Transport Phenomena of Non-Newtonian Fluids
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Lecture - 38
Quasi- Steady Analysis of Simultaneous HT and MT - III

Welcome to the MOOCs course transport phenomena of Non-Newtonian fluids the title of today’s lecture is Quasi Steady Analysis of Simultaneous Heat and Mass transfer, part 3. We have been discussing considering several problems where both heat and mass transfer occurring, but in addition to that one there is a time dependent behaviour also. So, but that time dependent behaviour we are taking only quasi steady analysis right.

So, under that category we are going to see one more problem today ok that is evaporation of a water droplet. A water droplet is suspended in a nitrogen atmosphere and that is being evaporated slowly it is evaporating slowly that is the reason we can consider the quasi steady analysis for this problem also.

So, how much time is required to completely evaporate the that droplet that is what we are going to see from the transport phenomena details that we have learnt in the course ok.

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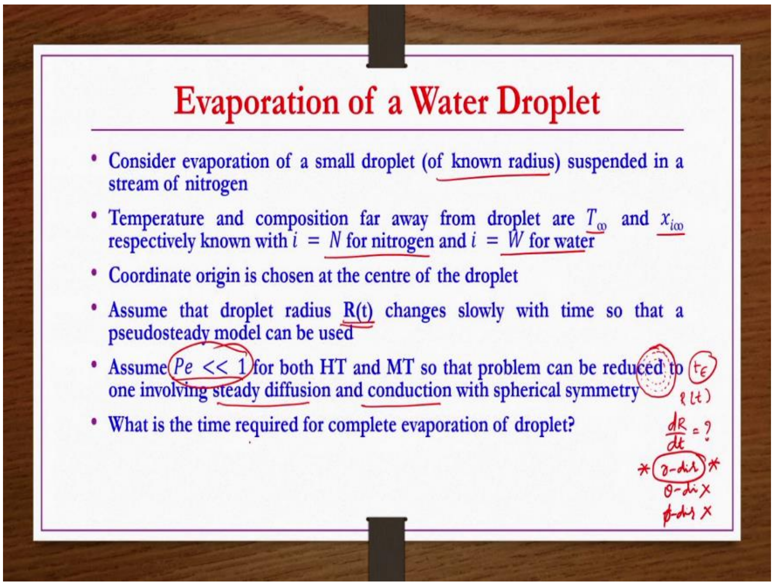

Evaporation of a Water Droplet

- Consider evaporation of a small droplet (of known radius) suspended in a stream of nitrogen
- Temperature and composition far away from droplet are T_∞ and $x_{i\infty}$ respectively known with $i = N$ for nitrogen and $i = W$ for water
- Coordinate origin is chosen at the centre of the droplet
- Assume that droplet radius R(t) changes slowly with time so that a pseudosteady model can be used
- Assume $Pe << 1$ for both HT and MT so that problem can be reduced to one involving steady diffusion and conduction with spherical symmetry
- What is the time required for complete evaporation of droplet?

t_E

$R(t)$

$\frac{dR}{dt} = ?$

$* \, r\text{-dir} \, *$

$\theta\text{-dir} \times$

$\phi\text{-dir} \times$

So, consider evaporation of a small droplet of known radius suspended in a stream of nitrogen. Temperature and composition far away from droplet are T_∞ and then $x_{i\infty}$. Now, here only two components are there so one is the water droplet, water component and then another one is the nitrogen component. So, water droplet is suspended in nitrogen. So, i can be W for water and then i can be N for nitrogen.

Coordinate origin is chosen at the centre of the droplet assume that droplet radius r (t) changes slowly with time so that a pseudo steady model can be used. Because let us say initially the droplet size is this much slowly what happens evaporation is taking place. So, then subsequently the size reduces, we are assuming that reduction of size is also in the spherical ok that is the simplicity.

So, gradually and by some more time it reduces to some more smaller size like this like that after some time t_E evaporation time let us recall. So, the droplet complete evaporates and then it is not in the liquid state it is in the vapour phase and then it is mixed with the nitrogen.

So, how much time required for this droplet to completely evaporate that is what we have to find out. So, this R (t) and then evaporation is slow so then that is the reason pseudo steady or quasi steady model we are adapting for this problem also.

So, what we basically need to understand or develop? We have to develop a reliable equation for $\frac{dR}{dt}$ and in from there we have to find out you know what is the time required for this R to become 0 ok, this R becoming 0; that means, droplet is completely evaporated.

Further assume Peclet number is very very less than one for both heat and mass transfer so that problem can be reduced to one involving steady diffusion and conduction with spherical symmetry. So, when we have this assumption we have already seen previously also previous problems also.

Then whatever the changes are there, changes would be there only in the radial direction. In the θ direction nothing would be there no changes would be there and then in the ϕ direction also there will not be any changes whatever the changes whether the velocity change or temperature change or flux etcetera whatever are there.

So, they are all, they all will be in R direction only and then this is true also from physics point of view also as long. As you provide or you maintain Peclet number is very very smaller than 1 ok. Then what is the time required for complete evaporation of droplet that is what you have to find out.

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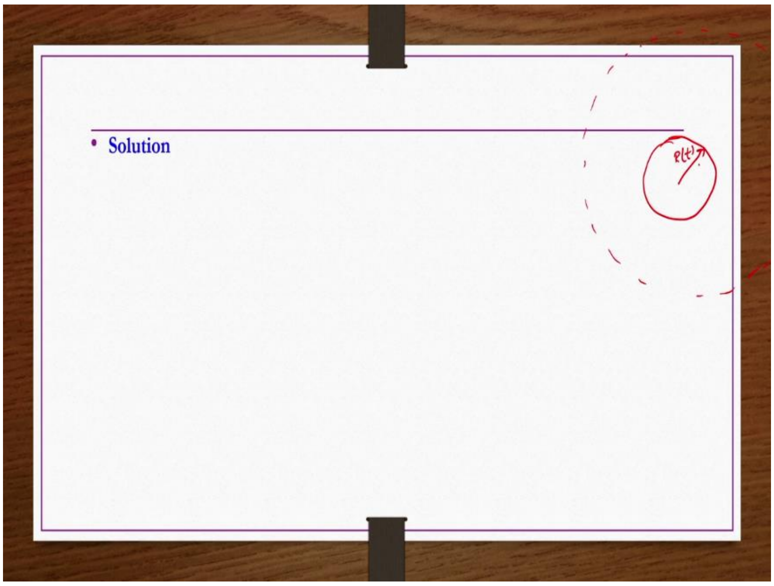

- Solution

Now, the solution what we have to do? We have to develop an equation for the flux which is function of both R and t that we have to develop and then that equation we have to solve. So, that we get a relation for $\frac{dR}{dt}$ as function of temperature or mole fraction or both right. So, that is what basically the mathematically mathematical object is that one basically right.

So, how do we do? This developing the flux that we already seen for other problems, but; however, we can have a recapitulation or the same thing we can discuss here also. So, let us say this is the droplet whose size is R and it is changing time t. So, now, within this spherical cavity, so within this circular object because now we are taking only r direction variation so then we can have a circle of radius R which is changing with time.

So, the outside domain also whatever the nitrogen is there that is also in the; that is also in the spherical object like this ok. So, that you know that is how the domain we take right. So, let us not worry about that one this R is decreasing with time ok.

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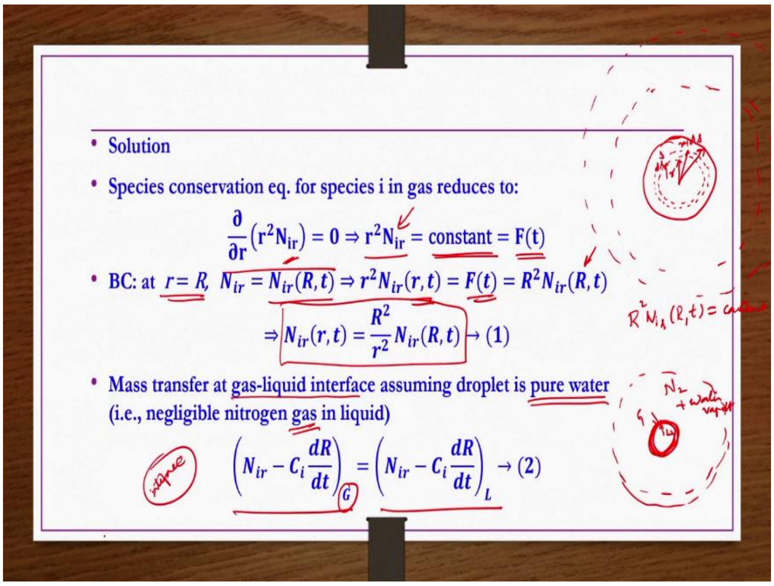

- Solution
- Species conservation eq. for species i in gas reduces to:

$$\frac{\partial}{\partial r}(r^2 N_{ir}) = 0 \Rightarrow r^2 N_{ir} = \text{constant} = F(t)$$

- BC: at $r = R$, $N_{ir} = N_{ir}(R,t) \Rightarrow r^2 N_{ir}(r,t) = F(t) = R^2 N_{ir}(R,t)$

$$\Rightarrow \boxed{N_{ir}(r,t) = \frac{R^2}{r^2} N_{ir}(R,t)} \rightarrow (1)$$

- Mass transfer at gas-liquid interface assuming droplet is pure water (i.e., negligible nitrogen gas in liquid)

$$\left(N_{ir} - C_i \frac{dR}{dt}\right)_G = \left(N_{ir} - C_i \frac{dR}{dt}\right)_L \rightarrow (2)$$

So, now, if you wanted to know the flux so what we do? We take a control volume after; a control volume of thickness d R. So, that is a in a circle whose radius is r we take and then there would be another circle whose radius is r + d R. So, that the circle thickness or the control volume thickness is dR right.

