Transport Phenomena of Non-Newtonian Fluids
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Lecture - 05
Capillary Viscometers

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids the title of this lecture is Capillary Viscometers. The question is that, how do we know that a given fluid is Newtonian or non-Newtonian? If it is non-Newtonian what kind of non-Newtonian it is? That we should know right, if we should know that information.

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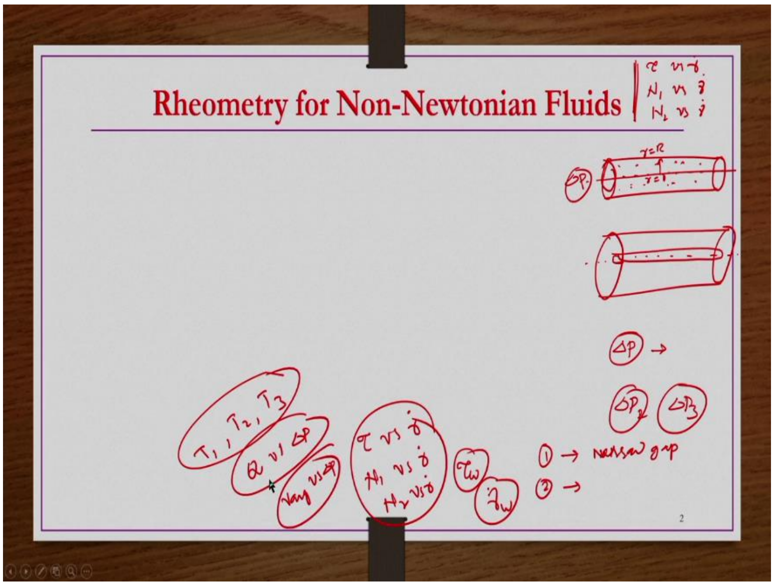

So, what information should we have about the fluid? We should have a information about a the so called shear stress versus shear rate and then normal stress differences versus shear rate, this information we should have right.

So, that means, with respect to the shear rate how the shear stress and normal stresses are varying that information we should have for a material. So, for a given shear rate that means, the material whatever the fluid for which we wanted to measure the rheology, we should allow that material to undergo some kind of deformation.

When it is undergoing deformation then only we have a deformation rate, then only corresponding shear stress or normal stress would be there or both shear stress and normal stresses would also be there depending on the nature of the fluid.

So, but that means, you know that means what? We have to have a kind of a rheology measuring device or system such a way that, we should allow that material to undergo certain kind of deformation right.

So, now, when we talk about deformation any geometry whether you take a flow through pipes, any geometry like you know flow through pipes you take or you know you take a concentric cylinder geometry right the material if it is undergoing some kind of flow. So, there would be deformation that is what we understand right.

So, now you know when the material is having you know are allowed to flow through certain geometry right, so then what happens? We understand that there would be a kind of relation pressure drop versus volumetric flow rate in general, if the flow is because of the pressure drop such kind of information we have. We also know that you know the velocity whatever the velocity of the fluid is there within the system that within the system whatever the fluid is flowing.

So, because of that flow the velocity of the fluid elements varies from point to point that is what we understand for any geometry that we take right. So, when the flow and when the velocity is changing there would be a kind of a change in velocity. So, then deformation would be there, so that means, deformation or you know shear rate of deformation or shear rate is also changing from point to point.

So, then accordingly corresponding shear stress and a normal stresses would also be changing, is not it? So, that means, you know we are not having one single value of shear stress or shear rate or normal stress for a given one single value for a ΔP.

Let us say you are taking the flow you know, you are allowing the flow to take place because of some pressure difference right because of that pressure difference you are making the flow to occur, but let us say if you maintain certain pressure drop and then flow is taking place. So, within this flow geometry at each and every location the velocity component you know all the velocity components would be having different values right.

So, now correspondingly at each and each and every location within the flow domain the shear rate and then shear stress and then normal stress differences would also be different obviously. So, now, which value should we take because your pressure drop is only one single value for given one particular situation for Other situation or other velocity you change again one some other pressure difference like ΔP_2 or ΔP_3 something like that that is a different thing.

But if you are maintaining certain pressure difference, so, then you are having one kind of one flow which is having certain average velocity or volumetric flow rate whatever. But within the system you are having different velocity at each and every locations. If you change the location within the system velocity is changing so shear rate is changing. So, the shear stress and normal stress would also be changing right. So, now, which one should we consider? So, that is the question.

We cannot consider these values at all locations. So, we should have a kind of device or rheology measuring system such a way that the flow domain or the region that is available for the fluid to flow that should be very narrow; that should be very narrow. So, that whatever the changes are there in shear stress or shear rate from one location to the other location they would be very small that those changes may be neglected.

So, that is one important thing that we have to observe before going into the different types of rheometers that we are viscometers that we are going to study in order to measure the rheology of a given fluid right. So, first thing is that one; narrow gap or flow region has to be narrow so that change in shear rate or shear stress or normal stress whatever is there that change should be very small from one extreme of the flow domain to the other extreme of the flow domain.

Let us say you know this pipe only if you take. So, if you change r value from 0 to r value R; R value 0 is nothing but centre of the pipe and then r value small r is equals to R is nothing but wall of the tube or cylinder wall circular tube whatever you have taken.

If this difference from 0 to r this difference is very very small maybe 1 mm or 2 mm. That means, it is in terms of meters it is having 10 power minus 3 meters. So, then obviously, the change in shear rate or shear stress from 1; from centre to the wall is going to be very small, is going to be very small. So, that can be neglected that is one point.

Second point is that in such kind of studies whatever geometry you take experimentally what are you measuring that is important, you are not measuring directly shear stress or shear rate directly is not it? So, whatever the information τ versus $\dot{\gamma}$ or N_1 versus $\dot{\gamma}$ or N_2 versus $\dot{\gamma}$ this information whatever is required for your analysis that should be you know having some relation with the measurable quantities because τ, $\dot{\gamma}$, N_1, N_2 etcetera you are not going to measure directly.

Let us say if you have a pipe flow, flow through a pipe. So, then what you can measure? You can measure the volumetric flow rate for a given pressure drop experimentally, that is what you can do. If you are varying the temperature then for different temperatures you can do these things right. So, let us forget about the temperature and all that for the time being.

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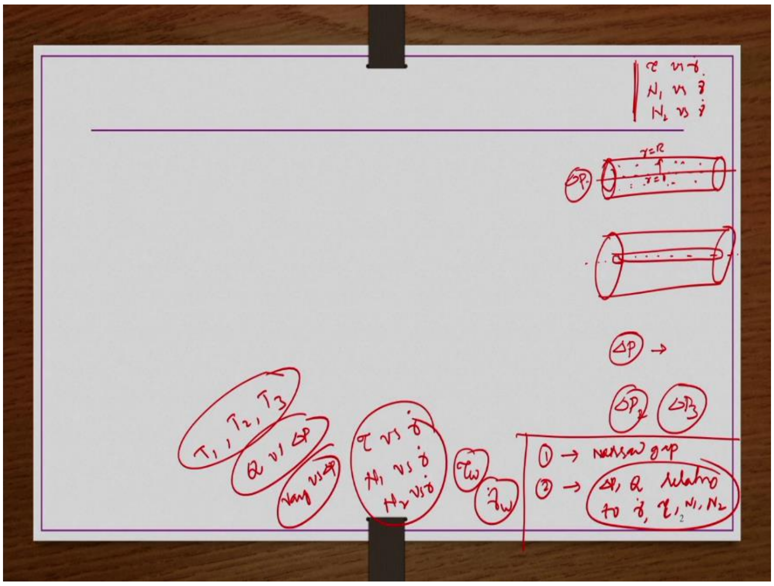

So, if you maintain some ΔP and then corresponding volumetric flow rate you get that means, v average velocity versus ΔP you are getting, that information only you are getting for this case let us say pipe flow case I am talking about right.

So, then what is the thing is that you have to have a kind of relation, where you can make a connection between this shear rate, shear stress with measurable quantities something like pressure drop and then volumetric flow rate etcetera. So, relating ΔP, Q to $\dot{\gamma}$, τ, N_1, N_2 etcetera that is the second point, that is these are the two important things one has to keep in mind before going into details of any type of rheometers or viscometers which are used for measuring the rheology of the fluids right.

So, now we are going to discuss different types of rheometers or viscometers in this and then coming couple of lectures ok. So, but primary target the geometry the flow first we have to have a kind of flow geometry where we can allow the fluid of unknown rheology to undergo certain kind of deformation right.

