

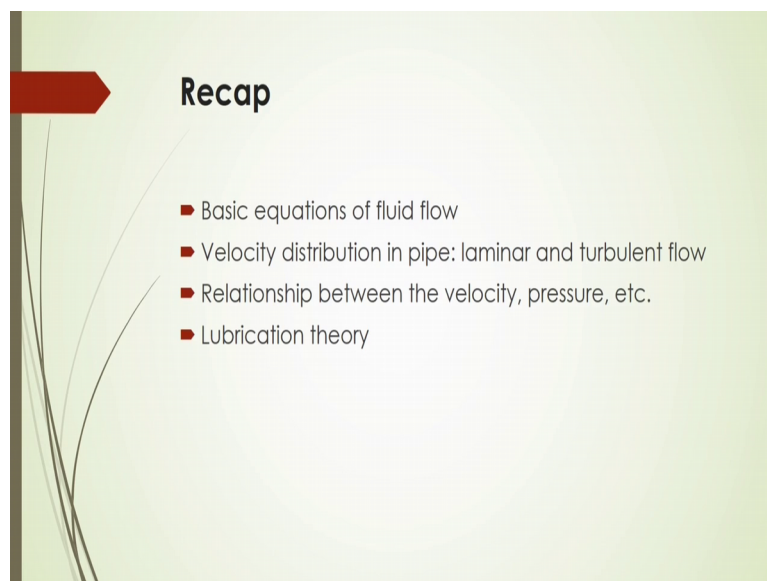
**Fluid Flow Operations**  
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**Module - 06**  
**Lecture – 15**  
**Different Losses in Pipes - Part 1: Frictional Resistance**

Welcome to massive open online course on Fluid Flow Operations. This is the lecture series number 15 under module 6. We will discuss in this module Different Losses in Pipes. In this part 1 we will discuss, Frictional Resistance whenever fluid will be flowing through the pipe.

And, in the previous lectures we have discussed to the basic equations of the fluid flow.

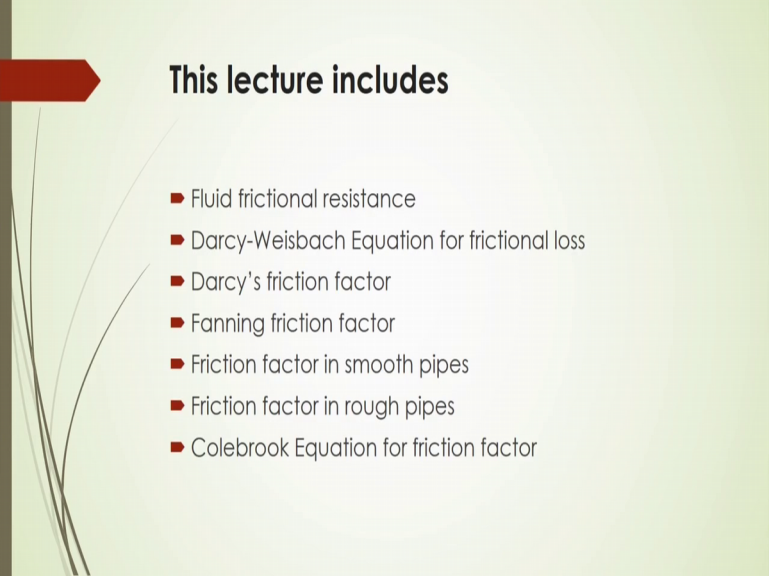
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And, velocity distribution in pipe that the velocity distribution under laminar or turbulent flow, how it will work and, the relationship between the velocity pressure etcetera. And, also the immediate previous lecture we have discussed to the lubrication theory, how the lubricants are acting, or what will be the resistance by the lubrication theory that offered by lubricant is has been discussed.

And, in this lecture we will discuss the fluid frictional resistance Darcy-Weisbach equation for the frictional loss Darcy's friction factor.

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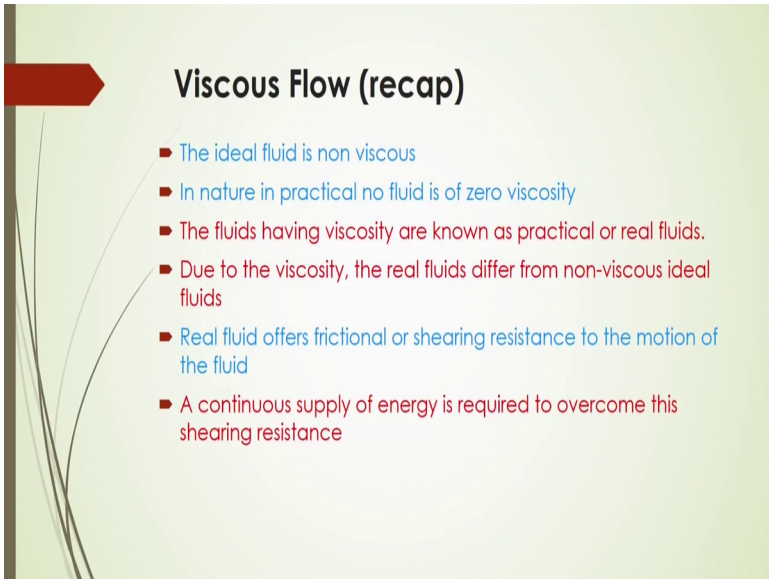


**This lecture includes**

- Fluid frictional resistance
- Darcy-Weisbach Equation for frictional loss
- Darcy's friction factor
- Fanning friction factor
- Friction factor in smooth pipes
- Friction factor in rough pipes
- Colebrook Equation for friction factor

What is the fanning friction factor how this fanning friction factor related to the Darcy's friction factor? And, also in the smooth and rough pipes how the friction factors can be calculated, and what are the different correlations developed by different investigators for this friction factor. In this regard we have to go behind of this fluid flow characteristics and also fluid whether this is ideal fluid or a viscous fluid.

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**Viscous Flow (recap)**

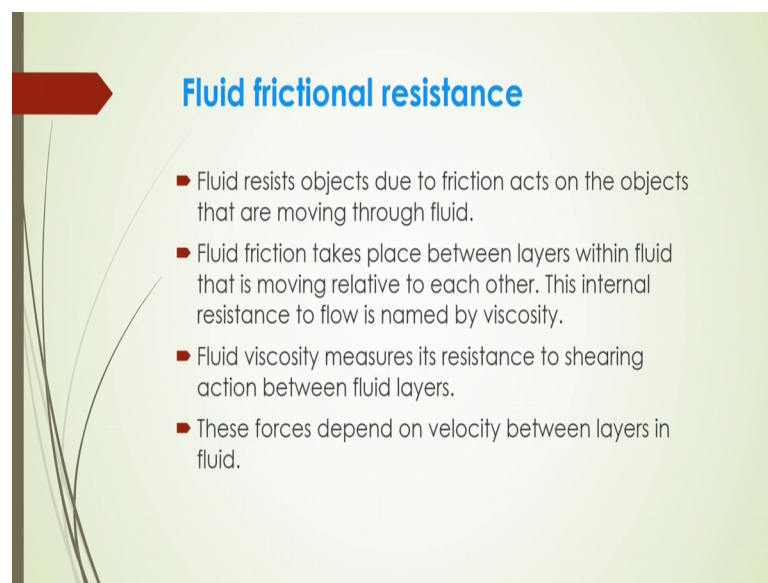
- The ideal fluid is non viscous
- In nature in practical no fluid is of zero viscosity
- The fluids having viscosity are known as practical or real fluids.
- Due to the viscosity, the real fluids differ from non-viscous ideal fluids
- Real fluid offers frictional or shearing resistance to the motion of the fluid
- A continuous supply of energy is required to overcome this shearing resistance

So, the ideal fluid is non-viscous that we have discussed and also in nature in practical no fluid is of 0 viscosity, the fluid having viscosity are known as practical or real fluids.

