Fluid Flow Operations
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Module – 05
Flow of Viscous Fluid - Part I
Lecture – 10

Flow of Viscous fluid-Introduction

Welcome to massive open online course on a Fluid Flow Operations. In this lecture, we will discuss something about flow of viscous fluid as a part 1. And we have, previously discussed about the basic principles of a fluid flow operations and what are the governing basic equations by which we can represents the fluid motion by mass conservation, energy conservation and momentum conservation equations and that the equation specifically used for non viscous fluid.

Now, if there is a viscous fluid, in case of real fluid that how this fluid will be behaving and also what should be the governing equations that we will discuss here. And in that case, one important equation it is called the Navier stokes equation. And how this Navier stokes equation can be derived and what will be the final form of the Navier stoke equation and how this Navier stroke equation can be used to represent the application of different fluid flow in a different processes.

So, let us have the derivation of this fluid flow operations for this viscous fluid flow in this lecture.

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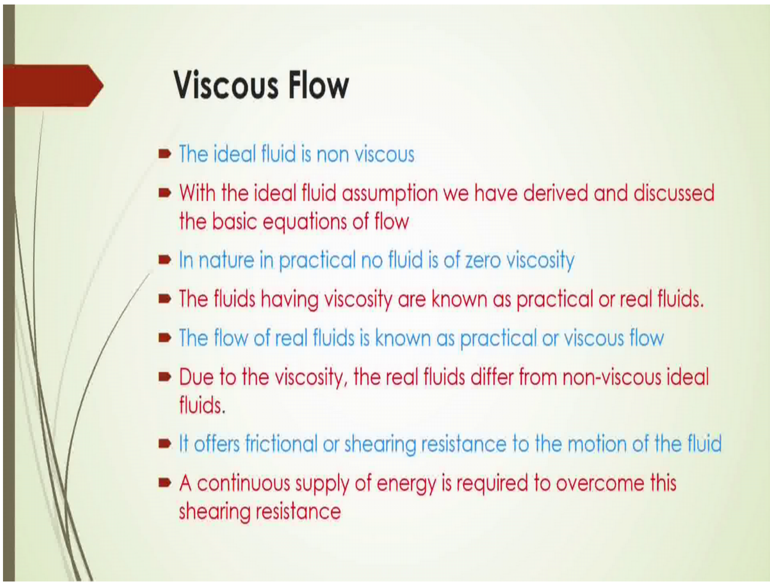

Before going to that, we have to know what is that viscous flow? Now, the ideal fluid is non viscous you know that we have already discussed in our pervious lectures. And with the ideal fluid that assumptions that we have made and discussed that the basic equations of the flow of mass conservation by equation, even energy equation that is mainly Bernoulli's equations and how it is derived actually we have discussed.

Also, momentum equation in the previous lectures we have derived the momentum equation as well as it is application in different processes like how is it pump is working on that momentum equations and also, how j depend what is that other applications of this momentum whenever fluid is thrusting on a solid surface, how that momentum will be changing and it is a fluid directions and also how this force will be balanced by that solid surface in the opposite direction that is in perpend; that means, opposite directions as per Newton's third law.

Now, in that case, of course, those are you can say that ideal fluid that is non-viscous. But in case of viscous fluid, that necessary in the practical that no fluid will be that non viscous. All the viscous, all the fluid in the real fluid it will be a viscous. So, there will be no fluid that will have 0 viscosity. And also, we can say that or we can called this fluid with which such that viscosity as a real fluids and the flow of that real fluids is known as practical or viscous flow. And due to this, viscosity of this real fluids that will differ from the non viscous ideal fluids and this viscous fluid will offer some frictional or shearing resistance to the motion of the fluid. Even whenever, it will be in motion, you will see there will be some angular deformations of the fluid element will occur.

So, whenever we will go to derive this Navier stroke equation there, so we will consider that what will be the shear stress, angular deformation, elongation, contraction and also mass conservation equation they are also when different forces that acting on the fluid body that will be discussed. And also, if we continuously supply energy which will be required to overcome this shearing resistance whenever any viscous fluid will offer some resistance to flow.

So, this is called viscous which will be really totally different in case of real fluid compared to the ideal fluid.

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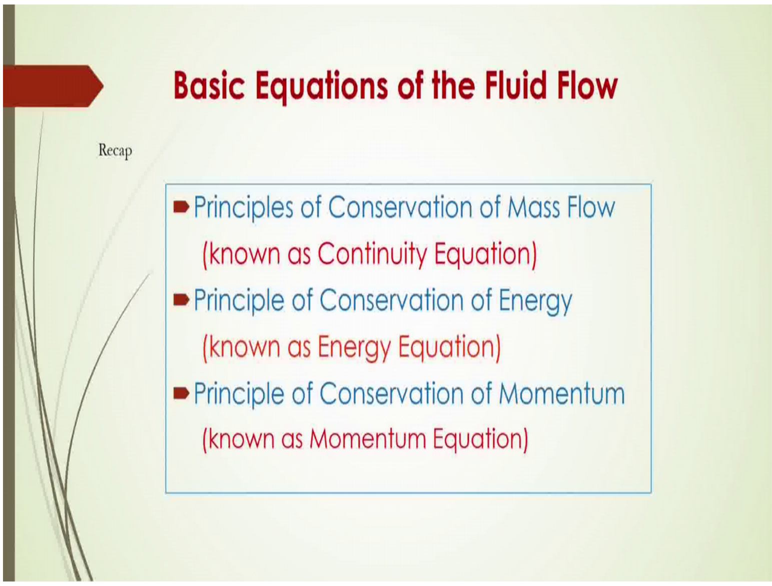

Now, we have discussed that what is that basic principles of that ideal fluid like continuity equation, energy equation and the momentum equation earlier. So, those basic equations of course will be required here to discuss this viscous flow and main viscous flow is called that is real fluid flow and which will be representing by the Navier stroke equation.

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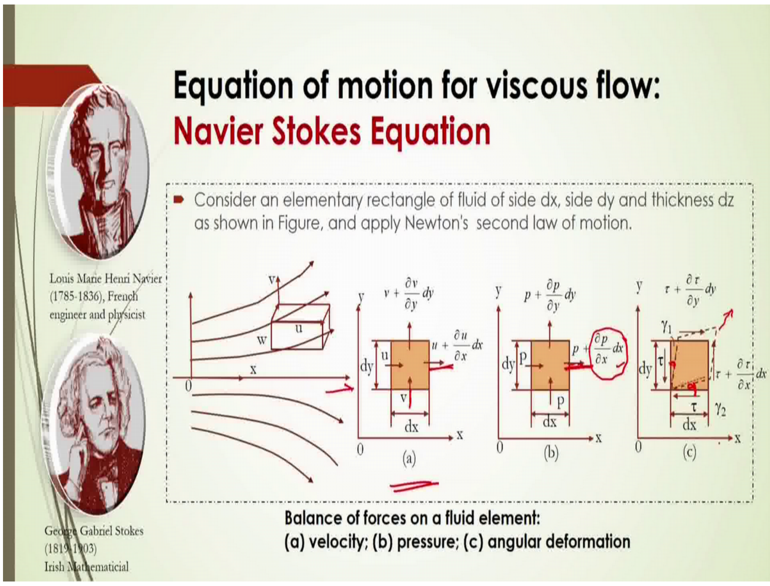

So, in that case, you have to consider some fluid element in the fluid flow to represent fluid flow even what should be the velocity, even how this velocity will be changing in the x, y and z direction and also how this pressure will be changing and how angular deformation will occur in this case. Now if you look at this figure like this a and this a figure will show you that some elementary rectangle of the fluid of side d x and d y and d x, d y. Accept this d x, d y, you can have other directions in the z direction whose length of course will be considered as d z which is here perpendicular to this slide.

So, if you consider that elementary rectangle of fluid of size d x, d y and d z, d z is as a thickness we can apply the Newton's second law of motion here. So, in this case, if we consider that a velocity of the fluid element as u in the x direction, then within a strip of this element; that means, of thick of side of length is d x, then from this side at a distance d x what should be the velocity change? That will be u plus do u do u x into d x. So, this is you change of velocity in the x direction.