And then that deformation should be such a way that the change in shear rate or shear stress or normal stress from one location to the other location or one extreme to the other extreme of the flow geometry should be very small. And then the second one is that whichever the geometry you consider you make a you know certain balance equations and then develop equations so that you can relate these measurable quantities like you know volumetric flow rate, pressure drop etcetera to the shear stress, shear rate etcetera ok.

So, these are the two things one has to keep in mind before starting details of a so called you know rheometry of non-Newtonian fluids. Of course, if the fluid is Newtonian then also the process is exactly same only thing that if it is a Newtonian fluid the shear stress versus shear rate curve would be you know straight line passing through the origin having the constant slope μ, viscosity μ ok otherwise the process is same. So, now we will be discussing rheometry for non-Newtonian fluids.

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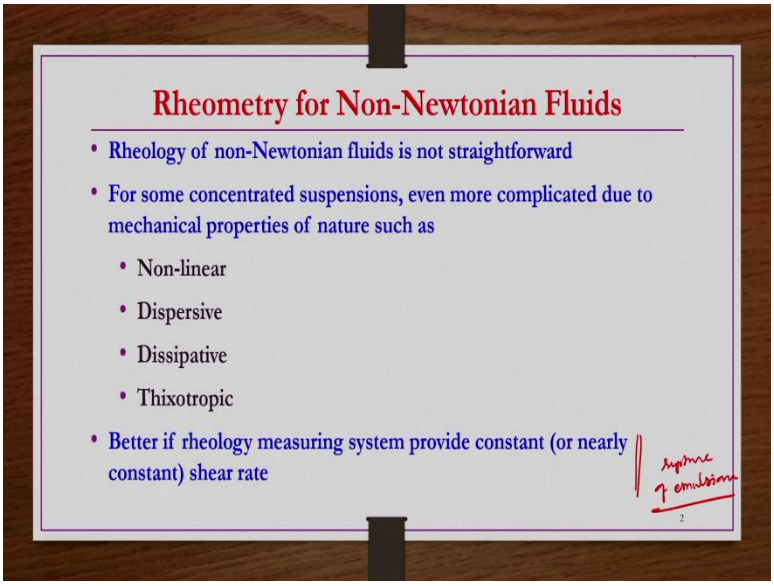

Rheology of non-Newtonian fluids is not straightforward obviously, because you do not know what is the nature of non-Newtonian behaviour that material is having. Is it shear thinning shear thickening or is there any plastic behaviour like in viscoplastic material or is there any elastic behaviour like in the kind of viscoelastic material etcetera.

So, those many complications are there in non-Newtonian fluids. So, then obviously, the rhelogy of these non-Newtonian fluids is not going to be straightforward, you need to have a proper device to measure the rheometry or the rheology of these non-Newtonian fluids. Further the complications may even increase for some kind of suspensions where the mechanical properties maybe of nature such as non-linear, dissipative, dispersive and thixotropic nature.

When the mechanical properties of some concentrated suspensions are having such kind of properties like you know our nature like non-linear nature dispersive, dissipative and thixotropic nature. Then rheological measurements of such kind of non-Newtonian fluids or concentrated suspensions which are non-Newtonian fluids that becomes even more difficult. So, now, before going into the details one thing I wanted to mention.

So, most of the non-Newtonian fluids are not single phase materials, majority of non-Newtonian fluids are a kind of multi-phase particulates systems like emulsions or suspension soups etcetera. These kind of materials where you know there are some kind of constituents within the solution that we have taken right. So, that is what we should realize that majority of it is not true that for all, that all the non-Newtonian fluid fluids are not single face fluids. So, there are you know some exemptions may be there, but majority of the non-Newtonian fluids or some kind of suspensions or particulate matters or you know emulsions like that ok.

So, now obviously, when you have such kind of constituents within the material right. So, you need to have a kind of narrow gap flow geometry for a rheology measurements as just explained. So, but when you are allowing or pushing or pushing through these concentrated suspensions or emulsions to pass through such narrow channels then what happens? The rupture of you know particulate systems or emulsions may take place. So, that is another problem one may have; so right.

So, though it is better to better if the rheology measuring system provide a constant or nearly constant shear rate you know we need to have a kind of case you know where the rupture, so called rupture of emulsions etcetera should not take place. Because if you if the emulsions or particulate systems etcetera some breakdown of this of the material is taking place. So, then again it is going to have a kind of you know change in properties.

So, it is obviously, better that you have a rheology measuring system that provide constant or nearly constant shear rate constant is not possible, but nearly constant share rate is possible anyway right. So, but and this constant, nearly constant shear rate is possible by using narrow gap shearing geometries such as co-axial cylindrical geometry, cone and plate geometry etcetera.

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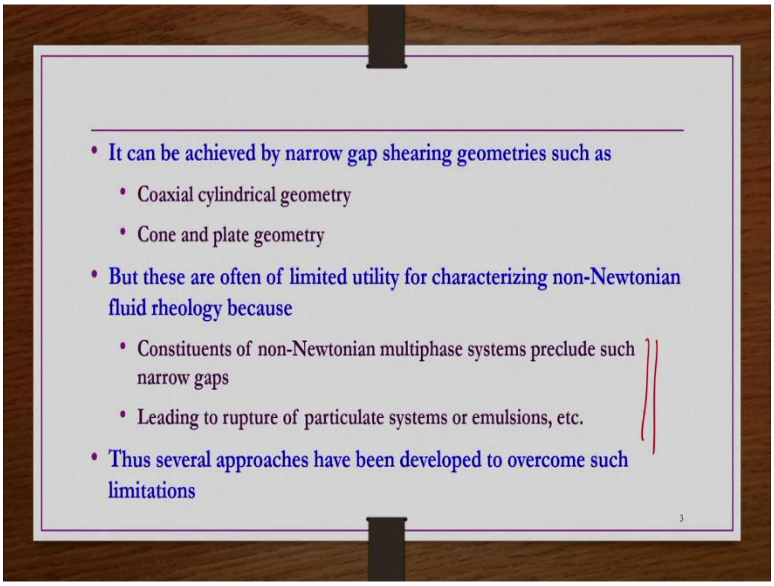

But the problem is that these are often of limited utility for characterizing non-Newtonian fluids because as mentioned constituents of non-Newtonian fluids non-Newtonian multi-phase systems precludes such narrow gaps leading to rupture of particulate systems or emulsions etcetera.

Because of such kind of reasons so then even the narrow gap shearing geometries even though theoretically they are good for a reliable measurement of rheology of such materials because of such kind of problems like rupture of particulates systems or emulsions etcetera these are you know these narrow gap measuring systems are preclude.

You know because of this non-Newtonian multi-phase system they precludes the use of such a narrow gaps ok. However, need not to worry when we go into the details we will be having certain situations wherever there is an inconsistency in measuring of rheological behaviour.

So, that means, inconsistency is then that means, there is some possible source, there is a source of error and then we can make. So, then we have to make adjustment or correction for those sources of error.

So, those things anyway we are going to see, but you know this is the problem one of the problem of a narrow gap shearing geometries if you are using for measuring the rheology of non-Newtonian fluids. So, but there are you know several approaches have been developed to overcome such limitations also. So, then we are going to see in detail one after another in coming couple of lectures.

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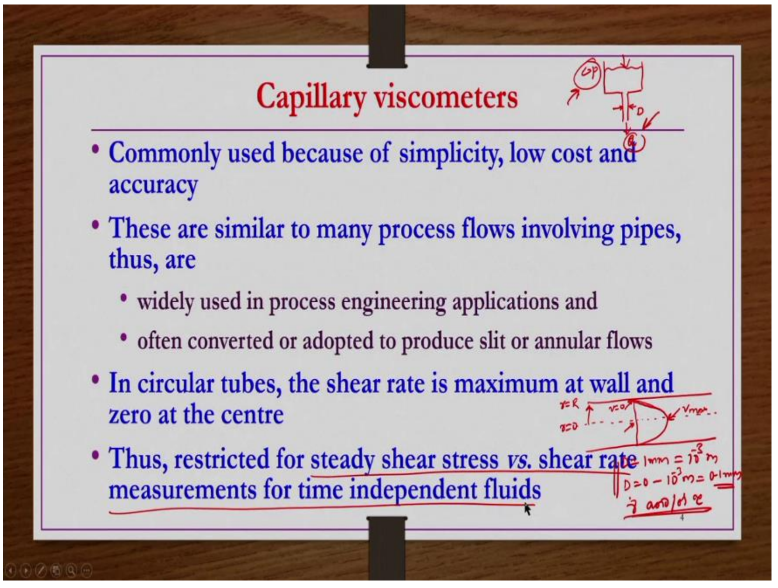

So, now we start with capillary viscometers ok. So, what is a capillary? Capillary is a straight tube of a very small diameter compared to the length of the tube right. So, that is large length by diameter ratio tubes whatever circular tubes are there those things we can regard as a capillaries.

