Transport Phenomena of Non-Newtonian Fluids

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Lecture - 33
Simultaneous Heat and Mass Transfer

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Simultaneous Heat and Mass Transfer. So, now, in this lecture we will be taking a situation where both heat and mass transfer are taking place. So, then how to obtain the mass transfer flux and then heat transfer or temperature distribution etcetera those things that we are going to see ok.

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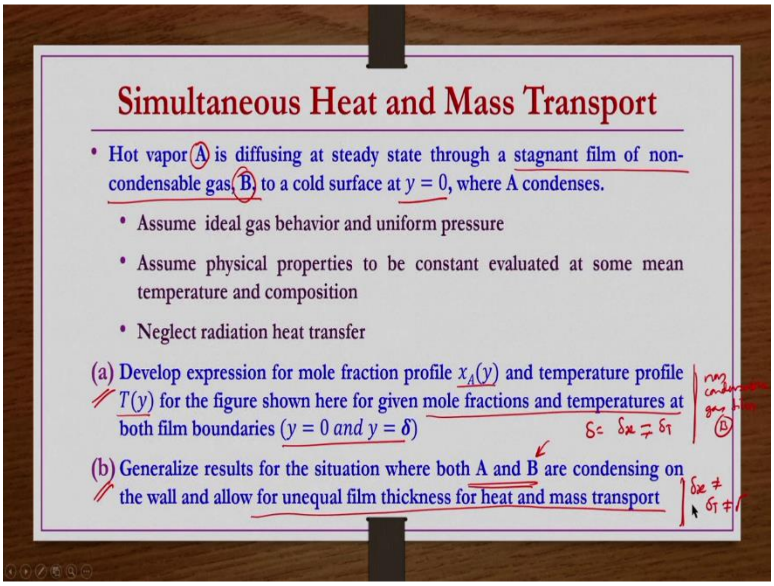

So, consider a hot vapor A is diffusing at steady state through a stagnant film of non-condensable gas B to a cold surface at y = 0 where A condenses. So, hot vapor A is condensing on a cold surface, but when it reaches the cold surface in between there is a stagnant film of non-condensable gas B and this A is diffusing into B when it reaches to the cold surface and condenses in that process ok.

So, then under such conditions what is the you know concentration profile of that component A and then what is the temperature profile that is what we have to find out. Assume ideal gas behavior and uniform pressure assume physical properties to be constant evaluated at some mean temperature and composition neglect radiation heat transfer right.

So, this is the simple base simple you know statement of the problem. So, this problem we can divide into two parts. First part as I mentioned you know developing concentration profile x_A as function of y temperature profile as function of y as shown be figure in the next slide as shown in the figure next slide right.

So, when mole fractions and temperatures at both film boundaries are known at y = 0 at y = δ. The thickness of non-condensable gas film is δ it is having two boundaries; one boundary location is y = 0 another boundary location is y = δ. At either or at both of these locations what we know? We know the temperature and concentration. So, boundary conditions are known.

So, this we are going to do when by taking non condensable gas film B. So, N_{Bz} that is what we are taking 0 here right and also when both heat transfer and mass transfer was taking place. So, then there would be different film thickness in general. So, δx δT, but what we are assuming that it is same here whether you know mass transfer or heat transfer the film thicknesses is same that is the assumption.

That for that for those conditions we are obtaining you know x_A as function of y and T as function of y. The second problem what we are doing we are generalizing the results for the situation where both A and B are condensing. Now not only A, B is also condensing on cold surface right. So, then that A and B are you know we cannot say that B is non you know non condensable and then its flux is 0 that we cannot say now because it is also you know condensing on to the cold surface ok.

And allowing for unequal film thickness for heat and mass transport then we are taking δx = δT ≠ δ. So, then what are the solutions that you know what are the concentration profiles and then temperature profiles that we are going to find out.

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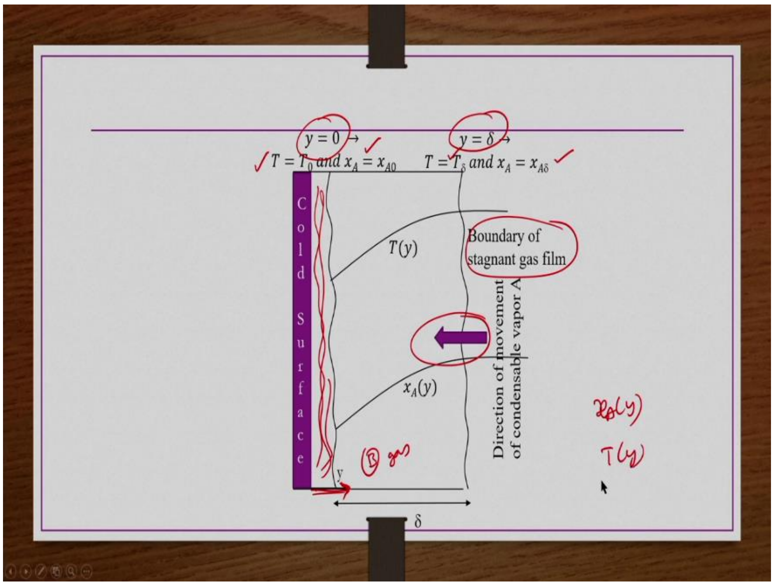

So, pictorially we have a cold surface here which is designated as y = 0 and then close to this one there is a boundary of stagnant gas film made up of B right. So, now the direction of diffusion or you know condensable vapor is coming in this direction and then reaching this cold surface and then condensing like this here right fine.

The coordinate system is taken such a way that the direction in which the change in concentration or temperature is occurring that is y film thicknesses is δ. So, at y = 0 T is T_0 and x_A is x_{A0} at y = δ T is T_δ and then x_A is $x_{A\delta}$. So, now, for this case we are going to find out this two.

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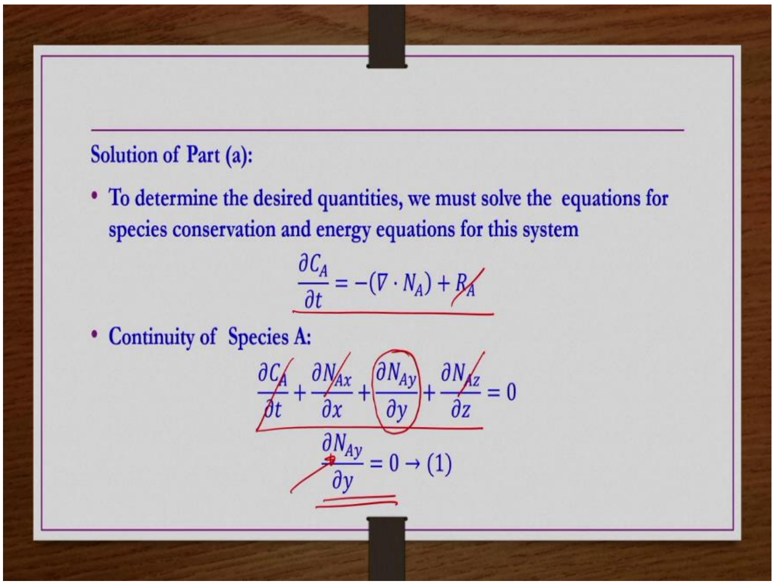

This is part a; to determine the desired quantities we must solve the equations for species conservation and energy equations for this system. So, then this is the species conservation equation in generalized form vectorial notation it is given. If you expand this one for species A you have this one $\frac{\partial C_A}{\partial t} + \frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z}$ and then reaction there is nothing occurring no reaction occurring. So, R_A is 0 right.

And then what we have seen diffusion are the concentration variations and temperature variations are only in y direction. So, only N_{Ay} would be there and that would be function of y whereas, N_{Ax} N_{Az} are 0 and then it is a steady state problem. So, this is $\frac{\partial C_A}{\partial t}$ is also 0.

Then we have this one; $\frac{\partial N_{Ay}}{\partial y} = 0$. So, now, if you know the N_{Ay} you can find out the concentration profile very conventional way in that we have been doing ok before getting the concentration profile what we do we try to look at the energy equation and its simplification as well as per the constraints of the problem.