Similarly, in the y direction, it will be changed if it is at this space the velocity is v and at a distance of d y of this strip, then what should be the velocity? It will be v plus do u v do u y into d y. So, this will be your representation of the velocity, how it will be changing for a smallest strip of a fluid element of side d x, d y and d z. And similarly, we can represent the pressure change in the x and y direction, like in the x direction, if we apply some pressure here as P, then in the other side of this stream what should be the pressure will be acting that will be increase of pressure or decrease of pressure, based on that fluid element the movement. So, it will be P plus do u p do u x into d x. So, this x direction at a strip length of d x, the change of pressure will be P plus do u p dou x d x.

Similarly, in the y direction, we can say that pressure will be changing from this P to P plus do u p dou y d 2 d y do u p; dou y is the pressure gradient in the y direction. Similarly, do u p do u x is the pressure gradient in the x direction. So, push total here in this phase what should be the pressure change that will be is equal to do u p do u x into d x this will be your pressure change; that means, how much pressure will be increasing or decreasing in this space.

Again, if we represent the angular deformation whenever fluid will be moving at a certain direction, you will see due to this viscous effect, you will see there will be a deformation of the fluid; that means, surface or you can say fluid layer over another one. So, in that case, if we suppose have these suppose due to the viscosity effect and it is flow, if there is an elongation like this in this direction, then you will see how this surface of this fluid element will be deforming from it is original position.

So, this is the angle at which this deformation of the fluid element will occurs at a certain velocity. So, it can be represented by this shear stress, even also you can say that will be pure shear strain also. So, shear stress will be changing in this direction that will be represented by tau. So, tau in this x direction will be changing as tau plus do u tau do u x into dx. And here, in this y direction tau will be reacting as tau plus do u t do u y do u d y.

And the angular deformation by if we consider that angles that have made after this deformation, it will be as in this side it will be gamma 1 and this side it will be gamma 2. Then, the rate of deformation of this angular angle that will be called as angular deformation, this angular deformation will be represented by do u gamma do u t or d gamma d t, how much angle will be changed with respect to time whenever it will be flowing within a short period of time that the elementary of element and it will be changed because of this viscous effect.

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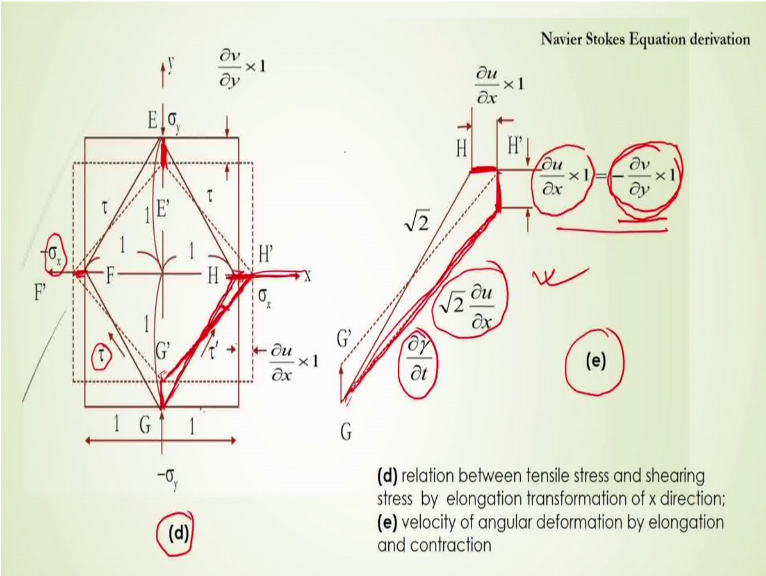

Now, if we see this diagram here, very interesting that there will be some elongation deformation in this x direction whenever you are stressing this fluid element in this x direction or y direction, you will see. In the x direction, if you are stressing that there will be a contraction in this y direction whereas, elongation will be in the x direction. Again if you suppose stress this y direction, the contraction in the x direction and elongation will be in the y direction.

Now, that elongation will be represented by this do u gamma do u t and in this case, see dotted line after elongation it will come. So, here if we represent this as unit of this length as 1, so if it is elongated, after elongation if the position is like this and before elongation if it is suppose like this, then what will happen? You will see there will be formation of this elongation of this time t. So, this will be represent by do u gamma do u t. We will show later on that how related with this velocity with this elongation, later on we will derive ok.

First of all, we try to understand what is this actually? So, here see in this direction x direction, there will be some force applying and because of which there will be this there will be a change of elongation, angular deformation, elongation given even something contraction also. Now, after this elongation or contraction if we represent this y this dotted line, then how we can represent it here, this to this length ok. It will be your root over 2. Why this will be root over 2 because this length and this length is considered as a 1 unit and 1 unit.

So, this one will be your root over 2; that means, root over 1 is plus 1, this will be root over 2. And of course, in this case due to this along elongation velocity, the velocity gradient is dou u dou x. So, this one will be coming as again root over dou u dou x because in this direction it will be, what is that in the y direction there will be dou u dou x and in this x direction it will be dou u d dou y. So, similarly, you can calculate what would be the portion of this length that will be your root over 2 dou u dou x.

Now, d represent here the relation between this tensile stress and the shearing stress. This sigma x is called tensile stress and tau is called shearing stress by this elongation transformation of in the x direction. Now, velocity of the angular deformation by the elongation is represented in figure e. And this case, if you are considering that contraction, that also you can have instead of y elongation.

So, in the same way, the same angle of deformation will be happened whenever it will be contracting by stressing and in the other way, it will be elongated in the same angle. Now, due to this elongation, there will be a contraction or elongation at a certain length. So, that length will be changing with respect to time that will be your dou u dou x. So, this will be your dou u dou x, here also negative of dou u dou x. Here also, this contraction if it happened, then here it will be your dou u dou v dou y, even this one also minus dou v dou y.

So, if we represent these suppose in the x direction, there will be a contraction that contraction will be represented by this here dou u dou x and here in the y direction, it will be dou v dou y. Since, the contraction in the y direction is in the negative direction. So, it will be actually represented by negative sign. So, it will be minus of dou v dou y. Now, the elongation and contraction both will be same. So, that is why, dou u dou x into one that will be Is equal to minus dou v dou y into 1. So, this will be same there, rho contraction and elongation both will mean the same in scalar size.

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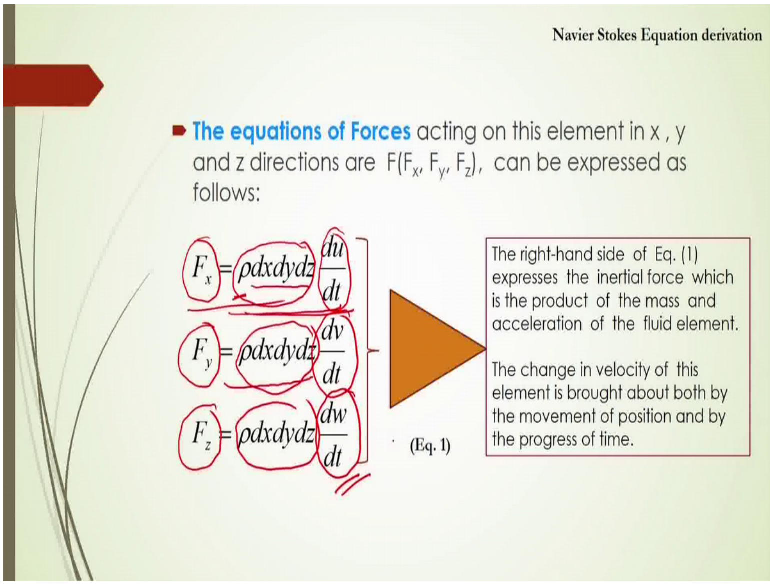

Next of course, whenever fluid will be flowing, that fluid element will rehearse some force there. So, what are those forces the acting on that element in x, y and z directions? So, those force will be represented by F. So, F in x direction that will be F x, F in y direction that will be F y and F in z direction, it will be F z. And this, the components of this force in x, y and z directions will be represented by this equation 1.

Now, F x will be is equal to you know that m into that is mass into acceleration. So, mass is rho into d x d y d z; rho is the density of the fluid; d x, d y, d z is the volume of the fluid. So, density into volume it will be mass and d u by d t, it will be your acceleration. So, mass into acceleration, that will be your force. So, F x in the x direction, the force will be like this.