That means, if you have straight pipe whose diameter is 1 mm, but the length is let us say 200 mm or 150 mm. So, then we can say that straight tube of 200 or 150 mm length and then 1 mm diameter that can be regarded as capillary in general ok.

Now, this capillary viscometers what happens in general? We have a container in general roughly like we are going to see details later anyway. So, this bottom of this container is having a provision to fix a capillaries of a different diameters. So, this capillary is having let us say some diameter D. So, whatever the material whose rheology you wanted to measure that you pour in the container and then you apply certain force or the pressure difference. So, that you can get the material flowing out.

So, then for ΔP versus Q information you can obtain when you are applying certain or when you are maintaining certain pressure drop the material that contain or whatever is said that will flow through the capillary once you open the opening at the bottom right.

So, you measure this volumetric flow rate with respect to different ΔP, that is the simplest way of doing this capillary viscometer or you know applying these capillary viscometers for rheological measurements. So, that is crudely that is what a, that is what a simple capillary viscometer right.

So, then what you see here it is setup is very simple you just need a container, you just need a capillary of a different diameters the container should have openings. Such a way that you know varying diameter capillaries can be attached to that one right, you should have a provision to measure volumetric flow rate, and you should have a provision to measure the pressure drop that is it. So, it is very simple and then cost also it does not required much cost for this and then it is also found to be reliable ok.

So, these capillary viscometers are commonly used because of their simplicity, low cost and accuracy and these are similar to many process flows involving pipes, because you know in general industries we have a situation like you know through pipes materials are flowing in general right.

So, that is having you know similar behaviour like you know flow through capillary kind of thing. So, they are having similarity these capillary viscometers are having similarity to many process flow situations involving pipes.

Thus, these are widely used in process engineering applications and often converted or adopted to produce slit or annular flows wherever required so that inline rheology of that material which is flowing through pipe can be measured without any difficulty ok. That is the one of the major advantage from the industry point of view that is the major advantage of capillary viscometer from industrial application point of view. In circular tubes the shear rate is maximum at wall and zero at the centre.

Because let us say what we understand from our previous transport phenomena course that we have studied in our UG courses that you know if there is a fluid is flowing through a tube circular tube what happens you know under the fully developed flow conditions the flow rate you know something like parabolic like this for Newtonian case for example, right. So, at the centre we have the maximum velocity at the tube wall we have the 0 velocity right.

So, then velocity is changing it is decreasing from r is equals to 0 to r is equals to R, R is the radius of the tube. So, as we increase as we move towards the wall from the centre the velocity is decreasing. That means, you know shear rate is increasing, shear rate is 0 at the centre, shear rate is 0 at the centre and then gradually it increases and then it attains a maximum value at the wall ok.

Now, let us say if the capillary is having diameter let us say 1 mm right, so that means, 10^{-3} meters ok. So, whatever the shear stress that is there you know that shear stress would be maximum at the wall and then minimum or 0 at the centre its it should be 0 at the centre and maximum at the wall.

So, that would be you know how much? If it is your D is changing from 0 to 10^{-3} meters or 0 to 1 mm such a small variation. So, that is not going to affect much in the values of shear rate and shear stress and you know and shear stress and shear rate and shear stress.

So, that is the advantage that we are taking this case anyway here. So, thus restricted for because of these variations such kind of capillary viscometers are often restricted for steady shear stress versus shear rate measurements for time independent fluids in general. So, time dependent fluids it may be not reliable to go for a capillary viscometers ok.

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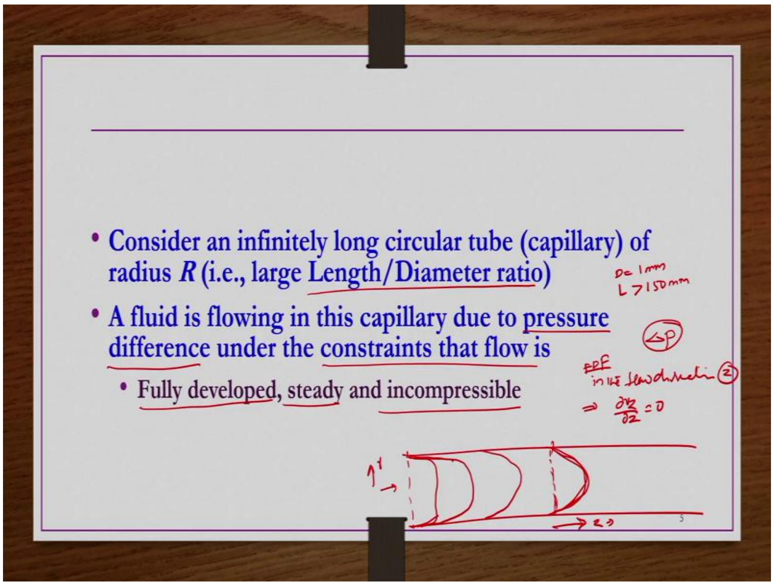

Now, what we do? We try to develop you know working principles for these capillary viscometers, developing working principles or equations for this capillary viscometer is nothing but you are going to develop a relation for shear stress in terms of measurable property something like pressure drop or volumetric flow rate or average velocity something like that.

Similarly, you are going to develop a relation for the shear rate $\dot{\gamma}$ as function of either pressure drop or volumetric flow rate or you know average velocity whatever that is what we are going to do now here ok. So, we see what are these relation exactly by development here.

So, now, consider an infinitely long circular tube which is a capillary of radius R. So, that you can have a large length by diameter ratio which is more than 150 or 200 something like that right. So, let us say if you take the capillary of diameter 1 mm.

So, the length should be at least 150 mm or higher minimum 150 mm it should be there. So, that $L/_D$ is going to be 150 or higher ok that is what mean by large $L/_D$ ratio length by diameter ratio. Consider a fluid is flowing in this capillary due to pressure difference due to pressure difference that is because of there is some pressure differences say because of that one pressure because of that pressure difference the flow is taking place under the constraints the flow is taking place because of the pressure drop.

But we are taking some constraints into the consideration. What are those considerations? That is the flow is fully developed, steady flow and incompressible fluid right. What do you mean by fully developed flow? So, that means, in the flow direction fully developed flow in the flow direction, whatever the let us say flow direction is z direction here and then there is velocity is in the z direction is nothing but v_z. So, $\frac{\partial v}{\partial z}$ should be 0 that is what mean by fully developed condition or pictorially.

So, you have something like this a tube like this. So, at the entrance if you draw a velocity profile let us say you have the uniform velocity you are given initially. So, then it develops something like this further you know it develop something like this and then gradually at certain locations it becomes parabolic like this. So, this parabolic profile I am talking about Newtonian fluid, it is not going to change even if you increase even if you go higher values of the z, this is z direction, this is r direction

So, this whatever the velocity that is the velocity component or you know stress etcetera are there. So, they are not going to change in the flow direction; they are not going to change in the flow direction that is what mean by fully developed flow.

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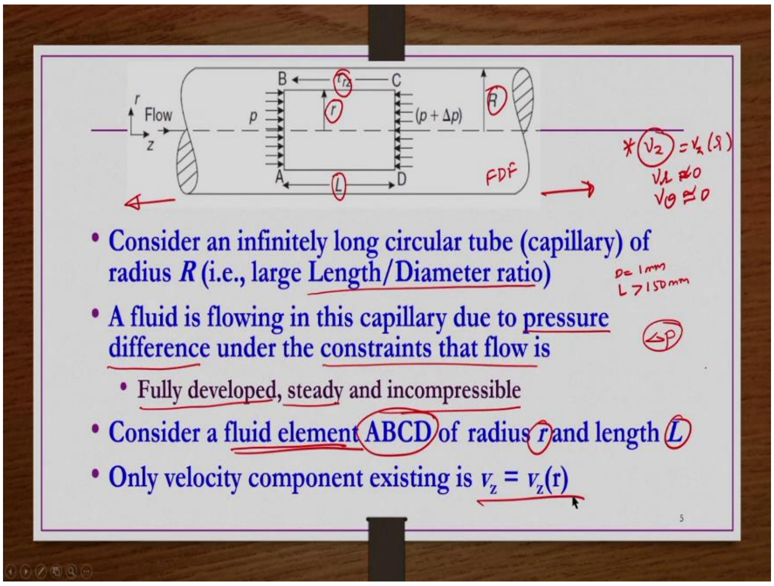

So, now, within this flow geometry consider a fluid element and then name it ABCD, the fluid element is having radius r and length L right pictorially if you see. So, it is an infinitely long cylinder.

