Transport Phenomena of Non-Newtonian Fluids
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Lecture - 40
Momentum Boundary Layer Thickness of Non-Newtonian Fluids

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of today's lecture is Momentum Boundary Layer Thickness of Non-Newtonian Fluids. Before going into the details of today's lecture, what we will be doing we will be having a kind of recapitulation of what we have seen in the previous lecture.

In the previous lecture we have discussed several basic aspects of a momentum boundary layer and then we have analyzed the momentum boundary layer how the velocity gradient is changing, how the velocity is changing from the solid surface to the you know far away distance gradually when we move in vertical direction all those things we have seen. Also what we have seen? We have seen how to develop the integral momentum equation for boundary layer flows right.

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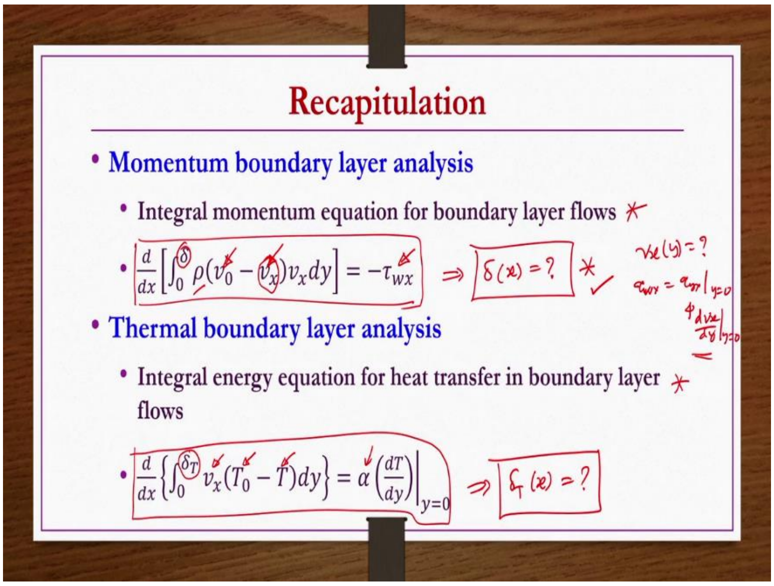

So, this integral momentum equation we have developed for the case of a fluid which is incompressible and flowing steadily. So, this equation whichever we developed that is integral momentum equation is valid for both Newtonian as well as non-Newtonian fluids.

Only constraint or restriction that we had in this development of integral momentum equation is the flow is steady and incompressible.

And then finally, we got this equation, here v_0 is the free stream velocity, v_x is the in the flow direction whatever the velocity component is that v_x and then it is function of y, ρ is the density of the fluid and δ is the momentum boundary layer thickness and then τ_{wx} is nothing but the wall shear stress at y = 0 right.

Then we have also seen a few basics of a thermal boundary layer and then we try to develop integral energy equation for heat transfer in boundary layer flows right. So, the here also the rheology of the of fluid is not coming into the picture and then finally integral energy equation that we got is this one.

So, here δT is nothing but the thermal boundary layer thickness v_x is nothing but the velocity of the fluid in the flow direction which is function of y that is v_x is function of y, y is vertical direction normal to the surface. T_0 is the free stream fluid temperature; T is the temperature which is function of y changing in y direction and then α is thermal diffusivity right.

So, now in this lecture what we are going to see? We are going to use this equation and then trying to find out what is this δ as function of x. In previous lecture we also found that this δ momentum boundary layer thickness or this δ over T even the thermal boundary layer thickness both of them are function of x they are increasing with increasing the x in the flow direction ok.

So, now this equation if we solve you can get expression for a momentum boundary layer thickness that is what we are going to see in this particular lecture. So, for that what we need to know, what is this v_x as function of y? That we need to know without that one we cannot solve the problem or we cannot simplify this equation. Once v_x is known everything is known because this τ_{yx} this τ_{wx} is nothing but τ_{yx} at y = 0 which is again having the contribution of $\frac{dv_x}{dy}|_{y=0}$.

So, if you know v_x as function of y then you can find out $\frac{dv_x}{dy}|_{y=0}$ as well. So, only thing that we need to have we need to have the velocity profile for that boundary layer flow and then use it in this equation to get the momentum boundary layer thickness. So, that is the aim of today's lecture.

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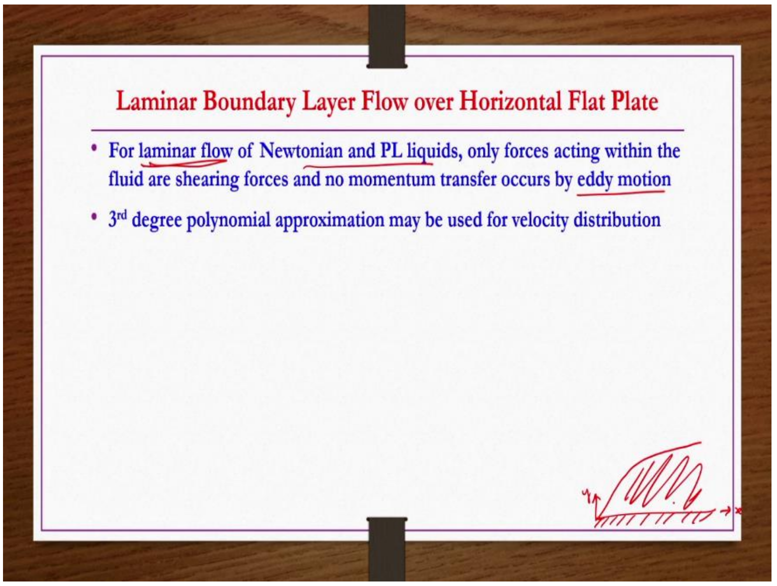

So, laminar boundary layer flow over horizontal flat plate we are considering that is the same scenario that we have consider in the previous lecture, where we developed integral momentum equation. Then for laminar flow of Newtonian and power law liquids, only force acting within the fluid are shearing forces and no momentum transfer occurs by eddy motion because we are assuming the laminar flow.

We have done the analysis only for the laminar boundary layer flow. So, we have not consider anything in the turbulent boundary layer flows ok. So, as long as the flow is laminar within the boundary layer then there will not be any momentum transfer by eddy motions right. So, now, we need to know the velocity profile.

So, under such conditions actually how this you know boundary layer is having the shape of this one right something like this is your x direction this is your y direction and then this is the plate right. So, now it is having certain this boundary layer flow is there only within this region right.