Similarly, in the F y; that means, in the y direction, what should be the force? That would be represented by again this mass into acceleration in the y direction that will be d v d t. Similarly, F z the force acting in the z direction that will be mass into acceleration in the z direction that will be your rho d x d y d z into a acceleration in the y direction. Here, u, v and w are the velocities in x, y and z directions respectively.

So, right hand side of equation 1 that will express the inertial force which is a product of the mass and the acceleration of the fluid element. And the change in the velocity of this element is brought about both by moment of position and by the progress of time. So, in this, way we can represent the force.

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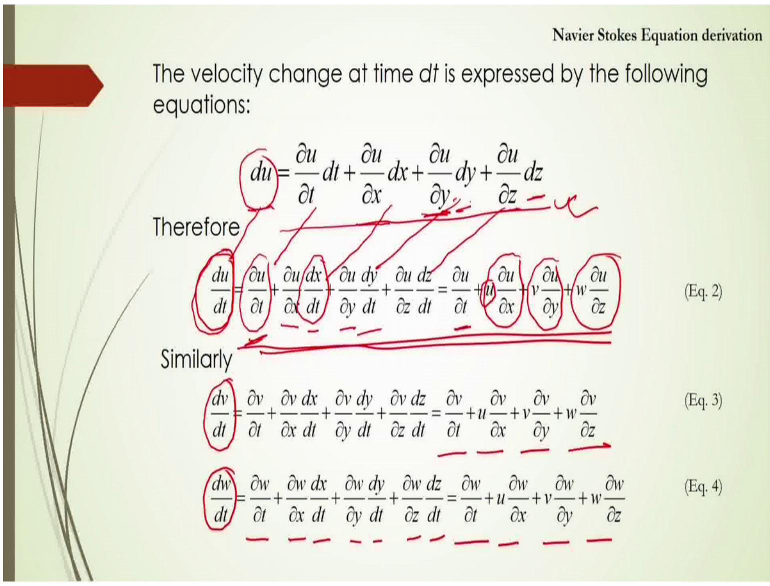

Now, if we consider this d u by d t here, what should be that d u by d t here? First of all, the velocity change at time d t can be expressed by this equation here. This d u will be is equal to the x, y and z direction are component and it will dou u dou t into d t plus dou u dou x into d x plus dou u dou y into d y plus dou u dou z into d z. So, this is your velocity change at time d t.

Now, if we divide both sides by d t, then we will get here d u d t that will be is equal to dou u dou t plus dou u dou x into d x by d t and dou u dou y into d y by d t here. So, it is here and this one this and this one here and this one here and this one, you are getting like this. So, finally, you can represent it as what is that dou u dou t here and in this case d x by d t will be represented by u; that means, in the x direction, what will be the velocity of this fluid.

So, here u into dou u dou x and this dou u dou x is the velocity gradient in the x direction there. Similarly, it will be v plus v into dou u dou y and w into dou u dou z. So, this is your from this dou expression, you can express what will be the acceleration in the x direction. Similarly, in the y direction, what should be the accelerations; so, it will be d v by d t. Similarly, you can have this dou v dou t plus u into dou v dou x plus v into dou v dou y plus w into dou v dou z by substation or by dividing this d v if you are representing d v by d t form this equation.

Similarly, in the z direction what will be the d w by d t; that means, the acceleration in the y direction that will be coming as dou w dou t plus dou w dou x into d x d t plus dou w dou y d y d t plus dou w dou z into d z d t. That will be is equal to dou w dou t plus u into dou w dou x plus v into dou w dou y plus w into dou w dou z.

So, this 2, 3 equation number 2, 3 and 4, you can have the representation of acceleration in the x, y and z directions respectively.

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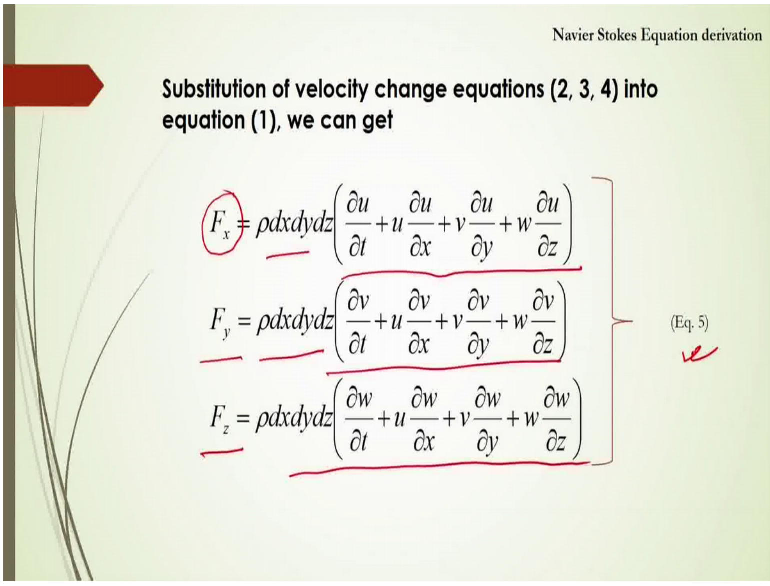

Now, after substitution of this acceleration terms dou d u d t d v d t and d w d t, we can have this force in the x direction as mass into acceleration. This acceleration terms we are substituting here and then, F y that will be is equal to mass into again that acceleration terms from equation number 1. Here, F z also this is the equations for the force balance here F z and this is your equation number 1.

If we represent this F x, F y and F z by substituting this d y d y d v d t this is acceleration terms, then we are getting this equation number 5 to express the force in x, y and z directions respectively.

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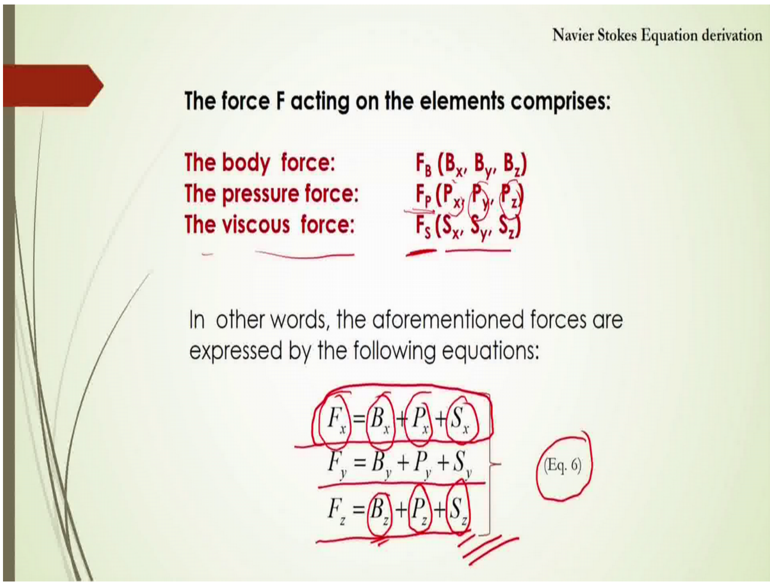

After that, now question is that force is applying. What type of force actually applied in the or acting on this fluid element. There are different types of forces that could be acting on the elements. That forces comprises of this body force, pressure force and viscous force.

The body force will be represented by here F B and this F B will have three components of this body force in the x, y and z directions that will be represented by F into B x, that is as a function of that is B x, B y, B z; that means, in the x, y and z directions the body force will be B x B y B z. Similarly, pressure force in the x direction, y direction and z direction, it will be P x, P y and P z and total force resultant pressure force will be F B there in the element. And the viscous force, this F S it will be the viscous force and again it will have three components in x, y and z directions. It will be a S x, S y, S z.

And in this case, very interesting then that in the x direction. Particularly in the x direction, what should be the total force acting on the fluid element. So, in the x direction this F x will be is equal to that summation of body force in the x direction, pressure force in the x direction and viscous force in the x direction. So, total in the x direction, the force summation of this body force pressure force and viscous force in this particular x direction.

Similarly, in the y directions, the summation of components in the body force, pressure force and a viscous force in y direction to be considered here; similarly, F z in the z direction what should be the summation of components of these x, y and z directional components out of which in the z directional components will be added a for this body force, pressure force and the viscous force there. So, in this way, equation by equation number 6, we can represents the total force in the x direction, y direction and the z directions they are respectively.