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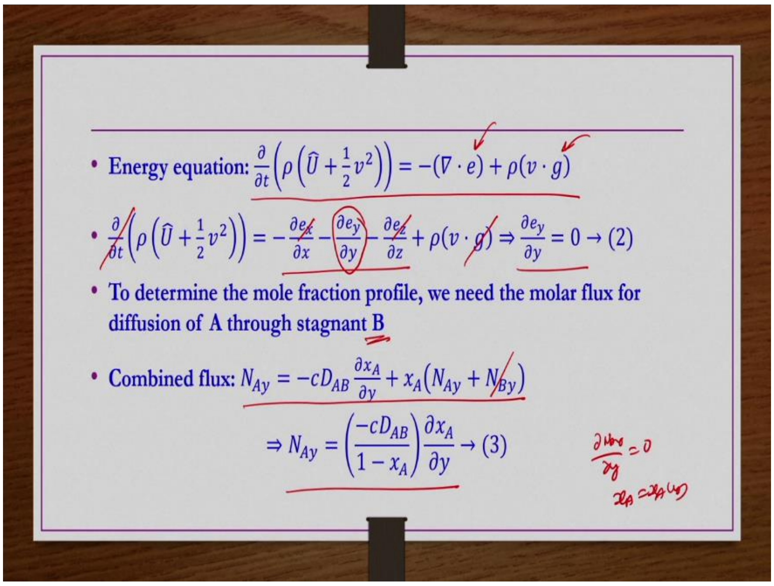

Energy equation we have derived in an equation we have derived in week number 4. So, this is what we have got. Whereas this e is there in all 3 direction x y and z direction and then each e and it consist of a contribution from the conduction convection work done because of the pressure you know viscous dissipation etcetera all those terms are included in all three directions right.

So, whatever the energy because of the gravity is this. So, then that part is in the last it is given right net less potential energy etcetera those things are coming from here. So, this equation we know ok. So, if you expand this equation only this part we are expanding because this e is having all the components like you know all the contributions from the conduction convection viscous dissipation work done because of the pressure etcetera all those things right, then we simplify it.

So, this if you expand you get $\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z}$. So, that is written here. So, now we apply the constraints of the problems steady state. So, then left hand side all terms are cancelled out gravity we are taking you know in the other direction.

So, in the direction of diffusion or temperature variations whatever are occurring in that direction there is no gravity. So, this we can take out 0 right. So, the variations in the energy or temperature are occurring only in the y direction and that is function of y. So, the remaining e_x e_z are 0. So, what we get $\frac{\partial e_y}{\partial y} = 0$ this is what we get.

Now, to determine the mole fraction profile we need the molar flux for diffusion of A through stagnant B. So, this how do we get? In general we write a combined flux equation. So, that is this one now in this equation N_{By} is 0 for part a of the problem.

So, if you simplify this equation by taking all N_{Ay} terms one side and then remaining terms other side you write you get $N_{Ay} = -\frac{cD_{AB}}{1-x_A}\frac{\partial x_A}{\partial y}$. So, now this you can substitute in $\frac{\partial N_{Ay}}{\partial y} = 0$ and then you get a concentration profile as function of y.

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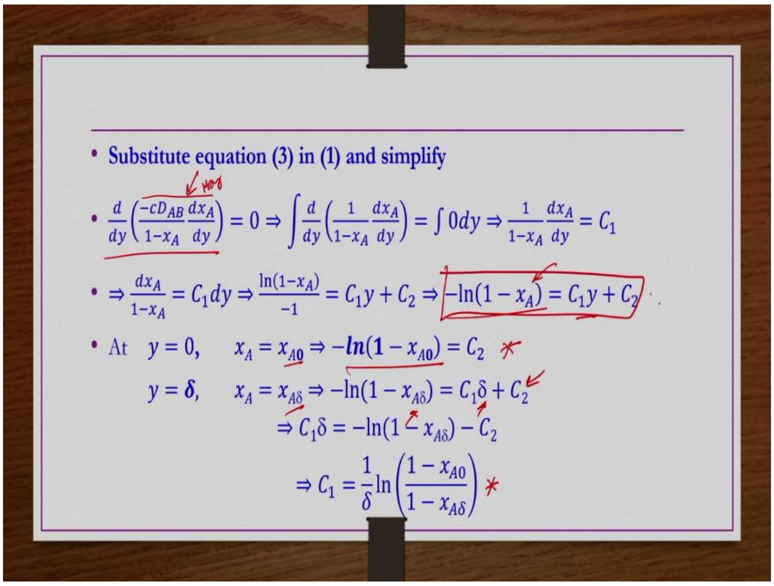

So, when you substitute you know this is nothing, but N_{Ay} term we substituted $\frac{\partial N_{Ay}}{\partial y} = 0$. So, N_{Ay} is this one now for this situation part a situation the concentration and then diffusivity we can take constant. Then we have a $\int \frac{d}{dy}\left(\frac{1}{1-x_A}\frac{\partial x_A}{\partial y}\right) = 0$ dy that is $\left(\frac{1}{1-x_A}\frac{\partial x_A}{\partial y}\right) = C_1$then further integrating you get $-\ln(1-x_A) = C_1 y + C_2$.

Now, we have our two-boundary condition at y = 0 $x_A = x_{A0}$. So, here if you substitute x_{A0} you get $-\ln(1-x_{A0}) = C_2$ because y is 0 in this boundary condition. So, C_2 you already got then other boundary condition at $y = \delta$, $x_A = x_{A\delta}$.

So, in place of x_A you write $x_{A\delta}$ in place of y you write δ and then C_2 you already got it as $-\ln(1-x_{A0})$. Then you get C_1 that is nothing but $\frac{1}{\delta} ln\left(\frac{1-x_{A0}}{1-x_{A\delta}}\right)$. So, this C_1 and then this C_2 we are substituting in this equation to get the concentration profile.

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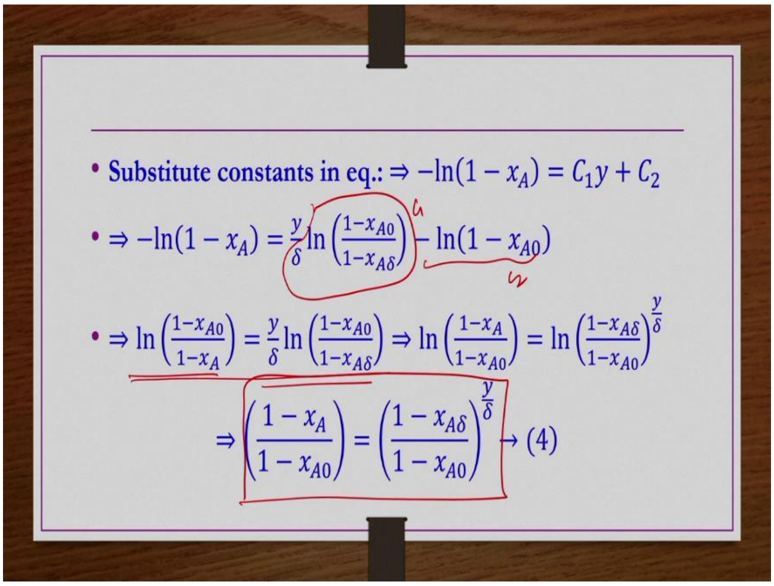

In this term is C_1 and then this term is C_2 right then if you further simplify in the sense what we do we take this $-\ln(1-x_{A0})$ to the left-hand side. So, that we can write $ln\left(\frac{1-x_{A0}}{1-x_A}\right) = \frac{y}{\delta}\, ln\left(\frac{1-x_{A0}}{1-x_{A\delta}}\right)$.

So, this is the final concentration profile. Further you can write by taking this $\frac{y}{\delta}$ as a power of the whatever $ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right)$ then you have this one if you take exponential both sides. You get this as a concentration profile right of course, any of these two equations you can use as a concentration profile right.

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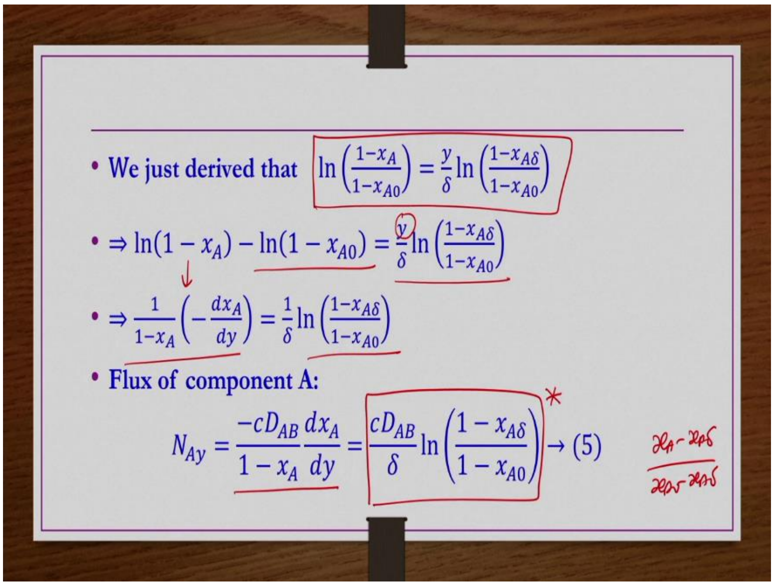

Now, what we do? We differentiate this equation with respect to y because we wanted to know the flux also, we have to find out the mass flux also for that we need to know $\frac{dx_A}{dy}$. So, that is the reason we are differentiating this equation with respect to y. So, so this from this term what we have $\frac{1}{1-x_A}\left(-\frac{\partial x_A}{\partial y}\right)$ this is constant. So, 0 derivative of constant is 0.

