will be designed and for how what should be the discharge for that based on that shear stress and drag force also. So, in this case the drag phenomena will be discussed for this typical shapes and how the components of the drag it will be used for that fluid flow phenomena analysis and also what should be the factor that affect on that drag and how it can be calculated. Even what are the components and based on which whether the drag will be contributed by the pressure or drag will be contributed by the shear stress that will be discussed here.

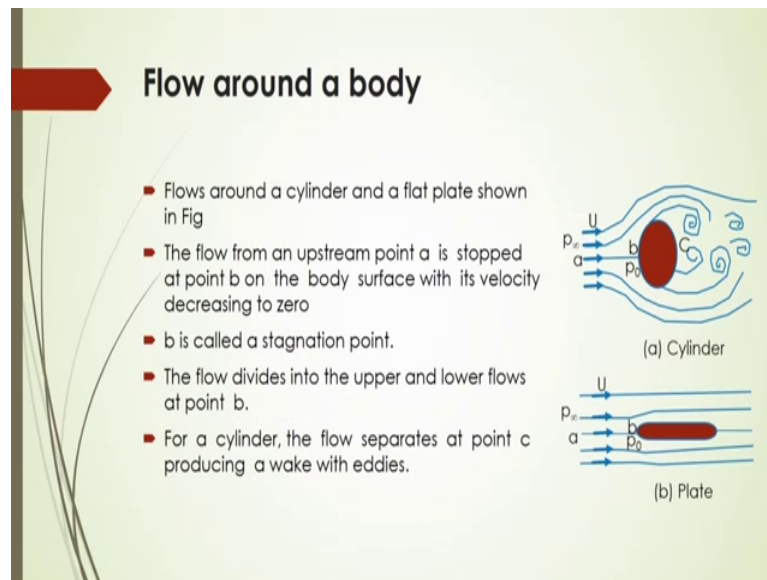
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Now if we consider the flow around a body here, now a body that is placed in a uniform flow and that flow develops in a thin layer along the body surface as shown here body surface see the figure and video also around this body fluid is flowing and whenever it would be cross this solid body. So, how this fluid streams are moving and flowing just by crossing the surface of the solid body here, around that body and that case the thin layer along the body surface with largely changing velocity here and the boundary layer due to the viscosity of the fluid is produced and in that case you will see the flow will separates behind the body and discharging a wake with a eddies here.

So, here eddies are shown in this video and here see this how eddies are forming beyond this object that is here around this position; that means, behind this object how eddies are forming. Here already we have shown this video also here that is see how eddies are forming what the size of eddies are forming here that you can observe. And this is the phenomena you can say that because of the velocity and formation of the boundary there will be a change of this turbulancy behind this what is that object and also there will be a pressure fluctuation and pressure variation behind this objects and because of which the distribution of pressure will give you the formation of this different shape of eddies there.

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And the flow of course, will be separating from this behind of this body and it will discharge a wake with this eddies there. So, in that case we can say that if flow around a cylinder and a flat plate as shown in figure here in the slides and the flow will form an upstream point a and which is stopped at point b here as shown figure. So, at point a that you will get that the flow will flowing with a certain velocity and at point b here whenever flow gets abstract and you will see that there will be a stagnation point and on the body surface here with it is velocity and there will be a decrease of velocity from this a to b here.

And then the you can say that the flow will form an upstream point a , that will be stopped at point b on the body surface with it is velocity by decreasing to 0. And this b is called stagnation point and of course, the flow divides at this point here into the upper and lower flows and for the cylinder the flow separates at point c as shown in figure and you will see producing a wake with eddies.

So, very interesting that so, whenever any solid objects will be placed on a fluid stream you will see that flow will get stucked at a surface of the body at a particular position as shown in here and the velocity will decrease to a 0 at that position and that position or that point is called the stagnation point.

And from that stagnation point the flow will be divided into upper and lower flows and in that case after that behind this object that flow whenever it will be just separating from

the upper part or lower part, it will be producing a circulation and that circulation will produce the wake with eddies.

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Pressure at stagnation point

- Let the upstream pressure at a, not affected by the body, is p_∞ ,
- The flow velocity is u and the pressure at the stagnation point is p_0 .

$$p_0 = p_\infty + \frac{\rho u^2}{2} \quad \text{(Eq. 1)}$$

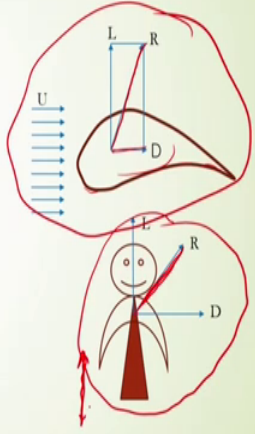
Now, what should be the pressure at that stagnation point? Let us consider that upstream pressure at point a, that will be not being affected by the body and which should be denoted by this p_∞ as shown in figure here p_∞ and the flow velocity is u and the pressure at the stagnation point is let it be p_0 .

And then this p_0 will be is equal to $p_\infty + \frac{\rho u^2}{2}$ by applying the Bernoulli's equation here energy equation. So, we can have this stagnation pressure or stagnation point the pressure will be p_∞ what will be the free stream pressure there that is called p_∞ plus, what will be the kinetic energy there under point. So, it will be is equal to $\frac{\rho u^2}{2}$. So, we can get this stagnation pressure by this equation 1.

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Realization of Drag and Lift

- A body realises a force from the surrounding fluid whenever it is placed in a flow of fluid.
- A flat plate realises a force only in the downstream direction when it is placed in the flow direction,
- However A wing realises the force **R** inclined to the flow as shown in the Figure
- In general, the force **R** acting on a body is resolved into a component:
 - **D** in the flow direction *u* **referred to as drag**
 - the component **L** in a direction normal to *u* is **referred to as lift**



Now, if we say that the body realises a force that is from the surrounding fluid and whenever it is placed in a flow of fluid. Let it be here this object in different shape this object and different shape here this object. So, in this case you will see that this body realises a force whenever it will be placed in a flow of flowing fluid and that force will be realised from the surrounding fluid.

And a flat plate in that case realises a force only in the downstream direction when it is placed in the flow direction there. However, this wing in this case say wing realises the force you will see **R** inclined to the flow as shown in the figure, that depends on the shape of the object. So, for the flat plate that realises a force only in the downstream direction. Whereas for the wing that it will realise the force **R** that will be inclined to the flow as shown in figure, here this is the force here it is realising.

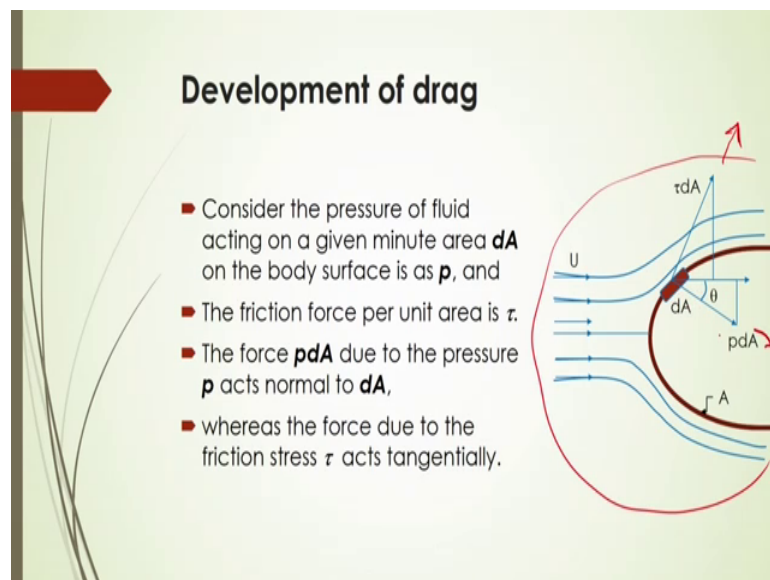
And in general this force **R** acting on the body that resolved into a components like here in the horizontal direction or at the surface the or in the flow direction then it will be considered as the drag, shown on the surface in the flow direction the component of this force acting on the body will be represented by this drag force and the component that the vertical direction in a direction that is normal to the velocity will be referred to as lift here.