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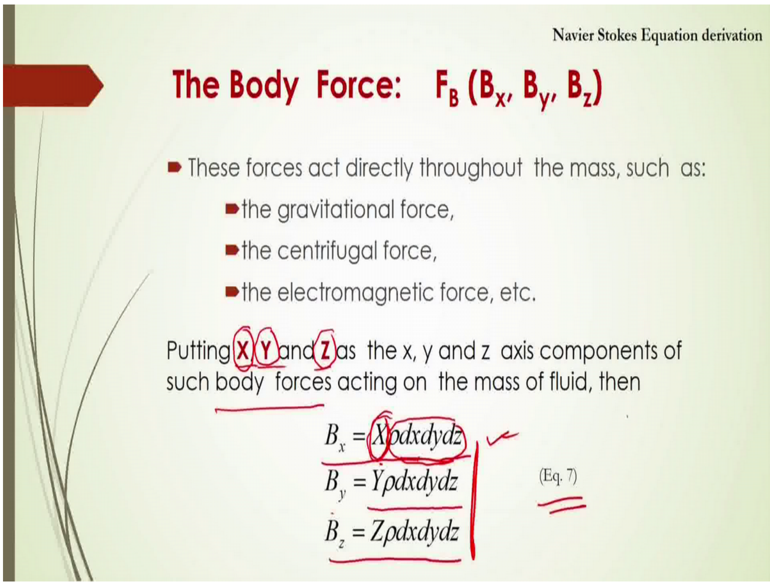

Now, what should be that body force, how this body force will be calculated? Now, this force actually act directly throughout the mass. So, such that the gravitational force it will be there, centrifugal force it will be there or other different types of forces electromagnetic forces and etcetera. Now, if we substitute this x, y, z as the this forces as the x, y, z axis components of such body forces. So, body forces will be represented as a here x, y and z in the z direction. Then, the B x can be represented as what is that here; x into rho d x, d y and d z. So, this will be your body force in the x direction.

What is this here, force into what is that this here rho into d x d y; that means, here mass into acceleration that will be your x. Similarly, here v y it will be y into rho d x d y d z. Similarly, B z will be B z rho d x d y d z. So, in this way, if we put this x, y, z as a acceleration of components in the x, y, z directions and then, body force will be calculated by this equation number 7 in this way.

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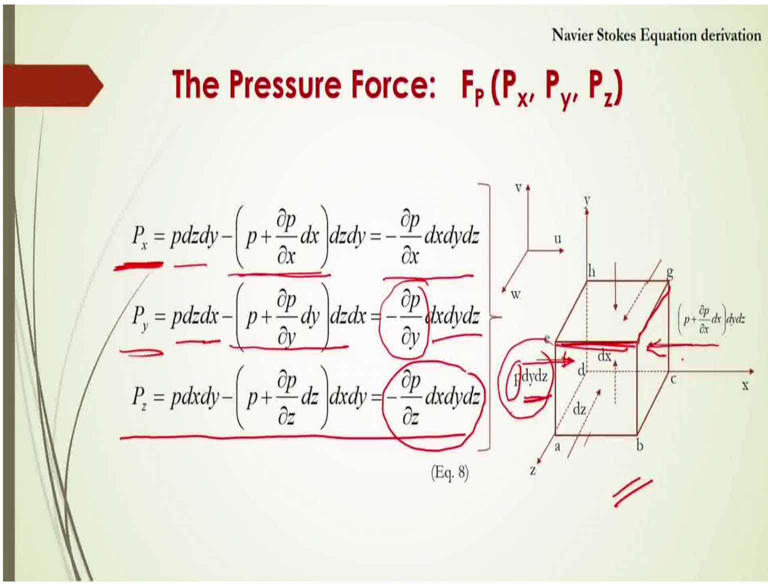

The pressure force again will have three components like P x, P y, P z. So, P x will be a calculated if you look at this picture here that the pressure force is acting on the surface of this fluid element in the x direction, then here in the x direction what will be the a force acting here, what is the cross sectional area that will be d y d z, then what will be the pressure then what will be that total force in this acting in this space, then it will be p into d y d z.

Similarly, the opposite side the pressure will be acting in the opposite direction it would be p plus dou p dou x dx into dy dz. So, this is your in the opposite side. Similarly, y direction in this you will get and here in this in this direction you will get this pressure difference is there. So, if we have this pressure; that means, here effective pressure in this x direction it will be P into d y d z minus of this; that means, here P plus dou p dou x into dx into d y, d z.

So, it will be here and after subtracting this components finally, it will be coming as minus of dou p dou x into d x d y d z. Similarly, in the y direction, if we have this balance of this pressure forces, it will be p y it will be is equal to p into d z d x here d z d x y because this one is your d z and this is your x.

So, this will be your d z dx. So, here pressure force will be in this direction, it will be what that p into d z d z, dx; similarly, from the other side of this y direction, it will be P plus dou p dou y into d y, then it will be in to d z d x. So, finally, after subtracting you can get this minus of dou p dou y into d x d y d z. So, similarly if you calculate the pressure force in the y direction, then you can gave this p z will be is equal to what minus dou p dou z into d x, d y, d z

So, in this way, we can easily calculate what should be the pressure force and in the x, y, z direction what will be their components. Now, considering that viscous force, this viscous force is a little bit con complicated because whenever the fluid will be flowing, there will be a frictional force and because of which viscous effect you will see there will be a angular deformation. And this deformation will be sometimes, you know that two types of air what is that we have discussed earlier that there are two components of this force; one is the called here that is I think you told that one is tensile stress and other is shearing stress.

So, we have discussed in this figure what will be the tensile stress and shearing stress. Now, if we first consider this forces sources are there because of this angular deformation of this fluid flow in the x direction, what should be the force.

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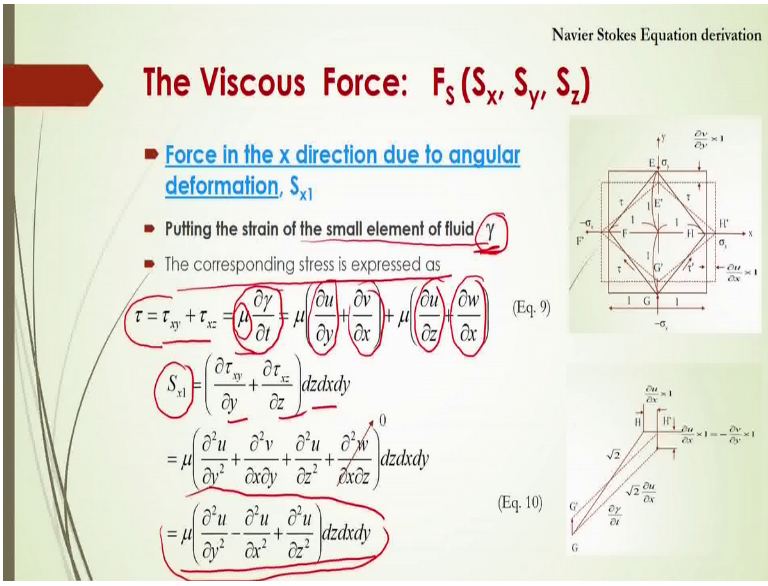

Now, putting the strain of the small element of fluid gamma, then here the corresponding stress will be expressed as by this. So, here it will be tau, will be is equal to tau x y plus tau x z. So, in this case this is in the x direction. So, it will be mu into what will be that rate of deformation here viscosity into here this is the coefficient of viscosity and d dou gamma dou t is the angular deformation here.

So, finally, you will get this how to calculate this dou gamma dou t here; dou gamma dou t it is actually by mu into dou u dou y plus dou v dou x and plus dou u dou z plus dou omega or d w d x. So, similarly here S x 1; that means, here what will be the angular deformations that will be is equal to dou tau x y dou y plus dou tau x z dou z into d x, d y, d z and final if you substitute this, tau x y tau x z there then you can have this. Now, at steady state condition or in the only two dimensional case you will see that in the z directions there will be no stresses or angular deformations in that case, this part will be equals to 0. And then finally, you are getting this mu into dou u dou y squared minus dou u square u y dou x square plus dou to u by dou z square into dx d y d z. So, this is your S x 1; that means, called the force the in the x direction that will be due to the angular deformation.

