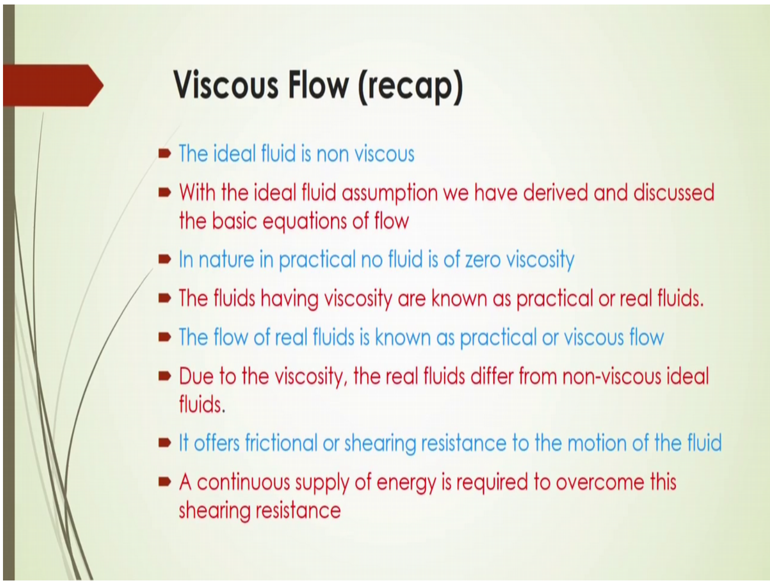
Fluid Flow Operations
Dr. Subrata K. Majumder
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Module – 05
Flow of Viscous Fluid - Part 2
Lecture – 11
Velocity distribution in laminar flow

Hello everybody, welcome to this massive open online course on Fluid Flow Operations. Here in this lecture we will discuss the flow of viscous fluid as a part 2. And based on the Navier-Stokes equation, we will discuss the flow or viscous flow through the pipe line and also other applications with examples. So, let us first consider that what exactly we have discussed in previous lectures.

In the previous lectures we have discussed something about the viscous flow and what is that viscous flow, that we have discussed that viscous flow should be a non ideal and also the based on the ideal flow of the fluid we have derived and discussed the basic equations of the fluid flow.

(Refer Slide Time: 01:45)

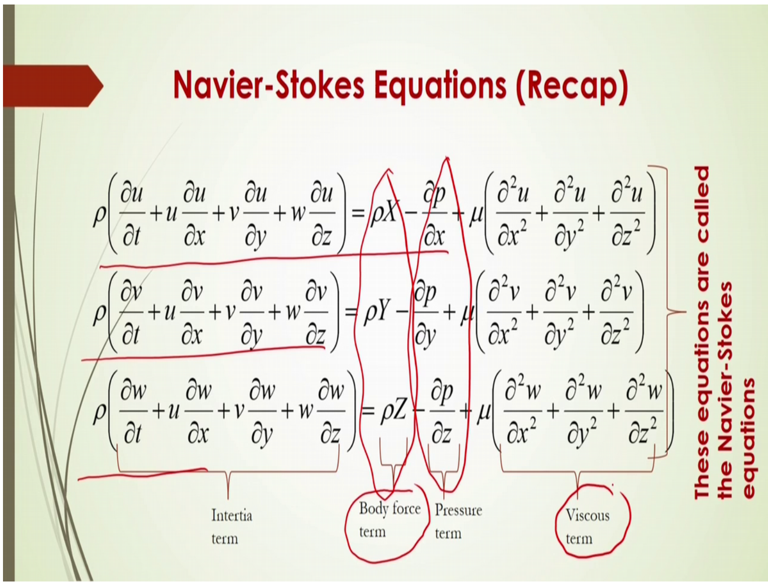


And also we know that there will be no fluid in the of nature that will have 0 viscosity, and the fluids having viscosity are generally known as real fluid and the flow of these real fluids should be called as practical or viscous flow. And due to this viscosity the real fluids differed from that non viscous ideal fluids of course.

And due to this viscosity effect of the fluid, the fluid will offer some frictional or shearing resistance to the motion of the fluid. And that is why a continuous supply of the energy is required to overcome this shearing resistance or frictional resistance. So, also we have discussed that how to derive the Navier-Stokes equation and the final form of this Navier-Stokes equation is what is that it is given in the slides that, here in the x and y and z directions what will be the governing equations for this Navier-Stokes equation

And in this case we observed that there are 4 terms of this Navier-Stokes equation, one is inertia terms that left hand side here.

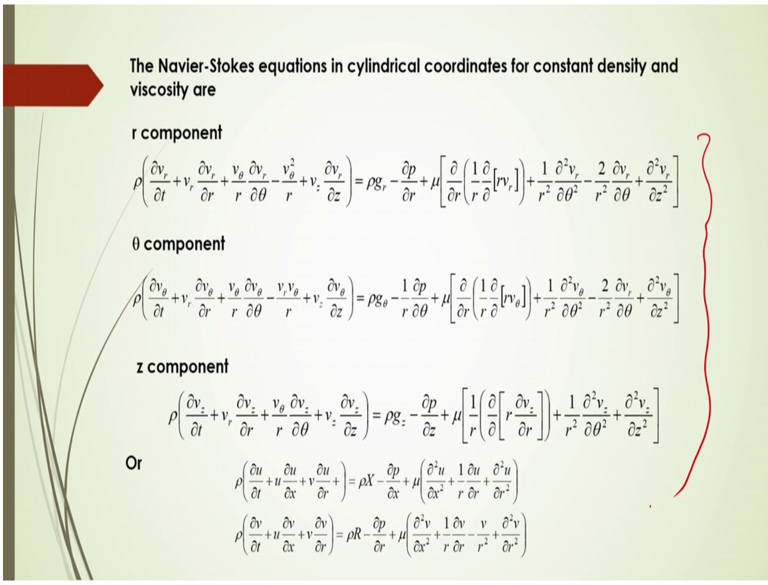
(Refer Slide Time: 03:11)



.

And in the right hand side this is the body terms; this portion is body terms and this portion is called pressure term and the other terms here, in terms of that is of second order derivatives of that velocity in x y and z directions flow with viscosity coefficient it is called as viscous terms.

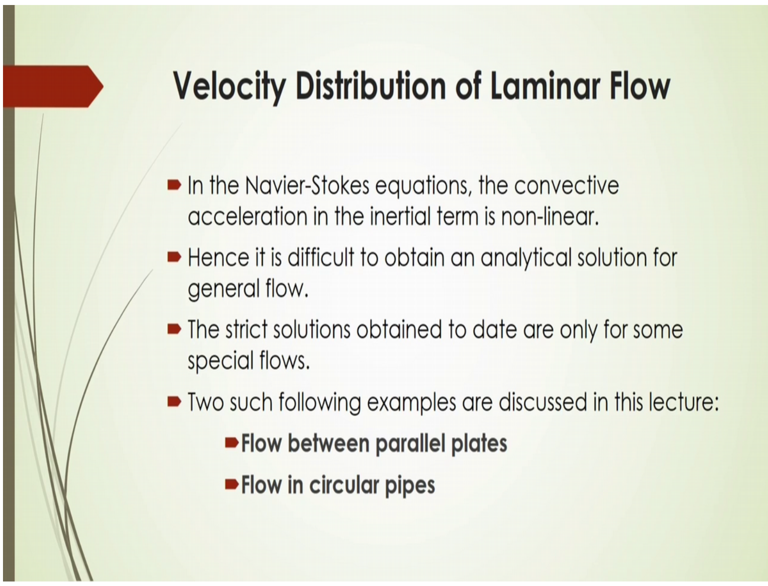
(Refer Slide Time: 03:39)



And also this Navier-Stokes equation in cylindrical coordinates or constant density and viscosity will have 3 components as r component, theta component and z component.

And these r component theta component z components are given in this slides respective equations and so, based on those equations that you can apply different aspects of fluid flow in that flow through the pipe whether it is in laminar flow or turbulent flow.

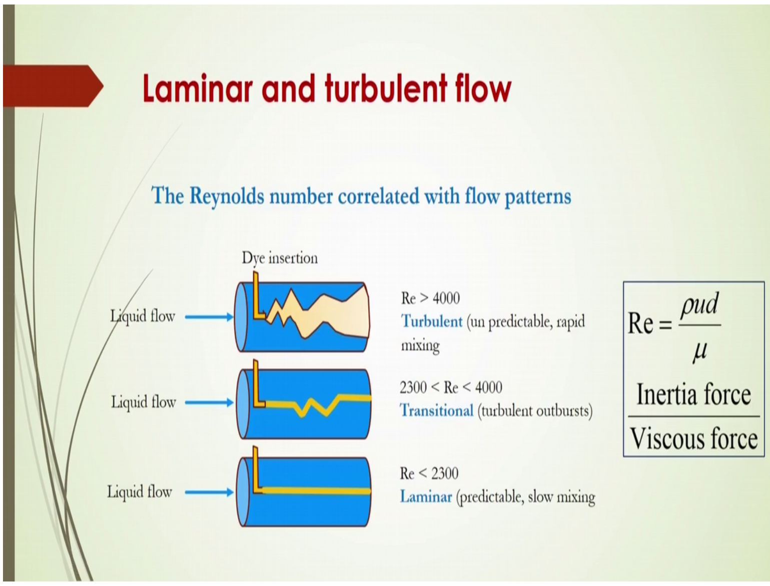
(Refer Slide Time: 04:01)



So, in this lecture we will discuss the velocity distribution of the laminar flow based on this simplification of Navier-Stokes equation with some assumptions.