If anybody that is placed in a fluid flow it will of course, realise a force and that force will have 2 components shown in the surface in the direction of the flow and another is

in the direction normal to the flow. So, which is that is parallel or you can see in the direction of flow it will be called as drag force and the component which is normal to the velocity that will be called as lift force.

So, in this case this one also this is the object here you will see a fluid stream is covered this bodies at a flow then here this object will get a or we will realise a force R that force will be again divided into 2 parts D and L there. And if this flow velocity is suppose in this direction then here also you can say that the drag force will be in this here opposite to this direction, nowhere in the normal direction there will be no what is that, that is called lift there.

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And development of drag in this case how it will be developed. So, consider this pressure of fluid that acting on a given minute area dA as shown in figure here. So, this is one object shown here in this figure in this object on the surface if we consider a small area dA on the body surface and the pressure acting on the given minute area is p then force will be is equal to that p into dA .

So, that force will be acting in this direction that is normal to this surface. So, here we can say that in the force pdA it will be due to the pressure force p that acts normal to this dA whereas, to the force due to the friction stress that is τ that will be acting tangential to this surface. So, here in this case we are getting this τ into dA in this direction it will be your shear force and here at this direction it will be your pressure force. And this

shear force will give a frictional shear stress whereas, this normal to this shear stress it will be called as that pressure force or pressure stress is there.

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Development of Drag

- The drag D_p , which is the integration over the whole body surface of the component in the direction of the flow velocity U of this force $p dA$, is called **form drag or pressure drag**.
- The drag D_f is the similar integration of τdA and is called the **friction drag**.

The drag (D) $\left\{ \begin{array}{l} D_p = \int_A p dA \cos \theta \\ D_f = \int_A \tau dA \sin \theta \end{array} \right.$ (Eq. 2)

The drag D on a body is the sum of the pressure drag D_p and friction drag D_f , whose proportions vary with the shape of the body.

And development this case drag this D_p which is denoted by D_p for the pressure drag the it will be donated by D_p and which is the integration over the whole body surface of the component in the direction of the flow velocity U of this force $p dA$. Now very interesting that if I have this $p dA$ that is dA is the small unit and p is acting in this direction and then this $p dA$ that is pressure force will have again components 2 components if I consider that this pressure force is making theta angle to this flow of this fluid then at this direction of the flow what should be the component of this $p dA$. It will be $p dA \cos \theta$.

So, this $p dA \cos \theta$ will be regarded as the drag due to the pressure; now this drag due to the pressure is called the form drag or pressure drag, now if I integrate this whole component of this pressure force with this area of this object surface area of this object then we will have the total drag force due to the pressure component it will be D_p that will be is equal to integration of under the area then it will be $\int p dA \cos \theta$.

Again the drag D_f that is here you will see another component in the same direction of the flow it will be coming from the shear force. So, the drag D_f is the similar integration of τdA if we if we consider that. In this case it will be of course, that this one will be is equal to $\int \tau dA \sin \theta$. So, we can say that D_f should be is equal to in this flow of direction it

will be as $\tau dA \sin \theta$ and it will be getting pull that amount of this fictional drag by integrating this whole surface area of the body here.

So, we can define these 2 components of the drag, but this drag will be that is in the direction of the flow here we are considering then we can say that D_p will be is equal to in this direction that is D_p will be defined as integration of $p dA \cos \theta$ and the drag force that is contributed by the friction it will be friction drag and it will be defined as D_f into integration of $\tau dA \sin \theta$.

Now this drag D on a body is the sum of the sum of this that is pressure drag D_p and the friction drag D_f whose proportions vary with the shape of the body. So, very interesting that you have to remember here that we have these 2 components of the drag, one is called form drag, another is called a friction drag, friction drag sometimes it is called skin drag also. So, there are 2 components here. So, if we sum up these 2 components of this drag due to pressure and drag due to friction, then we will get this total drag here.

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Contributions of D_p and D_f for various shapes

Shape	Pressure drag (D_p) (%)	Friction drag (D_f) (%)
	0	100
	≈ 10	≈ 90
	≈ 90	≈ 10
	100	0

So, this is the basic concepts of what is that drag here and contributions of this drag force by pressure and the friction for the various shapes are shown in this table. If you consider this shapes of these here it will say that a pressure drag will be almost 0 and whereas, friction drag will be 100 percent. Now it depends on that how oriented this objects to the liquid or fluid flow there and sometimes if this objects if you are just oriented like this in the horizontal directions laying down like that and if we place this object in vertically

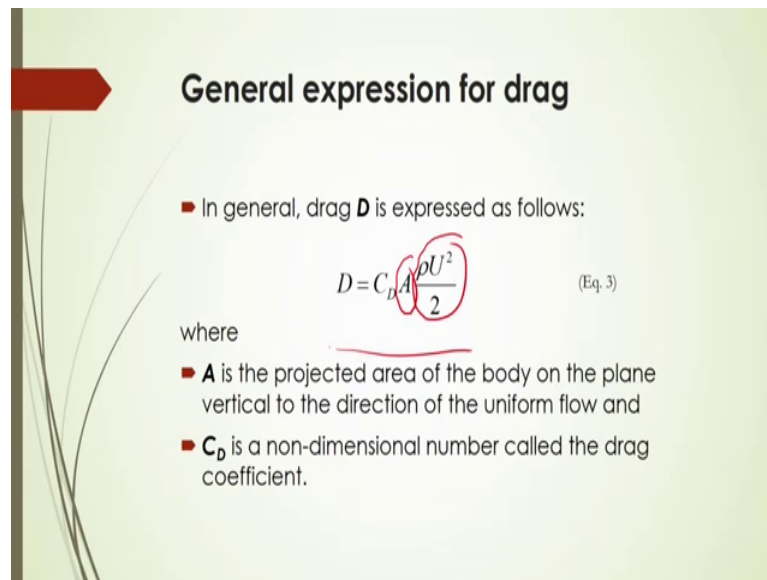
then your ahthat means, the friction drag will be less whereas so, the pressure drag be higher there.

So, accordingly the contribution due to the pressure and the friction that depends on the object surface which is coming in contact with the fluid. Now in this object you will see that pressure drag will be 10 percent whereas, friction drag will be 90 percent. Here maximum portion maximum surface area of this object is come in contact with the fluid with the surface and there will be a friction that is why frictional drag will be maximum this case.

Whereas for this cylinder you will see that only 10 percent of the friction drag will be there because only small portion of the surface of this cylinder is coming in contact with the fluid whereas, the diameter distance of this cylinder will get to contact with the fluid and the fluid will thrust on this surface and that drag will be contributed by the pressure and it will be maximum. And according to this here it maybe 90 percent or remaining is 10 percent there.

Whereas this type of object you will see they are very small; that means, almost negligible amount of surface is in coming with the contact of fluid that is why we can say that there will be negligible amount of a friction drag almost it will be 0; whereas, 100 percent of drag will be contributed by the pressure here. So, in this case we are having the different contribution of the pressure and form drag and summation of this 2 drag will be total drag there.

