Transport Phenomena of Non-Newtonian Fluids
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Lecture - 06
Capillary Viscometers - Errors and Corrections

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Capillary Viscometers Errors and Corrections. However, before going into the details of today's lecture what we will have? We will have a recapitulation of what we have studied in the previous lecture and then we see an example problem based on what we have seen in the previous lecture ok.

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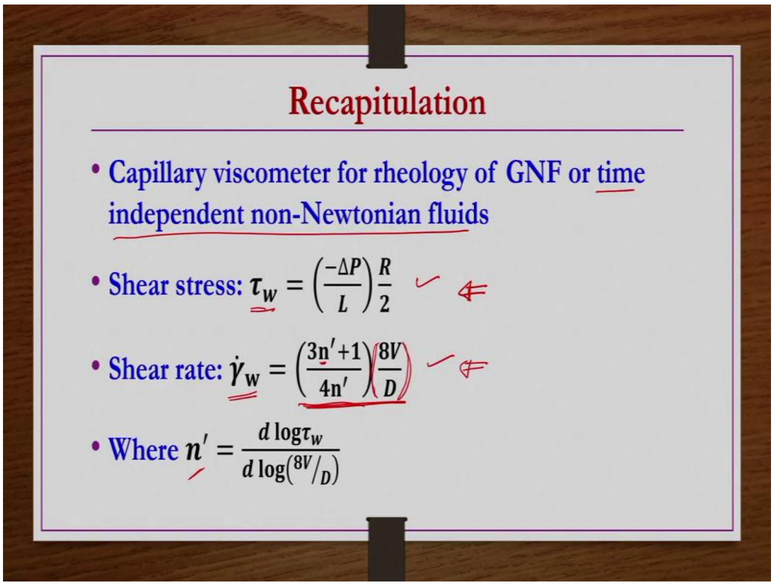

Recapitulation

- Capillary viscometer for rheology of GNF or time independent non-Newtonian fluids
- Shear stress: $\tau_w = \left(\frac{-\Delta P}{L}\right)\frac{R}{2}$
- Shear rate: $\dot{\gamma}_w = \left(\frac{3n'+1}{4n'}\right)\left(\frac{8V}{D}\right)$
- Where $n' = \frac{d\log\tau_w}{d\log\left({}^{8V}/_{D}\right)}$

So, in the previous lecture we have seen different details of capillary viscometers and then when we apply or when we use such capillary viscometers for measuring the rheology of non-Newtonian fluids. Especially for time independent non-Newtonian fluids or generalized Newtonian fluids we have developed what are the corresponding equations for shear stress and shear rate that is what we have seen.

So, what we have found by doing the simple analysis and derivation? We have obtained shear stress for the fluid whose rheology is tried to measure using the capillary viscometer is nothing but $\tau_w = -\left(\frac{-\Delta p}{L}\right)\frac{R}{2}$ and then shear rate $\dot{\gamma}_w = \left(\frac{3n'+1}{4n\prime}\right)\frac{8V}{D}$ that is what we have seen.

Remember this shear stress and then shear rate we are measuring at the wall and then we have seen the details why at the wall. And then what about the variation with respect to the R position and all those things we have seen and then we develop these equations ok. So, in this shear rate equation n prime is nothing but $n' = \frac{dlog\tau_w}{dlog\left(\frac{8V}{D}\right)}$ or when you plot τ_w versus $\frac{8V}{D}$ on a log-log plot and then get the slope of that line that slope is nothing but n' that is what we have seen in previous lecture right.

Otherwise what we further understood like you know if you wanted to know the shear stress of the unknown fluid that have been tested using capillary viscometer can be obtained if you know the pressure drop that has been applied to get a certain flow rate right.

So, that is if you know the pressure drop you can easily calculate the wall shear stress. Similarly if you know the volumetric flow rate for that pressure drop then that volumetric flow rate from there you can find out the average velocity. And then once you know the average velocity you can find out the wall shear rate.

And this shear rate whatever $\left(\frac{3n'+1}{4n\prime}\right)\frac{8V}{D}$ is there it is nothing but the true shear rate for a time independent non-Newtonian fluids or generalized Newtonian fluids. Remember, all this analysis we have done for time independent non-Newtonian fluids only ok.

And then we have also seen this whatever $\frac{8V}{D}$ is there that is nothing but the nominal shear rate or apparent shear rate which is a true shear rate for Newtonian fluid, but not for time independent non-Newtonian fluids. For time independent non-Newtonian fluids $\frac{8V}{D}$ is apparent or nominal shear rate and then if you multiply that $\frac{8V}{D}$ by a correction factor $\frac{3n'+1}{4n\prime}$ then you get the true shear rate this is what we have seen.

Now, we take an example problem; however, before taking the example problem we see a few modes of operation of a capillary viscometers. Now what are you measuring experimentally in capillary viscometers? You are simply varying the pressure drop and then obtaining the volumetric flow rate those are the measurable quantities right.

Those measurable quantities you are further modifying or using such a way that you can get the shear stress and shear rate information that is all different things. But in order to have a kind of constant flow rate or constant pressure drops so, you can have a different modes of operations right.

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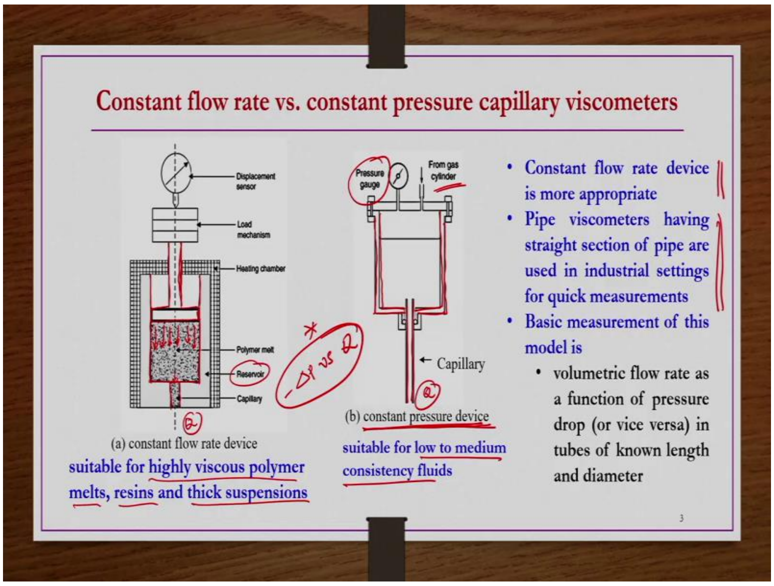

Constant flow rate vs. constant pressure capillary viscometers

(a) constant flow rate device

suitable for highly viscous polymer melts, resins and thick suspensions

(b) constant pressure device

suitable for low to medium consistency fluids

- Constant flow rate device is more appropriate
- Pipe viscometers having straight section of pipe are used in industrial settings for quick measurements
- Basic measurement of this model is
    - volumetric flow rate as a function of pressure drop (or vice versa) in tubes of known length and diameter

So, that is what capillary viscometers constant flow rate versus constant pressure capillary viscometers that is what we see what are the difference. And then which one is more reliable, which for what kind of fluid which kind of mode of capillary viscometer is better those things we see now.

So, constant flow rate devices, here what we have? We have a container right or reservoir or vessel whatever you call so right. So, here to this bottom there is an opening is there that opening is such a way that that can be adjusted that can be adjusted and then a capillary can be fixed of different diameters right. It is not a fixed opening it is an adjustable opening here. So, at the bottom you can attach a capillary right. To this container there is you know load mechanism is provided.

So, that you can provide certain pressure or certain load you can provide. So, that to get the required you know flow rate right this is one mode of the operation. So, now, you apply certain load and then you get obviously certain flow rate right. So, now, in this device what you want? You want to have a constant flow rate and then accordingly the load you will be changing. Why? Because let us say gradually when you are doing experiments.

So, then you know the fluid whatever that unknown fluid that you have taken in the reservoir or the vessel you know that will be you know consumed and going out as a flow rate as a fluid right. So, then obviously, this quantity volume is going to reduce so then obviously, the load if you are keeping fixed, so then the flow rate is going to change.

So, the monitoring or operation you will be doing such a way that you will maintain the constant flow rate and accordingly you change the load right. So, that is what this constant flow rate devices very simple and these are suitable usually for highly viscous melts, resins and then thick suspensions right.

Obviously, you know that if you are maintaining constant load or constant pressure drop you cannot have a constant flow rate. Because the material, whatever the head is there the material head is there that is going to decrease gradually. So, then accordingly your flow rate is going to decrease. So, accordingly there is another mode of operation is possible you maintain constant pressure, constant pressure you maintain. And then whatever the flow rate is there that flow rate you can be you can collect it right.

So, for here you have a pressure gauge and similarly entirely operation is very similar what you have? You have a reservoir right. And then to that bottom of that reservoir you have an opening that opening is adjustable opening. So, that you can change, so that you can fix a capillary of different diameters like this right.

But in addition you will be having a pressure gauge and a pressure gauge in that you know you will be using some gas cylinder so that to maintain the pressure required pressure. So, here what you are doing? You are maintaining constant pressure and then whatever the flow rate you are getting that you are noting down.

So, whether you use the constant flow rate device or constant pressure device ultimately what you are getting? You are getting pressure drop versus volumetric flow rate information only you are getting. In case, in one case you are fixing pressure drop fixed in one case you are keeping pressure drop fixed another case may be volumetric you are keeping fixed right.

So, but anyway this is the ultimate data that you are going to get which you will be using to get the shear stress and shear rate as we have already discussed. Then it is constant pressure devices are usually suitable for low to medium consistency fluids. However, it has been found that constant flow rate devices are more appropriate for different operational reasons.

And then pipe viscometers having straight section of pipe are used in industrial setting for quick measurements as I already discussed in the previous lecture that you know pipe flow is a very common phenomena in any of the process industries wherever the fluids are flowing.

So, if you wanted to know instantly the rheology or the viscosity or apparent viscosity of the fluid under operating conditions or under pressure or for a given pressure drop and then volumetric flow rate. You know you can have a kind of slit kind of thing within that pipe at different locations. And then periodically you can collect the flow rate and then measure the rheology for a given Δ p versus flow rate information ok.

Of course depending on the nature of the fluid, safety, precautions and all those things anyway will be coming into the picture. So, but that is basically it is possible to have pipe viscometers in real life industrial settings as in situ viscosity measurement devices. Let us say in your case you know your fluid viscosity has to be maintained.

Sometimes if you are you know allowing suspensions to flow through pipe what happens? Some of the particles may be settling at the bottom and those kinds of things those kinds of such kind of problems may be there. And then because of that one you know local viscosity may be different from the bulk viscosity right. So, you have to have a monitoring of such viscosity if required, if it is essential property in your process streams or in a product final product depending on situations.