For this control volume if you do the balance right mass balance species conservation equation for this for this control volume if you write. And then apply the differential principles by taking delta r or dR tends to 0, then what you get? You get this equation. $\frac{\partial}{\partial r}(r^2 N_{ir}) = 0$ that is what we get. This is i stands for any component nitrogen or you know water it can be anything right.

So, because it is valid for the outside also, outside the outside circle like you know outside of the droplet also we can have you know circle like this and then develop the equation. Because the domain also outer confined fluid whatever are outer nitrogen gas whatever is there that also spherically confined that is what mathematically we are taking. So, it is valid for both so then generalized one we have written right.

Now, when you integrate you get $r^2 N_{ir}$ is equals to constant it is constant with respect to r, but it is dependent on the time. Because we know this flux is also function of time in addition to the function of radial distance r right. So, that is because the quasi steady analysis we are doing it is slowly evaporating so we can apply quasi steady analysis right.

So, this constant is function of time, but it is independent of r that is what it mean by. So, what is that constant? We do not know. So, let us say if you obtain this flux at r = R that is at the interface of the droplet then N_{ir} would be called as, $N_{ir}(R, t)$. Whereas, this $N_{ir}(r, t)$ is there that is for N_{ir} that is for N_{ir}. So, that means when you apply this boundary condition you will get $R^2 N_{ir}(R, t)$ is also constant.

So, both the constants are same for the either of the cases. So; that means, we can write $r^2 N_{ir}(r, t)$ = $R^2 N_{ir}(R, t)$ we are calling function of (R, t) because this r is also function of time ok.

So, now these two are equals to this constant F (t). So, this is going to be useful in our you know subsequent calculations though we are not able to find out what is this F (t) by applying this boundary condition here ok. Now; that means, from here $N_{ir}(r, t)$ we can write it as $\frac{R^2}{r^2} N_{ir}(R, t)$. So, now, what we do? Interfacial balance equations we are going to write for both the components nitrogen and then water.

So, mass transfer at gas liquid interface assuming droplet is pure water; water is pure water that is negligible nitrogen gas in the liquid. So, this is the water droplet right and then it is in a surrounding you know nitrogen atmosphere. So, this water droplet is pure water, there is no nitrogen and then surrounding is the nitrogen, but here some water may also be there water vapours because of the evaporations right whereas, no nitrogen is present in the liquid droplet ok.

Then at the interface, at this interface what we are calling we are calling G if it is towards the nitrogen gas side and then we are calling it L if it is towards the liquid droplets side ok. So, that is $N_{ir} - C_i \frac{dR}{dt}$ towards the liquid side should be balanced by $N_{ir} - C_i \frac{dR}{dt}$ towards the gas side right. So, they are not same, but they are evaluated at the same location of interface droplet interface ok. Now, this we have to write for individual components for nitrogen as well as the water ok.

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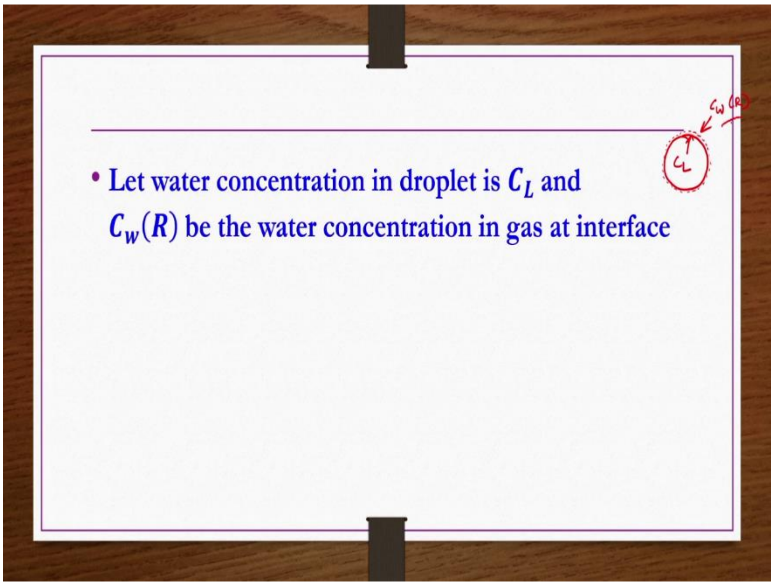

- Let water concentration in droplet is C_L and $C_w(R)$ be the water concentration in gas at interface

Let us assume whatever liquid concentration is there water concentration in the droplet is there. So, that we take as C_L, but the same water vapours whatever the first layer towards the gas side whatever is there that concentration or the interface concentration or concentration in gas at the interface that we call it C_w function of R. Why function of R? Because the size of the droplet is decreasing because of evaporation and then that R is function of time. So, there is a possibility that it may also change keep on changing.

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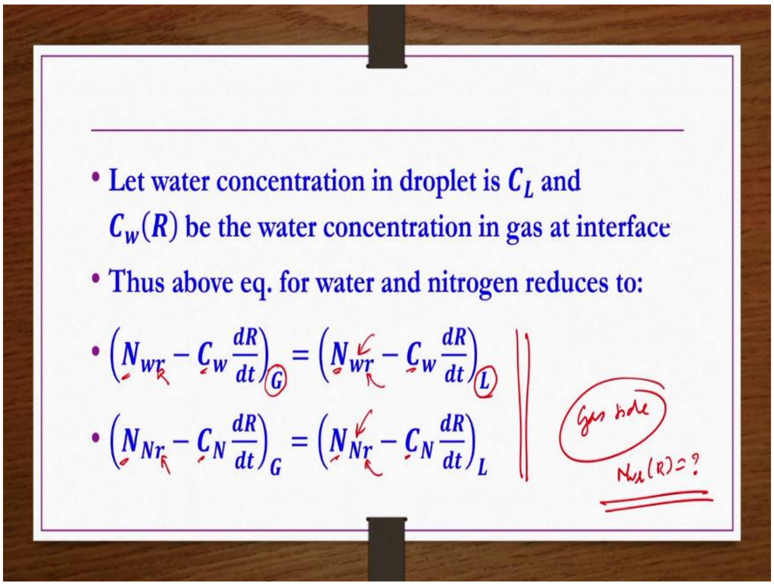

- Let water concentration in droplet is C_L and $C_w(R)$ be the water concentration in gas at interface
- Thus above eq. for water and nitrogen reduces to:
- $\left(N_{wr} - C_w \frac{dR}{dt}\right)_G = \left(N_{wr} - C_w \frac{dR}{dt}\right)_L$
- $\left(N_{Nr} - C_N \frac{dR}{dt}\right)_G = \left(N_{Nr} - C_N \frac{dR}{dt}\right)_L$

Gas side

$N_{wr}(R) = ?$

Now, the above equation previous slide whatever we have written for you know generalized species I if we write for water and nitrogen then for water $\left(N_{wr} - C_w \frac{dR}{dt}\right)|_G$ should be balanced by $\left(N_{wr} - C_w \frac{dR}{dt}\right)|_L$. L is towards the liquid side G is towards the gas side, but both of them are at the interface.

Similarly, for nitrogen $\left(N_{Nr} - C_N \frac{dR}{dt}\right)|_G = \left(N_{Nr} - C_N \frac{dR}{dt}\right)|_L$. This N stands for the combine flux that we know C stands for the concentration that also we know right and then this w stands for now water and then N stands for the nitrogen the r stands for the direction in which the flux is taking place or the change in mass transfer is taking place that is r direction. So, that this notations we already know ok.

And then $\frac{dR}{dt}$ is the change in size of the droplet with respect to the time that is integrated by this so now, these are the generalized one. So, now these things we are going to write towards the gas side so that to get what is this N_{wr}(R) because this we wanted to know.

