Fluid Flow Operations
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Module – 05
Flow of Viscous Fluid - Part 3
Lecture – 12
Velocity distribution of turbulent flow

Welcome to massive open online course on Fluid Flow Operations. In the previous lecture we have discussed something about a velocity distribution in a laminar flow and the how the flow is flowing, when there will be a plate over this fluid a moving in the same and opposite directions. And what will be the Poiseuille flow, what will be the Couettte flow and what are the different phenomena of the fluid flow whenever it will be moving through the pipe and fluid will be viscous in this case.

So, how the velocity of the fluid will be showing based on the governing equation like Navier-Stokes equation even other phenomena also we have discussed. Now, in this lecture we will discuss the Velocity distribution of a turbulent flow for this viscous fluid. Now, if we go back that what is that viscous flow.

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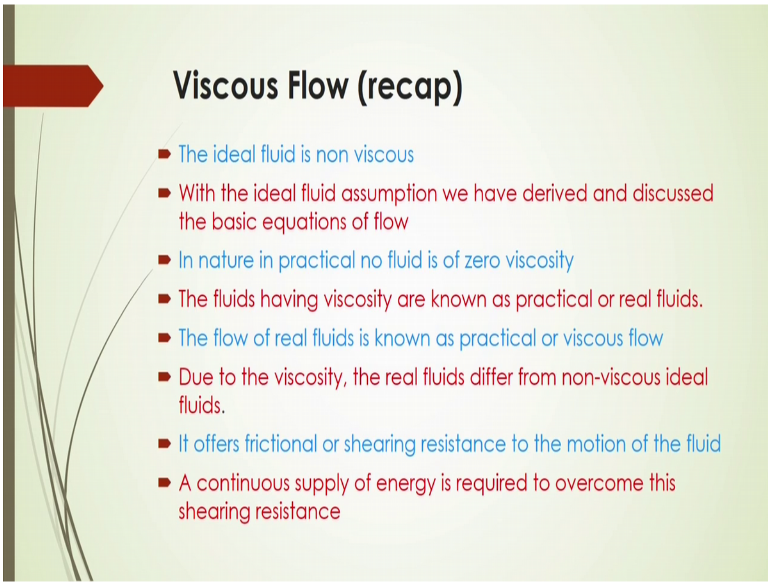

That viscous flow is the ideal fluid, which is non viscous in that case we cannot consider that it will be ideal if it is there is a viscous phenomena there. And for ideal flow, I think we have no viscosity in that case we will be calling as that the ideal flow ideal fluid and the flow is called as ideal flow. And also the ideal fluid assumptions we have derived and discussed the basic equations of the flow, and we also you know that in the nature there will be no practical fluid that has 0 viscosity.

So, all the fluids in the nature will be considered as the viscous flow and the flow of that viscous fluid is called the viscous fluid, and all the viscous fluid will be flowing as a turbulent flow, and also as some extent it will be laminar flow if there is a higher viscosity, and that depends on the velocity of the fluid and also geometry of the conduit through which the fluid will be a flowing.

And also the fluid having viscosity, that are known as practical or real fluids those who have that viscosity. And the flow of that real fluids is known as the practical or viscous flow and we have also discussed that the due to this viscosity the real fluids of course, differ from the non-viscous ideal fluids. Because, their governing equations that will be different and also we have discussed that energy balance equation where that for ideal flow, there will be no head loss is there.

But in the viscous fluid there will be friction and because of which there will be some head loss and because of that conservation of energy, we will be able to that what will be the head loss there because of this friction and because of this viscous flow. And viscous flow of course, will offer that frictional or shearing resistance to the motion of the fluid. We will be discussing again that what will be the universal resistance or universal resistance law, there in the that is fourth coming lecture. And also a continuous supply of energy is required to overcome this shearing resistance whenever this viscous fluid will be flowing through the conduit.

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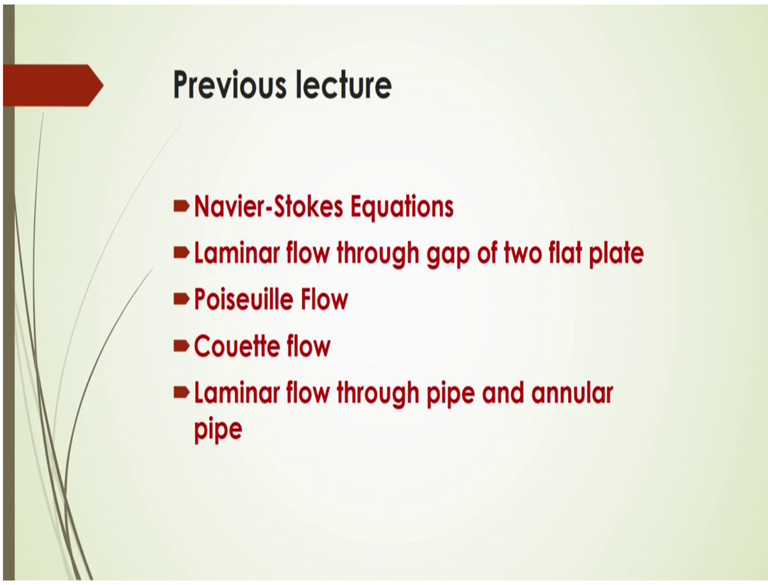

And we have discussed that Navier-Stokes equation, laminar flow through the gap of 2 flat plate, Poiseuille flow, Couette flow laminar flow through the pipe and annular pipe also.

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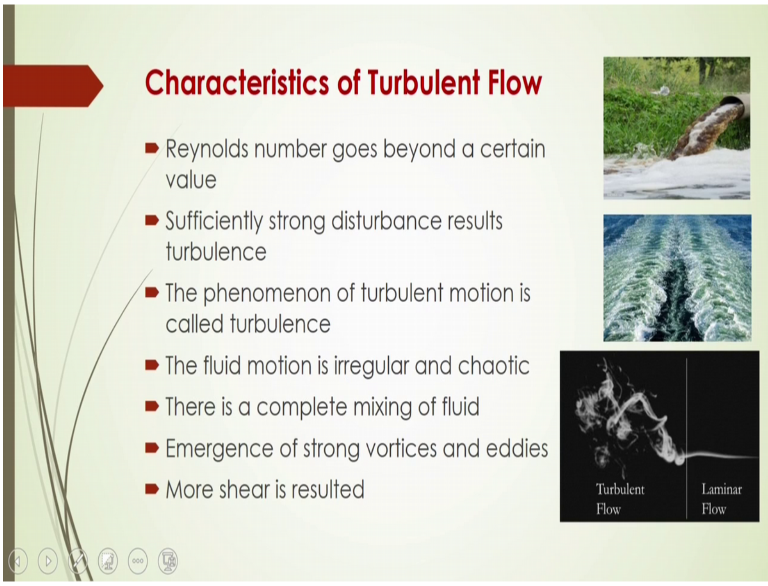

So, in this lecture we will be discussing that what will be the turbulent flow and what will be the different characteristics feature of turbulent flow. So, in this case Reynolds number if it goes beyond a certain value, we will be considering the flow will be as turbulent flow. We have already discussed in the earlier lecture also there will be a classification of this laminar flow and turbulent flow based on the Reynolds number. And in that case if Reynolds number is less than 230 0 that is for circular pipe flow in that case we are getting the laminar flow, and if this Reynolds number is greater than 4000s and it will be called as a turbulent flow.

So, in that case in the turbulent flow sufficient strong turbulence will be resulted based on this high velocity and because of which there will be a Reynolds number high and up. And also the phenomena of turbulent motion is called the turbulence is there. So, will be having what will be the degree of turbulence will be having the degree of turbulence and as well as the magnitude of the turbulence is there and based on this velocity.

And the fluid motion is irregular and in this case it will be chaotic of this see this picture there. In this picture it will see that there is a chaotic nature of the fluid flow and it will be called as turbulent flow and also whenever any sheep is moving on the surface of the any river or sea there behind this sheep, how this actually the fluid particles will be getting turbulence. Because, of that impeller a velocity and impeller rotation, and how fluid particles as a parcels it will be displacing from its main stream there.

So, based on which this turbulence will be creating there. And there is a complete mixing of the fluid also during this turbulence of course, whenever we are just pouring some liquid in a suppose in a cup of different types of liquid there, you will see because of that jet energy the liquid will be making turbulence inside the pan of the other liquid there. So, in that case the mixing of the fluid will be there and that depends on the how you are that supplying the liquid and what will be the velocity of that liquid in that particular operation there.

And also emergence of strong war vortices also will be there in the turbulence flow, and you will see some eddies also will be forming; whenever fluid will be flowing through the pipe, whenever it will be getting some disturbance at the point of disturbance you will see the fluid particles will be getting distorted. And, also may it will be a deviating from its or it will be just changing its position from the mainstream and hapozoidally and in the chaotic in nature so, that the strong vortices and eddies will be forming there.

And in that case you cannot expect that there will be laminar flow, of course, a high shear will be produced and because of which there will be a phenomenon that is called the degree of turbulence will be creating.