So, then what we are doing? We are not drawing the locations near the entry and then near the exit, but we are taken somewhere in between the location where fully developed flow is established, where the fully developed flow is established and then flow is steady right.

So, now, here within this one we have taken a fluid element ABCD right, the length of the fluid element is L the radius of this fluid element is r the radius of the tube is R, pressure at the inlet is p pressure at the outlet is p + Δp.

So, now here in this case we have taken the geometry such a way that you know only velocity component existing is v_z or velocity component in the flow direction is dominating, it is very dominating compared to the velocity components in the other directions. So, v_θ v_r are going to be 0 or approximately 0 they are very small compared to the values of v_z whereas, v_z is function of r only.

So, now that is what the constraints we are having. We are having only v_z component of velocity which is changing only in the radial direction that is in the r direction. So, because only v_z is existing and its function of r.

So, then only stress that is going to exist here this shear stress τ_{rz} other component of the shear stress would not be there and then normal stresses we are not considering here in this possible way ok, because we are doing this analysis for a time independent non-Newtonian fluids. So, only velocity component exist is v_z which is function of r.

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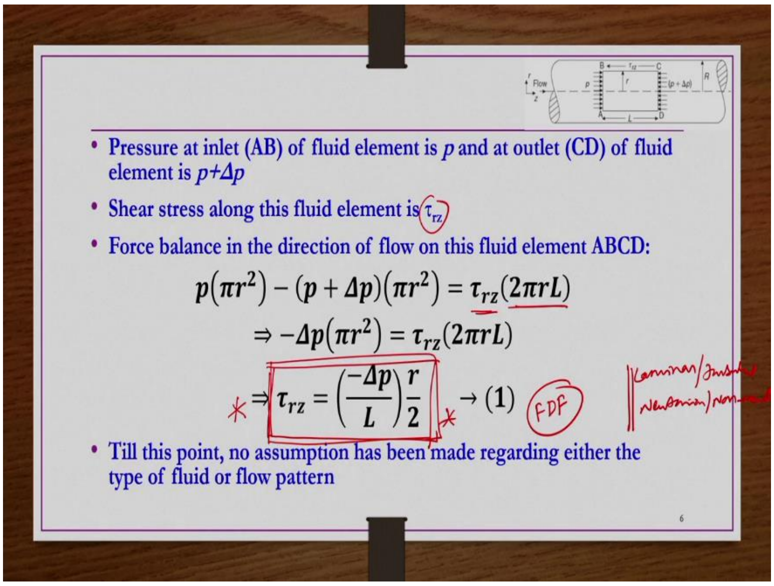

The same picture is shown here again the pressure at inlet AB is p and then pressure at outlet CD is the p + Δp. So, that you know we can do the balance subsequently the shear stress along this fluid element is only τ_{rz} is existing that we have seen right.

Now, if you make a balance; if you make a balance like you know whatever the force balance that you do let us say the force due to this pressure whatever is there or pressure difference that should be balanced by force whatever is there because of the shearing action. So, that is p(πr^2) is the force at the inlet of this fluid element.

Because the fluid element is having the radius r remember this, this is a circular tube we are drawing two dimensional thing and then this fluid element is also cylindrical the fluid element is also cylindrical whose radius is R, but length is L. So, cross section area of this fluid element is πr^2. So, pressure at the outlet is p + Δ p multiplied by the cross section area of the outlet is again πr^2.

So, difference if you do or difference of these two if you do that should be balanced by the shearing force. So, shear stress is τ_{rz} and then if you multiply by the surface area of that element whichever the element fluid element ABCD you are taken. So, then you get the shearing force; the surface area of that ABCD fluid element is nothing, but 2 π r L.

So, now, that means, if you do the simplification you will get $\tau_{rz} = -\left(\frac{-\Delta p}{L}\right)\frac{r}{2}$. So, for a given case when you are maintaining constant pressure drop to get a certain flow rate. So, the shear stress is found to increase linearly with r radial coordinate radial position. So, that is at centre it is 0.

And then as you move towards the wall gradually the shear stress increases and when you reach the wall of the capillary the shear stress is going to be maximum because r is equals to R that is the maximum value in the radial direction.

So, then that value of shear stress would be the maximum that is known as the wall shear stress right. Remember this equation is only for this capillary viscometer for different geometries you may take slightly different expressions you may get, for this geometry what we see? This τ_{rz} is directly proportional to the radial position r ok.

So, till this point what we have done did we consider anything about the nature of the flow or the nature of the fluid? Nature of the flow in the sense whether it is laminar or turbulent did we consider? No. Did we consider Newtonian or non-Newtonian? No, we did not consider. Only thing that we consider the flow has to be fully developed flow and then it has to be incompressible steady, incompressible, fully developed flow that is only we have considered.

So, even turbulent flow also it is possible that we may have a fully developed flow conditions right. So, laminar it is anyway it is possible to have a fully developed flow conditions that we have already seen in transport phenomena courses right. Whether the fluid, whatever the fluid element that we have taken we did not say whether it is a Newtonian fluid element or non-Newtonian fluid element.

So, that means, indirectly this equation whatever developed for the capillary viscometers. Whether you take the Newtonian fluid or non-Newtonian fluid, the flow is whether laminar or turbulent you are going to have this relation valid.

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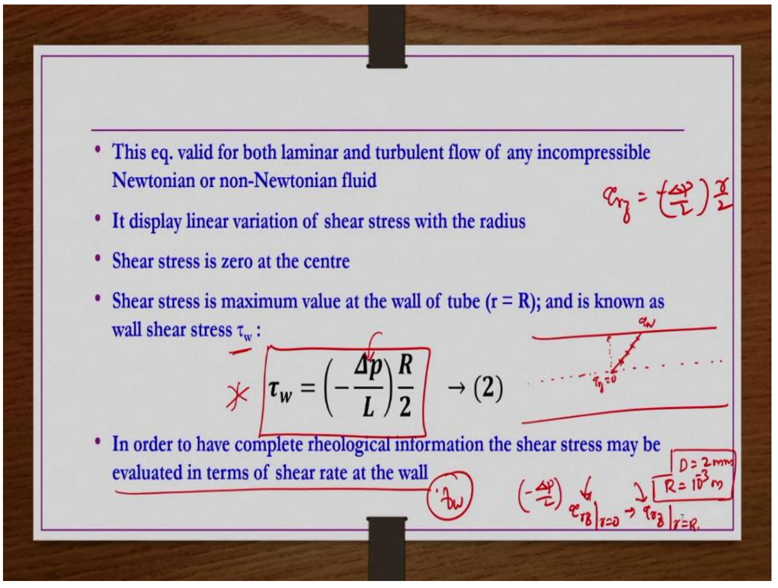

So, this equation valid for both laminar and turbulent flow of any incompressible Newtonian or non-Newtonian fluid as long as the flow is fully developed flow, as long as the flow is fully developed flow. And then also it display linear variation of shear stress with the radius that means, shear stress is 0 at the centre because at the centre r is equals to 0. If you substitute r is equal to 0 in previous equation $\tau_{rz} = -\left(\frac{-\Delta p}{L}\right)\frac{r}{2}$ you will be getting τ_{rz} = 0 and it is maximum at the tube wall that is at r = R.

Because R is the maximum value possible in this geometry and is known as this whatever the maximum value of shear stress is known as the wall shear stress. That means, τ_w is nothing but $\left(\frac{-\Delta p}{L}\right)\frac{R}{2}$ right.

So, in order to have a complete rheological information the shear stress may be evaluated in terms of the shear rate at the wall. Because, now we have seen this shear stress let us say we take a example which value of shear stress should we take that is the problem right.

So, let us say now this is the t cube. So, what we have at the wall? It is having τ w maximum value at the centre it is having τ_{rz} is equals to 0 value right, it is linearly increasing as you move from centre to, you know wall of the tube. Now should you take τ_{rz}value at this location, at this location, at this location, at this location or at the wall or at the centre which one should you take? For that region only if you have a very small capillary.