And, in these real fluids you will have this the frictional resistance. And due to the viscosity the real fluids differ from non-viscous ideal fluids. And, real fluid offers frictional or shearing resistance to the motion of the fluid and how the shearing resistance, or frictional resistance works whenever fluid will be flowing through the pipe.

And, what are the characteristics factor for that it will be discussed here. Also a continuous supply of energy is required to work on this shearing resistance or frictional resistance and how it can be calculated by these frictional resistance characteristics. ah.

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So, we have to discuss now. What is that fluid frictional resistance? We will see the resistance which resists the objects due to the friction acts on the subjects that are moving through fluid. And, fluid fraction takes place between layers within fluid that is moving relative to each other.

And, this internal resistance to flow is named by viscosity, that has been given by a Newton's, that is Newton's law of viscosity from which will we are able to calculate the viscosity, that from the relationship of a shear stress and shear rate. And, this fluid viscosity measures it is resistance to shearing action between the fluid layers.

And, this resistance of course, whenever fluid will be flowing over the flat surface, or over the solid surface any shapes with the of any shapes. And, how this viscosity can be measured and that was already been discussed by developing different velocity

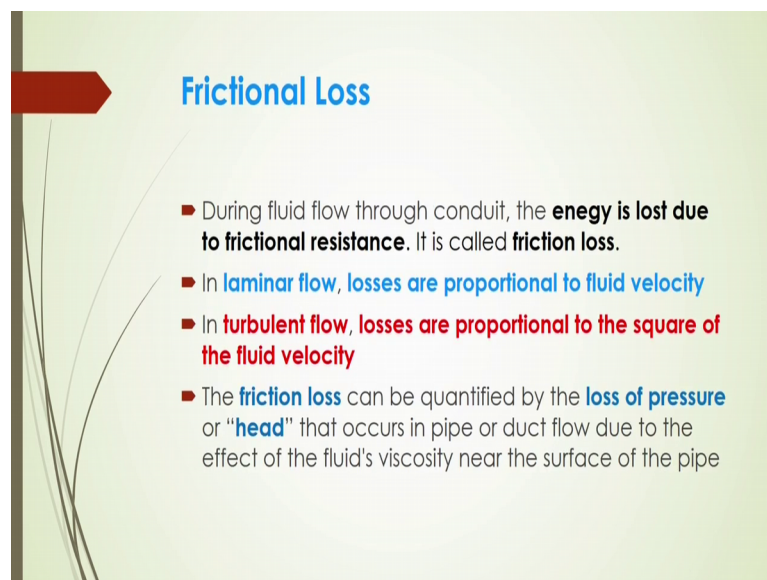
distribution coefficient and based on which, we can calculate we know the it is velocity what should be the viscosity from that.

And, how can Poiseuilles equation is one of the important equation from which we can calculate the viscosity, whenever fluid will be flowing through the pipe. And, also this force is depend on a velocity between layers and the fluid, and whenever there will be frictional forces, and based on that frictional forces we will be able to calculate or we have already discussed the velocity distribution between the layers. And what will be the boundary layer theory based on which the laminar and turbulent flow. How this frictional resistance between the layer acts that has already been discussed here.

And, so, in this case what is that frictional resistance so, we have to think about that there will be some resistance whenever fluid will be flowing through the pipe and whenever it will be flowing over the surface of solid. And suppose the fluid is flowing through the pipe, then through the pipe wall there will be some resistance whenever fluid will be flowing. Now, this resistance will be acting opposite to the flow of flow direction.

And, in that case the shear stress is one important factor, if you know the shear stress then you will be able to calculate what will be the friction factor.

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**Frictional Loss**

- During fluid flow through conduit, the **energy is lost due to frictional resistance**. It is called **friction loss**.
- In **laminar flow**, **losses are proportional to fluid velocity**
- In **turbulent flow**, **losses are proportional to the square of the fluid velocity**
- The **friction loss** can be quantified by the **loss of pressure** or "**head**" that occurs in pipe or duct flow due to the effect of the fluid's viscosity near the surface of the pipe

Now for that frictional resistance there will be some loss of energy and during this fluid flow through the conduit. The energy is lost due to frictional resistance and it is called



the friction loss. In laminar flow the losses are proportional to the fluid velocity, but in turbulent flow losses are proportional to the square of the fluid velocity. So, there is a difference here for laminar flow when you will see there will be a Reynolds number, that we have discussed that Reynolds number will tell you, whether the fluid is following whether in laminar flow or turbulent flow.

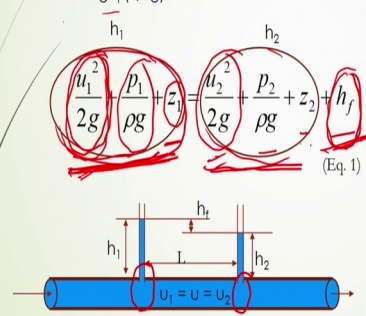
If Reynolds number is less than 2100, we have defined that the flow will be laminar whereas, the Reynolds number if it is greater than 4200 or greater than 4000s we can call it as a turbulent flow. So, in the turbulent flow the loss of energy due to this resistance, it will be proportional to the square of the fluid velocity.

And, in the laminar flow the losses simply is proportional to the fluid velocity. And, the friction loss that can be quantified by the loss of pressure or head there are several terms also how you can actually represent this friction loss. Either by loss of pressure or head that or some other terms also it can be expressed different investigates different books, they have expressed in different way this frictional loss. So, we will follow certain fashion here that how to express this friction loss, that occurs in pipe or duct flow due to the effect of fluid viscosity near the surface of the pipe.

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**Frictional Resistance Law (Laminar Flow):  
Darcy-Weisbach and Fanning's Equations**

- Bernoulli relates changes in the total energy of a flowing fluid to energy dissipation expressed either in terms of a head loss  $h_f$  (m) or specific energy loss  $gh_f$  (J/kg).



$$\frac{u_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{u_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_f \quad (\text{Eq. 1})$$

For  $z_1 = z_2$  and  $u_1 = u_2 = u$  for uniform cross-section and horizontal pipe

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{\Delta p_f}{\rho g} \quad (\text{Eq. 2})$$

If  $\tau_0$  is the wall shear stress, it can be obtained from pressure drop by force balance as

$$\tau_0 = \frac{\text{Frictional force}}{\text{Surface area}} = \frac{(p_1 - p_2)(\pi/4)d^2}{\pi d L} = \frac{d \Delta p_f}{4 L} \quad (\text{Eq. 3})$$

Now, here see in these slides it is seen that frictional resistance law, here Darcy-Weisbach and Fanning's Equation is very important for representing this frictional losses.

Whenever we are applying this Bernoulli's equation we have discussed earlier that Bernoulli equation that, total energy will be constant and based on which the Bernoulli relates to the changes in the total energy of a flowing fluid to energy dissipation, which can be expressed either in terms of a head loss or specific energy loss. Now, head loss that is denoted by  $h_f$  here  $h_f$  and specific energy loss can be represented by  $g$  into a  $h_f$ , that is joule per kg, that is specific energy, that is energy loss per unit weight or unit kg unit mass of the fluid, that is represented by the specific energy loss here.

So, if we write the Bernoulli's equation between 2 points of 1 and 2 here in this pipes we can write here the  $u_1^2$  by  $2g$  plus  $p_1$  by  $\rho g$  plus  $z_1$ , that will be equals to  $u_2^2$  by  $2g$  plus  $p_2$  by  $\rho g$  plus  $z_2$  plus  $h_f$  here.

