Fluid Flow Operations

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Module – 05
Flow of Viscous Fluid - Part 4
Lecture – 13
Boundary Layer Theory

Welcome to massive open online course on Fluid Flow Operations. In this lecture, we will discuss about the Flow of Viscous Fluid, for the part of Boundary Layer Theory

In the previous lecture, we have discussed about the turbulent flow and the velocity distribution over the flat plate at its laminar and turbulent flow conditions. And also what will be the drag force local drag force, even based on the based on different aspect of velocity components of a turbulent flow condition we have derived the velocity distribution as well as the what will be the shear stress acting over on the surface of flat plate and also what will be that in a circular tube. And also, we have given some example how to calculate that shear stress, drag force, even you can say that local skin friction factor all those things.

Now, in this lecture we will discuss about the boundary layer, what should be that characteristic feature of the boundary layer whenever fluid will be a flowing over a solid surface?

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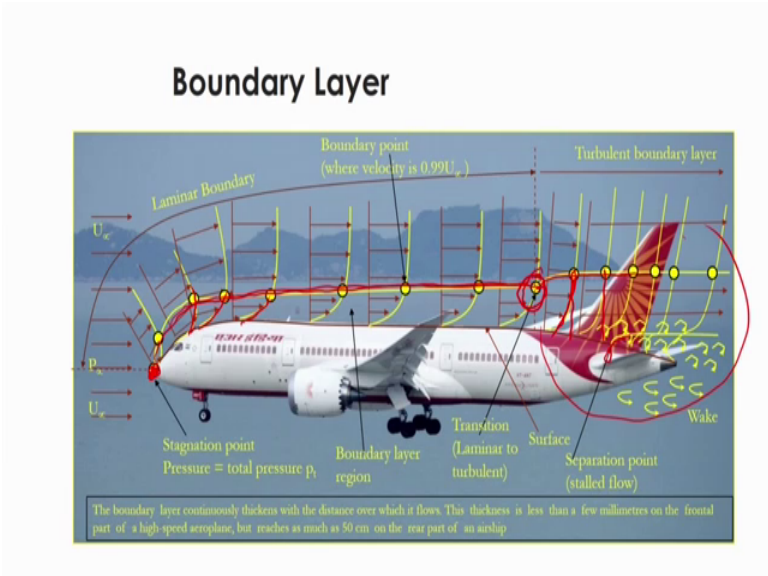

As an example, you can say that if suppose an aeroplane is moving with the high speed you will see whenever it will be moving surrounding the surface that is over the surface of this aeroplane there will be a flow of air opposite to the aeroplane at the same speed of this aeroplane.

Now, during this flow we are having some velocity over the surface of this aeroplane and you will see that velocity will be changing whenever you are going away from the surface of this aeroplane. And very near to the surface you will see there will be a some deviation of the velocity of the air due to some frictional forces acting on the surface and. Since, there will be a viscous effect of this air at this high speed we can say that the velocity gradient over the surface of this plane will be changing.

Now, during the velocity gradients change you will see it will be changing a based on this height of this or length of or you can say that a normal distance from the surface of this plane. And it will be changing in such a way that you will see the velocity of will be reaching to a free surface velocity where that means, it is called infinite velocity or uniform velocity there that is denoted by U infinity here in this picture. And yellow line are showing that there is a boundary yellow line here in this case you will see at this points the velocity from this surface of the aeroplane will be reaching to about 99 percent of the free stream velocity of the air there.

So, at a certain length from the edge of this aeroplane that is from the you can say end of this plane, at a certain length the axial direction that is in if it is horizontal then in the x direction you can consider. And then at a certain length of this x you will see this velocity will be reaching about 99 percent of the free stream velocity. Now, this point will be noted down here. Again, if you go certain extent of this length that is another point if you consider here and you will see from this point on ward there will be a again velocity change.

So, at each point you will see there will be a velocity change in such a way that the gradient will be reaching to what is that free stream velocity gradient there. So, it will be nullify when the free stream velocity will be that means, this velocity will be changing up to a free stream velocity their velocity gradient will be 0. So, when this velocity gradient will be having 0, those point and the distance between those point to the free surface of this solid surface here in this case the surface of the aeroplane, this distance will be called as a boundary layer thickness.

Now, what is that boundary layer? That means, if you add these points when the velocity will be reaching 99 percent of the free stream velocity and if you add those points and you are getting this profile like this, ok. So, this is called the boundary layer. That means, up to a certain distance from the solid surface of this object, there will be a velocity where it will be equal to 99 percent of the free stream velocity. So, this is called boundary layer.

Now, this boundary layer of course, the thickness of this boundary layer will be changing based on the velocity of the stream. Now, if the velocity is so high that means, if you are considering that it is a turbulent flow. We have earlier defined the turbulent flow and laminar flow. In the laminar flow you will get some distance of this that is boundary layer to the solid surface. So, it is called laminar boundary layer thickness.

Similarly, in the turbulent flow if suppose Reynolds number is greater than 4000s in the even above that you will see there will be a. That means, the flow stream will not be laminar in condition that will not give you the what is that uniform fashion of flow, where it will be making some eddies when there will be inter mixing of the fluid. And there in that case you will see to get the 99 percent of the free stream velocity you have to reach beyond some distance what is actually we are obtaining in case of laminar flow.

So, in that case that laminar flow regimes that boundary layer thickness what will be there it will be more than that laminar flow boundary layer thickness.

Now, in this turbulent region, here see in this turbulent region you will see there will be two sub region one is called laminar sub region another is called turbulent sub region. Now, laminar sub region it is sometimes called viscous sub li region or it is called viscous sub layer, and within that layer you will see there will be a more frictional force acting on the surface, whereas above that viscous layer relatively less friction we will observe. And but still there will be a viscous effect and it will cross the distance of the laminar boundary layer based on its turbulent condition.

And at this region there will be a formation of weight, weight means here the fluid particles as a parcel. It will move arbitrary in arbitrary direction and also you can say that this movement will be in such a way that the fluid particles will get intermixing and they will interact with each other and they will form some the chunk of fluid parcels and it will be called as weight. Now, that weight movement will be haphazardly and that means, in there will be a arbitrary direction there will there will change their direction as since the flow velocity is higher and also there will be a in high interaction of this fluid layer and because of which this direction of this weight will be changing.

Now, in this case we are observing that there will be a point where this laminar boundary layer will be changing to the turbulent boundary layer. Now, why where is that point? That here we are observing this point will be denoted as the separation point, where this laminar zone will be converting to the turbulent zone. Even this separation point you can observe in case of laminar sub layer to the turbulent sub layer region. So, that point also will be called as separation point but this will be called as stalled flow. So, in that case you will see the boundary layer thickness will be different from the laminar boundary layer thickness.

Now, observing this figure we are having another point important that is called stagnant point. This stagnant, at this stagnant point that is at this face of this aeroplane this is called stagnant point at this stagnant point the stagnation pressure will be is equal to total pressure where whatever the pressure will be exhorting by this free stream velocity whenever it will be that is push to this neck or that is for mouth of this aeroplane. And they are the stagnation point or stagnation pressure will be equals to here total pressure whatever giving by this free stream.

And you will see there will be a also a change of certain velocity gradient according to the length of this crossing over the velocity over the length of this aeroplane or ordinarily solid surface rather than aeroplane whatever we are observing.

So, in that case the boundary layer continuously thickens with the distance over which it flows and this thickness will be less than a few millimeters on the frontal part of the high speed of this aeroplane here. But it will reach as mass as 50 centimeter on the rear part of the air cheaper aeroplane. So, this you have to remember. So, this is called boundary layer.

So, what we are actually getting here that whenever a fluid will be flowing over a solid surface and adjacent to this solid surface the velocity will be having some gradient and this gradient will be changing according to the vertical direction over the surface of the solid surface. And it will be reaching almost equals to the free stream velocity. And up to which this velocity gradient will be 0 that distance will be called as boundary layer thickness. This boundary layer thickness will be changing according to what is that Reynolds number and also it will be changing based on the surface roughness there. So, if there it is a very smooth there is no friction there will be one laminar, of course, there will be a thickness will be less whereas, the roughness surface there you will get more boundary layer thickness.

Now, we will be discussing here how to calculate all this boundary layer and also how this velocity distribution will be there over the surface will be discussing in this lecture.

