Fluid Flow Operations
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Module - 07
Flow in Water Channel-Part 1
Lecture - 18
Flow Velocity and Optimum Shape

Welcome to massive open online course on Free Flow Operations. In this lecture under Module 7 as a Part 1, we will discuss the Flow Velocity and Optimum Shape whenever flow in water channel. So, in this lecture we will discuss the flow in open channel with constant section and the constant flow velocity.

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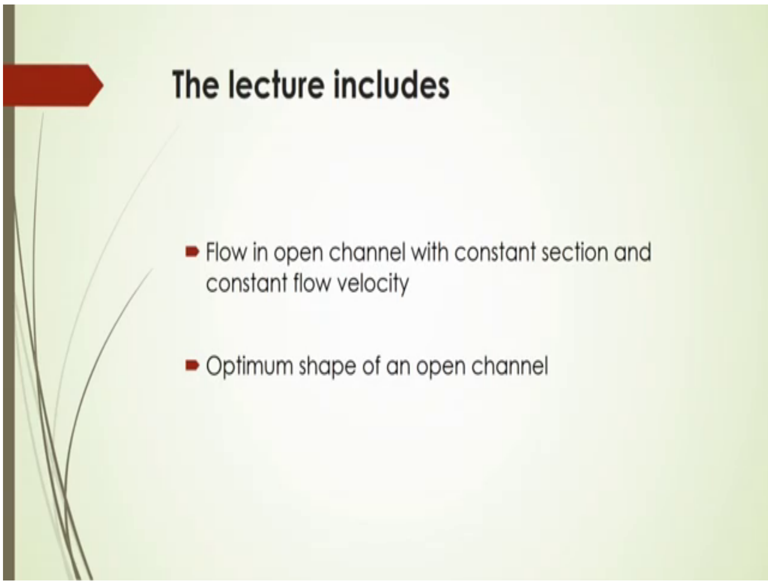

And what will be the optimum shape of an open channel based on the optimum flow velocity and the results through the channel.

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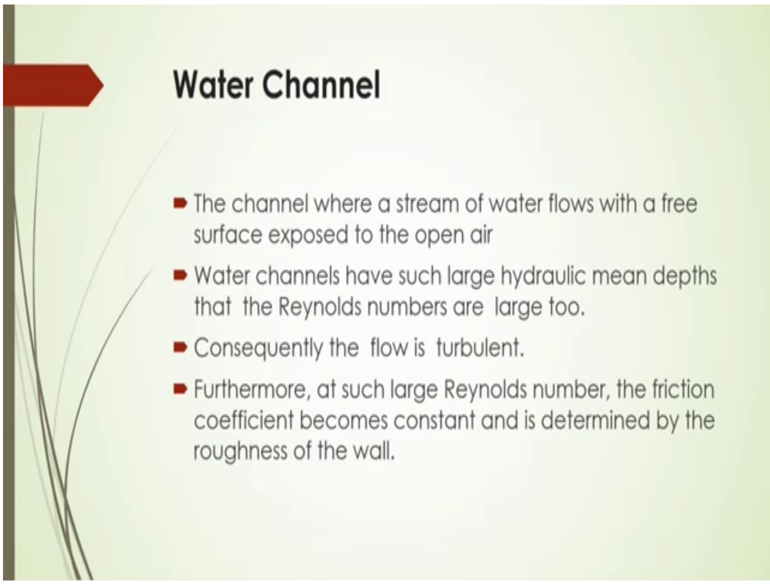

And before going to that of course we have to know what is that water channel. So, the channel that is a passage and where is a stream of water will flow with a free surface which will be exposed to the open air and other channels have of course large hydraulic mean depth where the Reynolds numbers will be too large. But in the laboratory scale, it may be the small in mean depth and their Reynolds number may be low. So, with a certain flow reserve our flow pattern we can have the water channel based on particular operation. And, for laboratory skill generally for estimating the optimum discharge flow velocity and what will be the optimum depth of the channel and after that if it is a scale up, then you will get the proper water channel with a certain hydraulic mean depth there.

So, if you water channel we will have large hydraulic mean depth and there had the Reynolds number is too large. So, in that case the flow will be turbulent and based on that channel diameter that is hydraulic diameter as well as the flow properties and also, we can say that at this high Reynolds number, the friction coefficient will be almost constant. As we have already discussed in our earlier lectures that within a certain range of a Reynolds number, we are getting different friction factor based on the Moody chart or different correlations that is suggested by different investigators.