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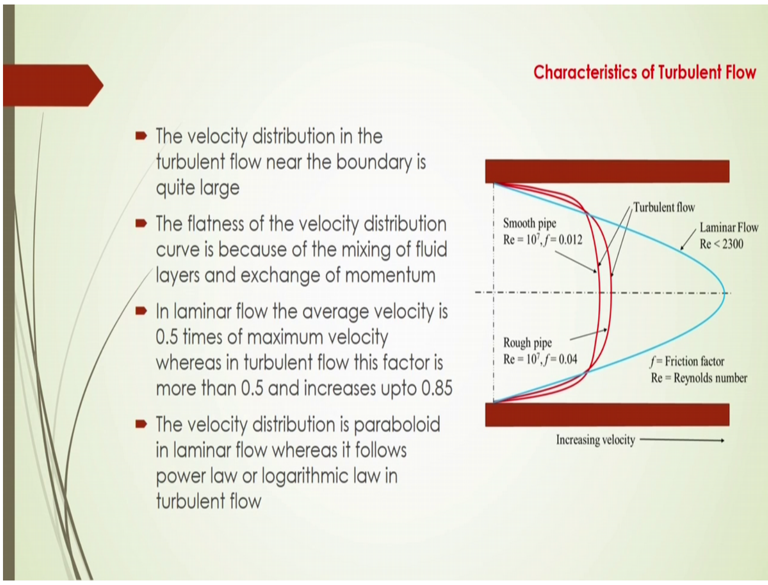

And other thing is that the velocity distribution in that case near the boundary will be quite large compared to the laminar flow. And the flatness of the velocity distribution curve will be there and this is because of the mixing of the fluid layers and exchange of momentum. And also we also derived that the in the laminar flow the average velocity is half of the maximum velocity and whereas, in the turbulent flow you cannot expect it will be half, it may be more than half and then it will be up to I think 0.85 this average velocity to the maximum velocity.

And also the velocity distribution is you will get, it will not be just in case of what is that we got for laminar flow it will be paraboloid, but in this case for turbulent flow this velocity distribution will follow the power law or what is called it is called logarithmic flow of or logarithmic law in turbulent motion. So, here in this picture you will see that nature of the distribution of the velocity, here in this case for laminar flow you are getting this type of paraboloid velocity distribution profile whereas, in the turbulent flow you are getting this type of velocity profile.

So, you will get a 2 types of profile for velocity distribution, one is for a smooth pipe and another is for pipe. You will see for rough pipe it would be like this here it is shown and whereas, for smooth pipe it would be more flat compared to that what is that rough pipe.

So, in that case you will see the viscosity nature also will be different and also there will be some friction, the nature of the fiction will be different in the case of rough pipe compared to the smooth pipe there that depends on the roughness of the pipe.

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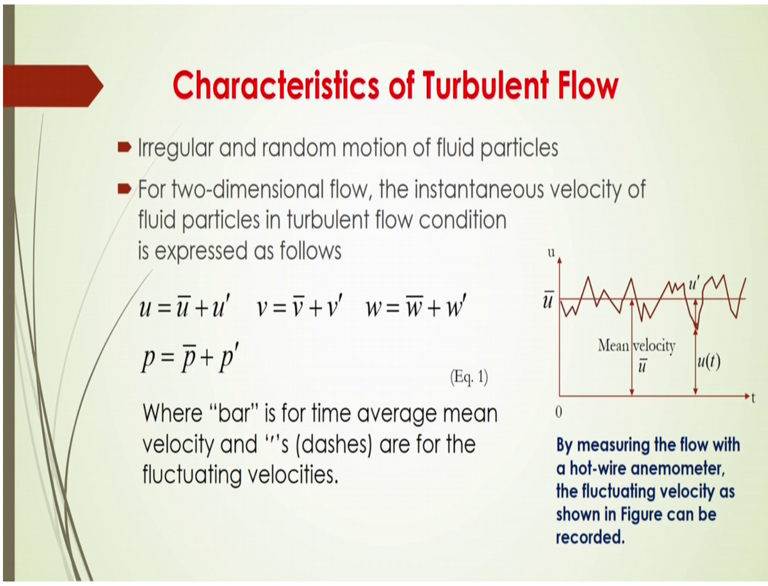

Now, in this case you will see that during this turbulent flow, there will be some irregular and random motion of the fluid particles. And a for 2-dimensional flow you will see the instantaneous velocity of the fluid particles in turbulent flow condition can be expressed by this u will be equals to u dashed plus u bar. Now, this u bar is called the time average mean velocity whereas, u dashed is called fluctuating velocities whereas, in the y direction this v will be is equal to v dashed plus v bar whereas, this v bar is called that average mean velocity in the y direction and v dashed is called the fluctuating velocities in this y direction.

Similarly, in the z direction w will be is equal to w bar plus w dashed and what should be the pressure components also similarly you can considered in the turbulent flow, that p should be p bar plus p dashed here. So, in this case bar should be considered as a time average mean velocity, where as dashed is considered as a fluctuating velocity. So, whenever we are observing any turbulent flow, you can say that velocity components will be such like that one should be the fluctuating components of the velocity there because, this fluid particles will not follow the regular fashion to flow in a conduit.

So, its path will be a zigzag way. So, that in each point its that is tangent or streamline coefficients will be of different and also because of this you will see the nature of the mixing of this fluid, there will be different compared to the laminar flow.

And in that case if you are having that the different components in the xy and z direction these velocity fluctuations, because of this turbulence and in that case you have to consider these fluctuating components of the velocity. Now, if you are getting the average or mean of these fluctuating components and if you are considering that velocity as a u bar here. So, it will be like this. So, this will be time average velocity; now this time average you have to consider this time is sufficiently enough so, that you will be having more accurate; that means, actual velocity to consider or to analyze the velocity distribution in the turbulent flow.

And also these fluctuating components you can measure by hot wire anemometer to get this fluctuating velocity as shown in this figure, you can record by this hot air anemometer to observe this fluctuation. And after that you have to take the time average so, that for further analysis you can get this other properties of the turbulent flow.

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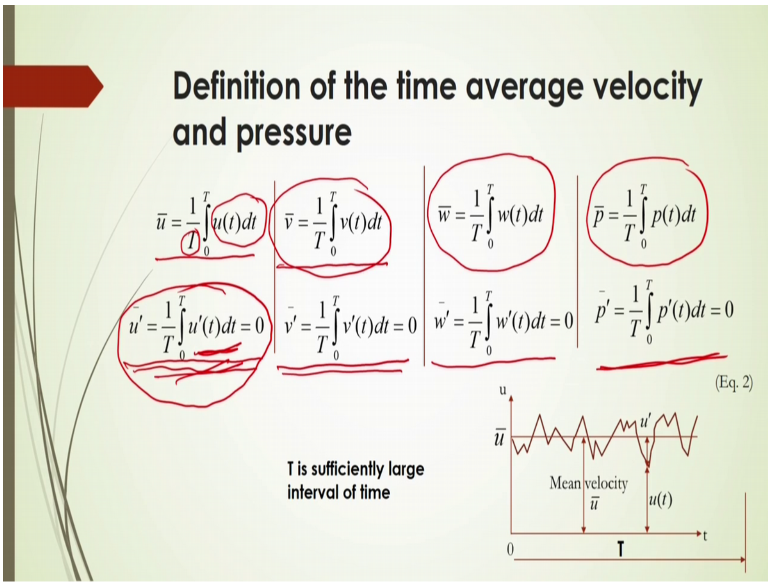

Now, what will be the definition of the time average velocity and that pressure. Now, time average velocity will be like this it will be defined like this here 1 by T into 0 2 t integration of this u t into d t. So, here in this case capital T is denoted by time range here 0 to may be infinite time also you can take, that is up to you that it should be sufficient large interval of time so, that you can get more accurate results to analyze. And also in the v directions it will be the like similarly time average in the y direction in the z direction it will be like this and also pressure average pressure will be again the time average.

Because, in this case whenever you are going to measure the pressure, by pressure transducer by sophisticated instruments there the pressure transducer will give you the force difference by the resistance like a voltage resistance, in that case you have to calibrate it. And, you will get the pressure and fluctuating pressure and with respect to time it will be there and after that you have to get the time average velocity of that resistance of the flow. And from v subtract calibration you can get the actual pressure components there.

Similarly what will be the fluctuating components average fluctuating components to be also considered by the time average a component here. So, you are here also you have to define that velocity fluctuating components in the x direction as u dashed, which will be 1 by T into 0 to t integration of this components here by this. Similarly in the y direction you have to calculate these velocity components by taking time average of this; similarly in the z direction and also what will be the fluctuating pressure components also will be taken care by taking the mean of this time average.

So, in that case we have to then consider for an further analysis by taking this time average velocity and pressure.