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General expression for drag

- In general, drag **D** is expressed as follows:

$$D = C_D A \frac{\rho U^2}{2} \quad (\text{Eq. 3})$$


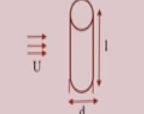
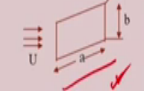
where

- A** is the projected area of the body on the plane vertical to the direction of the uniform flow and
- C_D** is a non-dimensional number called the drag coefficient.

Now, what will be the general expression for the drag? So, in general the drag force **D** is expressed as here as the given in equation number 3 that will be proportional to the kinetic energy and also surface area that is that surface area will be the projected area of the body on the plane that will be vertical to the direction of the uniform flow.

So, this **D** will be is equal to then **C_D** into **A** into ρU^2 by 2, here **C_D** is called a non dimensional number and it will be referred to as drag coefficient. So, here we are having this general expression for this drag force in terms of kinetic energy and the that depends on the also the surface which is a in contact with the fluid and this area should be considered as the projected area of the body on the plane.

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Body	Dimension ratio	Datum area, A	Drag coefficient C_d
Cylinder (flow direction) 	$l/d = 1$	$\frac{\pi}{4}d^2$	0.91
	2		0.85
	4		0.87
	7		0.99
	∞		1.20
Cylinder (right angles to flow) 	$l/d = 1$	dl	0.63
	2		0.68
	5		0.74
	10		0.82
	40		0.98
	∞		1.20
Oblong board (right angles to flow) 	1	ab	1.12
	2		1.15
	4		1.19
	10		1.29
	18		1.40
	∞		2.01

Now, in this case you will see in the table as shown in here, in this case the cylinder if you consider the cylinder in the flow direction then what should be the l , what should be the d .

Here that is length of the cylinder diameter of the cylinder and there would be a uniform flow here that is U . So, based on these if I considered this l by d is equal to 1, what should be the drag coefficient C_d , that will be 0.91 whereas, if we consider this l by d ratio is 2 then you will see there will be a what is that drag coefficient will be different.

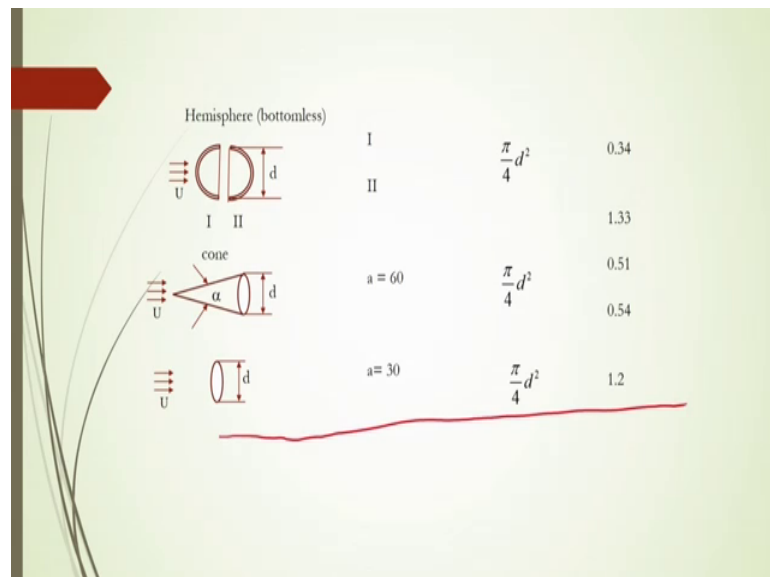
So, for this if we consider this l by d ratio change then you will see that cross sectional area will be almost same that is projected area will be same, but the drag coefficient will be changed because of this what is that a diameter change with respect to l there and this l by d it is called the dimension ratio or sometimes it is called shape factor or it is called what is that a scale ratio there.

And cylinder in this case right angles to the flow here in this case cylinder is shaped that is oriented right angles to the flow in that case you will see that l by d if it is consider 1. Then the area should be is equal to $d l$ whereas, this l by 2 if it is 2 then of again datum area will be the same. Similarly if I change this l by d accordingly whether this what is that projectional area will be almost same in that case the drag coefficient will be varying from this 0.63 to 1.20 if you change the l by d ratio from 1 to infinity there.

Now, another shape if I consider that oblong board right angles to the flow it is a oriented as shown in figure here, where you will have this dimensions a and b. So, if we change these dimensions of a by b accordingly you will see that the surface area will be a b our projected area will be a b and accordingly the drag coefficient will be changing as shown in here in that table.

So, mainly the message is that that if you change the orientation of the object to a fluid flow whether it is normal or whether it is the in the direction of the flow and the change the geometry of that object of course, you will get the different value of drag coefficient.

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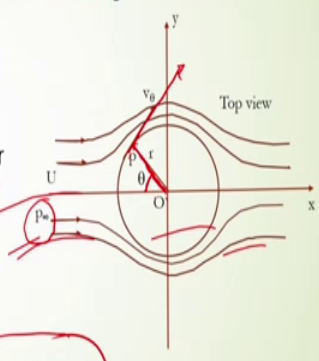


And here also some other values of drag coefficients are given as per object that is hemisphere bottomless here and also for cone how it will be and what will be the value of drag force and U cylinder oval shaped it is a if you can get it what is the value of what is that drag force here. So, from this table you can get the value of C d according to the object.

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Drag for a cylinder (In case of ideal fluid: d'Alembert's paradox)

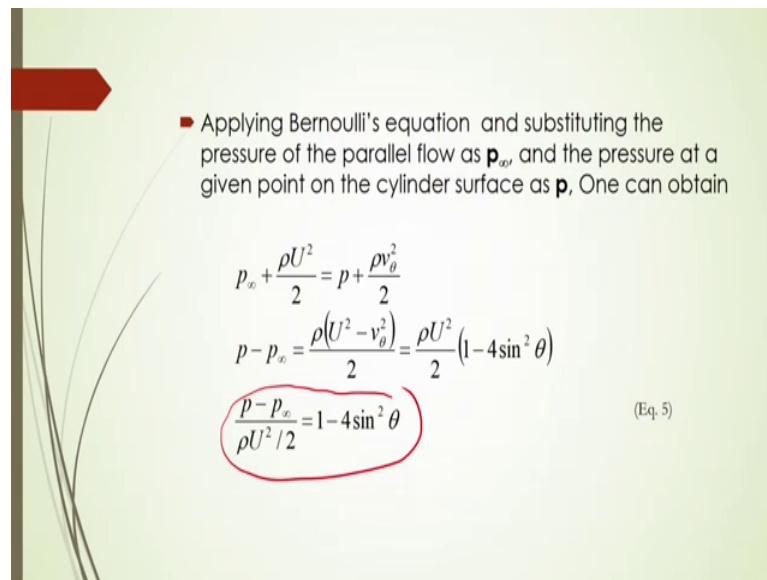
- Let us consider a cylinder placed in a non viscous ideal fluid
- The flow around a cylinder placed at right angles to the flow U of an ideal fluid
- The velocity v_θ at a given point on the cylinder surface is as follows


$$v_\theta = 2U \sin \theta \quad (\text{Eq. 4})$$

Now, if you consider a cylinder in case of ideal flow and what should be the drag, let us consider the cylinder that is placed in a non viscous ideal fluid. Now if the fluid is flowing around a cylinder that is placed at right angles to the flow U of an ideal fluid the velocity v_θ at a given point here on the cylinder surface will be as v_θ will be equal to $2U \sin \theta$.