And based on that correlations, you can calculate the friction coefficient and if the Reynolds number is too high, it is greater than 10 to the power 5, then you will see that the friction coefficient will be almost constant and that friction factor of course depends on the roughness of the wall whether it will be whether the flow will be through the channel or to the pipe there. So, based on that concept you have to consider that at high Reynolds number what should be the friction factor. And once you know that friction factor what should be the other frictional resistance or force for resistance even how that resistance will be utilized to design that channel that is you have to know.

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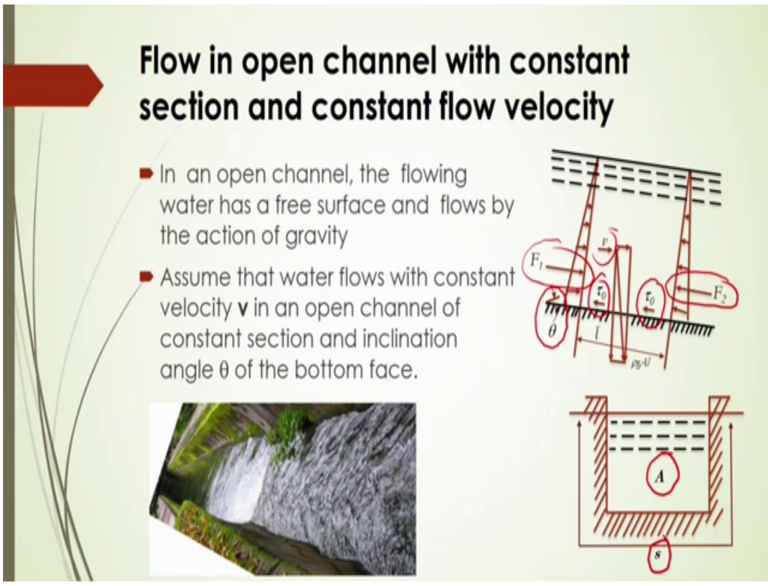

Now, if we consider the open channel with constant section and constant flow velocity where in an open channel the flowing water has a free surface and flows by the action of gravity. If we consider that here as shown in this figure that the channel is inclined at a certain angle with the horizontal that is theta and water is flowing through the channel under the gravity with a certain velocity.

And in this case, the gravitational force will be acting along that is the direction of the flow. So, what will be the that is gravitational force in that particular direction that you have to calculate. After that you have to balance those forces with other shear forces there. So, in that case if we assume that water flows with a constant velocity b in this open channel of constant section here and at an inclination angle of theta of the bottom phase here as shown in figure.

The other forces acting on this here F1 is one force is acting during this flow and opposite force also will be acting traces to this flow and what will be the shear stress? This is denoted by tau 0 velocity at which the water is flowing is denoted by v and the angle at which this bottom of the channel is inclined with this horizontal axis will be theta and the cross-sectional area of this channel is denoted by A and the wetted surface area of the channel to be denoted by s.

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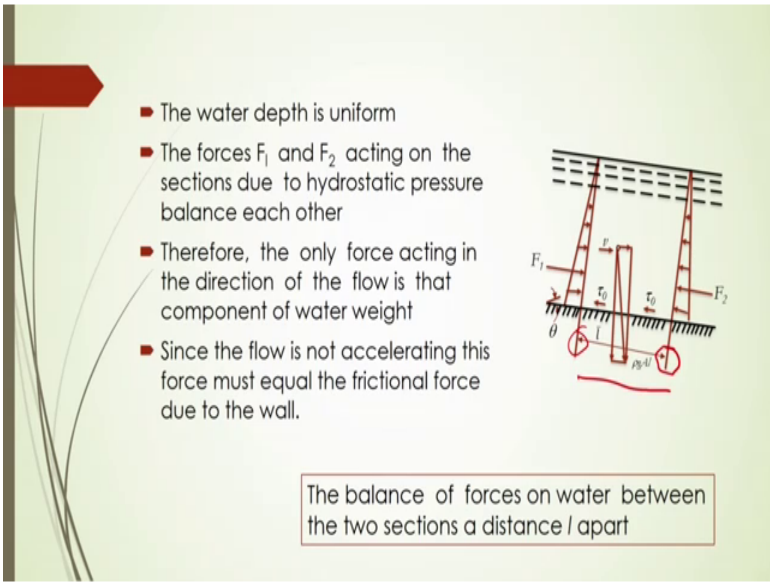

So, based on these terms we will be able to find out the force how it will be acting on this water for this flowing and what should be the frictional force due to this wall there. Now in this case you have to consider that the water depth will be uniform and the force that F1 and F2 acting on the sections due to the hydrostatic pressure and those hydrostatic pressure that the components of that hydrostatic pressure will give you this force F1 and F2 and this F1 and F2 will balance each other there during this flow.