Now, this equation except y everything is constant. So, $\frac{1}{\delta} ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right)$ multiplied by 1 right. So, now, flux of component A what we have $-\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dy}$ this is what we derived so; that means, this equation if you multiplied by cD_{AB} you will get flux that is given by this equation right.

Now, what we got? We got an expression for the concentration profile; we got an expression for the flux also. So, then what we try to do? We try to write these expressions you know concentration profile in a in a very conventional way of writing like what in general what we write $\frac{x_A-x_{A\delta}}{x_{A0}-x_{A\delta}}$ in this form we try to write. So, that is what we are doing in the next slide.

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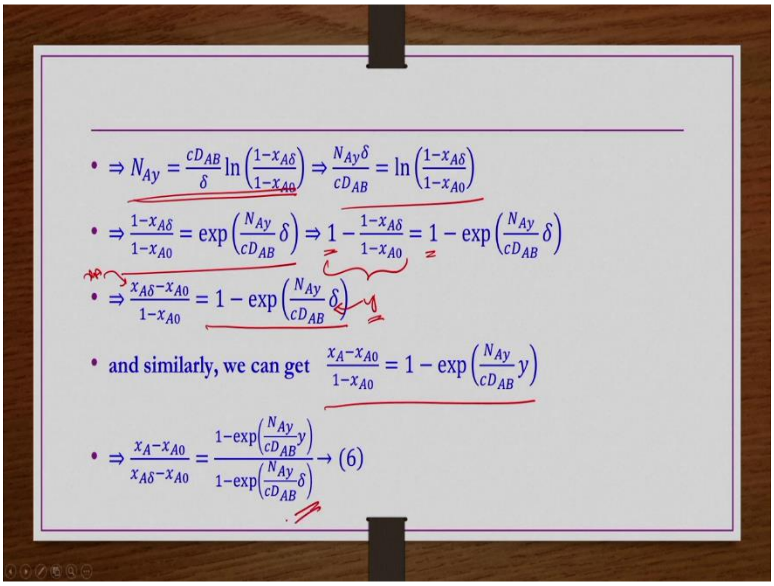

So, N_{Ay} is equals to this one we have. So, from this equation I can write you know $\frac{N_{Ay}}{cD_{AB}}\delta$ one side and then ln terms other side. Then I am taking exponential here then I am multiplying by minus 1 either side and then adding plus 1 either side then in the left hand side what I am doing? I am trying to do LCM. So, that I have $\frac{x_{A\delta}-x_{A0}}{1-x_{A0}}$ is equals to right hand side term as it is right.

If you wanted to know $\frac{x_A-x_{A0}}{1-x_{A0}}$ expression so, then what you have to do? Simply you have to replace this $x_{A\delta}$ by x_A and then here whatever δ is there that you have to replace by y because at y = δ $x_A = x_{A\delta}$ at y is equals to some unknown y x_A is equals to some unknown x_A right. So, that if you do you get this expression.

Now, if you divide this equation by the above equation, then you get $\frac{x_A-x_{A0}}{x_{A\delta}-x_{A0}} = \frac{1-exp\left(\frac{N_{Ay}}{cD_{AB}}y\right)}{1-exp\left(\frac{N_{Ay}}{cD_{AB}}\delta\right)}$ this is what we have right.

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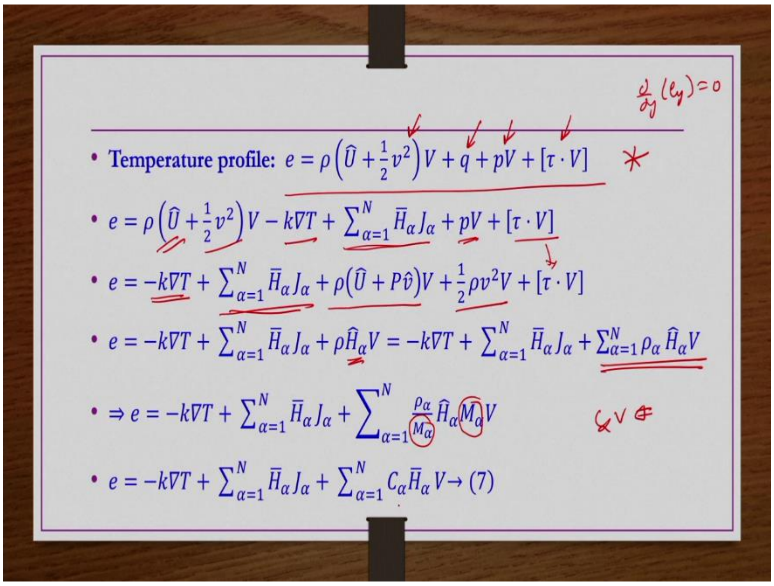

Now, we try to get the temperature profile. So, what we got by simplifying the energy equation we got $\frac{\partial e_y}{\partial y} = 0$, but what is the e_y we have not simplified in that slide. So, that we are doing here. So, the e is having the contribution from the conduction convection then any work done because of the pressure. And then any molecular stress you know energy because of the molecular stresses like you know viscous dissipation etcetera those terms are included.

But now we have to in incorporated term because of the mass transfer also. Remember when we are deriving the energy equation, we had incorporated most of the common contributions which are you know important from chemical engineering applications specific to the transport mass transport problem or heat transport problem like that specific to the heat transfer problem only.

But at the same time what we have mentioned, if any additional contributions are coming because of the reaction or mass transfer etcetera. So, those terms should be added up in this expression or final energy equation that we have derived either way we have to do that is what we have discussed.

Because in a generalized manner we cannot include each and every terminology like nuclear power, energy associated with the electrochemical reaction reactions, energy associated because of the you know hydrodynamic magneto hydrodynamics etcetera all those things we cannot include because they are specific to the problem.

So, now specific to this problem we have to incorporate term related to the mass transfer how it is affecting this e or the heat transfer problem. So, if you can understand clearly, you can simply add you know $\sum N_\alpha \bar{C}_p$ δ T that you can do it right. So, but if you are not able to understand how it is coming so, then we have to do this one.

So, before adding we will be adding that mass transfer part here, but also we are doing some kind of readjustment. Readjustment in the sense this conduction term we are bringing here right and then this whatever the mass transfer because of the mass transfer the contribution should come in e that is nothing, but $\sum \bar{H}_\alpha J_\alpha$.

J_α is nothing but a molar flux because of the diffusion and then $\bar{H}_\alpha$ is nothing but the enthalpy of component molar enthalpy of that component α there are N number of components right.

So, now in the next step from this P v and then this ρ v we are combining such a way that ρ $\hat{U} + P\hat{v}$ and then remaining $\frac{1}{2}\rho v^2$ V we are writing here [τ.V] is as it is in place of minus whatever the $-$k δ T is there that we are writing first and then this flux terms we are writing as a second one.

Why are we writing? So, that this $\hat{U} + P\hat{v}$ we can write H. So, now, here $\hat{H}$ we are using tildes so, we are writing for only one component. So, there are N number of components would be there. So, then we have to write a summation remaining two terms are as it is ok. Now next step what we are doing? This term we are multiplying by the molecular weight of that component alpha and then dividing by the same.

Why are we doing? Because we can write this $\frac{\rho_\alpha}{m_\alpha}$ we can write C_α and then that C_α V we can have a relation for in terms of the flux. So, that is the reason. So, $C_\alpha \bar{H}_\alpha$ V we are having here.