Let us say if you have D is equals to 2 mm only so that means, r = 10^{-3} meters. So, if for a constant $\frac{\Delta p}{L}$ for a constant pressure gradient you know if your diameter of capillary is very small.

So, then your shear stress variation in the shear stress from τ_{rz} at r = 0 to τ_{rz}at r = R is going to be very small. The difference between these two values is going to be very small. That is the reason we are insisting on using a very narrow gap shearing geometries that is the reason we are insisting on very narrow gap shearing geometries for measuring the rheology.

So, now one thing out of the shear stress versus shear rate one thing is clear, shear stress information we already got in terms of measurable quantities like. Now, here in the case of here we are if you know the pressure drop. So, then you can easily find out the shear stress because R that is the radius of capillary is known, L length of the capillary is also known. So, that means, if you know the pressure drop and then dimensions of the capillary then you can measure the shear stress without any difficulty.

So, but now we need to have corresponding shear rate at the wall that information also we should have. Then your τ_w versus $\dot{\gamma}_w$ information is available then you can know the rheology of the material. So, for that purpose what we do now?

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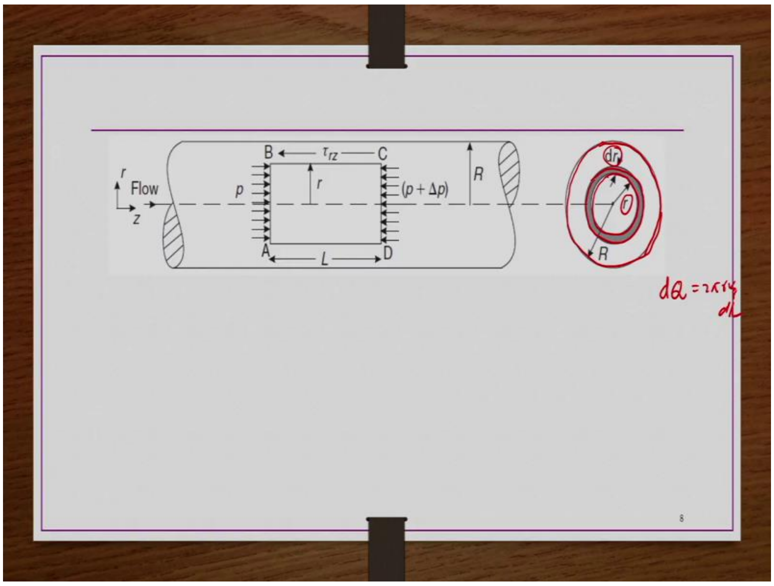

We take the cross section of geometric capillary whatever is there, we take the cross sectional view now we take ok. So, within this one we take one channel or you know one annulus right whose inside diameter is r and then outside diameter is r + dr right.

So, then if you wanted to know change in volumetric flow rate dQ that is taking place in this considered annulus of inner radius r and then outer radius R + dr then you can do dQ $=2\pi r v_z dr$ if you do you can get the volumetric flow rate information.

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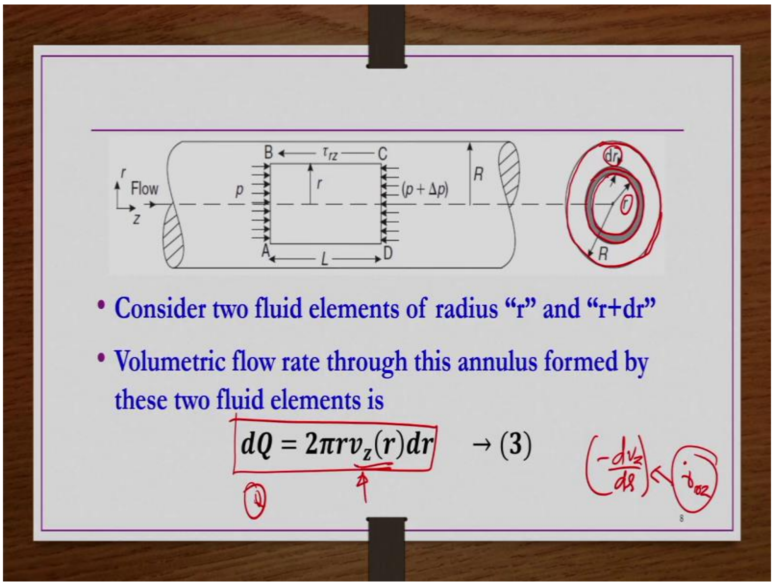

Consider two fluid elements of radius r and r + dr. So, the volumetric flow rate through this annulus formed by these two fluid elements would be dQ = $2\pi r v_z dr$, v_z is function of r.

Now, we are not interested in finding out what is v_z because for us if you integrate your left hand side obviously, you will get volumetric flow rate for the entire system that capillary right. So, that is experimentally you know, but we are not interested in v_z.

So, then what we do? We further play with this equation such a way that in the right hand side we get something like $\left(\frac{-dv_z}{dr}\right)$ which is nothing but $\dot{\gamma}_{rz}$which is corresponding to τ_{rz} ok. So, that is what we are going to do; that means, indirectly your shear rate may be related to the volumetric flow rate of the fluid passing through the capillary ok.

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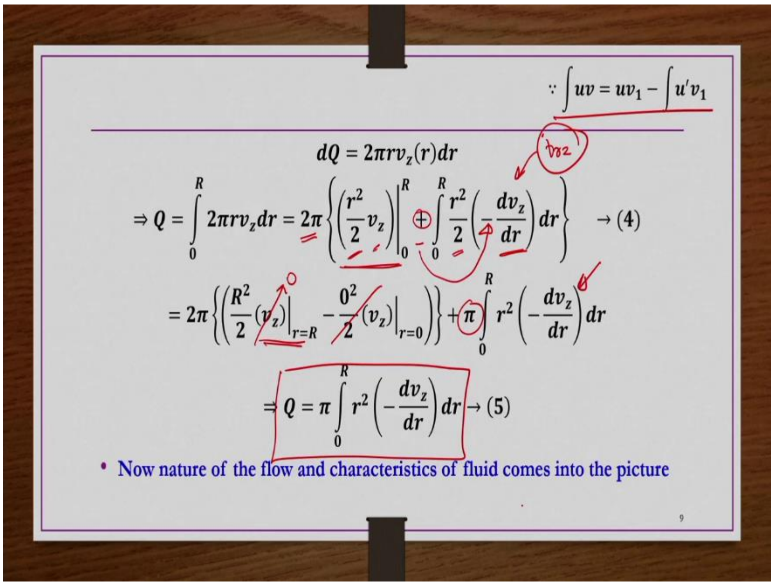

So, now this equation when you integrate you use this you know integration rule $\int uv = uv_1 \int u'v_1$. So, now, that when you apply 2 π is constant. So, integration of r is $\frac{r^2}{2}$ and then v_z we are keeping as it is.

Then minus integration of again integral of integration of r is $\frac{r^2}{2}$ and then differentiation of v_z is $\left(\frac{dv_z}{dr}\right)$. So, then what we I am doing? I am taking this minus here and then writing +

here because this $\left(\frac{-dv_z}{dr}\right)$ is nothing but $\dot{\gamma}_{rz}$ right. So, do not worry about that value substituting $\dot{\gamma}_{rz}$ right now here we will be doing later on.

So, first we will be trying to substitute the limits 0 and then R within this expression. So, then what we see first expression $\frac{R^2}{2}$ and then v_z at r = R, r = R is nothing but the tube wall at the tube wall velocity is anyway 0 because of the no slip velocity and then minus at centre r = 0. So, $\frac{0^2}{2}$ v_z so that means, again this is this quantity is also 0.

So, that means, altogether whatever this $\frac{r^2}{2}$ v_z, if you substitute upper and lower limit. So, then over all you are going to get 0 anyway + this 2 π multiplied by $\frac{r^2}{2}$. So, then 2, 2 cancels out. So, then I can have only π here $\int r^2 \left(\frac{-dv_z}{dr}\right)$. So, we do not know what is $\frac{dv_z}{dr}$ as of now. So, then we cannot we cannot substitute this 0 and then r values here and then indeed we do not wanted to substitute. So, what do you get? You get $Q = \pi \int_0^R r^2 \left(\frac{-dv_z}{dr}\right) dr$ this expression right.