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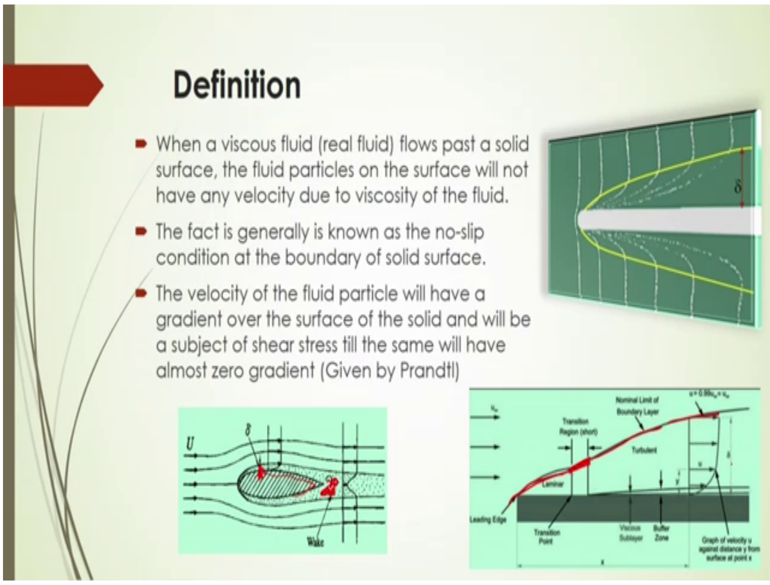

Now, let us first define that this boundary layer here. In this case, when a viscous fluid of course this boundary layer concept will be only a based on the visco viscous fluid flow. So, when this viscous fluid flow will be a flowing first a solid surface the fluid particles on the surface will not have any velocity due to the viscosity of the fluid.

And the fact is generally is known as no slip condition at the boundary of the solid surface that means, at the boundary there will be no velocity of the fluid. And the velocity of the fluid particles of course, will have a gradient over the surface of the solid and will be a subject of shear stress; still the same will have almost 0 gradient there. So, it is actually given by the Prandtl, in previous lecture we have also discussed that the boundary layer and also there will be a layer intermixing by mixing length theory there.

And in this figure, you will see there one coil there that over this surface how this boundary layer that is velocity profile how it will be changing. And the distance it is called boundary layer thickness it is denoted by delta. And here at this region it is forming a weight and here over the flat surface here see how the laminar boundary layer and here this is the transition where there will be a mixing of this laminar and turbulent conditions.

And at this case there will be a again you will get this boundary layer due to this viscous effect of the fluid. And then after that you will see there will be a normal limit of boundary layer here, and then you will see that velocity will be reaching at this 99 percent of the free stream velocity.

So, at this point what will be the boundary layer thickness that you can obtain.

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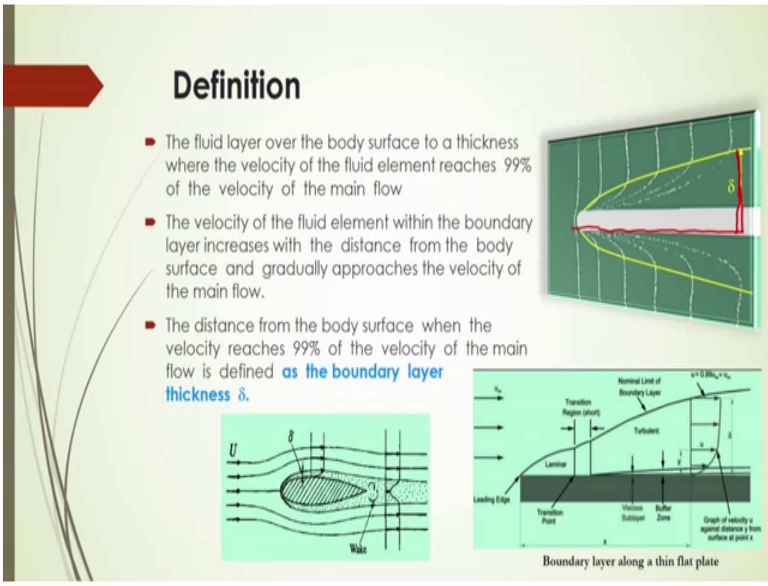

Now, let us to other things that if the fluid layer over the solid surface to a thickness, where the velocity of the fluid element reaches 99 percent of the velocity of the main fluid it will be called as this boundary layer thickness. And the velocity of the fluid element within the boundary layer that will increases with the distance from the body surface that we have already discussed. And also, it will gradually approaches to the velocity of the main flow or free stream it is called. The distance from the body surface when the velocity reaches that 99 percent of the velocity of the main flow is defined as the boundary layer thickness that is delta it is denoted by delta.

So, here in this picture we are observing where this boundary layer thickness, here in the top of this picture here by yellow line and at a certain distance x from this, here you can get this boundary layer thickness over there. So, this boundary layer thickness will be changing according to the x axis or length of the x or if it is in the y direction then it will be in the y the function of y.

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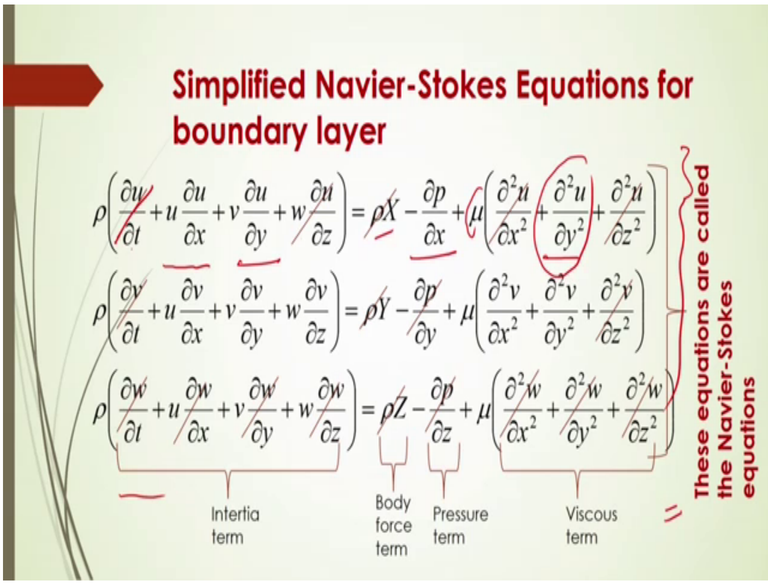

And now, what should be the actually Navier-Stokes equation simplified Navier-Stokes equations for boundary layer here. Now, if we follow that Navier-Stokes equation earlier we have derived in the earlier lectures. So, these are the equations of Navier-Stokes.

So, in this case what are the components will be neglected that are given here this cut mark here, this part will be neglected that is with respect to time it will not be changing of velocity that means, steady state of operation if we consider. And there will be no velocity component in the z direction or there will be no velocity in the z direction you can say. So, in that case the velocity gradient in the z direction will be 0 whereas, in the u and v direction you will observe the velocity gradient there.

So, only these two components u into dou u dou x and dou u dou y this components will be there. And pressure of course, will be there in the x direction there will be change of pressure, and other parts here for viscous effect of course, in the y direction there will be a viscosity because this strange here strange will be in the y direction there the velocity gradient in the y direction. So, there because of which you can get the viscous effect or viscous force there. So, in that case what will be the components? Mu into dou 2 u by dou y square. So, this component or this part will be of course, be considered.

And other part in the x direction there is no velocity and the z directions there will be velocity. So, the velocity gradient will be in 0 there. So, ultimately other than this one, this one and this one, this one, also this one remaining part will be considered as this simplified form in the x direction. If you are considering the y direction, similarly you can observe in the z direction also you can calculate here in this way. So, these are the simplified Navier-Stokes equation for the boundary layer.

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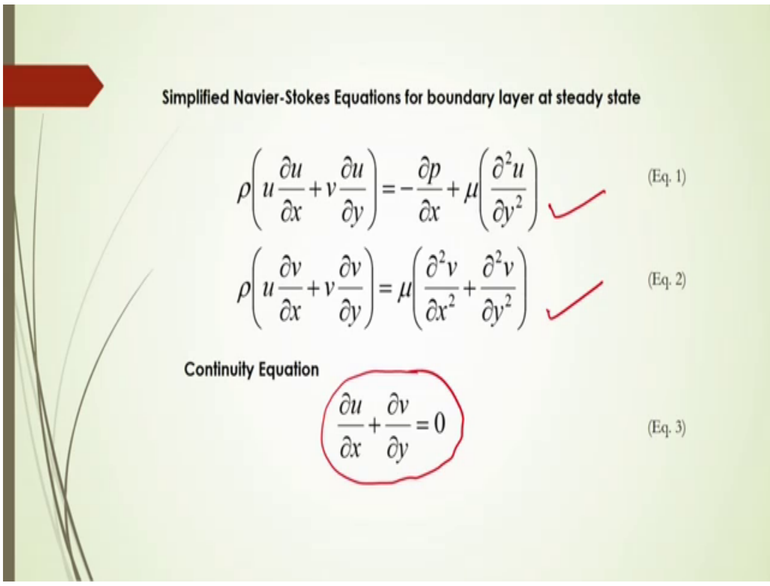

Now, finally, we are getting this rho into u dou u dou x plus v dou u dou y that will be is equal to minus dou p dou x plus mu into dou 2 U by dou y square. So, this will be your in the x direction and this will be your y direction the simplified Navier-Stokes equation for steady state operations. And similarly, can have the continuity equation also the z components will be here 0. So, based on which we are getting this dou u dou x plus dou v dou y that will be equals to 0.