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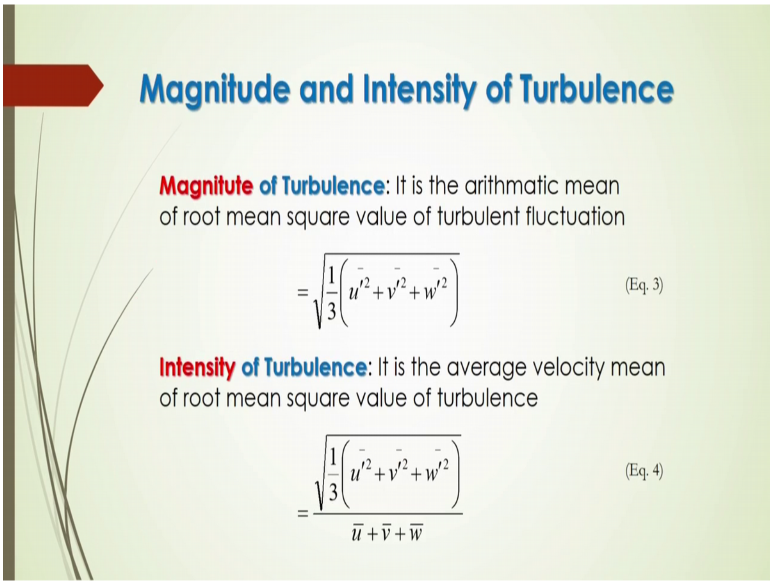

Now, what should be the magnitude and intensity of the turbulence there? Now magnitude of the turbulence will be defined as the arithmetic mean of root mean square value of turbulent fluctuation and it is defined and it is given in equation number 3 how it will be there. So, you have to sum up these fluctuating components, square of this fluctuating components and then divided by 3 that is mean value and then after that you have to take the square root of this then you will get the magnitude of the turbulence.

And what should be the intensity of the turbulence? It is also that the average velocity mean of root mean square value of the turbulence here. What should be the root mean here? Magnitude of the turbulence then, you have to taking the average based on this summation of this average velocity in x y and z direction.

So, by equation number 4, you will be able to calculate what should be the intensity of the turbulence. If you know the velocity components in x y and z direction and the average velocity, then easily you can calculate what should be the intensity of the turbulence and also magnitude of the turbulence there.

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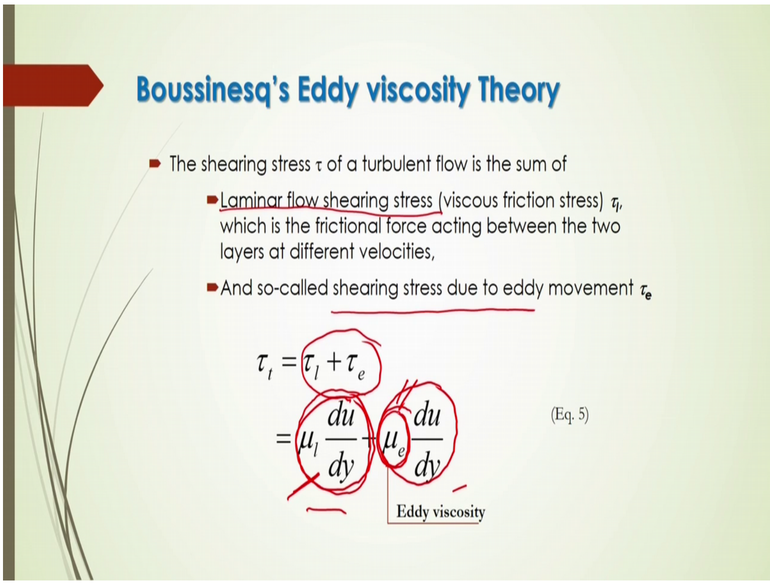

Now, another important theory to be discussed for the turbulence flow, that is called Boussinesq's Eddy viscosity theory. Now in this case the shearing stress in the laminar case also we have discussed the shearing stress. So, in the turbulent flow what should be the shearing stress? So, in this case the shearing stress of this turbulent flow, is the sum of this laminar flow shearing stress and also shearing stress due to the Eddy movement in this turbulent flow, because of the viscous effect and also what will be the roughness of the pipe there.

So, because of which you will get the shear stress will be something different from the laminar flow. So, what should be the shear stress in the laminar flow? What should be the shear stress in case of Eddy producing because of this turbulent flow. So, you have to sum up this 2 shear stress. So, it will be called as total shear stress in this turbulent flow. So, that will be defined as what is the turbulent flow. In the turbulent flow the shear stress should be mu e into d u by d y whereas, in the laminar fluid it will be mu l into du by d y. Mu l l for laminar and e for eddies here Eddy it is coming only for this turbulent flow there.

So, you have to sum up these 2 components to get the total shear stress of a turbulent flow. So, it will be here in this way only thing is that viscosity of Eddy to be a considered here instead of what is that laminar flow. So, in the laminar flow we got this mu l that is Newton's viscosity Newton's law of viscosity based on which we are defining that shear stress in the laminar flow as mu l into du by dy whereas, in the Eddy viscosity we are getting we are defining another viscosity coefficient it is called Eddy viscosity or Eddy viscosity coefficient mu e and it will be later on will be discussed also how it will be a define for that. Now so, as per Boussinesq's Eddy viscosity we can say that the summation of this 2 shear stress of laminar and Eddy will be considered to get the total shearing effect of the turbulent flow.

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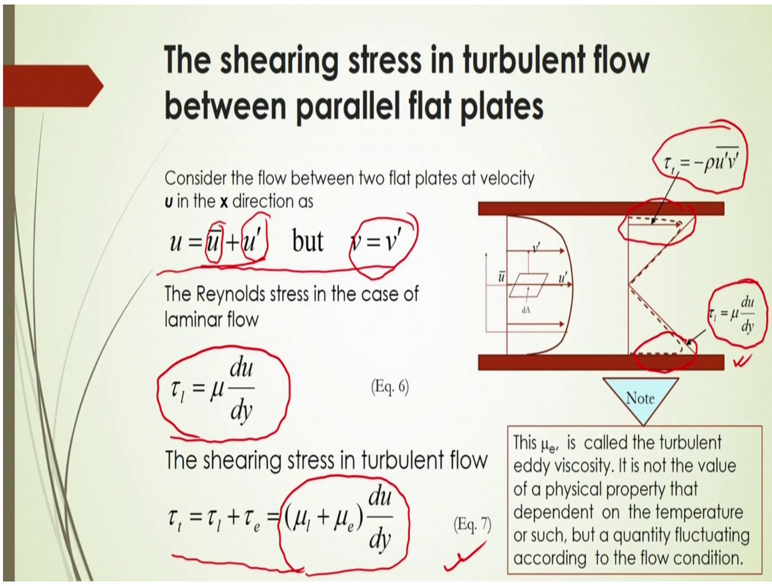

And the shearing stress in the turbulent flow if we consider that 2 parallel flat plates, then how this velocity distribution will be there and how this shear stress will be considered in that case. So, if we consider this pro between 2 parallel flat plates at velocity u in the x direction, as u will be is equal to u bar plus u dashed since it is a turbulent flow. So, we have to consider this average time average mean velocity, that is u dashed u bar and u dashed also fluctuating components and whenever you are considering the x direction, then in that case only v will be is equal to v dashed only that fluctuating components in the y direction to be considered.

And the Reynolds stress in that case in the laminar flow. In the case of laminar flow will be defined as tau l will be is equal to mu into d u by d y. And the shearing stress in the turbulent flow will be defined the summation of this 2 that is laminar and turbulent that we have already discussed. So, it will be here as equation 7 how this shearing stress in turbulent flow have to be considered. Now important point here that mu e whatever we have discussed here, see the mu e is called the turbulent Eddy viscosity it is not the value of the physical property, that depend on the temperature or such, but a quantity fluctuating according to the flow condition.

So, we have to remember it is not the turbulent it is not the physical property, but it is a phenomena that depends on the temperature and also it is not the temperature, but it is a quantity that according to the flow condition to be defined here. And also in this case you will see if we see the shear stress distribution here, how this turbulent flow in this case it will be there. So, this is the components of the turbulent flow, this is that components of the dash line is considered as a turbulent flow shear stress distribution whereas, the formula is considered as the laminar flow or shear stress distribution. So, this shear stress will called the Reynolds stress here for the turbulent flow, and Reynolds states in the turbulent laminar flow it will be considered as this.

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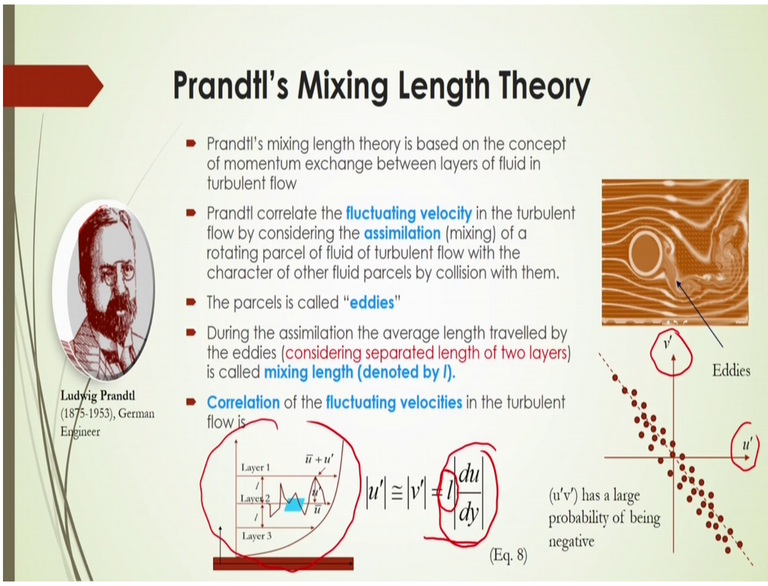

And in this case Prandtl's given a widely accepted theory to analyze the velocity components are based on the mixing length of the fluid layer, whenever it will be flowing through the conduit. So, in that case Prandtl's given the theory based on the concept of momentum exchange between the layers of fluid in a turbulent flow. And he actually correlate the fluctuating velocity in the turbulent flow by considering the assimilation or you can see that mixing of a rotating parcels of fluid whenever it will be flowing, see here in this video they are how fluid particles are having mixing by in that case the rotating parcels of the fluid of turbulent flow with the character of the other fluid parcels by collision with them how it will make this eddies there.