So, in this direction what will be the velocity you can have this if suppose there is this velocity components in this direction and at the point is making to this horizontal that is flow direction is θ and at this point p at radius, r that is cylinder radius, then you can say that the v_θ this component of this velocity in this direction it will be $2U \sin \theta$.

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■ Applying Bernoulli's equation and substituting the pressure of the parallel flow as p_∞ and the pressure at a given point on the cylinder surface as p . One can obtain

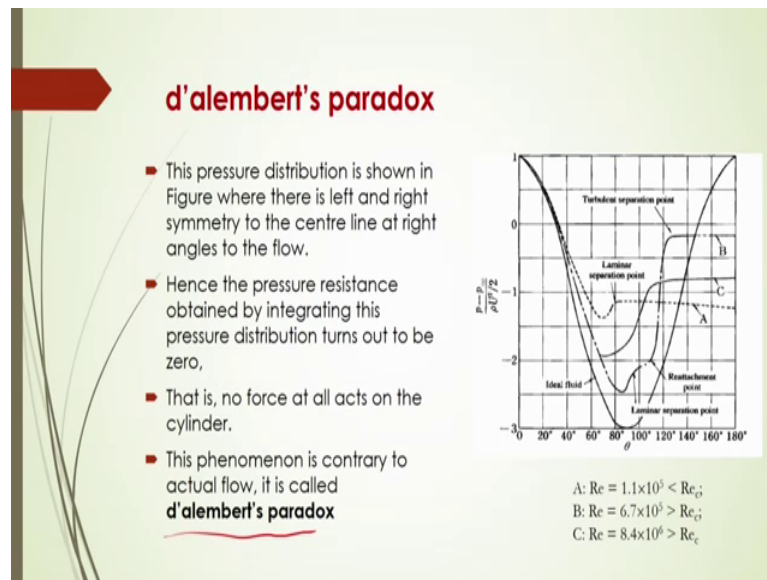
$$p_\infty + \frac{\rho U^2}{2} = p + \frac{\rho v_\theta^2}{2}$$
$$p - p_\infty = \frac{\rho(U^2 - v_\theta^2)}{2} = \frac{\rho U^2}{2}(1 - 4\sin^2 \theta)$$
$$\frac{p - p_\infty}{\rho U^2 / 2} = 1 - 4\sin^2 \theta \quad (\text{Eq. 5})$$

Now, applying this Bernoulli's equation and substituting the pressure on that parallel flow as p_∞ and the pressure at a given point of the cylinder surface as p then we can have this equation as $p_\infty + \frac{\rho U^2}{2} = p + \frac{\rho v_\theta^2}{2}$ and after simplification we can get this equation as $p - p_\infty = \frac{\rho(U^2 - v_\theta^2)}{2}$ that would be equal to $\frac{\rho U^2}{2}(1 - 4\sin^2 \theta)$.

So, if you know this component v_θ then what should be the pressure at that particular point p , there you can easily calculate, but you have to know this p_∞ stream pressure here and also the velocity uniform velocity of the fluid at which it will be crossing this cylinder and making the drag there. And so, this pressure is actually depending on this θ ; that means, here at point, if we consider this point here then θ would be equal to what is that 90 degree and accordingly $\sin 90$ degree that will be equal to 1 then simply you can get this value here and also if suppose θ is equal to 0 then $\sin \theta$ would be equal to 0.

Then p_θ would be equal to 0. So, accordingly you can say $p - p_\infty = \frac{\rho U^2}{2}(1 - 4\sin^2 \theta)$ that will be equal to $\frac{\rho U^2}{2}$. So, simply that pressure difference would be equal to kinetic energy of the fluid there.

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And this pressure distribution is shown in figure here where there is a left and right symmetry to the centre of the central line at the right angles to the flow. Hence in this case the pressure resistance obtained by integrating this pressure distribution that will turn out to be 0 here and that is there would be no force at all that acts on the a cylinder. So, this phenomena is a contrary to the actual flow that will be actually observed by this researcher that is call d' alembert's and according to his that is observation it is called d' alembert's paradox here.

So, very interesting that you will see this pressure distribution how it will be there. So, it is shown in this figure and there will be a some symmetry of this pressure are to the central line at right angles to the flow and it depends on the Reynolds number of course. If your Reynolds number is there then how this pressure distribution would be there according to this equation number 5 and if you change the theta then accordingly it will come now at around 18 degree angle theta we will see there will be a separation of this symmetry here. And then what will happen for this different condition if Reynolds number if of course, it would be if it is less than critical Reynolds number.

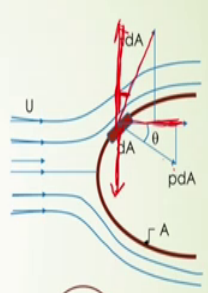
Then we will see the symmetry almost here it is shown in dotted line whereas in B and C how this symmetry would be breaking at an angle theta here. So, this type of a phenomena is a contrary to the actual flow and this is called the d' alembert's paradox here.

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Development of Lift

- The lift L_p , which is the integration over the whole body surface of the component in the direction normal of the flow velocity U of this force pdA , is called **pressure lift**.
- The lift L_f is the similar integration of τdA **normal to the flow** and is called the **friction lift**.

The lift (L) $\left\{ \begin{array}{l} L_p = \int_A pdA \sin \theta \\ L_f = \int_A \tau dA \cos \theta \end{array} \right.$



(Eq. 6)

The lift L on a body is the sum of the pressure lift L_p and friction lift L_f , whose proportions vary with the shape of the body.

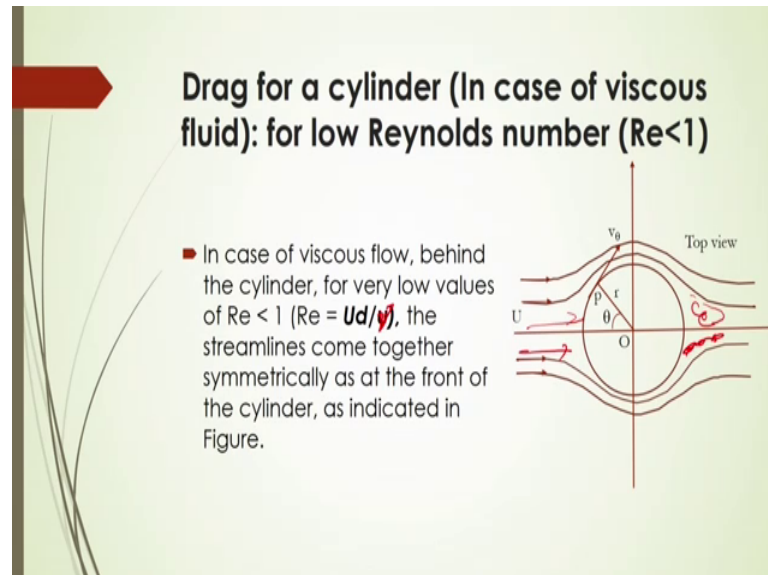
And how then lift it is developed there we will discuss the this lift more details about that lift in the next lectures also, but in this lecture let us see what is that lift other than this drag component. Now, this lift this is the integration over the whole body surface of the component in the direction that will be normal to the flow that we have already told in the previous or earlier slides they are normal to the flow.

If I consider this pressure force and this what is this value this is in the direction of the flow this pressure force will be considered as drag whereas, if we consider this shear stress in this case you will see in this direction what will be the component that is also will be a component of drag. But here if we considered that this shear stress component in this normal direction of this then it will be here, it will be here $\cos \theta$, then this will this, this components will be regarded as lift force because of this shear stress and whereas, for this shear this pdA component in this direction it will be regarded as also lift force due to the what is that pressure force.

