Transport Phenomena of Non-Newtonian Fluids
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Lecture - 08
Rotational Rheometers

Welcome to the MOOC's course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Rotational Rheometers. Before going into the details of rotational rheometers what we will be having now? We will be having a recapitulation of what we have studied in last three classes.

In last three lectures what we have seen? When we use capillary viscometers how to obtain the equations for the shear stress and shear rate in terms of a measurable parameters, measurable quantities such as pressure drop and then volumetric flow rate and those things we have seen.

And then also we have seen possible sources of errors when we apply or when we use the capillary viscometers in order to know the rheology of an unknown fluid. So, when you apply the capillary rheometer for a rheology measurement of time independent non-Newtonian fluids then you can obtain shear stress using this equation.

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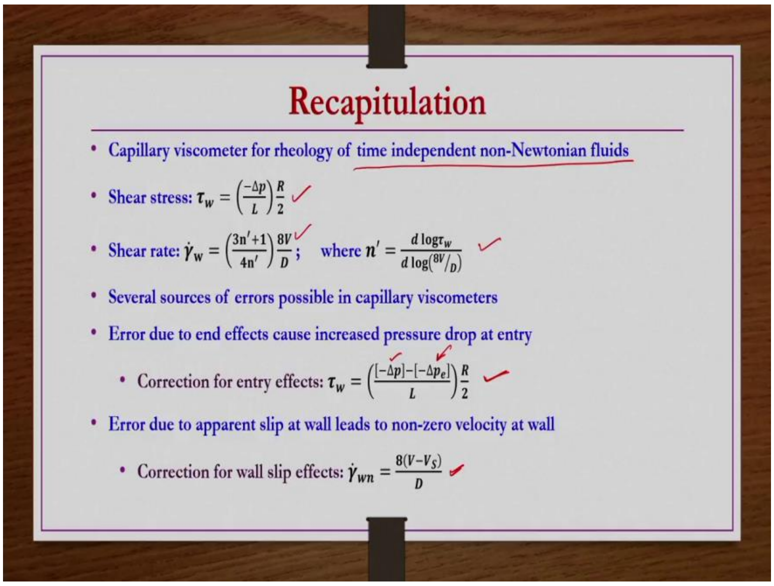

If you know the pressure drop using this equation you can know the shear stress; if you know the volumetric flow rate or average velocity you can know the apparent shear rate $\frac{8V}{D}$ and then from that true shear rate you can get it as $\left(\frac{3n'+1}{4n'}\right)\frac{8V}{D}$, where n' is nothing but $\frac{d\log\tau_w}{d\,log\left(\frac{8V}{D}\right)}$.

Remember, this is all for time independent non-Newtonian fluids we have done right. Then n number of sources of errors are possible in using the capillary viscometers for rheological measurements of unknown fluids.

Some errors you know can be avoided or reduced by just doing some kind of adjustment in operational or design conditions; however, errors due to end effects cause increased pressure drop at entry; entry is nothing but the connecting point of a capillary to the barrel. To the barrel bottom of the barrel we are connecting the capillary of a certain diameter.

So, then this capillary diameter is very small compared to the diameter of the barrel. So, because of that one sudden contraction in the cross section area is taking place because of that one pressure drop increases at the entry that is what known as the entry effect right.

So, then we have seen a method to find out how much pressure drop has increased at that entry that we found out using certain method that we have already discussed and then that pressure drop we are subtracting from the measured pressure drop of $-\Delta$ p.

Then, we are using the same relation $\tau_w = \frac{-\Delta p}{L}\frac{R}{2}$, but only change that in place of $-\Delta$p we will be using $[-\Delta p]-[-\Delta p_e]$ subtracting the increased pressure drop at the entry ok. This takes cares about the entry effects; end effects are found to be very negligible to worry about. So, that we have not discussed then error due to apparent slip at the wall leads to non-zero velocity at the wall.

At the wall in general what we assume? We assume no slip velocity, but you know for the case of non-Newtonian multi phase suspensions emulsions etcetera; at the wall there would be substantial velocity, slip velocity would be there. Because of that one whatever the shear rate that you measure you get lesser than the expected shear rate when you assume no slip boundary condition that is because of the slip of the fluid at the wall, right.

So, we have seen a method how to find out slip velocity at the wall and then that velocity we are subtracting from the average velocity to get the correction for wall slip effects in apparent shear rate. This is also apparent shear rate only ok in the apparent shear rate we are making this correction and then later on we can follow the same method of finding out that n prime and all that to get the true shear rate. So, this is what the summary of our last three classes.

Why are we discussing? Because this content is also related to the topics that we are going to discuss in this week. In this week, we are going to discuss rotational rheometers where the deformation is caused by allowing the flowing to rotate; the geometry has been taken such a way that you know the rotation of the fluid is taking place.

Because of that rotation deformation is taking place and then once deformation is there, then you can find out the shear rate and then shear stress; those things we are going to discuss. Because of the rotation there will be there would be torque also.

So, if you experimentally measure the rotational velocity versus torque then that information you may use in order to get required D in order to get the required shear rate and shear stress respectively. In addition, we will also be having some discussion on how to measure the normal stress differences as well which we have not done in the case of capillary viscometers. So, one of the earliest rotational viscometer was having the concentric cylinder geometry. So, we start with that one Concentric Cylinder Rheometer.

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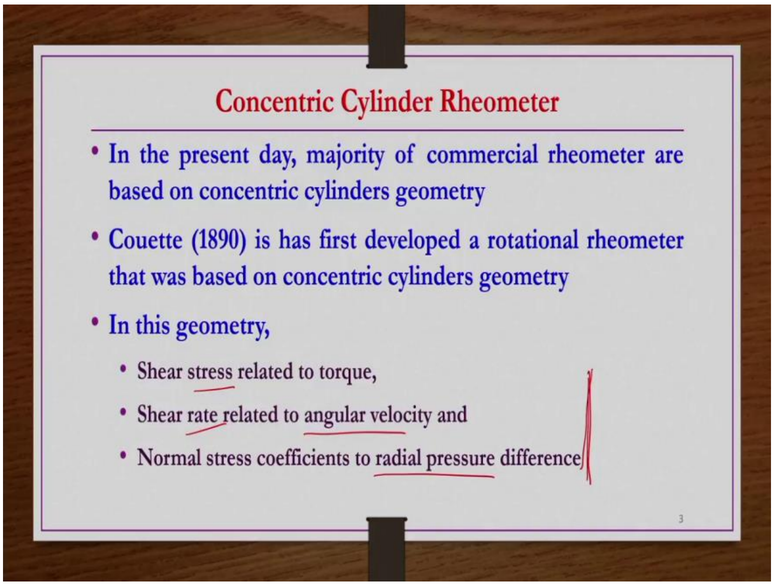

This is one of the earliest probably the first rotational viscometer developed by the Couette which is based on the concentric cylinder geometry; however, the present day, majority of commercial rheometers are also based on the concentric cylinders geometry ok.

So, here in this particular lecture what we will do? For this geometry, we will have an analysis where we can obtain the shear stress, shear rate and then normal stress differences using the measurable parameters such as the rotational velocity and then torque, etcetera those kind of thing that is what we are going to see in this lecture.