You will see for ideal flow there if there is no loss, then  $h_f$  should be equals to 0. Otherwise this energy will be in the left and right hand side in this 2 sections. Of course, this total energy will be constants, but in the real fluids ah. So, you will see whenever it will be flowing through the pipe you will see there will be a some loss of energy, that loss of energy to be considered here, that is represented by equation 1 here.

So, that energy loss if it is  $h_f$  this energy loss will be considered here. So, that is why in the right hand side of this equation 1, this  $h_f$  terms is added. Now, between these 2 points we are having this energy at these points if the velocity is  $u_1$  at the section 1.

And at section 2 a velocity is 2, then we can write the  $u_1^2$  by  $2g$  this is your energy, that is kinetic energy and this in the sections 2, that will be  $u_2^2$  by  $2g$ , that is kinetic energy for the velocity  $u_2$ . Now, here pressure energy it is what is that  $p_1$  by  $\rho g$  and this  $z_1$  and  $z_2$  are the elevation height.

So, this actually represented as a head because this  $u_1^2$  by  $2g$  the unit as a meter. So, we can write this as  $h_1$  as a head that is in terms of meter. So,  $u_1^2$  by  $2g$  or  $p_1$  by  $\rho g$  plus  $z_1$ . So, this is head loss this is head in terms of head.

So, we can write for the section 1 or the summation of energy as per Bernoulli's equation this and summation of energy as per Bernoulli's equation for the section 2 and this is the  $h_f$  energy loss. So, from this if we subtract this and if we denote it this left hand side as  $h_1$  and right hand side as  $h_2$ , then we can write a  $h_f$  will be equals to  $h_1$  minus  $h_2$ . This is the head loss due to the friction due to the friction. If in the case where the velocity

suppose in the section 1 and 2 are same, that is  $u_1$  is equal to  $u_2$  if it is equals to  $u$ , then you can say these terms this terms will be equals to 0.

So, in that case we can have this  $h_f$ ,  $h_f$  will be is equal to only  $p_1$  minus  $p_2$  by  $\rho g$  and this will be simply  $\Delta p_f$  by  $\rho g$ . So, frictional pressure difference between these 2 points 1 and 2 per unit what is that mass that will be is equal to specific energy loss here. So, if  $\tau_0$  is the wall shear stress, it can be obtained from the pressure drop by force balance by a force balance.

So,  $\tau_0$  will be equals to frictional force divided by surface area. So, it will be simply  $p_1$  minus  $p_2$  into frictional force ; that means, here frictional pressure into area this is your cross sectional area. So,  $\pi$  by 4 into  $D$  square  $d$  is the diameter of the pipe.

So, this is your cross sectional area of the pipe this is your pressure difference. So, this pressure difference in to cross sectional area you will get the frictional force and divided by surface area. Surface area means, here in this case the fluid, the surface at fluid contact; that means, the surface area at which the fluid will be contact with the wall. So, in that case this surface area will be equals to  $\pi$  into  $d L$ .

So, this will be your total surface area at which this water or fluid will be flowing. So, if you divide this frictional force by this surface area, you can have this shear stress [noise]. Those shear stress it will be coming as then after simplification  $d$  by 4 into  $\Delta p$  a by  $L$ . So, this will be your shear stress.

So, if you know the frictional pressure drop then what will be the shear stress that you can calculate by this equation number 3? Again the head loss that is  $h_f$  can be calculated by from that frictional pressure drop that will be  $\Delta p_f$  by  $\rho g$  by equation number 2.

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Therefore from Eqs. (2) and (3)

$$4\tau_0 = \frac{h_f \rho g d}{L} \quad (\text{Eq. 4})$$

Dividing both sides by  $\rho \bar{u}^2 / 2$

$$\frac{4\tau_0}{\rho \bar{u}^2 / 2} = f_D = \frac{h_f \rho g d}{L \rho \bar{u}^2 / 2} \quad (\text{Eq. 5})$$

or  $h_f = \frac{f_D L \bar{u}^2}{2 g d}$  **Darcy-Weisbach equation** (Eq. 6)

or  $\Delta p_f = \frac{(1/2) f_D \rho \bar{u}^2 L}{d}$  (Eq. 7)

The equation (6) can further be expressed as

$$h_f = \frac{4 f_F L \bar{u}^2}{2 g d} \quad (\text{Eq. 8})$$

$$\Delta p_f = \frac{2 f_F \rho \bar{u}^2 L}{d} \quad (\text{Eq. 9})$$

**Fanning's Equation**

The fanning friction factor ( $f_f$ ) is one-fourth of Darcy's friction factor ( $f_D$ )

$$f_F = \frac{1}{4} f_D = f(\text{Re}, \epsilon/d) \quad (\text{Eq. 10})$$

Now, from this equation number 2 and 3 we can have this  $4\tau_0$  is equal to  $h_f \rho g d$  by  $L$ .

So, shear stress can be expressed in terms of this head loss. Now, dividing both sides by  $\rho \bar{u}^2$  by  $2\rho \bar{u}^2$  of this equation number 4, then we can have this  $4\tau_0$  by  $\rho \bar{u}^2$  and that will be equal to  $f_D$ . And,  $f_D$  is nothing, but the friction factor this friction factor is called the Darcy-Weisbach friction factor. And, that will be equal to  $h_f \rho g d$  divided by  $L \rho \bar{u}^2$  that is given in equation number 5.

So, this is after simplification of equation 4 by dividing  $\rho \bar{u}^2$  we can get this one. So,  $f_D$  is a factor you can say that will be called as Weisbach Darcy Weisbach friction factor.

So, finally, from equation number 5 we can have this  $h_f$  will be equal to in terms of friction factor as  $f_D L \bar{u}^2 / 2 g d$ . So, this is called Darcy-Weisbach equation of frictional resistance or head loss. So, from which you can have this  $\Delta p_f$  will be equal to  $(1/2) f_D \rho \bar{u}^2 L / d$ , this is called Darcy-Weisbach equation of friction factor.

Now, the equation number 6 here can be further expressed by this head loss, or frictional head loss it is by  $4 f_F L \bar{u}^2 / 2 g d$  here.

Another term is  $f_F$  here this  $f_F$  is called the Fanning friction factor. This Fanning friction factor. So, the head loss frictional head loss is based on the Fanning friction factor it will be  $4 f_F L u^2 / 2 g d$ . Here in this case after substitution of  $h_f$  here then we can get this  $\Delta p_f$  will be is equal to  $2 f_F \rho u^2 L / d$ . So,  $2 f_F \rho u^2 L / d$  this. So, this  $1/d$  is called the Fanning's equation for friction factor and  $h_f$  is equal to  $4 f_F L u^2 / 2 g$  it is called the Fanning equation for head loss.

So, the Fanning friction factor how this related to this Darcy-Weisbach friction factor. Generally this Fanning friction factor if we compare this equation number 9, with this equation number 7, then you can have that the Fanning friction factor is one-fourth of the Darcy's friction factor; that means, here  $f_F$  will be is equal to  $1/4$  of  $f_D$ .

And this friction factor generally depends on the how the fluid is flowing, that is way the whether this is turbulent flow or laminar flow. And also if there any roughness of the pipe exist or not and also what is the pipe diameter that is very important in that case.

So, this friction factors depends on the Reynolds number and roughness factor and the diameter of the pipe. Of of course, the other terms the fluid properties is very important here.