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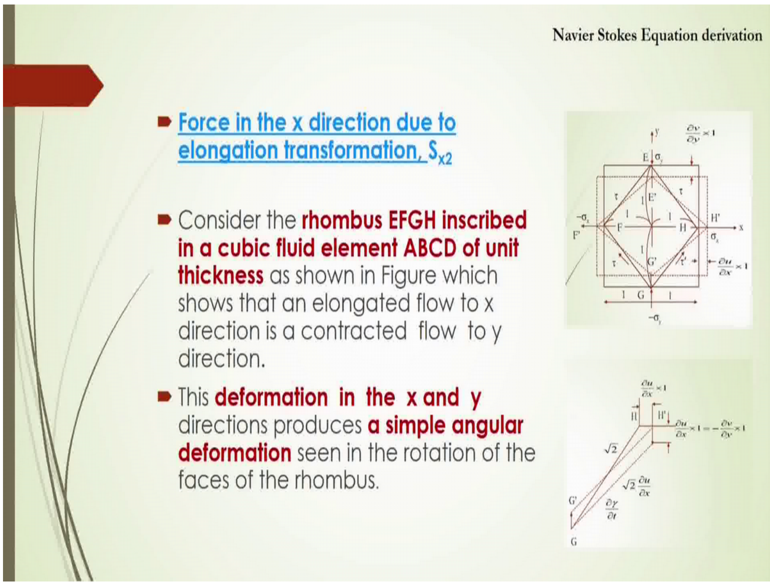

And, what should be the force in the x direction due to the elongation transformation there due to the stress; that means, here elongation or contraction.

Consider the rhombus shear of E, F, G, H that in striped in a cubic fluid element of A, B, C, D in this figure of unit thickness if you are considering as shown in this figure and which will show you an elongated flow to the x direction and also parallelly a contraction flow in the y direction. So, this deformation in this x and y directions that will produces a simple angular deformation that is seen in the rotation of the phases of this rhombus shear. So, whenever it will be rotating, then of course, you will get this deformation because of this elongation and contraction. So you have to consider what should be that elongation and how this elongation or contraction rate or angular deformation rate will be changing in the x and y directional components of the fluid velocity.

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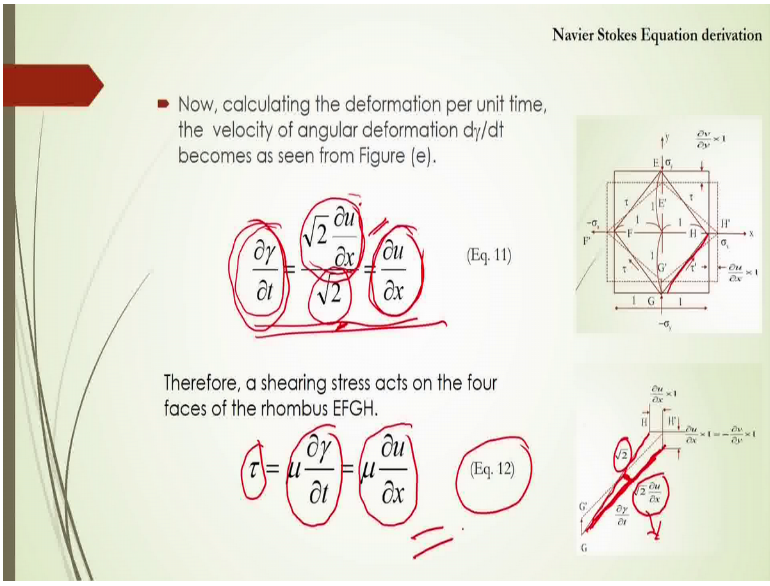

Now, in this case, calculating this deformation per unit time the velocity of angular deformation; that means, the d gamma by d t that will become here this 2 dou u dou x divided by root over 2. See here in this case finally, you can get dou u dou x.

So, this is your what is that here in this direction, what would be the value for this? Here, it is given this direction here it is given what should be that root over dou, root over 2 into dou u dou x and here this is your root 2. So, this where this is a your before elongation or after elongation what is that, how from this, what will be the angular deformation from this, how it will be coming from a. So, after if it is suppose in this direction, in this direction there will be an elongation. Now, after elongation this will be changing a root over 2 dou u dou x and this is this was this was your initial length. So, a final divide by initial that is increment and this, so it will get the elongation angular deformations which will be dou u dou x. Therefore, a shearing stress that acts on the four phases of the rhombus efgh shear. So, in that case, tau should be represented as a mu into dou gamma dou t. So, that will be equals to mu into dou u dou x, that will be based on this angular deformation. So, by equation 12 you can represent this shear stress.

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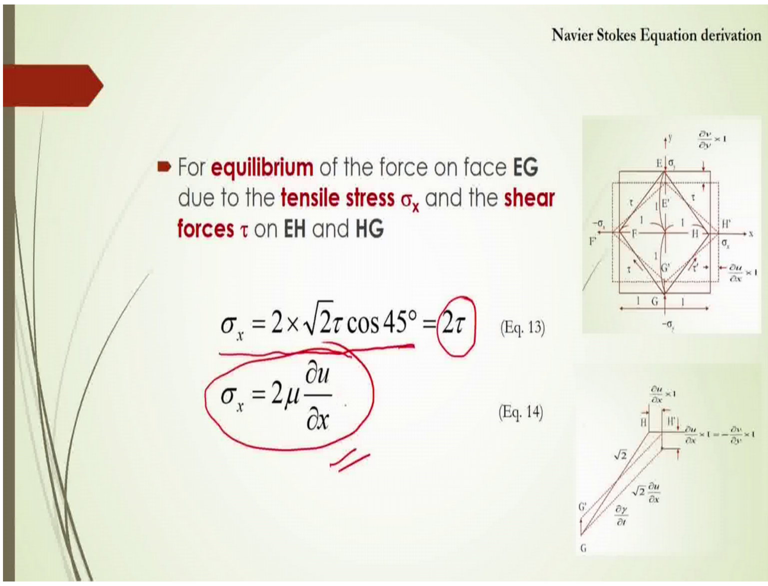

Similarly, for equilibrium of the phase that force that acting on this phases, to equilibrium condition; that means, for equilibrium of the force on the phases that easy due to the tensile stress sigma here as per this diagram shown, so in the shear force tau on this E H and here c E S in this figure E H E H. Then, this shower that is shear force tau on the E H and H G how it will be there what will be value in that case. Sigma x will be the representation of this equilibrium tensile stress. So, this tensile stress in this x direction it will be is equal to 2 into root over 2 tau cos 45 degree as punctuate here, it will be coming as 2 tau. So, sigma x will be is equal to what 2 mu in to dou u dou x here. So, this is your tensile stress.

So, we got this shear stress here as mu into dou u dou x whereas, this tensile stress is 2 into mu into dou u dou x, that will be 2 times of this shear stress here.

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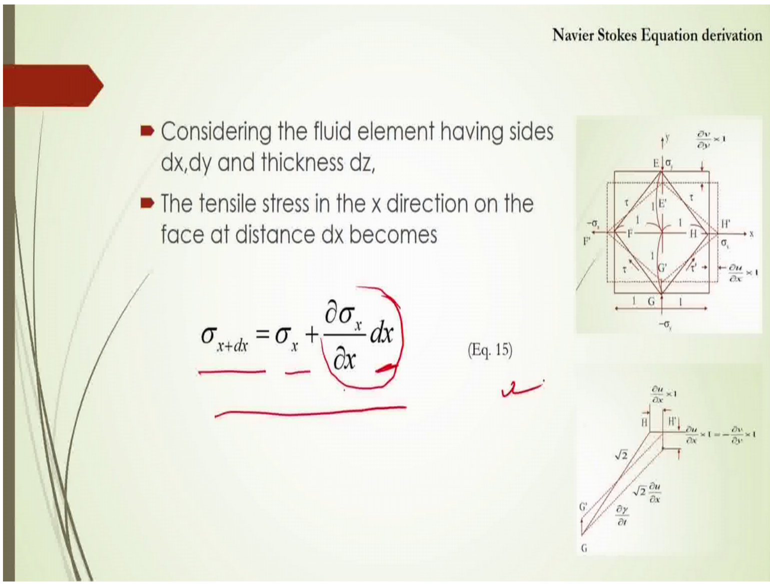

And if we consider the fluid element that having sides dx, dy and thickness d z and the tensile stress in the x direction on the phase at a distance d x, that will become a what is that sigma x plus d x that will be is equal to sigma x plus dou sigma x into dou x into dx. This will be your incremental a what tensile stress. So, at this x plus dx, what will be the tensile stress, you can obtain from this equation number 15.