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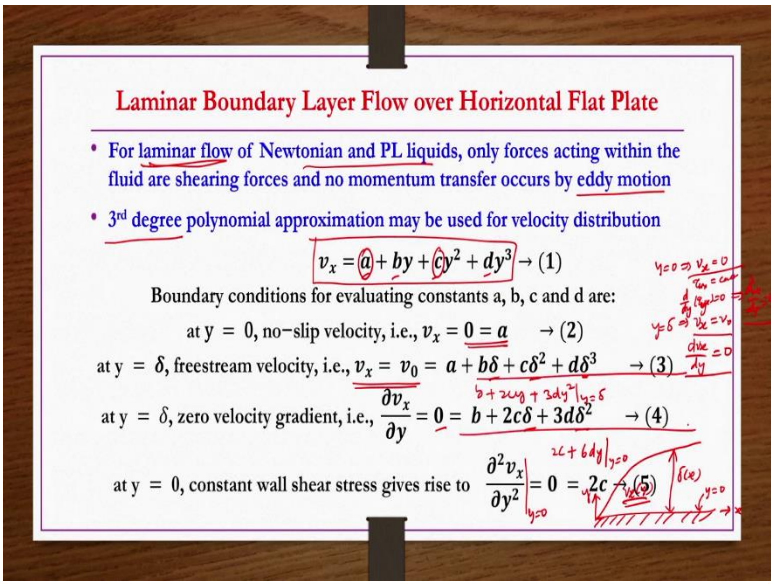

So, this region the thickness of boundary layer is nothing but δ it is function of x right. So, this is all δ which is varying with x. As x increasing δ is increasing. So, this velocity profile can be best approximated by higher degree polynomial. So, then what we are assuming?

We are assuming the velocity profile here v_x whatever function of y is there that can be best represented with 3rd degree polynomial right. Then what we have? We have this $v_x = a + by + cy^2 + dy^3$ where this a, b, c, d are constants fine. So, now we need four boundary conditions to find out these constants. So, what are they, we see.

So, one is that y = 0 location. At y = 0 location, what we have? $v_x = 0$ because of the no slip boundary condition right at the solid surface because of no slip velocity is 0. So, v_x is 0 and then other boundary is that y = δ. We have only two boundaries, but we need four boundary conditions. So, for each boundary we have to assign two boundary conditions ok.

So, at y = δ and then beyond y = δ, what we have? v_x is approximately = v_0 that is velocity is equals to the free stream velocity beyond the boundary layer within the boundary layer it changes with respect to y direction right.

And then also at boundary layer the velocity gradient becomes approximately close to 0 after boundary layer onwards what there will not be any velocity gradient. In fact, the boundary layer designation has been done such a way that the flow region is divided into

2; one within this envelop the velocity gradients are existing after that from y = δ onwards you know this velocity gradients are not existing. So, two boundary conditions we are having here.

Now, at y = 0 that is solid surface τ_{wx} is constant actually; constant shearing force is there. So, then what we have? $\frac{d\tau_{wx}}{dy}$ or $\frac{d\tau_{yx}}{dy}$, now we take τ_{yx} = 0 so; that means, τ_{yx} is nothing but τ_{yx} is proportional to $\frac{dv_x}{dy}$ right; that means, $\frac{d}{dy}\frac{dv_{xy}}{dy}$ is nothing but $\frac{d^2v_x}{dy^2} = 0$ at y = 0.

So, now at y = 0 that is at the solid surface we have two boundary condition that is v_x is 0 and $\frac{d^2v_x}{dy^2} = 0$ and then at y = δ which is other boundary v_x is v_0 and $\frac{dv_x}{dy} = 0$. So, when you apply these boundary conditions we can find out this a, b, c, d constants. At y = 0 no slip velocity that is in this equation 1 if you substitute y = 0 you get v_x = a and then v_x is 0. So, a = 0 you get here.

At y = δ v_x is v_0. So, in this equation 1, wherever y is there you substitute δ. So, then this equation number 3 you get. And then at y = δ we also have 0 velocity gradient that is $\frac{dv_x}{dy} = 0$. So, $\frac{dv_x}{dy}$ is nothing but $b + 2cy + 3dy^2$ and then you substitute y = δ here.

So, that is $b + 2c\delta + 3d\delta^2 = 0$. And then at surface y = 0 constant wall shear stress is there. So, $\frac{d^2v_x}{dy^2} = 0$ at y = 0. So, $\frac{d^2v_x}{dy^2}$ is nothing but $2c + 6dy$ and then you substitute y = 0 here.

So, 2 c = 0 you get so; that means, c is also 0 out of four constants a and c are 0 already then remaining two constants b and d you can find out by solving this equation number 3 and then equation number 4.

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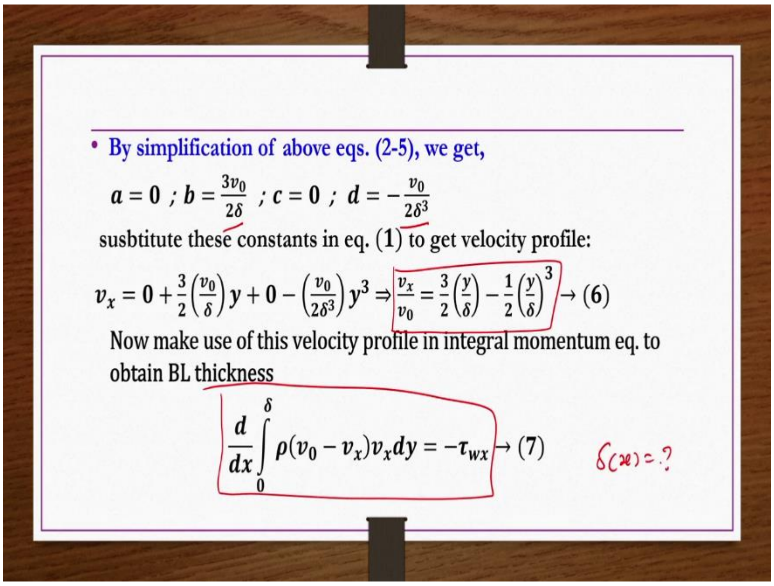

When you do that you get $b = \frac{3v_0}{2\delta}$ and $d = -\frac{v_0}{2\delta^3}$. Now, these constants you substitute in equation number 1. So, $v_x = 0 + \frac{3v_0}{2\delta}y + 0 - \frac{v_0}{2\delta^3}y^3$ that is $\frac{v_x}{v_0} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$.

So, now, v_x you got it. So, you can find out $\frac{dv_x}{dy}$. So, then that those things you can substitute in integral momentum equation this equation and then solve it to get boundary layer thickness as function of x that we are going to do.

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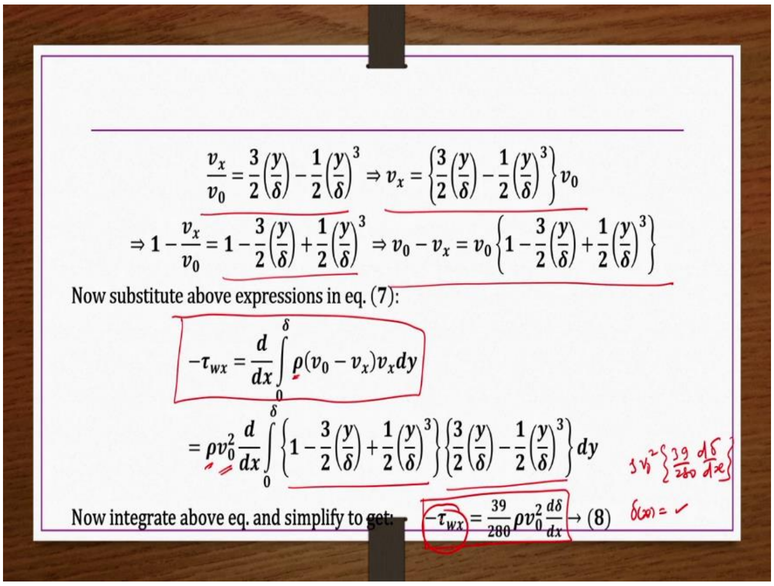

So, before directly going for the power law case what we are trying to do? We are trying to take a simple Newtonian case. So, $\frac{v_x}{v_0}$ we got this one. So, v_x is this one the v_0 we have taken to the right-hand side we also need $v_0 - v_x$ to substitute in integral momentum equation. So, this equation what we do? We both sides we multiply by minus 1 and then add + 1. So, $1 - \frac{v_x}{v_0}$ is nothing but $1 - \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$.