Because this if you find out from the combined flux we get this relation, another relation for this N_{wr}(R) in terms of the mole fractions right x_W or x_∞ etcetera. So, that is what we are going to see now.

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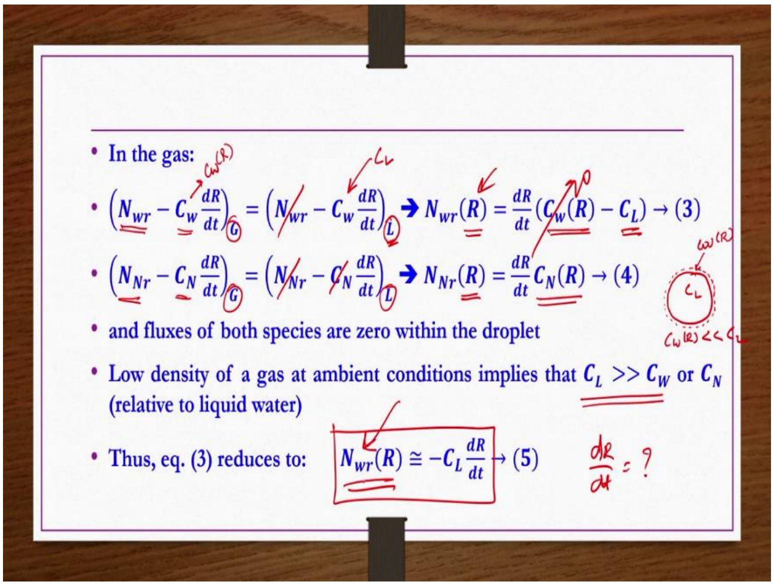

- In the gas:
- $\left(\underline{N_{wr}} - \underline{C_W}\frac{dR}{dt}\right)_{G} = \left(\cancel{N_{wr}} - C_W\frac{dR}{dt}\right)_{L} \Rightarrow N_{wr}(R) = \frac{dR}{dt}(\cancel{C_W(R)} - C_L) \rightarrow (3)$
- $\left(\underline{N_{Nr}} - \underline{C_N}\frac{dR}{dt}\right)_{G} = \left(\cancel{N_{Nr}} - \cancel{C_N}\frac{dR}{dt}\right)_{L} \Rightarrow N_{Nr}(R) = \frac{dR}{dt}C_N(R) \rightarrow (4)$
- and fluxes of both species are zero within the droplet
- Low density of a gas at ambient conditions implies that $C_L >> C_W$ or C_N (relative to liquid water)
- Thus, eq. (3) reduces to: $\boxed{N_{wr}(R) \cong -C_L\frac{dR}{dt}} \rightarrow (5)$

In the gas side$\left(N_{wr} - C_w \frac{dR}{dt}\right)|_G = \left(N_{wr} - C_w \frac{dR}{dt}\right)|_L$, so this is for the water ok. So, now in the liquid side water concentration whatever is the C w is there that is pure water. So, that concentration is C_L right, but in the liquid side that is the droplet concentration is same C_L is not changing from the centre of the droplet to the interface of the droplet, surface of the droplet. So; that means; obviously, that flux would not be there right.

And then this C_w is nothing, but is nothing, but the water concentration at the interface, but towards the gas side. So, that we calling we are calling C_w as function of R and this is the water flux towards the gas side at the interface, but at the interface. So, we can write $N_{wr}\ (R)$, this G and L are designating which side of the interface liquid side or gas side that is it.

But they are at the interface at the R only at the R, though this R is changing with respect to time so that we take care ok. So, the now what you get $N_{wr}\ (R) = \frac{dR}{dt}$ this is coming to the right hand side. So, C_w (R) and $-$ this C_w is nothing, but C_L as per the notations given in the problem or that initially we have taken ok.

Now, similarly for nitrogen you write $N_{Nr} - C_N \frac{dR}{dt}$ towards the gas side should be balanced by $N_{Nr} - C_N \frac{dR}{dt}$ towards the liquid side right. So, now, here in the liquid side in the droplet there is no nitrogen in the droplet there is no nitrogen that is what it is given. So, then C_N is 0 and then since there is no nitrogen in the droplet side or liquid side so then; obviously, its flux would now would also be not there.

But in the gas side there is a nitrogen and then there is a concentration variation of nitrogen because for N component water is being added up as the water is being evaporated progressively with respect to time slowly. So, it would be there and then; obviously, C_N would also be there.

So, N_{Nr} (R) is nothing, but $\frac{dR}{dt}$ C_N (R) ok. Remember this R as function of time we are not writing every time because we know it ok. So, next low density of gas at ambient conditions that implies that C_L is very very greater than C_w right.

So, C_w this this is the water droplet and then its concentration is C_L. So, C_w is nothing, but the very first layer or the in a towards the gas side at the interface whatever the water vapour concentration is there that we are calling $C_w\ (R)$. So, this is going to be $C_w\ (R)$ is going to be very very smaller compared to the C_L. So, from this equation number 3 we can take off so that N_{wr} is nothing, but approximately $-C_L \frac{dR}{dt}$.

So, now we have an expression for $\frac{dR}{dt}$ actually we have this is what our primary aim right. So, now if you know this, what is $N_{wr}(R)$ you know perfectly in measurable quantity so then; obviously, we can simplify this equation and find it out right. But, we do not know what it is, we know this is like you know $\frac{r^2}{R^2} N_{ir}(r,t)$ so that is if you use that equation that is equation number 1 it is not going to be helpful anyway.

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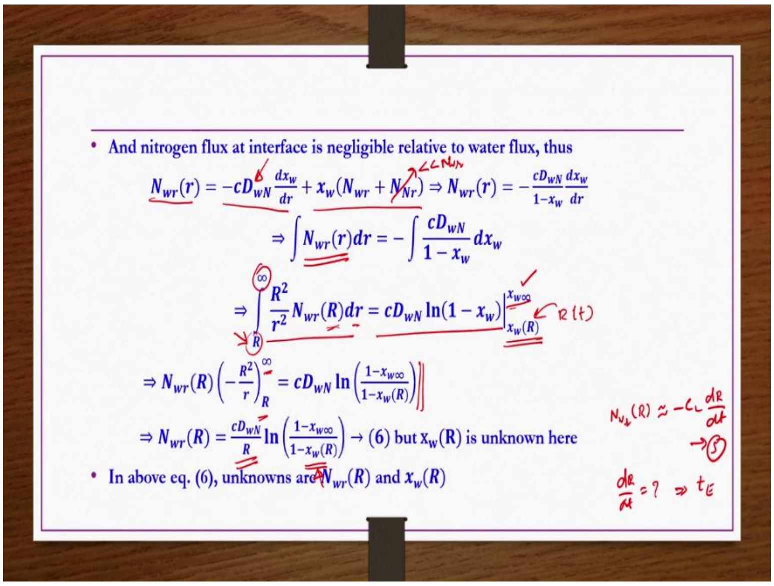

- And nitrogen flux at interface is negligible relative to water flux, thus

$$N_{wr}(r) = -cD_{wN}\frac{dx_w}{dr} + x_w(N_{wr} + \cancel{N_{Nr}}) \Rightarrow N_{wr}(r) = -\frac{cD_{wN}}{1-x_w}\frac{dx_w}{dr}$$

$<< N_w$

$$\Rightarrow \int N_{wr}(r)dr = -\int \frac{cD_{wN}}{1-x_w}dx_w$$

$$\Rightarrow \int_R^{\infty} \frac{R^2}{r^2}N_{wr}(R)dr = cD_{wN}\ln(1-x_w)\Big|_{x_w(R)}^{x_{w\infty}}$$

$R(t)$

$$\Rightarrow N_{wr}(R)\left(-\frac{R^2}{r}\right)_R^{\infty} = cD_{wN}\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)$$

$$\Rightarrow N_{wr}(R) = \frac{cD_{wN}}{R}\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) \rightarrow (6) \text{ but } x_w(R) \text{ is unknown here}$$

- In above eq. (6), unknowns are $N_{wr}(R)$ and $x_w(R)$

$N_{wr}(R) \approx -C_L \frac{dR}{dt}$

→ ③

$\frac{dR}{dt} = ? \Rightarrow t_E$

So, what we do? This combined flux we go back to the basics of combined flux so we write a combined flux equation right. When we write combined flux equation that is $N_{wr}(r) = -cD_{wN}\frac{dx_w}{dr} + x_w(N_{wr} + N_{Nr})$ right. Here D_{wN} is nothing, but diffusivity of water vapour in the nitrogen gas ok.

Now, here it has been said that the nitrogen flux it interface is negligible related to the water flux right, its very small because it is not getting into the water droplet nitrogen is not getting into the water droplet. And then its concentration variations at the interface are very small compared to the you know that of a water vapour.