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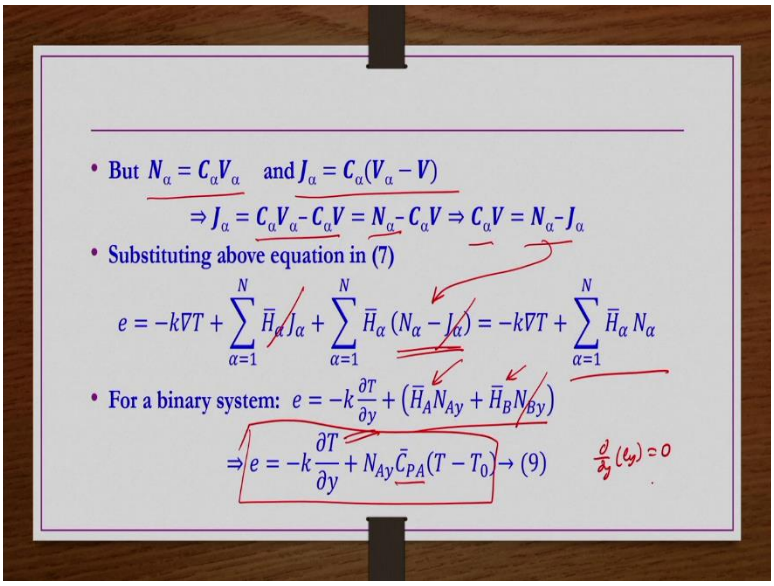

But we know that $N_\alpha = C_\alpha V_\alpha$ and $J_\alpha = C_\alpha(V_\alpha - \mathrm{V})$ this we have already seen in the beginning in the first lecture of mass transfer part right. Now, this J_α if I expand $C_\alpha V_\alpha - C_\alpha V$.

So, in place of $C_\alpha V_\alpha$ I can write N_α now from here I can write $C_\alpha V$ is nothing, but $N_\alpha - J_\alpha$. So, this we are going to use in equation number 4 here what we had $C_\alpha V$. So, in place of $C_\alpha V$ we are writing $N_\alpha - J_\alpha$. So, that this $\bar{H}_\alpha\ J_\alpha$ and then this $\bar{H}_\alpha\ J_\alpha$ cancelled out and then we have only sigma $\bar{H}_\alpha\ N_\alpha$ right.

Now, this if you write for the binary component, you have $\bar{H}_\alpha$ you have $\bar{H}_A N_{Ay} + \bar{H}_B N_{By}$ and then conduction is also only in the y direction. So, $-k\frac{\partial T}{\partial y}$. Now this $\bar{H}_A$ and then $\bar{H}_B$ we can replace by you know $\bar{C}_{PA}\ \delta$ T, but N_{By} is anyway 0 in this problem.

So, that $N_{By}\ \bar{C}_{PB} \delta$ T we don't need to write. So, now this equation you can substitute in $\frac{\partial e_y}{\partial y} = 0$ and then simplify to get the required temperature profile.

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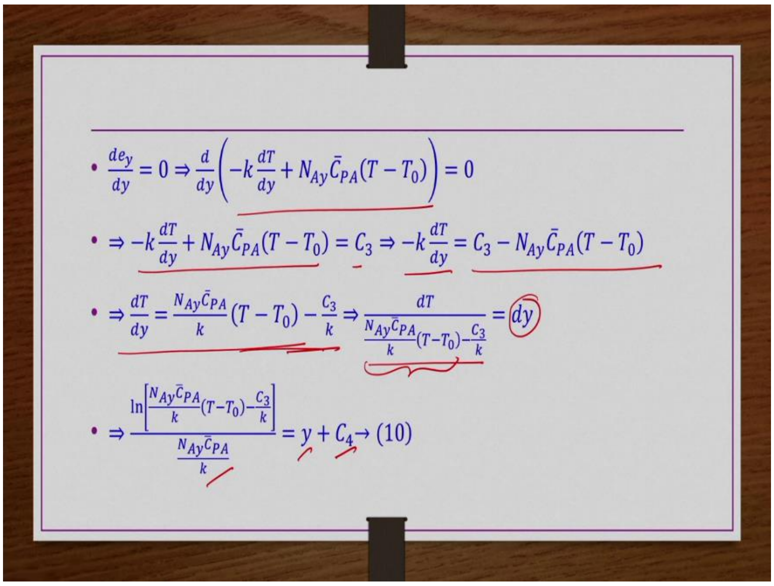

So, $\frac{\partial e_y}{\partial y} = 0$ this we integrated one. So, this term as it is equals to C_3 now next step we are keeping $-k\frac{\partial T}{\partial y}$ one side all other terms we are taking other side. next step, we are what we are trying to do? We are writing $\frac{dT}{dy} = \frac{N_{Ay}\bar{C}_{PA}}{k}(T - T_0) - \frac{C_3}{k}$.

So, next step dy term we are keeping one side all other terms we are taking the other side. So, dT by and this term you will be having right now what we can do? You can integrate. So, when you integrate you get ln of this term divided by the differentiation of you know T terms. So, then that is $\frac{N_{Ay}\bar{C}_{PA}}{k}$ you will get as a denominator right side you get y and then constant C_4 right.

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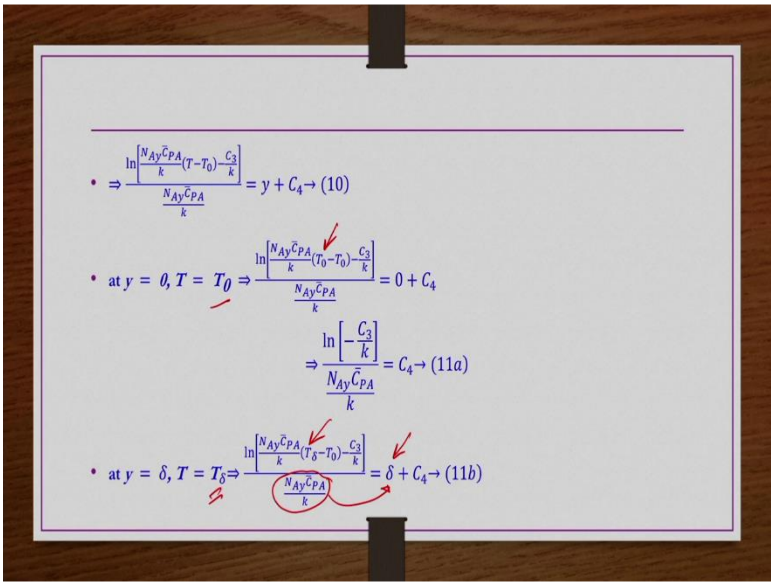

So, now we have two boundary conditions two constants we can evaluate. So, at y = 0 T = T_0 so, then here when you substitute T/T_0. So, then you get this term is 0 then; that means, what you get C_4 is nothing but $\frac{ln\left[-\frac{C_3}{k}\right]}{\frac{N_{Ay}\bar{C}_{PA}}{k}}$.

Now, we apply the other boundary conditions y = δ at y = δ T = T_δ. So, then here T_δ we are having in place of y we are having δ right. So, now, what we do? We take this term to the right hand side. So, that in the right hand side we have this one right.

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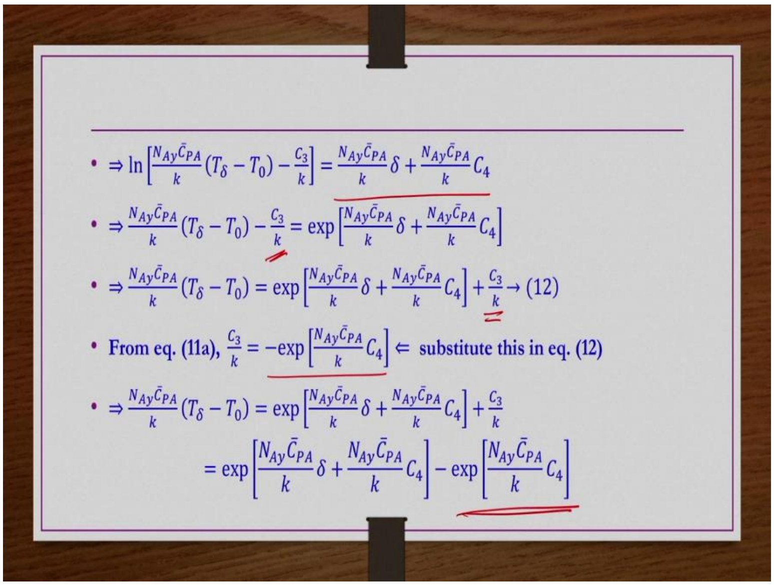

Next step what we are trying to do? We are writing exponential rather writing in you know ln terms. So, then this is what we have. Next step what we are trying to do? We are taking this $\frac{C_3}{k}$ to the right hand side right.

Next step, what we are writing in place of $\frac{C_3}{k}$ we will be writing minus exponential of $\frac{N_{Ay}\bar{C}_{PA}}{k}C_4$ which we just derived in the previous slide by applying the first boundary condition this is what we got. So, that you substitute here as this term remaining all terms are same right. So, that the same equation is written once again here.