So, from here we get $v_0 - v_x$ is nothing but v_0 multiplied by $1 - \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. So, now, we substitute this one in this equation $-\tau_{wx} = \frac{d}{dx}\int_0^\delta \rho(v_0 - v_x)v_x dy$ right.

Now, here $\rho(v_0 - v_x)$ is nothing but v_0 multiplied by this one and then v_x is nothing but v_0 multiplied by this one. So, v_0 multiplied by v_0 is v_0^2 and this ρ also we have taken outside. So, $-\tau_{wx}$is ρ $v_0^2\,\frac{d}{dx}$ of this one.

Now, this you expand and then do the integration and then substitute the limits 0 to δ. So, right hand side what you get? You will get ρ v_0^2 integration part you will be getting $\frac{39}{280}\frac{d\delta}{dx}$. So, that is what this one. So, now, you get δ as function of x some relation you got it right, but that you can solve only when you get what is this τ_{yx} τ_{wx}.

What is this τ_{wx} when you know then you can find out what is δ as function of y final solution. That you can get what is this τ_{wx} from the nature of the fluid that is from the rheological nature of the fluid.

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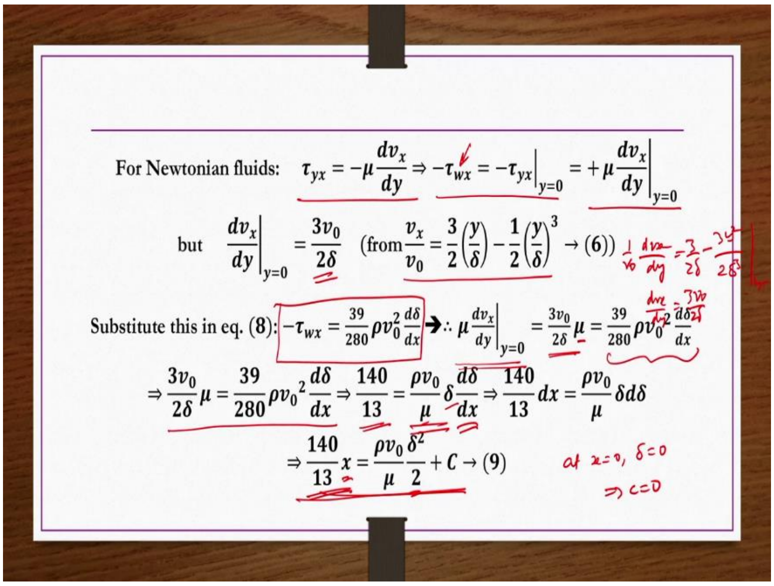

For a Newtonian fluid we take a simple case of. Now, what we do? Before going into the details of a complicated case of a momentum boundary layer thickness of a non-Newtonian fluids, we take simple Newtonian fluids as a starting problem. So, for Newtonian fluids τ_{yx} is nothing but $-\mu \frac{dv_x}{dy}$.

So, $-\tau_{wx}$ is nothing but $-\tau_{yx}$at y = 0, w we are indicating wall. So, wall location is y = 0. So, at whatever the $-\tau_{yx}$ is there in that one you substitute y = 0 that will provide you $-\tau_{wx}$. So, this is $-\tau_{wx}$is nothing but $\mu \frac{dv_x}{dy}$ at y = 0; v_x we have this one this is what the velocity profile we got.

So, from here $\frac{1}{v_0}\frac{dv_x}{dy}$ is nothing but $\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}$. So, now, you substitute y = 0 here. So, you get $\frac{dv_x}{dy}$ is nothing but $\frac{3v_0}{2\delta}$. So, that is what here. So, τ_{wx} is also known. So, now, $-\tau_{wx} = \frac{39}{280}\rho\, v_0^2 \frac{d\delta}{dx}$ is nothing but our equation number 8 that we have previously derived.

Now, here in place of τ $-\tau_{wx}$ we have to substitute $\mu \frac{dv_x}{dy}$ at y = 0 that is nothing but $\frac{3v_0}{2\delta}\mu$ mu and then that should be balanced by the whatever the right-hand side term as it is. Now, this equation what we do? This $\frac{39}{280}$ I take it to the left-hand side. So, $\frac{280}{39}$ I will be getting and then that is multiplied by $\frac{3}{2}$ that is nothing but $\frac{140}{13}$ and then remaining $v_0\, \frac{\mu}{\delta}$ whatever is there.

So, that I am taking to the right hand side. So, that I have $\frac{\delta}{v_0}$ v_0 δ is there v_0 was there and then, but here in the numerator $v_0{}^2$is there. So, the square and then divided by v_0 cancel out. So, then we have only $\frac{\rho v_0 \delta}{\mu}$ here $\frac{d\delta}{dx}$ as it is.

So, now I keep this equation like $\frac{140}{13}$ dx one side and then remaining terms other side an on integration, integration of dx is x integration of a δ d δ is nothing but $\frac{\delta^2}{2} + C$, but at x = 0 δ = 0 that we understand already in the previous lecture. So, C should be 0 when you substitute this limiting condition here boundary condition when you substitute here you get C = 0.

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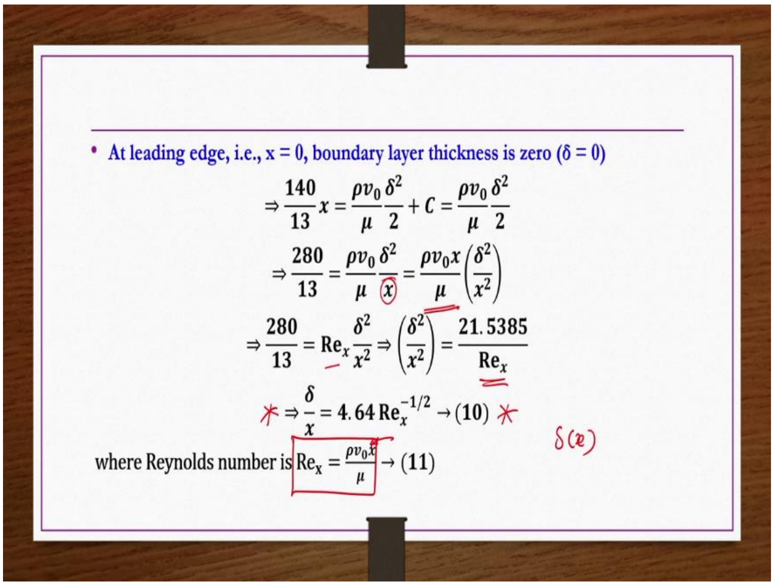

So, we have $\frac{140}{13}$ x = $\frac{\rho v_0}{\mu}\frac{\delta^2}{2}$ that is what we are having, because C is 0. Now, this 2 I take to the left hand side. So, $\frac{280}{13}$ and then this x I have taken to the right hand side. Next step I am multiplying and dividing by x in the right hand side. So, that I have $\frac{\rho v_0 x}{\mu}\frac{\delta^2}{x^2}$. So, this is nothing but Reynolds number local Reynolds number Re_x.