So, if it is required, so then you can have a pipe viscometers in line you know in process flow situations. And whether it is a constant flow or constant pressure device you operate. Ultimately what you get? You get volumetric versus pressure drop or vice versa data for different lengths and diameter of the pipe ok. That you convert in terms of you know given so called shear stress and shear rate and then you plot them and get the rheology of the fluid.

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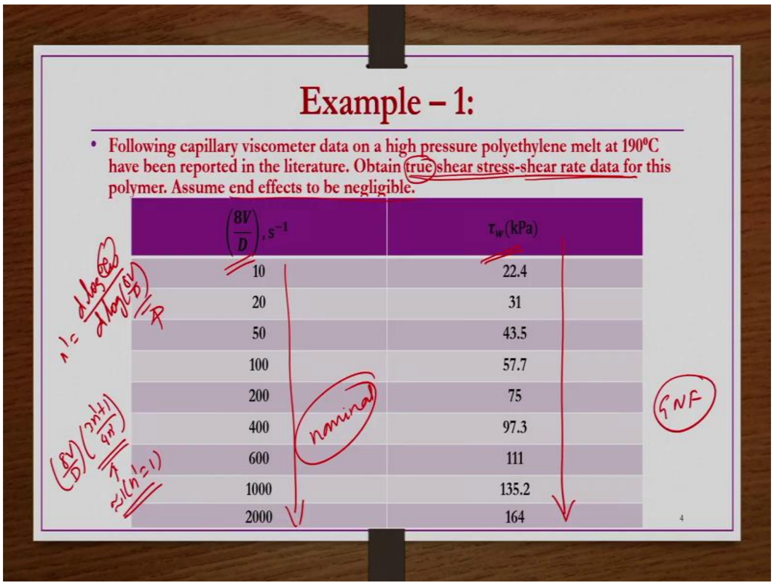

Example – 1:

- Following capillary viscometer data on a high pressure polyethylene melt at 190°C have been reported in the literature. Obtain true shear stress-shear rate data for this polymer. Assume end effects to be negligible.

$\left(\frac{8V}{D}\right), s^{-1}$	τ_w (kPa)
10	22.4
20	31
50	43.5
100	57.7
200	75
400	97.3
600	111
1000	135.2
2000	164

Now, we have an example problem to understand how to use these equations of you know shear stress, shear rate in order to know the rheology of a fluid if you are using capillary viscometer right. So, that is what we are going to see now. So, following capillary viscometer data on high pressure polyethylene melt at 190°C have been reported in the literature.

Obtain true shear stress versus true shear rate data for this polymer. The true word is specifically used because we know there is a case like you know apparent or nominal shear rate information that you may get in general from this info from the capillary viscometer.

But we do not want apparent shear rate or nominal shear rate, but we want true shear rate. And then also mentioned assume end effects are negligible. What are the end effects? That flow has to be fully developed, in order to flow has to be fully developed you know we have to have large $\frac{L}{D}$ ratio capillaries those things we have seen. And then under such conditions also at the entrance and an exit you may not have the fully developed flow.

So, that may lead to some kind of error. So, it has been stated in the problem that you avoid or neglect such kind of effects. In fact, our next topic is to incorporate and then make necessary corrections for such kind of end effects only. Data is given $\frac{8V}{D}$versus τ_w alright.

So, for generalized Newtonian fluids or time independent non-Newtonian fluids whatever we have studied in the previous class τ_w is nothing but the true shear stress. So, we do not need to make any change in these values that is already given. So, that is a given τ_w information is a true shear stress information.

But $\frac{8V}{D}$information whatever is given it is nominal or apparent shear rate for the case of generalized Newtonian fluids or time independent non-Newtonian fluids. It is $\frac{8V}{D}$ is true shear rate only for the Newtonian fluids. So, what we have to do? We have to do $\frac{8V}{D}$ multiplied by $\frac{3n'+1}{4n'}$ we have to do. That means, for each data we have to find out what is n’ and then multiply this $\frac{8V}{D}$ values with $\frac{3n'+1}{4n'}$ correction values.

If it is a Newtonian fluid; so then this correction is going to be close to 1 approximately close to 1 if the fluid is Newtonian. But if it is non-Newtonian, if it is time independent non-Newtonian fluids then definitely this n’ is not going to be coming close to 1. So, then obviously, this correction is also not going to be close to 1.

So, that we have to check now. So, how do we get this n’? n’ prime is nothing but $\frac{dlog\tau_w}{dlog\left(\frac{8V}{D}\right)}$.

So, τ_w $\frac{8V}{D}$ information has already been provided. So, on a log-log plot what you can have? You can have τ_w versus $\frac{8V}{D}$curve whatever the data is given and then at different $\tau_w$values you draw a line. You draw a line tangent and then find out the slope of the curve at that particular point that we are going to see now here.

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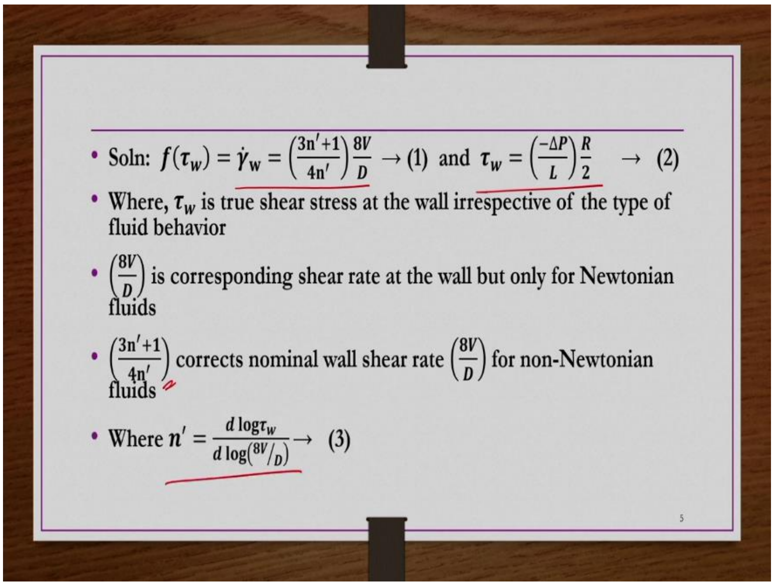

- Soln: $f(\tau_w) = \dot{\gamma}_w = \left(\frac{3n'+1}{4n'}\right)\frac{8V}{D} \rightarrow (1)$ and $\tau_w = \left(\frac{-\Delta P}{L}\right)\frac{R}{2} \rightarrow (2)$
- Where, τ_w is true shear stress at the wall irrespective of the type of fluid behavior
- $\left(\frac{8V}{D}\right)$ is corresponding shear rate at the wall but only for Newtonian fluids
- $\left(\frac{3n'+1}{4n'}\right)$ corrects nominal wall shear rate $\left(\frac{8V}{D}\right)$ for non-Newtonian fluids
- Where $n' = \frac{d\log\tau_w}{d\log\left(^{8V}/_{D}\right)} \rightarrow (3)$

So, $\tau_w$is this one that is true shear stress for a GNF and then true shear rate for GNF is this one we have already seen right. So, n’ this correction we need to make in order to get the now in order to get the true shear rate or correct the nominal shear rate we need to have this correction $\frac{3n'+1}{4n'}$ and then n’ is nothing but this one.

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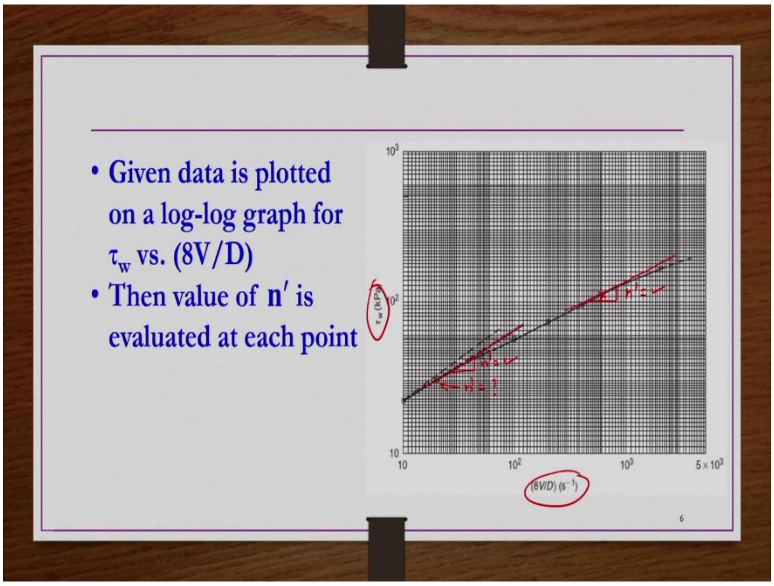

- Given data is plotted on a log-log graph for τ_w vs. (8V/D)
- Then value of n' is evaluated at each point

So, what we are going to do? We are going to have a plot log-log plot and τ_w versus $\frac{8V}{D}$. Let us say at this point of τ_w if you wanted to measure what is the n’ value then what you do? You draw a tangent at this point like this right and then you find out the slope of this tangent line that is nothing but your n’.

Let us say you wanted to measure the n’ value when the tau w value is coinciding at this point. So, at this point what you have to do? You have to draw a tangent like this very carefully and then find out the slope of this line. So, that would be giving your n’ value for this τ_w.

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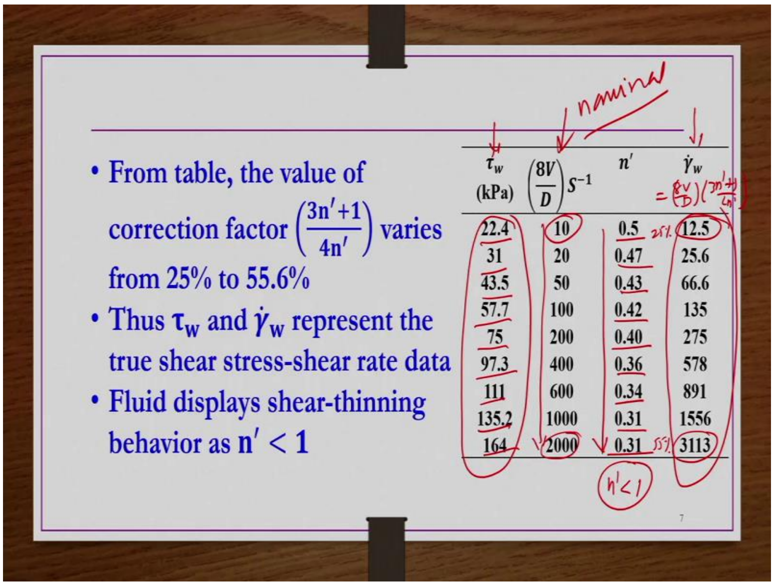

- From table, the value of correction factor $\left(\frac{3n'+1}{4n'}\right)$ varies from 25% to 55.6%
- Thus τ_w and $\dot{\gamma}_w$ represent the true shear stress-shear rate data
- Fluid displays shear-thinning behavior as $n' < 1$

τ_w (kPa)	$\left(\frac{8V}{D}\right) S^{-1}$	n'	$\dot{\gamma}_w$
22.4	10	0.5	12.5
31	20	0.47	25.6
43.5	50	0.43	66.6
57.7	100	0.42	135
75	200	0.40	275
97.3	400	0.36	578
111	600	0.34	891
135.2	1000	0.31	1556
164	2000	0.31	3113

nominal

$= \left(\frac{8V}{D}\right)\left(\frac{3n'+1}{4n'}\right)$

25%

55%

$n'<1$

So, that you do. So, then for each tau w value n' has been found out that corresponding to each τ_w value what we have done? We have a plotted a tangent at that point on the graph of τ_w versus $\frac{8V}{D}$ curve and then corresponding n prime value we have found for all the cases.