To be specific, we will be relating shear stress to torque and then we will be relating shear rate to angular velocity and then normal stress coefficients to radial pressure differences. How? That is what we are going to see.

Now, like in capillary viscometers, shear stress was related to the pressure drop; now here it is related to the torque; in the case of capillary viscometer, shear rate was related to the volumetric flow rate or average velocity, but now here in the concentric cylinder geometry or rotational geometry shear rate is related to the angular velocity whereas, the normal stress differences or normal stress coefficients in the case of rotational geometry are related to radial pressure differences.

So, that is what we are going to see. So, now before going into derivation of that equation, what we will be having? We will be having a geometry and then see how these cylinders are placed and then what is the gap between the cylinders etcetera all those things we see schematically.

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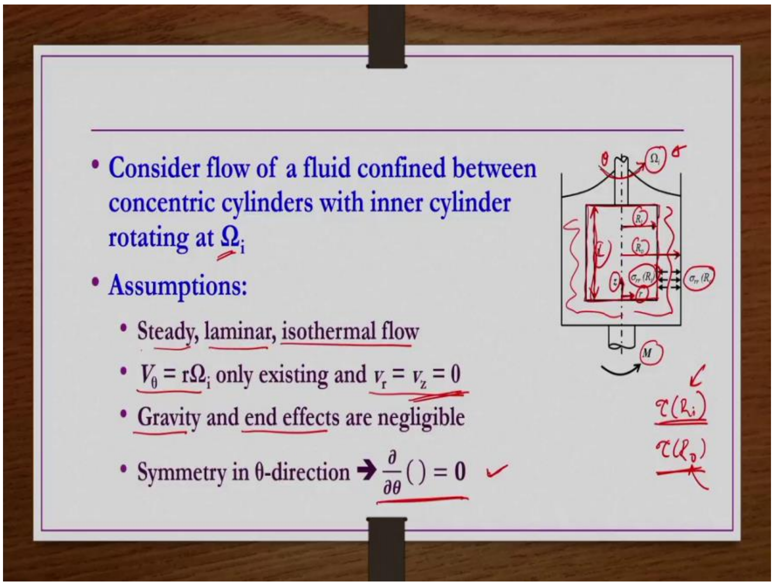

Consider flow of a fluid confined between concentric cylinders with inner cylinder rotating it Ω_i. What we have done? We have taken two cylinders right; they are arranged confined in such a way that there is a concentric analyse region is there within that concentric analyse region the flow is fluid is rotating.

How it is rotating? Because inner cylinder is rotating at Ω_i velocity whereas, the outer cylinder has been kept stationary like this. So, this is your inner cylinder inner box this one we are having two dimensional diagram here. So, this is your inner cylinder; the radius of inner cylinder is R_i; the radius of outer cylinder is R_0 ok; the height of the cylinders is L right, fine.

So, the inner cylinder is rotating it Ω_i. So, that arc is measured on this inner cylinder is M or M_i let us take ok. So, whatever the fluid is there that is confined between these two cylinders right.

Now, this coordinate system are selected such a way that this is r direction and then vertical direction is z direction the rotation is taking place in the θ direction so that the V_θ component of velocity would be there the rotation in the θ direction is taking place at rotational velocity Ω_i ok.

Let us say, stress at the inner cylinder is a σ_{rr}, the same at outer cylinder is σ_{rr} at R_0. So, now, in this lecture so many places we may be having $\tau(R_i), \tau(R_0)$ like this. So, this is not like you know kind of function because R_i, R_0 are constant, but it indicates that they are measured at so and so locations at R_i and R_0 locations ok.

So, further assumptions that we have, the flow is steady laminar flow and then isothermal flow conditions are there; temperature variations have not been taken into the consideration and then what we have seen the inner cylinder is rotating in θ direction. So, the predominant velocity component is the V_θ component right.

Out of V_θ v_r, v_z V_θ magnitude wise V_θ would be very much higher compared to v_r, v_z. So, then what we can say? In comparison with V_θ v_r v_z both of them are equals to 0. In some cases, if you have a kind of viscous I mean viscoelastic fluid, so then you the rising of the fluid in the z direction may also be taking place. So, then under such conditions v_z may not be 0. So, those things also we are going to see anyway.

So, only component of velocity is existing V_θ component which is nothing but r Ω_i gravity and end effects etcetera we are not taking under consideration right. Further symmetry in theta direction; so, $\frac{\partial}{\partial\theta}()$ of anything is 0, that is what we are taking.

Symmetry in the sense symmetry in θ direction in the sense between 0 to 180° whatever the flow distribution or velocity distribution etcetera is there the and exactly the same velocity distribution would be there from theta is equals to 180 to 360° as well ok.

So, now these conditions we apply to continuity and momentum equation in order to get a some simplified equations which we can use to obtain the shear stress, shear rate and the normal stress differences right. So, before going into these details what we have in the case of capillary rheometer?

We have done a balance, proper balance for the pressure distribution like you know for the you know we have done a proper balance by taking a fluid element A B C D like that and then we applied what are the pressure forces at the entry, what are the pressure forces at the outlet and then the difference is balanced by the shearing forces like that we have done and then we obtained the required expression for the so, for the shear stress; that is what we have done.

And then for the shear rate what we have done? We have found you know $dq = 2\pi R v_z dr$ and then we have integrated like that we got it right. So, now they say we need not to go such kind of analysis everywhere for every geometry because it is a PG level course what we will be doing? We will be using the momentum equation.

Continuity and then momentum equation and then when we do take the heat transfer problem we will be using the energy equations because these continuity, momentum, energy equations or species concentration equations whatever we have seen in your UG level transport phenomena, they are generalized balance equation.

So, they are valid for any problem, but according to problem you have to apply the constraints limitations of the problem and then simplify those equations to get the final simplified equation, usable equation right. So, that approach mostly we are going to follow to solve the problems in the coming weeks.

So, but; however, we start from now onwards for this lecture itself we start a using momentum equation continuity equation to simplify them as per the requirement of the problem and then solve those equation to get the required shear stress distribution or whatever required information ok. So, let us start simplifying the continuity and then momentum equation in cylindrical coordinates.

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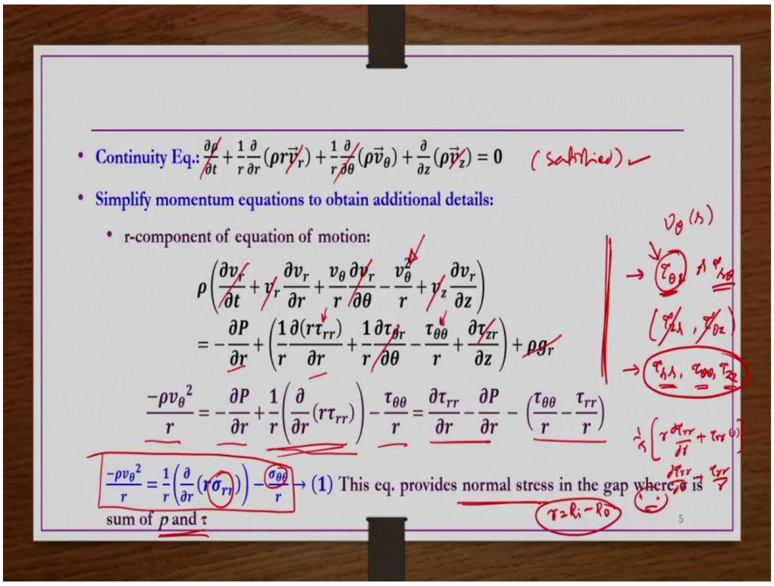

When you write continuity equation in cylindrical coordinates we have $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r \vec{v}_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \vec{v}_\theta) + \frac{\partial}{\partial z}(\rho \vec{v}_z) = 0$. So, if your velocity profile whatever you develop by solving the problem that should satisfy this continuity equation then only we can say the developed solution is reliable otherwise not; that is one point of simplifying the continuity equation.