And this parcels of fluid particles it is called the eddies and during the flow, whenever it will get the obstructions over any object, then beyond this object you will see the streamline how it will be broken down, and it will be splitting and because of the energy distribution over there and then how this fluid particles getting mixing intermixing by just breaking the breaking this what is that streamline of this fluid particles there. And so, in this case we can say that whenever there will be assimilation, then the simulation length travelled by the eddies will be considered as the mixing length here.

So, which will be denoted by l. So, in this picture also you will see that if there is a flow, what will happen the fluid layer will be having layer 1 layer 2 layer three and you will see what will be the length between these 2 layers layer 1 and layer 2 it will be considered as the mixing length here. So, considering the separate length of 2 layers, it will be called as mixing length and then this fluctuating components that is a fluctuating velocities in the x or y direction how it is correlated with this mixing length and also the velocity gradient that is given by this Prandtl's here. So, it is called Prandtl's mixing length theory.

So, u dashed that will be equivalent to v dashed, that will be is equal to l into that is what is called du by d y. So, this velocity component fluctuating components is related to the shear called shear strain. So, this shears strain or you can say shear rate here. So, the professional constraint is considered as a mixing now. So, this mixing length is nothing, but the separated length of the 2 layers whenever fluid will be following as a streamline there.

So, at higher velocity you will see that streamline will be broken down, and then there will be a gradient of this velocity and then velocity gradient will give you how these fluctuating components will be estimated there. So, in this case you will see if you consider that u dashed and v dashed that is velocity fluctuating component in the x and y direction, and if you are plate plotting this you see there will be almost same. Here in this case has a large probability of being a negative also there both these. So, and because of this you will see Reynolds stress will be defined as that rho into minus rho into u dashed v dashed. So, we will come to that point.

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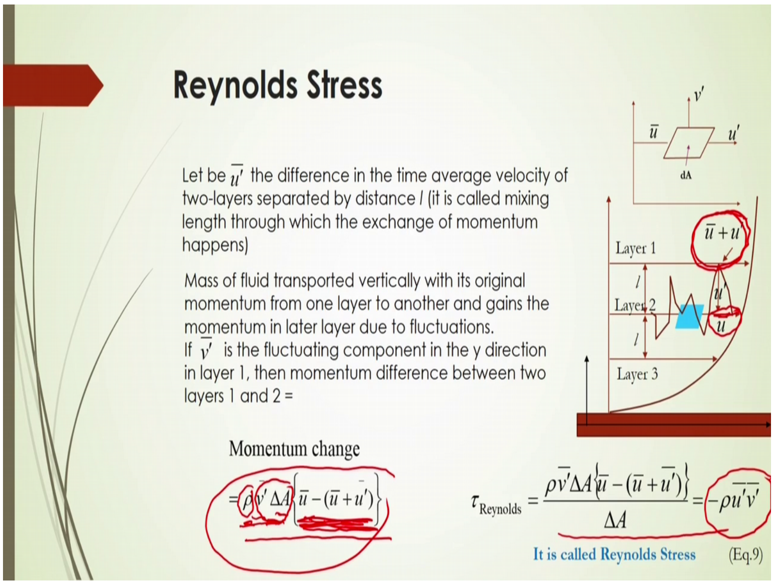

Now, how this Reynolds thrust Reynolds stress then to be actually estimated or to be analyzed here? Now if we consider that u dashed bar that is average fluctuating components in the x direction and it will be the difference in the time average velocity of the 2 layers separated by distance here in the figure it is shown, it is called the mixing length through which the exchange of the momentum happens. So, in that case if we consider that this 2 layers now what should be the mass of fluid that transported vertically with its original momentum from one layer to the another layer, and how the momentum can be a gained in the later layer due to the fluctuations.

And by in this case you have to know the average velocity for the other directions also there a velocity components in the directions like y directions, then if it is considered as v dashed bar then in the layer 1, then momentum difference between this 2 layers of 1 and 2 can be considered as rho v dashed into delta into u bar minus u plus u dashed bar here. You just see consider here this is the average velocity and in this case this is the what is that, velocity fluctuating components u dashed.

So, this in this case in this layer the velocities u bar, u bar plus u dashed stand here in this layer the velocity is u. So, what will be the difference between these 2 layer, what will be the velocity difference this is simply u dashed that is velocity fluctuating components in the x direction and then what will be the momentum there? Momentum it will be is equal to rho into v dashed into A; that means, mass is rho into v dashed into what is that delta

A it is your what is that in the y direction what will be the volume. So, this volume flow rate into density, it will be coming as mass. So, mass into here u that will be your momentum your what is the u; that means, what is the difference of velocity between 2 layers. So, that will be considered as the momentum change.

So, this total momentum changed will be rho into v dashed into delta A into u bar minus u dashed plus u dashed bar. So, finally, if it is a momentum then what will be the shear stress, then momentum divided by a cross sectional area then it will be considered as the stress. So, this is called the Reynolds stress, this Reynolds stress then finally, you are getting that minus of rho u dashed v dashed bar. So, Reynolds stress will be actually taken as the momentum change whenever in the turbulent flow, liquid layer having velocity difference based on the fluctuating component there.

So, we can calculate what will be the momentum exchange between 2 layers by this deviation of the velocity, and also in the y direction if there is a velocity component fluctuating component has v dashed. So, in the y direction if we consider that velocity, and also cross sectional area is delta A. So, we are having this that is volume, then rho into volume that will be mass into then what will be the in x direction, what will be the what is the deviating that is velocity that is called fluctuating components.

So, if we multiply this we are getting the momentum exchange and by divided by cross sectional area, we can get the Reynolds stress there.

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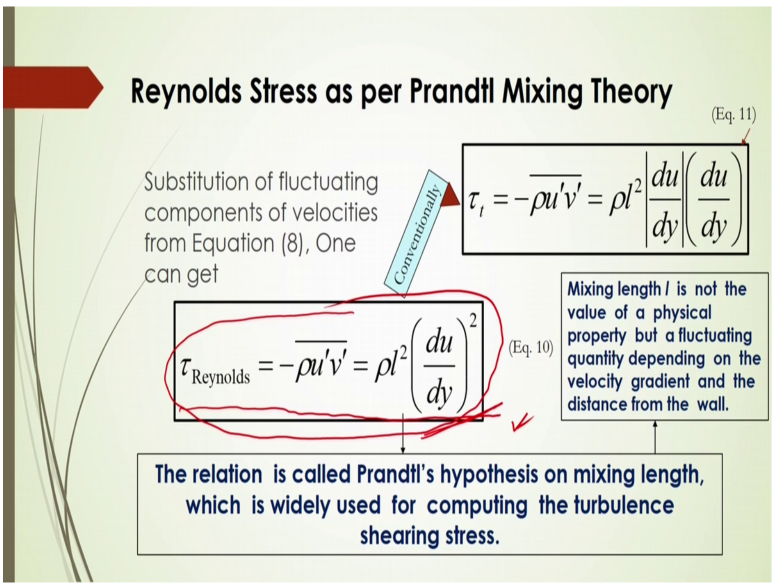

Now, as per Prandtl's mixing theory, what should be the Reynolds stress then? Now if we substitute the fluctuating components of velocity from equation number 8 that, here the fluctuating components are defined by this l into d u by d y given by the Prandtl's. So, as per the Prandtl's mixing theory if I substitute this velocity h fluctuating components here, then we are having this tau Reynolds; that means, Reynolds stress will be is equal to rho l square this one rho l square into du by d y whole square.

So, this is your Reynolds stress based on this Prandtl's mixing theory. Now if I consider that Reynolds mixing length is there and what should be the relation between this Reynolds stress and the mixing length then you can get by this equation number 10 there Reynolds stress. So, the relation is called the Prandtl's hypothesis or mixing length, which is widely used for computing the turbulence shearing stress here. And in this case remember that mixing length l is not the value of a physical property, but a fluctuating quantity that depending on the velocity gradient and the distance from the wall.

So, you have to remember this. So, very simple that you have to remember this Reynolds stress as minus rho u dashed v dashed and based on this Prandtl mixing length theory, if you substitute that u dashed and v dashed, you can get ho l square d u by d y whole square. So, this one is very important for computing the turbulence shearing stress, for any chemical engineering operation or by simulation of the process there in the turbulent flow condition.