So, $\frac{280}{13} = Re_x \frac{\delta^2}{x^2}$ we get. So, that is $\frac{\delta^2}{x^2}$ is nothing but 21.5385/Re_x that is $\frac{\delta}{x}$ is nothing but 4.64/$Re_x^{-1/2}$. So, this you might have studied or remember in any of your fluid mechanics courses sometime before during UG classes and then derivation is this one right.

So, here this Re_x is nothing but the local Reynolds number, it is not the overall Reynolds number it is local Reynolds number because x $Re_x = \frac{\rho v_0 x}{\mu}$ is there, so, which is Re_x ok. If you substitute x = L that is the length of the plane then it will become overall Reynolds number Re_L ok, but we have to write in terms of Re_x because we understand this δ is function of x ok. It is not a constant value.

So, that is about the boundary layer thickness for a Newtonian fluid flowing over a horizontal plate ok.

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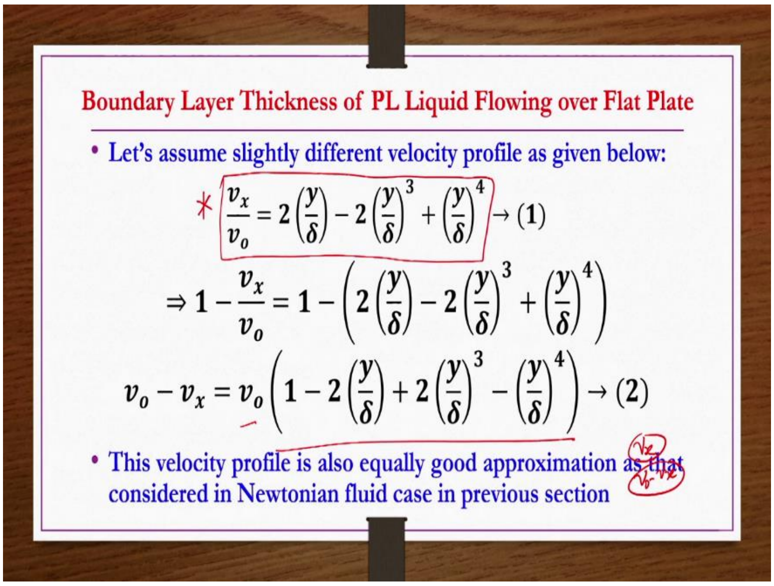

So, now we take boundary layer thickness of power law liquid flowing over flat plate. So, let us assume slightly different velocity profile. So, now, rather obtaining the velocity profile by taking 4th degree polynomial and then applying boundary condition and all that, we already assume the velocity profile is having this form and this is consistent with some of the experimental results. In fact, this results has been taken from one of the experimental research work ok.

So, $\frac{v_x}{v_0} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$ right. So, what we do? We multiply by minus 1 either side then we add 1 either side. So, $1 - \frac{v_x}{v_0}$ we are having this one so; that means, $\frac{v_0}{v_x}$ is $v_0\left(1 - 2\left(\frac{y}{\delta}\right) + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right)$.

Because we not only need v_x, we also need $v_0 - v_x$ in order to solve the integral momentum equation to get the boundary layer thickness ok.

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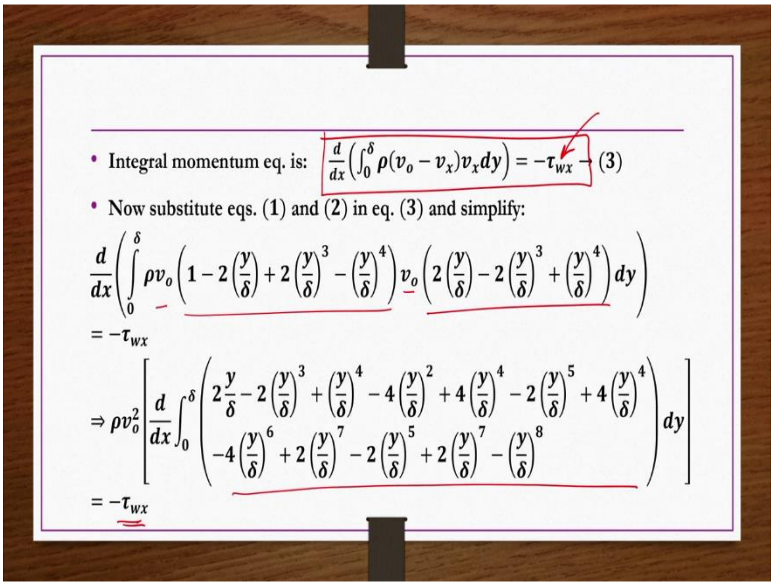

So, integral momentum equation is this one which is same for the fluid whether it is Newtonian or non-Newtonian it does not change ok because it has been developed a generalized one. It is not so, specific to any fluid ok the nature of the fluid will come into the picture through this τ_{wx} information ok.

Now substitute equation 1 and 2 they are nothing but v_x and then $v_0 - v_x$ in this equation. So, $v_0 - v_x$ is nothing but v_0 multiplied by this one and then v_x is nothing but v_0 multiplied by this one right. So, $\rho\, {v_0}^2$ I can take common and then $\frac{d}{dx}$ of this one I can expand like this and then right hand side $-\tau_{wx}$ as it is right.

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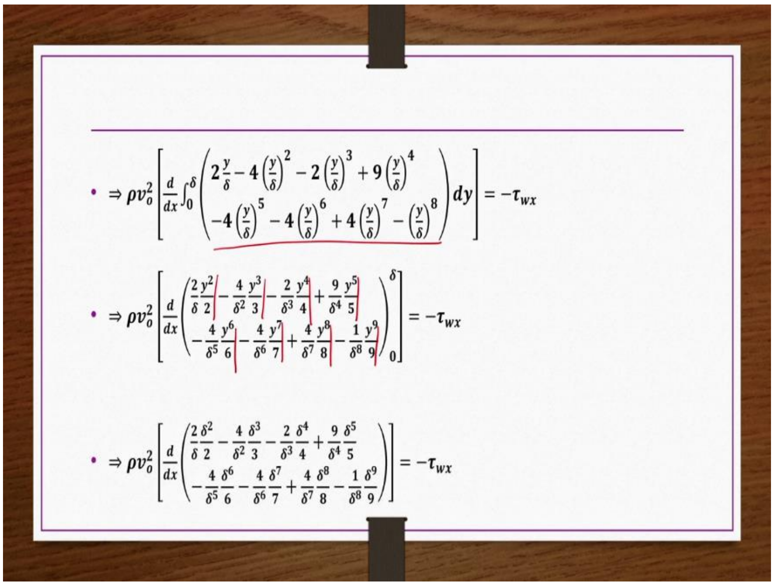

Now, this right hind side further simplified in order to have you know the power terms y^1, y^2, y^3, y^4 like that you know terms have been written like this then now you integrate this one. So, then you get this expression integration of y is $y^2/2$ next integration of y^2 is $y^3/3$ like that integrations we have done for all the terms right. And then we are substituting 0 to δ limits rest all the other terms are remaining constant remain does not change.