So, in this case the Navier-Stokes equation there I think we have that the convective acceleration terms in the inertial term, which will be non-linear and hence it is very difficult to obtain an analytical solution for general flow. And the strict solutions obtained to date are only for some special flows in that case. So, 2 such following examples are discussed in this lecture then what should be the flow between parallel plates and also flow in circular pipes.

(Refer Slide Time: 05:11)



So, we will discuss here, whether it will be laminar or turbulent; first of all we will just consider the laminar flow through the pipe or plates or through parallel plates.

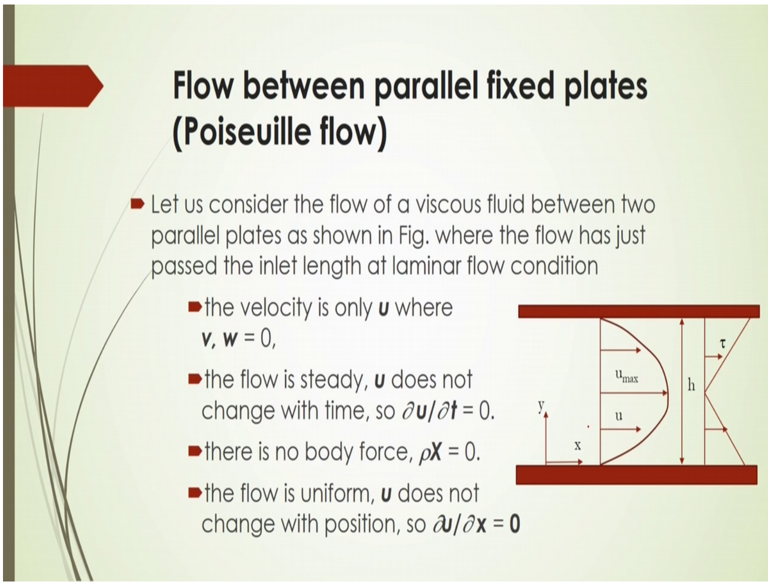
Now, what is that laminar and turbulent flow? We have already discussed in our earlier lecture that a laminar flow in that case the flow that the flow when Reynolds number will be less than 2300 that will be called thus laminar flow. Here liquid flow in such a way that the Reynolds number after calculation it will come this less than 2300. Here Reynolds number is defined as rho ud by mu this is inertia force to the viscous force. So, based on these if you have some particular pipe diameter through which the fluid will be flowing.

And if it has the particular cross sectional area and at a certain velocity if you are maintaining, and then by the viscosity of the fluid if you know then you can easily calculate what should be the Reynolds number. If the Reynolds number is coming to 2300 suppose water is flowing in that case, water is flowing as suppose one meter per second then what will be the flow what that is pattern of flow whether it will be laminar or turbulent flow that you can easily calculate here.

Just take the properties of the fluid and then substitute the density of the fluid, viscosity of the fluid here in this case water and the column diameter if you are taking so, you can get. Now this laminar flow you can I think have added I think different Reynolds number if it is less than 2330 based on if you change the diameter of the column even if you use different type of fluids then you will get the respective Reynolds number and whether it will be laminar or turbulent you can say. And for a turbulent flow you will have that turbulent flow that if it is the Reynolds number is greater than 4000s.

So, in that case it will be called as turbulent flow. Now in between that it will be called as the transitional flow here mixture of this turbulent and also what is that laminar will be there. So, basic think that whether the laminar or turbulent flow in this lecture then we will consider that laminar flow where the Reynolds number will be less than 2300. So, we have to select the variables in such a way that or Reynolds number should be less than 2300.

(Refer Slide Time: 07:43)



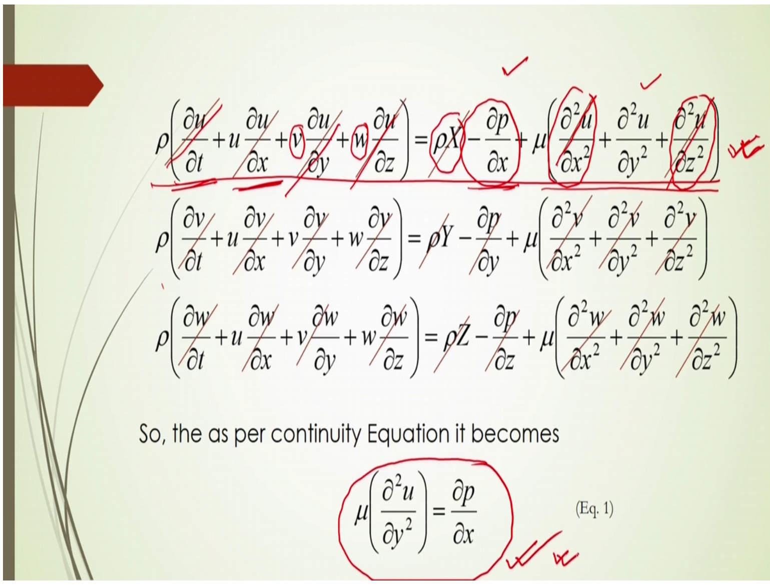
Now, let us consider the flow between parallel fixed plates here and the governing equation, final equation after that you will see sometimes it will be are called as Poiseuille equation

And the flow whenever it will be flowing through parallel fixed plates it will be called as Poiseuilles flow. And let us consider in that case the flow of a viscous fluid between 2 parallel plates as shown in figure here these are 2 plates this is one and this is another one 2 plates in between the liquid is flowing at a certain velocity u. And the height of these plates or this gap between these plates it is denoted by h here and the velocity is only in the u, where in the y direction velocity will be 0 and also in the z direction velocity will be 0.

So, only in the x direction the velocity will be considered here, and the flow will be considered here as a steady state it will not change with respect to time. So, that is why dou u dou t will be considered as 0 here. And also during this flow will not have any body force here like as per that terms we have used in that case rho x in the x direction there will be no a body force. So, in the x that row x will be equals to 0. And also the flow should be uniform that you have to consider otherwise you not have that laminar flow though the flow is uniform.

So, u does not change with the position also and. So, uniform flow means that velocity will not be changing with the distance here. So, that will be called that uniform flow. So, you know in this uniform flow conditions dou u dou x will be equals to 0. So, under these assumptions we will try to derive what should be the equation for a velocity distribution or other things.

(Refer Slide Time: 10:03)



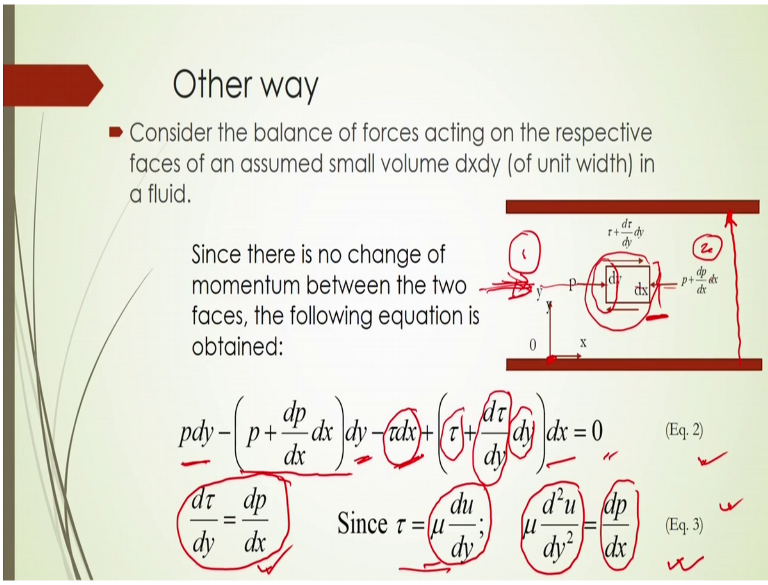
Now, if we apply the Navier-Stokes equation under these conditions, under these assumptions what will happen you will see, dou u dou t will be equals to 0 because here in the first equation see this terms this it will be dou u dou t will be equals to 0.

And because dou t equals because this is steady state conditions we have considered and dou u dou x dou u dou x will be here 0 because there will be no velocity gradient in the x direction because it is uniform flow condition is assumed. And also in the y direction there will be no velocity. So, that is why v will be equals to 0. So, this terms will be cancelled and also in the y in the z directions this w will be equals to 0. So, this terms will be cancelled out. And we have told that this rho x in the x direction there will be no body force. So, rho x will be comes to 0 only the pressure will be changing in the x direction.

So, there will be a pressure gradient in the x direction. So, it will be dou p dou x and here other terms in the viscous terms here, in this case since there will be no velocity in the x direction that is uniform velocity this terms will be cancelled out, and here also in the j direction there will be no change of velocity. So, this terms also will be canceled out. So, only these terms will remain and these terms will be remained in this equation. So, simply we can say here as per continuity equation the equation number 1 here in this case we will have from this equation this is from this part of this Navier-Stokes equation.

So, now and other parts here of course, will be 0 because the other directions in the y and z direction there will be no velocity and also the body force here acting here in the x directional condition uniform flow. So, finally, we are getting this equation, now what you have to do? You have to I think integrate this equation and then you will get the velocity distribution now without using these Navier-Stokes equation how can we derive this equation? Equation number 1.

(Refer Slide Time: 12:23)



Let us do that let us consider the balance of the force whenever fluid is flowing that and between 2 what is that parallel plates.