Other important point of continuity equation, sometimes we may not able to get some kind of conditions directly from the flow geometry like whether the flow is symmetry or not whether the flow is fully developed or not those kind of things we may not be able to realize quickly from the geometric given schematic or given the problem statement.

So, then one or other conditions you may get from by simplifying the continuity equation also, but; however, now in our case that is not required. So, because of the steady state first term is 0 compared to $\vec{v}_\theta$ $\vec{v}_r$ and $\vec{v}_z$ are 0 and then because of the symmetry in theta direction $\frac{\partial}{\partial\theta}$ of anything is 0; that means, all terms are negligible.

So, then we continue. So, then we can say that the continuity equation is satisfied; continuity equation is satisfied right. So, now, what we do? We simplify momentum equation, different components of momentum equations like r component, θ component and z component of momentum equation.

So, r component of equation of motion $\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial P}{\partial r}+\left(\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta}-\frac{\tau_{\theta\theta}}{r}+\frac{\partial\tau_{zr}}{\partial z}\right)+\rho g_r$ this is what we have.

So, this continuity and then momentum equations we are going to derive them in subsequent week anyway, but; however, let us take them for granted because we know these equations already from our previous UG level TP course Transport Phenomena course. Now, before simplifying this equation further the geometry that whatever we have seen. So, v_θ is only existing and then it is function of r and then what we have seen?

We have seen that only shear stress existing is $\tau_{\theta r}$or $\tau_{r\theta}$ this is the only shear stress is there and then both of them are same for the case of laminar flow anyway. So, then we do not need to worry whether it is $\tau_{\theta r}$or $\tau_{r\theta}$. So, if it is a flow is laminar and then symmetric then both of them are same ok.

So, now this is the only shear stress existing that is τ_{zr} and then other components of $\tau_{\theta z}$ etcetera these things are not existing, but normal stresses τ_{rr} rather saying the normal stresses normal viscous stresses let us call them extra stresses extra component of shear stress. So, τ_{rr} $\tau_{\theta\theta}$ τ_{zz}we are not cancelling out because we are also doing this analysis for the case of viscoelastic fluids; if the fluid is having some elasticity.

So, then these components would also be important. So, we cannot cancel out right. So, blindly we cannot cancel out these extra stresses and then this shear stress $\tau_{\theta r}$ ok. So, why this is important now? This comes from the schematic, this comes from the problem statement and then schematic then you can realize it ok. So, these conditions we apply to the momentum equation so that to simplify them ok. So, now, first term is 0 because of the steady state condition v_r is 0, then v_θ is not 0.

But $\frac{\partial}{\partial\theta}$ of anything is 0 because of the symmetry conditions; v_θ is existing so, we cannot cancel it out ok, v_z is 0 compared to v_θ. So, that term is anyway 0. So, left hand side you are we are having only one term $-\rho\frac{v_\theta^2}{r}$, pressure we do not know anything.

So, let us keep it as it is then the normal stresses or the extra stresses for the case of non-Newtonian fluids τ_{rr} $\tau_{zz}$and then $\tau_{\theta\theta}$ we cannot say whether they are negligible or not unless if you know the nature of the fluid. So, we cannot cancel out this term right and then $\tau_{\theta r}$ is existing it is non-zero quantity, but because of the symmetry $\frac{\partial}{\partial\theta}$ for anything is 0. So, that this term is 0.

So, $\tau_{\theta\theta}$ should be there because we are also doing the simplification for the fluids which are having the elastic behaviour right; τ_{zr} this component of shear stress is not there for the flow geometry that we have taken and then we mentioned at the beginning that we are not taking gravity and end effects into the consideration.

So, then what we have? This equation $-\rho\frac{v_\theta^2}{r} = -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} - \frac{\tau_{\theta\theta}}{r}$. So, this if you expand you can write it as like you know $\frac{\partial\tau_{rr}}{\partial r} - \frac{\partial P}{\partial r} - \left(\frac{\tau_{\theta\theta}}{r} - \frac{\tau_{rr}}{r}\right)$ when you differentiate this equation you have $\frac{1}{r}\left[r\frac{\partial\tau_{rr}}{\partial r} + \tau_{rr}(1)\right]$.

So, then we are having $\frac{\partial\tau_{rr}}{\partial r} + \frac{\tau_{rr}}{r}$. So, $\frac{\partial\tau_{rr}}{\partial r}$ is combined with $-\frac{\partial P}{\partial r}$ as one term and then whatever the $\frac{\tau_{rr}}{r}$ is there that is combined with $\frac{\tau_{\theta\theta}}{r}$ to write it as $\frac{\tau_{\theta\theta}}{r} - \frac{\tau_{rr}}{r}$.

Why we are writing? Because we know this whatever. So, called τ and p are there summation of these two we can write them as a sigma that is what we have already seen right; so, in the first lecture.

So, this equation we can write it as $\frac{-\rho v_\theta^2}{r} = \frac{1}{r}\left(\frac{\partial}{\partial r}(r\sigma_{rr})\right) - \frac{\sigma_{\theta\theta}}{r}$ right. So, now this equation we are having $\sigma_{rr}$and $\sigma_{\theta\theta}$ term. So, then this equation would provide some information about the normal stresses in the gap between r is equals to R_i to R_0 ok.

So, that is what we are going to see further anyway whereas, σ is nothing but sum of p and τ here fine ok. So, let us take this let us keep this equation as of now as it is without going further simplification we do it later subsequently.

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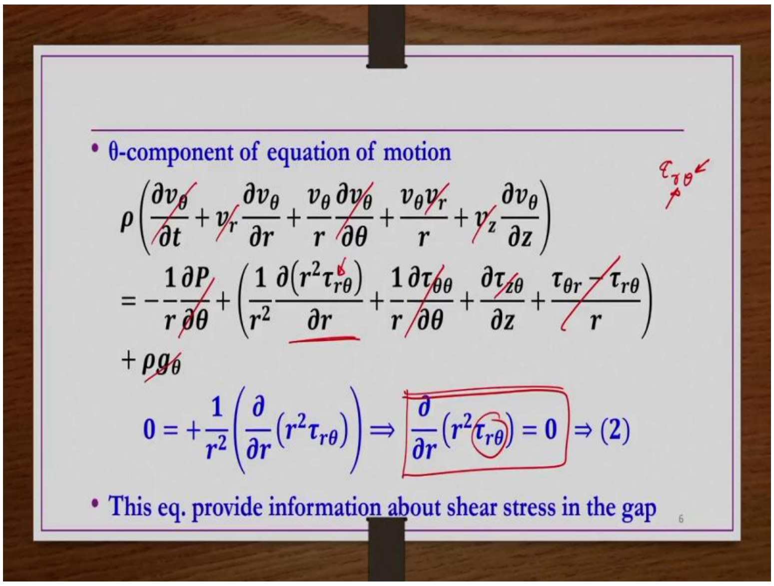

So, now, θ component of equation of motion if you write down these are available any of the transport phenomena books. So, now here first term is 0 because of the steady state condition compared to v_θ, v_r is 0, v_θ is not 0, by $\frac{\partial}{\partial \theta}$ of anything is 0 because of the symmetry v_r is 0 and then v_z is 0. So, the left hand side all the terms are negligible right.