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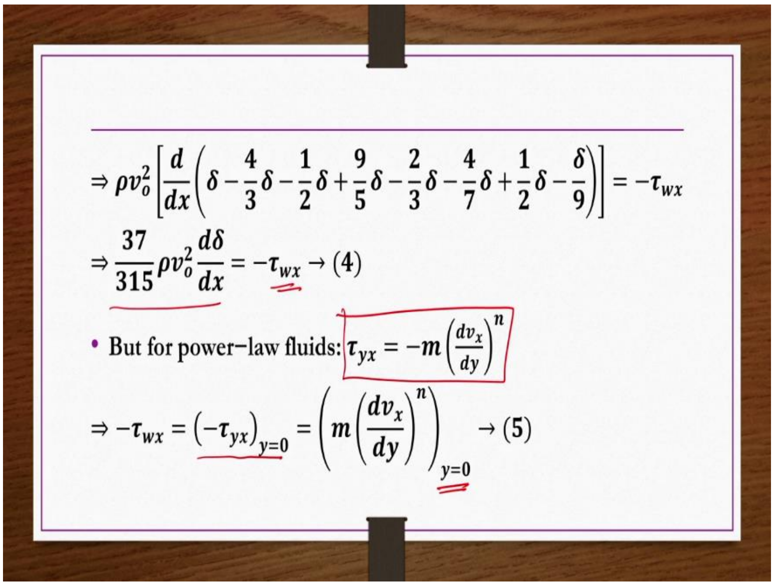

So, then we have after substituting the limits of 0 to δ we get $\frac{37}{315}\frac{d\delta}{dx}$. So, that is $\frac{37}{315}\rho v_0^2 \frac{d\delta}{dx} = -\tau_{wx}$. Now, if you know the τ_{wx} you can simplify this equation to get the boundary layer thickness for power law fluids. But for power law fluid τ_{yx} is $-m\left(\frac{dv_x}{dy}\right)^n$ whereas, $-\tau_{wx}$ is nothing but $-\tau_{yx}$ at y = 0. So, that should be $m\left(\frac{dv_x}{dy}\right)^n$ at y = 0.

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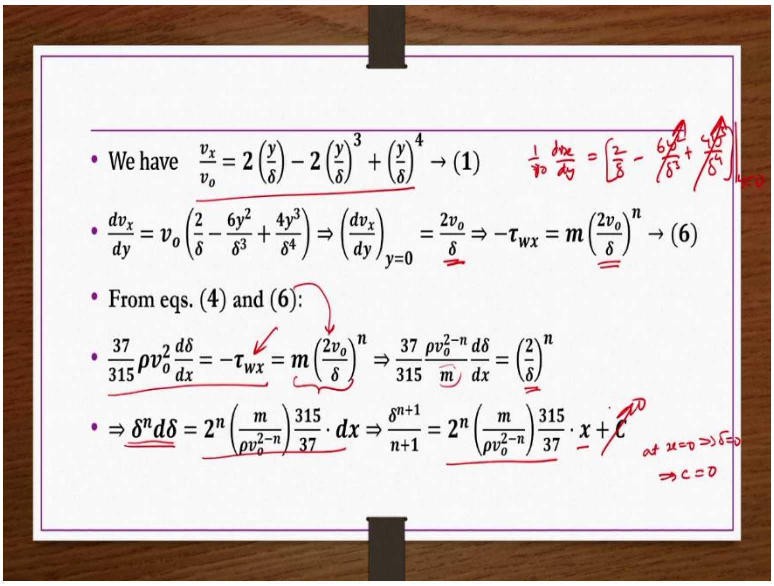

So, now what you have to do? You have to find out what is $\frac{dv_x}{dy}$ from v_x expression and then substitute y = 0. So, v_x is this one. Now from here $\frac{1}{v_0}\frac{dv_x}{dy} = \frac{2}{\delta} - \frac{6y^2}{\delta^3} + \frac{4y^3}{\delta^4}$. And then now you substitute y = 0 in these two equations. So, you get this term is gone this term is gone because of y terms are there. So, you get $\frac{2v_0}{\delta}$ as $\frac{dv_x}{dy}$ at y = 0.

So, $-\tau_{wx} = m\left(\frac{2v_0}{\delta}\right)^n$. Now, this equation number 4 and 5 this is equation number 4. In equation number 4 in place of $-\tau_{wx}$ we are substituting $m\left(\frac{2v_0}{\delta}\right)^n$ from this equation number 6 right.

So now here again, what we are doing? We are keeping $\left(\frac{2}{\delta}\right)^n$ in the right hand side remaining terms we are bringing to the left hand side. So, already left hand side v_0^2 is there. Now, from the right hand side v_0^n if you bring it we get v_0^{2-n} and then whatever the m was there. So, now, that would be dividing by m is there ρ is already there remaining things are as it is.

So, now, next step δ^n d δ we are keeping one side and then other terms we are keeping other side. So, that when you do the integration easily you can get $\frac{\delta^{n+1}}{n+1}$ and then this is all constant integration of d x is x + C. Now, here also at x = 0 δ = 0. So, the constant C = 0 you get.

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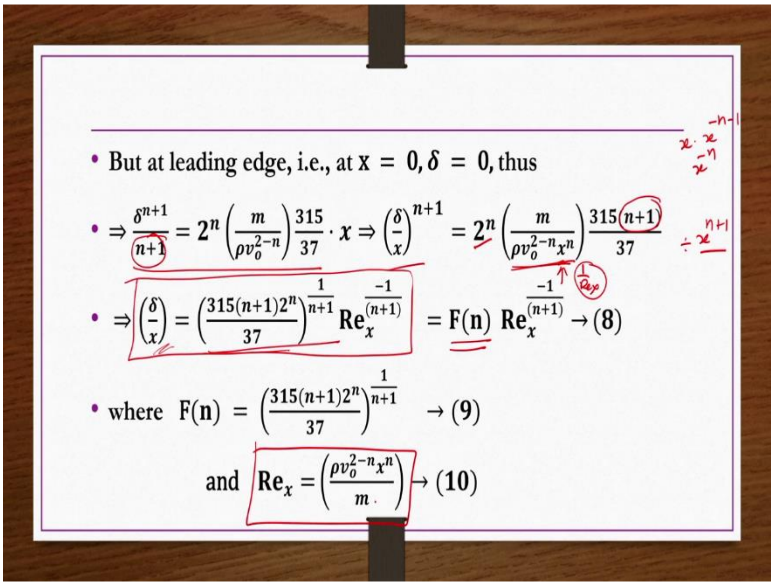

So, then finally, what we have? We have only this term right. Now next step what I am trying to do? I am taking this n + 1 of left hand side to the that whatever n + 1 term in the left hand side that I have taken to the right hand side and then both sides I have divided by x^{n+1} both sides.