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Therefore from Eqs. (2) and (3)

$$4\tau_0 = \frac{h_f \rho g d}{L} \quad (\text{Eq. 4})$$

Dividing both sides by  $\rho \bar{u}^2 / 2$

$$\frac{4\tau_0}{\rho \bar{u}^2 / 2} = f_D = \frac{h_f \rho g d}{L \rho \bar{u}^2 / 2} \quad (\text{Eq. 5})$$

Or  $h_f = \frac{f_D L \bar{u}^2}{2 g d}$  (Eq. 6) **Darcy-Weisbach equation**

Or  $\Delta p_f = \frac{(1/2) f_D \rho \bar{u}^2 L}{d}$  (Eq. 7)

The equation (6) can further be expressed as

$$h_f = \frac{4 f_F L \bar{u}^2}{2 g d} \quad (\text{Eq. 8})$$

$$\Delta p_f = \frac{2 f_F \rho \bar{u}^2 L}{d} \quad (\text{Eq. 9})$$

The Fanning friction factor ( $f_f$ ) is one-fourth of Darcy's friction factor ( $f_D$ )

$$f_F = \frac{1}{4} f_D = f(\text{Re}, \epsilon/d) \quad (\text{Eq. 10})$$

Now, let us do an example in this case by this head loss. Now, suppose the water is flowing through a 10 meter long circular pipe of diameter 0.05 meter at an average



velocity 2.50 meter per second. And, in this case you have to find out what should be the head loss, what will be the frictional pressure drop and the shear stress. You can apply the Darcy's friction factor is 0.115 here.

So, as per equation here  $h_f$  that is frictional head loss is represented by  $f D L u^2 / 2 g d$ . Here in this case you are given  $L$ ,  $f D$  also is given to you,  $g$  known to you and  $d$  also is known to you.

So, finally, after substitution of these values, you can calculate from this equation what should be the  $h_f$ .  $h_f$  should be is equal to 7.36. This is your head loss and what will be the friction or pressure drop, this friction or pressure drop you can either calculate by the Darcy's equation or by fanning's equation.

If, we consider the Darcy's equation then, it is simply as like this here  $\Delta p_f$  by  $L$ , that will be is equal to half of  $f D \rho u^2$  by  $d$ .

So, in this case  $\Delta p_f$  will be is equal to here  $7.2 \times 2 \times 1.04$  Newton per meter square. And, similarly we can calculate what will be the  $\tau_0$ , the shear stress the shear stress will be equal to  $d$  by 4 into  $\Delta p_f$  by  $L$ , after substitution we can get this 90.263 Newton per meter square.

So, in this case here the  $\Delta p_f$  this will be is equal to actually  $\Delta p_f$  will be is equal to here  $L$  should be calc multiplied by here. So, here it is corrected here in this  $\Delta p_f$  will be is equal to half of  $f D \rho u^2$  by  $d$   $2 L$  that will be is equal to this.

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Therefore from Eqs. (2) and (3)

$$4\tau_0 = \frac{h_f \rho g d}{L} \quad (\text{Eq. 4})$$

Dividing both sides by  $\rho v^2/2$

$$\frac{4\tau_0}{\rho \bar{u}^2 / 2} = f_D = \frac{h_f \rho g d}{L \rho \bar{u}^2 / 2} \quad (\text{Eq. 5})$$

Or  $h_f = \frac{f_D L \bar{u}^2}{2gd}$  (Eq. 6) **Darcy-Weisbach equation**

Or  $\Delta p_f = \frac{(1/2) f_D \rho \bar{u}^2 L}{d}$  (Eq. 7)

The equation (6) can further be expressed as

$$h_f = \frac{4f_F L \bar{u}^2}{2gd} \quad (\text{Eq. 8})$$

$$\Delta p_f = \frac{2f_F \rho \bar{u}^2 L}{d} \quad (\text{Eq. 9})$$

The fanning friction factor ( $f_f$ ) is one-fourth of Darcy's friction factor ( $f_D$ )

$$f_F = \frac{1}{4} f_D = f(\text{Re}, \epsilon/d) \quad (\text{Eq. 10})$$

And, now we will consider what should be the universal friction factor in the laminar flow turbulent flow that, we have to actually calculate from the well known actually velocity distribution. Now, in this case we know for laminar flow as described in lecture 11 that by equation 27 the mean velocity, that will be is equal to  $r_0$  square by  $8\mu$  into  $d p$  by  $d x$ , that will be is equal to  $d$  square by  $32\mu$  into  $\Delta p$  by  $L$ .

So, this is this are already been actually discussed how this equation is coming whenever fluid will be flowing through the pipe. So, it has been derived please go through the lecture 11 once again to actually have this equation number 11. Now, if we know the equation number 11 for this mean velocity, then from this equation number 9 and 11 we can get  $f_F$  will be is equals to  $16$  by  $\rho u d$  by  $\mu$ , that will be equals to  $16$  by  $R e$ . So, fanning friction factor will be is equal to  $16$  by  $R e$ .

Hence,  $f_D$  Darcy-Weisbach friction factor will be is equal to  $64$  by here  $\rho u d$  by  $\mu$  because, this Darcy Weisbach friction factor is 4 times of this fanning friction factor. So, it will be  $64$  by  $R e$ . So, from this equation number 12 or 13 if we know the Reynolds number what should be the fanning friction factor or Darcy Weisbach friction factor we can easily calculate. A from the experiment if you know the velocity and the frictional pressure drop, you can easily calculate velocity from there and also what will be Reynolds number from which you can calculate this friction factors.

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**Universal Friction Factor (Turbulent Flow)**

As per Equation (5)

$$f_D = \frac{4\tau_0}{\rho \bar{u}^2 / 2} \quad (\text{Eq. 14})$$

For turbulent Flow we know

$$v_* = \sqrt{\tau_0 / \rho} \quad (\text{Eq. 15})$$

So Eq. (14) can be written as

$$v_* = \bar{u} \sqrt{f_D / 8} \quad (\text{Eq. 16})$$

Now, as per equation number 5 we can express this Darcy Weisbach friction factor as  $4\tau_0$  by  $\rho u$  square by 2.

And, in this case we can express this friction factor for the turbulent flow. Now, for turbulent flow, we know that the friction velocity; that means, here  $v$  star we have already discussed in the lecture 11, that this will be is equal to root over  $\tau_0$  by  $\rho$ . So, from this equation number 14 can be written as  $v$  star will be is equal to  $u$  into root over  $f_D$  by 8.

And, from this equation number 16, we can easily calculate the friction velocity if we know the velocity and the Darcy Weisbach friction factor.

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### Universal Friction factor for turbulent flow through smooth pipe

- For turbulent flow through smooth round pipe the average velocity ( $V$ ) is given by (as discussed in previous lecture)

$$\frac{V}{v_{*}} \Big|_{smooth} = 1.75 + 2.5 \ln \left( \frac{\rho v_{*} R}{\mu} \right) \quad (\text{Eq. 17})$$

Substituting the shear velocity ( $v_{*}$ ) from eq. (16) we can get after simplification and rearrangement as

$$\frac{1}{\sqrt{f_D/8}} = 1.75 + 2.5 \ln \left( \frac{\rho \bar{u}}{\mu} (\sqrt{f_D/8}) \frac{d}{2} \right) \quad \text{or} \quad (\text{Eq. 18})$$
$$\frac{1}{\sqrt{f_D}} = -0.913 + 0.884 \ln(\text{Re} \sqrt{f_D})$$

Applicable for  $\delta_{laminar\ sublayer} > 1.7\epsilon$

Now, for the turbulent flow through the smooth round pipe, the average velocity  $V$  is given by as discussed in the previous lectures that  $V$  by  $v$  star, or you can sometimes use  $\bar{u}$  by  $v$  star sometimes  $v$  is the average velocity is sometimes expressed in different way.

So,  $V$  by  $v$  star that will be is equal to 1.75 plus 2.5 ln into  $\rho v$  star  $R$  by  $\mu$ . So, in this case equation number 17 we are having this what will be the ratio of the average velocity to the friction velocity, that is also depends on the Reynolds number here,  $\rho v$  star  $R$  by  $\mu$  it is called Reynolds number based on the friction velocity. So, it will be 1.75 plus 2.5 into ln  $\rho v$  star  $R$  by  $\mu$ , and  $R$  is the radius of the pipe. So, substituting these shear velocity  $v$  star from equation number 16.