Therefore, we can say that the force that are only acting in the direction of the flow is that component of the water weight and the flow will not be accelerating this force that must be equal the frictional force that due to the wall and here this due to this frictional force, you will see there will be a friction that is shear stress will be acting opposite to the direction of the flow. So, this force will balance the other forces on the water between these two sections here as mentioned at a distance of l here. So, the distance between these two sections are l. So, within this l you have to do this balance of forces on water based on these forces F1 and F2 also. What is the shear stress there?

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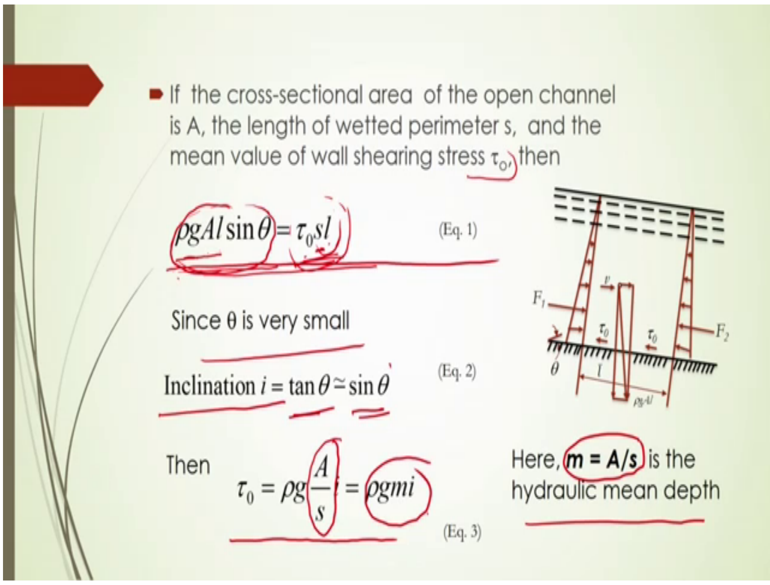

Now, if we consider that the cross sectional area of this open channel is A and the length of wetted perimeter is s and the mean value of the wall shear stress that is denoted by tau 0, then we can say the balance equation will be forming that is denoted by or represented by equation number 1 here. So, in this case this is your component of the weight of the water that is flowing through the channel. What is that? Rho into g into al; so, this is your mass into g. That means, here weight so rho into a into l with this volume.

So, rho into volume that is mass into z this is your gravitational force and since this surface of the bottom of this channel is inclined with theta, so in the that is in the vertical direction. What should be the gravitational force component? It will be coming as rho g a l into sin theta and this force of course will be balanced with the shear force. So, that we will water will not accelerate further. So, in this case what would be the shear force? The shear stress into area surface area at which the surface will be weighted by the liquid.

Now, in this case tau 0 the shear force and s is the surface area into l. So, this is your total area, this is s is perimeter into l that is your total surface area. So, tau 0 into surface area that will give you the shearing force and so, based on the balance of this equation we can simplify for this tau 0 here. Now, since the theta is very small we can denote this by i which is called inclination and this inclination i will be equals to tan theta which will be almost equals 2 sin theta for this small angle theta, then after substitution of this sin theta will be equals to i there.

Then we can represent the shearing force as tau 0 would be equals to that is rho g into A by s into i. Now, if we define this A by s that is cross section of the channel and the wetted perimeter s, the ratio of these two if it is represented by m, then we can have tau 0 is equal to rho g m i. Now, this m is equal to A by s is the hydraulic mean depth. It is called hydraulic mean depth. So, tau 0 you can express in terms of this hydraulic mean depth as rho g m into i as given in equation number 3.

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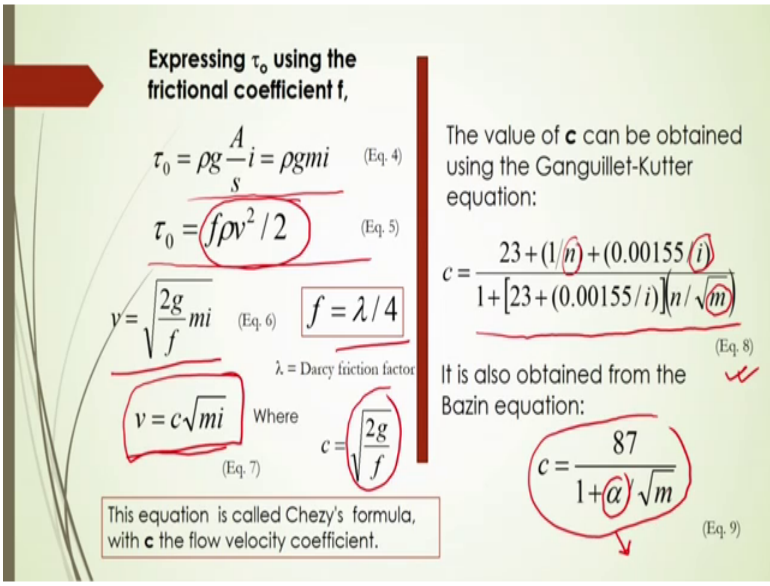