So, we are balanced you have to do the balance that acting on the respective faces of the assumed here a small volume dx dy. So, in this case we are considering there will be unit width that is perpendicular to the slides here that is to the flow, and then in this case there is no change of momentum between the 2 faces and then the following equation can be obtained here. Now in this case in the x direction what will be the force acting over this surface here that will be equal to p into d y into 1. 1 here what? Because we have considered that one width in the perpendicular to this flow.

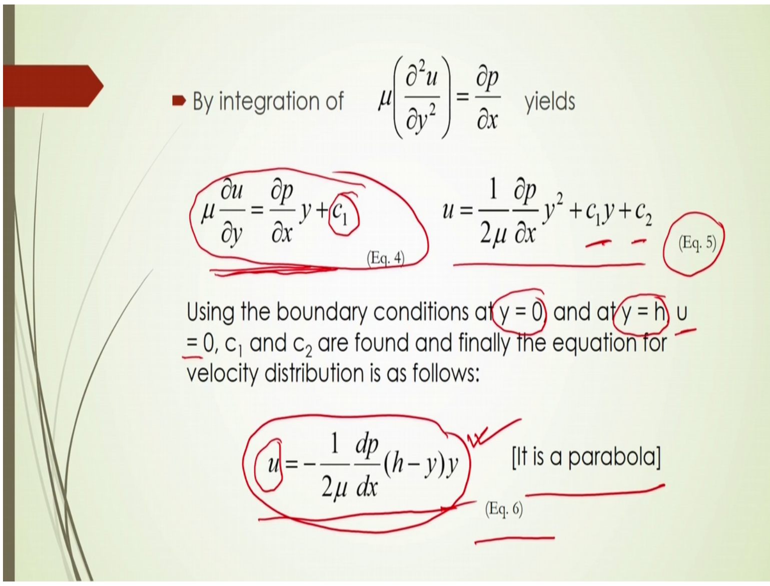
So, that is a p into d y and here at this face here this pressure will be is equal to p plus and dp dx into dx again into what will be the area cross sectional dy into 1, that will be is equal to dy here. And then what is that other part that there will be some shear stress acting whenever fluid will be flowing viscous fluid.

So, on the plate what the surface of this plate there will be a some shear force acting on that. So, what will be that shear force, it will be tau into dx into 1 that will be is equal to here tau d x tau is the shear stress and and the other part there will be what is that at distance dx this tau will be tau plus d tau into dy into dx that will be equals to 0. Now this d tau d y after simplification of this equation number 2, we can get here this d tau by d y that will be is equal to dp a dx. So, dp by dx.

Now, we know that the shear stress as per Newtons law, that shear stress will be is equal to upon Newtonian fluid that shear stress will be is equal to mu into d u dy. So, after substitution of this tau value here in this equation and deriving with respect to y we are getting mu into d 2 u by dy square that will be is equal to dp dx. So, this equation number 3 is exactly same as that we have obtained from Navier-Stokes equation as equation 1. So, we can derive it from directly Navier-Stokes equation or by just doing the force balance whenever fluid will be flowing through a conduit by taking some control volume with some what is the thickness width and what is that length.

So, this is the way by which you can calculate or you can derive also the governing equation of the fluid flow in between 2 parallel plates. Now after integration of this equation number 3 or you can see equation number 1 that we have derived from this Navier-Stokes equation that will be yield after first integration you will get.

(Refer Slide Time: 15:49)



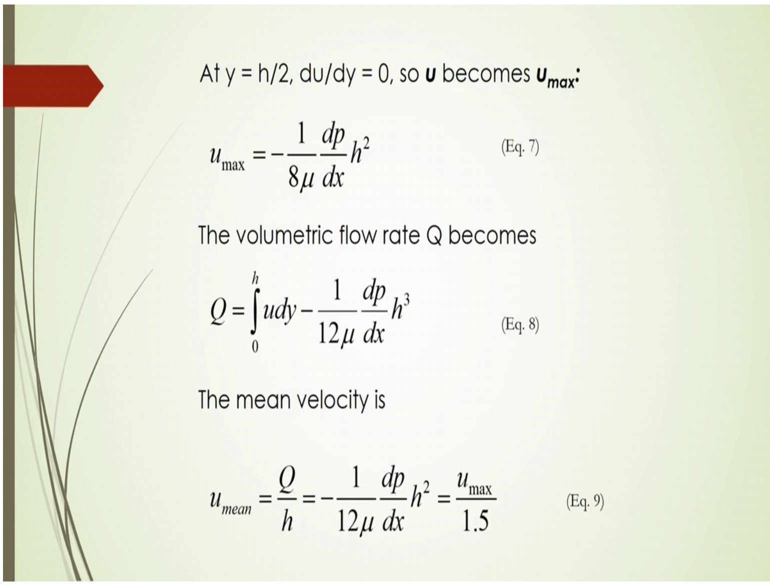
Mu into dou u dou y that will be is equal to dou p dou x into y and some constant of integration that is represented by c 1 and again if you do the integration for this velocity this equation number 4 then you can have this u will be is equal 1 by 2 mu dou p dou x into y square plus c 1 y plus c 2 here c 2 again the constant of integration.

Now, to solve this equation for c 1 and c 2 using some boundary conditions, then you have to consider that boundary condition as y will be equals to 0 and at y is equal to h u should be equals to 0 u should be equals to 0; y that at y is equal to 0 here you will see that y is equal to 0 there will no velocity. And at y is equal to a and y is equal to as up to this here there will also only I have no velocity of the fluid; because the fluid will be adjusting to this that is attaching to the solid surface.

So, there will be no slip there will be no velocity here. So, that is why we can use these boundary conditions of these that y will be equals to 0 and y is equal to h, y u should be is equal to 0. So, based on this boundary condition you can solve for c 1 and c 2 and you can again after finding out this c 1 and c 2 you substitute the value in equation number 5, after then rearrangement you can obtain this u will be equals to what is the minus 1 by 2 mu into dp dx into h minus y into y.

So, this is one type of parabola; that means, equation number 6. So, the velocity profile will be a parabola type velocity there. So, we can have this velocity distribution after just integration of the governing equation that obtained from the Navier-Stokes equation.

(Refer Slide Time: 18:07)



Now again if we consider that what should be the velocity, if y is equal to h by 2; that means, what will be the velocity at the center of this conduit flow or you can say the flow through the parallel plates.

So, what will be the velocity at the central part of this law? So, here that will be your maximum velocity and because there d u by dy will be equals to 0 there. So, to get this maximum velocity we have to derive it for u with respect to y, then you will get d u by dy and after substitution of this d u by dy is equal to 0 and solving for u you will get this is the u maximum there. So, u maximum is coming here negative of 1 by 8 mu into dp by dx into h square. So, this is this equation number 7 will give you the maximum velocity at the centre of this 2 parallel plates.

And here for these you have to know the pressure drop whenever fluid will be flowing through this plate and that case velocity gradient should be considered here dp by dx, and you will see in the section 1, if we consider the section 1 and here section 2 then the pressure will be reducing from this position to this position. So, there will be a drop of pressure. So, that is why negative term is coming here in equation number 7. And the volumetric flow rate Q will become in that case Q should be like this you have to integrate this u into dy over a height of this h.

So, here Q should be calculated in this way that it called volumetric flow rate and after again integration of this u with respect to y, then you can get this value, this will be is equal to what is that minus 1 by 12 mu into dp dx into h cube. So, this is your volumetric flow rate and the mean velocity what will be the mean velocity because everywhere there may be change of velocity over the y direction, then you have to calculate what should be the mean velocity over there.

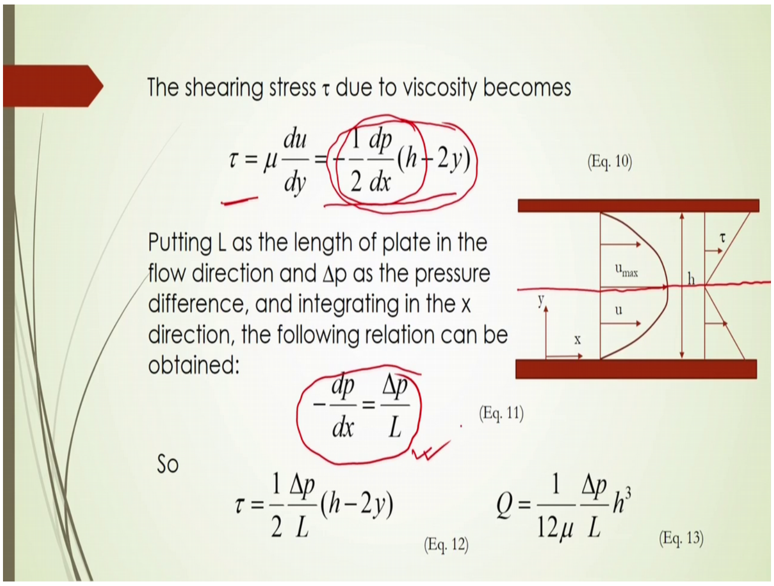
So, mean velocity will be is equal to what will be that volumetric flow rate upon what will be the height of these plates there or what will be the gap between these 2 plates. So, u mean should be calculated as like this Q by h. So, it will be simply if you substitute the Q value from equation 8 and divided by h you are getting here this 1 minus 1 by 12 mu dp by dx into h square.

