Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 39
Momentum and Thermal Boundary Layer Flows

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of today's lecture is Momentum and Thermal Boundary Layer Flows. We are in the last week of the course. In this week we will be discussing different aspects of boundary layer flows and their analysis. Then how to obtain momentum equation and energy equation for boundary layer flows; especially in integral form that is integral momentum equation and integral energy equation for a boundary layer flows.

Then we will be discussing how to obtain the boundary layer thickness for both the momentum boundary layer and then thermal boundary layer, those kind of aspects we will be discussing in this week ok.

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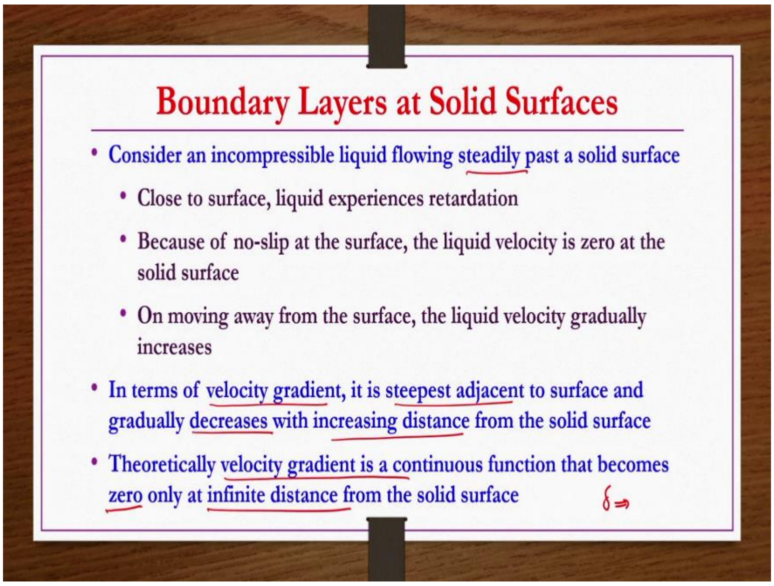

So, first we see a few basics of a boundary layers at solid surface. Let us consider an incompressible liquid flowing steadily past a solid surface. Then what happens, what we understand because of the no slip at the surface will be having 0 velocity right. And then as we move away from the surface gradually the velocity increases. Further the velocity gradients are very steep at the surface and then velocity gradients gradually decreases as we move away from the solid surfaces right.

So, this velocity gradient become 0 usually theoretically at infinite distance from the solid surfaces or the velocity becomes equals to the free surface or free stream velocity at infinite distance from the solid surface, but in general in reality in physical situations we cannot have a geometry of infinite size.

So, we have to have a kind of trade off to consider up to which region the velocity variations are significant or the velocity gradients are significant and then separate that region from the region where the velocity is almost equal to the free stream velocity or the velocity gradients are almost equals to 0. The separation of the flow in these two region that is what we are doing by boundary layers.

So, within the boundary layer the velocity distribution would be there and then velocity gradients would also be there. And then beyond the boundary layer the velocity is almost equal to the free stream velocity and then velocity gradients are almost equals to 0. They are equals to 0 velocity gradient becomes equals to 0 theoretically only at infinite distance, but we cannot afford that infinite distance.

So, we are finding some δ distance under which the velocity gradients are significant. And then after which the velocity gradients are negligible right. That is what basically we are going to see in this boundary layer flows and then more details we keep on seeing in subsequent slides.

So, when we have a liquid steadily flowing past a solid surface then what happens? Close to the surface liquid experiences the retardation because of the no slip that is because of no slip at the surface the liquid velocity is zero at the solid surface. Then on moving away from the surface the liquid velocity gradually increases, in terms of velocity gradient it is steepest adjacent to the solid surface and gradually decreases with increasing distance from the solid surface ok.

So, theoretically the velocity gradient is a continuous function that becomes zero only at infinite distance from the solid surface. So, that infinite distance we cannot afford. So, we find out some distance δ under which the velocity gradients are significant and then beyond a distance velocity gradients are almost equals to 0 ok.

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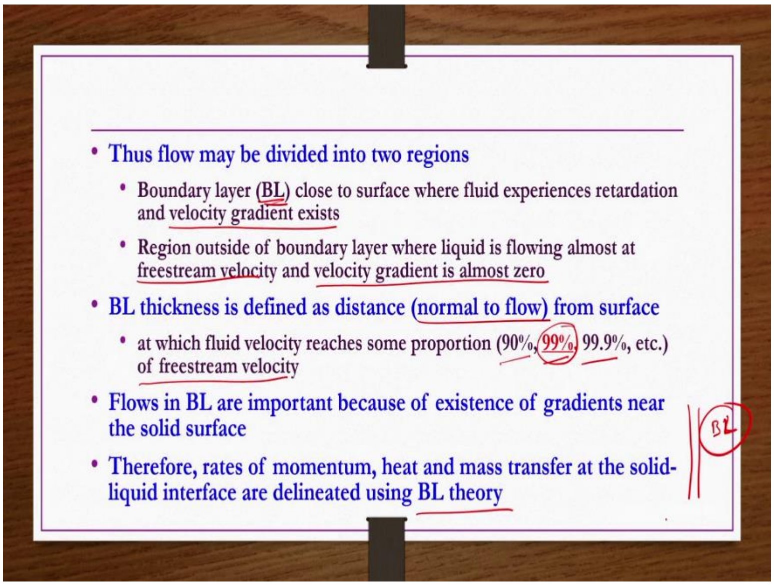

So, that is the flow may be divided into two regions; the boundary layer close to the surface where the fluid experiences retardation and velocity gradients exist and then region outside of boundary layer where the liquid is flowing almost at free stream velocity and velocity gradient is almost zero.

So, the flow region whatever is there above the solid surface that can be grouped in two regions; one region in which the velocity distribution occurs and then velocity gradients are significant or existing. Another region the velocity is equals to the free stream velocity or the velocity gradients are almost equal to 0 right. So, that region whichever is the flow domain is being separated by these two regions.

So, the interface between these two regions whatever are there, that is known as the boundary layer ok. So, boundary layer thickness is defined as distance normal to flow from the surface at which fluid velocity reaches some proportion of a free stream velocity; usually that proportion may vary some people say 90 percent, some say 99 percent, some say 99.9 percent right.

However mostly people take it 99 percent is of 99 percent of free stream velocity is significant to define the boundary layer thickness ok. Flows in boundary layer are important because of existence of gradients near the solid surface. What happens? Whatever the transport rates are there whether it is momentum transfer or heat transfer or mass transfer, they are affected by the gradients corresponding gradients; that is velocity gradient it maybe, thermal gradient it may be or concentration gradient it may be right.

So, depending on which kind of transport processes is occurring right. So, that is affected by this velocity gradient. So, that is the reason the information about the region in which velocity gradients are existing; you know that is going to be play that is more essential especially from the transport phenomena view point ok.

So, rates of momentum and heat and mass transfer at the solid liquid interface are delineated using boundary layer theory right. So, that is the reason we need to study; why we need to study boundary layer theory is the main reason is this one only right. So, every time you cannot have you cannot afford to do numerical solution for the entire domain. So, then what you do?

You divide the flow region such a way that you know one region within the boundary layer region that is where the gradients are existing which are essential for the evaluation of the transfer transport properties. So, whether the friction coefficient or drag coefficient or the heat transfer rate or in the mass transfer rate whatever you wanted to find out Nusselt number at the solid surface etcetera. You know for all that you need to know the corresponding gradients, corresponding gradients are essential these gradients are more significant in boundary layer.

So, if you take the flow region only boundary layer region and then do the analysis we will be sufficiently you know having sufficient this thing to you know information to solve the problems without needing to go into the numerical part. The same thing pictorially we see now by taking an example parallel flow over thin flat plate boundary layer analysis.