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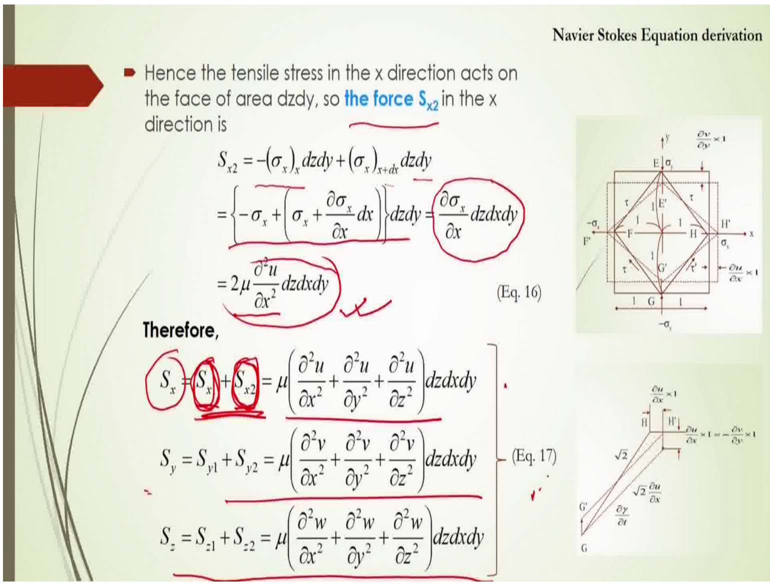

Now, after getting this equation number that is equation for what is the tensile stress and shear stress, then tensile stress in the x direction acts on the phase of area that is d z, d y.

So, finally, we can calculate the force S x 2, that is called a tensile stress in the x direction, so what will be is equal to minus sigma x into d z d y plus sigma x plus d x, d z, d y. Then, what will be the after subtracting, it will be coming as dou sigma x dou x into dx d y, d z. So, it will be is equal to 2 into mu into dou u dou 2 u dou x square 2 d x d y d z. So, this is your final S x 2. S x 2 is what? S x 2 what we have represented this S x 2 we called it that force in the x direction due to the elongation transformation, elongation transformation. So, this elongation transformation, elongation transformation, you can have a from this equation number 16 the x direction.

So, we can summarize it in the x direction. It will be what is that S x 1 and S x 2, what is S x 2. This is S x 1 is the elongation transformation and S x 2 is the what is that angular transformation or angular deformation elongation transformation and other S x 1 is the what is that angular deformation. So, S x 1 is the angular deformation that is force acting in the x direction. And this is your elongation transformation is S x 2. So, here S x 1 and S x 2 angular and elongation, this two transformation to be added in the x direction to get this tensile stress here.

So, this tensile stress, so what in x, the direction we are getting this equation here. Similarly, this here and for z direction, this will be your equation. So, equation 17 will give you that a S x that will be is equal to a x plus S x S x 1 plus S x 2. So, finally, you are getting these things.

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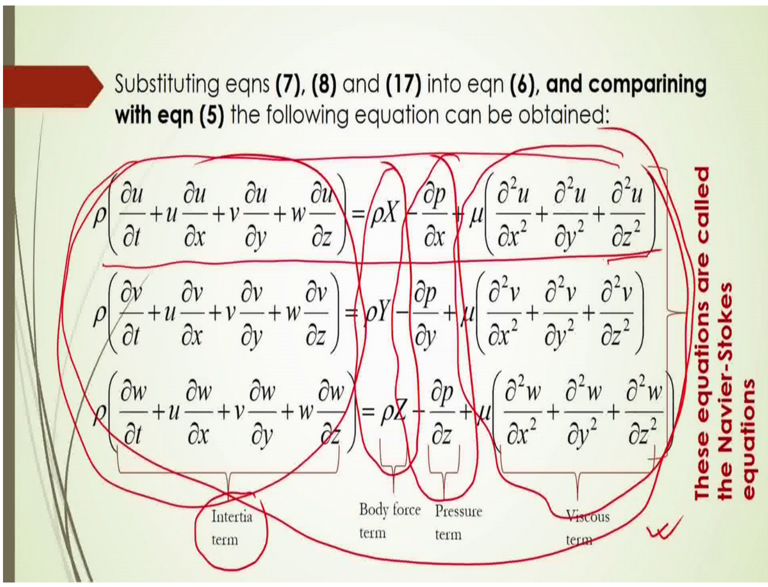

Now, after substitution of this body force and pressure force and tensile, what is that force due to this angular deformation, if we substitute all those components in equation number 1, then we can get what is that we can finally, represent it by this equation here rho into dou u dou t plus u dou u dou x plus v dou u dou y plus w dou u dou z is equal to rho x minus dou p dou x plus mu into dou 2 u dou x square plus dou 2 u dou y square plus dou 2 u by dou y z square.

So, this is your equation based on this substitution of S x 1 and S x 2 and also this pressure forces and then body forces and this body forces. Then, after substitution all this equations here in equation number 5, then we can have this final form of equation here like this one. So, this whole equations will be called as what is that Navier stokes equation. These equations is used for actually assessing different fluid motion in this viscous flow. And these equations have think four components here. This part is called the inertia terms, here this part is called inertia and this part it is called that body force term and this terms is called the pressure term and these are called as what is that viscous term.

So, this Navier stokes equation will have total four terms, that is one is inertia, another is body force, pressure force and the viscous force. So, all those things will be considered whenever you are representing any fluid motion in a general way that you have to calculate based on the assumptions whether these components will be there or not. Based on these equations, you can finally get some equation to represent the fluid motion.

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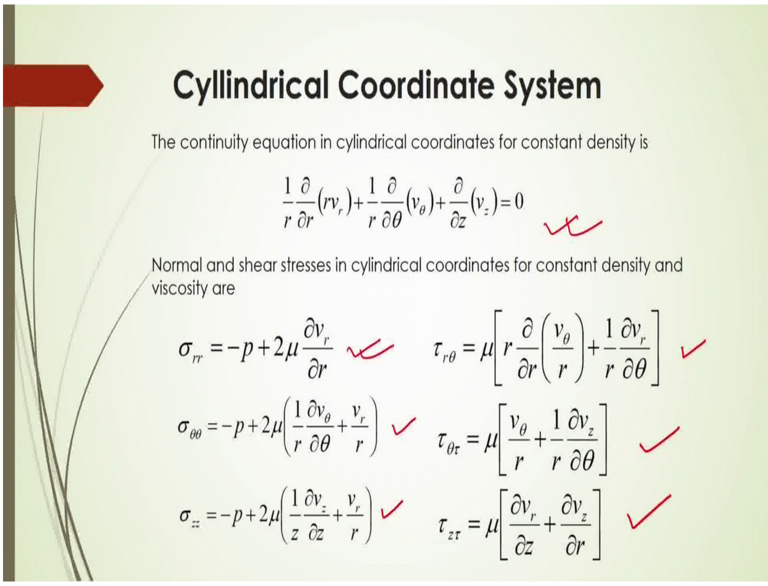

And this Navier strokes equation can also be represented by other system like cylindrical coordinating system. So, in that case, it will be represented by this equation. And in that case sigma r in the direction that is cylindrical coordinates and this one sigma theta sigma z and tau r theta tau theta tau and what is that tau z tau.

So, these are the final equations for the Navier stroke equation in the cylindrical co-ordinate system and you can apply also this cylindrical systems for the particular operations either by Cartesian coordinates and even by cylindrical coordinates.

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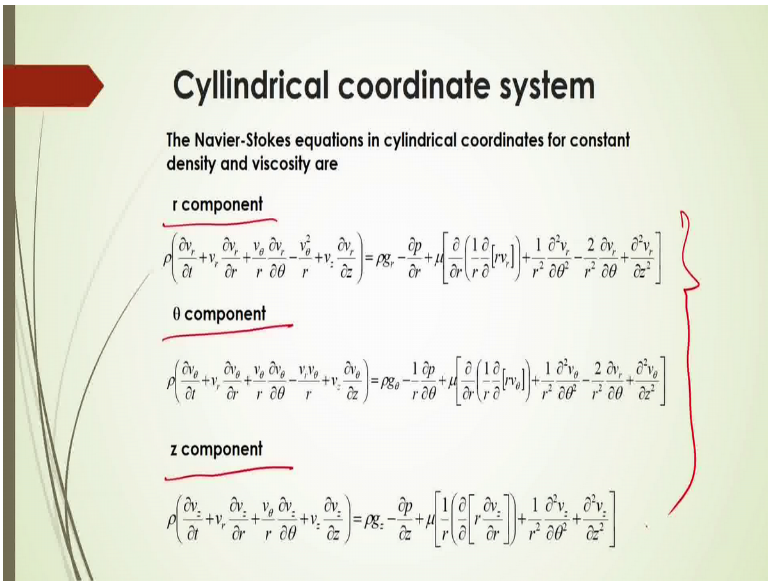

And other components of this here cylindrical coordinates for this Navier stokes equation is are component, theta component and the z components will be represented by this equations.