And then you can see here the n' is less than 1 and then as τ_w increasing it is further decreasing; that means higher shear stress. If you are increasing the shear stress or increasing the shear rate it is becoming more shear thinning kind of thing. n' less than 1 that means, it is a shear thinning. And then that shear thinning behaviour is increasing with increasing the shear rate or shear stress.

Further you can see the correction. So, this $\dot{\gamma}_w$ is nothing but $\frac{8V}{D}$multiplied by the correction factor $\frac{3n'+1}{4n'}$. So, without taking this correction into the account this shear rate is 8, but now after incorporating this correction it is 12.5. You may be thinking 2.5 s^{-1} only.

But it is nothing but 25 percent difference $\frac{12.5-10}{10}$ that is 2.5 by 10 multiplied by 100, right. So, $\frac{12.5-10}{10}$ ok multiplied by 100, so, then it is coming roughly 25 percent. And then if you go to the other extreme 2000 is the apparent shear rate, but the true shear rate is 3113 second inverse such huge difference. You can see it is almost like more than 50 percent increased ok.

So, it is almost like 55 percent is increased here shear rate ok. So, now, whatever this tau w versus gamma dot w this information is there that is the true shear rate versus true shear stress information. Whatever this $\frac{8V}{D}$ is there it is nominal one because n' is not equals to 1 or even close to 1 it is very different from 1 value of 1 ok.

So, this is how you can analyze you know information that you get from the capillary viscometers ok. Now, we see sources of errors while using the capillary viscometers for measuring the rheology of a non-Newtonian fluids; especially non-Newtonian fluid that is what we are going to say we are going to see. So, actually for that we have to do what we have to do? We have to enlist, are they these the sources of errors whatever we are calling are they important? Really, so much different.

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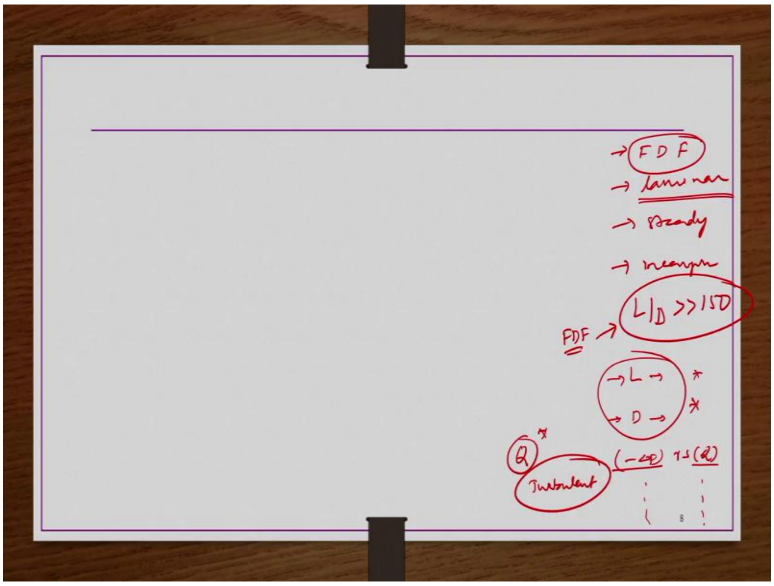

What we have taken assumption is the fully developed flow, laminar flow, steady flow right and then incompressible fluids these things we have taken into the consideration. While taking this fully developed flow, what we have taken that $\frac{L}{D}$? We have taken large value more than 100 or 150 something like that.

So; that means, if you take $\frac{L}{D}$ or individual L and D such a way that if $\frac{L}{D}$is less than this thing, FD, Fully Developed Flow is not going to maintain. So, is it really going to is it really going to have a kind of you know influence on the data that we have to see.

That means, if you change the L by keeping D constant or by keeping L constant and then you change the D or simultaneously when you keep changing L and D. So, are you getting are you going to get minus Δ p versus Q information very different from one case to the other case that is what we have to see.

If you are getting different information different data by changing the length of this thing; that means, it is having you know this parameter is very much essential and then we have to make a corrections for this L and D issues right and then another thing that we are saying that laminar flow only. So, what if the capillary viscometer the pressure drop you have maintained such a way that the volumetric flow rate is very large?

So, then the flow may not be laminar always. So, it may be turbulent also, is it going to incur some kind of error that is what another thing that we have to see, so flow rate right. So, what we understand from these especially this fully developed flow in laminar conditions? We this L this D and then this Q are having you know large influence and the overall rheological measurements that we are going to do that we are obtaining by using the capillary viscometer.

So, how much are they important? Do we really need to worry? Are these the only things that are you know important for us there may be other errors also possible. So, what are the possible sources of errors in capillary viscometers that we first enlist and then out of which what we do? Qualitatively we check which is more important which is less important so, that we can neglect.

There may be other some errors which are causing very less influence on the overall rheology of the system. So, then such a kind of errors we may oversee overtake right we can just avoid. There may be some errors which may not be you know small. So, then we have to make appropriate corrections, so that is what we are going to see now.

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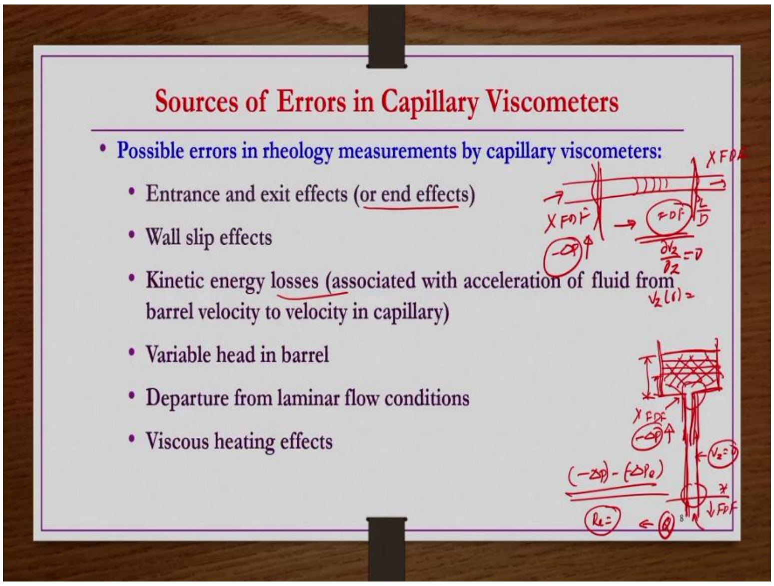

Sources of Errors in Capillary Viscometers

- Possible errors in rheology measurements by capillary viscometers:
  - Entrance and exit effects (or end effects)
  - Wall slip effects
  - Kinetic energy losses (associated with acceleration of fluid from barrel velocity to velocity in capillary)
  - Variable head in barrel
  - Departure from laminar flow conditions
  - Viscous heating effects

Sources of errors in capillary viscometers; possible errors in measuring the rheology of a unknown fluid by capillary viscometers may have a several errors right. When you try to measure the rheology of an unknown fluid rheologically unknown fluid to know its rheology by using capillary viscometers, then several number of errors are possible. What are they? Entrance and exit effects or end effects.

How much it is important? So, whatever the fully developed flow, what we have seen that you know we have taken this capillary. We have taken a capillary as a kind of you know circular pipe that is how we have taken.

How we have taken? We have taken $\frac{L}{D}$ large and then we said fully developed flow. This fully developed flow may not be there at the entry and then exit levels. At the entry the flow may not be will not be fully developed flow and then at the exit also the flow will not be fully developed flow.

But all our analysis is based on this one fully developed flow right. So, fully developed flow in the sense, in the flow direction the velocity is not going to change whatever the velocity component. Let us say z component that $\frac{\partial v_z}{\partial z}$ is going to be 0 or the pattern v_z as function of r that if you plot it is not going to change whatever the pattern is there that is going to remain same at different locations in the z direction. You know within the fully developed region, within the fully developed region.

So, this we know from our transport phenomena also so, but you know your analysis you are taking only for this consideration. But there are some reasons you know an, you know in that reason this constraint is not valid so; obviously that what is the effect that we have to see.

So, one effect is that when the flow is not fully developed usually pressure drop increases pressure drop increases at the entrance, at the entrance pressure drop may increase if the flow is not fully developed indeed it is not fully developed. So, what does mean by entry? So, you have this capillary viscometer arrangement like this right.

So, at this location at the entry location you are not going to have fully developed flow right and then this capillary is very large that is what we are saying though we are drawing you know very small one. So, this at the exit level again the flow is not going to be fully developed flow. These are the locations you know exit locations.

So, when at the entry location obviously, because of the sudden contraction of the flow area it is because this is all you know flow that is the fluid present here suddenly the flow whatever the cross section is here there when you open the wall to get the flow rate coming out through the capillary. So, then sudden contraction of this cross section is taking place because of; obviously, the pressure drop is going to be increased at this location.

So, how much it is increasing that we have to find out and then you know from your true Δ p you have to subtract that correction whatever the entry level the pressure drop increased pressure drop. So, let us say 15 Pascal it is showing, it is the entry at entry level the pressure drop is 15 Pascal more than the average pressure and the fully developed region.

So, whatever the data that Δp versus you know Q that you get from the Δp you have to subtract this one that is what we are going to see right. So, since this fully developed flow it is causing you know some kind of error in pressure drop. So, corresponding changes would occur in the shear stress calculations right shear rate is not going to be change because in shear rate we are going to have this V velocity which is coming from the volumetric flow rate.

So, correction is required only in pressure. So, then correction is required only in the shear stress that is the one. But exit also the flow fully developed flow may not be there, but you know that contribution of this you know decrease in pressure drop at the exit point is not going to have a much influence on the rheology of the fluid. Because it has already passed through the fully developed region mostly right, whereas, the entry point it has not yet entered the fully developed region ok.

So, that is the reason it is more effective entry point you know pressure effects or increased pressure drop is going to be have a more influence on this you know end effects. End effects is a combined term, one is entrance effect another one is exit effect. At the entrance what is the pressure drop, at the exit what is the pressure drop and then how much different are these values from the fully developed region pressure drop.