So, pressure we can we do not know anything, but what we can say because of the symmetry this is also 0 and then this $\tau_{r\theta}$ is existing or $\tau_{\theta r}$ is existing $\tau_{r\theta}$that we are taking because θ is the direction of the flow, r is the surface normal to the θ direction. So, $\tau_{r\theta}$ is existing right, but it is function of r as well. So, then we cannot cancel out this term.

So, it will be as it is $\tau_{\theta\theta}$ we cannot cancel out because of symmetry we can cancel out $\frac{\partial \tau_{\theta\theta}}{\partial \theta}$. Similarly, what we have? We do not have other component of shear stress other than tau r theta. So, this term is 0 and these two terms are equal to each other at least for the laminar symmetric laminar flow conditions ok and then there is no gravity. So, last term is also 0.

So, what we get here? We get $\frac{\partial}{\partial r}(r^2 \tau_{r\theta}) = 0$, that is what we get from this equation. And then this equation if you solve what you will get? You will get a relation for a $\tau_{r\theta}$ or you get an equation for $\tau_{r\theta}$ that is shear stress is required; $\tau\ \dot{\gamma}$ and then n_1, n_2 only we are going to find out. So, this equation will give you information about the shear stress.

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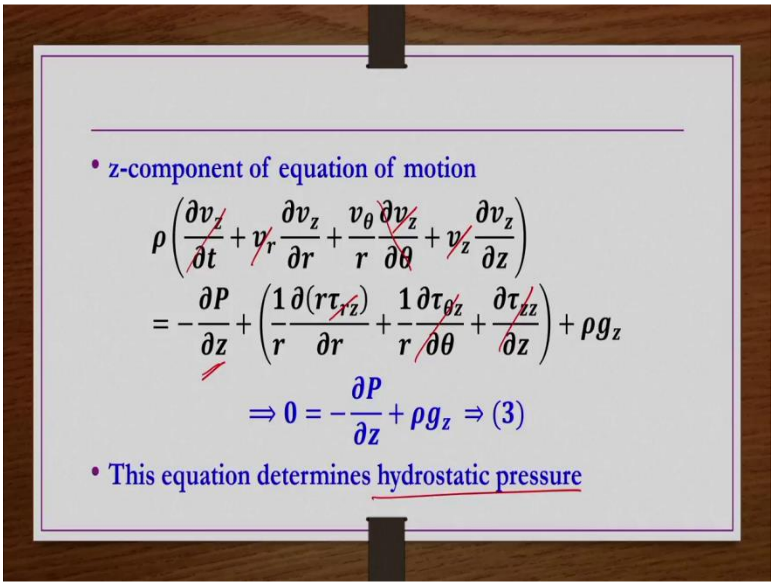

Now, similarly z component of equation of motion if you take the first term is 0 because of steady state assumption right. So, the second term v_r is 0 compared to v_θ; v_θ is existing, v_z is not existing or it is 0 compared to v_θ or $\frac{\partial}{\partial \theta}$ of anything, any velocity component is 0 because of the symmetry and then v_z is anyway 0.

So, pressure we do not know anything about the pressure τ_{rz} is not existing $\frac{\partial}{\partial \theta}$ of anything is 0 right. So, combinedly we do not want it to make a normal stress and then pressure combined together in the z direction, but we wanted to combine in the radial distribution direction that we by the equation 1.

So, now this term is also 0 that we are not taking. So, now, we get $\frac{\partial p}{\partial z} = \rho g z$; this will give the information about the hydrostatic pressure right. Now, by simplifying equation of motion, three components of equations of motion in cylindrical coordinates we got three equations, we use those three equations to get some more information ok in term some more information on shear stress, shear rate and then normal stresses.

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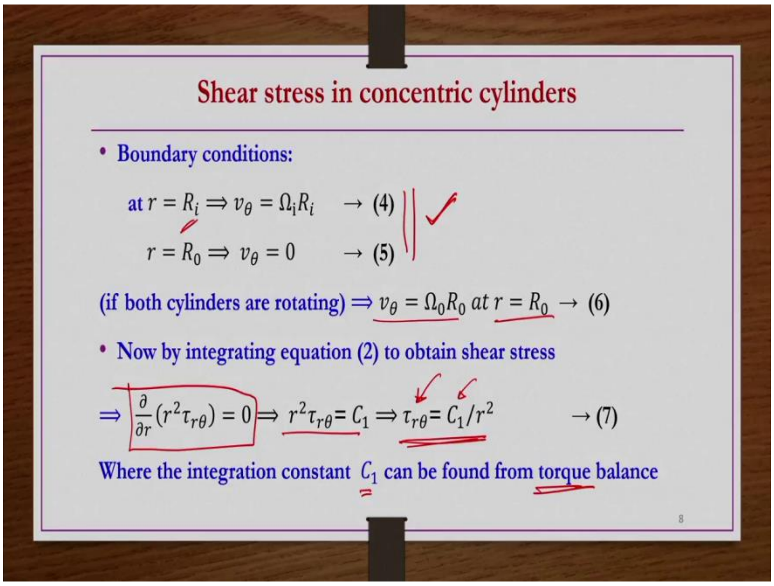

So, shear stress in concentric cylinders. So, boundary condition what we have inner cylinder is rotating inner cylinder is rotating at Ω_i angular velocity. So, then at r = R_i, v_θ should be R_i Ω_i, but outer cylinder at r = R_0, but outer cylinder is stationary. So, at r = R_0 v_θ should be 0, but it is also possible that both the cylinders may also be rotating.

So, let us say outer cylinder is also rotating at a different velocity than Ω_i, let us take outer cylinder if it is rotating at Ω_0 then the boundary condition at r = R_0 should be $v_\theta = \Omega_0 R_0$, but we are taking only this condition for this problem right. So, let us not worry about the both cylinders were in case.

Now, equation 2 what we had? We had $\frac{\partial}{\partial r}(r^2\tau_{r\theta}) = 0$; if you integrate this equation $r^2\tau_{r\theta} = C_1$ that is $\tau_{r\theta} = \frac{C_1}{r^2}$. So, now, you have an expression for $\tau_{r\theta}$, but unfortunately you cannot use directly because this C_1 is not known and then boundary conditions also we cannot say what is the shear stress at R_i what is the shear stress at R_0 we cannot know alright.

So, we know that some maximum shear stress or minimum shear stress like, but how much it is not that we could not know, but that is not an issue. So, let us not worry about that constant; we can find it out through experimental results. So, C_1 can be found from the torque balance that we know from torque because torque we find it from the experimental results.