So, the total lift force will be summation of this L_f and L_p now this lift L_f is the similar integration of this τdA normal to the flow and L_p again the integration over the whole body surface of the component in the direction normal to the flow velocity U . So, the $pdA \sin \theta$ and if we are integrating over this area of this object then you will get this pressure component of this lift force and the frictional component of the lift force will be by defined as this integration of $\tau dA \cos \theta$. Here also you have to remember that

lift L on a body is the sum of the pressure lift L_p and the friction lift L_f whose proportions vary with the shape of body.

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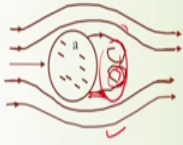
Now, drag for a cylinder in case of viscous fluid if you consider for low Reynolds number what will happen. In case of viscous flow behind the cylinder for very low values where the Reynolds number is less than and this Reynolds number is defined as Ud by ν ν is that is kinematic viscosity that will be dynamic viscosity by density.

So, in this case the streamlines come together symmetrically as at the front of the cylinder and that is indicated in this figure here. So, very interesting that the in case of viscous fluid you will see the how the fluid will be changing its direction from its surface of the cylinder and it will generate the eddies over this here at this location and this is happened only because of this viscous effect and the pressure distribution and this is different from the ideal flow, in the ideal flow we are not considering the viscosity affect there.

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Drag for a cylinder (In case of viscous fluid: for $Re = 2 - 30$)

- If Re is increased to the range 2 - 30 the boundary layer separates symmetrically at position a and two eddies are formed rotating in opposite directions (see the Ref. 1)
- Behind the eddies, the main streamlines come together.



1. Streeter, V.L., Handbook of Fluid Dynamics, (1961), McGraw-Hill, New York.

The diagram illustrates the flow of a viscous fluid around a cylinder. The flow is from left to right. The boundary layer separates symmetrically at position 'a' on the cylinder's surface. Two eddies are formed behind the cylinder, rotating in opposite directions. The main streamlines are shown coming together behind the eddies.


Whereas if we consider that drag for a cylinder in case of viscous fluid for Reynolds number is 2 to 30 in that case for higher that is little bit higher Reynolds number there would be a separation of the boundary layer symmetrically at position a and a 2 eddies are formed rotating in a opposite directions in this reference shear, how this eddies are forms or the flow is getting separates at this location from this cylinder.

And behind eddies the main streamlines come together and making a vortex here. So, at this Reynolds number of flow condition we will see that the boundary layer separations is occurred. Whereas for low Reynolds number where we get here that the boundary layer will be formed, but it will not be separated at this particular Reynolds number.

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Drag for a cylinder (In case of viscous fluid: for $Re = 40-70$)

- With an increase of Re (at $Re = 40 - 70$), the eddies elongate
- A periodic oscillation of the wake is resulted.
- These eddies are called twin vortices.



Streeter, V.L., Handbook of Fluid Dynamics, (1961), McGraw-Hill, New York.
Y. Nakayama, R. F. Boucher Introduction to Fluid Mechanics, Butterworth
Heinemann Publishers

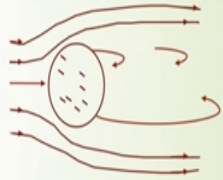
Whereas if Reynolds number if it is further increased what will happen that at around 40 to 70 of this Reynolds number the eddies will try to elongate and there will be a periodical oscillation of the wake and that eddies would be called as vortices and you will see at this particular Reynolds number of range of 40 to 70 you can expect or you can get the eddies formation like that, two vortices will be there.

And that will be almost similar in shape, but a flow directions or circulation direction would be different from each other, but they are almost same in shape and that is why it is called that twin vortices. So, at this Reynolds number there will be a formation eddies and eddies will be like a vortices and two vortices would be formed beyond this or behind this cylinder and it will be called as twin vortices.

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Drag for a cylinder (In case of viscous fluid: for $Re > 90$)

- When Re is over 90, eddies are continuously shed alternately from the two sides of the cylinder



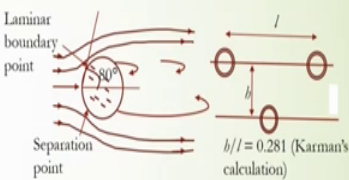
The diagram shows a cylinder in a flow field. Streamlines approach from the left and curve around the cylinder. On the right side, two eddies are shown being shed alternately from the top and bottom surfaces, representing a Karman vortex street.

And if you again this increase of this Reynolds number more than 90 what will happen that eddies are continuously will be shed and alternatively pop the 2 sides of the cylinder there.

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Drag for a cylinder (In case of viscous fluid: for $10^2 < Re < 10^5$)

- The separation occurs near 80° from the front stagnation point
- This arrangement of vortices is called a Karman vortex street.

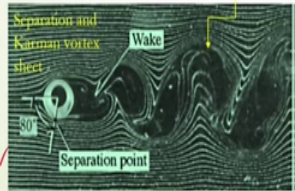


The diagram shows a cylinder in a flow field. The flow is laminar. The separation point is marked at approximately 80° from the front stagnation point. The flow behind the cylinder is characterized by a Karman vortex street. The distance between the cylinders is labeled l and the distance from the cylinder to the vortex street is labeled b . The relationship $b/l = 0.281$ (Karman's calculation) is noted.

$Re < Re_c (= 3.8 \times 10^5)$

At Re_c the boundary layer becomes turbulent

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The microscopic image shows a Karman vortex street. Labels include: Separation and Karman vortex street, Wake, Separation point, and 80° .

And you will see the Reynolds number in this case is more higher so in that case boundary layer will be separated and in such a way that the flow behind this cylinders will get more space to move and you will see there will be a continuous formation of shed that will come alternatively from the 2 sides of the cylinder. Here see this figure if

you consider that even for the increase of Reynolds number from 100 to 10 to the power 5 you will see the because of this viscous effects and the high velocity of the fluid in this case inertia force will be a little bit higher considered to this viscous force. So, the separation occurs near about 80 degree as shown in here in the figure.

In this figure as well as this snap shot figure of experimental condition and you will see that at around 80 degree from the horizontals that is flow directions if we consider the point then you will see there will be complete separation of this flow and also from this statement point of this here at this point there will be separation. And this will occur only for this 80 degree angle and complete separation will form and this arrangement of the vortices whenever it would be formed at this condition and that arrangement will be called as sometimes called the Karman vortex street.

So, how this you will see in this figure, how from the cylinder behind, how this flow gets separates at this Reynolds number range and you will see that will be the vortices will be coming in this way and this we will see this flow or vortices whatever it is formed or wake there those are forms like it will follow a certain what is that fashion of wave and this waves arrangement or path arrangements of this wake formation it is called Karman vortex street.

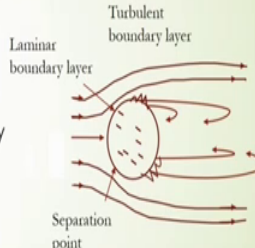
And it is happened only you will see that if the critical number of Reynolds number if it is less than there of 3.8 into 10 to the power 5 and it is of course, the depends on the shape of the cylinder and size of the cylinder also.

And you will see that at critical Reynolds number where the boundary layer becomes turbulent or gets separates and getting the or a forming the wake street like this as shown in figure you can say that the boundary layer no more will exist in that case and flow gets intense mixing beyond that.