Now, again this shear stress you can express based on the frictional coefficient that frictional coefficient is denoted by f. There are two frictional coefficient, two types of frictional coefficient. One is Fanning friction coefficient and another is Darcy waste based friction coefficient or friction factor and the Fanning friction factor that will be equals to one by fourth of Darcy friction factor that we have already discussed in our earlier lecture.

So, here you can express this shear force whenever liquid is flowing through the channel and in that case we can have this tau 0 will be equal to rho m rho g m i. Now, this is the then tau 0 will be is equal to f into rho v square by 2 as per definition where this v m will be is equal to then after equating this equation number 4 and 5, we can say that v will be is equal to root over 2 g by f into m i and here f will be equals to lambda by 4 where lambda is called Darcy friction factor.

So, finally we are getting this v will be is equal to c into root over m i where c again is denoted by here or defined by root over 2 g by f. Now, this c is called the velocity coefficient and then, v will be is equal to c into root over m i. This equation number 7 is called the Chezys equation. Here c is signified by the flow velocity coefficient. There this flow velocity coefficient is inversely proportional to the frictional coefficient and it is defined by this root over of 2 g by f.

Now, based on the experimental observation this c will be changing with respect to this m that is called Hydraulic depth and also angle of inclination there of the bottom surface of channel. Now based on the experimental observation Ganguillet-Kutter, they have done some experiment on that and they have suggested one correlation for this prediction of c or estimation of c as it is given in equation number 8 that will be c equals to 23 plus 1 by n plus 0.00115 by i divided by 1 plus 23 plus 0.00115 by i into n by root over m as shown in equation 8 here.

So, in this case this i is one factor, n is another constant and m is called hydraulic depth. So, if you know this inclination angle and m and this parameter n, then you will be able to find out what should be the c. Now this n value of course will be obtained based on the experimental results and this c also can be obtained from the Bazin equation. Bazin also he has done extensive work on this channel flow of water and based on his experimental work he has suggested that c should be equal to 87 by 1 plus alpha by root over m. Here alpha is another constant. He has defined this alpha will be obtained according to the experimental result.

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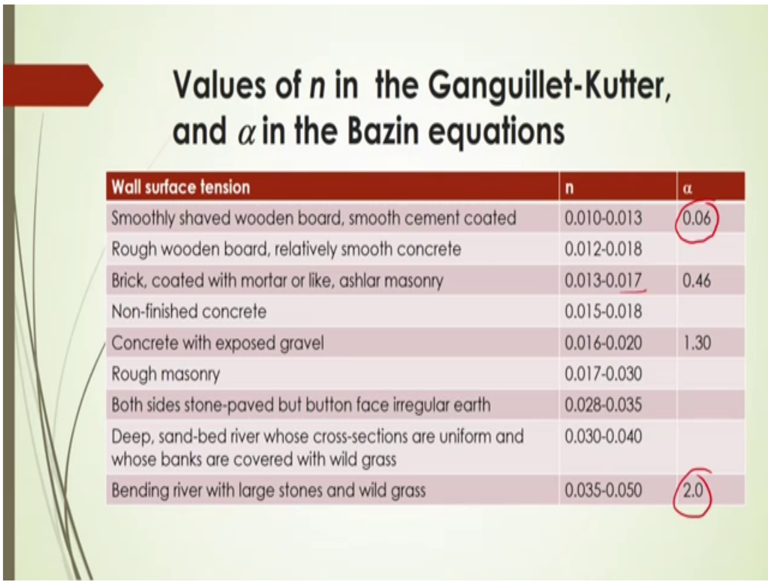

Wall surface tension	n	α
Smoothly shaved wooden board, smooth cement coated	0.010-0.013	0.06
Rough wooden board, relatively smooth concrete	0.012-0.018	
Brick, coated with mortar or like, ashlar masonry	0.013-0.017	0.46
Non-finished concrete	0.015-0.018	
Concrete with exposed gravel	0.016-0.020	1.30
Rough masonry	0.017-0.030	
Both sides stone-paved but button face irregular earth	0.028-0.035	
Deep, sand-bed river whose cross-sections are uniform and whose banks are covered with wild grass	0.030-0.040	
Bending river with large stones and wild grass	0.035-0.050	2.0

Now, in this table some values of n and alpha are given. Now if the experiment is done on smoothly shaped wooden board, smooth cement coated channel, then you will see that n should be within a range of 0.012 0.013 whereas, alpha should be 0.06. So, once you know this n and alpha and if you substitute the value of n and alpha there, then you can easily calculate what should be the value of c that is flow coefficient flow velocity coefficient there and once you know this flow velocity coefficient, then you will be having what should be the velocity of the liquid or water through the channel once you know the perimeter and the cross section of the pipe and also angle of inclination there.