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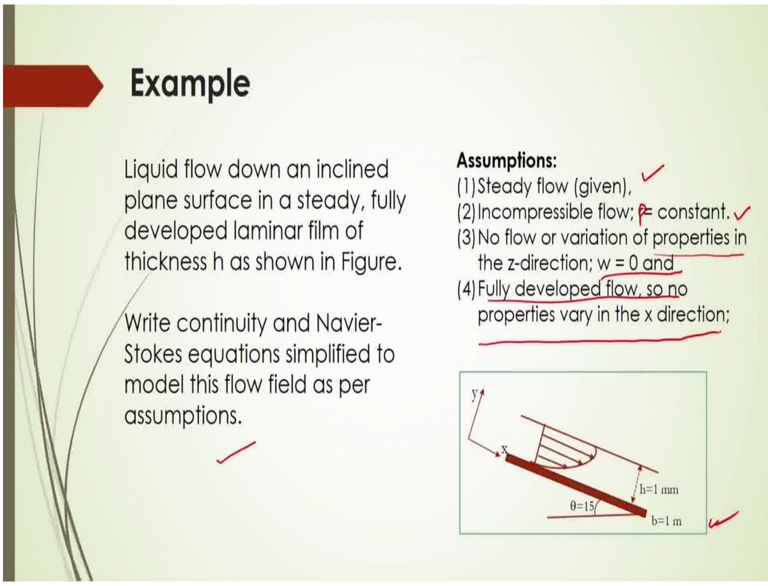

And like one examples, let us have an example like if any liquid flow down ward through an inclined plane of surface in a steady fully developed laminar film of thickness is as shown in figure here in this. And in this case, can you write the continuity and Navier stokes equation simplified to model this flow field as per assumptions here; what is that assumptions here? It is a steady state flow and incompressible flow in that case of course, density will be constant and here no flow or variation of properties in the z direction. So, w is equal to 0 and fully developed flow. So, no properties vary in the x directions there.

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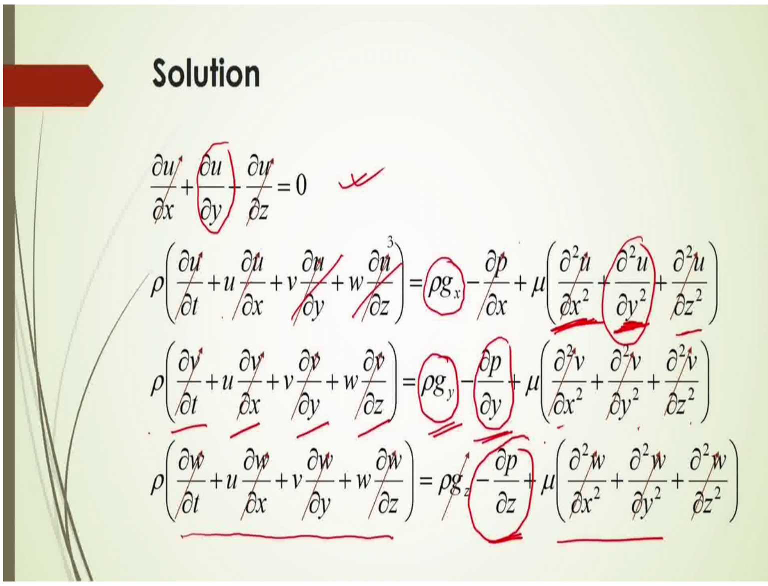

So, if we apply this Navier strokes equation based on these assumptions,we can have this dou u dou x will be equals to 0 and dou u dou z will be equals to 0. So, only this terms will be remain there in the continuity equation. So, it will be coming dou u dou y will be equals to 0. Other terms in this case here you will see this terms will be 0, this terms will be 0, this terms will be 0, this terms will be 0 as per assumption. So, here only thing is that rho g x in the x direction, what will be the gravitational force components.

And here, dou p dou x will be equals to g r here 0 because there will be no change of pressure in the x direction. And there will no what is that here viscous force in this x direction, but there will be viscous force acting over this y direction; that means, here there will be velocity gradient in the y directions due to this viscosity and that is why this components will be remain same in the x direction.

And other, in the y and z directions here, all other terms are here like this one will be cancelled, this one will be cancelled, this one will be cancelled and in the y direction of course, there will be a gravitational force. So, you have to consider this and also in the y direction, in this case there will be a in the y direction, there will be a pressure gradient so that you have to consider. Other terms will be neglected.

And here in the z direction, no other things will be considered here, but in the z direction, dou p dou z will be considered here in the z directions. Here also, you can say in the z direction, there will be a certain change of pressure. So, this one will be cancelled out if you are not having any pressure gradient in the z direction, then you can also neglect this dou p dou z here.

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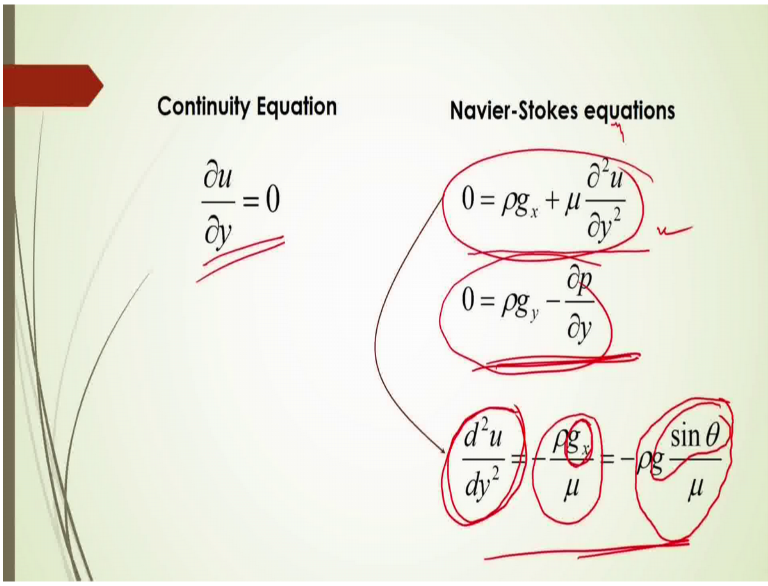

So, finally, continuity equation you are getting this one, dou u dou y will be equals to 0 and as per Navier stokes equation. After simplification, we are getting here rho g x plus mu in to dou u dou y that will be equals to 0. Similarly, here 0 will be is equal to rho g y minus dou p dou y this one. So, this two equations based on the Navier stokes equation. And finally, from this equation, you can have this dou d 2 y by d y square that will be minus rho g x mu.

Now, this g x in the x direction as per diagram here, the x direction what will be the here, we are considering this x direction in this here the that is parallel to the flow and perpendicular to the flow that is y direction. So, based on this, what should be the components in the y direction, what will be the velocity components or pressure components or gravitational force components there that you have calculate. If I consider this what is that g x here, in this x direction what will be the g x that will be g into sin theta they are.

So, ultimately, we are getting rho into g into sin theta by mu that will be your Navier stokes equation based on this x directional flow. So, this will be your change. So, from this equation, further you can calculate to have the velocity distribution and from this you can have the a pressure also by this equation.