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Drag for a cylinder (In case of viscous fluid: for $Re \sim 3.8 \times 10^5$)

- The boundary layer becomes turbulent The separation position is moved further downstream to near 130° from the front stagnation point
- For the rear half of the cylinder, just like the case of a divergent pipe, the flow gradually decelerates with the velocity gradient reaching zero.
- This point is called the separation point, where downstream flow get reversals and developing eddies
- The fluid particles in and around the boundary layer mix with each other by the mixing action of the turbulent flow



$Re > Re_c (= 3.8 \times 10^5)$

And drag for cylinder in case of viscous if it is the Reynolds number is almost 3.8 into 10 to the power 5; that means, at this critical Reynolds number. Then you can say that the boundary layer becomes turbulent and the separation position is moved further downstream to near 130 degree from the front stagnation point.

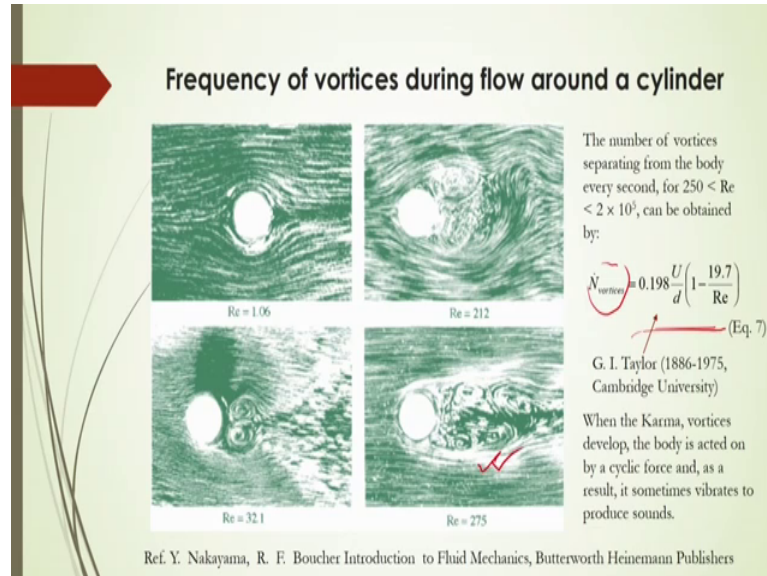
And for the rear half of the cylinder just like the case of divergent pipe here if you consider the flow gradually decelerates with the velocity gradient that will be reaching to 0 and the point where these velocity gradient reaching to 0 will be called a separation point and where the downstream flow get reversals also and the developing eddies there.

The fluid particles in and around the boundary layer that will mix with each other by the mixing action of the turbulent flow there so, at this Reynolds number of critical point that is 3.8 into 10 to the power 5 we are getting that to the rear half of the cylinder in that case just like a divergent pipe the flow will decrease and gradient will almost become 0.

And this point and there the point would be called as that separation point and beyond the separation points particles of fluid particles will be getting intense mix and it will come to each other and there will be a formation of circulation cell. In that way that circulation cell will be called as eddies and that eddies will generate the turbulence in the flow and also the what is that if there is a suppose the application of the chemical or biochemical processes based on this mixing characteristics of this flow, then you can

have the idea of this hydrodynamics where this mixing can happen and when this Reynolds number to be considered for that particular mixing there.

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And also frequency of vortices during the flow around a cylinder, if we consider that there is a formation of what you say at that particular Reynolds number. Then you will see sometimes twin vortices you will see at a Reynolds particular Reynolds number of around I think 32.1 here, if Reynolds number is 275 how this vortices are flowing and how the fluids are mixing mixed and here what is that at Reynolds number 212 how this vortices are formed and if it is a 1 only then there will be no vortices are formed. Now based on this Reynolds number and also the velocity and also what is the diameter of the shape you can have this number of vortices formed at the particular condition.

Now, how many vortices will be formed per seconds that you can calculate, you can estimate also, now this number of vortices that separating from the body every second for Reynolds number is 250 to what is that 2 into 10 to the power 5 that can be obtained by this correlation that is given by Taylor. And in this case then number of vortices will be number rate or it is called number of vortices per unit time that will be is equal to 0.198 into U by d into 1 minus 19.7 by Reynolds number. So, it depends on velocity of the fluid and also fluid properties and the diameter of the cylinder.

And when the Karman, vortices developed the body is acted on a cyclic force and what is as a result it sometimes vibrates to the product or sometimes it will produce the sound

there. So, by this case we have this Karman vortex, vortex sheet; now when this Karman vortex sheets are formed you will see the body is acted on by a cyclic force and as a result you can say that will be a vibration occurs and because of which there be a formation of or production of sound because of this cyclic force.

(Refer Slide Time: 42:19)

Example: A 2 mm dia wire stands erect in air. It starts vibrating with a velocity of 90 km/h when the temperature of air was 20 °C. Find the number of vortices separating from the wire every second. Density of air 0.00125 g/cc and kinematic viscosity = 0.15 stoke

■ **Solution:**
 After calculation $Re = 3.3 \times 10^3$ ✓
 The number of vortices separating from the wire

$$\dot{N}_{\text{vortices}} = 0.198 \frac{U}{d} \left(1 - \frac{19.7}{Re} \right) \quad (\text{Eq. 7})$$

$$= 2460 \text{ cycles/s}$$


Now, let us do an example, like if we consider a 2 millimetre dia wire stands erects in air in that case it starts vibrating with a velocity of 90 kilometre per hour when that temperature of air is 20 degree centigrade then in that case what should be the number of vortices separating from the wire every second.

Now density of air is given to you and kinematic viscosity is also given to you, now in this case you will see Reynolds number will be is equal to 3.3 into 10 to the power 3 after calculation and the number of vortices separating from the wire you can use this equation number 7 for calculating in this case U is known to you d is known to you and also Reynolds number you have calculated as 3.3 into 10 to the power 3.

So, finally, you are getting these 2460 cycles per a second. So, this number of vortices you can expect whenever this you will see the suppose this is an wire, it is placed and the air is flowing at a 90 kilometre per hour. So, in that case from this that is wire you can have this type of vortices here. So, this vortices will how many vortices that will be coming as 2460 cycles per second. So, this is by which you can calculate the vortices that is formed because of this velocity of the fluid.

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Streamline shape

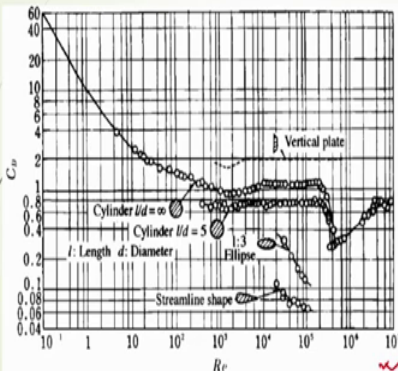



- Most of the contribution to produce drag is due to a separation of stream behind a body which develops vortices and lowers its pressure.
- Therefore, in order to reduce the drag, it suffices to make the body into a shape from which the flow does not separate. **This is the so-called streamline shape.**

Now, another important point is that what is the shape for this streamline. So, streamline shape it is called and in this case most of the contribution to produce drag is due to the separation of stream that behind a body which develops vortices and lower it is pressure. Therefore, in order to reduce the drag it suffices to make the body into a shape from which the flow does not separate. So, in that case this type of streamline of this shape is called streamline shape.

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Drag coefficients for cylinders and other column-shaped bodies



When $Re = 10^3 - 2 \times 10^5$, $C_D = 1 - 1.2$ or a roughly constant value; but when $Re = 3.8 \times 10^5$ or so, C_D suddenly decreases to 0.3.