Then if you simplify it or rearrange it you can get it u max by 1.5, because u max you know that it will be 1 by 8 mu into dp by dx into h square. So, comparing to this equation number 7 and this equation number 9, we can have that what is that u mean should be is equal to u max by 1.5; that means, maximum velocity will be 1.5 times of what is that mean velocity there.

So, this is very interesting that. So, we are getting what would be the flow rate what will the velocity distribution whenever fluid will be flowing through the space between 2 parallel plates, and how to derive this from the Navier-Stokes equation and also what should be the velocity distribution, that will be parabolic in nature like this here, this will be parabolic in nature. So, as per equation number here 6 and also what should be the maximum velocity that we can get it from equation number 7, what should be the volumetric flow rate, how can you calculate if you know the pressure drop over the sections, then you can calculate what will be the volumetric flow rate.

Even if you know the maximum velocity based on the pressure drop, then you will be able to calculate what will be the mean velocity they are. So, mean velocity will be 1 upon 1.5 into u max or maximum velocity should be 150 percent of the mean velocity that can be obtained from this equation number 9.

(Refer Slide Time: 22:41)



Now, what should the shear stress whenever fluid will be flowing through this parallel plate? So, in that case the shearing stress denoted by tau due to the viscosity that will become here tau will be is equal to mu into d u dy as per Newton's law of viscosity.

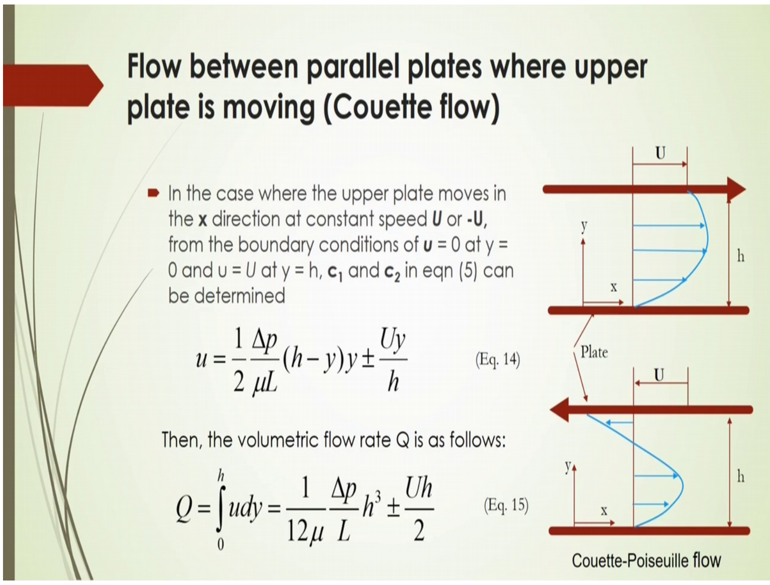
So, that if you substitute here d u by dy or derive the velocity distribution with respect to y, then after simplification you can get this tau that is the shear stress will be is equal to here. Then shear stress then depends on this pressure drop as well as h minus h also what will be the height of this 2 plates or gap between these 2 plates, and also at a certain distance what should be the shear stress there you can calculate.

So, when y will be equals to 0, then shear stress will be only this part and if suppose y is equal to a half a piece then you can say there will be no shear stress; that means, at the central part you can say at maximum velocity there will be no here shear stress. And also we can say that if you consider the length of this plate as L and in the flow directions and if we know the pressure drop that is delta p as the pressure difference and if we integrate this in the x direction, then we can have the relation between this pressure drop and the pressure gradient over this length L.

So, in that case the minus dp by dx we can represent by this equation number 11 in this way by considering the length of this plate here the x direction. This is the length of the plate. Now in this case if we now consider this dp by dx is del p by L, then after substitution you can get this equation number 12. And again from this distribution velocity distribution we can calculate what will be the volumetric flow rate they are. So, it will be here by equation number 13 and then if we know that velocity volumetric flow rate, then you can calculate what should be the pressure drop and for that particular gap.

And then from that pressure drop what should be the shear stress that you can easily calculate here. So, only thing is that if there is a flow rate, very interesting that only you have to do the volumetric flow rate to get the shear stress here. In this case if you know the volumetric flow rate then what will be the pressure drop that you can calculate from this equation number 13 and after substitution of this pressure drop over this length, the after substitution here in the equation number 12 you can get easily a tau value at a certain y distance there and this is a y. And find so, these are called Poiseuilles equation and this based on this momentum equation or you can say that Navier-Stokes equation or by the force balance equation you can easily calculate what will be the velocity distribution shear stress and the volumetric flow rate maximum velocity and mean velocity there.

(Refer Slide Time: 26:19)



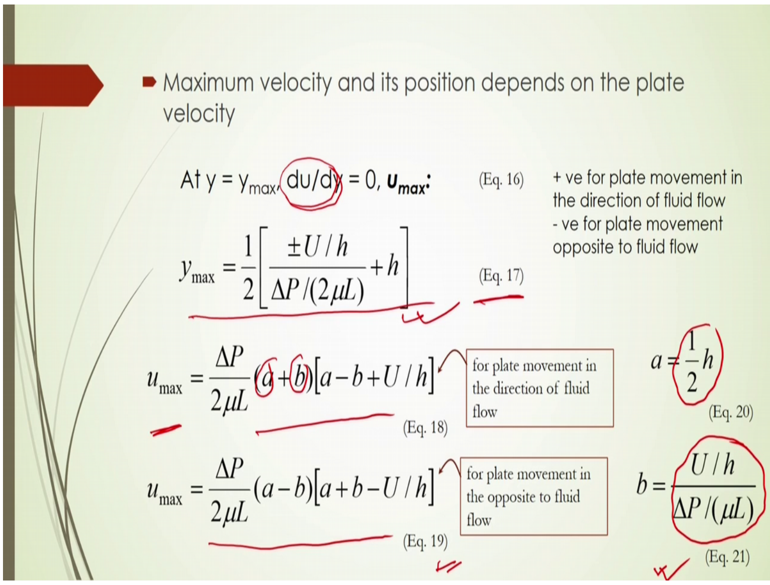
Now, another problem is there that if suppose the parallel plate, one of that parallel plates are moving in the same direction of fluid. So, in that case what will be the velocity pattern or velocity distribution that you have to find out and also how to obtain the volumetric flow rate and also what will be the shear stress there. Now, if suppose one of the plate is moving in the same direction. So, the fluid moving at that particular condition it will be called as a Couette flow.

And in the case where the upper plate moves in the x direction at a constant speed u or in the negative directions also minus u were then from the boundary condition of u is equal to 0 at y is equal to 0 and u is equal to u at y is equal to h. If you use, then we can have this c 1 and c 2 value that we have got in equation number 5 and after getting this constant c 1 and c 2 we can easily calculate what should be the velocity at a certain y direction length. So, they are from equation number 14 we can have this velocity when one of the plate is moving in the same direction or in the opposite direction there.

Suppose this upper plate is moving in this x direction and in that case the velocity of this upper plate is capital U, and if it is moving in the opposite direction it will be considered as minus u. Now you can use the Navier-Stokes equation or you can derive it from your force balance exactly the same as what we have shown earlier. So, in that case you are getting this final equation of u; here plus minus is for what is that in the positive direction you can get the if plate is moving and the p plate is moving in the positive x direction and if plate is moving in the negative y direction then you have to consider here minus u.

Then volumetric flow rate Q will be is as follows like Q will be is equal to again that integration of this u into dy height h here. So, then after simplification finally, you can get it 1 by 12 mu into delta p by L into h cube plus minus u h by 2 here. So, this equation 15 will give you this volumetric flow rate based on this Couette flow. And again you can calculate the maximum velocity and its position also depends on the plate velocity. So, at y is equal to y max we have again that d u dy equals to 0 and then you can calculate the maximum velocity there.

(Refer Slide Time: 29:39)

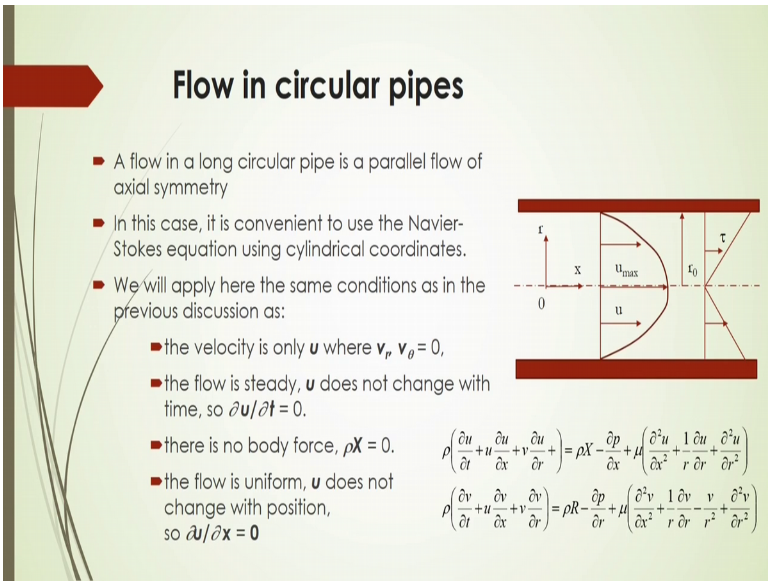


So, first of all if we do the derivative of this velocity with respect to y and getting it equals to 0 after solving then we can have the maximum length in the y direction what should be that or gap at which this the uniform velocity will be obtained; that means, d u by dy equals to 0. So, upon this y max you have to calculate what will be the u max there. So, what will be the u max? Then you substitute the value of y max that is given in equation number 17 and then after simplification you can get this u max as this as equation number 18 and equation number 19.