Now, here the nature of flow and characteristics of fluid will be coming into the picture in the at this point onwards ok at this point onwards whatever the nature of the flow and characteristics of the fluid comes into the picture how and those details we are going to see. Because if the nature of the fluid is changing so obviously, shear rate is going to change velocity profile is going to change. So, then obviously, shear rate is going to change ok.

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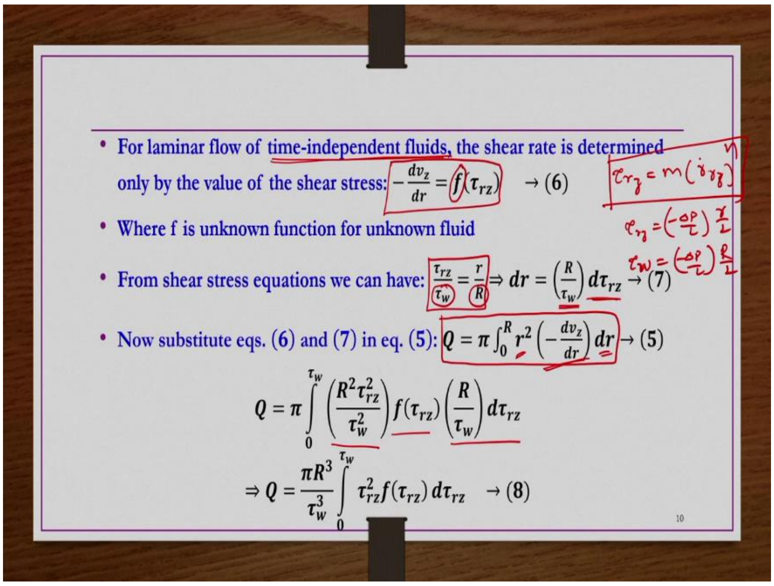

So, for laminar flow of time independent fluid. So, this is all we are doing for the time independent fluids analysis from this point onwards, the shear rate is determined by the value of the shear stress something like this $\frac{-dv_z}{dr} = f(\tau_{rz})$.

So, this f function is not known for an unknown fluid. Let us say if you know the fluid shear thinning behaviour. So, then you have whatever the $\tau_{rz} = m(\dot{\gamma}_{rz})^n$ this is the function this function is nothing but this one if it is a shear thinning fluid right.

So, but we do not know the nature of the fluid that is what we are measuring. So, this f is unknown function for an unknown fluid ok. Further what we have seen? We have seen these $\tau_{rz} = \left(-\frac{\Delta p}{L}\right)\frac{r}{2}$ equation number 1 and then equation number $2\tau_w = \left(-\frac{\Delta p}{L}\right)\frac{R}{2}$. So, if you divide $\frac{\tau_{rz}}{\tau_w}$ then you will be having $\frac{r}{R}$. So, from here τ_w is constant R is constant $\tau_{rz}$is varying as r is varying.

So, if you differentiate either side $dr = \left(\frac{R}{\tau_w}\right) d\tau_{rz}$ you are going to have because as r changing from 0 to R $\tau_{rz}$is also changing, though the change is small in our capillaries cases it is small, but it is there ok. So, then you cannot take it as a constant right.

So, now, this equation number 6 and 7 we are going to make use in our equation number 5; equation number 5 is nothing but this volumetric flow rate expression. So, in place of r you are going to write $\frac{\tau_{rz}}{\tau_w} R$, in place of dr you are going to write $\left(\frac{R}{\tau_w}\right) d\tau_{rz}$and then in in place of $\frac{-dv_z}{dr}$ you are going to write $f(\tau_{rz})$from equation number 6.

So, then you have $\left(\frac{R^2\tau_{rz}^2}{\tau_w^2}\right) f(\tau_{rz}) \left(\frac{R}{\tau_w}\right) d\tau_{rz}$that is what you are getting. So, now, this $\frac{R^3}{\tau_w^3}\pi$that etcetera which are constants you take to the left hand side. So, that in the next slide you will be having $Q\frac{\tau_w^3}{\pi R^3} = \int_0^{\tau_w} \tau_{rz}^2 f(\tau_{rz}) d\tau_{rz}$ ok.

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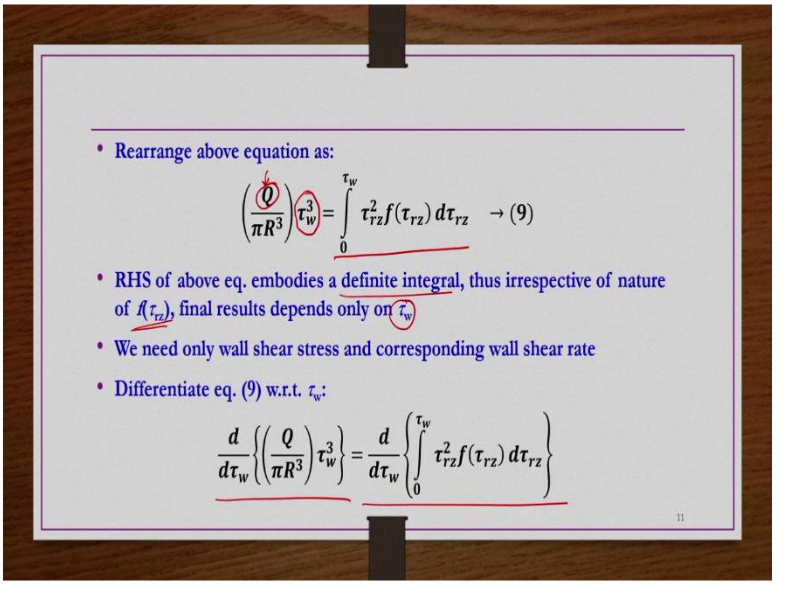

So, this is the rearrangement that we have done; after doing the integration whatever if you know the let us assume if you know that f of τ_{rz}function.

So, then if you do the integration and then substitute the limits. So, then you will be getting the value in terms of τ_w only irrespective of the; irrespective of the fluid whichever fluid it is ok. So, that is RHS of above equation embodies a definite integral thus irrespective of nature of $f(\tau_{rz})$.

That means, irrespective of nature of $f(\tau_{rz})$in the sense in irrespective of the nature of the fluid whether it is shear thinning shear thickening or viscoplastic the final results is going to depend only on τ_w.

Obviously because the limits are you know 0 and τ_w are the two limits lower and upper limits for this integration. So, now, what we do? This equation number 9 we are going to differentiate with respect to τ_w. So, now, left hand side you have to; you have to see the τ_w is there.

So, then the differentiation you can easily do. But Q whatever is there can you take it independent of τ_w? No, you cannot because it is Q is related to the shear rate or velocity change in is related to the velocity gradient and then velocity gradient is related to the shear stress.

So, then obviously, wall shear stress is going to be affected because of this k wall, I mean they are interrelated they are interrelated. So, then that should be taken care appropriately. So, $\frac{d}{d\tau_w}\left\{\left(\frac{Q}{\pi R^3}\right)\tau_w^3\right\} = \frac{d}{d\tau_w}\left\{\int_0^{\tau_w}\tau_{rz}^2 f(\tau_{rz})d\tau_{rz}\right\}$ this is what we are having.

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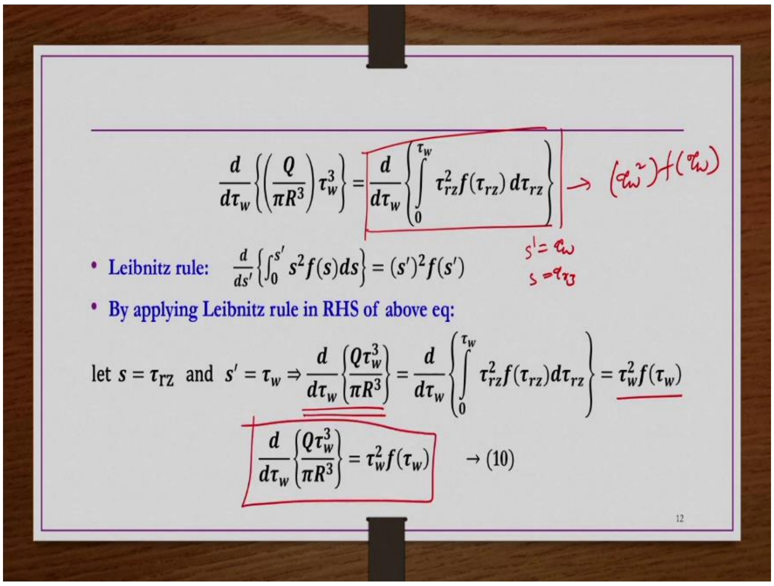

So, now, this equation what you understand right hand side? It is having Leibniz form it is having Leibniz form that is $\frac{d}{ds'}\left\{\int_0^{s'} s^2 f(s)ds\right\} = (s')^2 f(s')$ that is what you have.