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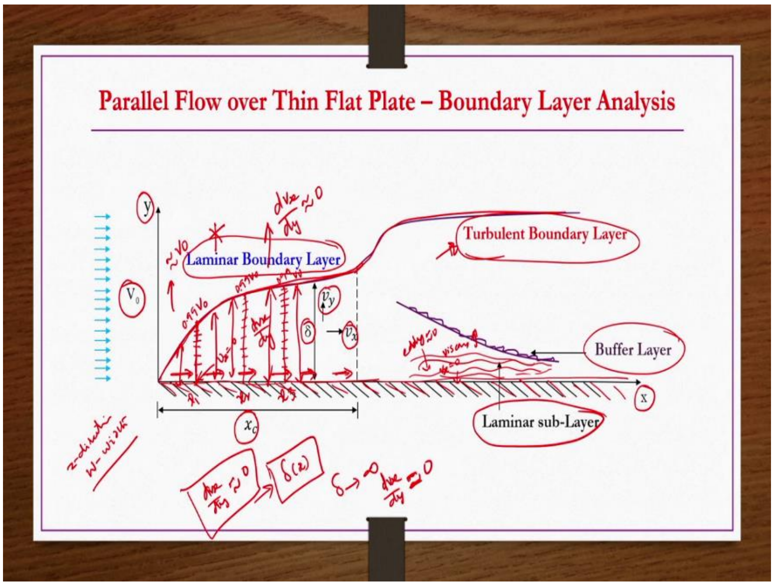

So, we have a flat plate in x direction this slanted lines whatever are shown here you know these lines so, that indicates the solid surface like this right. So, here the coordinate system we have taken such a way that horizontal axis is x axis vertical axis is y axis. And there is a fluid which is coming or flowing at a free stream velocity V_0 right and then velocity component in the horizontal is v_x, velocity component in the vertical y direction is v_y ok.

So, now, here when the fluid comes and hits the leading edge of the plate ok now the plate you have to visualize in three dimension also; in the z direction we are not able to draw here. So, in the z direction whatever the width is there that we are taking W right. So, now, here what happens? When the fluid comes and hits the leading edge of the plate within the board or within this you know screen you have to see.

So, then what happens you know the fluid elements experience the retardation only for infinitesimally small time negligible time. For negligible time the fluid elements experiences the retardation, but as you move further in the flow direction, in the flow direction only, but along the solid surface beyond the leading edge when you move you know like this you know what happens? The fluid element experience more and more retardation right.

So, then gradually its flow whatever is there that you know slows down ok. So, for example, let us say at this location you take this I am calling x_1 location right. So, if I draw the velocity profile let us say the velocity if I draw the velocity profile you know how does it look like that we are going to see. At the surface you know $v_x = 0$, v_y is also 0 let us take only one component horizontal component or velocity component we take let us not worry about v_y and all that right.

As you move up away from the surface in the normal direction the velocity gradually increases right gradually increases like this; we understand and then the point at which the velocity becomes 0.99 times the free stream velocity or 99 percent of the free stream velocity that you locate it.

Similarly, you take another location x_2 here let us say right. And then here also as you move away in the normal direction from the solid surface so, the gradually velocity increases like this. Is not it? Velocity increases and then you find out the location where the velocity becomes 99 percent of V_0 you take another location like that let us say x_3 here.

So, here also as you move away in the normal direction to the surface gradually velocity increases and then the point at which the velocity becomes 99 percent of free stream velocity V_0 that you locate. Like that for different x values you will locate those points and then you combine them draw them like this.

So, that will indicate, that will indicates the region in which the velocity variations are there and then region after which the velocity is approximately equals to the free stream velocity. Or in terms of gradients the region the enclosed region in which the velocity gradients are existing and then beyond that region whatever is there their velocity gradients are almost approximately 0.

So, this layer the points which we are joining then we got this curve that curve is known as the boundary layer and then flow within this boundary layer is known as the boundary layer flow right. So, as you increase or move in the flow direction with increasing x direction what happens? You can see now the boundary layer thickness now you see this boundary.

This is nothing but all this boundary layer thickness only a different x values what you understand? The thickness of the boundary layer is a function of x. So, that thickness of boundary layer we are calling δ, δ is now function of x that is what we understand from here and then it is increasing with increasing x. But there is certain x value which we calling x_c after that the boundary layer thickness abruptly increases or the slope of the boundary layer thickness curve changes abruptly right.

So, that point x_c up to that point x_c whatever is the boundary layer is there that we are calling laminar boundary layer and then after that whatever the boundary layer is the that we are calling turbulent boundary layer ok right. Now, in the turbulent boundary layer also what we have? Close to the surface the velocity or at the surface velocity is anyway 0, but the closed surface the velocity is there, but that velocity is very small actually literally will be very small.

So, that here the eddy contribution are you know eddy diffusion contribution is negligible in this boundary layer close to the surface; whereas, the viscous contribution is very high. So, this region close to the surface there is a small region. So, that region in which the viscous contribution is very higher compared to the eddy contribution in overall transport phenomena or momentum transfer that layer we call it laminar sub layer.

Though in the main core the flow may be turbulent, in the main core the flow may be turbulent, but close to the surface you know because of the no slip velocity at the surface and then velocity gradients are steep at the surface. So, the eddy contribution is very small there and then viscous contribution is more at the surface.

So, that region we call its laminar sub layer and then this turbulent core and the laminar sub layer are being separated by the buffer layer. So, all our discussion we are going to limit for a laminar boundary layer case only we are not going into the details of turbulent boundary layer in this course ok. So, now, this is about the basics about the boundary layer flows and then drawing the boundary layer how does it look like.

So, now from this profile you understand this δ. δ is actually you know theoretically it when it is ∞ or close to ∞ then only the velocity gradients are you know negligible or becomes 0 in theoretically right. But you know considering the practical difficulties we are finding this region δ region under which gradients are existing and then beyond which $\frac{dv_x}{dy}$ are approximately 0 not equals to 0 approximately 0 negligible right.

So, why we are doing separation? Because in this boundary layer most of the gradients are existing, if it is momentum transfer then velocity gradient if it is heat transfer then thermal or temperature gradient if it is concentration boundary layer, then concentration gradient are existing within the boundary layer. And these gradients are going to affect the transport process right rate of transport. So, that is the reason we are doing this process the boundary layer analysis ok.

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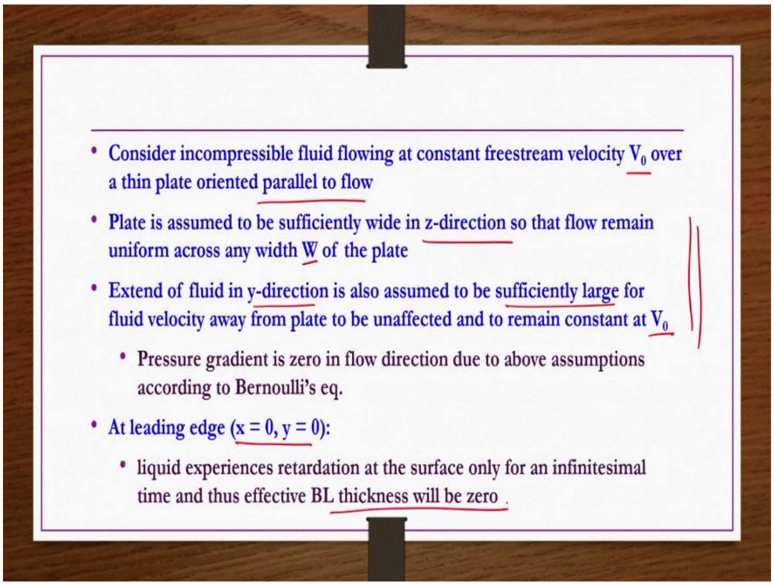

So, whatever we discussed here I have provided as a notes here right. So, considering incompressible fluid flowing at constant free stream velocity V_0 over a thin flat plate oriented parallel to flow and then plate is assumed to be sufficiently wide in z direction. So, that flow remain uniform across any width W of the plate; in the z direction width of the plate is W.

And then extend of fluid in y direction; in y direction is also assumed to be sufficiently large for the fluid velocity away from the plate to be unaffected and remain constant at V_0 ok. And then pressure gradient is zero in flow direction due to these two assumptions when you apply the Bernoulli equations then you get it. So, at the leading edge what we have? The liquid experiences retardation at the surface only for infinitesimal small time and thus boundary layer thickness will be zero at leading edge.