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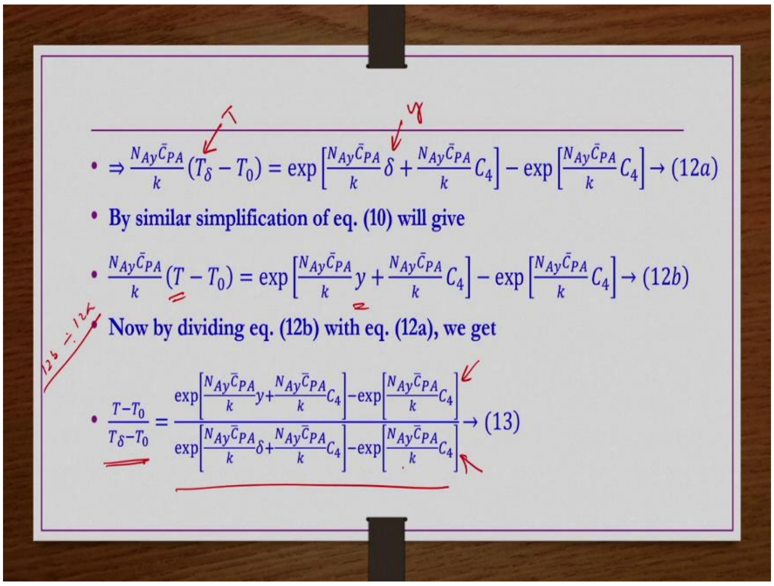

So, now what we do? We need to have $(T - T_0)$ also. So, that in place of T_δ we write T and in place of δ we right y right. Because at y = δ T = T_δ and then at y is equals to some unknown y T is equals to some unknown T right so, that if you do you get this equation.

So, now, you do 12 b divided by 12 a equations then you get $\frac{T-T_0}{T_\delta - T_0} =$ this one. Next step what we do? We take this term common from both numerator and denominator. So, that you know that can be cancelled out and then we have this particular term.

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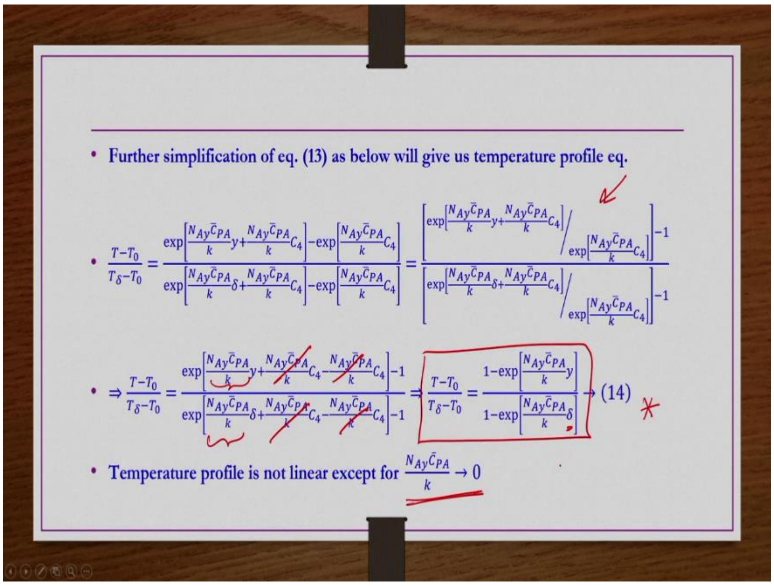

Now, this is exponential of A by exponential of B form. So, we can write exponential of A minus B. So, that you write. So, then this would be cancelled out. So, in the exponential we will be having only 1 term both in numerator and denominator terms that we can right now $\frac{T-T_0}{T_\delta-T_0} = \frac{1-exp\left[\frac{N_{Ay}\bar{C}_{PA}}{k}y\right]}{1-exp\left[\frac{N_{Ay}\bar{C}_{PA}}{k}\delta\right]}$. So, this is the temperature profile right.

Now, we try to find out the rate of heat transfer as well right. So, before going into the finding out the rate of heat transfer what we understand from here until and unless $\frac{N_{Ay}\bar{C}_{PA}}{k}$ is tending to 0 this temperature profile is not linear it is non-linear ok it can be linear only when this particular term tends to 0 right.

That is what it means by when the conduction is very much dominating compared to the mass transfer then only we can have a linear profile; obviously, if there is only conduction. So, then temperature profile is linear that we know ok.

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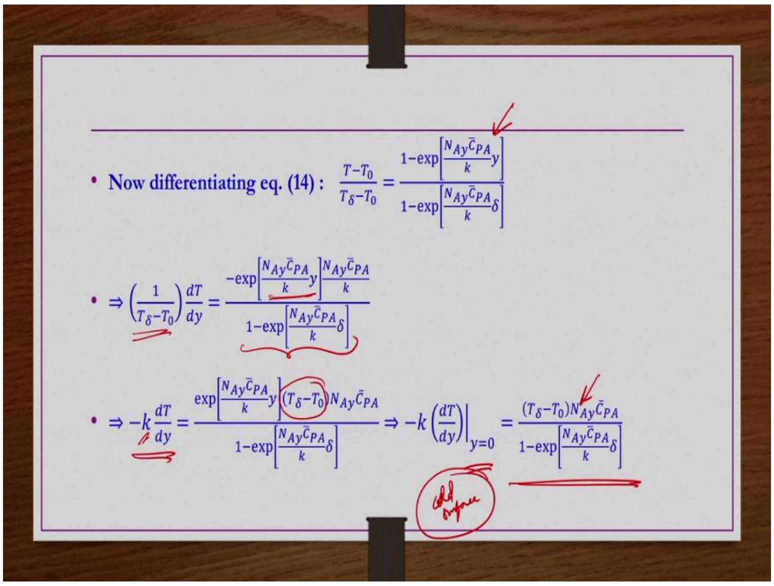

Now, differentiating with respect to y so, in the left hand side $\frac{1}{T_\delta - T_0}$ is you know constant then $\frac{dT}{dy} - 0 =$. And then in the right hand side except this particular term all other terms are independent of y. So, in the denominator we are keeping as it is numerator when you are differentiating this term exponential of x is exponential of x on differentiation.

So, you get the same thing minus exponential of same term and then differentiation of whatever this factors being multiplied by y. So, that is $\frac{N_{Ay}\bar{C}_{PA}}{k}$ right. So, now, we wanted to know $-k\frac{\partial T}{\partial y}$. So, $-k\frac{\partial T}{\partial y}$. So, then what we are doing this $T_\delta - T_0$ we brought to the right hand side ok and then this whatever this 1/k was there. So, that we have taken to the left hand side. So, then we have k here.

So, now this we wanted to do at the cold surface at the cold surface because at that point how much heat transfer is taking place because that is the location at which condensation is taking place right. So, in this equation if you substitute y = 0. So, the exponential of something multiplied by 0 is 0. So, exponential of 0 we take 1. So, remaining terms are anyway constant. So, they remain as it is.

Now, this is the rate of heat transfer when there is a mass transfer when there is a mass transfer N_{Ay} is there; that means, N_{Ay} associated with the mass transfer. So, mass transfer contribution has also brought into the temperature profile and then subsequent heat transfer.

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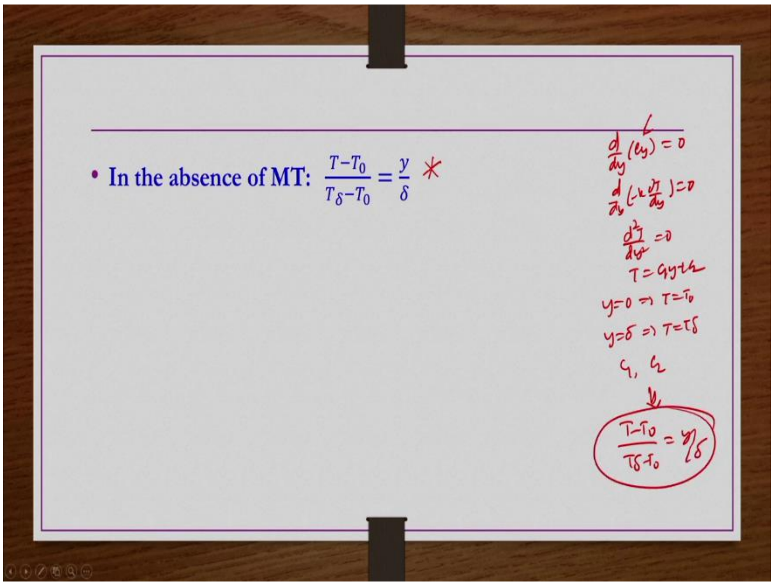

But if there is no mass transfer, if there is no mass transfer then in the absence of mass transfer temperature profile is linear that is this one how do you get? So, if there is no mass transfer $\frac{\partial e_y}{\partial y}=0$ and then e_y would be having only $-k\frac{\partial T}{\partial y}$ term only we will be having right.