So, compared to that one we using that statement we can say this N_{Nr} is very much smaller compared to the N_{wr} so then we can take off. That means, N_{wr} from this equation we get $\frac{-cD_{wN}}{(1-x_w)}\frac{dx_w}{dr}$. So, this equation we can integrate right by taking N_{wr} dR one side and then remaining terms to the other side.

Integration limits are R to ∞ because from the surface from the surface to faraway distance we are finding the flux at the gas side because the liquid side there is no flux because it is a pure water. Also, from the physics of the problem water is evaporating and then getting into the gaseous phase side ok.

So, surface interface radius is R and then faraway distance is ∞ N_{wr} we have already N_{wr} (r) we have already found it as $\frac{R^2}{r^2}$ N_{wr} (R) dr this we already know so, dr is as it is. Right hand side this you integrate you get cD_{wN} $\ln(1-x_w)$.

So, minus 1 and then minus it is a +. So, cD_{wN} $\ln(1-x_w)$ you get. Limits are at the surface, what is the water concentration we do not know right because it is changing interface is changing. So, we do not know it is function of time actually x_w is also function of time, but we are writing x_w (R) because we know that this R is again function of time. So, indirectly x_w is function of time we are writing.

Faraway concentration this is known $x_{w\infty}$ is known, $x_{i\infty}$ and then $t_{i\infty}$ are known far away from the droplet so this is known. So, now this left hand side when you integrate you get minus 1/r limits R to ∞ you substitute. Then here right hand side this limits when you substitute you get this one.

Now here you get you know $\frac{R^2}{R}$ so that N_{wr} (R) multiplied by R you would be having. So, that R in the left hand side that you bring it to the right hand side. So, then N_{wr} $(R) = \frac{cD_{wN}}{R} ln \frac{1-x_{w\infty}}{1-x_w(R)}$ right.

Equation number 5 we already got that N_{wr} (R) is approximately $-C_L\frac{dR}{dt}$. Now, you may be thinking this equation this equation you can combined and then find out what is this $\frac{dR}{dt}$ expression and then find out what is this t_E when R becomes 0 that you can you may be thinking. That we can do indeed there is no harm, but in this equation here what is this x_w (R) that we do not know.

It is changing, it is changing with the size of the droplet so it changing with the time. Had it been constant so then we could have equated equation 5 and 6 and then get this $\frac{dR}{dt}$ relation and then find out the t_E total evaporation time right.

So, now we have to do some more analysis to find out what is this x_w (R). Either as function of R we have to find out or we have to find out as function of the temperature because the evaporation is taking place because of the non isothermal conditions being maintained ok.

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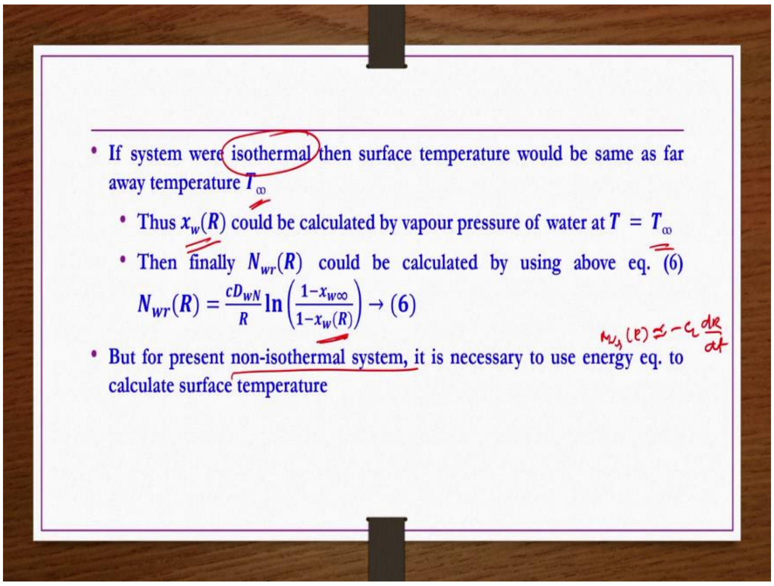

- If system were isothermal then surface temperature would be same as far away temperature T_∞
  - Thus $x_w(R)$ could be calculated by vapour pressure of water at $T = T_\infty$
  - Then finally $N_{wr}(R)$ could be calculated by using above eq. (6)

    $N_{wr}(R) = \frac{cD_{wN}}{R} \ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) \rightarrow (6)$

$N_{wr}(R) \simeq -C_L \frac{dR}{dt}$

- But for present non-isothermal system, it is necessary to use energy eq. to calculate surface temperature

Now, if system were isothermal then surface temperature would be same as far away temperature T_∞ then; obviously, x_w we could have got it by vapour pressure of water evaluating at $T = T_\infty$. And then that x_w (R) we could have substituted in this equation number 6 and then also we could have found what is this N_{wr} (R) and then equate to the equation number 5 that is N_{wr} (R) = or approximately equals to $-C_L \frac{dR}{dt}$.

And then also we could have got, but the system in our case it is not the isothermal it is non isothermal system. So, we cannot use this equation so; obviously, we have to use the energy equation to calculate the surface temperature and then that energy equation we have to do for this you know domain as well as for the interfacial energy balance we have to do.

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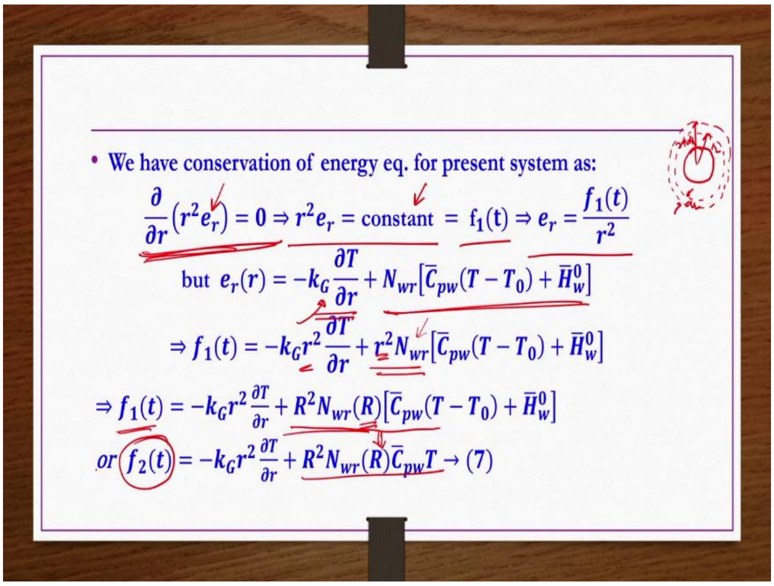

- We have conservation of energy eq. for present system as:

$$\frac{\partial}{\partial r}\left(r^2 e_r\right) = 0 \Rightarrow r^2 e_r = \text{constant} = \mathrm{f}_1(\mathrm{t}) \Rightarrow e_r = \frac{f_1(t)}{r^2}$$

$$\text{but } e_r(r) = -k_G \frac{\partial T}{\partial r} + N_{wr}\left[\bar{C}_{pw}(T - T_0) + \bar{H}_w^0\right]$$

$$\Rightarrow f_1(t) = -k_G r^2 \frac{\partial T}{\partial r} + r^2 N_{wr}\left[\bar{C}_{pw}(T - T_0) + \bar{H}_w^0\right]$$

$$\Rightarrow f_1(t) = -k_G r^2 \frac{\partial T}{\partial r} + R^2 N_{wr}(R)\left[\bar{C}_{pw}(T - T_0) + \bar{H}_w^0\right]$$

$$or\ f_2(t) = -k_G r^2 \frac{\partial T}{\partial r} + R^2 N_{wr}(R)\bar{C}_{pw}T \rightarrow (7)$$

So, we have conservation of energy equation for present system as this because here also the same thing, the droplet you have taken right. Now, you take a within the domain you know you take a you know one circle of radius r unknown r and then another circle adjacent to it whose radius is r + dr. so that this thickness is dr ok so that is so that this thickness is dr and then you do the energy balance so, then you get this one.

Or heat transfer equation whatever we have developed in Cartesian coordinates the same similar things we have we can also develop further you know spherical coordinates and then do this one ok. So, better you we do this kind of balancing by taking e_r; e_r includes the all the modes of heat transfer conduction, convection in and all that whichever are you know important associated with this problem all of them are included.

Rather writing you know heat entering because of the conduction, heat entering because of the convection etcetera and then heat leaving because of the conduction and then heat leaving because of the convection and conduction etcetera all those things writing and then equating to accumulation that becomes complicated.