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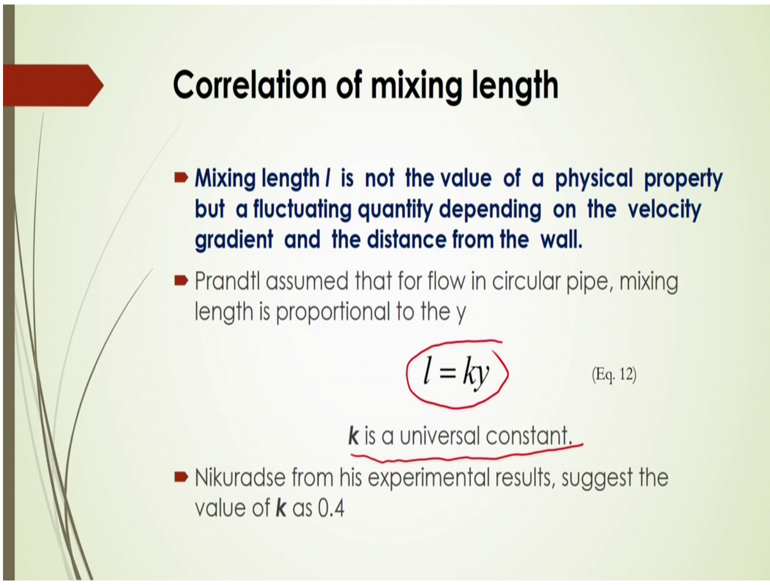

Now it is very difficult to actually calculate or estimate this mixing length by what is that by substituted instrument, what will be the distance between 2 layers. So, in that case Prandtl some other investigators, they have estimated this mixing length by experiment by relating 2 other components by indirect way. So, they have given different correlations. So, what should be that correlation? So, mixing length we know that it will be not the physical property, but it is a fluctuating quantity which will be measured or estimated by what is that velocity gradient and the distance from the wall. Now, Prandtl he has assumed that the flow in circular pipe, mixing length is proportional to the y.

So, the have I think Prandtl he has just assumed in this way; if it is suppose the professional to the y then we can write here l is equal to k y, which is k is the which is a constant and k is a universal constant this is called and Nikuradse say from his experimental results, he got this or he suggested the value of this universal constant as 0.4 just by considering l is equal to k y. And, from the velocity distribution and just messing the equation with experimental data or fitting the experimental data with that model and what should be the parameter k and based on which see he has suggested that k should be as 0.4 there.

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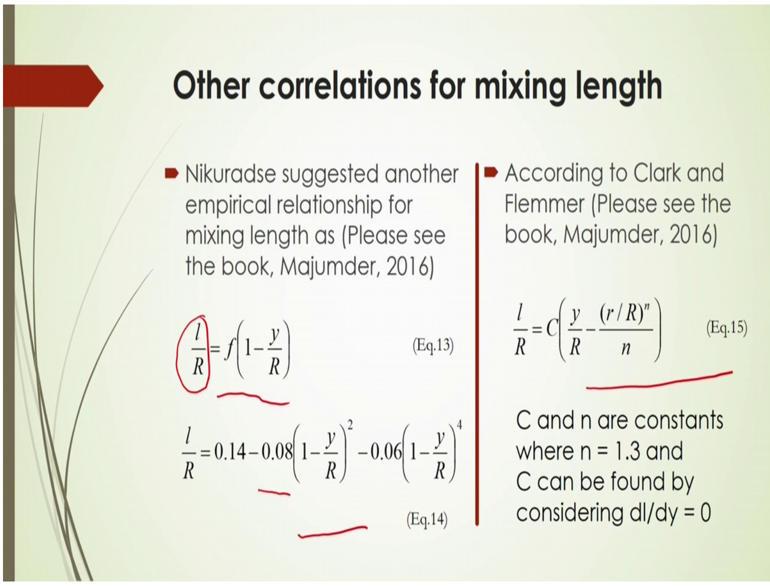

Now, other correlations also available for this mixing length; Nikuradse suggested another empirical relationship for mixing length as here l by R that is R is the radius of the pipe. So, it will be a function of 1 minus y y r and it is k it is actually imperially related like this 0.14 minus 0.08 into 1 minus y by y by R whole square minus 0.06 into 1 minus y by R whole to the power 4 this is the polynomial function.

So, from which you can calculate what should be the mixing length there. It is a function of just what is that radius of the pipe and also in the y direction what will be the velocity component there. According to Clark and Flemmer, also this mixing length can be calculated based on this correlations equation number 15. Now l by R will be is equal to some constant of y y R minus y by R a to the power n divided by n here; n and c are constants in this case n they have suggested n should be 1.3, whereas, thus C will be actually estimated based on the experimental data or you just get the dl by d y is equal to 0, from which you can calculate what should be the maximum what is that C value there.

So, you have to consider this what should be the C. So, C and n if you know then you can easily calculate what should be the mixing length by this equation number 15.

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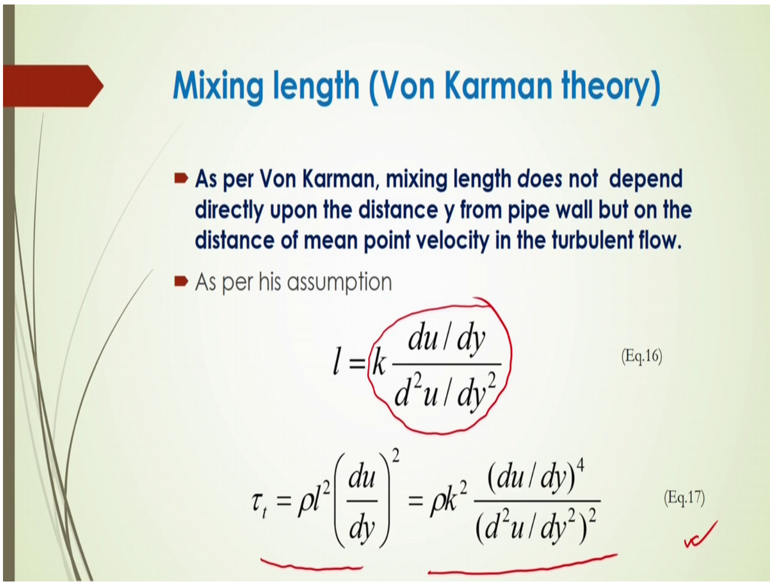

According to Von Karman theory, this mixing length he told that mixing length does not depend directly upon the distance y from the pipe whereas, this Nikuradse or what is that Prandtl they have suggested that, it will be a function of y. But, here is this Von Karman they told that their team they told that also it will not be a function of y, but on the distance of mean point velocity in the turbulent flow there.

So, as per his assumption that l should be is equal to k into d u by d y divided by d 2 u by d y square. So, in this case if we substitute this l value here, then shear stress should be is equal to rho into k square into d u d y to the power 4, y d 2 y u y d y square square. So, by equation number 17 you can have this turbulent shear stress there.

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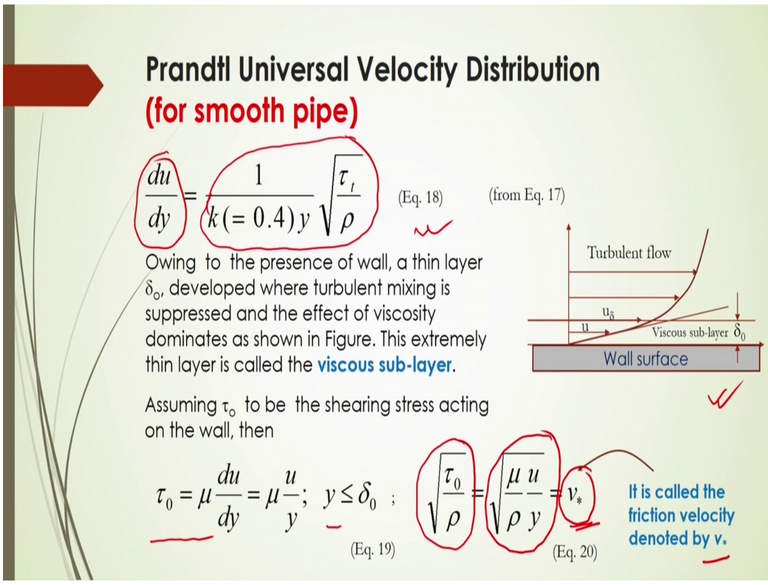

Now, let us derive or let us having this Prandtl universal velocity distribution bend, based on their mixing length theory. So, in this case we know that d u by d y will be is equal to what is that, this based on this is from equation number 17 we can get, here after what is that simplification this equation number 17 here this is the equation number 17. So, if we consider here this tau t rho k square, if we substitute this value of velocity gradient after derivate derive then we can get this one this equation number 18.

Now going to the presence of wall thin layer delta 0 as per this figure shown here, if we developed there this delta 0, and where turbulent mixing is suppressed and the effect of viscosity dominates as shown in figure, then the extremely thin layer is called that semi viscous some sub layer. So, in this case of course, within this viscous sub layer, there will be a shear stress that acting whenever fluid will be flowing over this wall surface there. So, if we assume that tau 0 to be the shearing stress acting on the wall, then we can have this tau 0 will be is equal to mu into d u by d y, that will be is equal to mu into h y by y r in this case, and this y will be considered as that less than equals to delta 0 that is very thin layer of the a liquid over this wall surface.