All this analysis we are trying to do such a way that we relate the shear stress and shear rate to measurable quantities like torque and an angular velocity or rotational velocity those kind of thing only right. So, now what we do? We do the torque balance so that to find out the expression for a shear stress.

Now, but one important thing that you can see here, this shear stress is inversely proportional to the square of the radial position right. So, whereas, in the capillary rheometers what we have found? Shear stress is directly proportional to the radial position ok. So, that is the region in previous week in all the lectures I was mentioning whatever the analysis that we are doing the shear stress, shear rate expressions are valid only for the capillary tubes only not for all the geometries ok.

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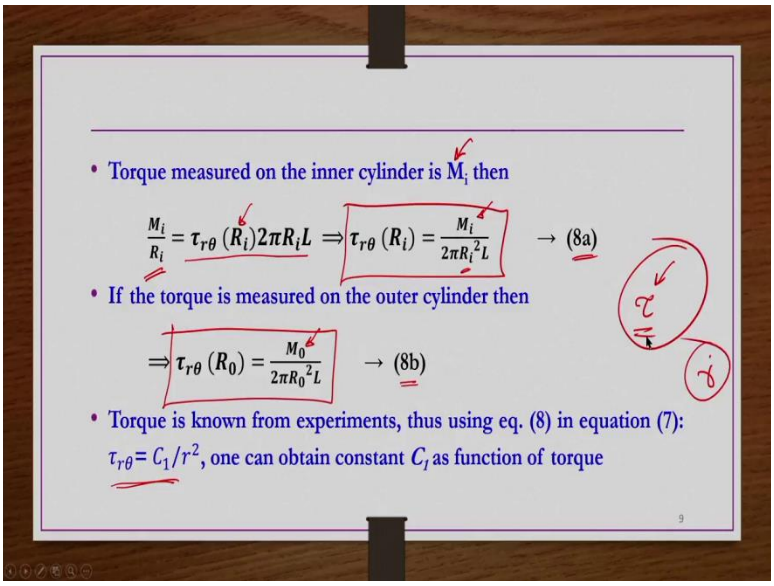

So, now the torque measured on the inner cylinder is M_i then whatever the force due to the torque is there that should be balanced by the whatever the force because of the shearing action ok. Shearing force you can get by multiplying the shear stress with the surface area of the cylinder inner cylinder or outer cylinder where are we measuring that shear that shear stress.

Let us say, if you are measuring shear stress on the inner cylinder because torque also we are able to measure only on the inner cylinder in this geometry because inner cylinder is only rotating. So, $\frac{M_i}{R_i} = \tau_{r\theta} 2\pi R_i L$, $\tau_{r\theta}(R_i)$ indicates that $\tau_{r\theta}$ is measured at the surface of inner cylinder whose radius is R_i ok it is not the multiplication ok.

So, then from here what we get? $\tau_{r\theta} = \frac{M_i}{2\pi {R_i}^2 L}$. So, here again we can see it is inversely proportional to the square of the radius of inner cylinder, because this we are measuring at the inner cylinder. And then M_i is also measured on the inner cylinder only ok so; that means, from the experiments, if you can measure the torque on in on the inner cylinder that torque you can use in order to get the shear stress on the inner cylinder by this equation.

Let us say torque is measured on the outer cylinder and then that torque if it is M_0 then $\tau_{r\theta}(R_0) = \frac{M_0}{2\pi {R_0}^2 L}$ similarly exactly same as this equation ok. So, now, torque is known from experiments thus this equation number 8, either 8a or 8b can be used in equation number 7 to find out that constants C_1 as function of torque that is even function can be found as function of torque.

So, now what we have by the end of this slide? We have information on shear stress, τ whether are you measuring on the inner cylinder surface or measuring on the outer cylinder surface we know how to measure it if you know the corresponding torque; torque on that cylinder right.

So, if the torque measured on the inner cylinder then we can use 8a equation if you are measuring the torque on the outer cylinder then you can use equation 8b. So, now we have the information about the shear stress. So, next step is to get an expression to get shear rate that is what we are going to do now.

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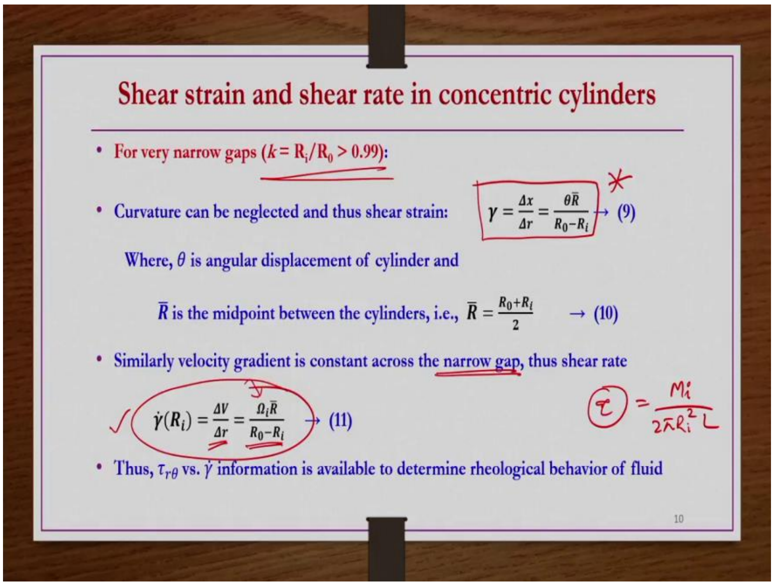

So, shear strain and shear rate in concentric cylinders. So, concentric cylinders we are taking the ratio between the radius of inner cylinder and then radius of outer cylinder plays an important role. So, if this ratio $\frac{R_i}{R_0}$ is very large; that means, both of them are almost touching to each other. So, if these two cylinders are. So, much close to each other. So, then what we can say the gap the gap is very very narrow very very narrow so, that curvature effects can be neglected right.

So, if the curvature effect is neglected then shear strain we can find out by taking $\frac{\Delta x}{\Delta r}$ and then Δx in this case is nothing but $\theta \bar{R}$. $\bar{R}$ is the position at which we are measuring this shear strain. So, $\bar{R}$ let us take the middle point between this R_i and R_0 ok. And then Δr is nothing but R_0 - R_i whereas, the θ is nothing but the angular displacement of cylinder.

Remember this is valid only that gap is very very narrow so that the curvature effect can be neglected. Here $\bar{R}$ is the midpoint between the cylinders that we are taking the average of R_0 and R_i similarly velocity gradient should also be constant across the narrow gap, if you are taking the gap is very very narrow then shear rate $\dot{\gamma}$ can be obtained by $\frac{\Delta v}{\Delta r}$; Δv is nothing but $\Omega_i \bar{R}$ ok.

Because why Ω_i? Because we are taking the inner cylinder rotating case in this case the equations are being developed for that case. So, Ω_i multiplied by some R location that R location we are taking midpoint we are taking midpoint between R_i and R_0 ok or within the concentric region whatever the midpoint is there that we are taking that is nothing but the average of R_i and R_0. Δr is again R_0 - R_i right.