Other examples, other experimental observations also it is noted here that if your channel is made a brick coated with mortar or like ashlar masonry say then you can have this n values in the range of this 0.013 to 0.017. Actually these are this range because of you will see that how you are coating or what will be the materials that you are using and what will the frictional it is called that roughness depth are there.

So, all those factor will be affecting on this value of n and alpha there. So, if you are using that concrete with exposed gravel, then you can have this n value 0.016 to 0.020 whereas, this alpha for this Bazin equation will be 1.30 there and for bending river with larger stones and wild grass if you are considering the experiment there, then you can have this n should be 0.035 to 0.050 whereas, alpha should be is equal to more than 1, it is called 2 here. So, based on this n and alpha value you will be able to calculate what will be the shear stress and order of flow velocity coefficient shear stress there.

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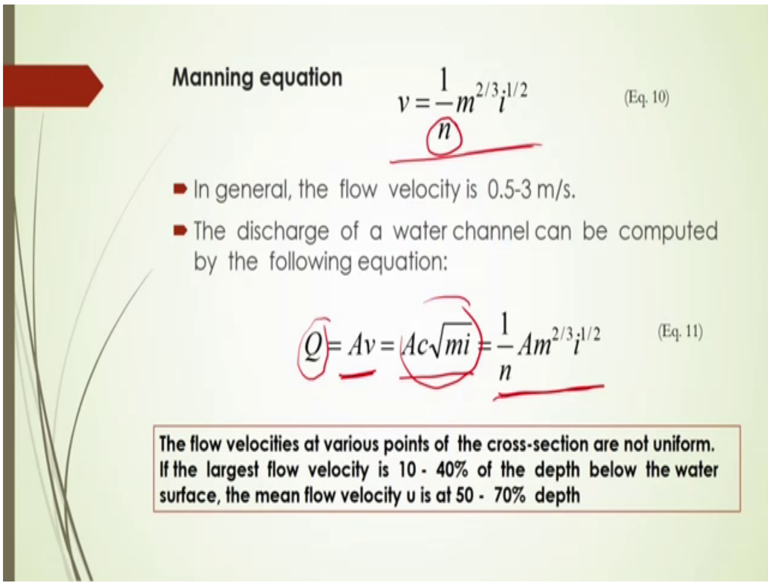

Another very interesting and important equation that is developed and suggested earlier the very beginning of the channel flow this called Manning equation. Now as per Manning he suggested that that v should be to the channel will be 1 by n into m is m to the power 2 by 3 into i to the power 1 by 2. So, here this again this coefficient n you have to know whereas, m is known to you that is a hydraulic depth channel depth and i is the angle of inclination.

Generally, flow velocity through the channel is considered 0.5 to 3 meter per second for the test and the discharge of the water channel which can be estimated by the following equation based on this velocity. So, this is should be is equal to cross sectional area into velocity to come then a into c root over m i and of course, this is as per equation given here earlier that in this case by equation number 7 that is the Chezys equation and this portion is based on the Manning equation. So, from this Chezys equation and the Manning equation, you can calculate the velocity of the water passing through the channel.

Once it is provided to the constants of n and alpha values are there for the calculation of flow velocity coefficient and also here it is important to note that the flow velocities at various points of the cross section are not uniform. So, if the largest flow velocity if it is considered as 10 to 40 percent of the depth below the water surface, the mean flow velocity u will do within a range of 50 to 70 percent of the depth. So, if we know that particular depth below the water and the other characteristics factor and respective coefficients, then you will be able to calculate what will be the discharge there through the channel.