And in this case then we can now, after simplification have this root over tau 0 by rho it will be is equal to root over mu u by rho y that will be is equal to v star. So, v star this is called as the fiction velocity, that is denoted by here in this case by equation number 20. So, after simplification of this equation 19, we are getting root over of root over tau 0 by rho it is called the friction velocity. So, interesting that whenever fluid will flowing over a smooth pipe at a velocity u, and if we consider the thin layer of this distance delta 0 over this wall surface. And, because of this viscous effect within this thin layer of the surface, there will be a some shear stress acting and this is shear stress will be then as per Newton's law that it will be considered as mu into d u by d y.

Now this d u by d y can be obtained by this mix what is that Von Karman theory of this mixing length, then after substitution of this and if we consider k will be equals to 0.4, then we can have this equation number 20 and it will be called as the friction velocity.

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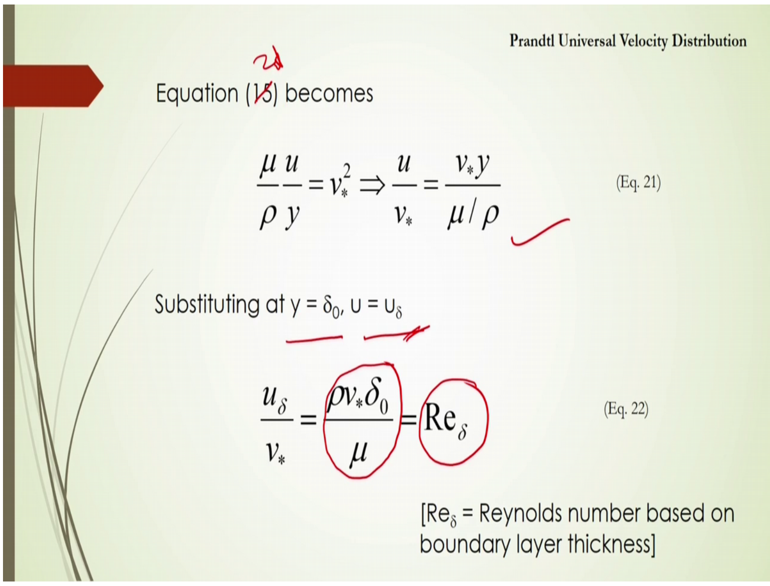

Now, this equation number 15 becomes mu u rho y, that will be is equal to v star square which will imply that u v u y v star that will be is equal to v star y y mu into rho. This will be equation number I think 20 equation number 20 becomes like this. Equation number 20 become like this. Now substitution substituting that y is equal to delta 0 and u is equal to u delta; that means, at over a thin layer of this a liquid what will be the velocity that will be considered u delta.

So, u delta by v star will be is equal to this rho v star delta 0 by mu, then it will be called as Reynolds number based on this thin layer of a liquid over this wall surface. So, delta is equal to Reynolds number based on the boundary layer thickness delta it is called. And now integrating this equation number 18 here we have shown here this case for the smooth pipe.

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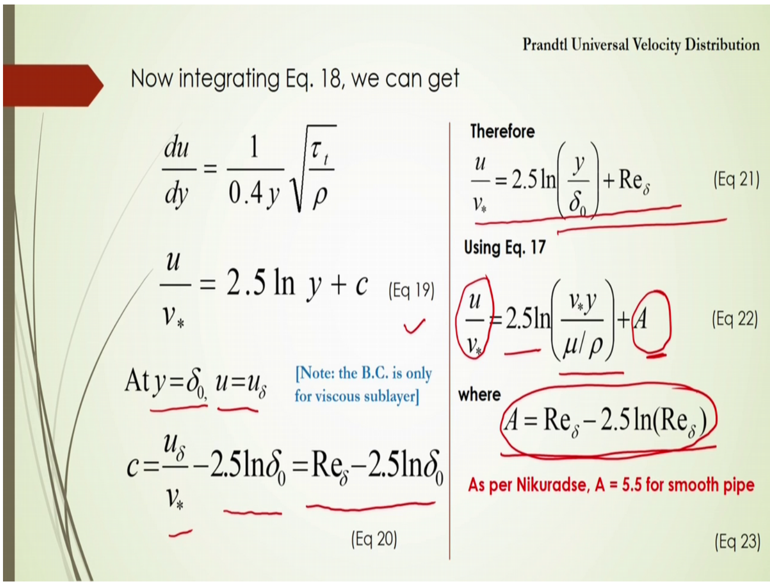

After integration we can have this equation number 19 here. So, here it is called u by v star that will be is equal to 2.5 l n y plus c and c is the constant of integration and after substitution of the boundary condition like y will be equals to delta 0 and u is equal to u delta, then we can have c will be equal to u delta by v star minus 2.5 l n of delta 0. And after simplification it will be coming as R e delta minus 2.5 into l n delta 0. Now this is the a constant of integration after substitution of this c in equation number 19, then we can have u by u star is equal to 2.5 into l n y by delta 0 plus R e delta. As this is called the velocity distribution for the turbulent flow whenever fluid will be flowing h over a smooth pipe surface.

Now, using equation number 17, then we can have u by star will be is equal to what is that? 2.5 l n into v star y by mu by rho plus A here. Now this A A is a parameters here that A will be defined as here in this case after simplification. This A will come here r e delta minus 2.5 into l n delta. So, as per this Nikuradse experimental observation this A is coming 5.5 for smooth pipe. So, h very interesting that we are getting this equation number 24 velocity distribution of this turbulent flow whenever fluid will be flowing over a flat surface the flat surface will be very smooth there.

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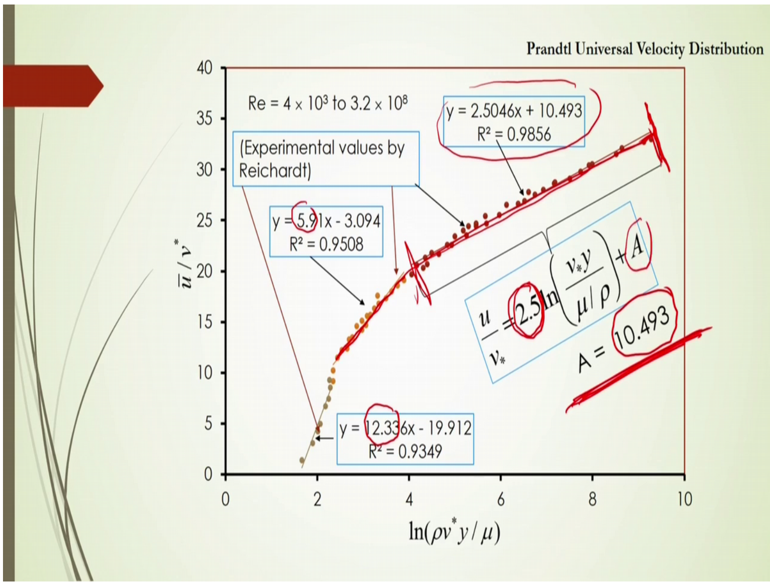

And if we compare these model with the experimental data of that is given by Reichardt, that case we are having this model here this different models here within a certain range up to this range here. We are fitting this experimental data based on that model here equation number 22 then in that case we are getting A is equal to 10.493 whereas. So, this Nikuradse that given a is equal to 5.5. Again here in this case within this range we are getting this y will be equals to what 5.9 x minus 3.094 in this case.

Here we are we are getting this for different A value and also this for this range also we are getting the differ difference a value corresponding to the con constant here 2.5, it will be coming different value like this. So, only thing is that this model based on this Prandtl mixing theory, we are having the almost; that means, almost similar trend within this range of experimental condition here. So, in that case A should be is equal to 10.493 whereas, it will be deviating from the Nikuradse results as 5.5.

So, anyway this will be actually obtained based on the experimental data, but this component is coming based on this what is that theory 2.5. So, only thing is that this A will be varying. So, this A depends on what is that? Reynolds number based on the boundary layer thickness and also what is that this one is called the Reynolds number here and other physical properties of the systems also. So, it is important to know that what should be the velocity distribution in case of turbulent flow, whenever fluid will be flowing over the flat surface there. So, you can easily calculate that things from equation number 22.