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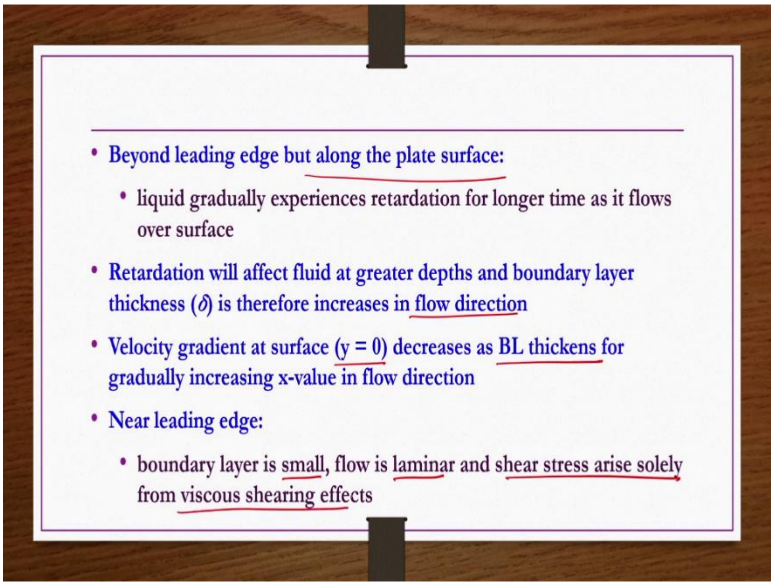

But beyond the leading edge, along the solid surface or plate surface liquid gradually experiences retardation for longer time as it flows over surface, thus boundary layer thickness increases in the flow direction. So, that is δ is function of flow direction now here in this case it is x.

So, velocity gradient at surface y = 0 decreases as boundary layer thickens for gradually increasing x value in the flow direction. Near leading edge boundary layer is small flow is laminar and shear stress arises only from the viscous shearing effect

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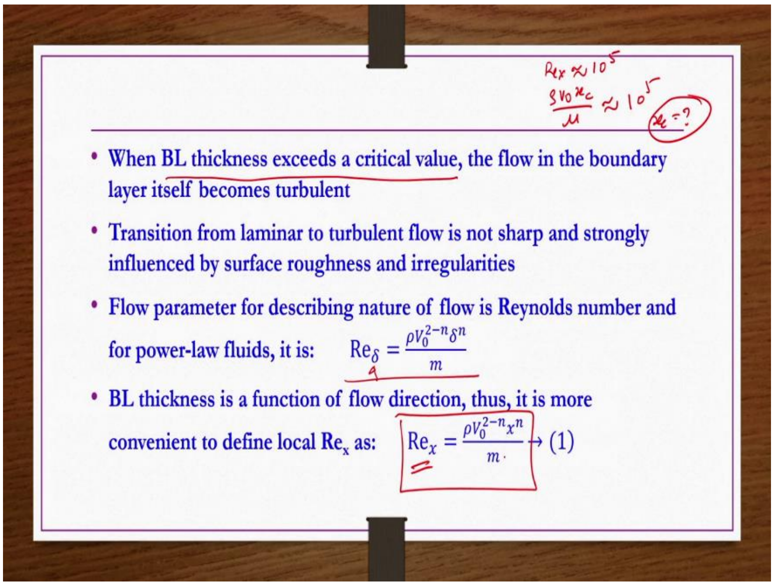

However, when boundary layer thickness exceeds a critical value at x c the flow in the boundary layer itself becomes turbulent. So, this transition from laminar to turbulent boundary layer occurs at Re_x approximately 10^5. So, that is if the fluid is Newtonian ok for Newtonian fluid $\frac{\rho V_0 x_c}{\mu}$ is approximately 10^5.

So, x_c value up to which boundary layer is laminar that if you have to find out you have to use this one. So, what you understand from here? This x_c value is dependent not only on the free stream velocity, but also on the density and then viscosity of the fluid ok. So, transition from laminar to turbulent flow is not sharp and strongly influenced by the surface roughness and irregularities.

Flow parameter for describing the nature of flow is Reynolds number and then for power law fluids we know this is defined like this; this is defined for a boundary layer thickness δ. But local Reynolds number if you define for a power law fluids so then you have to have Re_x that is $\frac{\rho V_0{}^{2-n} x^n}{m}$ ok.

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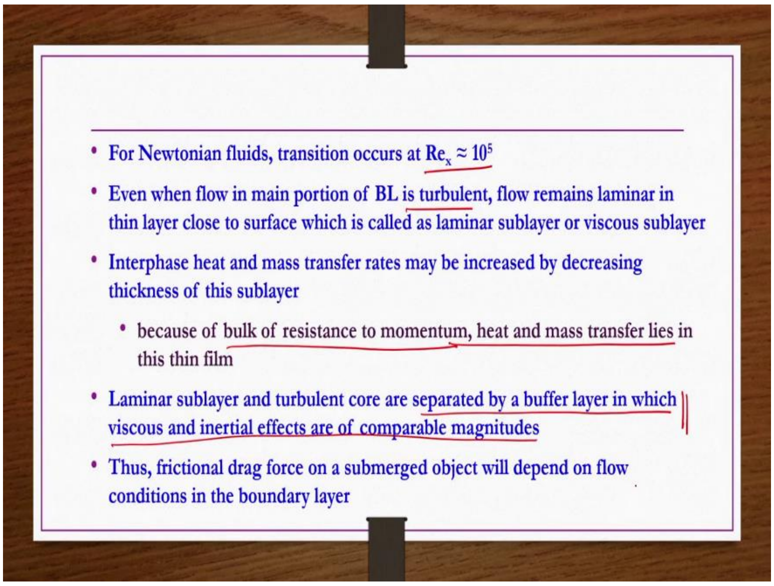

So, for Newtonian fluids, transition from laminar boundary layer to turbulent boundary layer occurs approximately at R_{ex} = approximately 10^5. Even when flow in main portion of boundary layer is turbulent flow remains laminar; in thin layer close to surface which is called a laminar sub layer or viscous sub layer. Interface heat and mass transfer rates may be increased by decreasing the thickness of the sub layer.

Because bulk of the resistance to momentum, and then heat transfer lies in this thin film and then laminar sub layer and turbulent core are separated by buffer layer in which viscous and inertial effects are of a comparable magnitudes. All these details, basic details we have all we have also seen in one of the previous week where we were discussing you know transition from laminar to turbulent flow in the case of flow through pipes.

So, basics are similar. Now the geometry is different, so the boundary layers are you know different you know analysis is different. So, frictional drag force on a submerged object will depend on flow conditions in the boundary layer.

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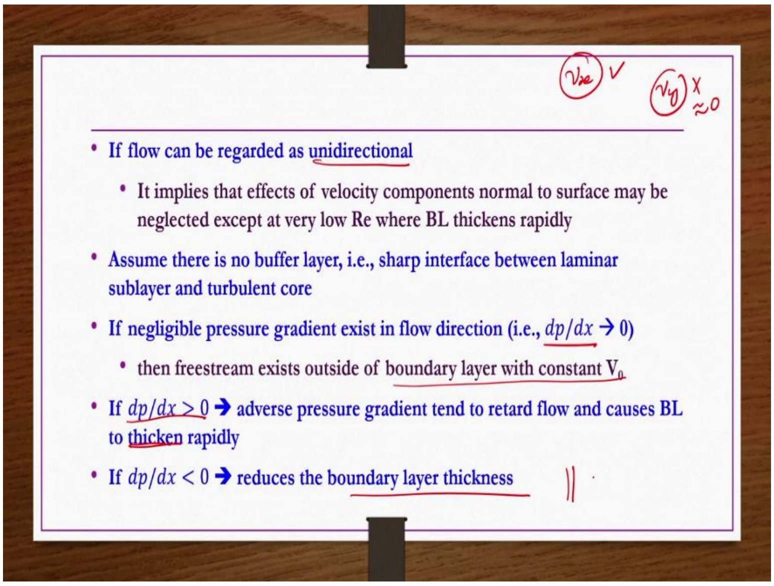

If we can regard as unidirectional flow; that is now we are going to develop integral momentum equation. So, that we are doing for unidirectional flow where we have only v_x that is in the flow direction velocity component v_x is there that is predominating. Whereas the other direction v_y is not there or approximately 0 because it is very small compared to the v_x right.

And this is true also until and unless the Reynolds number is not very small. If the Reynolds number is very small then the velocity would also be there in the normal direction that is in v_y that is in y direction. So, v_y component will also be there, but; however, this boundary layers are very significant in general at larger Reynolds numbers or higher Reynolds number flows.

So, under such conditions we can safely disregard the contribution of v_y which is very small compare to the v_x. Assume there is no buffer layer that is sharp interface is there between laminar sub layer and turbulent core. If pressure gradient is negligible in the flow direction, then free stream exists outside of the boundary layer with constant v_0.