So, now here in this case s' is nothing but τ_w, s is nothing, but τ_{rz}. So, the right hand side whatever is there if you apply the Leibniz rules. So, you will be getting $\tau_w^2 f(\tau_w)$ you are going to get. So, right hand side $\tau_w^2 f(\tau_w)$ you are getting left hand side we are not doing a differentiation as of now we will be doing subsequently right. So, now, we will be doing the differentiation of the left hand side also.

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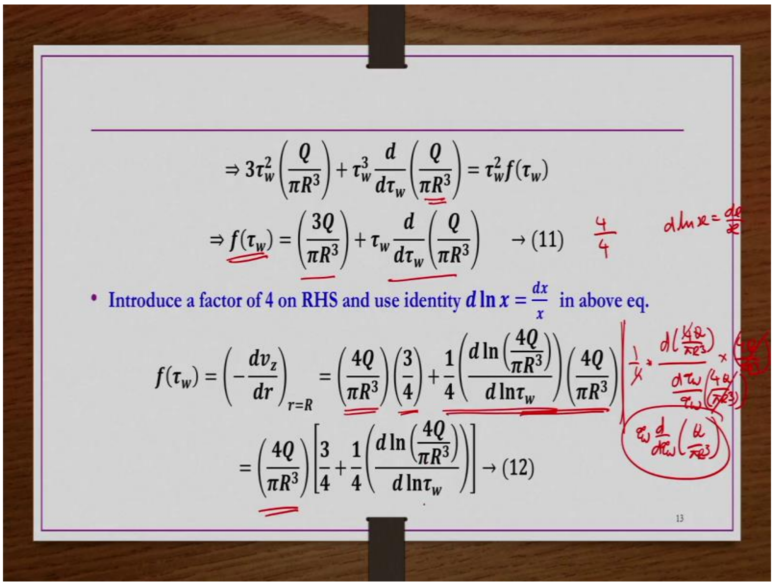

When you do it you will get $3\tau_w^2\left(\frac{Q}{\pi R^3}\right)+\frac{\tau_w^3}{\pi R^3}\frac{d}{d\tau_w}$. But this πR^3 I am keeping within the differentiation because of some reason subsequently we are going to realize it right. So, now, $f(\tau_w)$ if you keep one side rest other terms you take to the other side.

So, then what you have? $\left(\frac{3Q}{\pi R^3}\right)+\tau_w\frac{d}{d\tau_w}\left(\frac{Q}{\pi R^3}\right)$. So, now, this equation what we are going to do? In the right hand side both the terms we are multiplying and then dividing by 4 and then for the second term we are using this $dlnx=\frac{dx}{x}$ form.

So, that we can write $4\left(\frac{4Q}{\pi R^3}\right)\left(\frac{3}{4}\right)$ as a first term in the RHS second term in the RHS we can write it as $\frac{1}{4}\left(\frac{dln\left(\frac{4Q}{\pi R^3}\right)}{dln\tau_w}\right)\left(\frac{4Q}{\pi R^3}\right)$.

So, then what we are having we can cross check let us say we have $\frac{1}{4}$ here whether are we getting the same term or not the. So, that should be $dlnx=\frac{dx}{x}$. So, $\left(\frac{d\left(\frac{4Q}{\pi R^3}\right)}{\frac{d\tau_w}{\tau_w}\cdot\frac{4Q}{\pi R^3}}x\frac{4Q}{\pi R^3}\right)$.

So, then this $\frac{4Q}{\pi R^3}$ and $\frac{4Q}{\pi R^3}$ is cancelled out. So, that this within this, whatever the within this differentiation 4 is there that 4 these 4 can be cancelled out. So, then we can have $\tau_w \frac{d}{\tau_w} \frac{Q}{\pi R^3}$.

So, that is nothing but this entire time is nothing but whatever the second term in the RHS. In the subsequent step what we are doing? We are taking $\frac{4Q}{\pi R^3}$ as a common term right. Then we have $\left[\frac{3}{4} + \frac{1}{4}\left(\frac{dln\left(\frac{4Q}{\pi R^3}\right)}{dln\tau_w}\right)\right]$.

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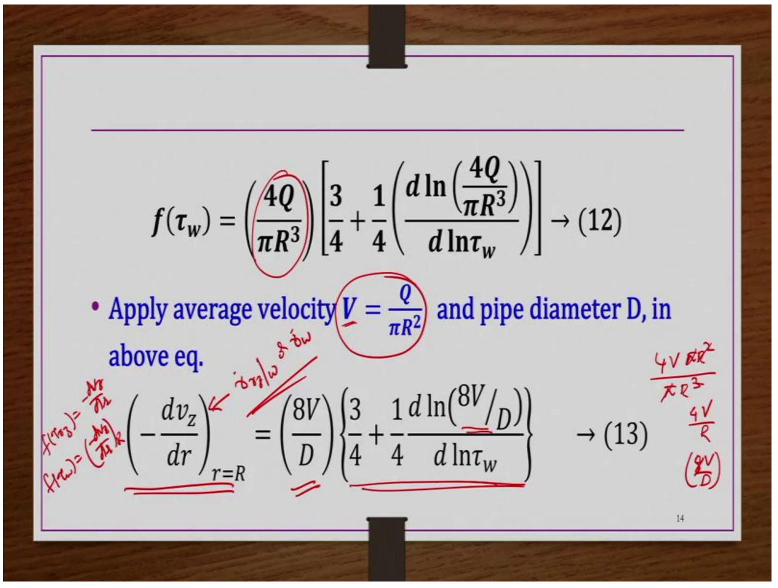

So, this is the equation once again we are written like this here ok. So, now, what you understand here? If you apply in terms of average velocity like whatever the Q in this above equation if you replace the average velocity terms then you know $\frac{Q}{\pi R^2}$. $\frac{Q}{\pi R^2}$is nothing but average velocity of the fluid that is flowing through the capillary for a given Δp.

So, that average velocity let us take as a V and then in place of a in place of R, what you can write? You can write $\frac{D}{2}$ then you have this equation let us say we will do this one only 4 in place of a Q we can write $\frac{V\pi R^2}{\pi R^3}$ we are having.

So, 4 V π, π is cancelled out square of here and then cube of here is gone. So, then R we are having. So, then further R if you write $\frac{D}{2}$ then $\frac{8V}{D}$we are having. So, $\frac{8V}{D}$ here $\frac{3}{4}+\frac{1}{4}\frac{dln\left(\frac{8V}{D}\right)}{dln\tau_w}$ again here also $\frac{\left(\frac{8V}{D}\right)}{dln\tau_w}$ this is what we are having.

Now, this is what this $f(\tau_w)$ is nothing but this is nothing but we have seen already $f(\tau_{rz})$is nothing but $\frac{-dv_z}{dr}$. So, $f(\tau_w)$ should be nothing but $\frac{-dv_z}{dr}$ at r = R. So, that is nothing but this one in place of $f(\tau_w)$I am writing $\frac{-dv_z}{dr}$ at r = R.

So, that means, this whatever the $\dot{\gamma}_{rz}$ at wall or $\dot{\gamma}$ wall information is that you already got it here; that you already got it here. But the problem is that what is we have to analyze? Wwe have to analyze subsequently and then said what is about what about this term what about this terminology and all that.

So, for the analysis what we do? We take a known fluid flowing through the capillary the known fluid is nothing but Newtonian fluid; if Newtonian fluid is flowing through a infinitely long circular pipe, then what we know? Pressure drop expression, pressure drop we can get for that case by Hagen Poiseuille equation that we know.