In the previous slides that equation number 16 here it is given if we substitute this equation number 16 here, in equation number 17, then we can express after rearrangement this  $1/\sqrt{f_D/8}$ , that will be is equal to 1.75 plus 2.5 into ln into  $\rho \bar{u}$  by  $\mu$  into  $\sqrt{f_D/8}$  into  $d$  by 2. That is given in equation number 18 or even after simplification you can have this  $1/\sqrt{f_D}$ , that will be is equal to minus 0.913 plus 0.884 ln  $\text{Re} \sqrt{f_D}$ .

Now, this equation is developed based on the average velocity whenever fluid will be flowing through the pipe and also the friction velocity. Now, so, it is interesting that only thing is that the Reynolds number if you know the Reynolds number, based on this

friction velocity what should be the Darcy weisbach friction factor you can easily calculate from this equation number 18. Now, this equation number 18 whenever you are going to find out this Darcy weisbach friction factor, you have to find out this  $f_D$  by trial and error method because this is the non-linear equation.

So, to actually equate this left hand side and right hand side just by assuming the  $f_D$  suitable  $f_D$  here. And, by trial and error method by changing this  $f_D$  whenever this left handed side and right side hand side will be equal almost equals then you can fix it up as so, what will be the Darcy friction factor there.

Now, it is applicable for remember that whenever this boundary thickness in the laminar sub layer if it is greater than 1.7 times of roughness factor of the pipe. So, it is applicable only for those conditions. So, we can have the universal friction factor turbulent flow through the smooth pipe and this friction factor is expressed here in this equation number 18.

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**Example:** Water is flowing through a 10 m long circular smooth pipe of diameter 0.05 m at an average velocity 3.076 m/s. Determine Shear velocity and Universal Darcy's friction factor

$$\frac{V}{v_*} \Big|_{smooth} = 1.75 + 2.5 \ln \left( \frac{\rho v_* R}{\mu} \right)$$

$$\Rightarrow \frac{3.076}{v_*} = 1.75 + 2.5 \ln \left( \frac{1000 v_* 0.025}{0.001} \right)$$

$$\Rightarrow v_* = 0.139 \text{ m/s}$$

$$\frac{1}{\sqrt{f_D}} = -0.913 + 0.884 \ln \left( \text{Re} \sqrt{f_D} \right)$$

$$\Rightarrow f_D = 0.01646 \text{ (by trial and error method)}$$

Now, let us do an example for this suppose that water whenever examples is given earlier, that water is flowing through a 10 meter long circular smooth pipe of diameter is 0.05 meter, at an average velocity of 3.076 meter per second. In this case what should be the shear velocity or friction velocity and universal Darcy's friction factor here?.



So, you just apply here whatever shear velocity, the shear velocity it is called sometimes friction velocity. So, shear velocity is calculated based on this equation here. So,  $V$  by  $v^*$  that will be equals to  $1.75 \ln \rho v^* R / \mu$ . So, in this case you know the  $V$  that is 3.076 only  $v^*$  is unknown to you, other terms are all known to you, if you substitute all other terms and you can finally, calculate  $v^*$  will be equals to here 0.139 meter per second. So, after substitution of this  $v^*$  you have to calculate what should be the Reynolds number.

So, Reynolds number after substitution of Reynolds number here. And, after trial and error method by assuming this  $f_D$  and just by changing this  $f_D$  and getting the almost equal of this left hand and right hand side then finally, we can have this  $f_D$  will be equals to 0.01646. So, this is trial and error method. So, you have to find out.

So, we are having this  $f_D$  will be equals to 0.01646 whenever water is flowing through a 10 meter long circular pipe of diameter 0.05 meter, if it is flowing as an at an average velocity of 3.076 meter per second. Again, if we consider the rough pipe then what should be the universal friction factor for turbulent flow through the pipe.

So, in this case for turbulent flow through the rough round pipe the average velocity is given by  $V$  by  $v^*$  that will be equals to  $4.75 \ln R / \epsilon$  here. What is epsilon? Epsilon is the roughness factor a substituting this shear velocity  $v^*$  or friction velocity  $v^*$  from equation number 16, then we can get after simplification and rearrangement as  $1 / \sqrt{f_D} = 4.75 \ln R / \epsilon$  or we can have this as  $1 / \sqrt{f_D}$  is equal to  $1.066 \ln R / \epsilon$ .

So, this one so, this friction factor depends on the roughness of the pipe here, and also the diameter of the pipe.

So, if you know the ratio of the diameter to the roughness of the pipe you can easily calculate what should be the Darcy's Weisbach friction factor, but this equation 20 is applicable only for if the boundary layer thickness at laminar sub layer, if it is less than  $0.08 \epsilon$ , that is the roughness factor of the pipe. Now, if you do another example for this.

So, in you can say that if water is flowing through a 10 meter long circular pipe it is rough pipe here of diameter 0.05 meter of the same diameter, an average velocity of the

same 3.076 meter per second, then what should be the shear velocity and the universal Darcy's friction factor and the roughness factor here.

Now, in this case if we consider again this equation velocity distribution based on the average velocity through the rough pipe, then we can have this  $V$  by  $v$  star will be equals to  $4.75 + 2.5 \ln R$  by  $\epsilon$ . So, in this case a  $v$  star will be equals to 0.136 and  $\epsilon$  will be equals to 0.02 millimeter here. This is by trial and error method you have to find out what should be the  $v$  star and  $\epsilon$  here. So, after substitution of this  $\epsilon$  and  $d$  here then after getting this logarithm of this then we can have finally,  $f_D$  will be equals to 0.01561.

So, this is your rough pipe whereas, in the smooth pipe we are getting 0.0164 whereas, for rough pipe it is having 0.01561. So, the for rough pipe the friction factor is less than that of a smooth pipe here.

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**Example:** Water is flowing through a 10 m long circular smooth pipe of diameter 0.05 m at an average velocity 3.076 m/s. Determine Shear velocity and Universal Darcy's friction factor

$$\frac{V}{v_{*smooth}} = 1.75 + 2.5 \ln \left( \frac{\rho v_* R}{\mu} \right)$$

$$\Rightarrow \frac{3.076}{v_*} = 1.75 + 2.5 \ln \left( \frac{1000 v_* 0.025}{0.001} \right)$$

$$\Rightarrow v_* = 0.139 \text{ m/s}$$

$$1/\sqrt{f_D} = -0.913 + 0.884 \ln \left( \text{Re} \sqrt{f_D} \right)$$

$$\Rightarrow f_D = 0.01646 \text{ (by trial and error method)}$$

Now, what will be the values of this roughness in terms of length we can say in millimeters for the different pipe materials, that have a look here in the table it is given, if we consider that asphalt cast iron made pipe, then this roughness factor to be considered as 0.120. If, you are considering the pipe is made of the cast iron then you have to consider this as 0.265. If, you are considering the commercial or welded steel or wrought iron made pipe e then it will be 0.045.