But if pressure gradient is positive then adverse pressure gradient tend to retard the flow and causes boundary layer to become more and more thicker rapidly; if the pressure gradient is negative then it reduces the boundary layer thickness ok. So, if you have strong gradients; that means, in such case we are go, we are expected to having the negative pressure gradients within the boundary layer.

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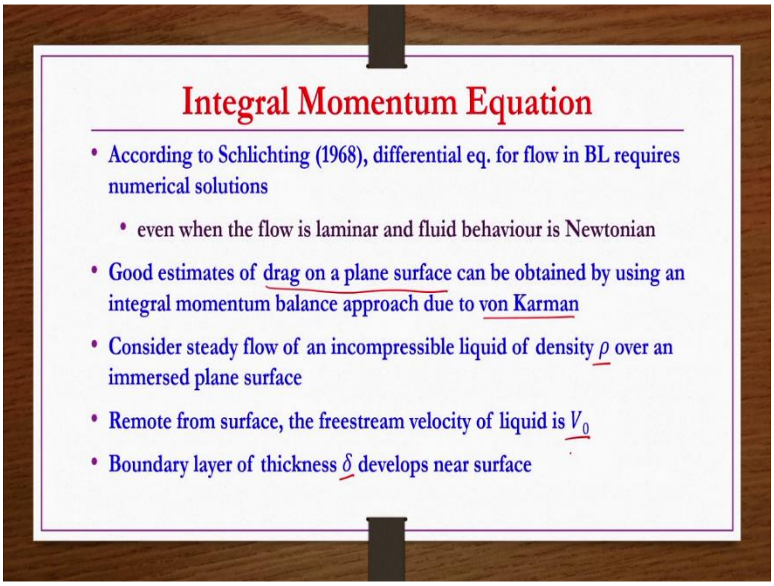

Now, we are going to develop integral momentum equation for this boundary layer flow, laminar boundary layer flow only right. According to Schlichting differential equation for flow in boundary layer requires numerical solutions even when the fluid is Newtonian and the flow is laminar.

So, that is the reason good estimates of drag on plane surface can be obtained by using an integral momentum balance approach due to von Karman. Actually if you wanted to find out the boundary layer thickness there are different approaches are there. So, we are taking one of them which is very simpler and the easier straightforward to get it that you can realize you know as we move subsequently to the next few coming slides or coming lectures.

Consider steady flow of an incompressible liquid of density ρ over an immersed plane surface, remote from the surface free stream velocity of liquid is V_0; boundary layer thickness δ develops near the surface. The details are similar, but now we are going to develop the integral momentum equation. So, that is the reason we have written them again.

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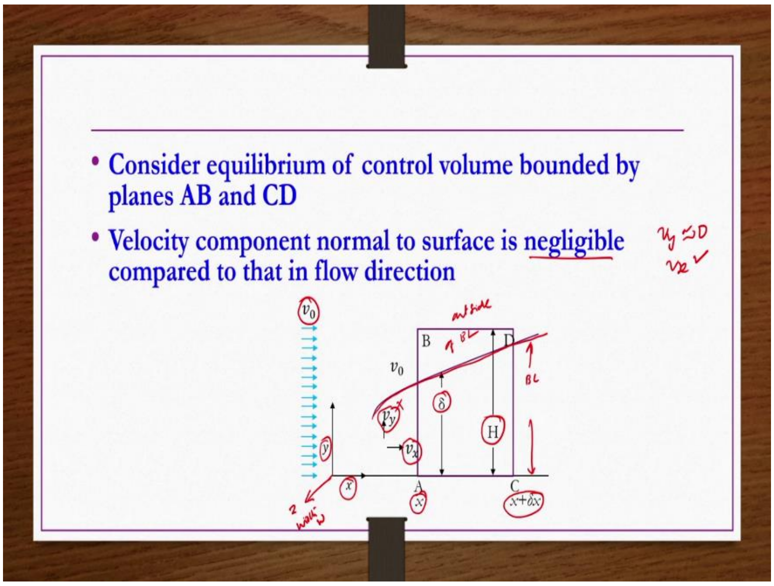

So, now we are considering equilibrium of control volume A, B, C, D right. Velocity component normal to surface is negligible that is v_y is approximately 0 v_x is only there compare to v_x v_y is approximately 0 very small. So, now, this is the control volume let us say, equilibrium of control volume.

So, A, B, C, D is the control volume that is designations are given. Flow direction is x direction here also vertical direction is y direction, in the z direction you know width is width of the plate is W, the fluid is coming at free stream velocity V_0 right. So, A is located at some x distance and then C is located at some x + δx or x + d x location this B and then D are outside of the boundary layer right.

So, whatever this line is there this indicates the boundary layer within this region only there right. So, the thickness of boundary layer is δ, the height of this control volume is H, this v_x component is existing v_y component is very small or 0 compared to v_x right. So, now, here for this case we have to develop the integral momentum equation that is what we are going to do now quickly right.

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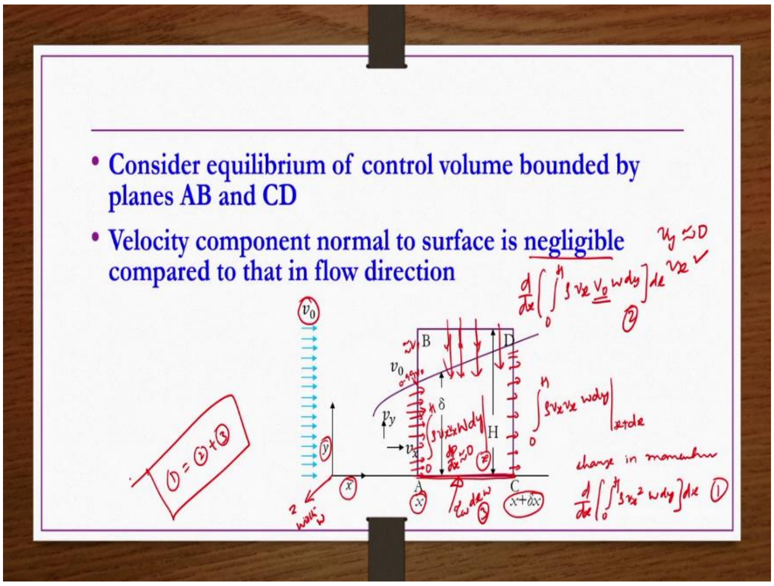

So, now, let us say what is the rate of mass in this control volume let us say AB phase is the inlet phase of the control volume. So, that is nothing but ρ v_x multiplied by the width multiplied by the area of the plate that is in the z direction since we are taking in the x direction normal to x direction or z and y direction, in the z direction size of the plate or width of the plate is W.

And then in the y direction it is height of the control volume is H right, but we are worried about this analysis within the, about this analysis within the boundary layer only, but some of the control volume is outside of the boundary layer. And then also the velocity is changing, it is 0 at surface at location A and then gradually it is increasing is not it? Gradually increasing at this point it is becoming 0.99 times V_0 and then after this approximately it is V_0 becoming right.

So, then this v_x is changing with y. So, that is the region, W dy we are taking as area and then we are integrating it over 0 to H region right. So, this is the rate of mass in. If you wanted to know the rate of momentum means, so, then this has to be multiplied by another v_x that is ρ v_x^2 W dy. This is occurring at x location.

Similarly at CD if you wanted to do, what is the momentum is going out? Then exactly $\int_0^H \rho v_x v_x \, w \, dy$ρ you will be having and then this is occurring at x + dx location. This is at x location this is at x + dx location, expression wise they are looking same right, but they are evaluated at different location.

So, at AB location AB phase within the boundary layer, the flow would be higher because the lower cross section it is having. As well the fluid is experiencing lower retardation compared to the at CD location. CD locations, CD phase the cross section area of this boundary layer is more so, that the flow rate would be small so; obviously, the momentum would also be small.

So, expression wise they are looking same, but the you know overall integral quantities at AB plane it would be higher at CD plane it would be lower because of the increasing cross section of the boundary layer as you move from x to x + δx location. So, from AB to CD when you move, what is the change in momentum right, in the x direction? That would be nothing but this minus this divided by δx and δx tends 0 that you do. So, that is nothing but $\frac{d}{dx}\int_0^H \rho v_x{}^2 w dy dx$ right.