So, now we also have the relation for the shear rate. So, previous slide we got a relation for a shear stress that is $\frac{M_i}{2\pi {R_i}^2 L}$; $\dot{\gamma}$ is nothing but $\frac{\Omega_i \bar{R}}{R_0 - R_i}$, $\bar{R} = \frac{R_0 + R_i}{2}$.

So, you can measure the rheological behaviour of that fluid right, but you can do that one only when this gap is narrow whatever the experiment that you are done. So, you have arranged the cylinder such a way that the gap is very very narrow that both the cylinders are almost touching to each other.

Or $\frac{R_i}{R_0}$ is greater than 0.99, then only we can say the curvature effect is negligible and then we can have this shear rate expression otherwise we have to do adjustment or you know corrections to get the reliable shear rate information ok that is what we are going to do, because for most of the commercial rheometers or the existing rheometers having k > 0.99 is not possible, but most of them are operated it k < 0.99.

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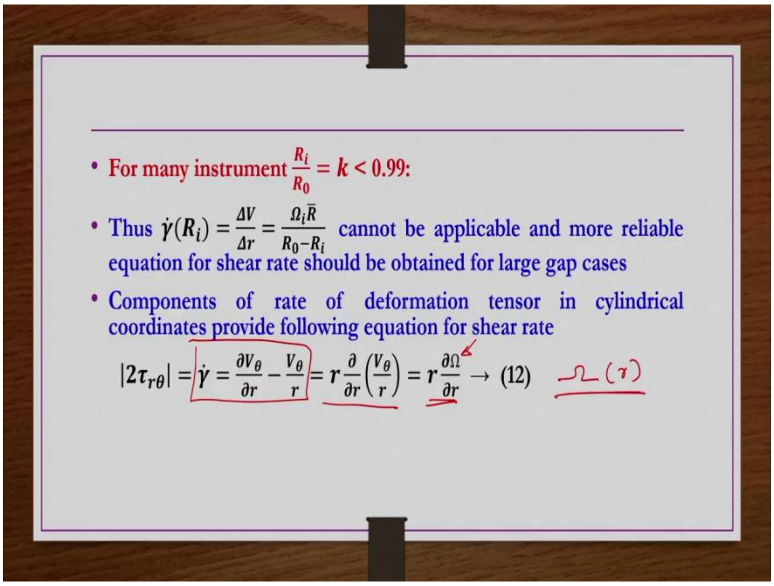

For many instruments $\frac{R_i}{R_0}$ that is k < 0.99 in general. So, whatever this $\dot{\gamma}\ (R_i) = \frac{\Omega_i \bar{R}}{R_0 - R_i}$ that we derived in the previous slide that cannot be used and more reliable equation for shear rate should be obtained for large gaps, because if the gap is larger so, then you cannot avoid the curvature effect; when you avoid the curvature effect then only you can use this equation.

So, what should we do for larger gap cases? We know the components of rate of deformation tensor in cylindrical coordinates. So, for that if you have transport phenomena books then we can get $\dot{\gamma}_{r\theta}$ is nothing but $\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}$.

The same thing we can write $r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$. So, $\frac{v_\theta}{r}$ is nothing but Ω or to be specific Ω_i in our case. So, this $\dot{\gamma}$ is nothing but $r\frac{\partial \Omega_i}{\partial r}$. So, now, what you got? You got an expression for $\dot{\gamma}$ also for the large gap cases which is good anyway.

But the problem with this equation is that you need to know what is this Ω as function of r, the velocity distribution, how the velocity is changing you know in the narrow gap that you should know then only you can use this equation.

Such analysis such you know such information you can get by the transport phenomena analysis, but in real life experimental conditions when you are doing the experiments you cannot measure them easily right. So, then we have to further modify this equation such a way that we do not need to worry about this omega as function of r. So, that is what we are going to do now.

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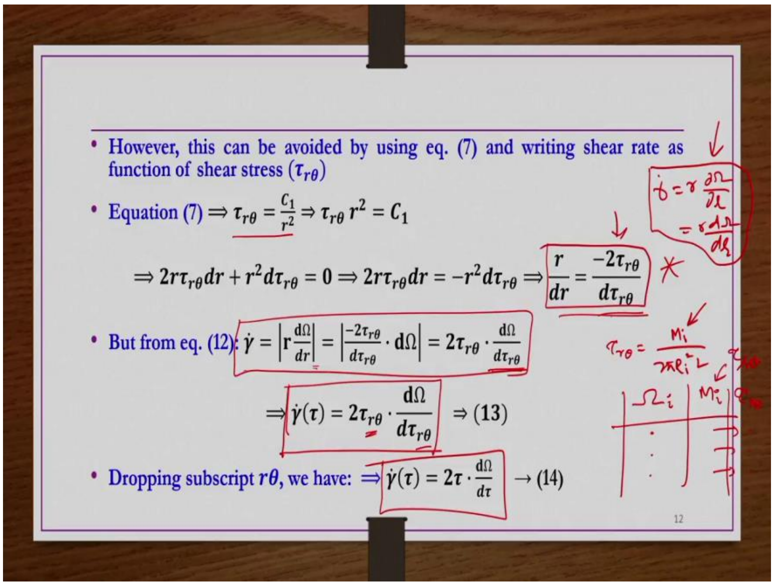

How do we do? We use equation number 7 right. So, that shear rate will we can write it as function of shear stress equation number 7 we know that $\tau_{r\theta} = \frac{C_1}{r^2}$ that is what the equation number 7. So, from here $\tau_{r\theta} r^2 = C_1$ constant; now r is also changing with radial position.

And then $\tau_{r\theta}$ is also changing with the radial position that we know. So, then what we do? We do the differentiation either side then we have right hand side 0 because that is a constant. So, left hand side $2r\tau_{r\theta}dr + r^2 d\tau_{r\theta} = 0$; that means, $2r\tau_{r\theta}dr = -r^2 d\tau_{r\theta}$ that is what we can rearrange $\frac{r}{dr} = \frac{-2\tau_{r\theta}}{d\tau_{r\theta}}$.

So, now this expression we make use in the previous equation previous slide where we have written $\dot{\gamma} = r\frac{\partial \Omega}{\partial r}$ that is $r\frac{d\Omega}{dr}$. So, in this equation wherever $\frac{r}{dr}$ is there we are going to write $\frac{-2\tau_{r\theta}}{d\tau_{r\theta}}$, but anyway this relation whatever we got we got for the modulus; $\dot{\gamma} = \left|r\frac{\partial \Omega}{\partial r}\right|$. So, in place of $\frac{r}{dr}$ I can write $\left|\frac{-2\tau_{r\theta}}{d\tau_{r\theta}}\right|$ from this equation and then $d\Omega$ as it is. So, that I can write it as after lifting the magnitude modulus then we can have $2\tau_{r\theta}.\frac{d\Omega}{d\tau_{r\theta}}$. So, this is more reliable equation because the your $\tau_{r\theta}$ you know it how do you know? You know by $\frac{M_i}{2\pi R_i^2 L}$.

So, this torque you know experimentally when you do when you operate concentric cylinder geometry what you measure? You measure this velocity and then this torque only from this torque you can get $\tau_{r\theta}$ or τ_{R_i} right and then. So, that you know. So, for different M_i corresponding $\tau_{r\theta}$ values we can find out so; that means, the for the given Ω_i value corresponding $\tau_{r\theta}$ value you know without knowing the velocity profile or without knowing the velocity distribution within the confined narrow gap.