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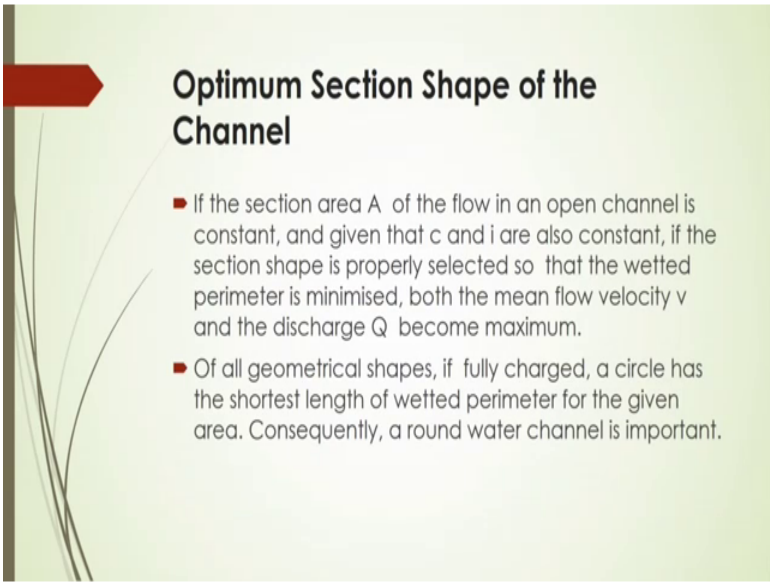

Now, what should be the optimum section shape of the channel? Now to remember that the intersection area A of the flow in the open channel if it is kept constant and the coefficient c and i inclination angle are also kept constant, then the section shape is to be properly selected, so that the weighted perimeter should be minimized to there and in that case both the mean velocity v and the discharge q will be having maximum there.

So, if the section area a of the flow in an open channel is constant and a given that c and i are kept constant and also if the section shape is properly selected, so that the wetted perimeter is minimized and both the mean flow velocity v and the discharge q should be maximum there and now of all geometrical shape if you fully charged of the channel a circle has the shortest length of wetted perimeter for the given area.

Consequently round water channel will give you the optimum discharge for that particular design and that case round water channel will be more important than compared to that certain rectangular or square shaped channel there and if we consider that circular water channel, then we can have the relationship between the water level flow velocity and discharge for the round water channel of the inner radius of r.

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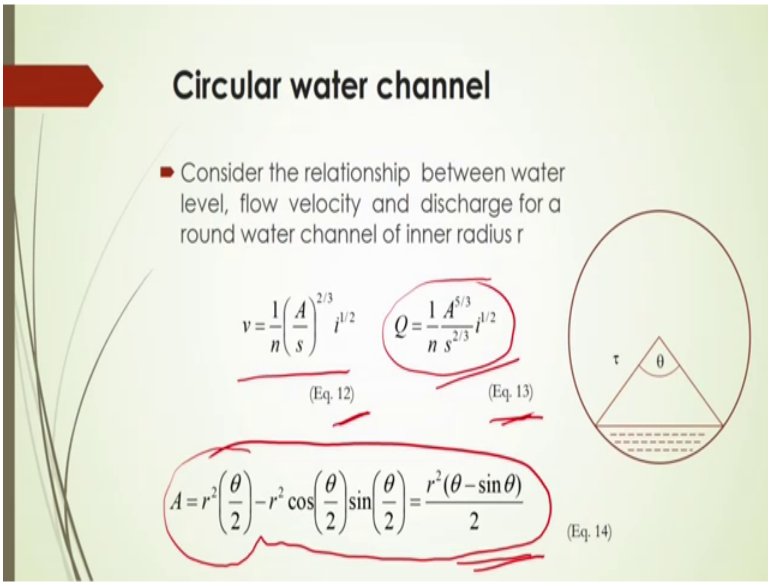

So, in that case the velocity should be is equal to as per that manning equation we can have this v will be equals to 1 by n into A by s to the power 2 by 3 into i to the power half. So, in that case the discharge will be is equal to just v into A, then it will be 1 by n into A to the power 5 by 3 by s to the power 2 by 3 into i to the power half as given in equation number 13. Now if we have this cross sectional area for this round water channel with a certain angle of theta here as shown in figure, now in this case A should will be is equals to that is r square into theta by 2 minus r square cos theta by 2 into sin theta by 2.

So, as per geometry you can have this relationship. Finally, you will get this r square into theta minus sin theta by 2. So, from this equation you have to calculate this cross sectional area for this round water channel of inner radius r. So, once you find out this A and substitute this A here in equation number 12 and 13, then you can respectively find out the velocity and the discharge of the fluid through the channel.