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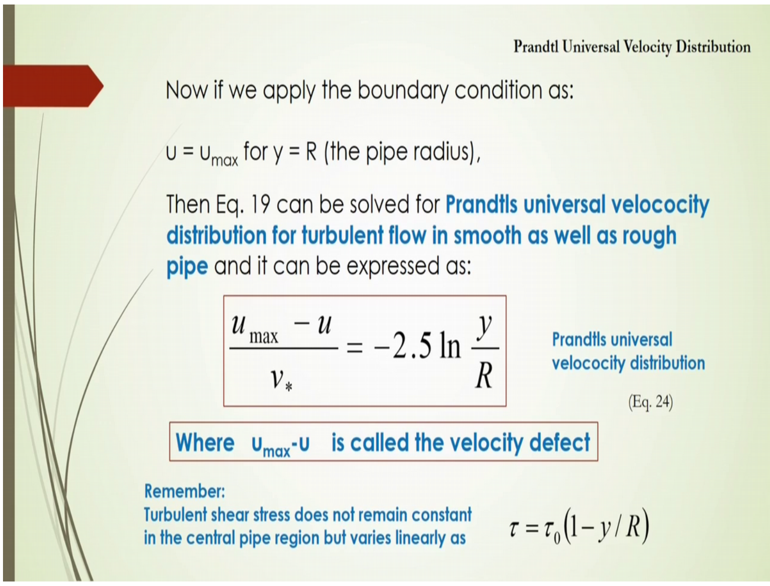

Now, if we apply the boundary condition as u is equal to u max for y is equal to r the pipe radius, then equation 19 earlier can be solved for Prandtl's universal velocity distribution for turbulent flow in smooth as well as rough pipe also. So, in that case that can be expressed as u max minus u by v star that will be is equal to minus 2.5 l n into y y r whereas, u max minus u it is called the velocity defect there. So, you have to remember that turbulent shear stress that does not remain constant here, and in the central pipe region, but whereas, linearly as tau is equal to tau into 1 minus y by r.

So, equation number 24 you can use for both turbulent smooth as well as rough pipe for this. So, in this case you have to consider what will be the maximum velocity the turbulent flow. So, that maximum velocity can be calculated from this equation number provide the value of y; that means, from the flat surface hat will be the distance, and also what will be the radius if you are considering the radius of the pipe, you are then you will be able to calculate what is the maximum velocity and also.

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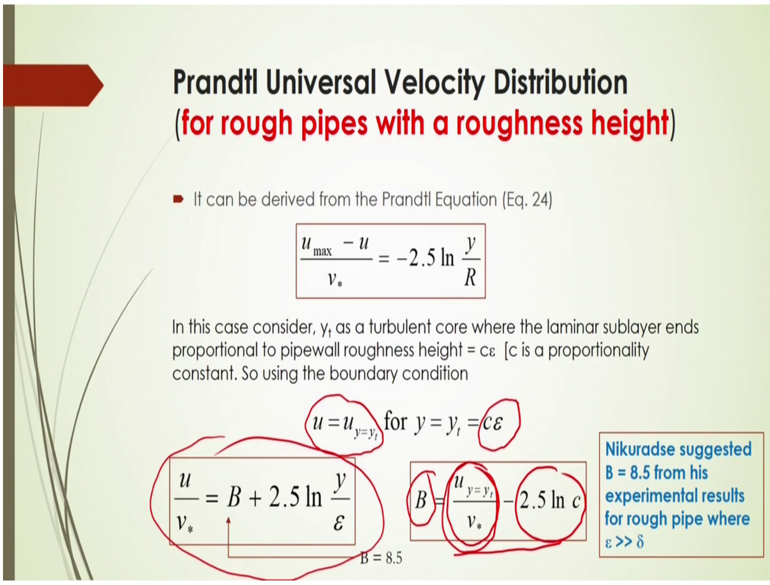

If you know the maximum velocity what will be the velocity distribution there and also Prandtl universal velocity distribution for rough pipes with a roughness height that can also be considered here. So, it can be derived from the Prandtl's equation that is given in equation number 24. So, based on that in this case you have to consider that y t; that means, up to which that turbulence; that means, here turbulent core will be there for laminar sub layer ends that, you have to consider. In that case beyond the laminar sub layer this distance will be professional to the pipe wall roughness height. So, it will be is equal to c into epsilon. So, c is a approximately constant.

So, using the boundary condition, if you consider that y is equal to u is equal to u that is y at to this y t and turbulent core here and y is equal to y t, that is that will be is equal to c into epsilon; epsilon is the roughness height then c is the constant. So, if you substitute this boundary condition based on this we can have this u by v star, that will is equal to v plus 2.5 l n y y epsilon. And from which we can have this mu will be is equal to what is that? u by v star u will be y is equal to at y is equal to y t and then if you are having this ratio you have to subtract this 2.5 into l n c.

So, only thing is that you have to know what will be the proportionality constant; that means, if you know the roughness height then according what will be the height over the surface if you are considering that turbulent core height if you are considering then it will be there. So, y t if it is suppose 1 into epsilon then accordingly it will come here what will be the c value c is 1 here. So, in that case l n c l n 1 will be equals to 0. So, this part will be neglect only b part will be here.

So, accordingly if you substitute this b here; so, you can get what will be the velocity distribution for the rough pipe. So Nikuradse suggested that this v value will be eight point five from their experimental results, and for rough pipe in this case epsilon should be is greater than equal to delta. So, delta is very thin layer; that means, if the roughness heights smaller than roughness height we consider there.

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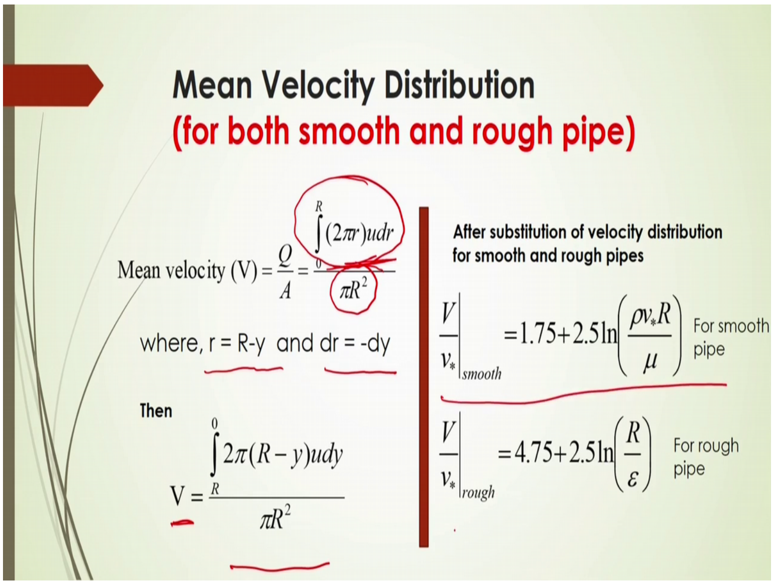

Now, what will be the mean velocity distribution for both smooth and rough pipes? Now mean velocity a to be defined as that is volumetric flow rate upon cross sectional area. So, it will be what is that this if we or having the control volume over that over a small elemental analysis then after integration of this 2 pi R u R d R and divided by this y r square, then you can get this mean velocity where R here will be is equal to capital R minus y this capital R is called radius of the pipe and d r is equal to minus d y.

Now, substituting this then you can get this v mean velocity will be is equal to this. And after substitution of this velocity distribution for smooth and rough pipes we are getting this velocity distribution here for smooth pipe and this one is for rough pipe.

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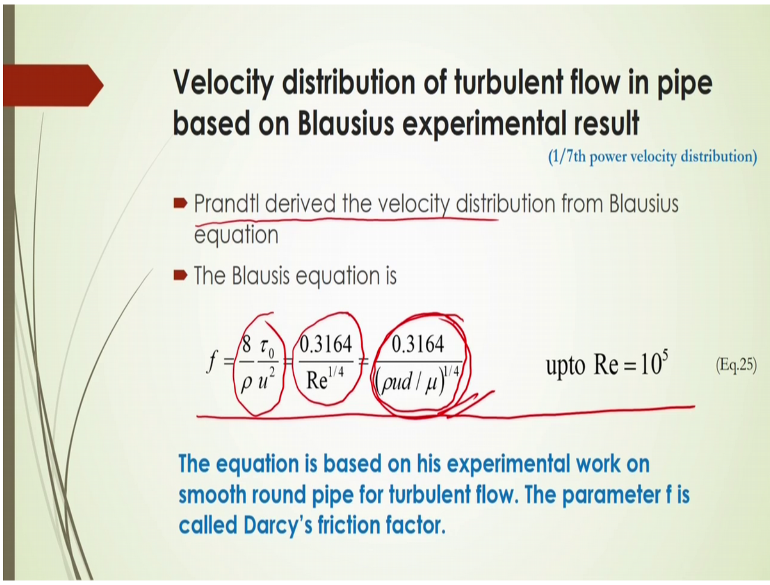

Similarly, velocity distribution of turbulent flow in best in pipe based on Blausius experimental results, then Prandtl derived the velocity distribution from Blausius equation. In that case the B equation is are having this f is equal to 8 tau 0 by rho u square. So, this will be is equal to this after simplification and then it is coming like this. Now, this equation is based on his experimental work on smooth round pipe for turbulent flow and the parameter f is called the Darcy friction factor here.

So, they have actually got the shear stress that is proportional to the kinetic energy and based on which they have got this a friction factor that is called Darcy friction factor, we have already defined in the in the pre earlier lectures that what will be the Darcy friction factor there. And this Darcy friction factor and also this friction factors from this friction factor after simplification or just rearrangement of this equation, then we are getting this. Now, this is applied only after Reynolds number 10 to the power 5.