This equation number 18 is for the plate when it will be moving in the same x direction of the fluid flow. And this equation number 19 is for a plate when it will move in the opposite direction of the fluid flow. And here is one parameter this a and b is actually defined a big for its simplification, where a is defined as half of the gap of the plate and b will be what is this? This is u by h divided by delta P by mu L.

So, this is defined for a and b respectively and after you substitute this a and b, you can easily calculate what will be the value for u max and the case of plate movement in the x direction and in the case when plate will be moving in the negative x direction there with the velocity of u.

(Refer Slide Time: 31:25)

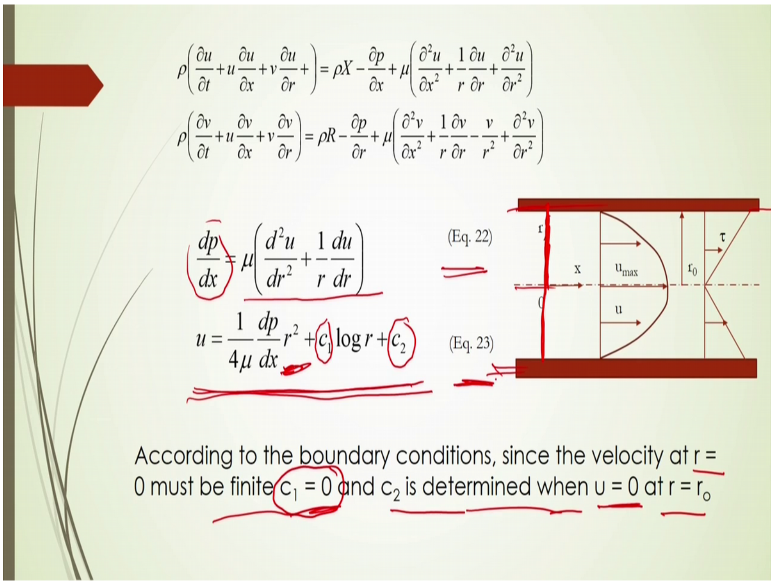


Now, we will consider another case, if suppose flow is moving in a circular pipes now in that case suppose there is a long circular pipe which is parallel flow of a axial symmetry. And in this case it will be convenient to use the Navier-Stokes equation using cylindrical coordinates, even accepts the Navier-Stokes equation you can also derive by force balance.

So, we will apply here the same conditions as in the previous discussion as your velocity is only u where v r and v theta will be equals to 0; that means, in r and v theta direction there will be no velocity. The flow is steady; that means, u does not change with that time and also there is no body force here in the X direction. So, it will be rho X that will be 0 and the flow is uniform. So, u does not change with position you can say. So, dou u dou x will be equals to 0.

Now, as per Navier-Stokes equation in the cylindrical coordinates you can have this equation here these 2 equations. So, based on this equation and upon assumption on of those, then we can get the simplified equation of the Navier-Stokes equation as given in equation number 22.

(Refer Slide Time: 32:35)

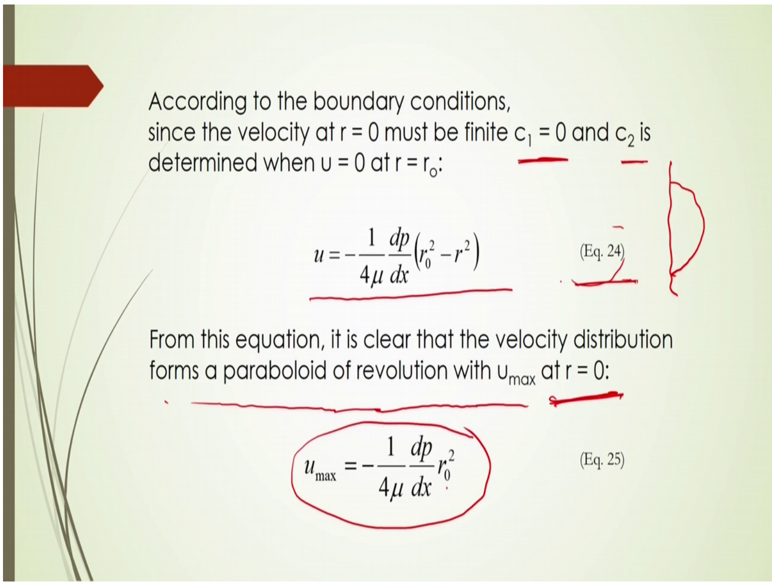


So, it will be dp by dx that will be is equal to mu into d square u by dr square plus 1 by r d u by dr. So, in this case after integration of this equation number 22 we can have this u will be equals to 1 by 4 mu dp by dx into r square that plus of c 1 log r plus c 2.

Here again after integration we are getting the c 1 and c 2 as constants of integration, and this r is what is that? Radius of this pipe this is circular pipe not in what is that plate here this is circular pipe. So, in this case r will be the radius of the radius that is r this radial directions you can say and this up to this it will be radius of this pipe and total it will be diameter of this pipe here. So, this r square so, you have to calculate the velocity at a particular r length there; and now this equation number 23 is having 2 constants unknown constant c 1 and c 2 you have to find out this unknown constants based on the boundary conditions.

What are those boundary conditions? Again we can have here at r is equal to 0; that means, here at the center line must be finite value c 1. So, c 1 will be equals to 0 will be considered and c 2 is determined when u will be equals to 0 at r is equal to r 0. This is your r 0 up to this; that means, this to this that is radius of the pipe. So, according to this boundary we can have this c 1 will be equals to 0 and also what is that c 2 will be equal to some value there.

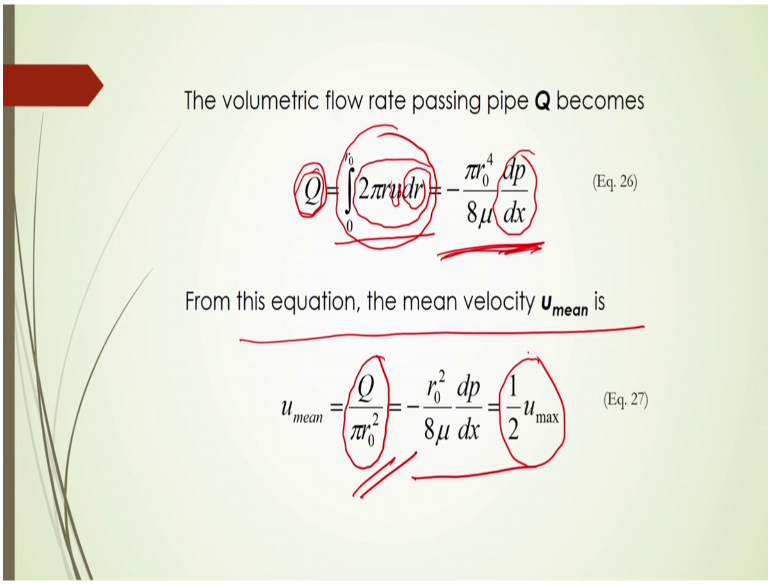
(Refer Slide Time: 34:39)



Now, if we substitute this boundary condition from this equation number 23 and then according you can get c 1 equals to 0 and c 2 will be is equal to some value and after substitution of those c 1 and c 2, you can get the final equation of the velocity distribution whenever fluid will be flowing through the pipe as given in equation number 24. Now from this equation it is clear see again that the velocity distribution forms a paraboloid of revolution with u max at r is equal to 0; that means, here again here you will get this type of parabolic velocity distribution there like this here this is the parabolic velocity distribution here again.

And at the center line you will get the maximum velocity and again the maximum velocity to calculate it you have to consider r is equal to 0 in this equation number 24. So, after substitution of r is equal to 0 here, we can get simply this u max will be is equal to minus of 1 by 4 mu into dp by dx into r 0 square this is. So, this equation number 25 will give you the maximum velocity whenever fluid will be flowing through the circular pipe there. And again the best way to calculate the volumetric flow rate based on this pressure difference.

(Refer Slide Time: 36:07)



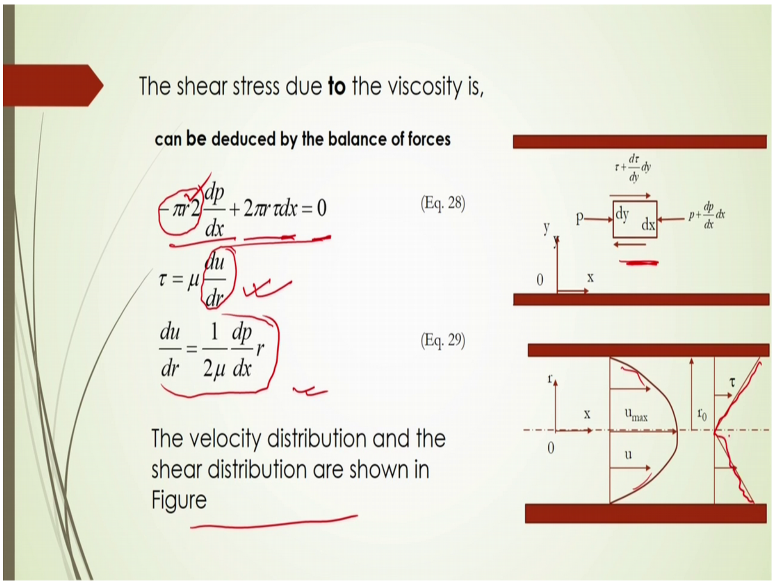
So, in that case if we know the velocity distribution, then the volumetric flow rate will be that which what will be the periphery and also what will be that u velocity and over up this radial distance of d r after integration of this 2 pi r u d r, then this will be your volumetric flow rate and this volumetric flow rate after substitution of u, you can get this simplified form of this velocity volumetric flow rate as pi r to the power 4 divided by 8 mu into dp by dx here. So, volumetric flow rate that depends on this what will be the r 0 and also what is the pressure difference is there.