So, this is that, from these equations you can derive further for velocity distribution of the fluid over the surface of the solid surface at this boundary layer thickness.

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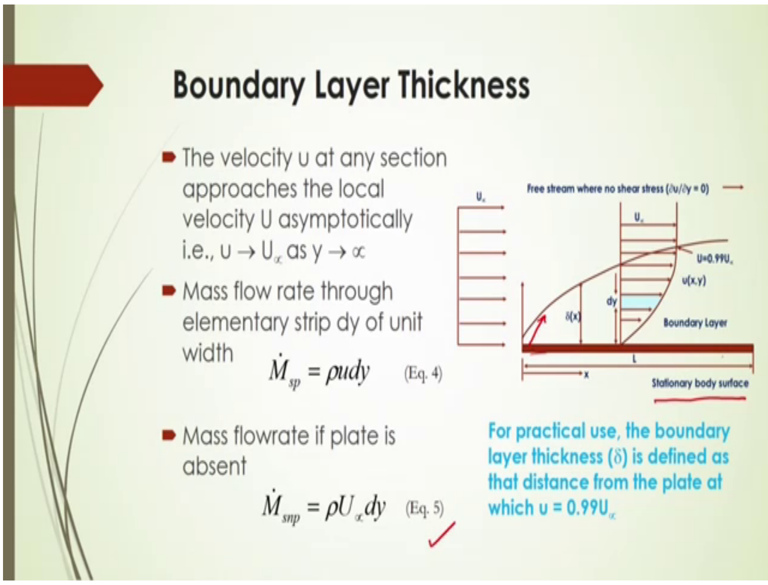

Now, what is that boundary layer thickness? Here, see in this picture it is shown that this boundary layer thickness here. This is a flat plate stationary body surface we can see, and here this one I think in this case here this figure. This is called that the how velocity that is your boundary layer profile here, this one, and this is how this velocity changing in the y direction at a particular x and also the velocity at any section approaches the local velocity U that asymptotically that is U tends to infinity as a when y tends to infinity there.

So, in this case you will see you can say that there will be mass flow rate through the elementary strip of dy if we consider here elementary strip, here in this case this one are shown in blue color. So, here mass flow rate through elementary strip dy of unit width. So, in that case it will be rho U into dy as given in equation number 4. And U infinity is called free steam velocity and u is the velocity at any length x there and x y you can there will be a velocity also there v. So, in that case mass flow rate plate is absent then you can observe another that is what will be the amount of mass is flowed through this strip of small thickness dy. So, it will be is equal to M s and p that will be equals to rho in p U infinity dy which is shown in a equation number 5.

So, mass flow rate through the elementary stream strip by dy of unit width we consider unit width, here in this case this is unit width, ok. So, we can say there will be mass flow rate through this elementary strip of this by equation 4, and also if there is no plate then what should be the mass flow rate through this elementary strip it is given in equation number 5.

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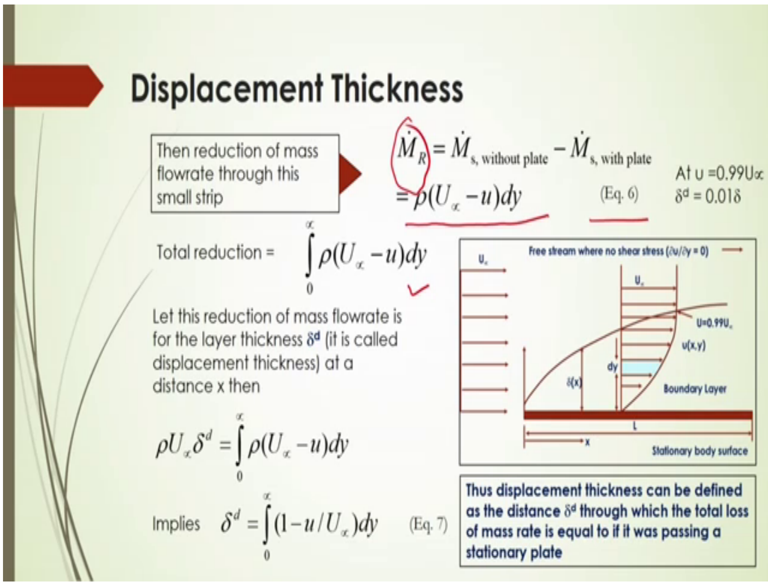

And in this case what should be the reduction of this the mass flow rate through this small strip? That means, if we consider the plate what will be the mass flow rate, if you are not considering the plate what should be the mass flow rate. Just subtracting these we are getting the reduction of mass flow rate when we are putting the solid surface or stationary body surface in the flow. So, then it will be calculated as M dot R that will be equals to rho into U infinity minus u into dy that is us equation number 6 it is shown.

Now, total reduction how it will be there you have to integrate over this infinite length in the y direction, then it will be 0 to infinity, then rho into U infinity minus u into dy this one. Now, if we consider that this reduction of this mass flow rate due to this placing of this stationary body surface in the flow for the layer thickness it is it will happen for the layer thickness of d to the power d that is called displacement thickness at a distance of x.

Then we can say we can write here this rho U infinity into delta d will be equals to 0 to infinity into rho U infinity minus u into dy which implies delta d that means, displacement thickness will be is equal to 0 into infinity one minus u by U infinity into dy. So, this is your displacement thickness.

Now, in this case to find out this displacement thickness you have to know what should be the local velocity to the free stream velocity of in the boundary layer region. So, that profile will give you the displacement thickness. Now, you will get the several laminar boundary a profile turbulent boundary layer, if you are substitute in this profile of that boundary layer you can easily calculate the displacement thickness.

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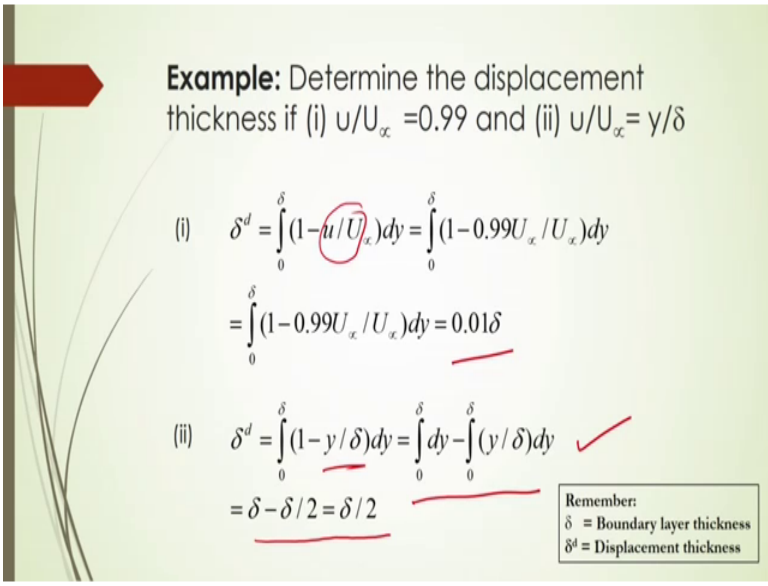

Like, example here that determine the displacement thickness if the velocity is 99 percent of the free stream velocity that means, u by U infinity is equal to 0.99 and also if this velocity distribution will be equals to y by delta into U infinity. So, at this two conditions for should be the displacement thickness.

Now, this displacement thickness of the substitution of this value of this ratio of u by U infinity there we are getting here this displacement thickness will be is equal to 1 percent of the boundary layer thickness there. So, this a displacement thickness will not be exactly the boundary layer thickness. This displacement thickness will be actually changing according to the x, but it may not be the case that that the thickness where this velocity will be reaching almost uniform free stream velocity there.

In the other cases if u by U infinity is a function of that is y then how it will there this displacement thickness the case two here. So, in this case if we substitute this u by U infinity as y by delta here, so you are getting after integration this value as delta by 2. So, very interesting that this displacement thickness will be 50 percent of the boundary layer thickness at this condition of y by delta. At any y you can say for this delta y by delta ratio we can have this 50 percent of the boundary layer thickness as a displacement thickness here.