So, overall pressure drop that we get in the capillary viscometer experimental study that is fully developed pressure drop. Because we are taking $\frac{L}{D}$ sufficiently large values ok that is one. Then wall slip effects. What we have seen? At the capillary wall at this inside locations the velocity whatever is there that we have taken as 0 in our analysis, is not it? Right.

But in general when you are taking you know non-Newtonian viscous fluids the velocity is not it is not you know 0, it may be having some different velocity compared to the 0 velocity. So, that is that also incurring some kind of effects in the rheology right. So, since it is showing effect in the velocity so, then corrections would be there in the shear rate not in the shear stress ok that is also in detail we see right. Then kinetic energy losses what happened you know?

When the fluid is accelerating from this barrel or vessel to get into the capillary and then get out of the capillary, so, there is a kinetic loss kinetic energy loss; obviously, how much it is important.

Then when the fluid is going out, so, the head is this fluid head is gradually decreasing. So, then fluid head when it is decreasing gradually, so, then how much is it going to have effect on the rheology that is the other thing. Then departure from laminar conditions. Your flow rate you are measuring, that corresponding to that one what is the Reynolds number, you are not measuring actually.

Because you cannot measure also because you do not know nature of the fluid if it is Newtonian, so, then Reynolds number definition is different. If it is shear thinning, so it is different and then different in the sense different parameters and n’ m’ would be coming into the picture right.

So, that is the reason you know. So, you have to see you have to see without calculating the Reynolds number how can you how can you have a kind of a priori information from simply -Δp versus Q plot so that you can understand whether the flow is in laminar conditions, or turbulent conditions because our all analysis is for the laminar condition only. If it is deviating from laminar condition Ss, that will again induce some kind of error.

So, that is another error that we are going to discuss and then viscous heating; obviously, as we discussed you know most of the non-Newtonian fluids are multi phase components, it is not a single component kind of thing. But most of the non-Newtonian fluids it is not true for all the cases. But majority of non-Newtonian fluids are a kind of multi phase flows where you know some particles or you know bubble straps are you know suspended like particulates systems, emulsion, suspensions etcetera.

So, when you have these kind of multi component system passing through a very narrow gap because narrow gap we are maintaining so, that the changes in the shear rate or shear stress should be negligible that is the reason we are taking the narrow gap. But when these material are passing through such narrow gap especially high viscous materials are passing through narrow gaps at high velocity, high viscosity, and high velocity then that means, high flow rate then obviously, viscous heating would be there.

So, are you accounting for the viscous heating or not? If you are not accounting that is also going to change the rheology of the fluid or that is also going to show a different rheological parameters compared to the true rheological parameters. So, these are the some of the possible errors right.

Now, before going into the details of each and every error, so what we see? Can we avoid some of them, right? So obviously, as engineering students we try to reduce our work. Can we avoid some of them, on what basis? Blindly we cannot avoid, we can have a kind of you know qualitative estimation which is important which is less important kind of thing, if you cannot avoid.

Can we make certain kinds of changes in the design or operational conditions so that certain kind of these errors may be reduced if not all? So, either of the options we can see ok. How we can change the design or operational parameters, so that some of these errors may be minimized so, that we can avoid them or neglect them without considering in the design of the capillary viscometers. So, that is the next step.

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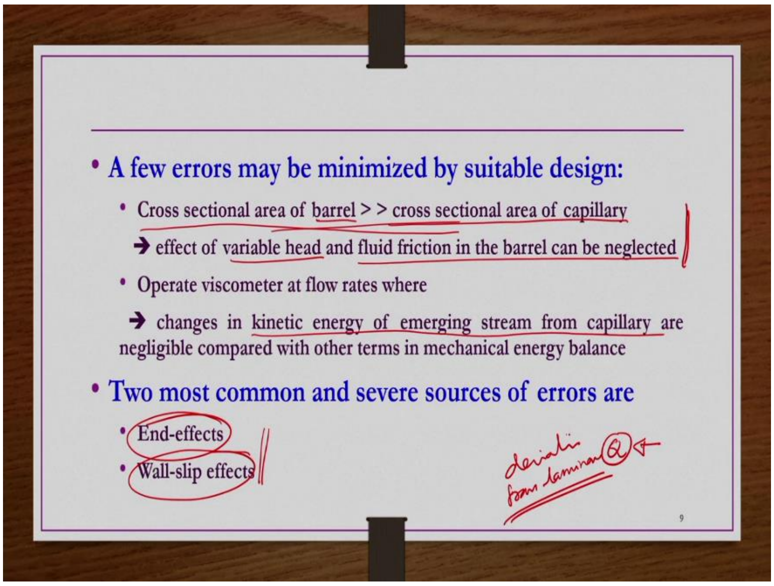

- A few errors may be minimized by suitable design:
  - Cross sectional area of barrel > > cross sectional area of capillary

    ➔ effect of variable head and fluid friction in the barrel can be neglected
  - Operate viscometer at flow rates where

    ➔ changes in kinetic energy of emerging stream from capillary are negligible compared with other terms in mechanical energy balance
- Two most common and severe sources of errors are
  - End-effects
  - Wall-slip effects

deviation from laminar @

So, if you have a cross section area barrel very much larger than the cross section area of capillary then what will happen? Effect of variable head and then fluid friction in the barrel can be neglected. So, that one that design point of view you make such kind of thing. That is true also most of the cases barrel diameter is very much larger compared to the capillary diameter, capillary maybe 1 mm, 2 mm diameter only in general you prefer to use right.

So, at a max 5 mm capillaries you 5 mm diameter capillaries you may be using, but barrels you may be having like you know diameter something like you know 50 centimeters or 20 centimeters something like that. So; obviously, this is taken care in most of the capillary viscometers by default without making much changes. But of course, if such conditions are not maintained in the designing, so then you can try to have this point into the consideration.

Then operate viscometer at flow rates where the changes in kinetic energy of emerging stream from capillary are negligible compared with other terms in the mechanical energy balance. You can do the mechanical energy balance for the barrel and then for the capillary. You can cancel out the term such a way or you can have the flow rates such a way that the kinetic energy losses because of the fluid emerging from the capillary is very much small compared to the other terms in the mechanical energy balance right.

So, that is the other thing. So, after this you know most of the things have been taken care. What are the things have been not taken care? You know end effects we cannot avoid we have to do some corrections, slip effects we cannot avoid we have to do some corrections and then in addition to this one so called deviation from laminar flow deviation from laminar flow that also you can avoid by operating in such a way that you know your flow rate is under the laminar condition.

But you do not know. So, that is the reason we will be seeing a few discussions like from Q versus Δp information can we have a kind of a priori you know a priori, can we have a kind of intuition whether the flow is in the laminar or you know turbulent condition? That is the other one we are going to see.

So, remaining lecture we are going to discuss on end effects. In the next lecture we will be discussing on wall slip effects and then deviation from laminar flow conditions. Viscous heating we are not taking into the consideration as of now.

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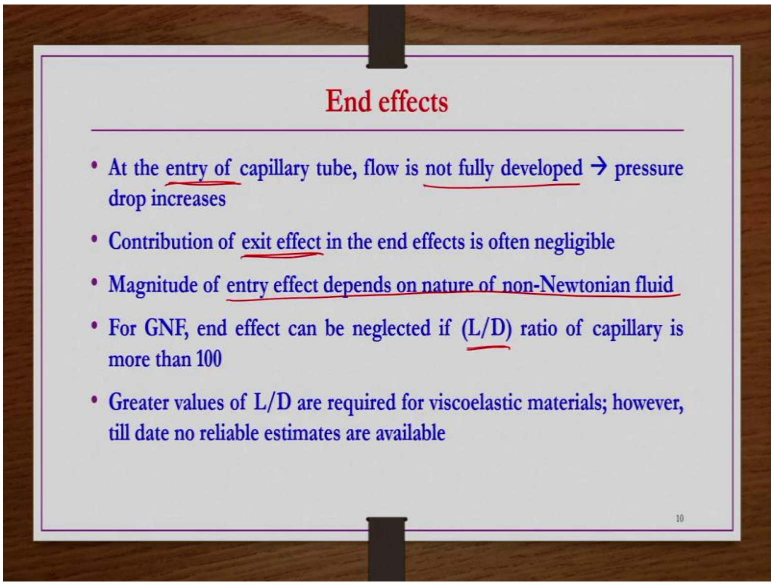

End effects

- At the entry of capillary tube, flow is not fully developed → pressure drop increases
- Contribution of exit effect in the end effects is often negligible
- Magnitude of entry effect depends on nature of non-Newtonian fluid
- For GNF, end effect can be neglected if (L/D) ratio of capillary is more than 100
- Greater values of L/D are required for viscoelastic materials; however, till date no reliable estimates are available

End effects, so at the entry of the capillary tube flow is not fully developed and then pressure drop increases. Because at the entry the cross section area flow cross section area suddenly contraction has taken place. So, then because of that you know; obviously, pressure drop would be higher at that entry point as I just explained right.

So, contribution of and further the contribution of exit effect whatever at exit the pressure drop is usually small slightly smaller than the fully developed flow pressure drop. So, that is not going to affect much in the overall rheology. So, that exit effect maybe you know neglected maybe neglected. Exit effect in the end effects overall end effects may be negligible, but entry effects cannot be neglected. So, then we have to do appropriate considerations.

So, magnitude of entry effect depends on nature of the non-Newtonian fluid that we are doing. So, but whatever the things that we are going to see now, they are primarily for a time independent non-Newtonian fluids. For generalized Newtonian fluids or time independent non-Newtonian fluids, if you have $\frac{L}{D}$ more than 100 or 150 it is more or less you know this end effects are taken care.

And then you can be safe you do not need to worry about these end effects. If you are maintaining $\frac{L}{D}$ more than 100 or 150, but for the other fluids like viscoelastic materials even 200 may not be sufficient and there is no concrete data available. How much $\frac{L}{D}$ value is sufficient for viscoelastic fluids that is not known yet, ok.

But it has been observed that even $\frac{L}{D}$ 200 or more may not be sufficient to avoid end effects in the case of a viscoelastic materials when they are flowing through capillaries ok. So, now, correction for end effects. Now, we understand that at the entrance or at the entry point the pressure drop increases because of the sudden contraction of the flow area right from which the material is passing through.

So, there is a pressure increased pressure drop increased. So, how much pressure drop has increased because of such sudden contraction that we have to estimate and then that amount we have to subtract from the Δp information that we are getting from the experimental info data ok. So, that is what we are doing.