So; that means, $\frac{\partial^2 T}{\partial y^2}=0$. So, $T = C_1 y + C_2$ you get you apply $y = 0$ $T = T_0$ $y = \delta$ $T = T_\delta$ you apply and then obtain this constant C_1 C_2 and then simplify. So, then you get $\frac{T-T_0}{T_\delta-T_0}=$ y/δ. So, that we have written directly right.

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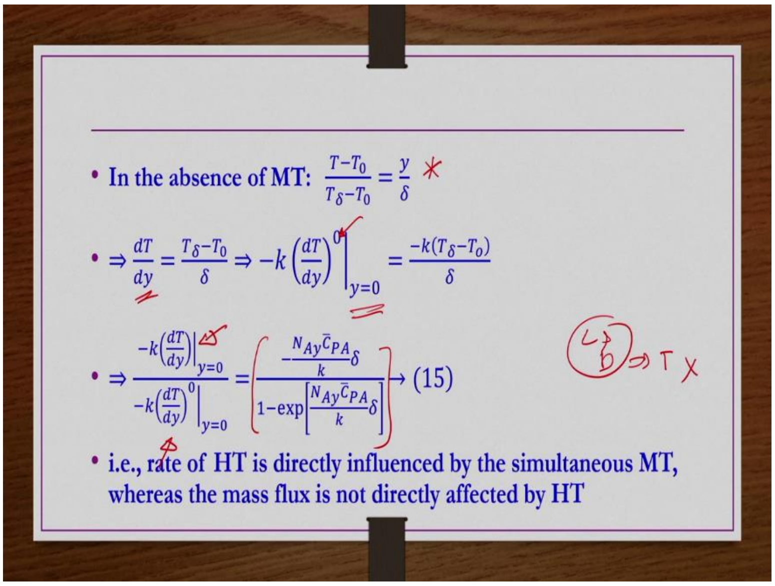

So, now in the absence of mass transfer we have this linear temperature profile for this case also we try to find out the rate of heat transfer. So, for that we get $\frac{dT}{dy}$ which is nothing, but $\frac{T_\delta-T_0}{\delta}$. And now we are multiplying by minus k either side and then substituting y = 0 and then this superscript 0 is indicating in the absence of mass transfer. So, that we can compare the rate of heat transfers in the absence and then in the presence of mass transfer.

So, that we do this is in the presence of mass transfer how the rate of heat transfer you know being affected. So, that is one this is in the absence of mass transfer. So, then this quantity will give the relative contribution of mass transfer and the relative heat transfer rates by you know including the mass transfer and by without including the mass transfer right.

So, what we understand here, the heat transfer part is clearly affected by the mass transfer or mass flux, but the mass transfer part there is no heat transfer effect why? That is because whatever the dC etcetera are there how are they related to temperature that is not given. So, that is the reason you know we did not bring this term you know effect of temperature in mass transfer ok.

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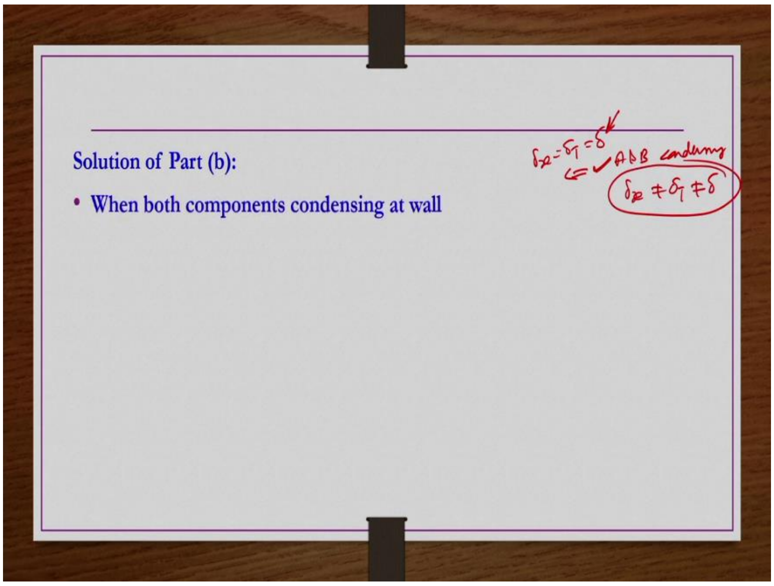

Now, we take part b part b is what? Both A and B are diffusing A and B are condensing. So, there is nothing like non condensable thing here and then this $\delta_x \neq \delta_T \neq \delta$ we have to generalize it. But first what we do?

We take only this thing and then we take δx = δT = δ one same constant film thickness is same whether mass transport or heat transport right; however, unequal film thickness we later on incorporate ok. So, when both components are condensing at wall so, pictorially the same thing is shown here right.

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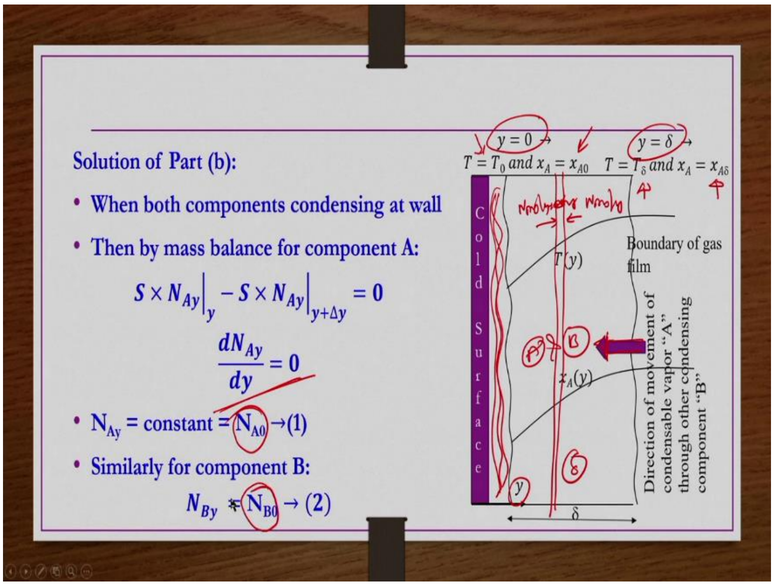

Same horizontal axis is y axis and then condensable vapors are coming in this direction. And then they are reaching the cold surface upon reaching the cold surface they are condensing on the cold surface. At cold surface that is at y = 0 T is T_0 x_A is x_{A0} and then at y = δ T is T_δ x_A is $x_{A\delta}$ because the film thickness is δ now both A and B are coming and condensing on the vapors.

So, so we cannot say that N_{By} is 0 here in this case. So, mass balance for component A if you wanted to do, but by taking a film in between of thickness δ y. So, whatever the flux N_{Ay} at y = y is entering flux and then whatever the flux that is leaving at y + δ y is N_{Ay} at y + δ y.

And then both of them are multiplied by the cross section area of the surface through which the channel of thickness δ y that is S we have taken. So, we can do balance this way or in the pathway we have directly taken this species conservation equation $\frac{\partial C_A}{\partial T} + \nabla . N_A = 0$ that we have taken. So, that way also we can do it ok.

So, now you apply limiting conditions after dividing this equation by S and δ y then you get this equation now this is N_{Ay} equals to constant. So, what is that constant we do not know. So, we are calling it N_{A0} right similarly if you do for component B you get N_{By} is equals to constant and that constant we do not know. So, we are calling it N_{B0}.