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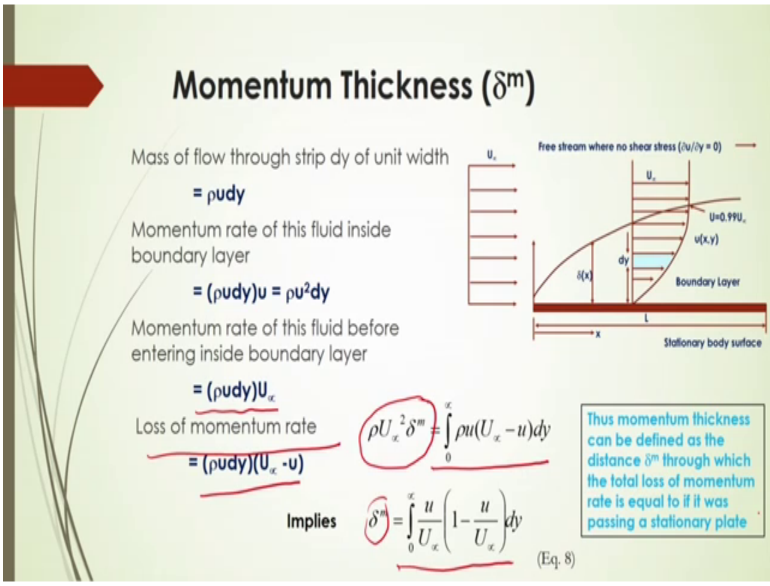

Now, what will be the momentum thickness similarly, if we observe that if I not place this a stationary body surface there. Then, what should be the mass of flow through strip dy of unit width? It will be rho U into dy. And then what should be the momentum rate of this fluid inside the boundary layer? It will be rho u dy into U is equal to rho u square dy. So, momentum rate of this fluid before entering inside the boundary layer that will be is equal to this rho u into dy into U infinity.

Now, loss of momentum rate that will be equals to here just after subtracting this two terms then we are getting the loss of momentum rate, then again defining this momentum thickness as delta M which will be equivalent to that distance through which the total loss of the momentum rate is equal to if it was passing a stationary plate. So, in that case it will be calculated by this again integration of to the infinite distance in the y direction after substitution of this reduction. So, which will implies that momentum thickness will be equals to this one as given in equation number 8. So, from this also you can calculate the momentum thickness based on the velocity profile of the boundary layer.

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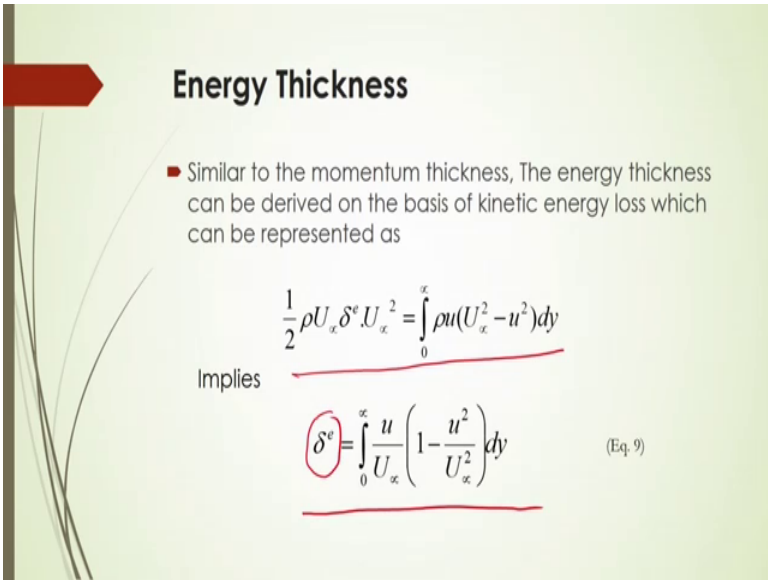

Now, energy thickness similarly to this momentum thickness we can derive also energy thickness and based on the kinetic energy loss which is represented by this equation here. And this after simplification integration will implies this equation number 9 and from which also you can get this energy thickness and this is also that equivalent thickness at which this after placing this stationary surface there what will be the change of energy there. So, from this equation 9 we can calculate what will be the energy thickness.

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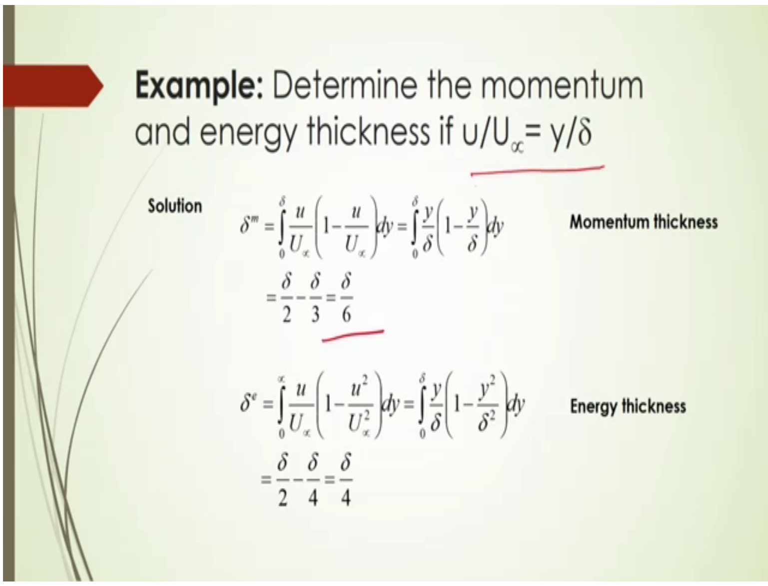

Now, let us do an example for this again with that a velocity profile of u by U infinity is equal to y by delta what should be the momentum thickness and energy thickness there. Now, if we substitute this velocity profile over that formula given earlier in equation number 8 and 9, we are getting respectively these momentum thickness as delta by 6 and energy thickness as delta by 4.

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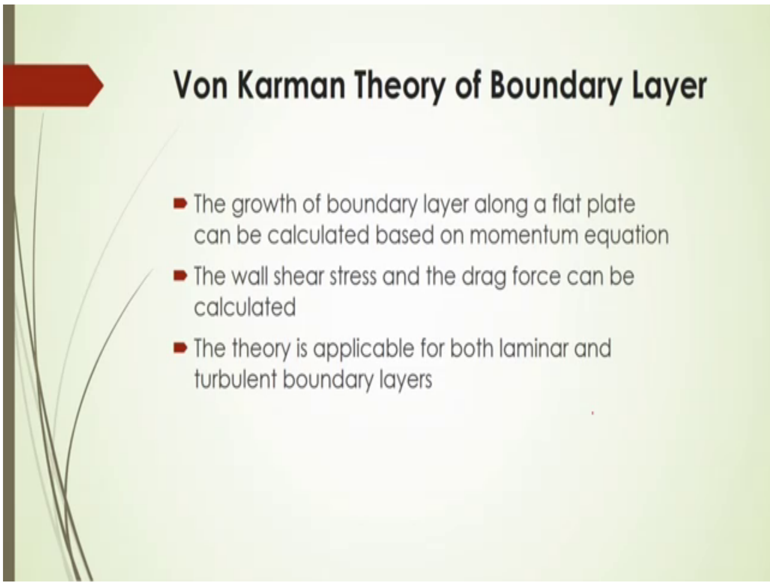

And again what should be the Von Karman Theory of boundary layer, that is again we can say that if we place any flat plate and then growth of the boundary layer how it will be there we can calculate based on the Von Karman theory. And the theory is applicable for both laminar and turbulent boundary layers here. And in this case wall shear stress and the drag force also can be calculated.

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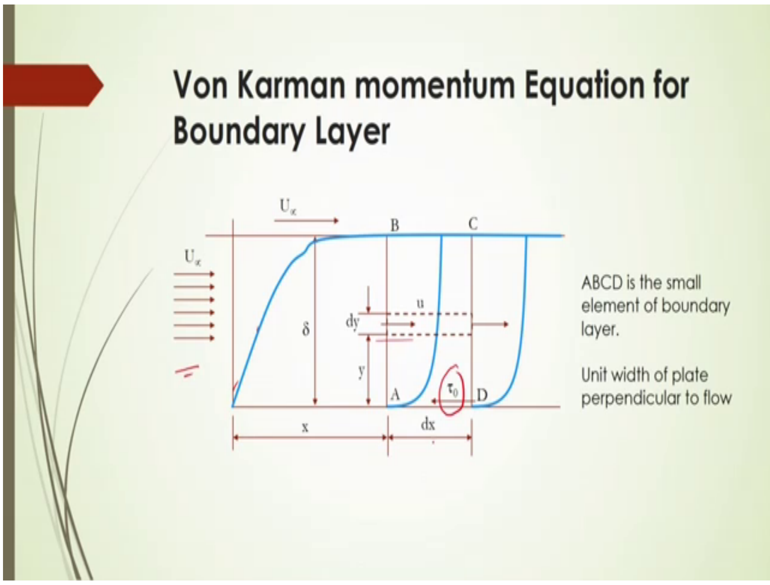

So, let us see that Von Karman momentum equation for the boundary layer here. Let us consider the A B C D as a small element of the boundary layer shown here. So, this is your boundary layer and within this boundary layer we are considering here A B C D as a small element of this boundary layer and here in this case we are considering unit width of the plate perpendicular to the flow and U infinity is the free stream velocity, and at a certain x we are considering this elementary boundary layer.