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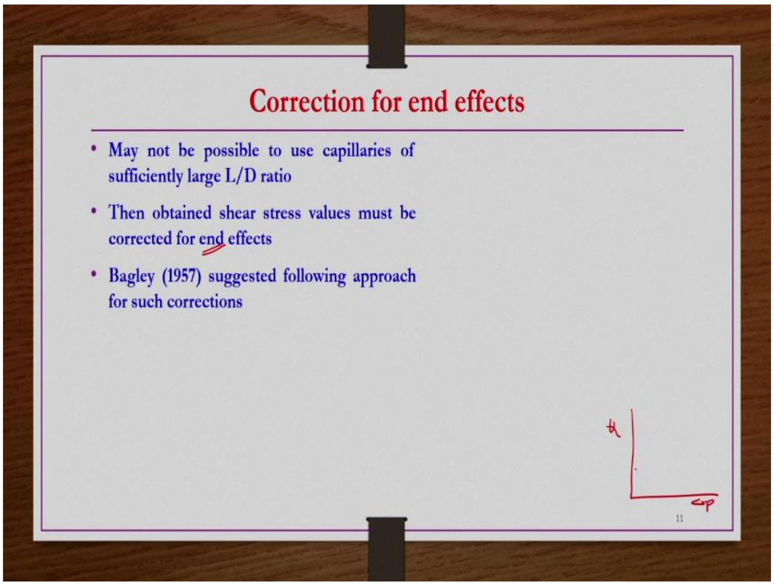

Correction for end effects

- May not be possible to use capillaries of sufficiently large L/D ratio
- Then obtained shear stress values must be corrected for end effects
- Bagley (1957) suggested following approach for such corrections

So, why it is required? Because often it is not possible that you can have large $\frac{L}{D}$ ratios ok, so obviously, having some kind of correction methods to make a correction. Let us say you have only one single capillary and then when you are measuring $\frac{L}{D}$ of that capillary is around 70 or 80 only. So, it may be difficult to have reliable rheological information from such small capillaries. Then what we do? Obtained shear stress values must be corrected for end effects.

Now, we can safely say rather end effects entry effect we can say, we got exit effects are negligible ok. So, Bagley suggested an approach for a corrections in the pressure drop. So, that pressure drop correction you know is you know when you take when you taken care of that pressure drop increased pressure drop and then subtracted from the pressure drop of experimental information.

So, then whatever the shear stress that you get that is not going to be you know different from the that is going to be reliable that is going to be true shear stress. Otherwise also simply how do you understand whether the obtained you know this information Q versus Δp also, you can simply do different you know you can get the Q versus Δp information you know by using different capillaries, different capillaries of different L and different D.

And then you plot all of them, if they are not superimposing onto each other or if they are not close to each other. So, then that means, they are incurring some kind of error in the rheology. So, that is what the simplest way that you can understand without going into the details whether the effects end effects are there or not or wall slip effects are there or not. If one any error is there so, then this information is going to deviate even if you are maintaining the large $\frac{L}{D}$ ratios also.

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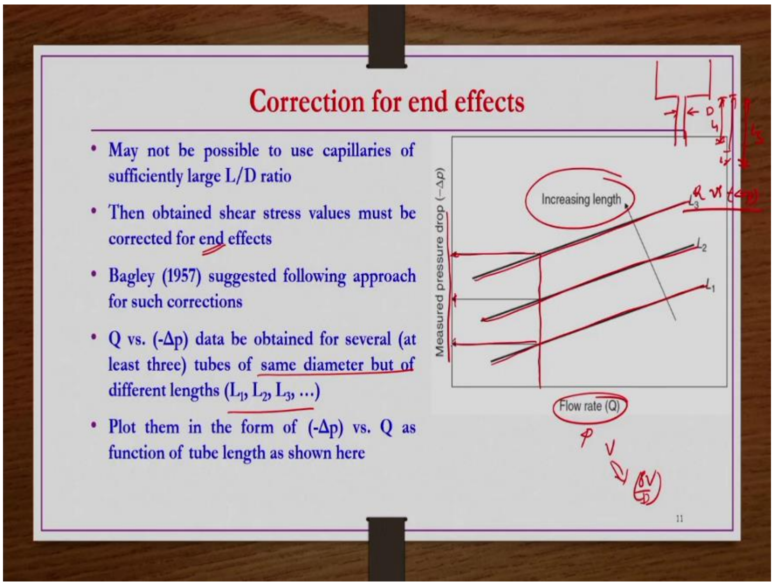

Correction for end effects

- May not be possible to use capillaries of sufficiently large L/D ratio
- Then obtained shear stress values must be corrected for end effects
- Bagley (1957) suggested following approach for such corrections
- Q vs. (-Δp) data be obtained for several (at least three) tubes of same diameter but of different lengths (L_1, L_2, L_3, …)
- Plot them in the form of (-Δp) vs. Q as function of tube length as shown here

So, Q versus Δp data can be obtained for several at least 3 tubes of same diameter, but different lengths like L_1, L_2, L_3 and so on and so, minimum 3 are required. The condition is that if you wanted to find out the correction for end effects you have to have at least 3 or more you know capillaries of different lengths, but the diameter has to be same because if your diameter if you change the diameter, so, then wall slip will also come into the picture.

So, you cannot handle 2 errors at a time or even if you try to do there may be some kind of errors. So, what we do? We take one by one ok. Now, whatever 3 capillaries you are using; that means, you are taking you know barrel like this and then different capillaries of and then you are maintaining same diameter.

But different times you are taking different length capillaries sometimes L, L_1, sometimes L_2 next time maybe further larger L_3 capillary like this you are taking, diameter you are keeping same and then you are trying to get the Q versus $-\Delta p$ information for each of 3 cases.

So, that if you plot measured pressure drop that is $-\Delta$ p versus flow rate if you plot then you can have a different lines like this. Different lines you may get if that length is not sufficient. If you have the even the lowest length whatever the capillary you have taken if it is sufficiently large value so, that $\frac{L}{D}$ is more than 100 or 150 then all 3 lines maybe you know very close to each other merging onto each other also, superimposing onto each other that is also possible.

Since the length is not sufficient so, then you get 3 different curves like this ok. So, then well with increasing length you what you see? For a constant flow rate the pressure drop also increases that is what you can understand with increasing the length. Now, what we do?

This volumetric flow rate information you get velocity from here. And then from this velocity what you get? $\frac{8V}{D}$. So, for each flow rate what is the corresponding $\frac{8V}{D}$ and then you draw a line here. And then you measure what is the pressures that information that you can you may require for further calculations.

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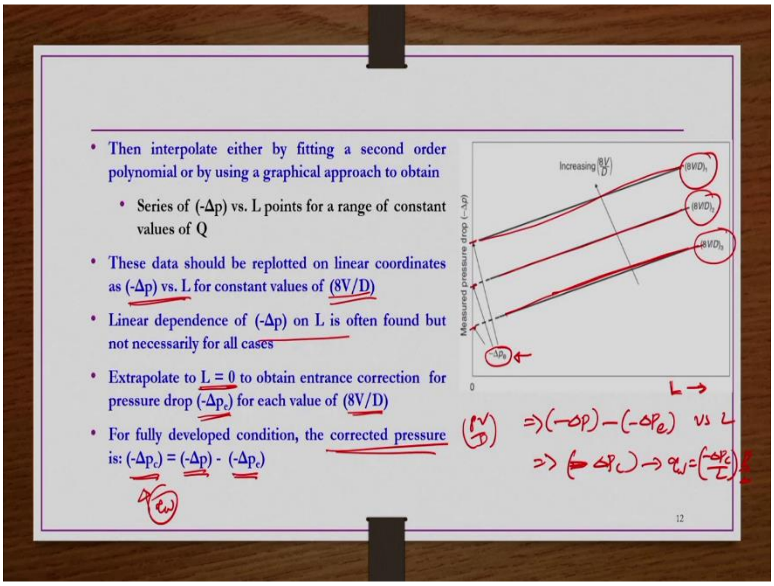

- Then interpolate either by fitting a second order polynomial or by using a graphical approach to obtain
  - Series of (-Δp) vs. L points for a range of constant values of Q
- These data should be replotted on linear coordinates as (-Δp) vs. L for constant values of (8V/D)
- Linear dependence of (-Δp) on L is often found but not necessarily for all cases
- Extrapolate to L = 0 to obtain entrance correction for pressure drop ($-\Delta p_e$) for each value of (8V/D)
- For fully developed condition, the corrected pressure is: $(-\Delta p_c) = (-\Delta p) - (-\Delta p_e)$

τ_w

$\left(\frac{8V}{D}\right) \Rightarrow (-\Delta p) - (-\Delta p_e)$ vs L

$\Rightarrow (-\Delta p_c) \rightarrow \tau_w = \left(\frac{-\Delta p_c}{L}\right)$

So, then interpolate either by fitting a second order polynomial or by using graphical approach to obtain a series of $-\Delta$p versus L points for a range of constant values of Q. Different Q values you take and then replot the data or you know interpolate the data and then get $-\Delta$p versus L data points. Then you plot this $-\Delta$p versus L data whatever by interpolation you have got it or by graphical approach.

So, that you plot for different Q values, Q values corresponding to Q is $\frac{8V}{D}$. So, why $\frac{8V}{D}$? Because we have to do $\frac{8V}{D}$ calculations anyway again. So, for that 8V by D is so, let us say one $\frac{8V}{D}$ value you know pressure drop versus L information whatever is there. So, that you are plotting here like this right.

So, another $\frac{8V}{D}$ for different L values you may get this curve like this. Another $\frac{8V}{D}$ for different L values you may get the $-\Delta$p information like this $-\Delta$p versus L for different $\frac{8V}{D}$ values you get different curves like this, because L is not sufficient sufficiently large enough.

If it is lengthy capillary then here again you may be getting the same lines or superimposing onto each other or these lines are very close to each other. Then what you do? You extend these lines to L is equals to 0 curve or to get the intercept on the y axis. So, these intercepts are nothing but they will give whatever that extra pressure or increased pressure at the entry point is there that extra pressure you get from this point from these intercepts ok.

So, linear dependence of $\frac{-\Delta p}{L}$ is often found, but not necessary for all cases that also depends on different situations. Then extrapolate to L is equals to 0 to obtain entrance correction for the pressure drop. So, $-\Delta$p for each value of $\frac{8V}{D}$, then from whatever the measured pressure drop you subtract this $-\Delta p_e$ for each $\frac{8V}{D}$ then you get this whatever the pressure correction right.

Whatever this pressure correction $-\Delta p_c$ that you get that you will be using in your τ_w calculation $\left(\frac{-\Delta p_c}{L}\right)\frac{R}{2}$. In this equation you will be using this one Δp_c you will be using rather than Δ p ok. So, for fully developed condition the corrected pressure would be $-\Delta p_c$ that is nothing but measured pressure drop minus this whatever the entrance correction for pressure drop $-\Delta p_e$.

So, if you subtract $-\Delta p_e$ from the measured pressure drop you will be getting corrected pressure drop corrected pressure drop for fully developed flow condition that you will be using for your tau w calculations. It is a bit lengthier, but simple process right.

Now, we take an example problem so that to understand how to do this one otherwise it becomes difficult to realize without doing an example problem ok. So, before taking example problem we see what are the drawbacks of this approach, ok.