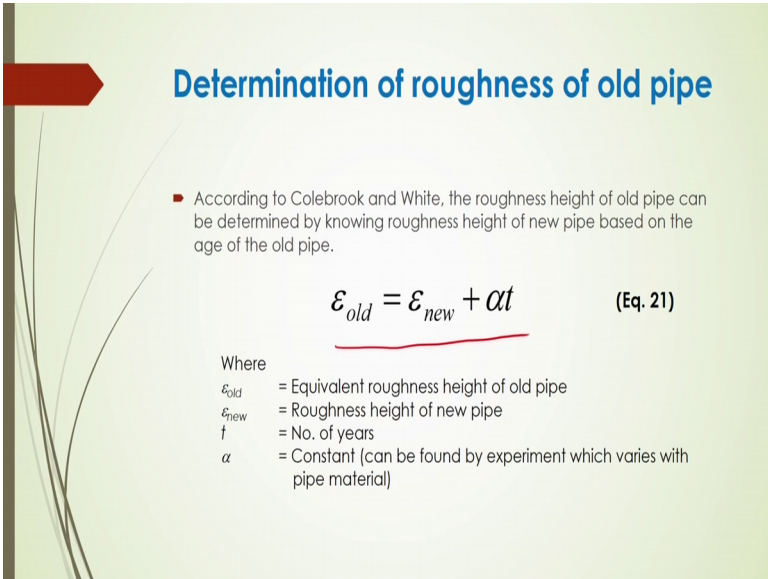
If it is a concrete material made a pipe then you can have a this roughness pipe of 0.3 to 3.0. And, depends on the concentration of this other materials in the pipe. Galvanized iron if you are considering that pipe made of galvanized irons in this case, that roughness factor to be considered as 0.15. Whereas, in the mostly you will see in the commercial or in the market the PVC pipe glass pipe other drawn tubing's are available in that case, you have to consider this PVC pipe would be for of the PVC pipe will be 0.0015.

So, whenever you are going to consider this friction factor by considering the flow through the PVC pipe, in that case roughness factor to be considered as 0.0015 whereas, in the pipe is made up sometimes the wood stage type of material. So, in that case the roughness factor to be considered as 0.09 to 0.18; whereas riveted steel made pipe; in that case the roughness factor to be 0.9 to 9.0.

So, whenever you are going to design any pipe through which the flow will be fluid for a particular operations stop for calculating the frictional resistance to get the less friction factor, or the flowing through the pipe will be of certain materials of high viscous materials or corrosive materials.

Then in that case you have to design the pipe in such a way that the roughness factor is the one important factor there and should be considered based on these values.

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**Determination of roughness of old pipe**

- According to Colebrook and White, the roughness height of old pipe can be determined by knowing roughness height of new pipe based on the age of the old pipe.

$$\epsilon_{old} = \epsilon_{new} + \alpha t \quad (\text{Eq. 21})$$

Where

- $\epsilon_{old}$  = Equivalent roughness height of old pipe
- $\epsilon_{new}$  = Roughness height of new pipe
- $t$  = No. of years
- $\alpha$  = Constant (can be found by experiment which varies with pipe material)

Now, if we consider the roughness of the pipe, you can calculate the roughness of the old pipe based on the roughness factor of the new pipe is there. So, according to Colebrook and white the roughness height of old pipe can be determined by knowing rather roughness height of the new pipe that based on the age of the old pipe. Now, this the relationship is given as equation a 21 as epsilon old, that will be is equal to epsilon new plus alpha t. Where epsilon old is called equivalent roughness height of the old pipe, and epsilon new is called roughness height of the new pipe, and t is the number of years.

Suppose, if it is kept for 5 years new is then what will happen the roughness factor before 5 years the of that pipe that you have to calculate. And, also alpha is equal to constant that can be a found experiment which varies with the pipe material of course.

(Refer Slide Time: 33:55)

**Example:** It is observed that a high viscous corrosive liquid is flowing through a pipe of roughness 0.08 mm. The pipe was kept unused for a five year. It was found noted that when it was used, the roughness of the pipe was 0.01 mm. What should be the roughness of the same pipe when it will be used after 5 years

$$\epsilon_{old} = \epsilon_{new} + \alpha t$$

$$\frac{0.01}{1000} = \frac{0.08}{1000} + \alpha(5 \times 365 \times 24 \times 3600)$$

$$\Rightarrow \alpha = -4.439 \times 10^{-13}$$

$$\epsilon_{new} = \epsilon_{old} - \alpha t$$

$$= \frac{0.08}{1000} - (-4.439 \times 10^{-13}) \times 5 \times 365 \times 24 \times 3600$$

$$= 0.00015 \text{ m} = 0.15 \text{ mm}$$

Now, if we see that if you consider an example here, it is observed that a high viscous corrosive liquid is flowing through a pipe of roughness 0.08 millimeter. The pipe was kept unused for a 5 years. Now, it was found noted that, when it was used the roughness of the pipe was 0.01 millimeter, then what should be the roughness of the same pipe when it will be used after 5 years.

So, you can use simply this equation here epsilon old equals to epsilon new plus alpha t. Now, you have to find out what could be the value of alpha by knowing the value. In this case the old pipe this is your 0.0 1 this epsilon, by 1000, that will be equals to 0.08 by

1000s, that is 1000 divided by 1000 means you have to considered in terms of meter, that is why you have divided here by 1000s.

So, after substitution of this then alpha is coming minus 4.439 into 10 to the power minus 13. So, again if we substitute this alpha value here, then we can have this epsilon new will be equals to epsilon old minus alpha t, that will be coming as after substitution it is coming 0.15 millimeter. So, earlier it was 0.08 and or 0.01. Now, after 5 years it is coming as 0.15 millimeter.

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**Colebrook Empirical Equation for Friction Factor**

- Colebrook recommended the following empirical equation for his experimental conditions of ranges:

$$\delta_{\text{laminar sublayer}} = 0.08 \text{ to } 1.7\epsilon \quad (\text{Eq. 22})$$

$$\frac{1}{\sqrt{f}} = 1.74 - 0.868 \ln \left( \frac{\epsilon}{R} + \frac{18.7}{\text{Re}\sqrt{f}} \right) \quad (\text{Eq. 23})$$

Or

$$\frac{1}{\sqrt{f}} = -0.868 \ln \left( \frac{\epsilon}{3.7d} + \frac{2.51}{\text{Re}\sqrt{f}} \right) \quad (\text{Eq. 24})$$

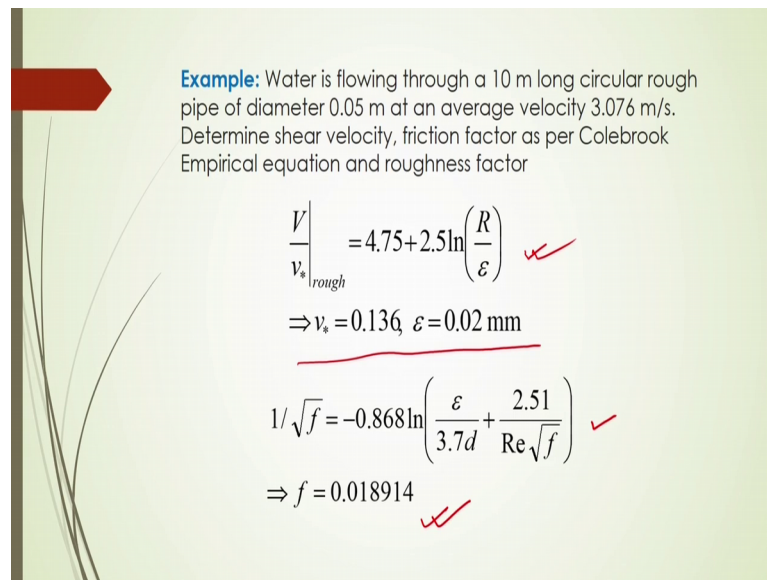
Now, different empirical correlations are given here to calculate the friction factor. And, in this case Colebrook recommended the following empirical equation for his experimental conditions of ranges here. If, suppose boundary thickness of laminar sub layer is equal to 0.081.7 epsilon.

Then, you have to calculate the friction factor as this is Darcy's Weisbach friction factor will be equals to 1.74 minus 0.868 l n into epsilon by R plus 18.7 by Reynolds number into root over f. And, or after simplification also you can have this one 1 by root over f into minus 0.868 into l n this one.

So,. So, this equation number 23 or equation number 24 you can use to calculate the friction factor, based on the empirical equation suggested by Colebrook from his experimental results.