And if we consider there will be dy of thickness for this then what should be the shear stress is acting over there? In the opposite direction of the flow the shear stress will be acting as a here it will be denoted by this tau 0. And at the y direction we are considering again small thickness of this boundary layer as a dy which will be flowing with a velocity U. And now, based on this we can apply the Von Karman theory for the momentum equation and which can be derived based on this shear stress equation that means, shear stress will be actually defined based on the what is that momentum equation.

Now, what will be that momentum? If we divide it by that kinetic energy terms there rho U infinity square then we can getting that shear stress will be equals to rho U infinity square that will be again it will be a d by dx into what is that u by U infinity 1 minus u by infinity into dy.

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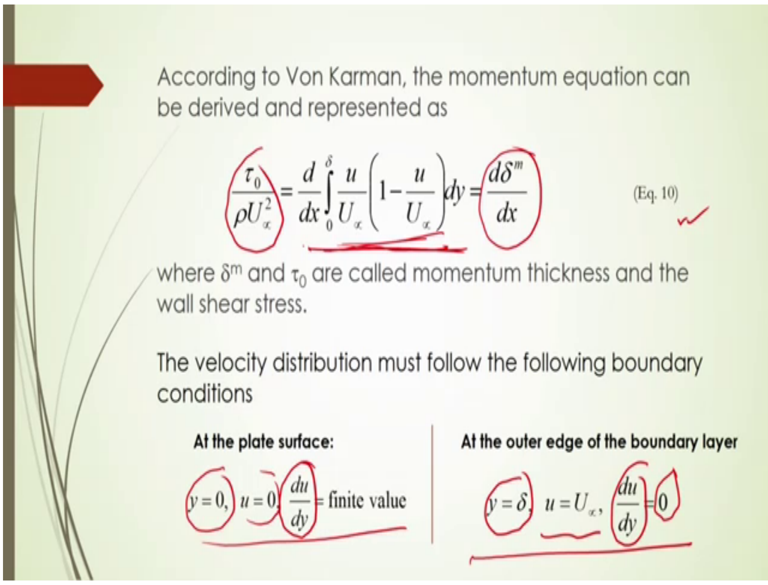

So, this from this equation you can easily calculate what should be the momentum equation for that. And in this case very interesting that what will be the momentum term what we have derived here. So, this is your momentum thickness based on which we can say that this is your profile by which you can calculate the momentum thickness, and if you are substituting that momentum thickness here we can have these the change of momentum thickness with respect to x.

So, according to that Von Karman this momentum equation can be represented by this equation number 10, where we can see what should be the shear stress over the surface which is acting opposite to the flow of the fluid on the surface of the solid. So, in that case you need to have the free stream velocity it depends on free stream velocity and also a fluid properties. So, the change of momentum thickness with respect to x will give you the shear stress over there.

And the velocity distribution then of course, it will be following the boundary conditions certain boundary conditions. So, at the plate surface if we consider that y is equal to 0, then U should be 0 because there will be no slip and then there will be no velocity gradient in the y direction then it will be a certain finite value. And over that surface, at that particular y is equal to 0 you will not get any velocity gradient because here there will be no velocity. But, very small thickness if you are considering the film thickness like that infinite this that means, very is you can say that infinitely small thickness if you are considering that then there will be a some value of gradient of velocity. So, that will be considered as the finite value.

At the other side you can say that if you are considering the outer edge of the boundary layer in that case of course, y should be is the boundary layer thickness that is y is equal to delta and u should be reaching to the U infinity that means, free stream velocity. And in that case since there we no change of velocity in the y direction that is called uniform velocity. So, in that case the du by dy that is called velocity gradient in the y direction will not be there, so it will be 0.

So, at this boundary, so two boundary conditions we are having at the surface where y is equal to 0, there will be a velocity gradient finite and at the outer edge of the boundary layer we can say there will be uniform velocity. So, there will be no gradient that will be is equals to 0 at the boundary layer thickness.

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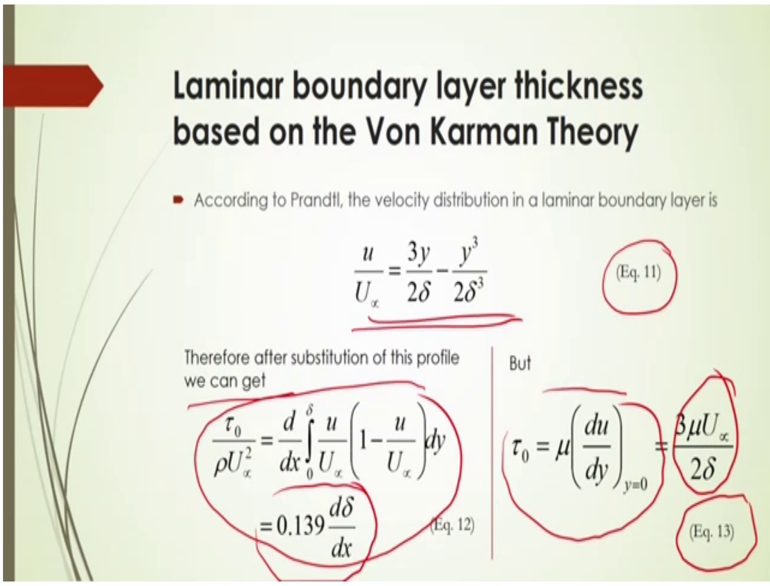

Now, after substitution all these things we can get this boundary conditions we can get this velocity distribution in a laminar boundary layer as this and therefore, after substitution of this profile we can get the momentum equation as this, here. So, this we are substituting this in equation number 10, this is momentum equation and after substitution and integration and rearrangement we can get this shear stress by rho U infinity U square is equals to 0.139 to d delta by dx.

But we have the definition of this shear stress at surface as tau 0 is equal to mu into du by dy, where y is equal to 0. Now, after substitution of this du by dy at y is equal to 0 from this equation number 11 we can get this shear stress as 3 mu U infinity by 2 delta after simplification, as equation number 13 here. So, we can then easily calculate what is the shear stress, if you know the boundary layer thickness and the free stream velocity. Of course, the properties will fluid should be known because here viscosity is the one important terms important physical properties of the fluid by which you can calculate the shear stress.

So, this shear stress will vary with respect to viscosity as well as that free stream velocity. So, if you are using high viscous fluid, like if you are using oil you can get more shear stress whereas, if you are using only simple water you can get less shear stress there because the while will have more viscosity than water.

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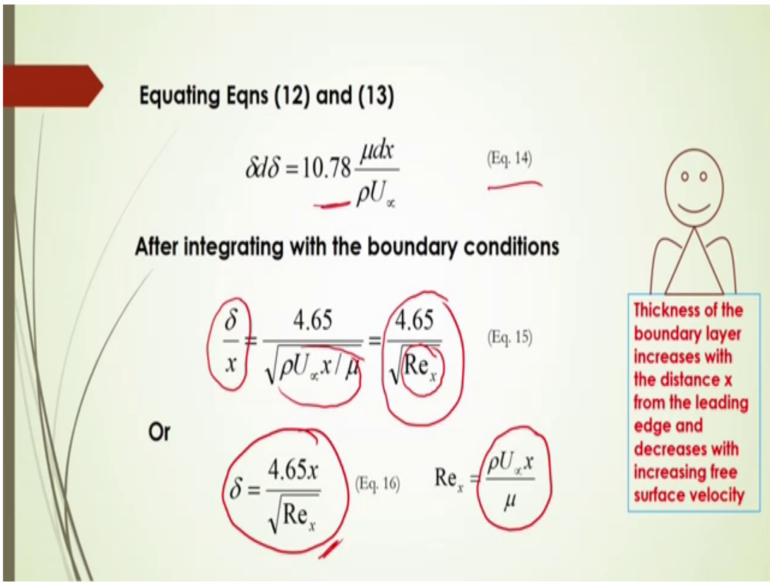

Now, I equating equation this 12 and 13, here shown then we are having this how this boundary layer thickness will be changing according or with respect to x there. So, we are getting delta into d delta will be equals to 10.78 into mu dx by rho U infinity as given in equation number 14.