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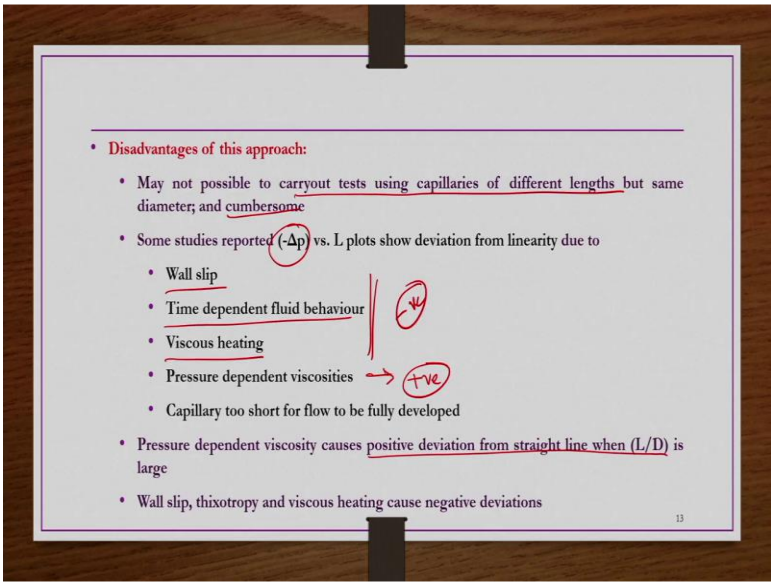

- **Disadvantages of this approach:**
  - May not possible to carryout tests using capillaries of different lengths but same diameter; and cumbersome
  - Some studies reported (-Δp) vs. L plots show deviation from linearity due to
    - Wall slip
    - Time dependent fluid behaviour
    - Viscous heating
    - Pressure dependent viscosities → +ve
    - Capillary too short for flow to be fully developed
  - Pressure dependent viscosity causes positive deviation from straight line when (L/D) is large
  - Wall slip, thixotropy and viscous heating cause negative deviations

-ve

First of all may not be possible to carry out test using capillaries of different lengths, but same diameter. It is very difficult to get the same diameter different capillary different length capillaries also it is very cumbersome. And then some studies reported whatever this $-\Delta p$ versus L curve that previous slide that we have seen, so, you may not get the linear line, straight line you may not get.

Sometimes you may get the non-linear curves, you may get ok that is another problem. And then that problem is coming maybe because of other errors like wall slip time dependent behaviour fluid may be having time dependent behaviour. There may be viscous heating and then viscosity may be pressure dependent because all these things are you know pressure is involved. So, the fluid viscosity is sensitive to the pressure difference.

So, then again that is also you know leading to some error and then altogether making this $-\Delta p$ versus L curve non-linear ok. So, sometimes capillary too short for flow to be fully developed flow, right. This pressure dependent viscosity cause positive deviation from straight line behaviour even when $\frac{L}{D}$ is large whereas, the wall slip thixotropy, thixotropy in this sense time dependent fluid behaviour and then viscous heating.

These three may be leading to negative deviation from the pressure drop right. And then the pressure dependent viscosity may be leading to positive deviation from the straight line not the pressure drop straight line behaviour ok.

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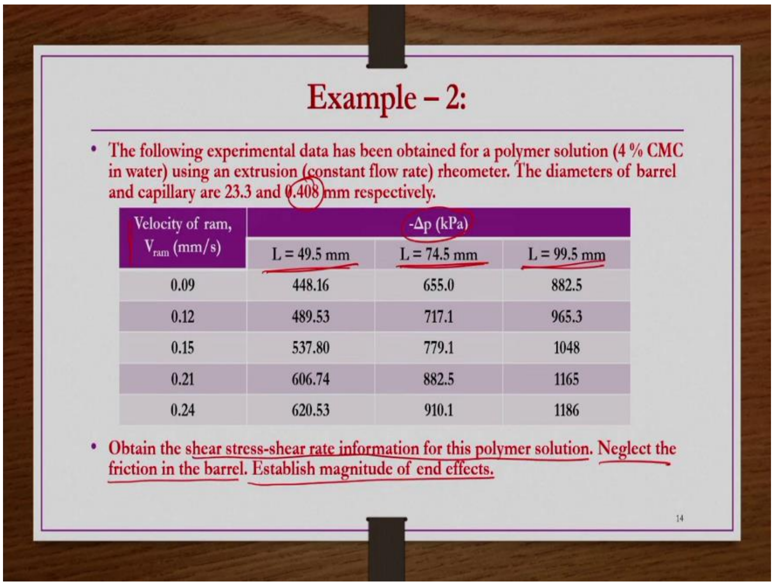

Example – 2:

- The following experimental data has been obtained for a polymer solution (4 % CMC in water) using an extrusion (constant flow rate) rheometer. The diameters of barrel and capillary are 23.3 and 0.408 mm respectively.

Velocity of ram, V_{ram} (mm/s)	$-\Delta p$ (kPa)		
	L = 49.5 mm	L = 74.5 mm	L = 99.5 mm
0.09	448.16	655.0	882.5
0.12	489.53	717.1	965.3
0.15	537.80	779.1	1048
0.21	606.74	882.5	1165
0.24	620.53	910.1	1186

- Obtain the shear stress-shear rate information for this polymer solution. Neglect the friction in the barrel. Establish magnitude of end effects.

So, now, the example problem. So, following experimental data has been obtained for a polymer solution of 4 percent CMC in water using an extrusion constant flow rate rheometer. The diameters of barrel and capillary are 23.3 and 0.408 mm respectively. Barrel diameter is 23.3 mm; capillary diameter is 0.408 mm ok.

So, that data is given in terms of velocity of ram or barrel velocity whatever the fluid that velocity say, that velocity is given that velocity versus pressure drop information is given when you use different length capillaries. Diameter you are keeping same, but length you are changing once you are taking only 49.5 mm, next time you are taking 74.5 mm and then 99.5 mm next time right.

So, now obtain the shear stress, shear rate information for this polymer solution. So, we have to obtain the true shear stress, true shear rate information not the nominal shear rate information. Neglect the friction in the barrel one source of error it is already stated in the problem that we can neglect and then establish magnitude of end effects ok. So, that is what we are going to see.

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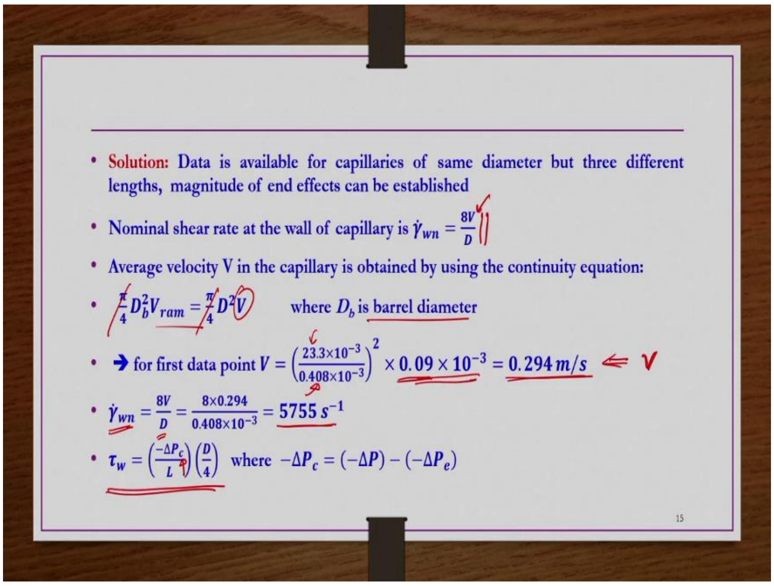

- **Solution:** Data is available for capillaries of same diameter but three different lengths, magnitude of end effects can be established
- Nominal shear rate at the wall of capillary is $\dot{\gamma}_{wn} = \frac{8V}{D}$
- Average velocity V in the capillary is obtained by using the continuity equation:
- $\cancel{\frac{\pi}{4}} D_b^2 V_{ram} = \cancel{\frac{\pi}{4}} D^2 V$ where D_b is barrel diameter
- → for first data point $V = \left(\frac{23.3\times10^{-3}}{0.408\times10^{-3}}\right)^2 \times 0.09\times10^{-3} = 0.294\, m/s \Leftarrow V$
- $\dot{\gamma}_{wn} = \frac{8V}{D} = \frac{8\times0.294}{0.408\times10^{-3}} = 5755\, s^{-1}$
- $\tau_w = \left(\frac{-\Delta P_c}{L}\right)\left(\frac{D}{4}\right)$ where $-\Delta P_c = (-\Delta P) - (-\Delta P_e)$

Since obviously, the data information whatever is given that is given for the 3 different length capillaries, but of same diameter. So, it is possible to establish entrance effects right. So, that is what we are going to see. So, we have seen that nominal shear rate is nothing but $\frac{8V}{D}$, but V is not given, V is not given, V$_{ram}$ is given or the battle velocity is given right.

V is nothing but the average velocity of the fluid when it is passing through the capillary not in the barrel, so that we have to find out. Average velocity V in the capillary is obtained by using continuity equation that $\frac{\pi}{4} D_b^2 V_{ram} = \frac{\pi}{4} D^2 V$. D$_b$ is nothing but barrel diameter, D is nothing but capillary diameter, V$_{ram}$ is nothing but the velocity of the fluid in the ram that is given, V is nothing but average velocity of the fluid in the capillary.

So, this V we have to find out. So, let us say for the first data point V$_{ram}$ is nothing but 0.09 is not it? Right that is given and then D$_b$ barrel diameter is given as 23.3 mm, D that is capillary diameter 0.408 mm that is also given. So, this $\frac{\pi}{4}$ $\frac{\pi}{4}$ is cancelled out. So, V is nothing but D_b^2 by D^2 multiplied by V$_{ram}$ that is given 0.09 mm per second. So, in meters 10^{-3} is there.

So, then overall you get V average or V in the capillary average velocity in the capillary is nothing but 0.294 meter per second, this is for the first data point. So, $\dot{\gamma}_{wn}$; n stands for here the nominal or apparent shear rate that is nothing but $\frac{8V}{D}$. So, V just now you got it for the first data point D is given. So, then 5755 second inverse you will get nominal or apparent shear rate. This is not the true shear rate it is the nominal or apparent shear rate it is true shear rate if it is a Newtonian fluid.

If it is time independent non-Newtonian fluid it is not the true shear rate. We have to make that correction $\frac{3n'+1}{4n'}$ all that also we will be doing. And then τ_w is nothing but $\frac{-\Delta p_c}{L}\frac{D}{4}$. Now we are taking the corrected pressure.

Because we have to find out we have to find out you know entrance effect. If the entrance effect is not there, so, all that pressure information for different capillary lengths L, L_1, L_2, L_3 you might be you may be getting same pressure drop, but that is not true.