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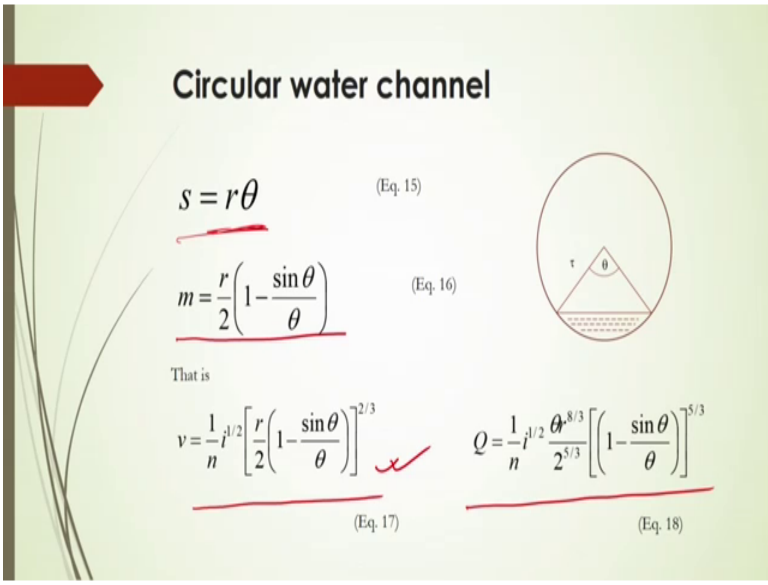

Now, what should be the perimeter? For that parameter will be is equal to r into theta in this case and m should be equals to then that is A by h, then finally it will come r by 2 into 1 minus theta by 2 by the equation 14 and 15. So, after simplification by putting this value this as m n they are in equation number 13.

We can have this 1 by n into i to the power half into r by 2 into 1 minus sin theta by theta whole to the power 2 by 3 here as given in equation number 17 and final discharge equation will be as for equation number 18 shown in here. So, discharge here again depends on the angle of inclination and theta that is as shown in figure and also the radius of the circular channel.

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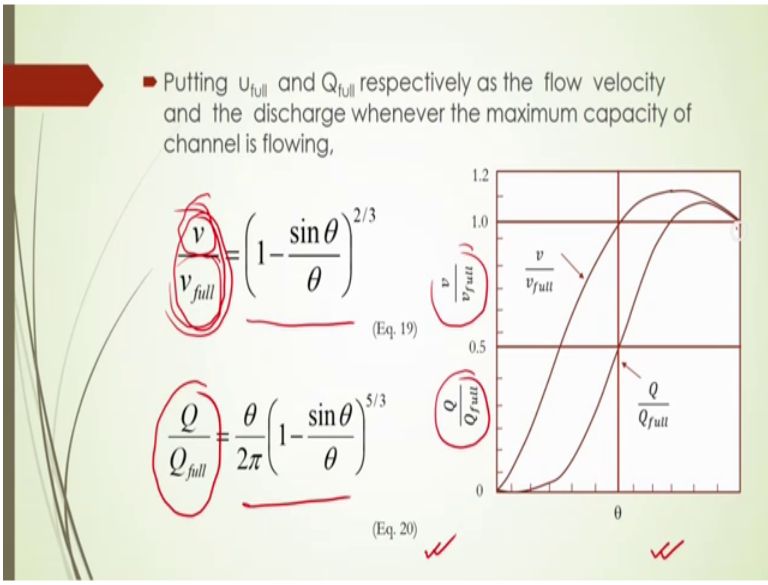

Now, putting this full that is flow velocity and the discharge whenever the maximum capacity of the channel is flowing there, so in that case v by v full that will because 1 minus sin theta by theta to the power 2 by 3. So, one condition is that there is no water that is fully occupied in the channel and another condition is that liquid or water whenever it will be occupied full cross section of this channel, then what will be the velocity and the ratio of these? Velocities that depends on this theta value that will be equals to 1 minus sin theta by theta whole to the power 2 by 3 as given in equation number 19.

And similarly the ratio of this Q by Q full that is discharge in the partial fulfil and the fully occupied the fluid, then again it will be calculated based on this equation number 20 after substitution of this Q by Q full here, then we are getting theta by 2 pi into 1 minus sin theta by theta to the power y by 3. So, here as per equation number 20 you will be able to calculate what will be the discharge at a certain angle theta.

If you know the fully discharged channel capacity there and here in this diagram, it is shown that how this Q by Q full is changing with respect to theta again v by v full is changing with respect to theta. So, initially it will be stiff and after a certain time whenever this v by v full will be equals to same, so in that case they are you will see that theta should be maximum there.