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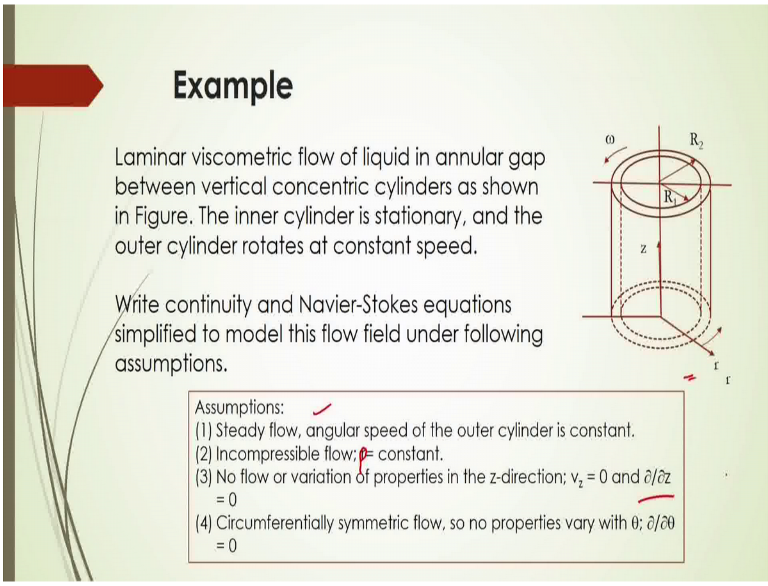

Another examples, if it is suppose cylindrical coordinates, how it will be there?

If suppose the any laminar visco matric flow of liquid in annular gap between vertical concentric cylinders as shown in figure there, the inner cylinder is to be considered as stationary and the outer cylinder rotates at a constant speed there in this case. While this flow of this outer cylinder, there will be a some viscosity acting over the surface of this inner cylinder. So, in this condition, you have to actually simplify the continuity and Navier stokes equation based on this assumption given below here.

It will be considered as a steady flow and angular speed of the outer cylinder is constant there and the flow is what is that incompressible; that means, rho is constant. And also, there will be no flow or variation of the properties in the z direction. So, v z and rho dou z will be equals to 0. Similarly, circumferential if you are considering that symmetric flow, then there will be no change of properties that will vary with the theta direction. And so, dou dou theta will be equals to 0.

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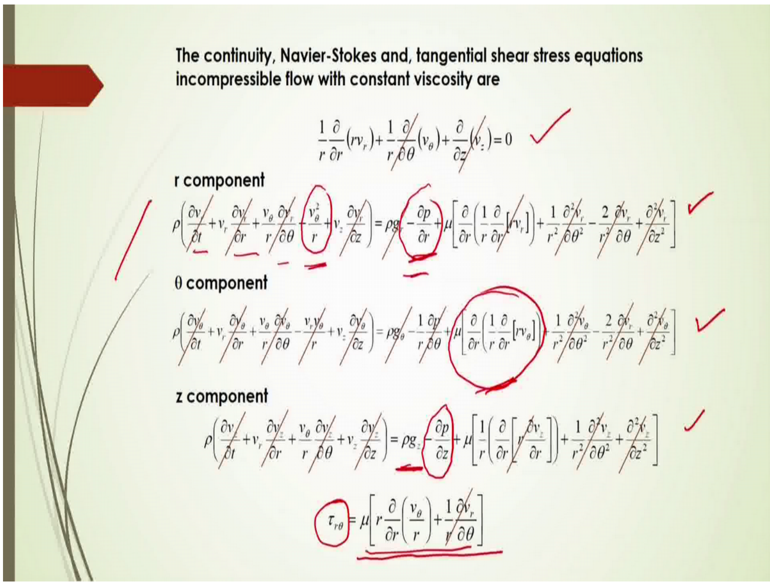

So, under these assumptions, we can have this based on the cylindrical coordinates of this continuity and Navier stokes equations will be continuity equation and this Navier strokes equation were the r component and this theta component and z this marks is actually can considering that here this marks this is 0 based on this assumptions. So, please go through this again and see. You will see how this rho v r dou t will be equals to 0 this based on this assumption.

But here, the theta direction there will be a velocity change with respect to r. So, you have to consider that things. And other persevere with respect to r, that the pressure also will be changing. So, that part also you have to consider here. And other things here in this components in the y theta components here, so this viscous terms will be considered here and in the z directions only this gravitational force and this what is that pressure force in the z directions will be considered.

And this shear stress in r and theta directions, then there will be shear stress here. So, based on which you can calculate further what will be the velocity distribution. Those problems this a velocity distribution pressure change all this things will be actually represented in the next class with an example also.

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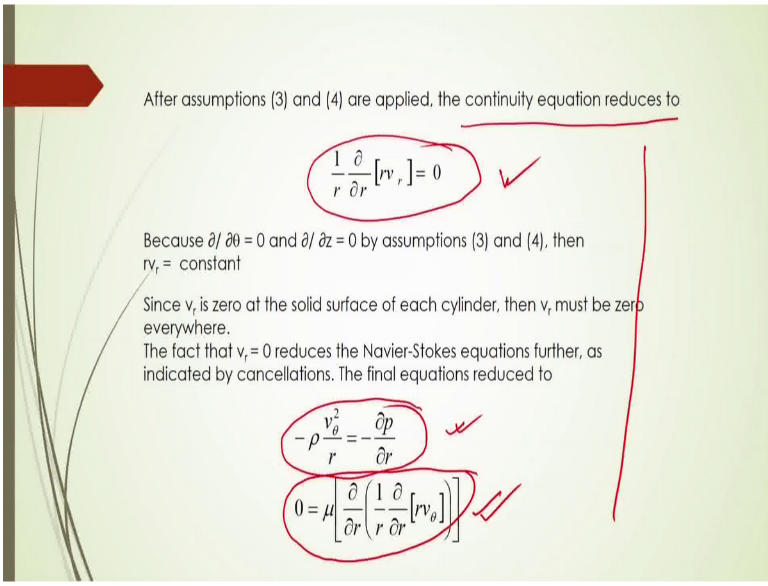

So, here finally, we can get the continuity equation that will be reducing after simplification this equation and this here this is the pressure and this will be your viscosity; based on viscosity, v theta component.

And here, based on this, we can then further calculate the what is that to obtain the governing equation for this what is that final form of equation for pressure and the components of the velocity in the r and theta direction.

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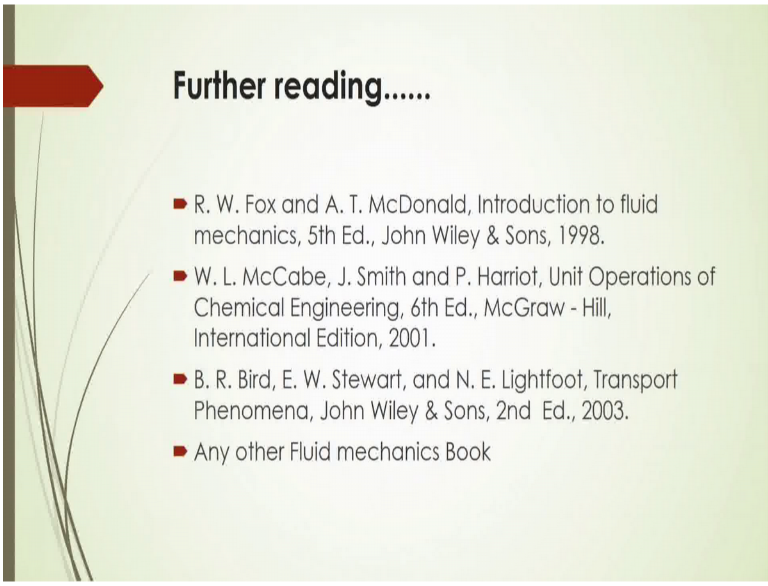

So, I will suggest to practice this lecture again and again to understand to understand the governing equations how it has come and also try to just imagine how you can actually write this Navier stokes equation here. Without just seeing all this here, just try to practice here, try to write yourself whether you can write the Navier strokes equation yourself or not.

And then, for a particular problem in the horizontal direction if there is a flow, then how this Navier stokes equation can be applied, what will be the components, whether you can write or not that you have to apply. And just based on your own understanding ok, just you can write you try to cancel that what is that part of velocity components or pressure components x, y and z directions. Then, you will get the simplified equation. After that, you can go further for calculation.

Bust, but based on these Navier stokes equation, you can I think I have this governing equation for any flow there, that will be flowing through the pipeline channel or any other things. So, this is basically, the lecture is basically the Navier strokes, how it is derived and what will be the form of this Navier strokes equation that will be here to be a learnt. So, I think you can read further for more understanding of this Navier stokes equation from this text books also and also practice it ok. So, that is for all today's lecture.

Thank you.