Also if you know the volumetric flow rate, what will be the pressure drop to be created over a certain length of distance dx, then now you can calculate the dp by dx also from this known value of Q. Now from this equation the mean velocity also can be calculated. So, mean velocity will be is equals to just divide this volumetric flow rate by cross sectional area of this pipe. So, it will be your what is that Q by pi r 0 square. So, it is called cross sectional area average velocity.

So, it will be is equal to simply the substitution of Q value in this equation then you can get finally, half of u max. So, u mean will be is equal to 50 percent of the maximum velocity. So, this very interesting that you can get the maximum velocity twice up mean or mean velocity will be half of maximum there whereas, in case of flat plate with a gap as in that case we are getting that u mean will be is equal to u max by 1.5; whereas, here in this case we are getting half of u max here.

So, there is a difference this that depends on cross sectional area and also the periphery of that pipe there and pressure drop how it also will be developed over there. So, in this way we can calculate. Now, again in the same fashion due to the viscosity effect what should be the shear stress that we can calculate here.

(Refer Slide Time: 38:27)

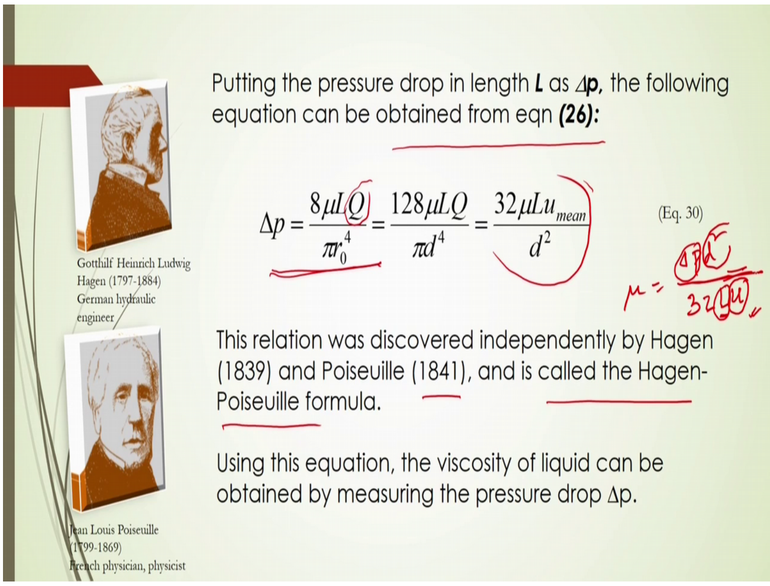


Now, this here if we consider that again this shear stress then we can do the force balance here this will be is equal 2 pi r square this pi r square into dp by dx plus 2 pi r tau into dx, that will be equals to this is called the force balance within this small strip of fluid element flowing through the pipe at a velocity u and if we do the force balance here then we can get this shear stress as here tau is equal to mu into d u by dr.

Again after substitution of this derivative of du by dr from the velocity distribution, you can get this equation number 29. So, it will be d u by dr will be equals to what is that? 1 by 2 mu into dp by dx into r. Now the velocity distribution and the shear distribution are shown in the figure as per equation number earlier it is given as what is that here velocity distribution 24 equation number 24 and the shear stress distribution as equation number here 29.

In this case then we can have this here and this velocity distribution and here it is the velocity distribution and this is your shear stress distribution over this cross section. So, we can easily calculate the shear stress and velocity distribution whenever fluid will flow in through the pipe, that can be obtained from the Navier-Stokes equation and also by the force balance also you can do.

(Refer Slide Time: 40:01)



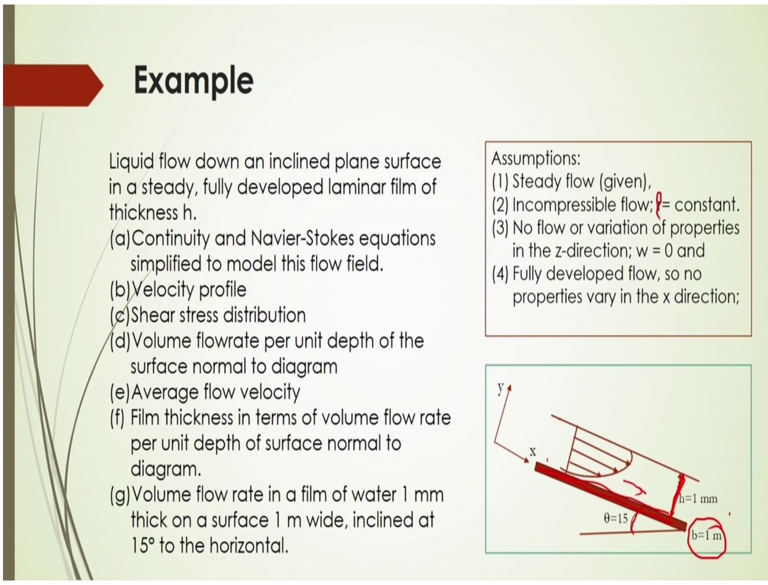
Now, putting the pressure drop in length L as delta p the following equation can be obtained from equation number 26 earlier it is this is the equation number 26 here, this equation number 26.

So, from this equation number 26 what should be the delta p that is pressure drop; that pressure drop if you substitute the Q value in terms of mean velocity, then you can have this here 32 into mu L u mean by d square here. So, this relation was discovered independently by Hagen and Poiseuilles Poiseuille in 1841 and it is called again Poiseuilles formula or equation using this equation the viscosity of the liquid can be obtained by measuring the pressure drop delta p here.

So, interesting that you can calculate the viscosity also if you know the pressure drop from this and also what is that if you know the diameter of the pipe and the length of the pipe, over which you are considering that pressure drop, then what will be the viscosity. So, from this equation number 30 you can calculate the viscosity here as what is that mu will be is equal to what? Mu will be is equal to in this case it will be delta P d square by 32 L into u mean u mean.

So, what you have to do the one you have to take the pipe whose diameter is known to you have to calculate the u mean velocity just by dividing the volumetric flow rate by its cross sectional area, then you can get u mean. And L certain distance of that certain length of the pipe to be considered for the pressure drop. So, you have to measure the pressure drop between 2 points with distance of L this is delta P then delta P by L what that you can calculate and after substitution you will get the viscosity of the liquid. So, from this Hagen Poiseuilles equation you can calculate what will be the viscosity ok.

(Refer Slide Time: 42:09)



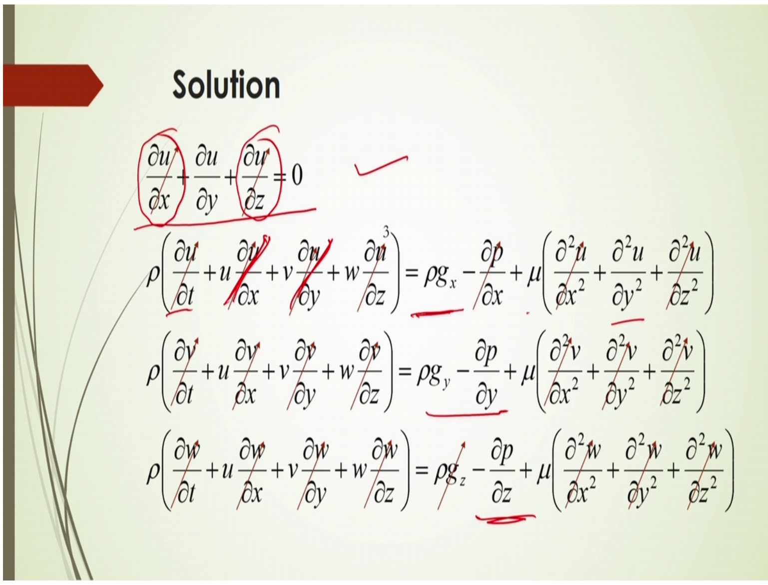
And let us do an example for this I think we have given earlier these examples earlier just to have the simplified Navier-Stokes equation, here also we will see what will be the mean velocity maximum velocity based on this problem and with assumptions. Now this examples is that if liquid flow downward at an angle suppose in a 15 degrees then the flow will be at steady state and fully developed and the laminar film of thickness say h is there.

So, in that case what should be the continuity and Navier-Stokes equation? That will be simplified to model this flow model and what should be the velocity profile, what will be the shear stress distribution and volume metric flow rate or a by unit depth of surface. So, that normal to the diagram here and what will be the average flow velocity, and how to calculate the film thickness in terms of volume flow rate per unit depth of surface normal to the diagram here. And volume flow rate in a film of water one millimeter thick if you are considering on a surface and 1 meter wide, if you are considering which will be inclined at an angle of 15 degree to the horizontal.