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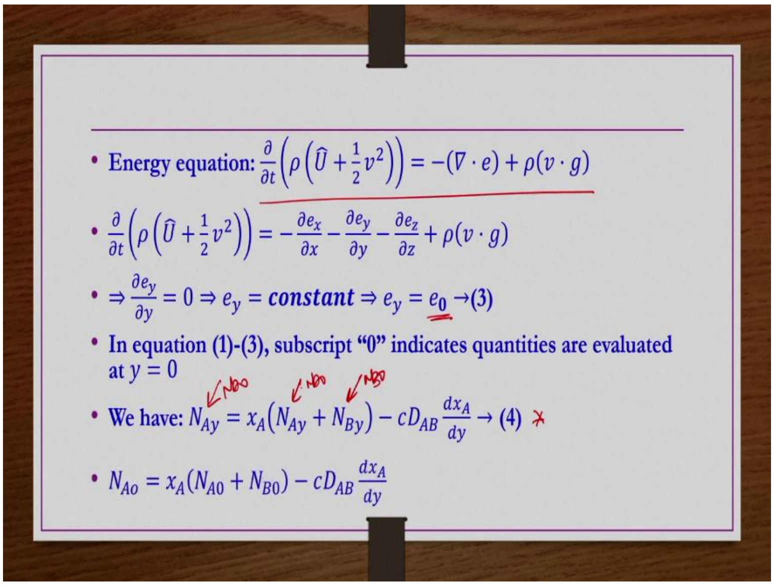

So, now energy equation simplification of energy equation is exactly the same as we have done previously right so, but that $\frac{\partial e_y}{\partial y} = 0$ and then $e_y = e_0$ at constant we do not know we are calling it e_0. Now, the combined flux equation is given here. So, now, here in place of N_{Ay} you have to write N_{A0} and then in place of N_{By} you have to write N_{B0} right. So, when you write you get this equation.

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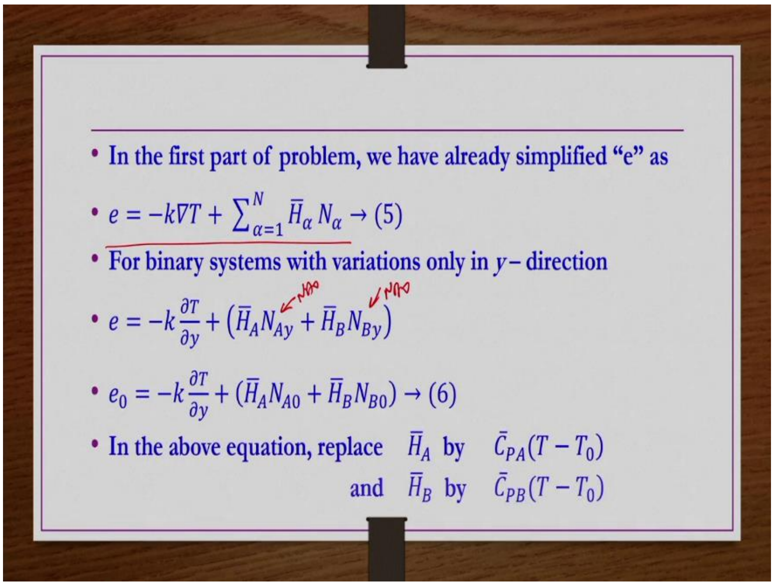

Now, here e up to this point we have simplified in the part a right. So, it is same here also exactly same right now what we do? This one we write for the binary component then we have $-k\frac{\partial T}{\partial y} + \bar{H}_A N_{Ay} + \bar{H}_B N_{By}$ right.

In the previous case we have taken N_{By} is 0 and then $\bar{H}_A$ we replaced by $\bar{C}_{PA}$ δT, but now we cannot do that one this N_{Ay} is N_{A0} and this N_A N_{By} is nothing, but N_{B0} and then $\bar{H}_A$ is $\bar{C}_{PA}$δT $\bar{H}_B$ is T $\bar{C}_{PB}$ δ T. So, that when you do you get final e expression this one right.

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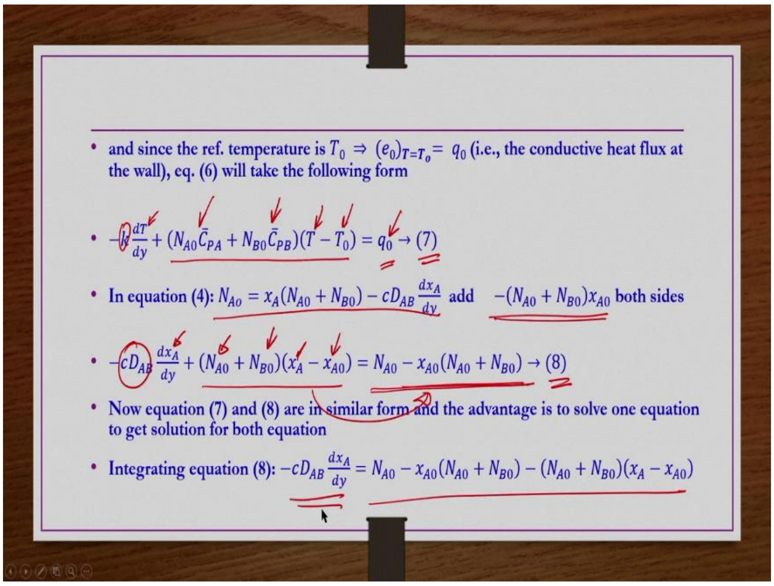

So, that is $-k\frac{\partial T}{\partial y} + N_{A0}\bar{C}_{PA} + N_{B0}\bar{C}_{PB}$ multiplied by $\delta T = e_0$ that e_0 we are evaluating it $T = T_0$. So, then at that point we have only conductive heat flux at the wall there is no mass transfer so, that we calling q_0 ok.

Now, in equation 4; this is the equation 4 that we had. So, we are adding $-(N_{A0} + N_{B0})$ x_{A0} both sides. So, there is a reason we are neither solving this equation number 4 or equation number 7 directly right. What we are trying to do? We are adding this term with the equation number 4 both sides.

So, then we have this component in the left hand side and then in the right hand side N_{A0} is already there. So, $-x_{A0}$ $(N_{A0} + N_{B0})$ right. So, the reason is that now this equation number 8 and then equation number 7 if you compare they are exactly similar right; only thing that k is replaced by c D_{AB}.

And then T is replaced by x_A then N_{A0} $\bar{C}_{PA}$ is replaced by N_{A0} and then $N_{B0}\bar{C}_{PB}$ is replaced by N_{B0} T is replaced by x_A. And then T_0 is replaced by x_{A0} and then q_0 is replaced by this particular term this particular term in the right side of an equation number 8.

So, that they are quite similar to each other. So, you can solve one equation we can solve any of these one equation and then analogously you can write the solution for the other equation right. So, what we do now? We solve this equation number 8. So, for that this particular term we take to the right hand side, but we do not simplify we do not simplify. So, then we have this thing and then left hand side we have only - c D_{AB} $\frac{dx_A}{dy}$.

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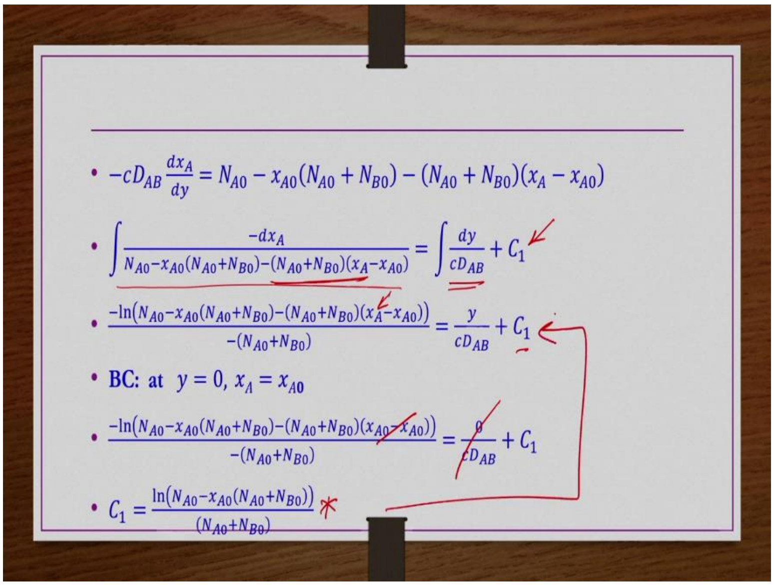

So, the same is written once again here. So, now, here $\frac{dy}{cD_{AB}}$ we are taking other side and the rest other terms dx_A by all these N_{A0} N_{B0} terms we are keeping in the left hand side and integrating. So, then integration constants C_1 is there right.

So, this one is minus ln of whatever this term divided by you know differentiation of the x_A terms. So, that is $-(N_{A0} + N_{B0})$ and the right hand side simply $\frac{y}{cD_{AB}}$ we are having plus this constant alright.

So, now we apply boundary condition y = 0. So, x_A should be x_{A0}. So, that if you substitute $x_A = x_{A0}$ here. So, the second term would be 0 within the logarithmic and then right hand side this y is 0. So, this term is 0. So, directly you get C_1 as this one. Now, this C_1 we are going to substitute here and then simplify.