So, that way this equation is final, you can use it anyway. However we are going to do some more simplification anyway. So, now, this what we have written the shear stress this equation whatever we had in the previous slide the same equation for the shear rate we have written as function of you know velocity distribution right.

But now the same shear rate we are writing as function of the shear stress because experimentally you cannot know velocity distribution you know the angular velocity you know the torque right you cannot know the distribution of the velocity across the r position by transport phenomena analysis you can get that is a different thing.

But all these analysis we are doing. So, that to get expression for shear stress, shear rate and the normal stresses in terms of measurable properties right like torque angular velocity etcetera. So, this equation is better compared to equation number 12 because here you need to know only Ω that is known from the experiment quantities.

And then you need to know τ also you know because torque is known from the experiment quantity from the experimentation. So, then using that torque you can find out the τ. So, this altogether information is known. So, this is the better equation for large gap concentric cylinder rheometers.

So, now this we know that only shear stress is occurring $\tau_{r\theta}$ components. So, other components are not existing. So, then we can take off this subscripts r θ. So, that we can have $\dot{\gamma}$ as function of $\tau = 2\tau. \frac{\partial \Omega}{\partial \tau}$.

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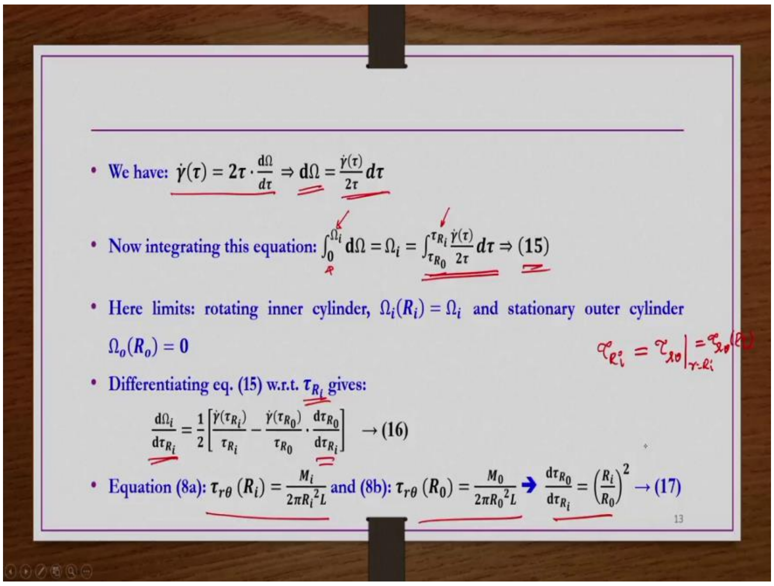

Now, we do some more simplifications. So, now, this equation if you integrate you what you do this equation you write such a way that d Ω is one side remaining terms are other side; then you do the integration from 0 to Ω_i, 0 for the outer cylinder stationary outer cylinder Ω_i inner cylinder rotating at Ω_i velocity corresponding shear stress are τ_{R_0} and τ_{R_i} right.

Then what you get? You get Ω_i is equals to integral; this integral we are not able to solve because we do not know what is this $\dot{\gamma}$ (τ). So, the next step what we are doing? We are differentiating with respect to τ_{R_i}, τ_{R_i} is nothing but $\tau_{r\theta}$ at r = Ri the same thing we have been writing as $\tau_{r\theta}$ (Ri) like this. So, they are same ok. So, different notations only are used ok.

So, now this equation 15, if you differentiate with respect to τ_{R_i}, then your left hand side $\frac{d\Omega_i}{d\tau_{R_i}} = \frac{1}{2}\left[\frac{\dot{\gamma}(\tau_{R_i})}{\tau_{R_i}} - \frac{\dot{\gamma}(\tau_{R_0})}{\tau_{R_0}} . \frac{d\tau_{R_0}}{d\tau_{R_i}}\right]$.

This is what we get simple straightforward, but now equation number 8a we got τ_{R_i} is nothing but $\frac{M_i}{2\pi {R_i}^2 L}$ and then τ_{R_0} is nothing but $\frac{M_0}{2\pi {R_0}^2 L}$so; that means, $\frac{d\tau_{R_0}}{d\tau_{R_i}}$ is nothing but $\left(\frac{R_i}{R_0}\right)^2$ right. So, this we are going to substitute here, but before that we do some more simplifications.

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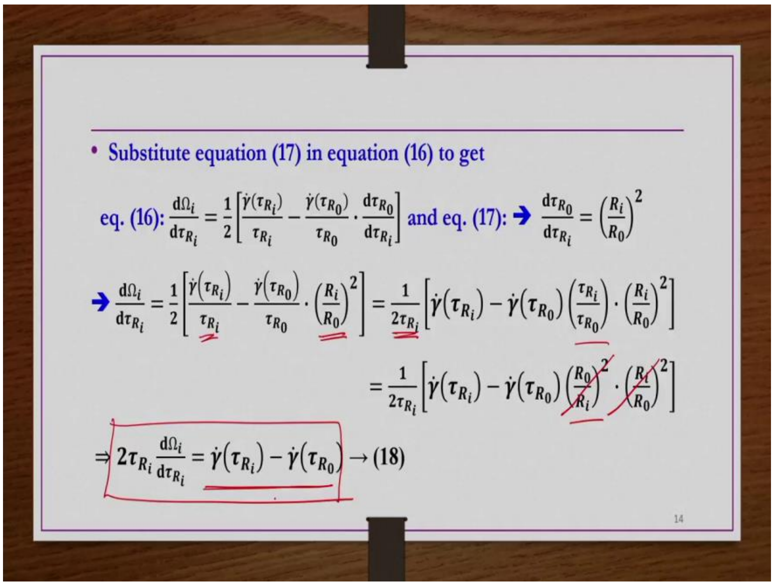

So, when you substitute here you get in place of $\frac{d\tau_{R_0}}{d\tau_{R_i}}$ you get $\left(\frac{R_i}{R_0}\right)^2$. So, next step what I am doing? I am taking τ_{R_i} common. So, that here $\frac{1}{2(\tau_{R_i})}$, I have out of the parentheses. So, that I can have $\dot{\gamma}(\tau_{R_i}) - \dot{\gamma}(\tau_{R_0})\left(\frac{\tau_{R_i}}{\tau_{R_0}}\right).\left(\frac{R_i}{R_0}\right)^2$.

But again we know that $\frac{\tau_{R_i}}{\tau_{R_0}}$ is $\left(\frac{R_i}{R_0}\right)^2$. So, then the two quantities when multiplied it will give 1 so; that means, we have $2\tau_{R_i}\frac{d\Omega_i}{d\tau_{R_i}} = \dot{\gamma}(\tau_{R_i}) - \dot{\gamma}(\tau_{R_0})$.

Now, since inner cylinder is rotating. So, at that position you know the gradients would be very strong. So, what about the outer cylinder? How much is important is $\dot{\gamma}(\tau_{R_0})$ compared to $\dot{\gamma}(\tau_{R_i})$ that is what we are going to see now.