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**Example:** Water is flowing through a 10 m long circular rough pipe of diameter 0.05 m at an average velocity 3.076 m/s. Determine shear velocity, friction factor as per Colebrook Empirical equation and roughness factor

$$\frac{V}{v_*} \Big|_{rough} = 4.75 + 2.5 \ln \left( \frac{R}{\varepsilon} \right) \quad \checkmark$$
$$\Rightarrow v_* = 0.136 \quad \varepsilon = 0.02 \text{ mm}$$

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$$1/\sqrt{f} = -0.868 \ln \left( \frac{\varepsilon}{3.7d} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad \checkmark$$
$$\Rightarrow f = 0.018914 \quad \checkmark \checkmark$$

Now, let us consider that water is flowing through a 10 meter long circular rough pipe of diameter 0.05 meter at an average velocity of 3.076 meter per second. In this case calculate the shear velocity friction factor as per Colebrook empirical equation and the roughness factor. So, again we will use this equation  $V$  by  $v$  star for the turbulent flow and the rough pipe, then we can have this  $v$  star and epsilon after trial and error method as 0.136 and 0.02 millimeter respectively.

Now, after substitution of this  $v$  star and epsilon and you have to calculate what should be the Reynolds number and after substitution all this Reynolds number and epsilon here in this equation, we can calculate what would be the  $f$  based on this Colebrook empirical equation. So, it is coming as 0.018914. It is almost equals to that earlier what you have calculated there is a very small error that is 0 point earlier we have got 0.016, here in this case in this equation here  $f D$  is equal to 0.01561 whereas, it is for this by Colebrook equation it is coming as 0.01894.

So, it is almost same there is a with a small error. And so, in this way we can calculate based on this Colebrook empirical equation what will be the friction factor?.

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**Blasius Empirical Equation for Friction Factor**

- In 1913, Blasius developed an empirical correlation based on his experimental work to calculate the friction factor in turbulent flow through smooth round pipe
- He proposed the following correlation

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = \frac{0.3164}{Re^{1/4}} \quad (\text{Eq. 25})$$
$$2300 \leq \left( Re = \frac{\rho\bar{u}d}{\mu} \right) \leq 10^5$$

Similarly, a Blasius also they have suggested based on their experimental work some friction factor co relationship. So, in 1913 Blasius developed an empirical correlation. Based on his experimental work to calculate the friction factor in turbulent flow, through smooth round pipe and he proposed this following equation number 25 as  $f$  will be equals to  $4\tau_0$  by  $\rho u$  square by 2 that will be equals to  $0.3164$  by  $Re$  to the power  $1.5$  by 4.

So, this  $f$  is nothing, but that Darcy Weisbach friction factor. In this case if it is suppose fanning friction factor it will be coming as you have to divided by 4, then it will come  $0.0794$  divided by Reynolds number to the power  $1$  by 4. That you can get in other text book also that fanning friction factor will be equals to  $0.079$  divided by  $Re$  to the power  $0.25$  exact to the same.

So, here it will be Darcy's Weisbach friction factor. So, this correlations given by blasius from his experimental results and they also suggested that this equation will be used only within the range of Reynolds number up to  $300$  to  $10$  to the power  $5$ .

So, this equations only valid within this range of Reynolds number.

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**Example:** Water is flowing through a 10 m long circular smooth pipe of diameter 0.05 m at an average velocity 1.05 m/s. Determine shear velocity, friction factor as per Blasius Empirical equation.

$$2300 \leq \left( \text{Re} = \frac{\rho \bar{u} d}{\mu} = 52500 \right) \leq 10^5$$
$$f = \frac{4\tau_0}{\rho \bar{u}^2} = \frac{0.3164}{\text{Re}^{1/4}} = 0.0209$$
$$\tau_0 = \frac{1}{8} \rho \bar{u}^2 \frac{0.3164}{\text{Re}^{1/4}}$$
$$= \frac{1}{8} (1000)(1.05)^2 (0.0209) = 2.881 \text{ N/m}^2$$

Now, again if we considered that examples the water is flowing through a 10 meter long circular smooth pipe of diameter 0.05 meter at an average velocity of 1.05 meter per second, then determine the shear velocity, as well as the friction factor as per Blasius empirical equation. So, you can get the shear velocity as per that equation from the velocity distribution as earlier. And, also calculate this Reynolds number whether it is coming within the range of or not we are having this Reynolds number as 52000s. So, it is coming within this range it is suggested by this Blasius.

So, as per blasius equation say f will be is equal to here after substitution of these values it is coming 0.0 to 09. And this shear stress is coming after substitution is this friction factor. So, it is coming as 1 by 8 into rho u square into 0.3 1 6 4 by r e to the power 1 by 4. So, after substitution of these values so, we are getting 2.881 in to Newton per meter square. So, this is your calculation for shear stress.

So, from this example we can have an idea what should be the Darcy's Weisbach friction factor can be calculated from the Blasius empirical equation ok.

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### Lees Empirical Equation for Friction Factor

- Lees in 1924 proposed an empirical correlation based on his experimental work to calculate the friction factor in turbulent flow through smooth round pipe as

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0072 + \frac{0.611}{\text{Re}^{0.35}} \quad (\text{Eq. 26})$$
$$4000 \leq \text{Re} \left( = \frac{\rho\bar{u}d}{\mu} \right) \leq 4 \times 10^5$$

Similarly, other empirical equations that is given by other investigators here is one important correlation, that is given by lees in 1924. And, he proposed this equation number 26 as 0.0072 plus 0.614, 611 divided by Reynolds number to the power 0.35 here. And, this he has proposed based on his experimental work and his experimental condition was this the Reynolds number within the range of 4000s to 4 into 10 to the power 5. So, these correlations will be valid only within this range of Reynolds number.

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**Example:** Water is flowing through a 10 m long circular smooth pipe of diameter 0.05 m at an average velocity 1.05 m/s. Determine shear velocity, friction factor as per Lees empirical equation.

$$4000 \leq \left( \text{Re} = \frac{\rho\bar{u}d}{\mu} = 52500 \right) \leq 4 \times 10^5$$
$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0072 + \frac{0.611}{\text{Re}^{0.35}} = 0.0208$$
$$\tau_0 = \frac{1}{8} \rho\bar{u}^2 \left( 0.0072 + \frac{0.611}{\text{Re}^{0.35}} \right)$$
$$= \frac{1}{8} (1000)(1.05)^2 (0.0208) = 2.868 \text{ N/m}^2$$

Now, in this case if we again consider that example of that water is flowing through a 10 meter long circular smooth pipe of the diameter 0.05 meter at an average velocity of 1.05. So, in this case what should be the friction factor and the shear stress based on this lees empirical equation. So, based on this say lees empirical equation we are having this Reynolds number is this. Of course, this is within the range of Reynolds number of his experimental conditions. So, f should be is equal to after substitution of this values it is coming 0.0208.

Similarly, tau 0 you can calculate based on this equation it is coming 2.868 Newton per meter square.

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**Schiller and Herman Empirical Equation for Friction Factor**

- In 1930 Schiller and Herman suggested an empirical correlation based on their experimental work to estimate the friction factor in turbulent flow through smooth round pipe which can be expressed as

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0054 + \frac{0.396}{\text{Re}^{0.30}} \quad (\text{Eq. 27})$$

$$2300 \leq \text{Re} \left( = \frac{\rho\bar{u}d}{\mu} \right) \leq 4 \times 10^5$$

And, Schiller and Herman empirical equation also is important they have given another empirical correlations based on their experimental work in 1930. And, they estimate the friction factor in turbulent flow through the smooth round pipe, and it is expressed by this equation number 27. And this is 0.005 4 plus 0.396 by R e to the power 0.30.

So, this is second applicable within the range of Reynolds number given here.