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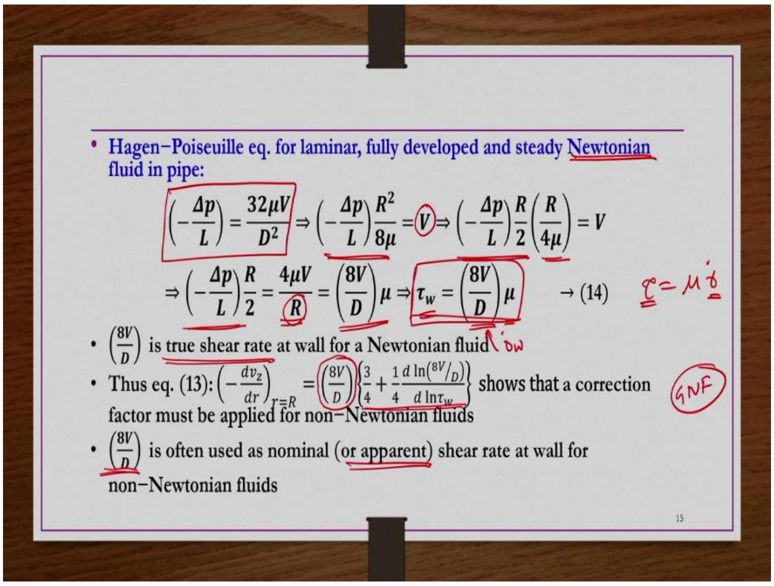

So, that Hagen Poiseuille equation is nothing but $\frac{-\Delta p}{L}=\frac{32\mu V}{D^2}$, remember this is for the Newtonian fluid only there is a point for writing this one we will be realizing in next step.

So, now, what I am doing? I am keeping V only in the right hand side rest of all the terms I am taking to the left hand side also I am converting that D in terms of R its not required anyway, but for some reasons.

So, then further left hand side what I am doing? $\frac{-\Delta p}{L}\frac{R}{2}$ I am writing as one term and then remaining $\frac{R}{4\mu}$ as the other term. So, this $\frac{R}{4\mu}$ I take it to the right hand side and then joined with V. So, that I can have right hand side $\frac{4\mu V}{R}$ and then left hand side $\frac{-\Delta p}{L}\frac{R}{2}$. This now what we do? Whatever these R is there again we will be writing $\frac{D}{2}$. So, that we can have $\frac{8V}{D}\mu$ in the right hand side, left hand side $\frac{-\Delta p}{L}\frac{R}{2}$ is there.

So, which is nothing but $\tau_w = \frac{-\Delta p}{L}\frac{R}{2}$is nothing but τ w which is 2 irrespective of the nature of the fluid whether it is Newtonian or non-Newtonian for this geometry for this flow geometry right hand side $\frac{8V}{D}\mu$ we are having.

So, now, what is the relation for the Newtonian fluid? T is equals to μ $\dot{\gamma}$ is not it? So, that means, from here what we understand by applying the newtons law of viscosity here and analogously if you see this$\frac{8V}{D}$ is nothing but $\dot{\gamma_w}$is not it? w because we are measuring at the wall and then geometry we have taken that it variations are very small as we move from you know centre to the wall ok.

So, now, $\frac{8V}{D}$is true shear rate is true shear rate at the wall for Newtonian fluid in the previous equation 13 we are going to see. What we are having? That $\frac{-dv_z}{dr}$ at R is nothing but $\frac{8V}{D}$multiplied by $\frac{3}{4}+\frac{1}{4}\left(\frac{dln\left(\frac{4Q}{\pi R^3}\right)}{dln\tau_w}\right)$ that is what we are seeing right.

So, in this equation whatever this additional term is there it should be a kind of a correction for the $\frac{8V}{D}$ value which is true for the Newtonian case only for Newtonian case $\frac{8V}{D}$ is shear rate true shear rate right.

Remember all these analysis we are doing for time independent non-Newtonian fluids only and then time independent non-Newtonian fluids are also known as the generalized Newtonian fluids. So, that is the reason we are making connection with the Newtonian case here right. So, now, for the Newtonian case $\frac{8V}{D}$ is the true shear rate, but it is not true shear rate for the non-Newtonian fluid.

So, then there is a because there is a correction if this correction whatever available in the, within the this parenthesis $\frac{3}{4}+\frac{1}{4}\left(\frac{d\ln\left(\frac{8V}{D}\right)}{d\ln\tau_w}\right)$ if it is close to 1 or equals to 1 directly we can say is the fluid is a Newtonian fluid unknown fluid is a Newtonian fluid.

If it is not equals to 1so, then we cannot say that the material is Newtonian fluid. So, that means, for non-Newtonian fluids for time independent non-Newtonian fluids this $\frac{8V}{D}$ is often regarded as apparent shear rate or nominal shear rate ok. So, we have to make this correction.

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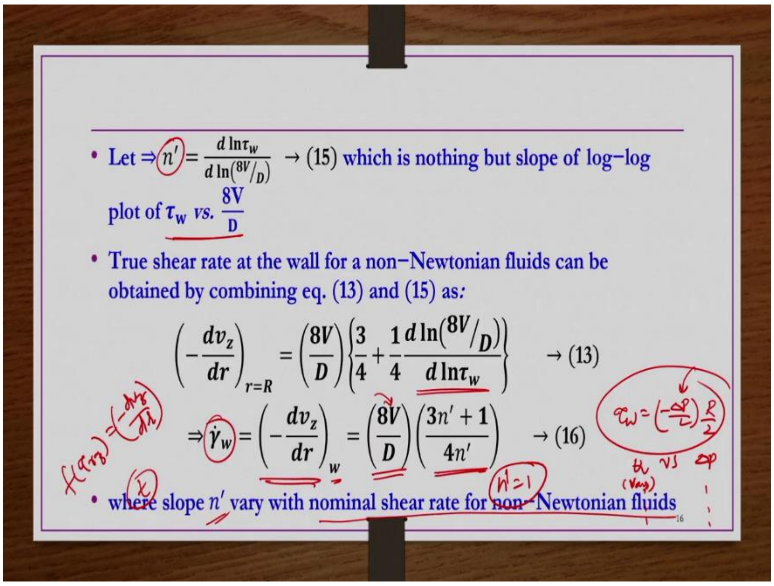

So, now further what we do whatever $\frac{d\ln\tau_w}{d\ln\left(\frac{8V}{D}\right)}$ is say that we take it as n' which is nothing but the slope of τ_w versus $\frac{8V}{D}$ plot on a log log graph sheet, on a log-log graph sheet if you plot out w versus $\frac{8V}{D}$ whatever is there.

So, whatever the slope is there that slope we are taking n'. So, that equation number 13 we can write in place of this we can write $\frac{1}{n'}$ so that we have $\frac{3n'+1}{4n'}\left(\frac{8V}{D}\right)$ is nothing but $\frac{-dv_z}{dr}$ at wall which is nothing but $\dot{\gamma_w}$.

So, for time independent non-Newtonian fluids $\tau_w = \frac{-\Delta p}{L}\frac{R}{2}$and then shear rate $\dot{\gamma_w}$is nothing but $\left(\frac{8V}{D}\right)$ $\frac{3n'+1}{4n\prime}$. If n' is come close to 1 then it is a Newtonian fluid because we this correction factor is 1, we will be having only $\frac{8V}{D}$. So, what we understand? If you have the pressure drop and then average velocity from your capillary rheometer experiments right.

So, you can change pressure drop and then get the different volumetric flow rate. So, then Q versus ΔP information you can get enough number of experiment you do and then Q you convert in terms of V average or V as we are using the symbol V here.

So, then corresponding τ_w and then $\dot{\gamma_w}$you can get from these equations and then when you plot them if you get a straight line passing through the origin. So, then you can say that material is a Newtonian fluid otherwise it is not a Newtonian fluid that is what you can understand.

So, where n' here vary with the nominal shear rate for non-Newtonian fluids. So, in the next lecture what we will be discussing? We will be discussing different modes of operating capillary viscometers and then we see a few example problems and then we see sources of errors. We are saying that narrow gap if you take the variation in the shear rate or shear stress as you move from the centre of the capillary to the wall of the capillary is small, but sometimes it may not possible.

So, then there may be variation. So, if there are inconsistencies how to incorporate? Inconsistency in the sense you use different capillaries of different length and diameter and then do the experiment. And then you whatever the Q versus Δp information that you get for different capillaries of different length and diameter you convert them in terms of τ versus $\dot{\gamma}$ information.

Whatever the Q versus Δp information is there you convert them in terms of τ versus $\dot{\gamma}$ and then you plot if all of them are not super imposing onto each other; that means, there is inconsistency, inconsistency maybe because of some possible source of errors. So, what are those sources of errors if the sources of errors are there we should able to make an appropriate corrections, those things we are going to see in the next lecture.

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So, the references for this lecture are given here. Majority of details have been taken from this book for this lecture ok, but however, similar analysis you may also find in other books provided here.

Thank you.