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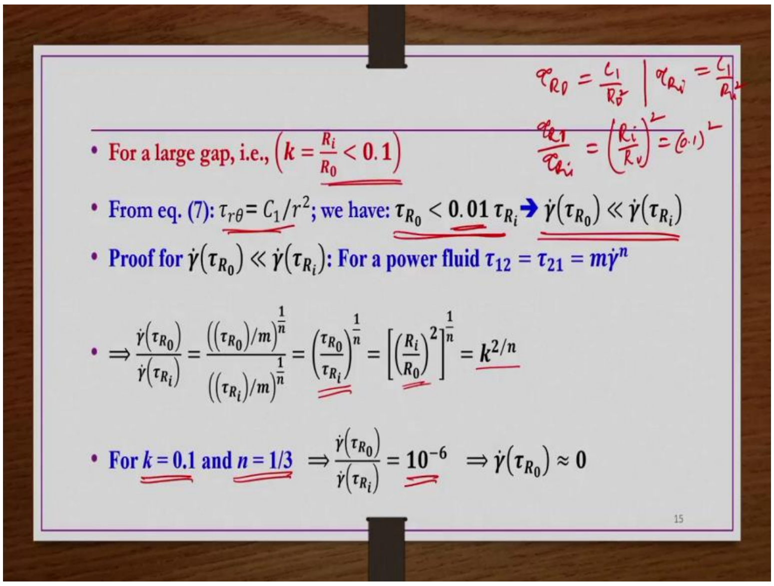

So, for large gap especially k < 0.1 right. So, when tau r when k that is very small. So, then we already know from equation number 7 $\tau_{r\theta} = \frac{C_1}{r^2}$; that means, $\tau_{R_0} = \frac{C_1}{{R_0}^2}$ and then $\tau_{R_i} = \frac{C_1}{{R_i}^2}$ so; that means, $\frac{\tau_{R0}}{\tau_{R_i}}$ if you do, you get $\left(\frac{R_i}{R_0}\right)^2$.

Now, $\frac{R_i}{R_0}$ is 0.1 the square of this one is 0.01; that means, τ_{R_0} is very much less than theτ_{R_i} because it is multiplied by the factor of 0.01. So, when it is valid, when it is valid? It is valid when $\frac{R_i}{R_0} < 0.1$ right.

So, similarly $\dot{\gamma}(\tau_{R_0})$ is going to be very much smaller compared to the $\dot{\gamma}(\tau_{R_i})$. How it is? Let us take a power law fluid. So, for which we have τ = m$(\dot{\gamma})^n$. So, $\dot{\gamma}(\tau_{R_0})$ I can write $(\tau_{R_0}/m)^{\frac{1}{n}}$. Similarly $\dot{\gamma}(\tau_{R_i})$ I can write $(\tau_{R_i}/m)^{\frac{1}{n}}$.

So, then since the fluid is same, so, this m and n would be same. So, then you are going to have $\left(\frac{\tau_{R0}}{\tau_{R_i}}\right)^{\frac{1}{n}}$ $\frac{\tau_{R0}}{\tau_{R_i}}$ is nothing but $\left(\frac{R_i}{R_0}\right)^2$. So, which is k. So, $(k^2)^{\frac{1}{n}}$. So, that is $k^{\frac{2}{n}}$; that means, if you take k 0.1 are lesser one and then n 1/3. So, then this ratio is going to be order of 10^{-6}; that means, $\dot{\gamma}(\tau_{R_0})$ is approximately 0 compared to the $\dot{\gamma}(\tau_{R_i})$.

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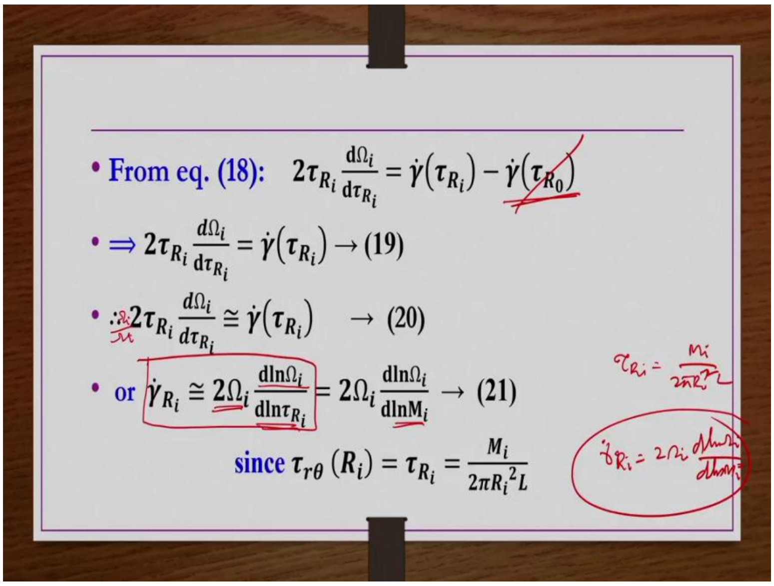

So, this condition if you use in equation number 18 here, so then this you can strike out. When you strike out, so you can have $2\tau_{R_i}\frac{d\Omega_i}{d\tau_{R_i}} = \dot{\gamma}(\tau_{R_i})$; that means, $2\tau_{R_i}\frac{d\Omega_i}{d\tau_{R_i}} \cong \dot{\gamma}(\tau_{R_i})$, this is what we are having.

So, this is the final equation or this equation you can also write it as $\dot{\gamma}_{R_i} \cong 2\Omega_i\frac{dln\Omega_i}{dln\tau_{R_i}}$, what we are doing this equation here? We are multiplying by Ω_i in the left hand side and then dividing by Ω_i.

So, d Ω_i by Ω_i I am writing $dln\Omega_i$ and then $2\tau_{R_i}$ is multiplied by 2 is multiplied by Ω_i 2 $\Omega_i$whereas, the $\frac{d\tau_{R_i}}{\tau_{R_i}}$, I am writing as $dln\tau_{R_i}$ so; that means, since $\tau_{R_i} = \frac{M_i}{2\pi R_i{}^2 L}$ so; that means, τ is directly proportional to M_i.

So, what I can write in place of $ln\tau_{R_i}$? I can write ln M_i. So, this is the final equation. This is the final equation for γ_i or γ_{R_i}= $2\Omega_i\frac{dln\Omega_i}{dlnM_i}$, because even the previous equation whatever the 14th equation number 14 whatever we have got, there also we have to convert that you know torque information has to be converted to the shear stress and then you can get the shear rate.

But now, because of this simplification shear stress directly you can get from the angular velocity or rotational velocity versus torque information whatever you are getting from the experiments directly right. So, this is. So, that is the reason this is the most reliable equation.

So, in the next class what we will be doing? We will be taking fairly narrow gap cases and then do the analysis. In this class what we have done? We have taken the cases where the gap is very narrow or gap is very large that is k greater than 0.99 and k less than 0.1 cases we have taken and then done the analysis to get the required shear stress and shear rate information right. What if the gap is not very large not very narrow in between then how to do the analysis that we will take care in the next class.

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The references for today's lecture are provided here. Most of the lecture has been prepared from this book Macosko that is Rheology: Principles, Measurements and Applications by Macosko. And then some details have also been taken from this reference book Chhabra and Richardson; however, similar analysis you may find in other references books provided here.

Thank you.