So, left hand side $\left(\frac{\delta}{x}\right)^{n+1}$ and then right hand side $2^n\left(\frac{m}{\rho v_0^{2-n}}\right)$. And then so, here in the right hand side already we are having x and then dividing by x^{n+1} that is x^{-n-1}. So, we are getting x^{-n}. So, that is in the denominator I am writing x^n.

Remaining $\frac{m}{\rho v_0^{2-1}}$ as it is 2^n as it is and then $\frac{315}{37}$ as it is this is nothing but our 1/Re_x right for power law liquid this is for the power law liquid. So, $\frac{\delta}{x}$ we can write whatever 315 multiplied by n + 1 multiplied by 2^n and then divided by 37 this all constant whole power 1/n + 1. And then Re_x^{-1} is there here.

So, then $Re_x^{\frac{-1}{n+1}}$ is there because this step we are both sides taking the power of 1/n + 1 so that we can get rid of power of n + 1 from the left hand side. So, now these all constant and it is function of n. So, that is F (n) we are writing.

So, boundary layer thickness we get this expression for the power law fluids $\frac{\delta}{x} = F(n)\, Re_x^{\frac{1}{n+1}}$ fine. Where Re_x is nothing but the local Reynolds number for the power law liquids that is $\frac{\rho v_0^{2-n} x^n}{m}$ ok.

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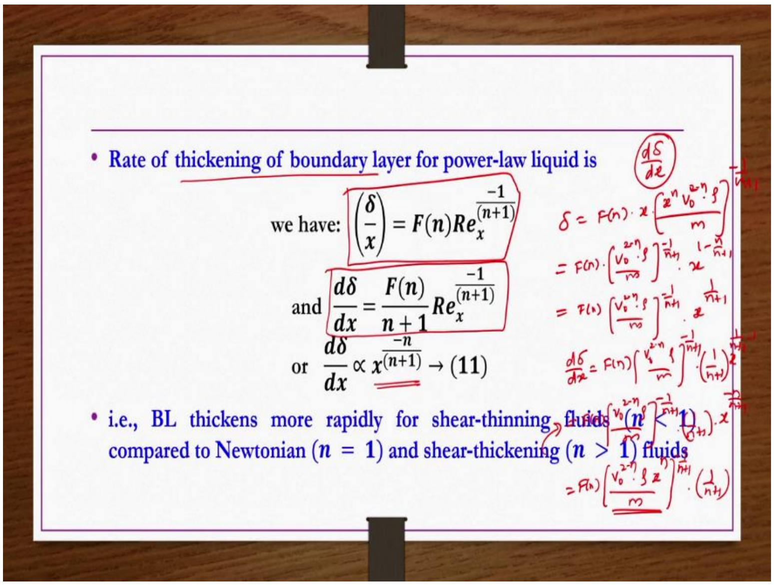

Now, we do some more simplification. Rate of thickening of boundary layer if you wanted to find out that means, what you have to do? You have to get $\frac{d\delta}{dx}$ right. So, for that we need to do some simplifications. So, in order to get $\frac{d\delta}{dx}$ we have to do some simplification by making use our relation $\frac{\delta}{x}$ is this one.

So, now, here δ I can write it as F (n) which is function of n only it is independent of x, x and then this Re is nothing but $\frac{\rho v_0^{2-n} x^n}{m}$ this is what I am having and this whole power $-\frac{1}{n+1}$ is there right. So, that next step I can have this F (n) and then $\left(\frac{\rho v_0^{2-n} x^n}{m}\right)^{-\frac{1}{n+1}}$ as it is whereas, the $x^{1-\frac{n}{n+1}}$.

So, that F (n) $\left(\frac{\rho v_0^{2-n}x^n}{m}\right)^{\frac{-1}{n+1}}$ and then $x^{1-\frac{n}{n+1}}$ that is nothing but $x^{\frac{1}{n+1}}$. Now, $\frac{d\delta}{dx}$ is nothing but this F (n) then $\left(\frac{\rho v_0^{2-n}x^n}{m}\right)^{\frac{-1}{n+1}}$ this is $\frac{1}{n+1}$ and $x^{\frac{1}{n+1}-1}$. So, that we get F (n) $\left(\frac{\rho v_0^{2-n}x^n}{m}\right)^{\frac{-1}{n+1}}$ and then $\frac{1}{n+1}$ and then x power what?

$\frac{1-n-1}{n+1}$ that is $\frac{-n}{n+1}$. So, that F (n) $\left(\frac{\rho v_0^{2-n}x^n}{m}\right)^{\frac{-1}{n+1}}$ then $\frac{1}{n+1}$ I can write, is not it. So, this $\frac{d\delta}{dx}$ is nothing but $\frac{F(n)}{n+1}$ and this is nothing but $Re_x^{\frac{-1}{n+1}}$ or from this equation from this step what you can understand, $\frac{d\delta}{dx}$ is proportional to the $x^{\frac{-n}{n+1}}$ ok.

So, what we understand? The boundary layer thickness increases rapidly for the case of shear thinning fluid compared to the share thickening and then Newtonian fluids that is what we can understand ok.

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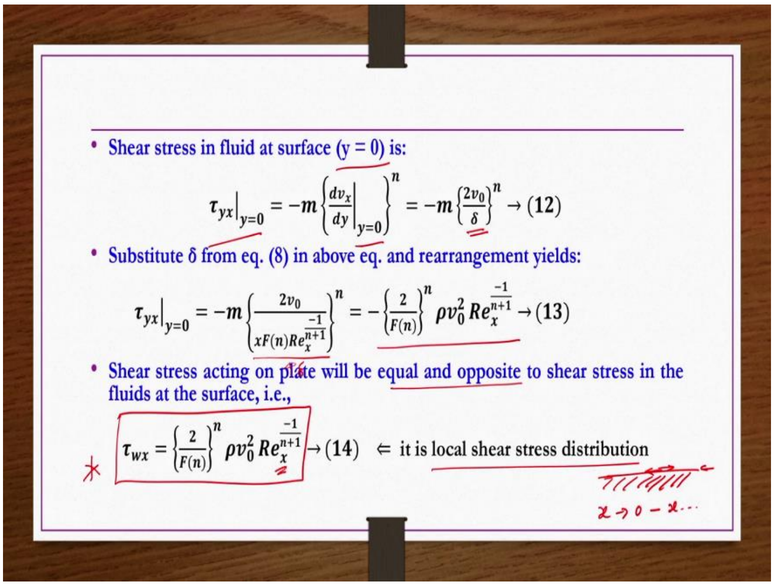

So, now shear stress in fluid at surface y = 0 if you wanted to find out. Why you wanted to find out? Because you wanted to find out the drag force and then finally, drag coefficient as well. So, for that you need to know what is the shear stress at the solid surface that is at y = 0 ok. So, τ_{yx} at y = 0 is nothing but $-m\left(\frac{dv_x}{dy}\right)^n$ at y = 0.