And now we understand more momentum is coming at AB, but less momentum is going at CD. So, the remaining has to be balanced through the BD plane. So, from the BD plane actually that momentum goes out right, but we wanted to get the what is the momentum coming into the boundary layer; because we wanted to find out the integral momentum equation within the boundary layer region.

So, through BD phase how much momentum is coming into the control volume that would be again $\frac{d}{dx}\left[\int_0^H \rho v_x V_0 w dy\right] dx$, why V_0? Because this BD surface is outside of the boundary layer and then outside of the boundary layer v_x is approximately equals to V_0 right.

Now at the solid surface also AC, at the solid surface also now this case what we have seen that $\frac{dp}{dx}$ is 0 within the boundary layer $\frac{dp}{dx}$ is approximately 0 there is no pressure gradient. So, only forces acting at the AC plane is the shearing force. So, that shearing force let us say τ_w we are calling.

And then area of this plane through which this shearing force is you know entering to the fluid that is nothing but the size of this plane in the x direction that is d x and then size of this plane AC in the z direction is nothing but w. So, τ_{wx} W dx, so, now, if this is quantity 1 this is quantity 2 this is quantity 3. So, if you do 1 is quantity 1 is equals to quantity 2 + quantity 3. So, that will give you the integral momentum balance equation right.

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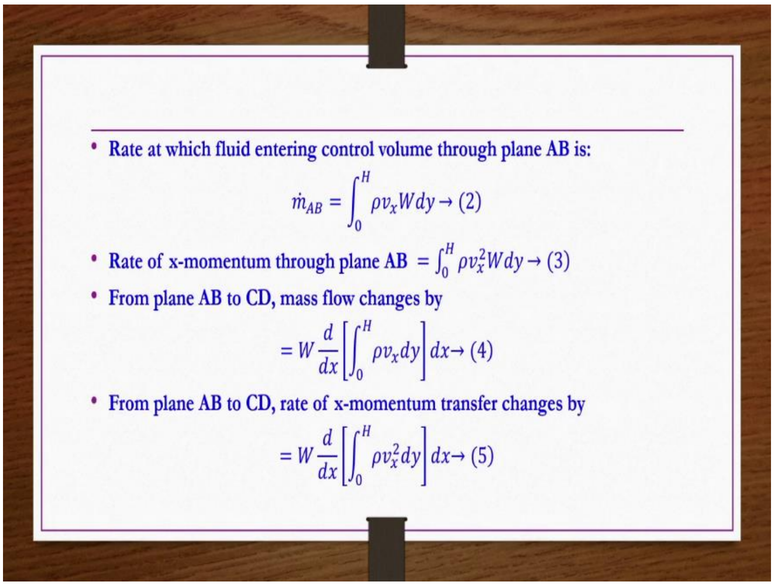

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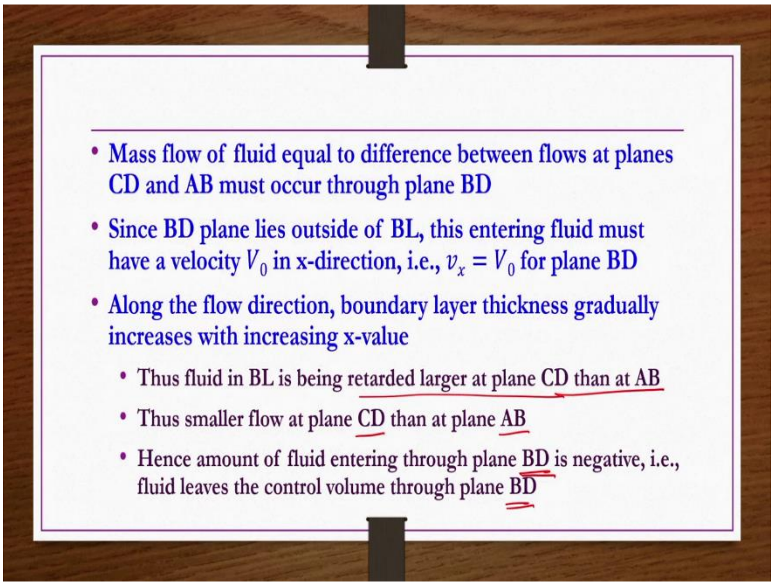

So, now mass flow of fluid equal to difference between flow at plane CD and AB must occur through plane BD. So, since BD plane lies outside BL this entering fluid must have a velocity V_0 in x direction. Along the flow direction boundary layer thickness gradually increases with increasing x value. So, because of that one fluid in BL is being retarded larger at plane CD than plane AB.

So, smaller flow at plane CD than at plane AB this way also you can see or you can see through the cross section area of the boundary layer at AB plane and then CD plane. At CD plane cross section is more so, then less flow would be there. So, hence amount of fluid entering through BD plane is negative that is fluid leaves the control volume through BD plane. So, we want how much is entering the momentum how much momentum is entering into the boundary layer. So, that is the reason we take the negative of this one.

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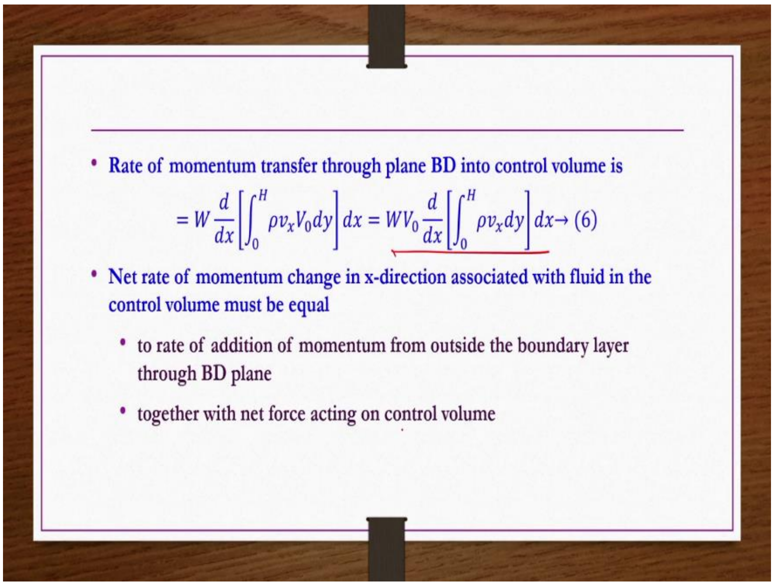

So, the rate of momentum transfer through BD plane into control volume would be nothing but $\frac{d}{dx}\left[\int_0^H \rho v_x V_0\, dy\right] dx$. So, net rate of momentum change in x direction associated with fluid in the control volume must be equal to rate of addition of momentum from outside the boundary layer through BD plane, together with the net force acting on control volume.

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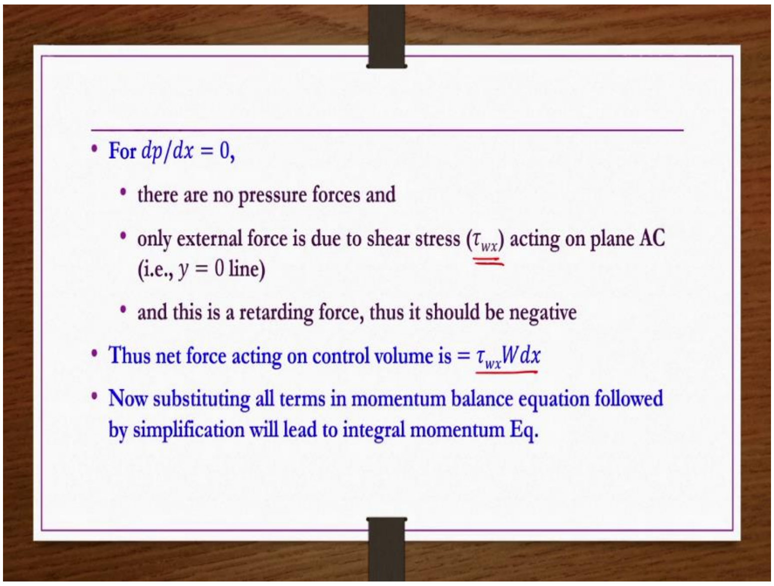

For $\frac{dp}{dx} = 0$ there are no pressure forces and only external force is due to shear stress that acting on plane AC that is at y = 0 line that we are calling τ_{wx}. So, and this is retarding force thus it should be negative. So, both at BD and then AC planes whatever the momentum is there that they are negative. So, thus net force acting on control volume would be τ_{wx} W dx as described in the figure.