This is due to the fact that the location of the separation point suddenly changes as it reaches this Re .

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Then drag coefficient, how you can obtain the drag coefficient, you can calculate or estimate the or you can point out or you can extract the drag coefficient for the cylinder and other column shaped bodies as shown in here figure. From this figure you can of course, extract the value of C_D and this C_D is that is drag coefficient depends on this Reynolds number at different Reynolds number this C_D is coming. Where Reynolds number is 10 to the power 3 to 2 into 10 to the power 5 that drag coefficient will be within the range of 1 to 1.2 or a roughly constant value.

But when this Reynolds number is 3.8 into 10 to the power 5 or so, then this drag coefficient suddenly decreases to 0.3 as shown in figure here and this is due to the fact that the location of the separation point that will suddenly changes as it reaches to this Reynolds number.

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Drag of a sphere

- Slow flow ($Re < 0.2$) around a sphere is known as Stokes flow.
- From the Navier-Stokes equation and the continuity equation the drag D is as follows:

$$D = 3\pi\mu U d = C_D A \frac{\rho U^2}{2} \quad (\text{Eq. 8})$$

where

$$C_D = \frac{24}{Re}$$

This is known as Stokes' Equation

Now, if we consider the drag of the sphere. Now if we consider the again the very slow flow rate where Reynolds number is less than 0.2 around this sphere then it will be called as stokes flow and from the Navier Stokes equation and the continuity equation the drag it is as also here as per this definition, then drag will be represented by this equation. In this case the same way that is defined C_D into A into ρU square by 2 where this C_D will be is equal to 24 by Reynolds number for this stokes flow and this equation is called the stokes equation.

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Drag and drag coefficient of a sphere

- For flow ($0.2 < Re < 5$) around a sphere, modification of Stokes equation by Oseen by partly taking account the effect of inertia terms and he found that

$$C_D = \frac{24}{Re} \left[1 + \frac{3}{16} Re \right] \quad (\text{Eq. 9})$$

The drag force

$$D = C_D A \frac{\rho U^2}{2} \quad (\text{Eq. 10})$$

Or

$$D = \frac{24}{Re} \left[1 + \frac{3}{16} Re \right] A \frac{\rho U^2}{2} \quad (\text{Eq. 11})$$

Whereas for the flow if Reynolds number is less than 5 whereas, it will be greater than 0.2 around a sphere you will see there will be a modification of a Stokes equation that is done by Oseen by partly taking into account the inertia terms and he found that that drag coefficient should be is equal to n this as shown in equation 9 and as per definition of this drag force. Then you can substitute this equation number 9 in equation number 10 then you can get this to drag force as equation number 11.

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Drag coefficient of sphere

- Within the range where Re is fairly high, $Re = 10^3 - 2 \times 10^5$
- The resistance is proportional to the square of the velocity, and C_D is approx. **0.44**
- As Re reaches 3×10^5 or so, the boundary layer changes from laminar flow separation to turbulent flow separation. Therefore, C_D decreases to 0.1 or less.
- On reaching higher Re , C_D gradually approaches 0.2.

C_D

Re

Now, again this drag coefficient of the sphere you can get from this figure or graph now within the range of this Reynolds number if it is where the Reynolds number is very high that is 10^3 to 10^5 . Then the resistance will be proportional to the square of the velocity and in that case the drag coefficient will be almost remain same or constant and it will be approximately 0.44.

Now, if we consider the Reynolds number that reaches to 10^3 or so, then the boundary layer changes from laminar flow separation to the turbulent flow separation. Therefore, the drag coefficient will be decrease and because of which these drag coefficient will be 0.1 or even less and even if we consider that high Reynolds number beyond these into 10^5 . Then you will see that the drag coefficient will be gradually approaching to value of 0.2 there.

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Example: An aeroplane passenger weight of 100 kgf comes down to the ground from an aeroplane with the help of a parachute against the resistance of air. The shape of the parachute is hemispherical of 2 m diameter. Find the velocity of the parachute with which it comes down. The drag coefficient $C_D = 0.5$, Density of air is 0.00125 g/cc; viscosity of air is 0.15 stoke.

■ Solution

Projected area $A = \pi/4 \times 2^2 = 3.14 \times 10^4 \text{ cm}^2$.

$$D = C_D A \frac{\rho U^2}{2} \quad (\text{Eq. 10})$$

F_D must be equal to the weight of the man falls down
 So equation this, the velocity U of the parachute will be equal to 31600 cm/s = 31.6 m/s

Now, let us so, an example for this an aeroplane passenger weight of 100 kg force comes down to the ground from an aeroplane with the help of a parachute against the resistance of the air. So, in that case the shape of the parachute is hemispherical if we consider of the 2 metre of diameter. So, in this case you have to find out the velocity of the parachute with which it will come down and the drag coefficient for this is considered as 0.5. Whereas density of the air is considered as that is 1.25 k g per metre cube and viscosity of the air it is 0.15 stokes.

So, in this case simply you have to apply this drag force equation, now you have to calculate first what should be the projected area. The projected area is 5 by 4 into that is diameter square of this hemispherical parachute and it is simply 3.14 into 10 to the power 4 centimetre square. And after substitution of this drag coefficient here C D in this equation 10 and this projected area A. And what will be the U? U is given to you. What is that velocity? Velocity is given to you and after that you can simply calculate what should be the drag force.

Now, F D must be equal to the weight of the what is that mean object or in this case here weight of the man that falls down. So, equation in this case what should be the velocity that you have to calculate, now this drag force will be equal to the weight of the man, now what should be the weight of the man it is given to you. So, based on this equation number 10 if you substitute this D value and C D, A, rho and then after simplification you can get this velocity U of the parachute that will be almost equals to 31.6 metre per second. So, when parachute is moving at this 31.6 metre per second you can say the drag force will be equal to this weight of this man there.

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Drag of a flat plate (contd..)

■ For **Laminar boundary layer** ($Re_l < 5 \times 10^5$) ✓

$$\tau_0 = 0.365 \sqrt{\frac{\rho \mu U^3}{x}} = 0.730 \frac{\rho U^2}{2} \sqrt{\frac{\mu l \rho}{U x}} \quad \text{(Eq. 14)}$$

Therefore

$$D = \int_0^l \tau_0 dx = 0.73 \sqrt{\mu \rho U^3} l b \quad \text{(Eq. 15)}$$

Defining

$$D = C_f l \frac{\rho U^2}{2} \quad \text{(Eq. 16)}$$

The friction drag coefficient $C_f = \frac{1.46}{\sqrt{Re_l}} \quad \text{(Eq. 17)}$ where $Re_l = Ul/\nu$

Now, what should be the drag of a flat plate when a uniform flow of velocity U that flows parallel to a flat plate of length l in this case, the boundary layer of steadily develops showing to the viscosity and the drag force is as described also earlier that it will be is equal to vr rho into u into U minus u, that is capital U is v stream velocity as

shown in figure here and τ_0 that is shear stress wall shear stress that will be dD by dx that will be simply this equation as shown in here 13.

And for laminar boundary layer you can say whenever Reynolds number within this range then shear stress will be is equal to this that already we have discussed earlier also boundary layer theory lecture. And therefore, we can say that drag force will be here denoted by this equation number 15 and defining this drag force here and equating this equation then you can say that what should be the C_f , C_f is called that friction drag coefficient and this C_f will be is equal to 1.46 by root over Re_l . This Re_l is defined as this Reynolds number based on the length of the flat plate here so Ul by μ here.