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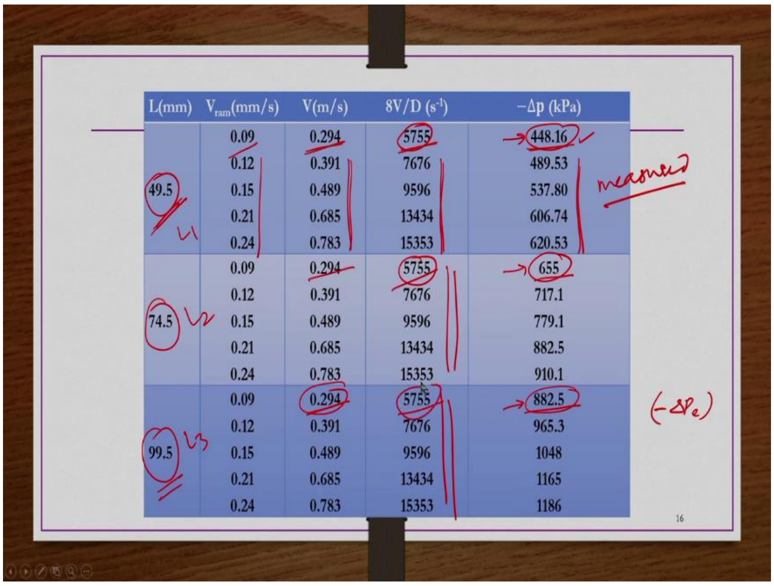

L(mm)	V_{ram}(mm/s)	V(m/s)	8V/D (s^{-1})	−Δp (kPa)
49.5	0.09	0.294	5755	448.16
	0.12	0.391	7676	489.53
	0.15	0.489	9596	537.80
	0.21	0.685	13434	606.74
	0.24	0.783	15353	620.53
74.5	0.09	0.294	5755	655
	0.12	0.391	7676	717.1
	0.15	0.489	9596	779.1
	0.21	0.685	13434	882.5
	0.24	0.783	15353	910.1
99.5	0.09	0.294	5755	882.5
	0.12	0.391	7676	965.3
	0.15	0.489	9596	1048
	0.21	0.685	13434	1165
	0.24	0.783	15353	1186

So, first capillary of 49.5 mm length, V_{ram} is given as 0.09, corresponding V average for the capillary is 0.294 meter per second just now we have found it. And then the $\frac{8V}{D}$ corresponding $\frac{8V}{D}$ is 5755 second inverse and then Δp for the first capillary first ram first V_{ram} value is 44.8, 448.16 kilo Pascal that is also given.

So, similarly for the same first capillary of 49.5 mm you obtain the V_{ram} is given. So, corresponding V average or capillary velocity you find out. So, then corresponding nominal shear rate you find out and tabulate them here and then measured pressure drop this is measured, this is all measured the pressure drop that is given. So, that you just note down.

Similarly, for the second capillary and third capillary also you can see. So, you can see the shear rates you know they are given like you know exactly same. Actually you do not get exactly same, but you get very close to each other when you do the calculation. So, then I have done the average kind of thing and then make and made it to the round figure.

So, then these values are given, but they are very close to each other ok. So, such kind of averaging for the pressure I cannot do because see pressure for the when one capillary you are using 49.5 and then you are maintaining the capillary velocity 0.294 meter per second. What is the pressure drop is required for the first capillary is 448.16 kilopascal.

For the second capillary to maintain the same velocity a capillary velocity you need 655 kilopascal. To maintain the same capillary velocity again if you use further longer capillary of 99.5 mm then you are getting you required to provide 882.5 pressure drop kilopascal's of pressure drop you have to 888 to 882.5 kilopascal of pressure drop you have to provide to get the same capillary velocity for a longer for a longer capillary.

So, then these are very different you know from L_1 let us say if you take this one L_1, this one L_2, this one L_3 from L_1 to L_3 it is almost becoming double. So, we cannot take that averaging. So, that is that why it is coming because of the entrance effects. So, then we have to find out Δp_e. So, now what we do?

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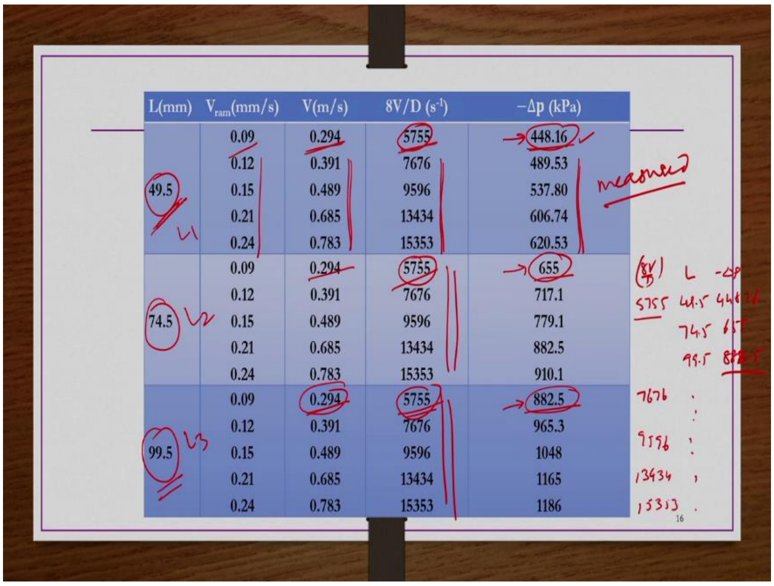

L(mm)	V_{ram}(mm/s)	V(m/s)	8V/D (s^{-1})	−Δp (kPa)
49.5 L_1	0.09	0.294	5755	448.16
	0.12	0.391	7676	489.53
	0.15	0.489	9596	537.80
	0.21	0.685	13434	606.74
	0.24	0.783	15353	620.53
74.5 L_2	0.09	0.294	5755	655
	0.12	0.391	7676	717.1
	0.15	0.489	9596	779.1
	0.21	0.685	13434	882.5
	0.24	0.783	15353	910.1
99.5 L_3	0.09	0.294	5755	882.5
	0.12	0.391	7676	965.3
	0.15	0.489	9596	1048
	0.21	0.685	13434	1165
	0.24	0.783	15353	1186

measured

$\left(\frac{8V}{D}\right)$	L	−Δp
5755	49.5	448.16
	74.5	655
	99.5	882.5
7676	⋮	
9596	⋮	
13434	,	
15313	.	

For each value of this thing $\frac{8V}{D}$ so, let us say first value of $\frac{8V}{D}$is 5755 then corresponding L value. So, let us say when L is 49.5 what is the −Δp measured pressure drop is 448.16. For the same $\frac{8V}{D}$ of 5755, but L is 74.5 then it is 655. Then again same 5755 shear rate, but L is 99.5.

Then what is the corresponding pressure drop or measured pressure drop? 882.5 kilopascal like that this is the tabulation. So, this $\frac{8V}{D}$, this is L, this is −Δp. So, similarly for other $\frac{8V}{D}$7676 you have to do like that. Likewise you know 9596, 13434 and then 15353. Like this you have to have these things.

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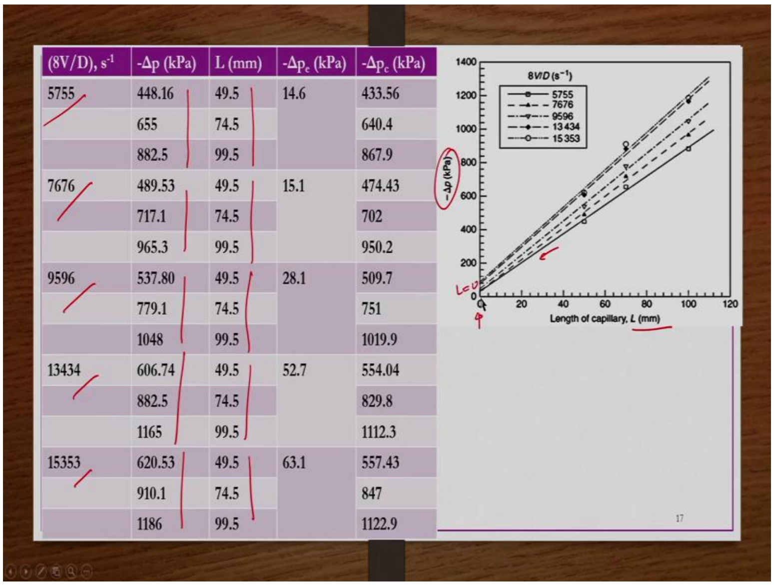

$(8V/D)$, s^{-1}	$-\Delta p$ (kPa)	L (mm)	$-\Delta p_e$ (kPa)	$-\Delta p_c$ (kPa)
5755	448.16	49.5	14.6	433.56
	655	74.5		640.4
	882.5	99.5		867.9
7676	489.53	49.5	15.1	474.43
	717.1	74.5		702
	965.3	99.5		950.2
9596	537.80	49.5	28.1	509.7
	779.1	74.5		751
	1048	99.5		1019.9
13434	606.74	49.5	52.7	554.04
	882.5	74.5		829.8
	1165	99.5		1112.3
15353	620.53	49.5	63.1	557.43
	910.1	74.5		847
	1186	99.5		1122.9

Why because, next what we are going to do? We are going to plot $-\Delta$p versus L for different $\frac{8V}{D}$ for that reason we need to do this one ok. So, that when you arrange when $\frac{8V}{D}$ is 5755 for different L values given pressure drop is this one measured pressure drop this one. So, other $\frac{8V}{D}$ similarly you have like this, this information.

So, whatever the previous slide you have calculated that we have rearranged here. We just rearranged here because for each $\frac{8V}{D}$ we wanted to plot $-\Delta$p versus L. When you plot you get the curve like this, different curves you get like this and then you extend these curves to L is equals to 0 point right.