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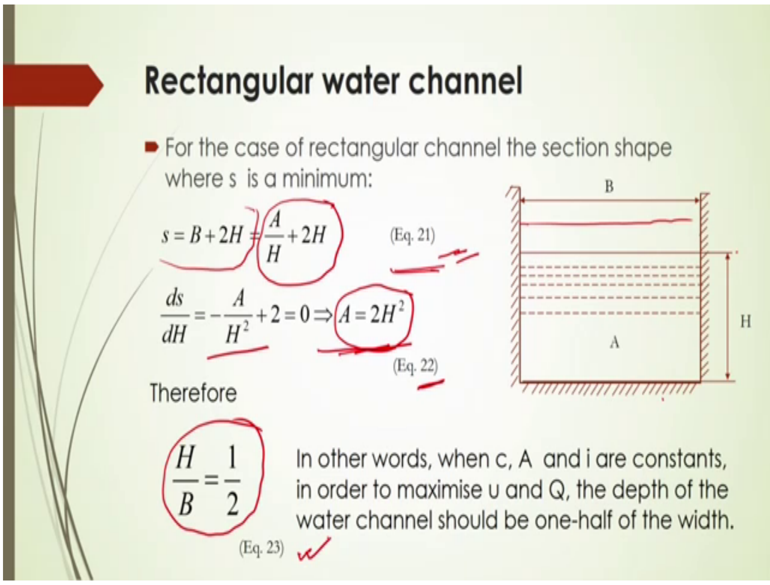

Now, if we consider the rectangular water channel for the case how the section shape s is minimum that is perimeter is many minimum. So, in that case s should be represented by this equation number 21. So, it will be coming as this s should be is equal to B plus 2 H here as shown here figure. What is the B? This to this length is B and cross section is A and this water height is H. So, H should be here, the wetted perimeter should be B, this B plus 2 into H.

So, it will be coming as A by H plus 2 H here and so, in this case it will be your minimum wetted perimeter and if we differentiate these perimeter with respect to water height, then we are getting minus A by H square plus 2 and for this optimum value, it should be 0 which will give you or which will imply that A will be equals to 2 H square. So, therefore we can say that here h by B should be is equal to half where if you substitute this A value here in equation number 21.

From equation 22 then after simplification you can get this H by B should be equal to what 1 by 2. So, here it is important to say that the when c A and i, that means flow velocity coefficient cross sectional area and the angle of inclinations are constants to get the maximize or maximum u that is maximum velocity and respective maximum discharge, the depth of the water channel should be one half of the width of the channel.

So, this is very important that to get the maximum velocity and discharge, you have to design or you have to consider the channel and such a way that water height there should be half of the width of the channel. So, in this way for maximum discharge and flow velocity at a constant flow velocity coefficient cross sectional area and inclination angle, you have to follow this equation number 23 for your suitable design of channel.

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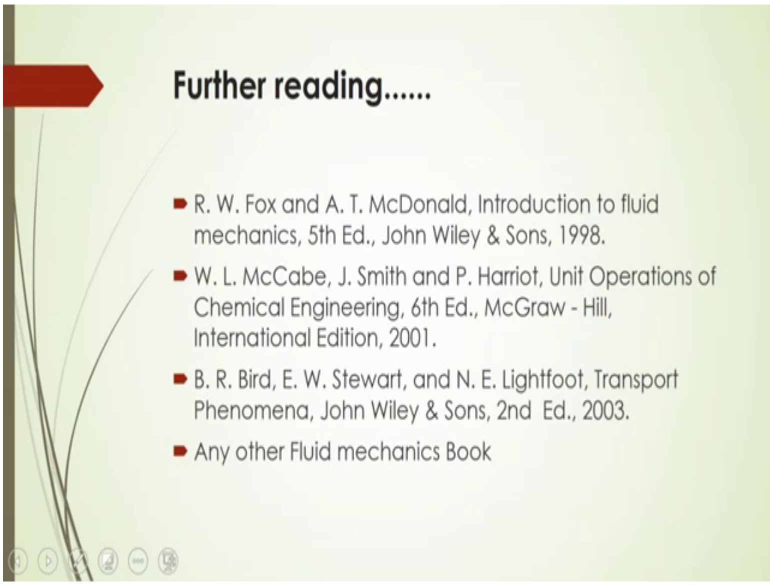

Now, I will suggest to go further for this reading and this textbook as shown in here for further reading and so, based on this literature we are having the how water is flowing through the channel and what should be the optimum that is flow velocity and the discharge through the channel and whether this rectangular channel or the round shaped channel will be the better results, a better that is flow velocity or not for the design. And, how the frictional resistance will be actually obtained from this channel flow and that can be obtained from the force balance there.

And also what will be the flow velocity coefficient that you can calculate from that different equation and what are those standard equation for the channel flow that is manning equation and says this equation there those are very important. So, based on this equation we will be able to design the water channel there for particular operation.

Thank you for this lecture today.