So, based on this equation number 14 we can calculate what should be the shear stress and if we know the shear stress. What should be the drag force that will be given by equation number 15 and based on the general definition of the drag force, we can find out what should be the drag coefficient due to this friction and this is called friction drag coefficient as given in equation number 17.

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Drag of a flat plate (contd..)

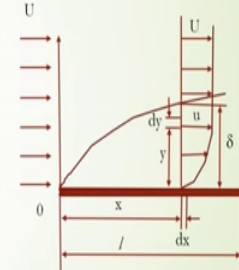
■ For **Turbulent boundary layer** ($5 \times 10^5 < Re < 10^7$)

$$\tau_0 = 0.029 \rho U^2 \left(\frac{\nu}{Ux} \right)^{1/5} \quad (\text{Eq. 18})$$

$$D = \frac{0.036 \rho U^2 l}{Re_l^{1/5}} \quad (\text{Eq. 19})$$

$$C_f = 0.072 Re_l^{-1/5} \quad (\text{Eq. 20})$$

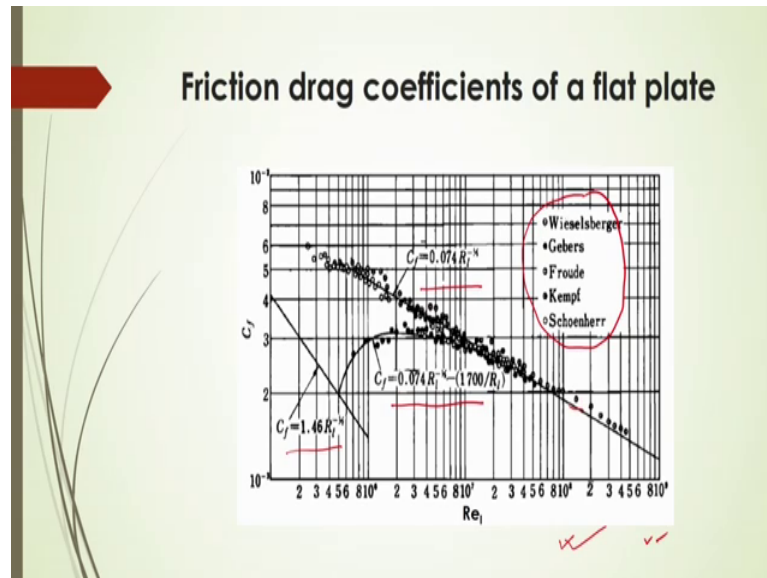
From experimental result

$$C_f = 0.074 Re_l^{-1/5} \quad (\text{Eq. 21}) \quad \text{where } Re_l = Ul/\nu$$


Now, if we consider these turbulent boundary layer, then again you have this shear stress after integration of this equation number 13 and then it will come here and D again it will come as per equation number 19.

And then comparing with the general definition of the drag force and equating this based on this equation you can get this C_f that is friction drag coefficient it will be equal to $0.072 Re^{-1/4}$. So, this is as per theory whereas, from the experimental results it is obtained as what is that $0.074 Re^{-1/5}$. So, there will be hardly actually a difference in this theory and this experimental results.

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So, we can have at a different that is laminar and the boundary layer condition what should be the drag force and to the drag coefficient based on this laminar and turbulent boundary layer and also the friction drag coefficients that C_f that is in equation number 21 you can calculate or you can estimate from this or you can take from this graph for further calculation.

And you will see this are developed from the different what is that experimental data obtained from the different sources of experiment that is the published by different investigators and based on the data that this form lines are produced as per equation here. Here this datas will give you the C_f values like this and this one is the C_f is like this and this is C_f is like this. So, several correlations develop based on the experimental data obtained by different investigators here.

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Example: A smooth flat plate 2.5 m wide and 5 m long moves through water at a relative velocity of 1.5 m/s parallel to its length. Calculate the drag force on one side of the plate: (a) for laminar boundary condition, (b) For turbulent boundary conditions over the entire plate. Density of liquid is 1000 kg/m³. Viscosity of water is 0.001 kg/m.s

Solution:
Area of plate in one side: $2.5 \times 5 = 12.5 \text{ sq. m}$ ✓
Reynolds number: $Re = 1000 \times 1.5 \times 5 / 0.001 = 7.50 \times 10^6$
For Turbulent boundary condition: $C_f = 1.46 \times (7.50 \times 10^6)^{-1/2} = 6.16 \times 10^{-2}$ by ✓
(Eq. 17) ✓
For Turbulent boundary condition: $C_f = 0.074 \times (7.50 \times 10^6)^{-1/5} = 3.12 \times 10^{-3}$ by
(Eq. 21)
Therefore Drag force:

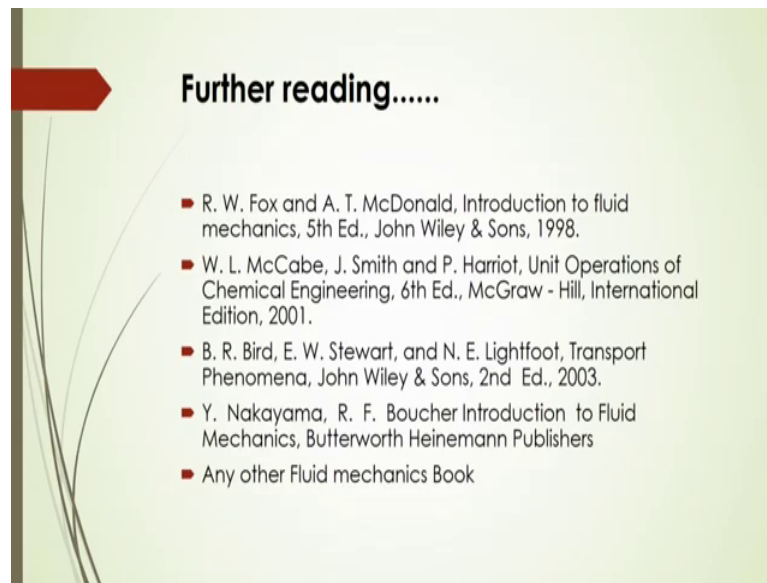
$D = C_f A \frac{\rho U^2}{2}$ $= 6.16 \times 10^{-2} (2.5 \times 5) \times \frac{1000 \times 1.5^2}{2}$ $= 866 \text{ N For Laminar boundary layer}$	$D = C_f A \frac{\rho U^2}{2}$ $= 3.12 \times 10^{-3} (2.5 \times 5) \times \frac{1000 \times 1.5^2}{2}$ $= 439 \text{ N For turbulent boundary layer}$
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Let us do an example of a smooth flat plate of 2.5 metre wide and 5 metre long moves through water at a relative velocity of 1.5 metre per second parallel to its length then in that case calculate the drag force on one side of the plate and for laminar boundary condition and for turbulent boundary conditions over entire plate and density of the liquid is given and viscosity also it is given.

So, in this case we have to calculate area of the plate in one side here given and what should be the Reynolds number you have to calculate as shown in here and also the turbulent boundary conditions. This is sorry this is laminar boundary conditions here, this is laminar boundary condition, the C_f will be is equal to this similarly for turbulent boundary conditions C_f will be is equal to this.

Then drag force will be as per this definition for this laminar boundary condition what should be the value and for turbulent boundary condition what should be the value, that is given and for laminar it is 866 Newton whereas, for turbulent it will be 439 Newton. So, for the turbulent boundary layer you will have the less drag force compared to the laminar boundary layer.

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So, I would suggest to read further for better understanding even more information of this drag force for different applications and you can get the informations from this textbooks even further that you can follow other textbooks for fluid mechanics.

Thank you for your attention for this lecture.