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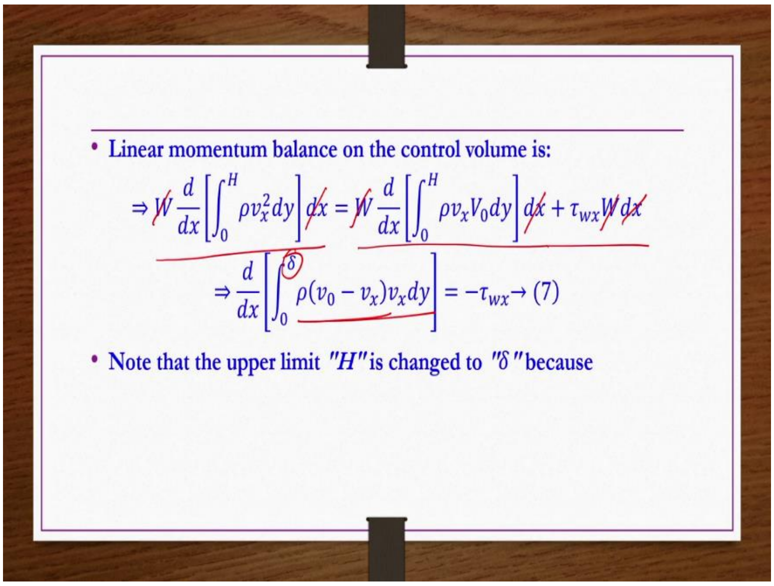

Now, substituting all these in momentum balance equation you know we get whatever the change in momentum rate from AB plane to CD plane that should be balanced by what is the momentum entering at BD plane and what is the momentum entering at AC plane that is wall.

So, when you take out this constant W's and then rearrange this equation you get $-\tau_{wx} = \frac{d}{dx}\left[\int_0^{\delta} \rho(v_0 - v_x)v_x\, dy\right]$ right. So, because this dx, these dx, this dx, are also cancelled out. So, now, here this upper limit also we change to δ because we wanted to have this integral equation only for the boundary layer region and then boundary layer region is in the y direction that is from y = 0 to y = δ only.

If you take y = H that is outside of the boundary layer and then outside of the boundary layer the velocity $v_x = v_0$. So, then integrand would become 0 that is the reason upper limit has been changed to δ right because of these two reasons; one is that we are interested about the momentum transfer within the boundary layer only that is between 0 to δ distance only. And then outside of the boundary layer $v_x = V_0$. So, integrand will become 0 if you take upper limit H ok.

So, this is about the momentum boundary layer analysis if the surface temperature and fluid temperature are different from each other there is also possible that thermal boundary layer would be developing because of the heat transfer. So, that part we see heat transferring boundary layer.

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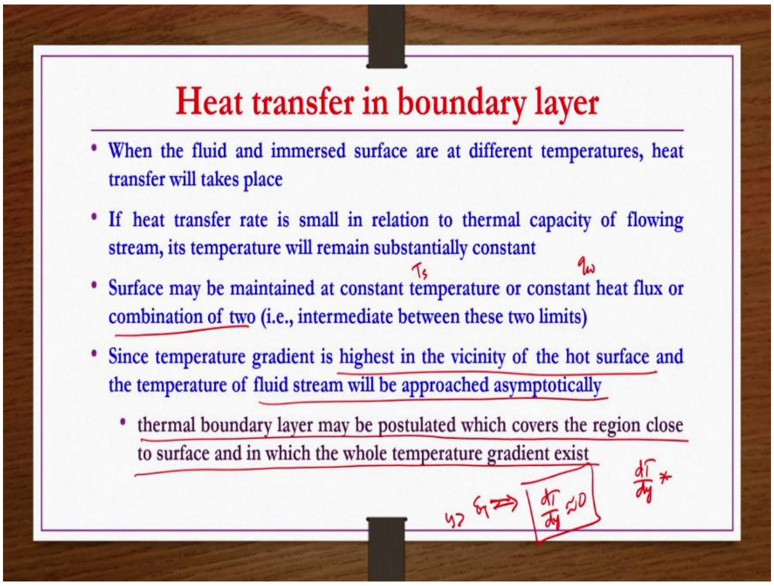

So, when the fluid and immersed surface are at different temperatures, heat transfer will take place. If heat transfer rate is small in relation to thermal capacity of flowing a flowing stream its temperature will remain substantially constant. Surface may be maintained at constant temperature or constant heat flux or combination of two.

So, we can have a constant temperature T_s or we can have a q_w at wall or combination of these two also possible in general. Since temperature gradient is highest in the vicinity of the hot surface and the temperature of fluid stream will be approached asymptotically, thermal boundary layer may be postulated which covers the region close to the surface and in which the whole temperature gradient existing.

Exactly, similar way or analogous to the momentum of boundary layer case if at all there is a heat transfer also. The flow region is divided into two regions; one region in which the temperature gradients are existing within that is within the boundary layer; another region in which temperature gradients are approximately 0 they are they become exactly 0 only at infinite distance.

But we can have some δ T distance in the y direction. So, after which you know when y is greater than δ T then $\frac{dT}{dy}$ is approximately 0 because physically infinite distance we cannot have theoretically we can have right. So, this is what exactly the similar way we are doing as we did for the momentum boundary layer case.

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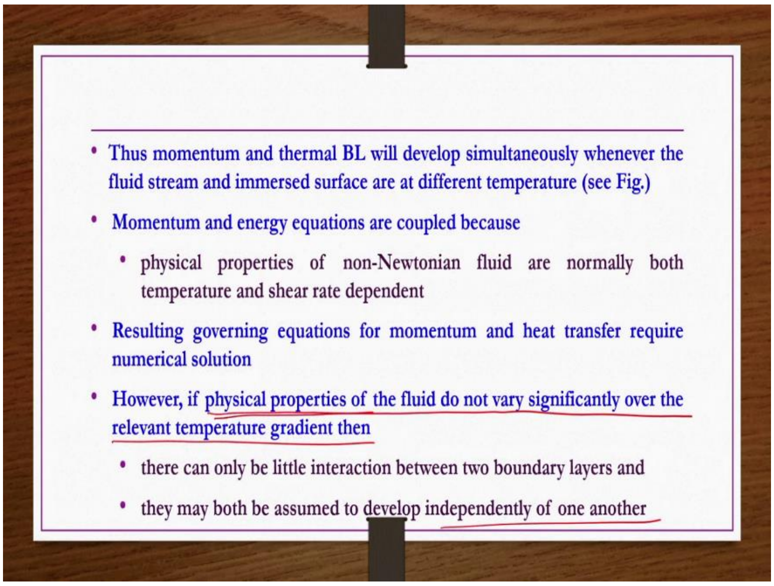

Thus, momentum and thermal boundary layer will develop simultaneously whenever the fluid stream and the immersed surface are at different temperatures, we are going to see the picture also. Momentum energy equations are coupled in general especially for non-Newtonian fluids.

Why because the non-Newtonian fluids the properties are physical properties not only shear dependent, but also they are dependent on the temperature gradients that is the reason ok. So, because of that one we will be having you know both momentum and thermal boundary layer together and then we may be requiring numerical solutions in general.

So, but that is not possible in general so; however, if we assume the physical properties of the fluid do not vary significantly over the temperature gradient of concerned, then we can assume that these boundary layers are you know forming independent of each other independent of each other. There can only be little interaction between these two boundary layers and then both may be assumed to be developing independent of one other if this physical properties are independent of a temperature gradient.