$\frac{dv_x}{dy}$ at y = 0 is nothing but $\frac{2v_0}{\delta}$ and then whole power n is as it is. δ just now we found it as x multiplied by F (n) $Re_x^{\frac{-1}{n+1}}$ ok this is nothing but δ right. So, whole power n is as it is. So, this in the similar way as we have done in the previous slide expanding the Re_x and then simplifying so, that we get this final expression as $\left(\frac{-2}{F(n)}\right)^n \rho v_0^2 Re_x^{\frac{-1}{n+1}}$.

It is simple straight forward simplification we can do it right. Now, shear stress acting on plate will be equal and opposite to shear stress in the fluids at the surface and then we wanted to know at the fluids at the fluid layer at the surface for that we wanted to know. So, then we should take the negative of this one this is at the wall at y = 0 right. So, at y = 0 that is at this surface on the surface whatever the shear stress acting let us say if it is acting in this direction because it is given minus.

So, with the very fast layer the fluid will be having the shear stress equal and opposite to this force, but in the other direction. So, we have to take the positive of that one. So, that will give $\tau_{wx} = \left(\frac{2}{F(n)}\right)^n \rho v_0^2 Re_x^{\frac{-1}{n+1}}$.

And this is nothing but local shear stress distribution that is how the shear stress is changing as x is increasing from 0 to whatever the x values you take ok. So, that is this local shear stress Re_x is local Reynolds number.

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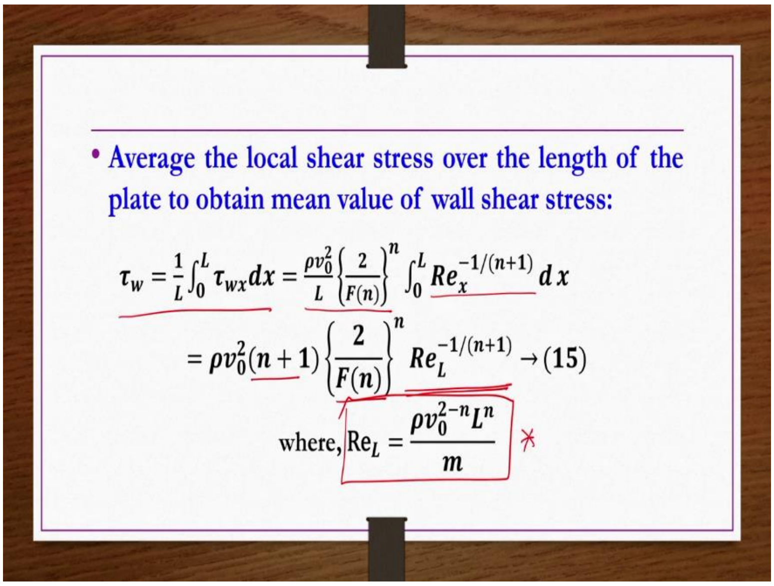

Now, if you wanted to find out the average shear stress over the length of the plate then what you have to do? You have to take the integral of a τ_{wx} from 0 to L and then divide by L. So, that τ_w that is the average shear stress along the length of the plate is integral τ_{wx} dx divided by integral dx limits 0 to L.

So, then we have $\frac{1}{L}\int_0^L \tau_{wx}dx$ that is ρv_0^2. And then this actually τ_{wx} we just found it as $\left(\frac{2}{F(n)}\right)^n \rho v_0^2 Re_x^{\frac{-1}{n+1}}$. So, other than this $Re_x^{\frac{-1}{n+1}}$ all other terms are constant. So, we are keeping outside of the integration.

So, now here again you expand and only x terms you do the integration and then substitute the limits you get ρ v_0 whole power you get ρv_0^2 n + 1. This $\left(\frac{2}{F(n)}\right)^n$ as it is and then after substituting the limits for x from 0 to L you get $Re_L^{\frac{-1}{n+1}}$ whereas, this Re_L is nothing but the overall Reynolds number based on the length of the plate. So, that is $\frac{\rho v_0^{2-n} L^n}{m}$ right. So, the average shear stress also we got fine.

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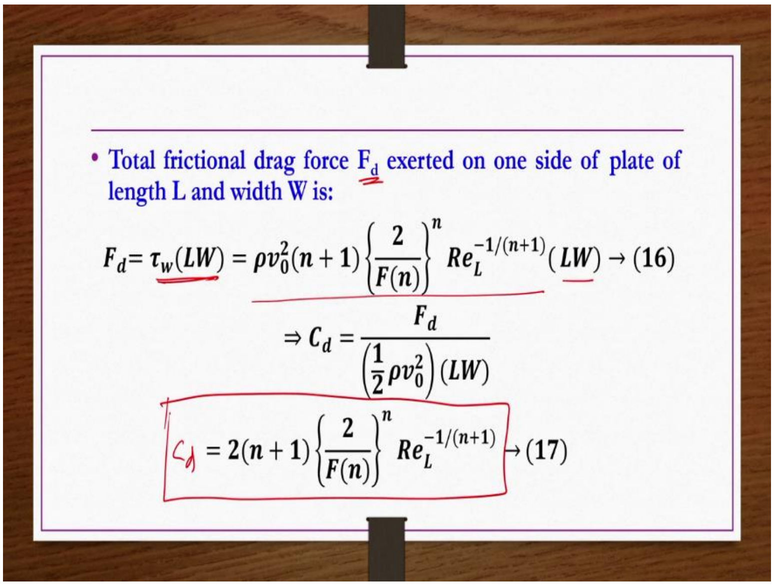

Now, total frictional drag force F_d exerted on one side of plate of length L and width W is nothing but τ_w (LW) because width of the plate in the z direction is W and then frictional force if you wanted to find out that τ_w should be multiplied by the surface area of the plate on which it is this frictional force is acting.

So, surface area of the plate is nothing but the length direction that L and then width direction that is in the z direction the width of the plate is W. So, LW fine so, τ_w already we had this expression in the previous slide multiplied by LW. Now, if you wanted to find out the drag efficient then $C_d = \frac{F_d}{\left(\frac{1}{2}\rho v_0^2\right)(LW)}$ you have to do. So, this F_d you substitute.

So, the $(LW), (LW)$ this $\rho v_0^2, \rho v_0^2$will be cancelled out. So, you have 2^{n+1} or you can simply write 2 multiplied by n + 1 and then remaining things are as it is, that is $\left(\frac{2}{F(n)}\right)^n$ and then $Re_L^{\frac{-1}{n+1}}$. So, this is nothing but the drag coefficient for a power law liquid flowing parallel to flat plate which is aligned in a horizontal direction right.