Now, after integrating with the boundary conditions that we have given here at the plate surface and at the outer range of the boundary layer, we can we can have this delta by x will be is equal to 4.65 divided by root over rho U infinity x divided by mu. And here this terms rho U infinity x by mu will be called as Reynolds number based on that is horizontal distance from the starting of the boundary layer. So, in that case we are having then delta by x will be is equal to 4.65 by root over Reynolds number. So, this delta we are getting it is a function of x now, so delta will be equal to 4.65 x by root over Re x, where Re x is defined as rho U infinity x by mu.

So, very interesting that for this laminar boundary conditions if we apply then simply we can calculate what should be the boundary layer thickness over this laminar boundary conditions here. It can be related to the Reynolds number that means, related to the velocity and the physical properties and also it will vary according to the axial distance in the x direction. Now, thickness, this thickness of this boundary layer it will increases then with the distance x from the leading is and the decreases with increasing free surface velocity there.

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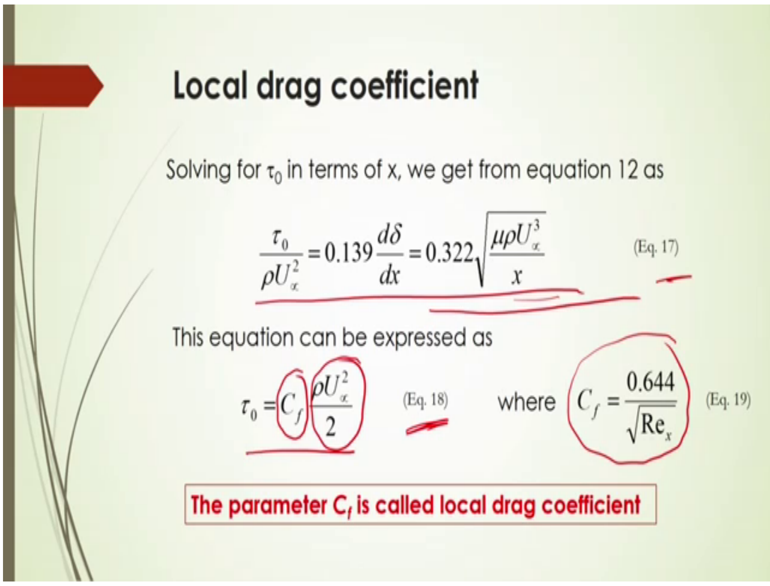

Now, how to calculate the local drag coefficient based on this boundary layer theory? Now, solving for this shear stress in terms of x we get from equation number 12 as tau 0 by rho U infinities square that will be equals to 0.139 into d delta by dx that will be is equal to 0.322 root over mu rho U infinity cube by x. So, as shown in equation number 13. So, from this equation you can simply calculate this shear stress.

Now, if we relate this shear stress with this expression of this tau 0 as a function of kinetic energy that it will be a function of kinetic energy like here rho U infinity square by 2 then the proportionality constant the C f will be called as local drag coefficient here.

Now, this if you compare this equation number 18 and with this equation number 17, then you can say what is that the local drag coefficient will be equals to that is 0.644 by root over Reynolds number. So, this parameter C f is called the local drag coefficient which is very important to model the flow over the flat surface at this boundary layer condition. And based on which the even other I think the flow device will also designed in such way that a for a range of high viscous flow to be flowed and it will be designed in that particular physical properties condition.

And what should be the drag coefficient that also, because this fictional drag will give you the various laws of the flow and energy loss during the flow that. So, you have to know this part for this local drag coefficient to calculate and also to calculate the energy economy of the flow process there.

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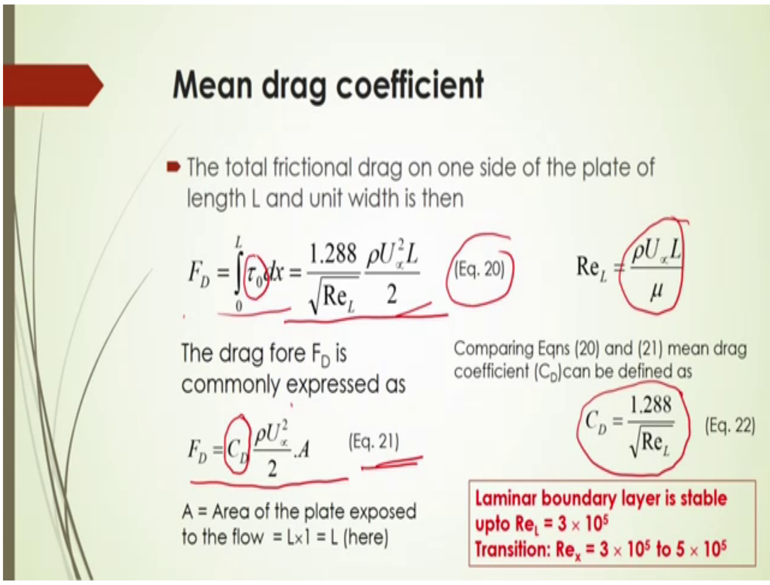

And a mean drag coefficient also one important aspect there whenever there will be frictional force acting over the surface at this boundary layer condition, then the total frictional drag on one side of the plate of the length if you considering a land you need to width if we consider then it can be defined as F D is equal to tau 0 into dz dx. Again you have to substitute this tau 0 here and finally, after simplification and integration and simplification this equation 20 we can obtain. And in this case we are defining Reynolds number based on the length of that is plate over which the fluid is flowing.

Now, this drag force generally expressed by this equation number 21. Again, it is a, it is related to the cross sectional area and also what is that kinetic energy of the flow. So, this F D will be is equal to C D into rho U infinity square by 2 into A, here C D is called again proportionality constant and this will be called as mean drag coefficient. And if we compare this equation number 20 and 21 then we can have this mean drag coefficient as cd is equal to 1.288 by root over Re L, Re L is the Reynolds number based on the length of the plate.

Now, this laminar boundary layer is a stable up to Reynolds number is equal to 3 into 10 to the power 5. You have to remember it and also the transition will occur there where this laminar boundary layer to be converted to the turbulent boundary layer that will be to Re x that will be to be 3 into 10 to the power 5 to 5 into 10 to the power 5. And the critical distance it will be there where Reynolds number will be is equal to 5 into 10 to the power 5. So, within this range of Reynolds number of 3 to 5 into 10 to the power 5 you can get this mean drag coefficient based on this laminar boundary theory.

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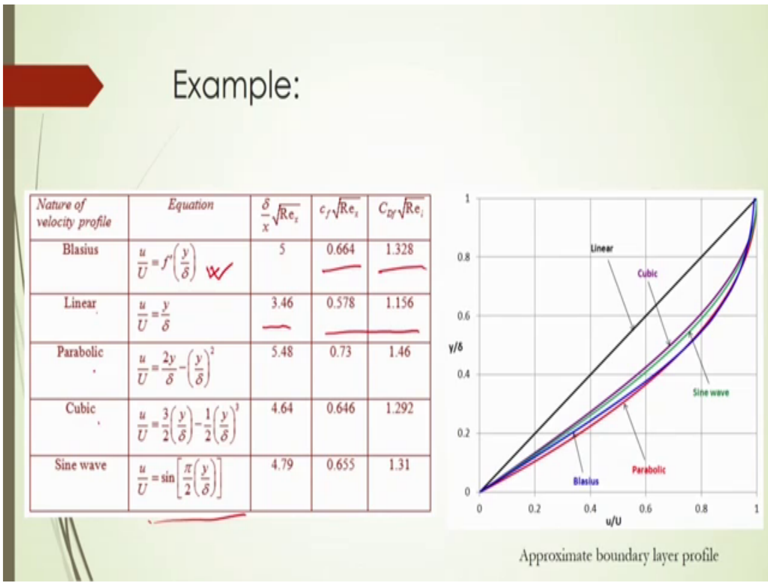

Now, some examples of this drag coefficient that is here by that is the local drag coefficient even mean drag coefficient and also the boundary layer thickness you can have a based on the different boundary layer profile.

Now, if you consider the Blasius profile it is called as Blasius profile u by U that will be is equal to f dashed into y by delta and based on these we are getting delta will be equals to 5 into x divided by root over Re x. Similarly, for C f it will be 0.664 divided by root over Re x, and also the schedule we considered as 1.328 into 8 divided by root over Re L. So, in this way you can get the different values of delta C f and C D for linear parabolic cubic and the sine wave profile of the boundary layer there. So, by this you can easily calculate. So, what should be the actual boundary layer thickness, that depends on the velocity profile over the surface.