So, we have included all of them in e_r and then written balance so then we get this equation by applying the differential quantities dr tends to 0 right. That means, when you integrate this equation you get further $r^2 e_r =$ constant, here also this constant is constant with respect to the r radial position, but it is function of time that time function we are calling f_1 (t).

So, from here $e_r = \frac{f_1(t)}{r^2}$, but this e_r is having several contributions; however, for our problems it is having only two contributions one is the conduction another one is the heat associated because of the mass transfer part. So, both of them we have written here. And then we are writing for water only for the time being so let us not worry about the nitrogen ok right.

So, e_r for water as function of r we write so then conduction term is $-k_G \frac{\partial T}{\partial r} + N_{wr}\bar{C}_{pw}(T - T_0) + \bar{H}_w^0$. T_0 is the reference temperature and this $\bar{H}_w^0$ is evaluated at reference temperature T_0 right. This suffix G is what? k is thermal conductivity of water only, but suffix G indicates towards the towards the gas side ok.

So, from here $f_1(t)$ is nothing, but $r^2 e_r$. So, this equation is being multiplied by r^2 here ok. Now, this $r^2\ N_{wr}$ we know it as $R^2\ N_{wr}(R)$. Explicitly if you are writing $N_{wr}(R)$; that means, that is at the interface right. If we are not writing explicitly $N_{wr}(r)$; that means, that is if now explicitly we are not writing function of r; that means, that is it is r any r value, R is at the interface right.

So, now here this equation what we do? We keep only $R^2\ N_{wr}\bar{C}_{pw}T$ here and then remaining terms whatever are there that is $R^2\ N_{wr}\bar{C}_{pw}(T - T_0) + \bar{H}_w^0$ all that we mix, all that we combined with $f_1(t)$ and then write it as $f_2(t)$ for simplicity ok.

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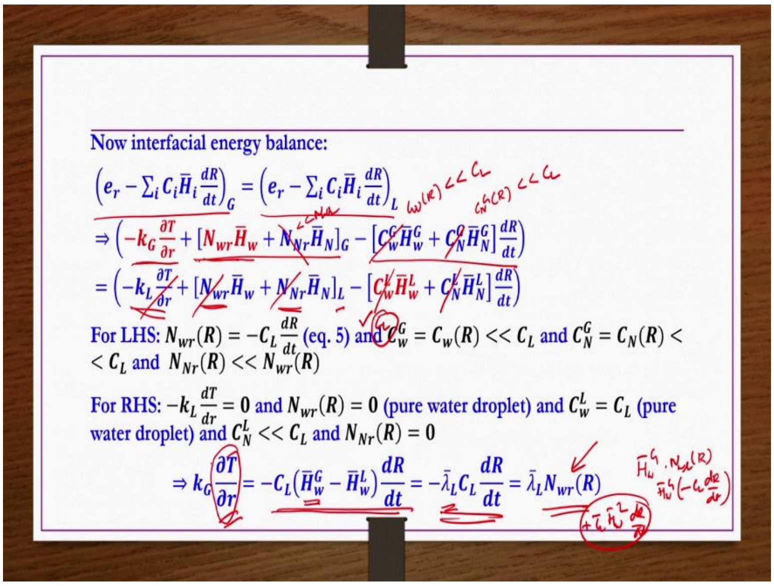

Now interfacial energy balance:

$$\left(e_r - \sum_i C_i \bar{H}_i \frac{dR}{dt}\right)_G = \left(e_r - \sum_i C_i \bar{H}_i \frac{dR}{dt}\right)_L$$

$C_w(R) << C_L$

$C_N^G(R) << C_L$

$<< N_w$

$$\Rightarrow \left(-k_G \frac{\partial T}{\partial r} + [N_{wr}\bar{H}_w + \cancel{N_{Nr}}\bar{H}_N]_G - [\cancel{C_w^G}\bar{H}_w^G + \cancel{C_N^G}\bar{H}_N^G]\frac{dR}{dt}\right)$$

$$= \left(-k_L \cancel{\frac{\partial T}{\partial r}} + [\cancel{N_{wr}}\bar{H}_w + \cancel{N_{Nr}}\bar{H}_N]_L - [\cancel{C_w^L}\bar{H}_w^L + \cancel{C_N^L}\bar{H}_N^L]\frac{dR}{dt}\right)$$

For LHS: $N_{wr}(R) = -C_L \frac{dR}{dt}$ (eq. 5) and $C_w^G = C_w(R) << C_L$ and $C_N^G = C_N(R) < < C_L$ and $N_{Nr}(R) << N_{wr}(R)$

For RHS: $-k_L \frac{dT}{dr} = 0$ and $N_{wr}(R) = 0$ (pure water droplet) and $C_w^L = C_L$ (pure water droplet) and $C_N^L << C_L$ and $N_{Nr}(R) = 0$

$$\Rightarrow k_G \frac{\partial T}{\partial r} = -C_L(\bar{H}_w^G - \bar{H}_w^L)\frac{dR}{dt} = -\bar{\lambda}_L C_L \frac{dR}{dt} = \bar{\lambda}_L N_{wr}(R)$$

$\bar{H}_w^G \cdot N_{wr}(R)$

$\bar{H}_w^G\left(-C_L \frac{dR}{dt}\right)$

$+\bar{\lambda}_L \bar{H}_w^L \frac{dR}{dt}$

So, that equation number 7 we have written once again here. So, now, what we do? We do interfacial energy balance that is towards the gas side whatever $e_r - \sum C_i \overline{H}_i \frac{dR}{dt}$ is there that should be balanced by the same $e_r - \sum C_i \overline{H}_i \frac{dR}{dt}$ towards the liquid side right.

Now, we are writing in the gas side $-k_G \frac{\partial T}{\partial r}$ and then whatever $[N_{wr}\overline{H}_w + N_{Nr}\overline{H}_N]_G$ this is because of the mass transfer contribution. The changes in the temperature because of the mass transfer contribution, the changes in the temperature because of the conduction is this one $-k_G \frac{\partial T}{\partial r}$. And then remaining $-\sum C_i \overline{H}_i \frac{dR}{dt}$ that we are writing this one, that is $[C_w^G \overline{H}_w^G + C_N^G \overline{H}_N^G] \frac{dR}{dt}$, this is gaseous side.

Same is true in the liquid side also that is $-k_L \frac{dT}{dr} + [N_{wr}\overline{H}_w + N_{Nr}\overline{H}_N]_L - [C_w^L \overline{H}_w^L + C_N^L \overline{H}_N^L] \frac{dR}{dt}$. This here now k_L we are writing, that is the thermal conductivity of the same species whichever we are taking right, but it is, but it is you know towards the liquid side it is G stands for towards the gas side ok.

Now, in the gas side there is a temperature variation from T_0 surface temperature to T_∞ surface temperature. So, $\frac{\partial T}{\partial r}$ is there that we cannot cancel out right and then in the gas side water flux N_{wr} is there. So, it should be there right, but N what happens this N_{Nr} in the gas side it may be there, but it is very very smaller compared to the N_{wr}.

So, we can in comparison with N_{wr} we can strike off N_{Nr} right. And then towards the gas side what is C_w^G C_w^G is nothing, but C_w (R) we have written and then it is very much smaller compared to the C_L. So, in comparison to that one we can cancel out this one also and then whatever C_N^G is there, C_N^G that is nothing, but C_N (R) is also very very less than compared to this C_L. So, that way also we can cancel out this one.

Now, coming to the liquid side; liquid side what we have temperature is not changing from the centre of the droplet to the interface of the droplet temperature is maintained same. So, there would not be any temperature gradient so then this conduction is not possible within the droplet so it is 0.

And then in the, within the liquid side only water pure water is there so then there will not be many flux. And then this nitrogen is not getting into the liquid droplet side that is mentioned in the problem so its flux in the liquid side would not be there or inside the droplet there will not be flux of either water or nitrogen.

Now, in the liquid side in the liquid side what is the concentration of water is nothing, but C_L so it is there so it should be there. But nitrogen is not penetrating into the liquid droplet whereas, the water droplet is evaporating and getting into the nitrogen. So, because nitrogen is not getting into the liquid water droplet so this C_N in the liquid is also 0.

So, now what are the terms are remaining only 1 2 3 terms which are presented in red font or are remaining. And then cancellation have been done because of this thing. So, this $k_G \frac{\partial T}{\partial r}$ I take to the right hand side so then $+k_G \frac{\partial T}{\partial r}$ is equals to what we have $-N_{wr}\overline{H}_w^G$. So, $\overline{H}_w^G$ and N_{wr} (r) this is what we are having + quantity.

So, this $\overline{H}_w^G$ this is nothing, but N_{wr} (R) is nothing, but $-C_L \frac{dR}{dt}$ that is $-C_L\overline{H}_w^G \frac{dR}{dt}$; and from the other side already what we have? We have this $-C_L\overline{H}_w^L$ is there. So, that we are taking to the other side so that is becoming $C_L\overline{H}_w^L \frac{dR}{dt}$.