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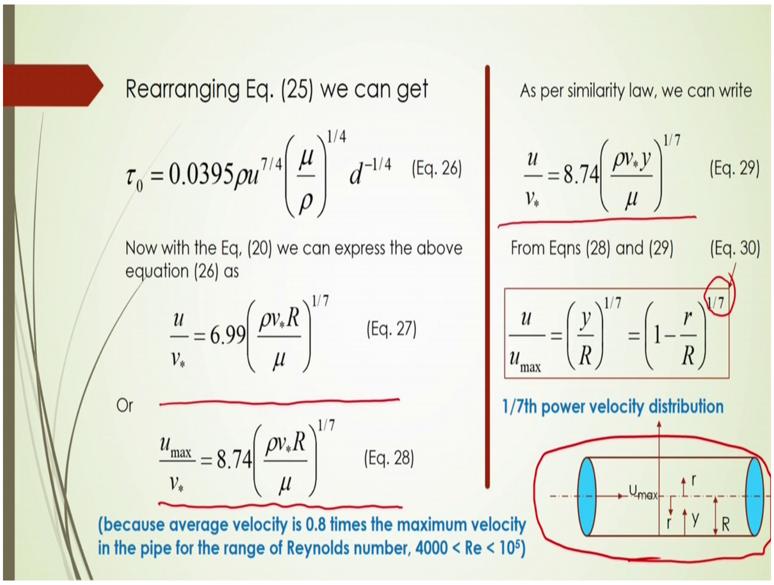

And now arranging this equation number 25 here, this equation number we are getting this tau 0 is equal to 0.0395, rho into mu mu to the power 1 by 4 into my by rho to the power 1 by 4 into d to the power minus 1 by 4. Now with the equation number 20 earlier we have given that we can express this above equation 26 here as v by e star will be is equal to this and or you can express this as a equation number 28 here. Now in terms of this maximum velocity and now because, this average velocity is 0.8 times the maximum velocity in the pipe for the range of Reynolds number of 4000 to 10 to the power 5.

In this case we are able to calculate what should be the velocity distribution based on this maximum velocity there. Now, this in this case as per similarity law we can write that u by u v star that will be is equal to 8.74 into rho v star u by mu to the power 1 by 7. Now from this equation number 28 and 29 we can get this equation number 30 as a general equation like u by u max, based on this maximum velocity that will be is equal to y y r to the power 1 by 7; that this is nothing, but 1 minus r by R to the power 1 by 7 for circular pipe. So, this power is 1 by 7 that is why it is called the one seventh power velocity distribution here

So, we are how this Y and R are related here in this case this circular pipe this shown. So, based on this we can have this velocity distribution, as a one seventh power law of velocity distribution. This is obtained based on the what is that Blausius experimental results.

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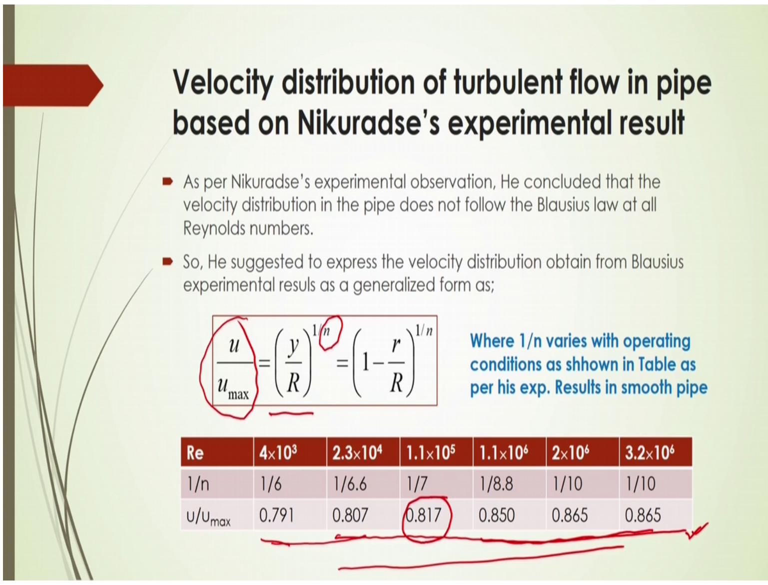

Similarly, the velocity distribution of the turbulent flow in pipe can be obtained from the Nikuradse experimental result. So, as per Nikuradse experimental observation, he concluded that the velocity distribution in the pipe does not follow the Blausius law at all Reynolds number. So, in that case he suggested another expression to express the velocity distribution that obtained from the Blausius experimental results as a generalized form

So, he suggested that the general equation for the velocity distribution will be is equal to u by u max, that will be is equal to y by R to the power 1 by n. Here instead of 7 here he suggested that it will be n; n is a constant this 1 by n that will vary with the operating conditions and here in the table it is sung given some results if suppose 1 by n is equal to 1 by 6, then it will be u by u max should be 0.791. Whereas, for other values of 1 by n it is shown in the table, that we are getting almost that here the u by u max never about to each other, but there will be a variation.

So, this because of this one by n value, this u by u max will change. So, it may not be only 1 by 7 there. So, 1 by 7 if we consider we are getting here 0.817 whereas, for other values it is changing here from a 0.817 to 0.791 0.806 like this. So, this u by u max changes with respect to n value here.

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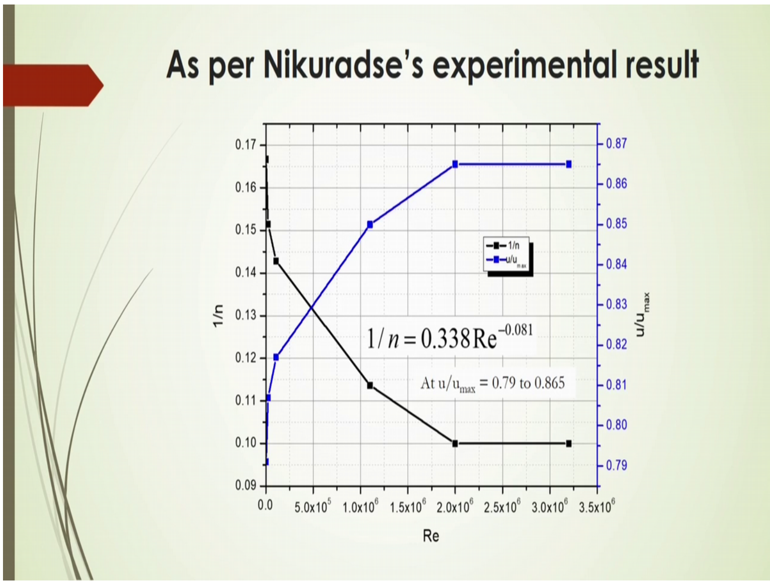

Now, as per Nikuradse experimental results, that 1 by n how this coefficient 1 by n changing with Reynolds number there. It is seen that this power is inversely proportional to the Reynolds number and the proportionality is there is power is 0.081. So, u by u max should be 0.79 to 0.0865 if the Reynolds number is ranges from 0 to what is that 3.5 into 10 to the power what is that 6. So, from this graph you can get what is the ratio of velocity to the maximum velocity, and with respect to what is that Reynolds number and also what will be the power based on this Reynolds number there.

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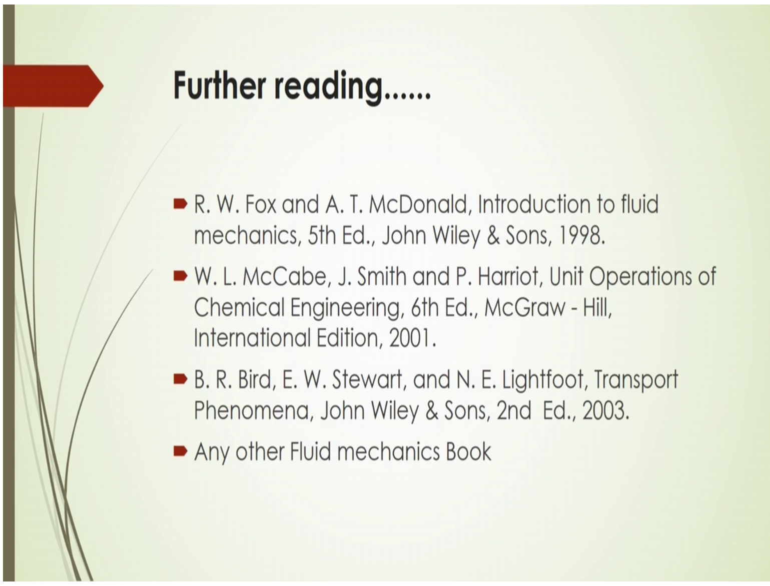

So, I suggest to for further reading this text book for getting more details. So, in this lecture we have given the concise way of the velocity distribution of the turbulent a flow. Over the surface and also in the what is that through the circular pipe. And this velocity distribution for the turbulent flow is analyzed based on the Prandtl's mixing length theory and also Von Karman mixing length theory, also the generalized equation for this turbulent flow velocity distribution is analyzed based on the what is that Blausius experimental observation and also Nikuradse experimental observations there.

So, that is for all today for this lecture.

Thank you.