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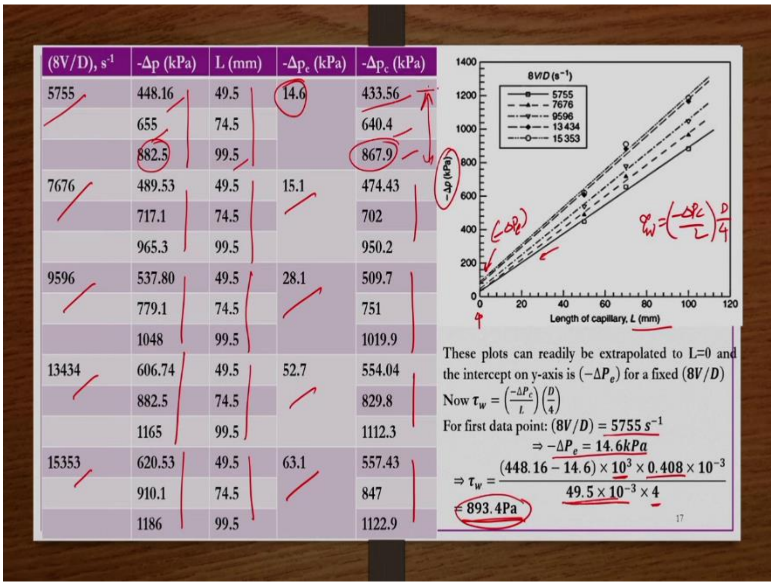

$(8V/D)$, s^{-1}	$-\Delta p$ (kPa)	L (mm)	$-\Delta p_e$ (kPa)	$-\Delta p_c$ (kPa)
5755	448.16	49.5	14.6	433.56
	655	74.5		640.4
	882.5	99.5		867.9
7676	489.53	49.5	15.1	474.43
	717.1	74.5		702
	965.3	99.5		950.2
9596	537.80	49.5	28.1	509.7
	779.1	74.5		751
	1048	99.5		1019.9
13434	606.74	49.5	52.7	554.04
	882.5	74.5		829.8
	1165	99.5		1112.3
15353	620.53	49.5	63.1	557.43
	910.1	74.5		847
	1186	99.5		1122.9

These plots can readily be extrapolated to L=0 and the intercept on y-axis is $(-\Delta P_e)$ for a fixed $(8V/D)$

Now $\tau_w = \left(\frac{-\Delta P_c}{L}\right)\left(\frac{D}{4}\right)$

For first data point: $(8V/D) = 5755\, s^{-1}$

$$\Rightarrow -\Delta P_e = 14.6 kPa$$

$$\Rightarrow \tau_w = \frac{(448.16 - 14.6) \times 10^3 \times 0.408 \times 10^{-3}}{49.5 \times 10^{-3} \times 4}$$

$= 893.4$Pa

So L is equals to 0 when you extend or extrapolate so then you get this $-\Delta$pₑ for each $\frac{8V}{D}$. So, for the first $\frac{8V}{D}$of 5755 second inverse what is $-\Delta p_e$ you get 14.6. So, this 14.6, if you have to subtract from 448.16, so, then you get 433.56, then also from 655 you have to subtract these 14.6. So, then you get 640.4 Δp_c for the second capillary. For the third capillary again from 882.5 you have to subtract this 14.6. So, then you get 867.9 for the third capillary.

So, these are the corrected pressures Δp_c ok. So, this Δp_c you will be using $\tau_w = \frac{-\Delta p_c}{L}\frac{D}{4}$ or $\frac{R}{2}$. So, like that for other values of $\frac{8V}{D}$ you find out what is $-\Delta p_e$ and then what is what are the corresponding Δp_c values.

So, let us say first data point if you take $\frac{8V}{D}$ 5755 second inverse Δp_e we got 14.6 kilopascals. So, corresponding tau w would be 448.16 minus 14.6 that is nothing but 433.56 kilopascal. So, 10^3; then multiplied by D; D is nothing but 0.408 mm capillary multiplied by 10^{-3}.

So, you get meters and then L is nothing but for the first case 49.5 mm first capillary. So, then that is how much it will be into multiplied by 10^{-3} you are doing because that is given in mm. So, we are getting in meters divided by 4 is there. So, as it is, so you will get 893.4 Pascals, you will get the 893.4 Pascals corresponding to this point actually.

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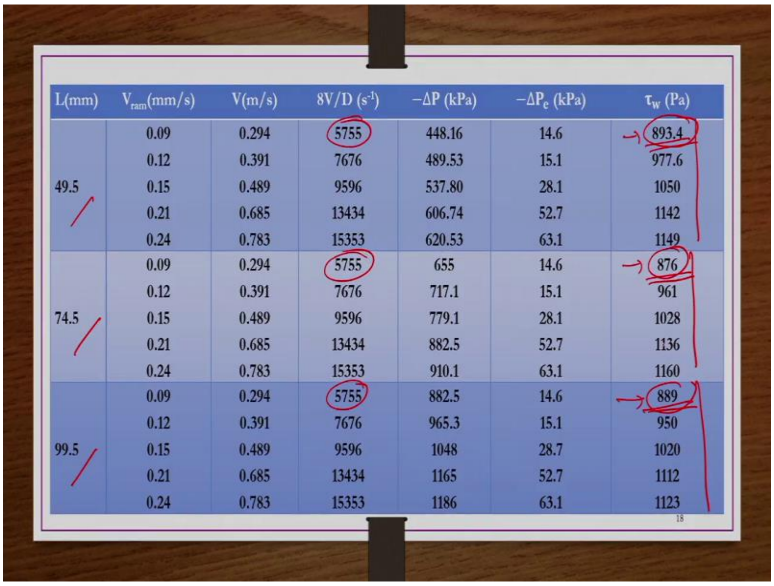

L(mm)	V_{ram}(mm/s)	V(m/s)	8V/D (s^{-1})	$-\Delta P$ (kPa)	$-\Delta P_e$ (kPa)	τ_w (Pa)
49.5	0.09	0.294	5755	448.16	14.6	893.4
	0.12	0.391	7676	489.53	15.1	977.6
	0.15	0.489	9596	537.80	28.1	1050
	0.21	0.685	13434	606.74	52.7	1142
	0.24	0.783	15353	620.53	63.1	1149
74.5	0.09	0.294	5755	655	14.6	876
	0.12	0.391	7676	717.1	15.1	961
	0.15	0.489	9596	779.1	28.1	1028
	0.21	0.685	13434	882.5	52.7	1136
	0.24	0.783	15353	910.1	63.1	1160
99.5	0.09	0.294	5755	882.5	14.6	889
	0.12	0.391	7676	965.3	15.1	950
	0.15	0.489	9596	1048	28.7	1020
	0.21	0.685	13434	1165	52.7	1112
	0.24	0.783	15353	1186	63.1	1123

So, likewise if you do for all other cases you will get for different capillaries you get τ_w values like this. Now you see when $\frac{8V}{D}$ is 5755 right even if you change the capillaries of length of the capillaries. After incorporating the correction after incorporating the correction what is the tau w that you can see, 890, 875, 890 like that. They are almost same they are not exactly same, but they are very close to each other.

As I said this $\frac{8V}{D}$ is also exactly they are not same. They are close to 8; 5750; 5760 like that they are close to each other. So, then we have done the averaging and written like this. So, τ_w also after incorporating this correction what we get? This τ_w values even if you are changing L value they are not changing.

Because we have already incorporated that correction that is subtracted $-\Delta p_e$ information from the measured pressure drop information ok that is it. So, this is how we how to find out. So, now again what we do? We do the averaging of these 3 values and then write it as one value here.

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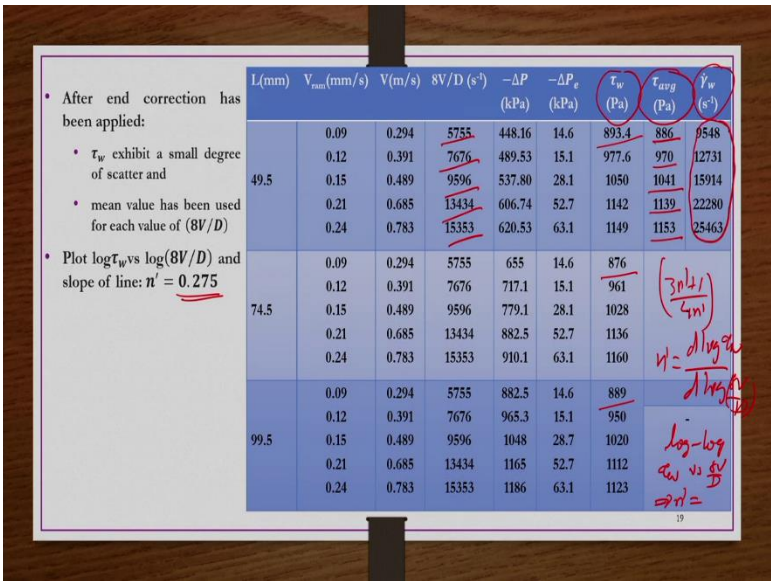

- After end correction has been applied:
  - τ_w exhibit a small degree of scatter and
  - mean value has been used for each value of $(8V/D)$
- Plot $\log\tau_w$ vs $\log(8V/D)$ and slope of line: $n' = 0.275$

L(mm)	V_{ram}(mm/s)	V(m/s)	8V/D (s^{-1})	$-\Delta P$ (kPa)	$-\Delta P_e$ (kPa)	τ_w (Pa)	τ_{avg} (Pa)	$\dot{\gamma}_w$ (s^{-1})
49.5	0.09	0.294	5755	448.16	14.6	893.4	886	9548
	0.12	0.391	7676	489.53	15.1	977.6	970	12731
	0.15	0.489	9596	537.80	28.1	1050	1041	15914
	0.21	0.685	13434	606.74	52.7	1142	1139	22280
	0.24	0.783	15353	620.53	63.1	1149	1153	25463
74.5	0.09	0.294	5755	655	14.6	876		
	0.12	0.391	7676	717.1	15.1	961		
	0.15	0.489	9596	779.1	28.1	1028		
	0.21	0.685	13434	882.5	52.7	1136		
	0.24	0.783	15353	910.1	63.1	1160		
99.5	0.09	0.294	5755	882.5	14.6	889		
	0.12	0.391	7676	965.3	15.1	950		
	0.15	0.489	9596	1048	28.7	1020		
	0.21	0.685	13434	1165	52.7	1112		
	0.24	0.783	15353	1186	63.1	1123		

$\left(\frac{3n'+1}{4n'}\right)$

$n' = \frac{d\log\tau_w}{d\log\left(\frac{8V}{D}\right)}$

log-log τ_w vs $\frac{8V}{D}$

$\Rightarrow n' =$

So, we have average τ_w is 886, 970, 1041 and then 1139, 1153 for different $\frac{8V}{D}$ values ok. So, length effect is entrance effect has been taken care. So, now true shear stress you got, this is true shear stress only ok. This even this also true shear stress, but we have taken average over all 3 L values. So, that is because averaging because they are very close to each other. Even if you take one case that is also not going to affect any much any anything on the rheology, but shear rate also we need to have true shear rate.

So, what we need to have? We need to have this correction $\frac{3n'+1}{4n'}$we need to have for each case right n’ is nothing but $\frac{d\log\tau_w}{d\log\left(\frac{8V}{D}\right)}$. So, for each $\frac{8V}{D}$ τ_w information you already got. So, that you plot on a log-log scale you plot τ_w versus $\frac{8V}{D}$ then you find out n’.

So, that n’ you will get that n’ is nothing but 0.275; that means, the material is highly shear thinning material whatever the 4 percent CMC in water that solution you got that is highly shear thinning. Because it’s n’ value is very small 0.275. Now, you get $\frac{3n'+1}{4n'}$ and then multiply that correction for with the corresponding $\frac{8V}{D}$ value.

So, then you get true shear rate information this right. This is how you have to incorporate corrections for the entrance effect in the capillary viscometer if at all if you do not have a large sufficiently large $\frac{L}{D}$ capillaries.

(Refer Slide Time: 61:00)

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References for this lecture - the entire lecture is prepared from this lecture from this reference book by Chhabra and Richardson. However, similar information you may also find in the remaining reference books.

Thank you.