So, these two terms are combined and dR taken common from these two term $-C_L$ and $\frac{dR}{dt}$ taken common. So, then we have $\overline{H}_w^G - \overline{H}_w^L$ so that we can write $-C_L \frac{dR}{dt} \bar{\lambda}_L$. So, that is latent heat of evaporation for the liquid water ok.

So, this we can write like this or again you can rewrite back in place of $-C_L \frac{dR}{dt}$ you can again write N_{wr} (R) right. Now, this N_{wr} as function of temperature gradient or the temperature profile we got a relation all this is what we are doing only for this one actually because N_{wr} (R) if you know.

So, we can substitute in equation number 6 or 5 and then find out what is $\frac{dR}{dt}$ ok. So, now that also we cannot do now, though we have an expression because what is this $\frac{\partial T}{\partial r}$ we do not know ok.

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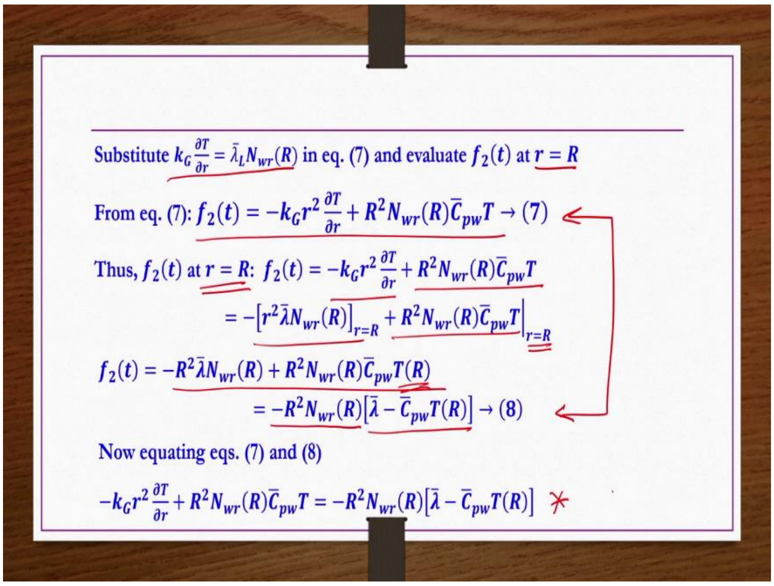

Substitute $k_G \frac{\partial T}{\partial r} = \bar{\lambda}_L N_{wr}(R)$ in eq. (7) and evaluate $f_2(t)$ at $r = R$

From eq. (7): $f_2(t) = -k_G r^2 \frac{\partial T}{\partial r} + R^2 N_{wr}(R) \bar{C}_{pw} T \rightarrow (7)$

Thus, $f_2(t)$ at $r = R$: $f_2(t) = -k_G r^2 \frac{\partial T}{\partial r} + R^2 N_{wr}(R) \bar{C}_{pw} T$

$$= -\left[r^2 \bar{\lambda} N_{wr}(R)\right]_{r=R} + R^2 N_{wr}(R) \bar{C}_{pw} T\Big|_{r=R}$$

$$f_2(t) = -R^2 \bar{\lambda} N_{wr}(R) + R^2 N_{wr}(R) \bar{C}_{pw} T(R)$$

$$= -R^2 N_{wr}(R)\left[\bar{\lambda} - \bar{C}_{pw} T(R)\right] \rightarrow (8)$$

Now equating eqs. (7) and (8)

$$-k_G r^2 \frac{\partial T}{\partial r} + R^2 N_{wr}(R) \bar{C}_{pw} T = -R^2 N_{wr}(R)\left[\bar{\lambda} - \bar{C}_{pw} T(R)\right] \ast$$

So, let us see what else we can do. Now, this expression we are substituting in equation number 7 and then finding out f_2 function of time at r = R. What is equation number 7 is this one. So, now, this equation we wanted to find out at r = R ok. So, here now this first quantity at R in place of $k_G \frac{\partial T}{\partial r}$ we have written $-\bar{\lambda}$ N_{wr} (R). You can have this suffix L r 0 it does not matter now, because we have already came across that step of interfacial balance it is only for the water droplet that we know and this part is as it is now ok.

So, this here, this is also at r = R, but there is no r term so need not to worry here. So, now from these 2 steps what we get $f_2(t)$ is this one $-R^2\bar{\lambda}N_{wr}$ (R) + $R^2 N_{wr}(R)\bar{C}_{pw}$ T (R) right. So, this T is now function of R at R that is at the interface right.

So, now this we if from these two if you take $-R^2 N_{wr}(R)$ as a common term then we get $\bar{\lambda} - \bar{C}_{pw} T(R)$. So, now this equation 6 and 7 you can equate to get this relation. Why are we doing? We can get an expression for $\frac{\partial T}{\partial r}$ that is the reason we are doing.

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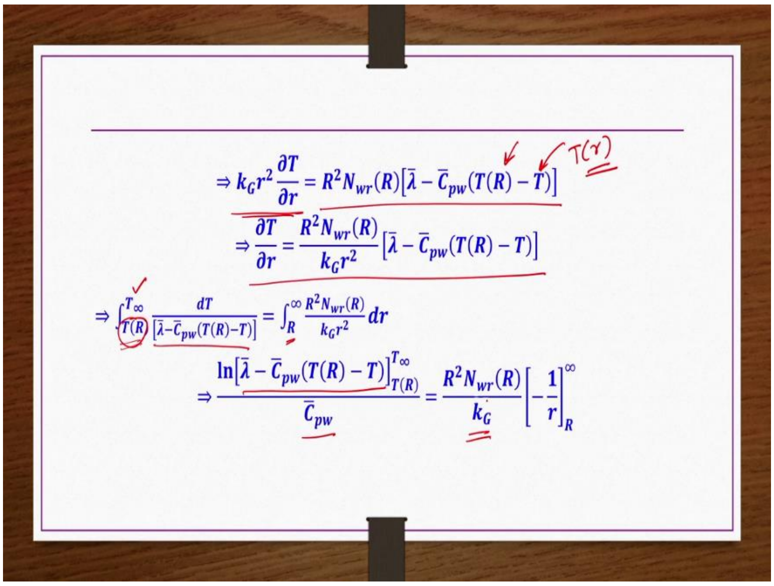

$$\Rightarrow k_G r^2 \frac{\partial T}{\partial r} = R^2 N_{wr}(R)\left[\bar{\lambda} - \bar{C}_{pw}(T(R) - T)\right]$$

T(r)

$$\Rightarrow \frac{\partial T}{\partial r} = \frac{R^2 N_{wr}(R)}{k_G r^2}\left[\bar{\lambda} - \bar{C}_{pw}(T(R) - T)\right]$$

$$\Rightarrow \int_{T(R)}^{T_\infty} \frac{dT}{\left[\bar{\lambda} - \bar{C}_{pw}(T(R) - T)\right]} = \int_R^\infty \frac{R^2 N_{wr}(R)}{k_G r^2} dr$$

$$\Rightarrow \frac{\ln\left[\bar{\lambda} - \bar{C}_{pw}(T(R) - T)\right]_{T(R)}^{T_\infty}}{\bar{C}_{pw}} = \frac{R^2 N_{wr}(R)}{k_G}\left[-\frac{1}{r}\right]_R^\infty$$

So, from that equation previous slide $k_G r^2 \frac{\partial T}{\partial r}$ we have written one side and all other terms we have written other sides ok right. So, this T without any you know r locations; that means, it is at any r location. This at R; that means, that is at the interface only, at small r we are not writing as function of r because that is for any r value starting from R to ∞.

So, from this equation $\frac{\partial T}{\partial r}$ we can write like this right. So, this equation we can write d T by whatever this constant terms and then other side we can write $\frac{R^2 N_{wr}(R)}{k_G r^2} dr$ and then integrate it. Integrating from r to ∞ at r = R temperature is T (R) at r = ∞ T is nothing, but T_∞ it is known, but it is not known T (R) is not known.

So, when you integrate this one this equation what we get left hand side we get ln of this particular term divided by minus of minus + $\bar{C}_{pw}$ and the limits T(R) to T_∞ we are substituting here. So, right side $\frac{R^2 N_{wr}(R)}{k_G}$ is constant, integration of $\frac{1}{r^2}$ is $-\frac{1}{r}$ limits R to ∞. So, when you substitute a limits here what you get? You get + $\frac{1}{r}$ you get and then that R and then square of this R you can cancel out.