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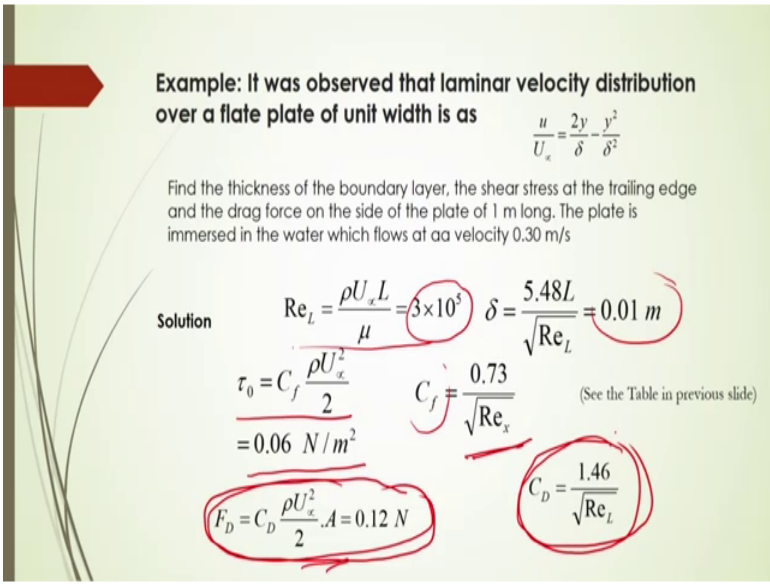

And let us do an example for this. It is seen that the laminar velocity distribution over a flat plate of unit width is as like u by U infinity is equal to 2 y by delta minus y square by delta square. So, in this case what should be the thickness of the boundary layer and what should be the shear stress at the trailing edge and drag force on the side of that is sub object or here in this case plate of 1 millimeter long. And the plate is immersed in the water where this water is water is flowing at 0.3 meter per second.

So, in this case you have to calculate first Reynolds number for the length of one meter and it is coming 3 into 10 to the power 5 as for this a problem. And in this case after substitution of this Reynolds number we are getting this delta will be equals to 0.01 meter. And shear stress again as per definition that is given in the previous slides that tau 0 will be equals to what; and then C f is the local drag coefficient then you can calculate is this local drag coefficient after that substitution of this local drag coefficient in this equation you can get this drag force will be is equal to 0.06 Newton per meter square.

Similarly, F D can be calculated based on this equation. The equation is shown in the earlier a slide, so here also you can have this. So, based on this equation you can calculate what will be the drag force according to this drag coefficient here.

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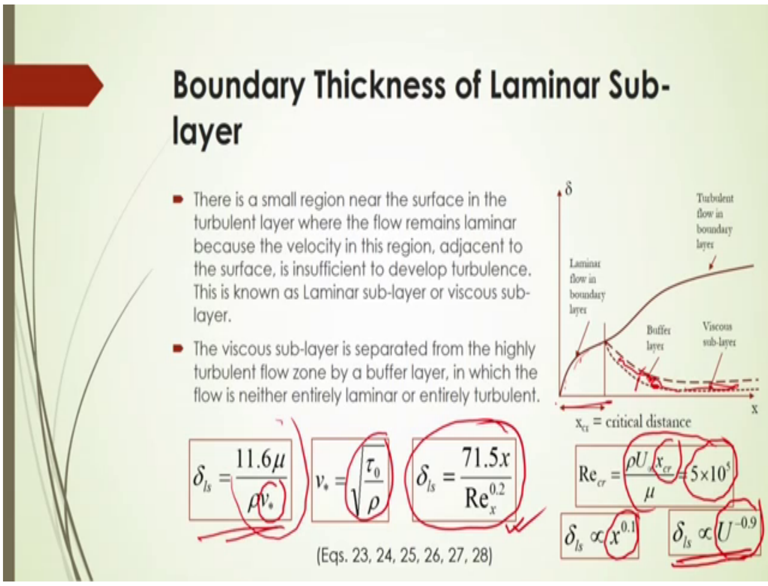

Now, what should be the boundary layer thickness of laminar sub layer here? So, we have already discussed that what should be the laminar sub layer or viscous sub layer. Here in this picture it is shown this is the boundary layer and this region is called what is that a laminar flow in boundary layer and this is the buffer layer where this viscous sub layer and the turbulent sub layer we will meet there. So, this region is called the buffer region and beyond this here it will be turbulent and before this it will be viscous sub layer.

Now, what will be that viscous sub layer thickness that is separated from this buffer layer? You can calculate based on this formula given here. In this case it depends on the fiction velocity that the defined as by root over tau 0 by rho that already we have discussed in the previous lecture in the turbulent flow condition what will be the friction velocity. And also based on this friction velocity you can calculate the laminar sub layer boundary layer thickness and also how it will be related to the x that you can calculate from this equation.

Now, what will the critical distance from this edge, where you can get the separation of this laminar to turbulent boundary layer? That is called critical distance that critical distance can be calculate from the critical Reynolds number that critical Reynolds number will be is equals to 5 into 10 to the power 5. So, based on this critical Reynolds number what will be the x critical that you can calculate, provided that you have to have the value of uniform velocity that means, free stream velocity and also physical properties of the system.

And it is observed that this laminar sub layer thickness is related to the distance from its edge that is x here, and it will be a power law that is related to the x to the power 0.1 and also it will be related to the inversely proportional to the free stream velocity. So, which will be represented by this delta ls will be equals to some constant of this free stream velocity to the power minus 0.9 there.

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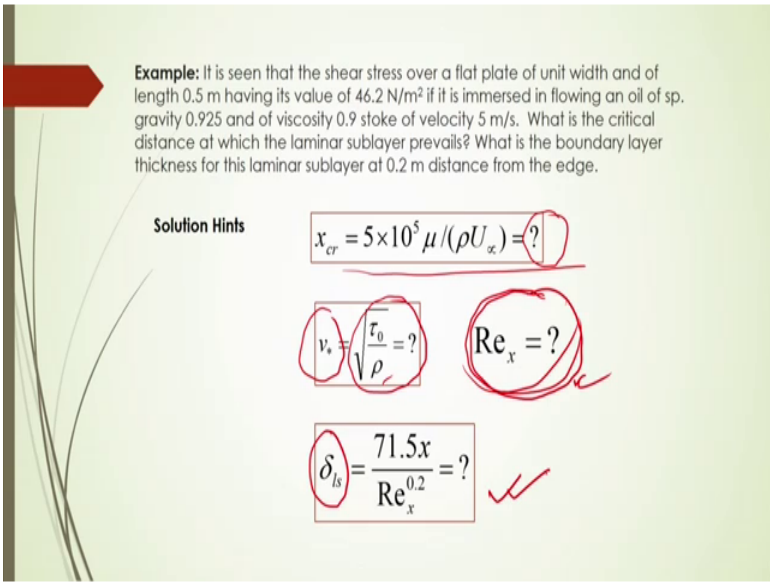

Now, let us do an example here again that, if you are observing that shear stress over a flat plate of unit width and of length of 0.5 meter which having its value of 46.2 Newton per meter square. And if you are merging this flat plate into an oil of specific gravity 0.925 and the viscosity as 0.9 stroke of velocity 5 meter per second. Then what should be the critical distance at which you can get the laminar sub layer? And also what will be the boundary layer thickness for this laminar sub layer at 0.2 meter distance from the edge?

Now, in this case we have already shown that what should be the critical a distance there. This critical distance can be calculated from this Reynolds number. In this case then it will be equals to 5 into 10 to the power 5 mu by rho U infinity. What should be that value? You can calculate easily. And here v star that means, a friction velocity that will be is equal to tau 0 to the tau 0 by rho, tau 0 is given to you rho is also is given to you then what should be the Reynolds number you can calculate at particular edge. Once you know this at length x Reynolds number you can easily calculate what will be the laminar sub layer thickness from this equation.