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### Nikuradse Empirical Equation for Friction Factor

- Nikuradse in 1932 developed an empirical correlation based on his experimental work to predict the friction factor in turbulent flow through smooth round pipe which can be expressed as

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0032 + \frac{0.221}{\text{Re}^{0.237}} \quad (\text{Eq. 28})$$
$$10^5 \leq \text{Re} \left( = \frac{\rho\bar{u}d}{\mu} \right) \leq 10^8$$

And, Nikuradse also suggested another empirical equation for the friction factor in 1932, and he developed this empirical equation based on his experimental work. And, he suggested this empirical correlation for the calculation of friction factor here as equation number 28. And, this correlation is valid only within the range of Reynolds number of 10 to the power 5 to 10 to the power 8.

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### Moody Empirical Equation for Friction Factor

- Moody later on in 1944 developed an empirical correlation for friction factor based on his experimental work on turbulent flow through rough round pipe which can be expressed as

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0055 \left[ 1 + \left( 20000 \times \frac{2\varepsilon}{d} + \frac{10^6}{\text{Re}} \right)^{1/3} \right] \quad (\text{Eq. 29})$$
$$4000 \leq \text{Re} \left( = \frac{\rho\bar{u}d}{\mu} \right) \leq 10^7 \quad \text{and} \quad 0 < \frac{\varepsilon}{d} \leq 0.01$$

Moody later on 1944 he developed another empirical correlation for the friction factor, from his experimental work on the turbulent flow through the rough round pipe.

In that case he considered that roughness factor. And, he got that empirical correlation as given in equation number 29 here  $0.0055 \text{ plus } 1 \text{ plus } 20000 \text{ into } 2 \text{ epsilon } d \text{ plus } 10 \text{ to the power } 6 \text{ by } R e \text{ to the power } 1 \text{ by } 3$ .

So, this equation is developed based on his experimental work. And, this is applicable only if the Reynolds number is  $4 \text{ into } 10 \text{ to the power } 3 \text{ to } 10 \text{ to the power } 7$ . And, the roughness a factor of this epsilon by d this ratio if it is lies within the range of 0 to 0.01. Then only you have to apply this correlations to find out the friction factor. And all these friction factor of course, in the turbulent regime or turbulent condition.

And, also we got the friction factor in the laminar condition. Now, what should be the friction factor in the transition?

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**Friction factor for Transition**

- According to **Desouky and El-Emam, 1989** for the transition region, the fanning's friction factor for Newtonian fluid flow can be represented as

$$f = \frac{4\tau_0}{\rho \bar{u}^2 / 2} = 0.50 \left[ 0.0112 + \frac{1}{\text{Re}^{0.3185}} \right] \quad (\text{Eq. 30})$$

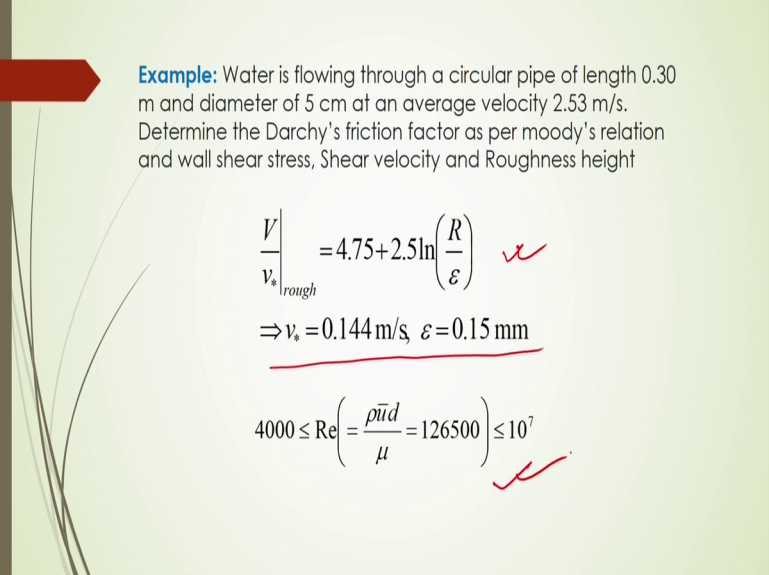
$$2100 \leq \text{Re} \left( = \frac{\rho \bar{u} d}{\mu} \right) \leq 4000$$

That means, if Reynolds number lies within the range of 2300 to 4000s. So, in that case the friction factor you can calculate from this Desouky and El- Emam, correlation that has been developed in 1989 from the experimental data with Newtonian fluid. So, in that case they have suggested this equation to calculate the friction factor for this condition of Reynolds number is 2100 to 4000s.

So, so equation number 30 can be used to calculate the friction factor, this is fanning friction factor, this is not the Darcy Weisbach friction factor, this fanning friction factor

will be calculated based on this. It will be is equal to 0.5 0 into 0.0 1 1 2 plus 1 by Reynolds number to the power 0.ah 3185.

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**Example:** Water is flowing through a circular pipe of length 0.30 m and diameter of 5 cm at an average velocity 2.53 m/s. Determine the Darcy's friction factor as per moody's relation and wall shear stress, Shear velocity and Roughness height

$$\frac{V}{v_*} \Big|_{\text{rough}} = 4.75 + 2.5 \ln \left( \frac{R}{\varepsilon} \right)$$

$$\Rightarrow v_* = 0.144 \text{ m/s}, \quad \varepsilon = 0.15 \text{ mm}$$

$$4000 \leq \text{Re} \left( = \frac{\rho \bar{u} d}{\mu} = 126500 \right) \leq 10^7$$

Now, if we use this equation here if we again considered this water is flowing through a circular pipe of length 0.30 meter and diameter of 5 centimeter at an average velocity 2.53 meter per second. Determine the Darcy's friction factor as per Moody's relations and the wall shear stress shear velocity and roughness height.

So, you have to calculate based on this here equation number equation for velocity distribution, by trial and error method you have to find out this v star and the roughness factor. And, if you are Reynolds number lies within this range of Reynolds number.

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So, Moody's Equation can be used

$$f = \frac{4\tau_0}{\rho\bar{u}^2/2} = 0.0055 \left[ 1 + \left( 20000 \times \frac{2\varepsilon}{d} + \frac{10^6}{\text{Re}} \right)^{1/3} \right]$$
$$= 0.13144$$
$$\tau_0 = f(\rho\bar{u}^2/2)/4 = 105.17 \text{ N}$$

And so, moody s equation if we used then we can have this friction factor as 0.131 4 4. And, the corresponding shear stress will be is equal to after substitution here in this equation; we can have this 10105.17 Newton.

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### Further reading.....

- R. W. Fox and A. T. McDonald, Introduction to fluid mechanics, 5th Ed., John Wiley & Sons, 1998.
- W. L. McCabe, J. Smith and P. Harriot, Unit Operations of Chemical Engineering, 6th Ed., McGraw - Hill, International Edition, 2001.
- B. R. Bird, E. W. Stewart, and N. E. Lightfoot, Transport Phenomena, John Wiley & Sons, 2nd Ed., 2003.
- Any other Fluid mechanics Book

So, in this lecture we have discussed what will be the frictional resistance what will be the friction factor. And how this friction factor can be calculated based on the different empirical correlations suggested by different investigators from their experimental work.

And, also how this velocity distribution can be used for the laminar and turbulent flow to calculate the shear stress and friction factor, we have actually discussed.

So, I think you will get this idea you can use these correlations to calculate or predict the friction factor for any pipe design, or any flow characteristics calculation, or any a simulation of flow through the pipe, where this friction factor or frictional resistance and also head loss is required to calculate that you can use those equations. And, I think you have understood on to calculate this friction factor, and how these equations are coming for this laminar and turbulent flow.

So, that is for all today thank you I will suggest you read further this text book for your better understanding. So, in the future we will discuss more about this frictional resistance as a part 2 so.

Thank you for this lecture.