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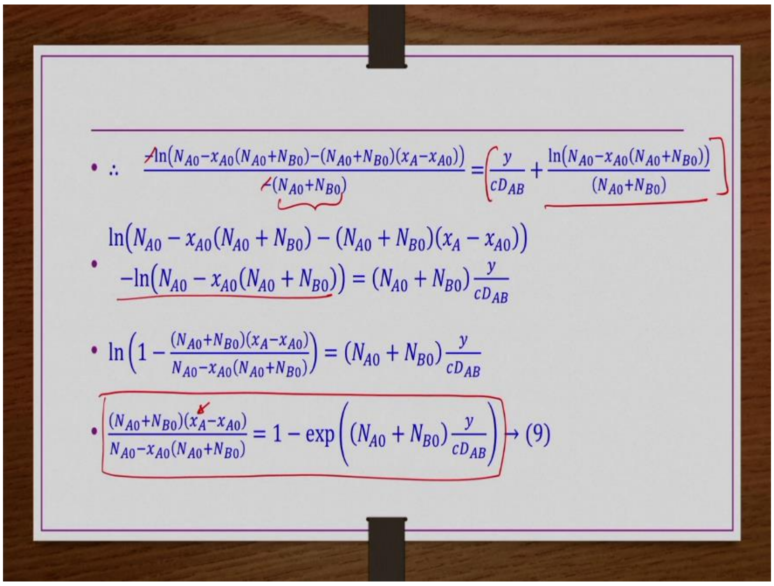

So, that C_1 we have substituted here now this minus this minus you can cancel out and then this $N_{A0} + N_{B0}$ you take it to the right hand side and multiply by these two terms. Then what you have? Left hand side this term and the right hand side $(N_{A0} + N_{B0})\frac{y}{cD_{AB}}$ should be there.

And then second term $(N_{A0} + N_{B0})$ is cancelled out only ln $N_{A0} - x_{A0}(N_{A0} + N_{B0})$ would be there. So, that we brought to the left hand side directly I have written.

So, now this is $lnA - lnB$ form. So, I can write $ln\frac{A}{B}$. So, that I write and then simplify I get $1 - \frac{(N_{A0}+N_{B0})(x_A-x_{A0})}{N_{A0}-x_{A0}(N_{A0}+N_{B0})} = (N_{A0} + N_{B0})\frac{y}{cD_{AB}}$ right. So, after you know removing this ln in terms of exponential issue right. So, this is the expression for the concentration profile x_A as function of y for the second case right.

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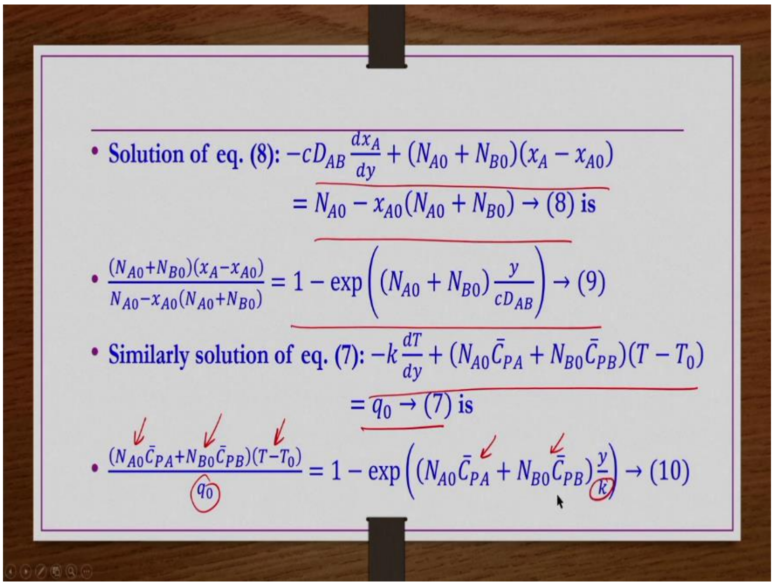

So, now this is for this equation number 8 equation number 9 is the solution right. Now our equation number 7 is similar as an equation number 8. So, its solution should be similar to the equation number 9 so, that you have here; N_{A0} has to be replaced by $N_{A0}\bar{C}_{PA}$ and N_{B0} has to be replaced by $N_{B0}\bar{C}_{PB}$ and δx should be replaced by δ T and whatever $N_{A0} - x_{A0}(N_{A0} + N_{B0})$ is that should be replaced by q_0. Right side also on the same thing we are doing and then c D_{AB} should be replaced by k alright.

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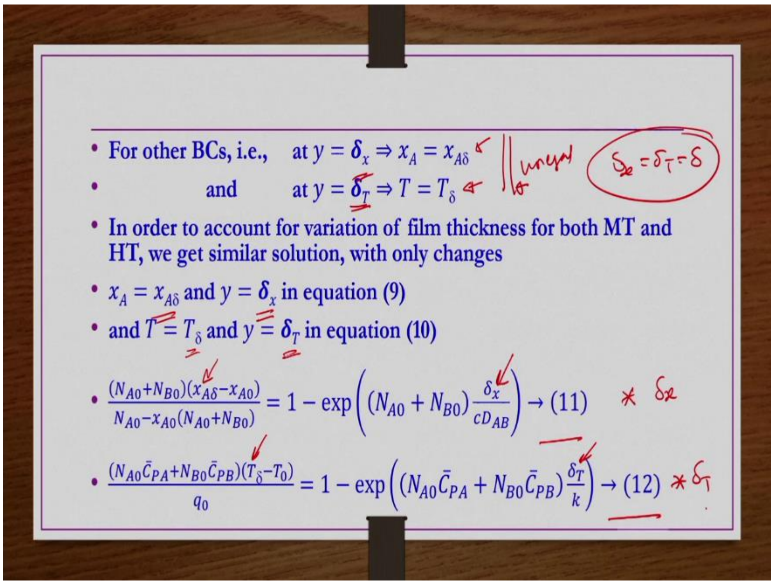

Now, what we do? The limiting conditions of you know unequal film thickness if the film thickness are unequal then at $y = \delta x$ $x_A = x_{A\delta}$ this is for the mass transfer part right. So, film thickness for the mass transfer part whatever is there δx.

So, under such conditions x_A is a $x_{A\delta}$ and then from and for the heat transfer part film thicknesses is δT. So, then $y = \delta_T$ $T = T_\delta$. After getting the solution of equation number 8 in the form equation number 9 what we have taken? We have taken $\delta_x = \delta_T = \delta$ right, but in the problem we have unequal film thickness. So, then accordingly we have two different boundary conditions right.

So, now in equation number 9 wherever x_A is there you replace by $x_{A\delta}$ and wherever δ is there wherever y is there you replaced by δ_x; similarly in equation number 10 which is the heat transfer solution. So, you replace T by T_δ and then y by δ_T then you have this equation number 11 and 12 as solutions. So, now, this is concentration profile when film thickness is δ_x for mass transfer and then this is the temperature profile when film thickness is δ_T for heat transfer right.

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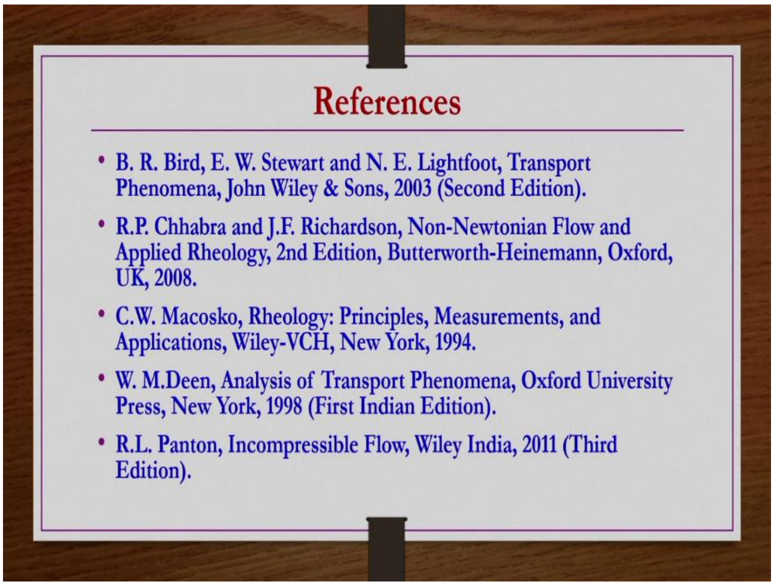

So, the references for this lecture are provided here.

Thank you.