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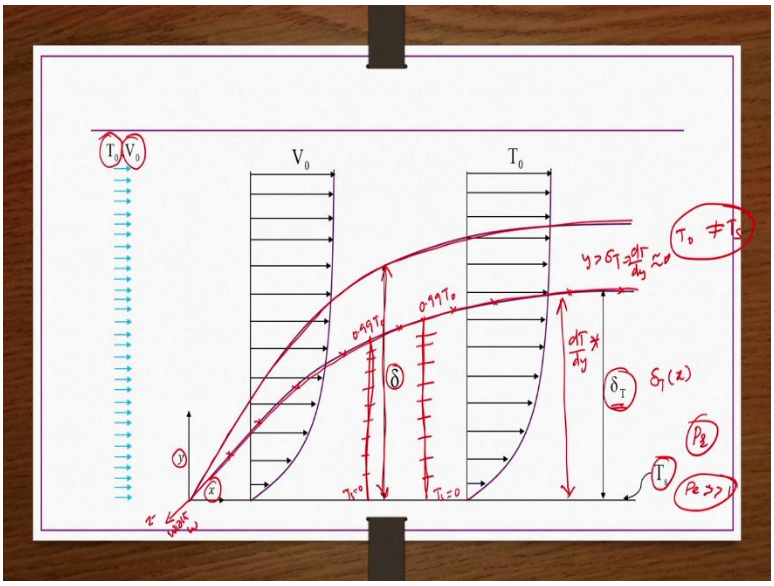

So, that case only we are taking. So, pictorially it is shown here. So, again the horizontal direction is x direction vertical direction is y direction and then z direction in the z direction width is nothing but w right. So, surface temperature T_s we are taking right; a free stream fluid a fluid moving with a free stream velocity V_0 at temperature T_0.

Now, this T_0 and T_s are not equals to each other they are different from each other. So, because of that one thermal boundary layer is also forming. So, for simplicity what we assume? We are temperature scale we take such a way that at the surface the temperature is 0 right and then as we gradually move up right at certain location like this.

So, temperature gradually increases right and then at certain location the temperature becomes 0.99 times the free stream temperature T_0; like that a different locations if you try to develop you know then here also $T_s = 0$. And then gradually as move up the temperature gradually increases and then find out the location where the temperature becomes 0.99 time the T_0.

Like that you may find out different points for different x values and then you find here also you know when you joining these lines you can have a region one region in which the temperature gradients are existing right. So, that region we are calling boundary layer region whose thickness is δ T and this δ T is function of x that is what we are saying. And in beyond this δ T value in the y direction when y is greater than δ T what you have?

You have the region where $\frac{dT}{dy}$ is approximately 0; the other one is the momentum boundary layer this already we have seen. So, that we are calling δ this we have already seen in the previous slide ok. So, in order to differentiate them they are of different thickness, thermal boundary layer thickness is written as δ T. Now, this δ T is less than δ or greater or greater than δ it depends on the Prandtl number or the Peclet number.

If it is very small then δ T is usually greater than δ, but that is true only when the small Reynolds number cases very small Reynolds number flows are there. However, but this boundary layer; however, this boundary layer flows are more important at higher Reynolds number and then higher Prandtl number case. So, then Peclet numbers would be greater than very very larger than one.

So, under such conditions the thermal boundary layers are thinner than the momentum boundary layer. So, for that case only we have drawn here ok. So, now for this case also we are trying to, we will be trying to develop an integral energy equation to conclude this lecture ok.

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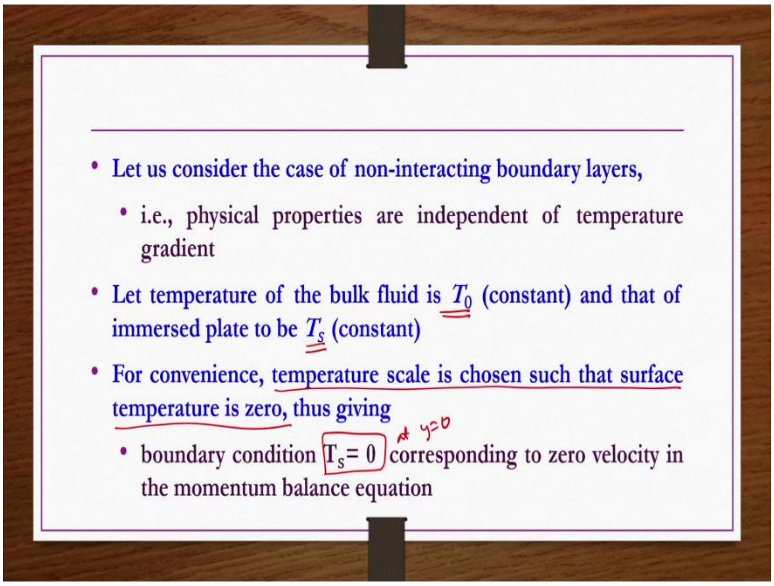

Let us consider the case of non interacting boundary layers that is physical properties are independent of temperature gradient. Let the temperature of bulk of fluid is T_0 which is constant and that of immersed plate at T_s that is also constant. For convenience temperature scale is chosen such that surface temperature is 0 thus giving a boundary condition $T_s = 0$ at y = 0 locations that is corresponding to 0 velocity along the plane. So, that we can have an analogous development and it will be easy right.

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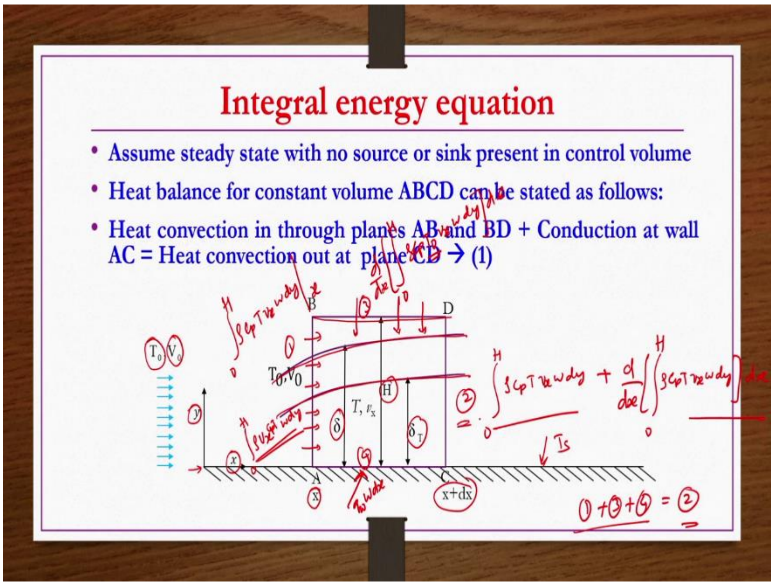

So, now we develop integral energy equation, here the same equilibrium of control volume we are taking. Now, here in addition to V_0 free stream velocity the temperature of free stream fluid is T_0 and the surface temperature is T_s right. So, flow direction is x direction vertical direction is y direction.

The size of the control volume you know in the x direction is δ x that is A is located at x and then C is located at x + δ x. Height of this control volume is H and then this BD plane is outside of the control volume. So, this is momentum, boundary layer δ and this is a thermal boundary layer of δ_T.

So, δ_T is function of x that also we understand. So, when the fluid comes and hits the surface at the leading edge what happens? The fluid experience retardation only for infinitesimal time. So, then the boundary layer thickness is 0 and then gradually as you move along the flow direction along the surface then fluid molecules experience more and more retardation. So, then gradually boundary layer thickness increases in the x direction.

So, δ or δ_T both of them are function of x. Now here what is the rate of heat in here that we have to find out. Similarly, we have to find out other planes also at this plane the $\int_0^H \rho v_x w dy$ is nothing but rate of mass in that is $\dot{m}$, $\dot{m}$ m $c_p dT$. So, now, this quantity if you multiply by $c_p T$, you get this one.

So, that is at the AB plane the rate of heat in is $\int_0^H \rho c_p T v_x w dy$ this is at x location. Similarly at CD plane that is x + d x location what we will be having? This 0 to H whatever is there in addition to this one there would be some change in rate of heat transfer that is in x direction; that we do not know let us say that we are calling $\frac{d}{dx}$ of whatever the entering one δ x.

So, this is entering and then + this much of change is occurring. So, that overall is going out right; how much is the heat is entering through the BD plane? That would be $\int_0^H \rho c_p T_0 v_x w dy$ and then $\frac{d}{dx}$ of this one; because this BD plane is outside of the boundary layer and outside of the boundary layer temperature is equals to T_0 right.

So, at the surface whatever the heat transfer rate is there that is q_w; let us say that multiplied by the area of the plane of this AC in the we have to take in the z direction also, in the x direction size of the plane is dx in the z direction width of the plate is w so, w dx. So, whatever the entering let us say this is entering is 1, leaving one is 2 and then entering from BD is 3 and then entering through all by conduction is 4. So, quantity is 1 + 3 + 4 should be equals to quantity 2.