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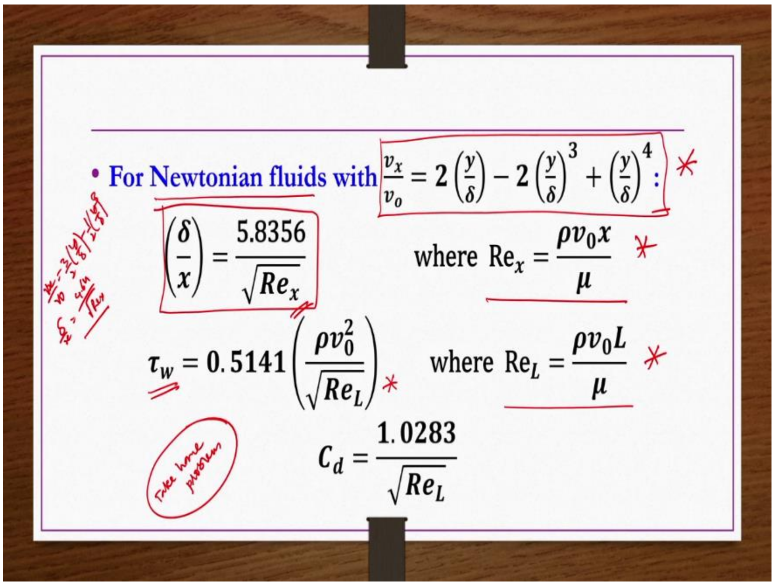

Now, this is this analysis all we have done in the previous slide when we taking this velocity profile right and then power law fluid. But the velocity profile you keep the same and then fluid nature if you change to Newtonian fluid then what you get $\frac{\delta}{x} = \frac{5.8356}{\sqrt{Re_x}}$.

Remember when you have taken $\frac{v_x}{v_0} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. What you got? $\frac{\delta}{x}$ you got $\frac{4.64}{\sqrt{Re_x}}$, but now when you change the velocity profile slightly by increasing the degree of polynomial that is by taking 4th degree polynomial you are getting this $\frac{\delta}{x}$ that is $\frac{5.8356}{\sqrt{Re_x}}$.

So, what we understand from here? The boundary layer thickness or dimensionless boundary layer thickness that is $\frac{\delta}{x}$ is inversely proportional to the square root of local Reynolds number that is what we can say ok. And then similarly if you do some more calculation so, then wall shear stress you get this thing as we do as we did similarly in the previous case.

And then drag coefficient you get $\frac{1.0283}{\sqrt{Re_L}}$ these things you can take exactly in the same way that we have done in an until the previous slide same approach you have to follow. In this case a Newtonian case Re_x is nothing but $\frac{\rho v_0 x}{\mu}$ whereas, Re_L that is the overall Reynolds number is $\frac{\rho v_0 L}{\mu}$ this is local Reynolds number this is the overall Reynolds number.

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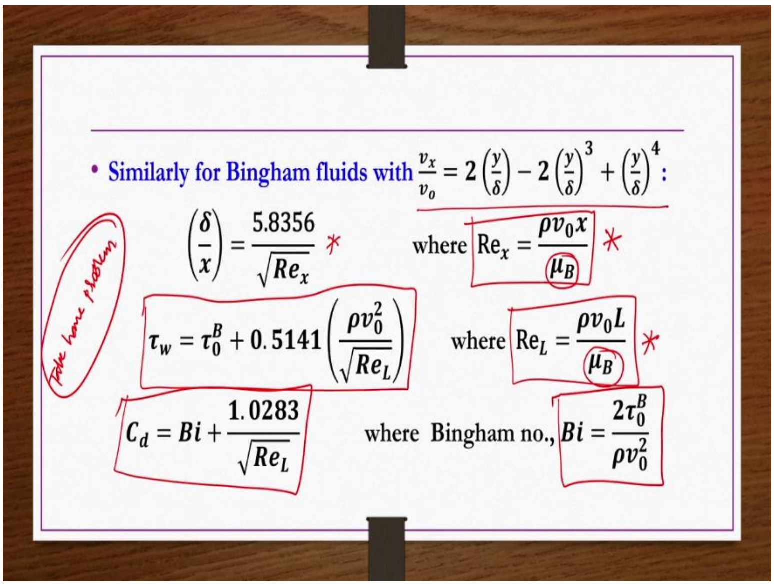

Now what if the fluid is a Bingham plastic fluid, but the velocity profile is provided by the same expression this one? Then we get $\frac{\delta}{x}$ is equals to same as a Newtonian expression that is $\frac{5.8356}{\sqrt{Re_x}}$, but now Re_x is $\frac{\rho v_0 x}{\mu_B}$ plastic viscosity, Bingham plastic viscosity.

And then average shear stress at the wall would be $\tau_w = \tau_0^B + 0.5141\left(\frac{\rho {v_0}^2}{\sqrt{Re_L}}\right)$, Re_L is nothing but $\frac{\rho v_0 L}{\mu_B}$ plastic viscosity, this is a local Reynolds number this is overall Reynolds number. And C_d you will be getting $C_d = Bi + \frac{1.0283}{\sqrt{Re_L}}$ whereas, Bingham number is nothing but, $\frac{2\tau_0^B}{\rho {v_0}^2}$.

So, this also you can take as a take home problem, you have to follow the similar approach that we have followed for the power law liquids in a couple of slides before right. So, in the next lecture we will be discussing now how to obtain the thermal boundary layer thickness when the fluid is a non-Newtonian fluid ok.

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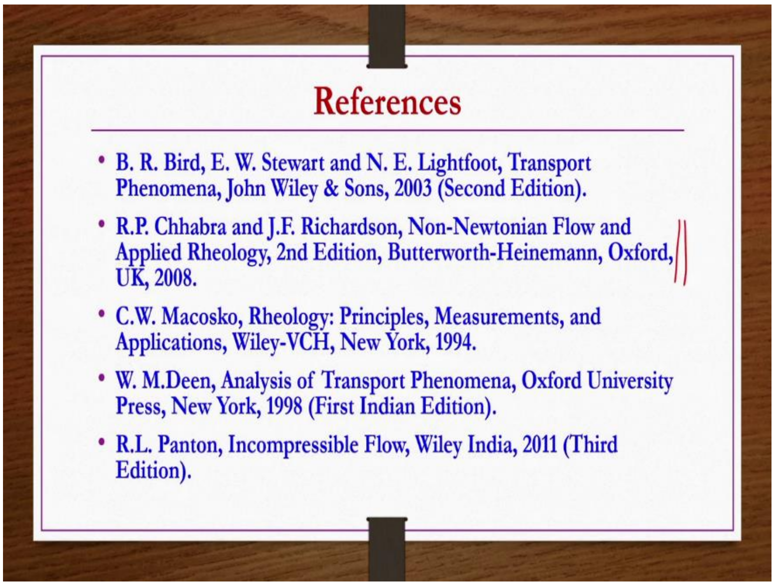

The references for this lecture are provided here. So, details you can find out in this reference book, but derivations you have to do yourself, you do not find anywhere any text books.

Thank you.