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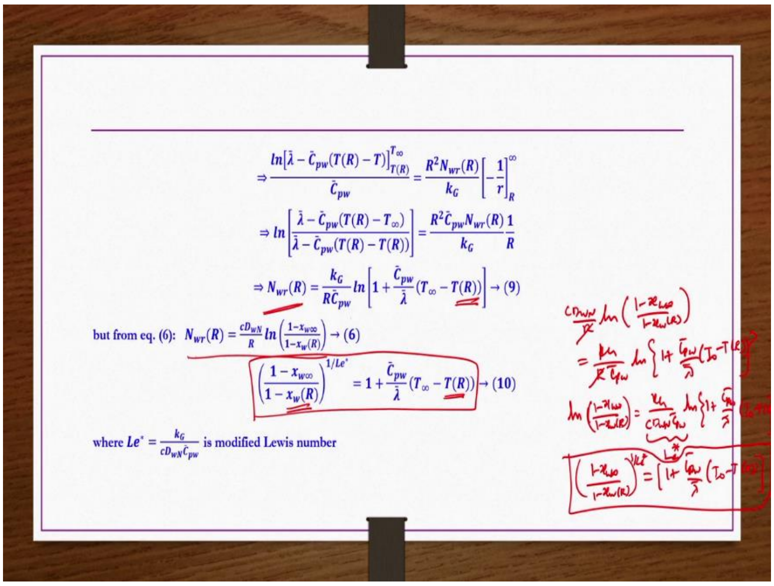

$$\Rightarrow \frac{\ln\left[\bar{\lambda} - \bar{C}_{pw}(T(R) - T)\right]_{T(R)}^{T_\infty}}{\bar{C}_{pw}} = \frac{R^2 N_{wr}(R)}{k_G}\left[-\frac{1}{r}\right]_R^{\infty}$$

$$\Rightarrow \ln\left[\frac{\bar{\lambda} - \bar{C}_{pw}(T(R) - T_\infty)}{\bar{\lambda} - \bar{C}_{pw}(T(R) - T(R))}\right] = \frac{R^2 \bar{C}_{pw} N_{wr}(R)}{k_G}\frac{1}{R}$$

$$\Rightarrow N_{wr}(R) = \frac{k_G}{R\bar{C}_{pw}} \ln\left[1 + \frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right] \rightarrow (9)$$

but from eq. (6): $N_{wr}(R) = \frac{cD_{wN}}{R}\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) \rightarrow (6)$

$$\boxed{\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)^{1/Le^*} = 1 + \frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))} \rightarrow (10)$$

where $Le^* = \frac{k_G}{cD_{wN}\bar{C}_{pw}}$ is modified Lewis number

$$\frac{cD_{wN}}{\cancel{R}}\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) = \frac{k_G}{\cancel{R}\bar{C}_{pw}}\ln\left\{1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right\}$$

$$\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) = \underbrace{\frac{k_G}{cD_{wN}\bar{C}_{pw}}}_{L_e^*}\ln\left\{1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right\}$$

$$\boxed{\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)^{1/L_e^*} = \left[1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right]}$$

Then you get this expression $\frac{k_G}{R\bar{C}_{pw}} ln\left[1 + \frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right]$ right. So, left hand side right hand side after substituting the limits this is what we get for the N_{wr}. Now, $N_{wr}(R)$ we got as function of surface temperature T (R) right.

From equation number 6 this is already we have right. So, now these two equations when you equate it you get this expression that is we do in one step. So, that is what we have $\frac{CD_{wN}}{R} ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) = \frac{k_G}{R\bar{C}_{pw}} ln\left[1 + \frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right]$.

So, this left hand side 1/R right hand side 1/R cancelled out. So, what we have? $ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) = \frac{k_G}{CD_{wN}\bar{C}_{pw}}$ and then L N of whatever this 1 + $\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))$as it is right.

So, now what we can take this entire thing as a Lewis number L_e^* so then we can write $\frac{1-x_{w\infty}}{1-x_w(R)}$ = one + $\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))$ this whole to the power of Lewis number we can right.

So, then what we can what we are writing we are writing that Lewis number to the left hand side term. So, one by Lewis number L_e^* we are writing so this is nothing but this equation number 10 is given here. So, now this equation also you know this x_w (R) T (R) are there, they are not known actually or you can find out what is T (R) if you know x_w (R) right. So, that is the problem so we can have some approach to overcome this one.

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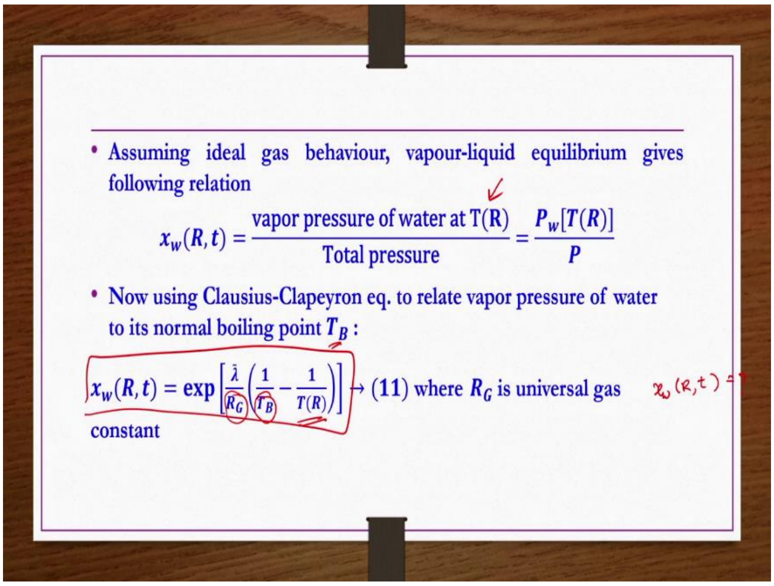

- Assuming ideal gas behaviour, vapour-liquid equilibrium gives following relation

$$x_w(R,t) = \frac{\text{vapor pressure of water at } \overset{\swarrow}{\text{T(R)}}}{\text{Total pressure}} = \frac{P_w[T(R)]}{P}$$

- Now using Clausius-Clapeyron eq. to relate vapor pressure of water to its normal boiling point T_B :

$$\boxed{x_w(R,t) = \exp\left[\frac{\bar{\lambda}}{R_G}\left(\frac{1}{T_B} - \frac{1}{T(R)}\right)\right]} \rightarrow (11)$$ where R_G is universal gas constant

$x_w(R,t) =$

Assuming ideal gas behaviour and then vapour liquid equilibrium gives as the following relation x_w (R, t) is nothing, but vapour pressure of water at T (R) divided by the total pressure ok. And now using the Clausius-Clapeyron equation to relate vapor pressure of water to its normal boiling point T_B we can have this expression.

x_w (R, t) is $exp\left[\frac{\bar{\lambda}}{R_G}\left(\frac{1}{T_B} - \frac{1}{T\ (R)}\right)\right]$. $T\ (R)$ is nothing, but surface temperature or temperature at the interface T_B is the boiling point temperature for this component and then R_G is nothing, but the universal gas constant. So, these things are known here.

So, now here if $T\ (R)$ is not depend on this not dependent on the size we can say that x_w is also not dependent on the side size of the droplet. See all these analysis after developing equation number 6 we are finding we are trying to do what is this x_w (R, t) is it constant or not.

If it is not constant which function it is if it is; obviously, it is dependent on the temperature how is it dependent on the temperature that we are trying to find out. Otherwise, there itself we could have got the solution of the problem ok.

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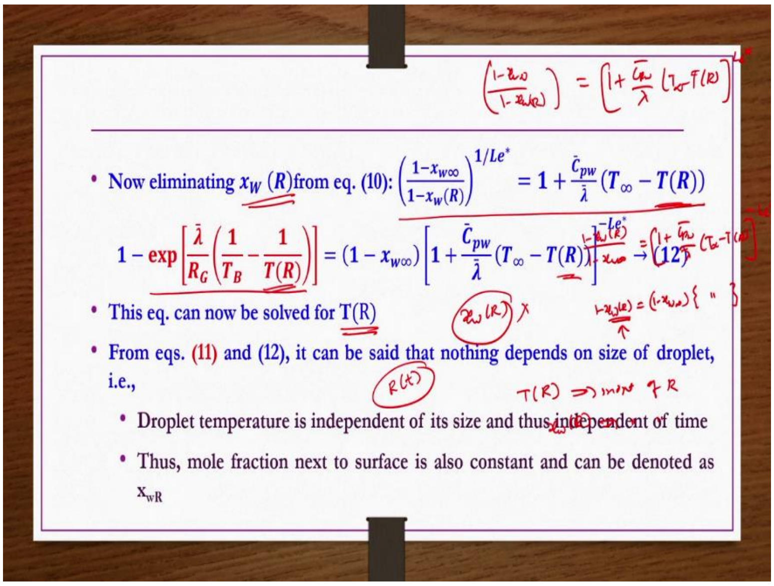