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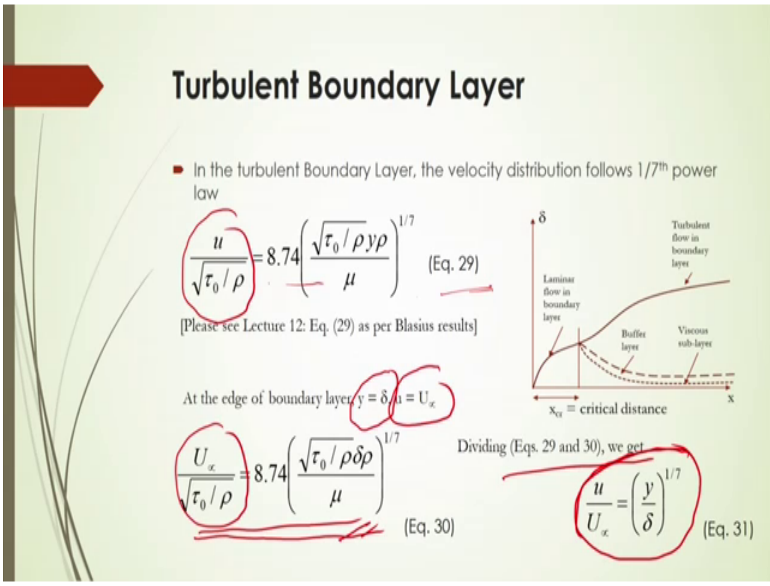

Now, let us consider the turbulent boundary layer how to calculate the turbulent boundary layer profile as well as what will be the boundary layer thickness at this turbulent condition.

Now, since we have observed in the previous lecture that that velocity distribution boundary, velocity distribution in this turbulent condition will follow the one 7th power law. So, based on that we can see that U by root over tau 0 by rho that will be is equal to 8.74 into root over tau 0 by rho y rho y mu to the power 1 by 7. Here instead of v star we are just a substituting root over tau 0 by rho So, that is given in equation number 29 here. So, follow this lecture twelve that is previous lecture equation number 29 as per Blasius results, we have obtained this equation number 29.

Now, at the edge of this boundary layer if we substitute the boundary layer condition as y is equal to delta, and where u is equal to U infinity then we can get this value that is given in equation number 30. So, this profile will give you the shear stress equation when the free stream velocity also you can calculate from this equation, once you know the friction velocity or a shear stress there. So, dividing equation number 29 and 30 this two equations we are getting this u by U infinity is equal to a y by delta to the power 1 by 7 here.

So, earlier we have observed that that velocity distribution in a circular pipe that will be is equal to what that, u by U max will be is equal to y by R to the power 1 by 7. Here in this case based on this boundary layer we can get this over u by U infinity that will be is equal to y by delta to the power 1 by 7. So, by equation number 31 you can get the bound turbulent boundary layer equation provided by boundary layer thickness known to you.

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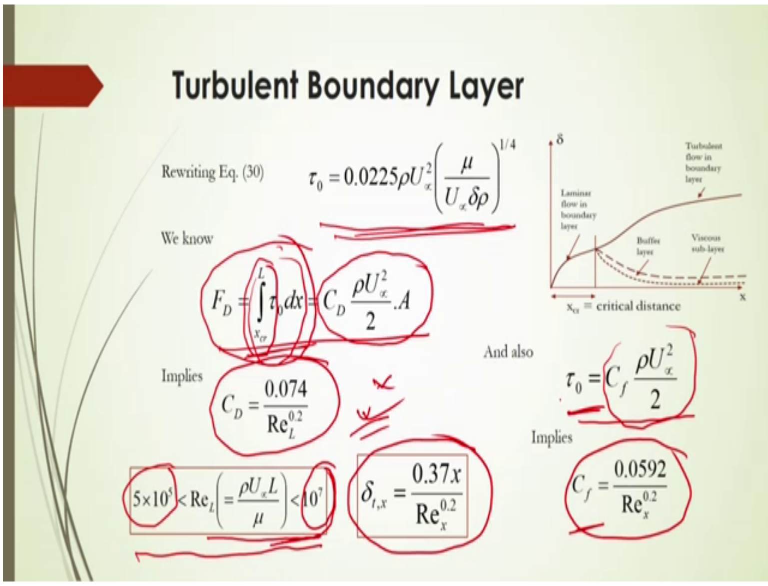

Now, relating this equation number 30 again we are getting this tau 0 will be equals to 0 this, and after substitution of tau 0 in this and integrating with this limit of x critical to this L because this is this limit is called that bound turbulent boundary region and from which you can calculate the what will be the total drag force over there. So, after substitution of tau 0 from this then you can have the total drag force there.

Again, this total drag force will be equating with this definition of drag force as C D into rho U infinity square by 2 into A and based on this a equality then we can get this what is that cd that means, drag coefficient it is called mean drag coefficient is like this. And also, then shear stress will be is equal to local drag coefficient will be this by defining shear stress and equating with this tau 0, then we can get this C f could be equals to this.

And after that if we know this Reynolds number based on this drag coefficient and also what is that boundary layer thickness we can observe what would be the boundary layer thickness based on this theory. And this equations or this correlations will be actually valid to the in this range of Reynolds number here, where this is the critical Reynolds number and this is your what is that up to this 10 to the power 7 up to this boundary condition of turbulent flow that you can calculate this shear stress, local drag coefficient, and average drag coefficient there.

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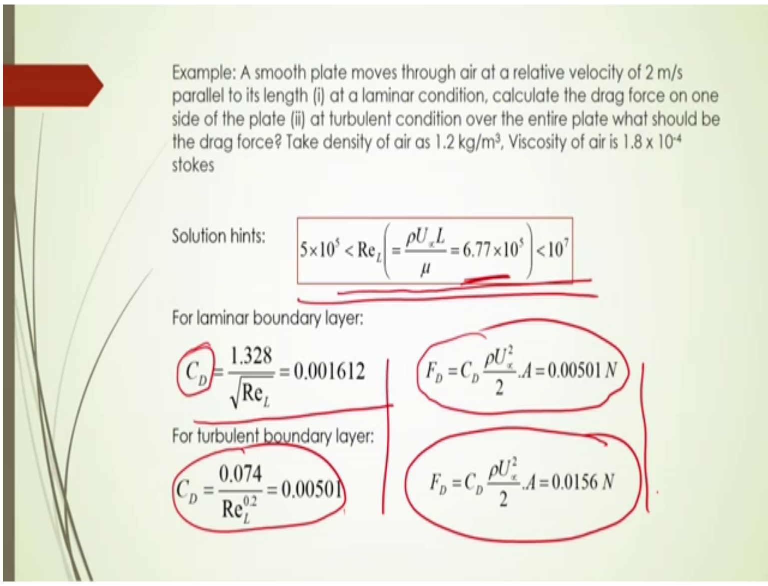

And let us do an example here which seems there this smooth plate moves through air at a relative velocity of 2 meter per second parallel to its length. Now, in this case at a laminar condition; in this case calculate the drag force on the one side of the plate and at turbulent conditions over the entire plate. What should be the drag force? And in this case, density of there is required viscosity of there is required it is given here.

And I am giving this solution hints here, so you have to first find out the Reynolds number. It is observed that (Refer Time: 56:22) 6.77 into 10 to the power 5, it is actually within the range of this turbulent conditions. Now, for laminar boundary conditions what will be that, if we substitute this Reynolds number here then we can calculate the C D and F D also by these equations, and for turbulent boundary conditions we are getting the C D and the F D here. So, the laminar and turbulent condition see the difference here. So, there will be change of drag coefficient in laminar and boundary laminar and turbulent conditions there based on this example.

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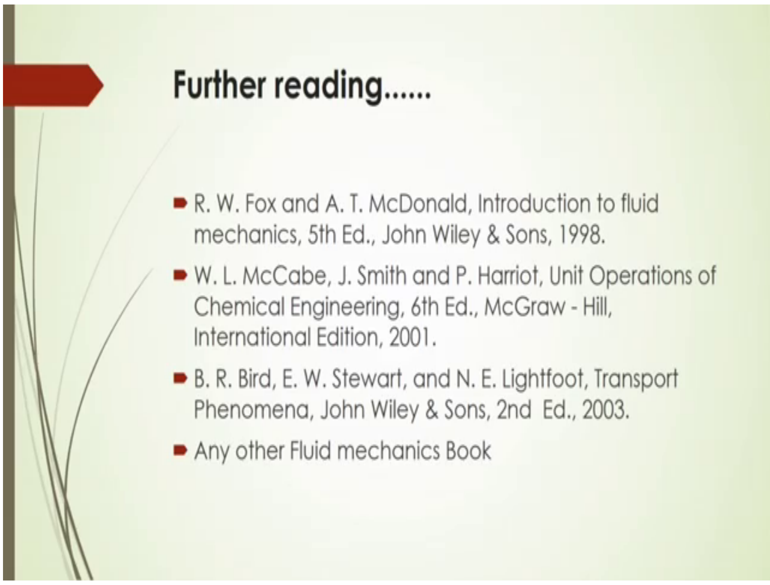

Now, I will suggest you to go for the reading by different text books of this and even you can follow this notes also. Just the following different fluid mechanics books here shown in the slides, and also try to practice this examples based on the theory.

Thank you.