And for this we are assuming here something that flow should be steady state and also the liquid will be using as a incompressible liquid and flow should be incompressible and there will be what is that density should be constant and no flow or a variation of properties in the z direction. And w should be equals to 0 and what is that fully developed flow to be considered; that means, no properties vary in the x direction in that case.

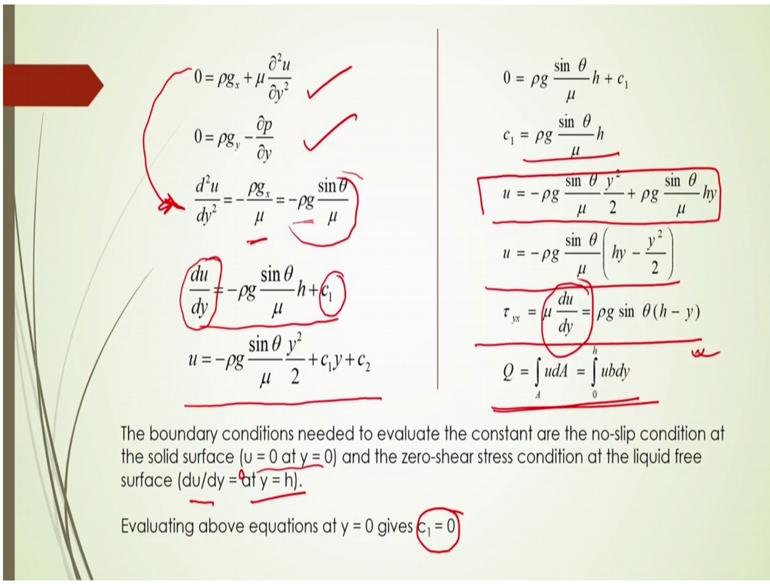
So, please look at this diagram here? Here in this case there will be a what is that surface inclined at 15 degree and over with some thin layer of liquid it will be flowing at a certain velocity and the width of this film it will be h equals to 1 and this is what is that depth is the h millimeter and width is 1 millimeter here. So, in that case if you are considering the direction in the x direction and y direction then you have to find out different velocity distribution shear stress distribution Navier-Stoke simplified equation.

(Refer Slide Time: 44:43)



So, let us do this first of all if we consider that a continuity equation. So, in that case this is the continuity equation and this d u by dx and d u by dz that will be equal to 0 as per assumptions. So, finally, only dou u dou y equals to 0 in this case. And other part here for steady state these terms will be negligible and finally, we are getting this when this marking is represented and it will be neglected and other parts will remain same here. So, as per a simplified Navier-Stokes equation we are getting these 2 equation here, here one will be is equal to rho gx plus mu into rho dou 2 u by dou y square that will be equals to 0 and in the y direction this will be equals to 0 and in the z direction this will be equals to 0.

(Refer Slide Time: 45:39)

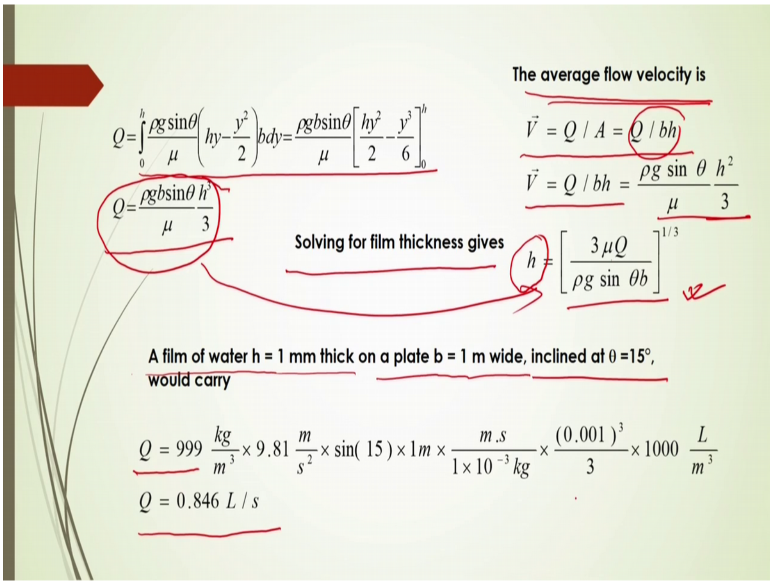


So, simplified this Navier-Stokes equation it will come here rho gx plus mu into dou 2 u by dou y square that will be equals to 0 in this equation, and the second equation it will be like this and then if we consider this equation here after simplification, we are getting here this d 2 u by dy square that will be equal to minus rho g x by d u what is that g x? That is in the gravitational acceleration in the x direction as per diagram we are shown in the x direction what should be the gravitational acceleration that will be considered here g.

So, rho g into g sin theta here rho g sin theta by mu it will come finally. So, g sin theta is the gravitational acceleration component and du by dy after that, after integration you can get this here with the constant of integration c 1, and again if you do the integration for this then you will get the velocity distribution with a constant of integration of c 1 and c 2. And if you consider the boundary condition as u is equal to 0 at y is equal to 0 and also d u by dy will be equals to 0 at y is equal to h.

Then you can have that c 1 is equals to here as this and c 2 will be equals to what here in this case, we can get this constants this here c 1 will be is equal to this and finally, we can have this distribution of velocity is like this and again by this velocity distribution we can calculate the shear stress as mu into d d u by dy then you can have this a shear stress distribution. And then Q also can be calculated by this equation here; now Q after substitution of this value of u and integration over.

(Refer Slide Time: 48:01)

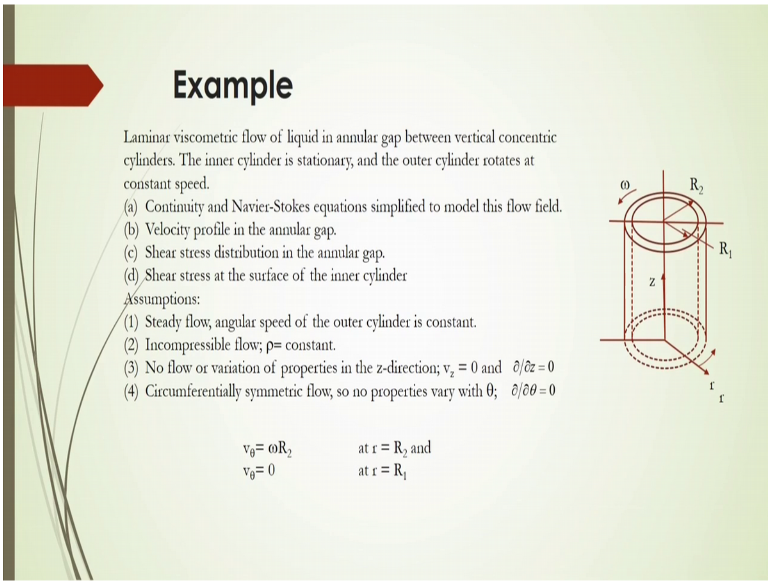


This then you can get Q will be equals to rho g b sin theta by mu into h cube by 3. And in this case solving for film thickness here you know that average flow velocity will be like this Q by A. So, Q if you know that this will be if you divided by a cross sectional area, you can get the average velocity and then average velocity after substitution of this Q value, you can get this value and then what will be the h value from this you can have this h will be equal to 3 mu Q by this theta b rho z sin theta b to the power 1 by 3 from this equation. And then you can easily calculate what should be the from this actually is h will be equal to this here.

So, this h is called the film thickness. So, film thickness will be here like this and then film of water if it is 1 millimeter thick, on a plate b is equal to 1 meter wide and inclined at theta 15 degree then Q will be is equal to this and here finally, it will be 0.846 a liter per second. So, this is as per equation that simply you have to calculate here by substituting the value, all these things and this is the procedure is given to you, you please go through this and practice it to have this type of equation for by just inclined conduit through which this liquid will be flowing as a film, then you have to calculate the film thickness by this formula.

And also in this case maybe with some other parameters you can try it you can consider Q value as an assumption you just assume it, and you assume it is a b value, you assume it some width value you assume it somewhat is that angle theta then accordingly how this Q shear stress velocity volumetric flow rate are changing also film thickness how it will be changing you can easily calculate based on this derived equation here.

(Refer Slide Time: 50:17)

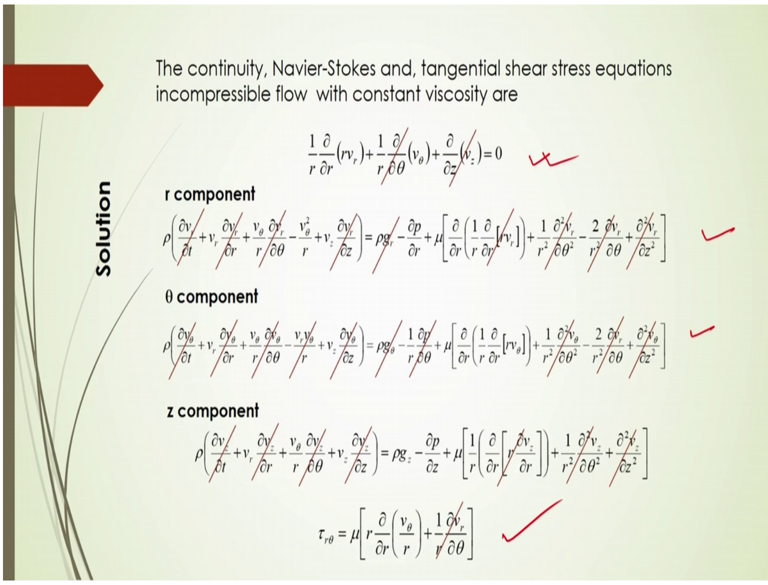


Now, another examples if you consider that the laminar viscometric flow of liquid in annular gap between vertical concentric cylinders are shown in the figure, and the inner cylinder is stationary in that case and the outer cylinder rotates at constant speed. So, in that case if we consider the continuity equation and also the Navier-Stokes equation, then you can have the velocity profile and also shear stress shear stress at the surface of the inner cylinder how it will be there.