$\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) = \left[1+\frac{\bar{C}_{pw}}{\lambda}(T_\infty - T(R))\right]^{Le^*}$

- Now eliminating $x_W(R)$ from eq. (10): $\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)^{1/Le^*} = 1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))$

$$1-\exp\left[\frac{\bar{\lambda}}{R_G}\left(\frac{1}{T_B}-\frac{1}{T(R)}\right)\right] = (1-x_{w\infty})\left[1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right]^{-Le^*} \rightarrow (12)$$

$\frac{1-x_w(R)}{1-x_{w\infty}} = \left[1+\frac{\bar{C}_{pw}}{\lambda}(T_\infty - T(R))\right]^{-Le^*}$

- This eq. can now be solved for T(R)

$x_w(R)$ ✗

$1-x_w(R) = (1-x_{w\infty})\{\ "\ \}$

- From eqs. (11) and (12), it can be said that nothing depends on size of droplet, i.e.,

$R(t)$

$T(R) \Rightarrow$ ind. of R

  - Droplet temperature is independent of its size and thus independent of time
  - Thus, mole fraction next to surface is also constant and can be denoted as x_{wR}

So, we will see that one also how we can play around with the equations to find out whether this x_w (R) is really you know function of you know droplet size. Similarly, $T\ (R)$ is really function of the size of the droplet that we are going to see.

So, this equation number 10 this is what we are derive previous slide what we are trying to do here. In this place now what I try to write, I try to take this $\frac{1-x_{w\infty}}{1-x_w(R)}$, x_w as R I am keeping one side and then whatever 1 + $\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))$ is there. So, this whole I am taking Lewis on the power star to this side right.

Next step what I am trying to do? I am trying to write the inverse of these two so; that means, I get $\frac{1-x_w(R)}{1-x_{w\infty}} = \left[1+\frac{\bar{C}_{pw}}{\bar{\lambda}}(T_\infty - T(R))\right]^{L_e^*}$ because inverse I have written right.

So, now this I can write $1-x_w(R) = 1-x_{w\infty}$ and then multiplied by right hand side whatever is there. And then this $x_w(R)$ just now we found from the Clausius Clapeyron equation in the previous slide as $exp\left[\frac{\bar{\lambda}}{R_G}\left(\frac{1}{T_B}-\frac{1}{T\ (R)}\right)\right]$ ok.

So, now here this equation we can solve to find out T ® because there is no unknown $x_w(R)$ here, we could eliminated it right. So, what we can say here this $T\ (R)$ this $T\ (R)$ here also, you see any of the terms are having R as function of time here in this equation number 12; no. So, whatever the solution that you are going to get for $T\ (R)$is going to be independent of the size of the droplet right.

So, if $T\ (R)$ that is the surface temperature is independent of size of the droplet so then; obviously, x_w (R) that is the water concentration at the interface towards the gaseous side that would also be independent of R. So, that equation number 5 and 6 we can directly use to find out the time required ok.

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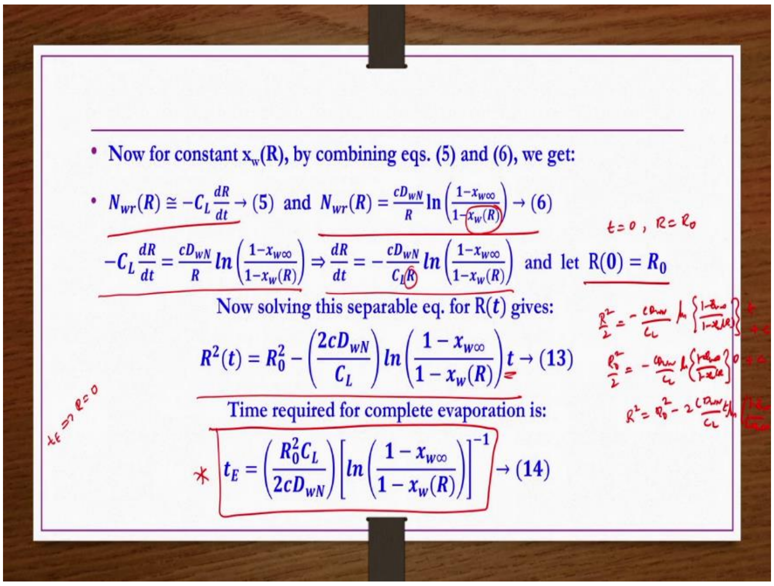

- Now for constant x_w(R), by combining eqs. (5) and (6), we get:
- $N_{wr}(R) \cong -C_L\frac{dR}{dt} \rightarrow (5)$ and $N_{wr}(R) = \frac{cD_{wN}}{R}\ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) \rightarrow (6)$

$t=0,\ R=R_0$

$-C_L\frac{dR}{dt} = \frac{cD_{wN}}{R}ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right) \Rightarrow \frac{dR}{dt} = -\frac{cD_{wN}}{C_L R}ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)$ and let $R(0) = R_0$

Now solving this separable eq. for R(t) gives:

$$R^2(t) = R_0^2 - \left(\frac{2cD_{wN}}{C_L}\right)ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)t \rightarrow (13)$$

$\frac{R^2}{2} = -\frac{cD_{wN}}{C_L}ln\left\{\frac{1-x_{w\infty}}{1-x_w(R)}\right\}t + c$

$\frac{R_0^2}{2} = -\frac{cD_{wN}}{C_L}ln\left\{\frac{1-x_{w\infty}}{1-x_w(R)}\right\}0 + c$

$R^2 = R_0^2 - 2\frac{cD_{wN}}{C_L}t\,ln$

Time required for complete evaporation is:

$t_E \Rightarrow R=0$

$$* \boxed{t_E = \left(\frac{R_0^2 C_L}{2cD_{wN}}\right)\left[ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)\right]^{-1}} \rightarrow (14)$$

So, now equation number 5 this is there and then this is there. So, all these analysis after the equation number 6 we did in order to find out what is this x_w (R). We found that it is independent of R that is independent of the size of the droplet so then we can directly equate these two equations and then do the integration to find out R as function of time.

So, when you equate these two equation this is what you have, you keep $\frac{dR}{dt}$ term one side and then initial condition at t = 0 whatever, R is there that is nothing, but R_0. So, now this equation what you do you integrate it then you get $\frac{R^2}{2} = \frac{cD_{wN}}{C_L R}$; C_L because this R we are taking to the left hand side then only we get $\frac{R^2}{2}$ on integration.

And then $ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)$ which is constant or independent of R and then multiplied by T + constant C, at is equals to 0 R = R_0. So, $\frac{{R_0}^2}{2} = -\frac{cD_{wN}}{C_L}ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)$ multiplied by 0 + c.

So, c is nothing, but $\frac{{R_0}^2}{2}$ so; that means, R square you can write $\frac{{R_0}^2}{2} - \frac{2cD_{wN}}{C_L} ln\left(\frac{1-x_{w\infty}}{1-x_w(R)}\right)$. So, this is what you get multiplied by time t right. Now, the total time for evaporation is nothing, but time E and then that is when R becomes 0.

So, in this equation you substitute t = t_E and then make this ${R_0}^2$ then you get expression for t_E as this one. So, this is the total time required for the droplet evaporation.

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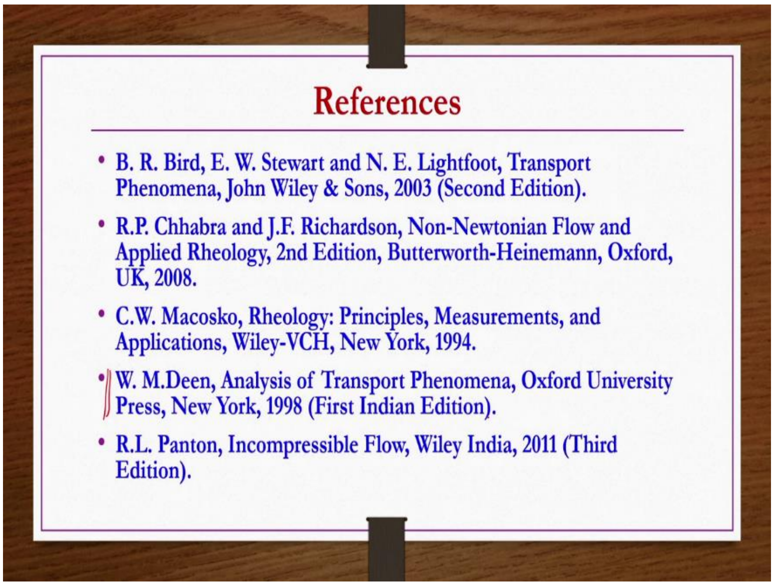

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References for this lecture are provided, but these problems are can be found from this book analysis of transport phenomena by Deen.

Thank you.