So, these quantities we substitute here and then simplify then we get integral energy equation ok. So, assume steady state with no source or sink present in control volume. Heat balance for constant volume ABCD can be stated as follows. Heat convection in through planes AB that is 1 and then BD that is 3 + conduction at wall at AC that is 4.

So, quantities or expressions written 1, 3 and 4 should be balanced by heat convection out at plane CD that is this 2 and this plane that is written expression 2 here right.

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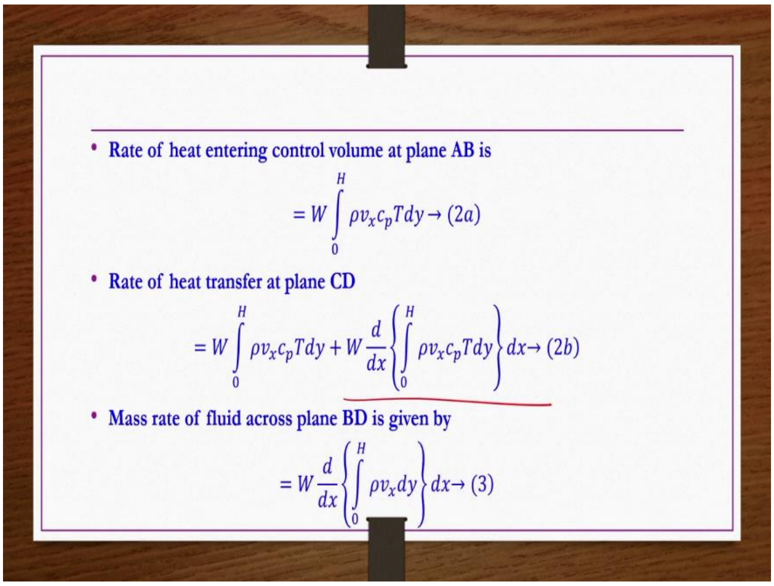

The pictorially I have explained whatever there, so, the same thing I have written here. Rate of heat entering control volume at plane AB is this one rate of heat transfer at plane CD is nothing but this one + $\frac{d}{dx}$ of that whatever 2 b expression right. And then mass rate of fluid a cross plane BD that we have already seen that is nothing but $W\frac{d}{dx}\left\{\int_0^H \rho v_x dy\right\} dx$ that we have seen.

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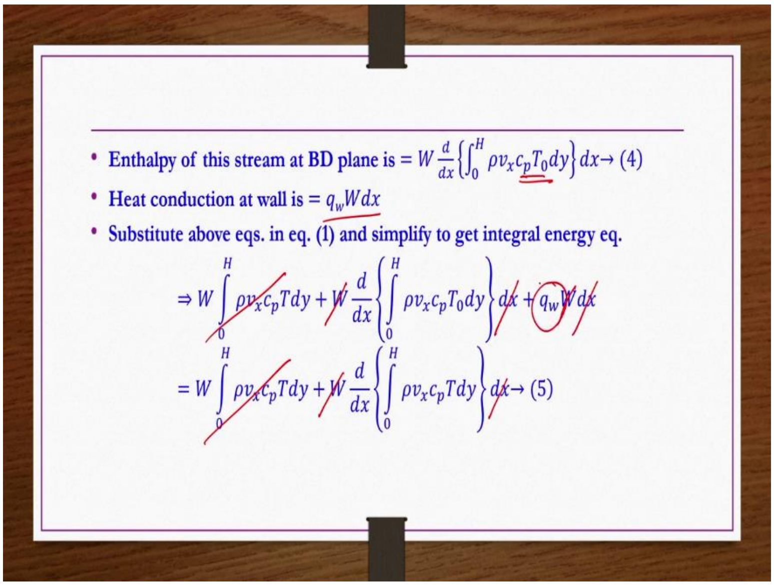

So, this if you multiply by ρ c_p T_0 you will be getting the rate of heat entering through plane BD ok. Then heat conduction at wall is nothing but q w multiplied by the area W dx W is in z direction dx in x direction ok. So, now, these things you substitute here in the equation number 1. So, this is what we are having.

So, this quantity and this quantity are exactly same then from remaining quantities W you can cancel out from all three quantities and then d x also you can cancel out. And then in place of q w you can write $-k\frac{dT}{dy}|_{y=0}$ because q dw is at the solid wall that is at y = 0 location and then rearrange that equation.

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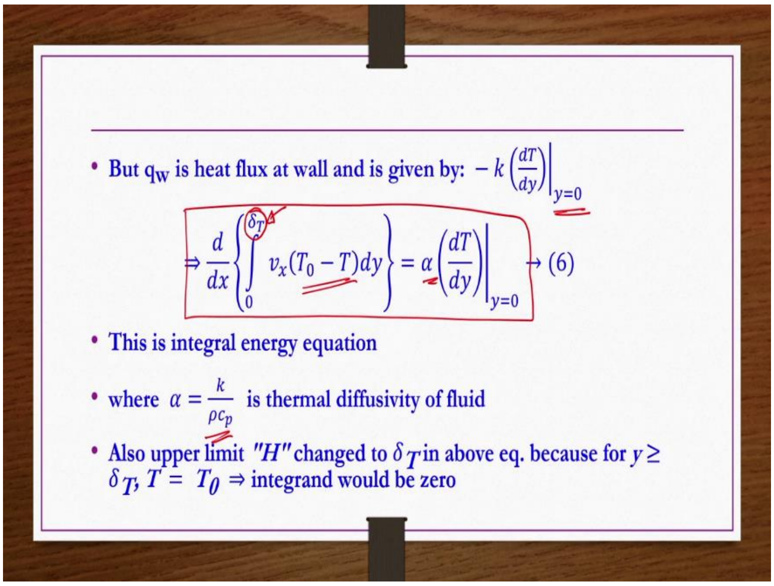

So, you get this x equation as integral energy equation integral form of the energy equation. Here also the upper limit of integration has been changed from H to δ_T; because we wanted to have this integral energy equation for the boundary layer region only within the boundary layer region only which is encapsulated between y = 0 to y = δ_T and it is function of x.

We do not want what is happening outside of the boundary layer because outside of the boundary layer gradients are approximately 0 or outside of the boundary layer temperature is equal to T_0. So, then integrand will become 0. So, that is the reason upper limit has been changed to δ_T from H right. So, here this alpha is nothing but $\frac{k}{\rho c_p}$ which is nothing but thermal diffusivity of the fluid.

So, now this integral momentum equation that we have developed few slides before and then integral energy equation developed in this slide here. These are going to be used in our next classes to find out the momentum and thermal boundary layer thickness for power law fluids and then Bingham plastic fluids in addition to the regular Newtonian fluids as well.

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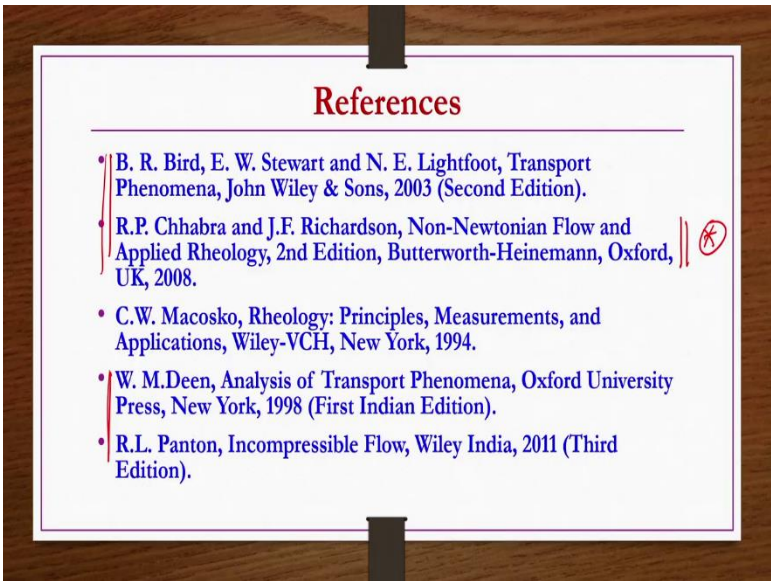

References for this lecture are provided here. So, you can find out details in three four books which are available. But primarily this lecture I have prepared from this book, but more concepts on boundary layer analysis you can find out from the remaining three books as well.

Thank you.