Based on the assumption shear assumptions are given the steady flow, angular speed of the outer cylinder is constant to be considered and also incompressible flow in that case rho should be constant and no flow or variation of the properties in the z directions to be considered. So, in that case v z and v theta should be considered 0 here and dou dou z all in the z direction should be considered 0 and also circumferentially symmetric flow. So, not so, that there will be no property change with the respect to theta. So, dou dou theta will be equals to 0.

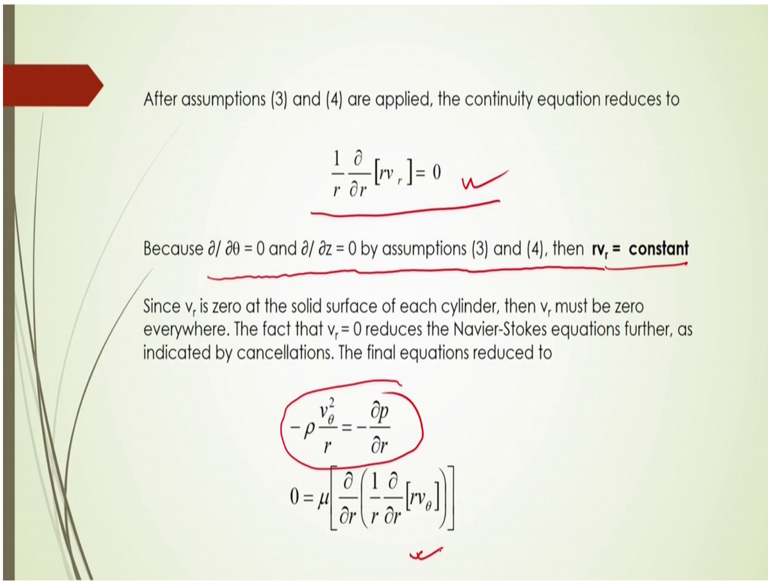
And also v theta the direction that should be omega into r 2; that means, omega at which this a cylinder is rotating and b theta will be equals to here v theta will be equals to 0. If on this the inner cylinder it will be 0 and; that means, here v theta will be equal to omega r 2 at up for outer cylinder inner cylinder since there will be no movement. So, it would be 0 simply and after simplification of the continuity equation you can get this.

(Refer Slide Time: 51:59)



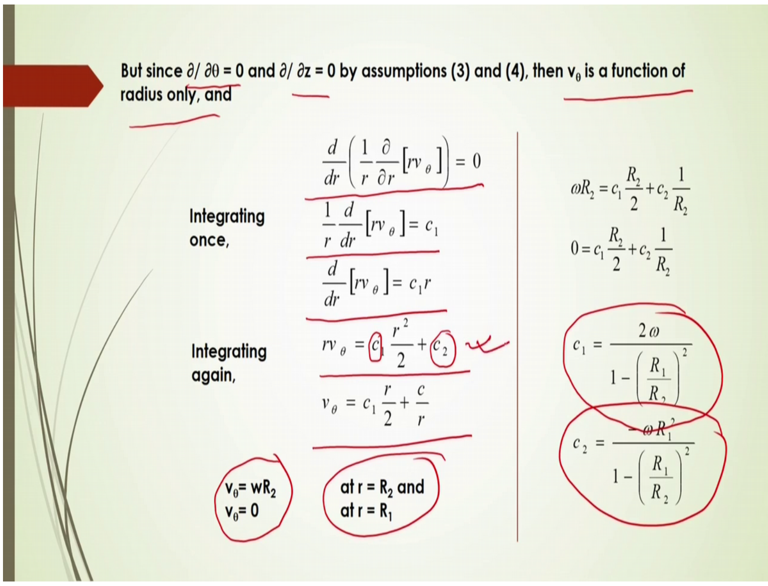
And for r component theta component and z components for this Navier-Stokes equation you can have by this formula.

(Refer Slide Time: 52:07)



And after getting the simplified this continuity equation, you can have this and then because of dou dou theta equals to 0 and dou dou z equals to 0 by assumptions of 3 and 4 then you can say that r v r will be equals to constant here. And also from the Navier-Stokes equation here you can say this equations should be simplified form and after substitution of v theta here, then you can get finally, this equation here.

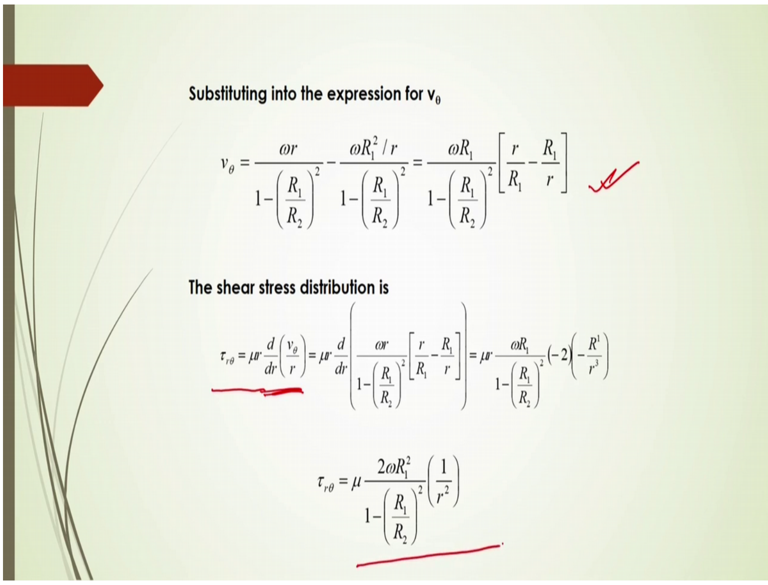
(Refer Slide Time: 52:41)



And from this 2 Navier-Stokes equation and with this boundary conditions of these then v theta can be obtained as a function of what is that radius only and finally, you can get these equations.

After integration you can get the equation with a constant of integration c 1 and here after that again if you do the integration over there, you will get the c 1 and c 2 integration constant and then simplified form of this equation as like this; and what is that? Here if you substitute these boundary conditions over there, then you can get this c 1 and c 2 after solving this and after that you have to substitute the c 1 and c 2 value in this equation and then you will get v theta what should the v theta there.

(Refer Slide Time: 53:31)



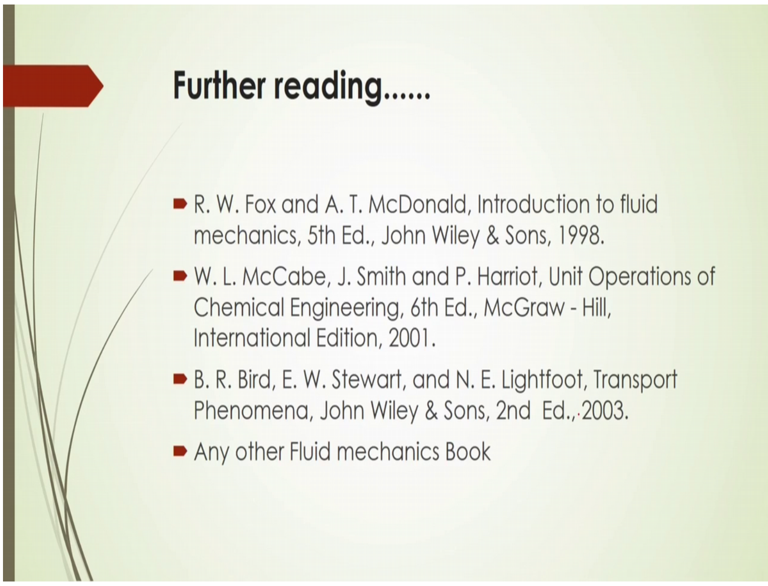
So, finally, that v theta will be is equal to this after a simplification. Then the shear stress distribution will be again this shear stress will be is equal to mu r in to d dr into v theta by r, now v theta if you substitute here and derive it with respect to r you can get this equation finally. And so, from the Navier-Stokes equation and also from the basic force balance equation, if fluid flow through the I think parallel plates or the cylindrical pipe how the velocity distribution and the shear stress distribution also volumetric flow rate and viscosity how can calculate we are actually now able to actually understand based on this lecture.

So, I suggest you to practice those equations for this laminar flow through the circular pipe as well as the plate between in which the fluid will be flowing based on this Navier-

Stokes equation. I think it will be helpful for your understanding further understanding, and you can have the idea whenever flow oil will be flowing through the pipe, how the velocity of the oil will be there inside the pipe.

Even if you know the pipe diameter, if you know the pressure drop from the pressure drop and pipe diameter cross sectional area how can you have the shear stress and velocity from that. And also if you want to measure some oil viscosity you can also do this by flowing this oil through the pipe and measuring the pressure drop then you can easily calculate, what is the viscosity of the fluid by Hagen Poiseuilles equation.

(Refer Slide Time: 55:37)



So, that is for all today's lecture I should suggest for further studying this textbook.

Thank you for this lecture today.