Transport Phenomena of Non-Newtonian Fluids
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Lecture - 32
Non-Isothermal Diffusive MT and Forced Convective MT

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian fluids. Today we will be discussing two different types of problems; Non-Isothermal Diffusive Mass Transfer and then Forced Convective Mass Transfer. So, in the case of non-isothermal diffusive mass transfer, we are taking a problem where diffusion through a non-isothermal spherical film is taking place, ok.

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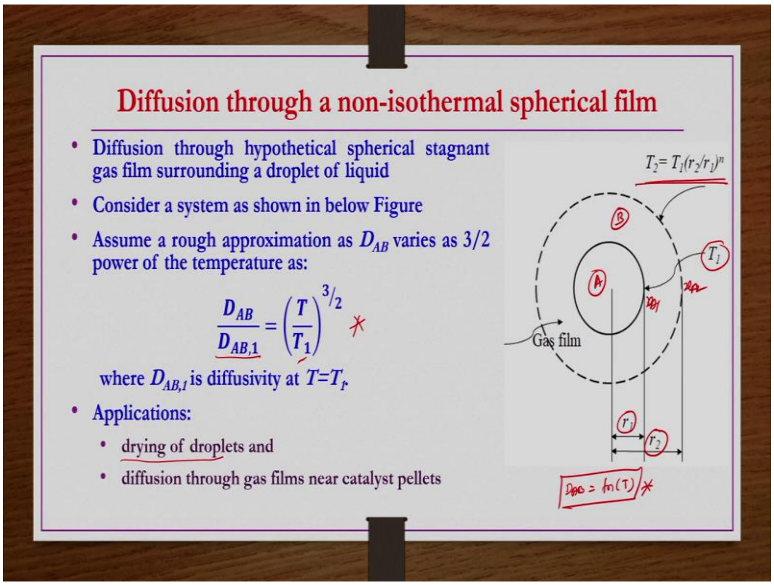

Diffusion through a non-isothermal spherical film

- Diffusion through hypothetical spherical stagnant gas film surrounding a droplet of liquid
- Consider a system as shown in below Figure
- Assume a rough approximation as D_{AB} varies as 3/2 power of the temperature as:

$$\frac{D_{AB}}{D_{AB,1}} = \left(\frac{T}{T_1}\right)^{3/2} \ast$$

where $D_{AB,1}$ is diffusivity at $T=T_1$.

- Applications:
  - drying of droplets and
  - diffusion through gas films near catalyst pellets

Diffusion through hypothetical spherical stagnant gas film surrounding a droplet of liquid; that is the problem you know statement. So, consider this system as shown here in the picture. So, we have a droplet which is made up of the component A and then gas film component B is there, the radius of droplet is r_1, the radius of gas film surrounding this droplet is r_2, ok.

The temperature of the droplet surface is T_1 temperature at the gas film is function of ratio between these two radius power n. So, that is T_2 which is given as $T_1 \left(\frac{r_2}{r_1}\right)^n$ right. So, now the component concentration x_A is x_{A1} at $r = r_1$ and then, component A concentration is x_{A2} at $r = r_2$.

Now, if you have to consider the diffusion through non-isothermal situations, what you need to have information? You need to have a diffusivity as function of temperature. That information you should have, then only you can solve the problem. If that information is not available, you cannot solve the problem. You can solve the problem only for isothermal conditions.

So, how the diffusivity is changing? By change in temperature. That information should also be provided if you wanted to consider non-isothermal diffusive mass transfer problems, ok. So, for that a rough approximation of D_{AB} which varies as 3 by 2 power of the temperature; that is $\frac{D_{AB}}{D_{AB,1}} = \left(\frac{T}{T_1}\right)^{3/2}$.

So, the diffusivity how is it changing with the temperature is given by this expression $D_{AB,1}$ is nothing but diffusivity of a droplet A component in gas film B component at temperature T_1 at fixed temperature T_1, right. D_{AB} is nothing but diffusivity at unknown temperature T. So, now the temperature is varying from T_1 to T_2 and then T_2 is again function of T_1 and then function of r_1 and r_2 as well as given this one, ok.

So, this problem we have to solve here ok. Then further applications if you wanted to know here, such kind of applications in general we have a drying of droplets. Where actually drying of droplets when it taking place usually the size of the droplet decreases as the sum of the components of droplet are evaporating that is in general possible. But in this case what we are trying to take, we are taking the droplet radius is fixed.

Later on, after a week or so we are going to discuss similar kind of problem where evaporation of droplet or evaporation of column, liquid, etcetera would take in place. So, there we are going to take change in the position; interface position that also we are going to take. So, that is in the later course.

But since we are at the beginning of the mass transfer part, so then what we are taking we are taking the droplet radius is fixed even though it is evaporating, right. Applications we see drying of droplets; are sometimes you know what we have catalysts fillets are there. So, near the catalyst surfaces, there would be some kind of diffusion of the component.

Let us say on the catalyst surface some reaction is taking place. So, that product whatever is forming on the surface of the catalyst that is passing through the gas film surrounding the catalyst surface and then reaching the bulk of the fluid. So, that is another kind of application.

So, such kind of application very common in chemical engineering, that is the reason we are taking this particular case to study under non-isothermal diffusion mass transfer.

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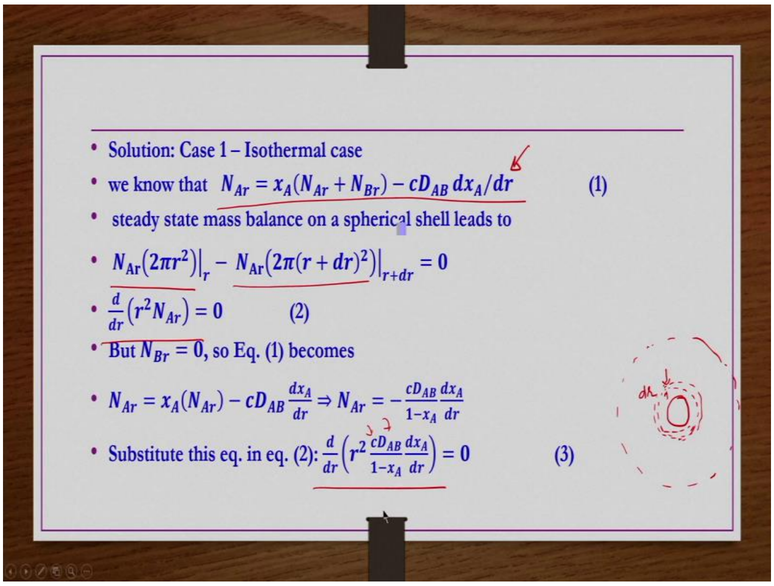

- Solution: Case 1 – Isothermal case
- we know that $N_{Ar} = x_A(N_{Ar} + N_{Br}) - cD_{AB}\, dx_A/dr$ (1)
- steady state mass balance on a spherical shell leads to
- $N_{Ar}(2\pi r^2)\big|_r - N_{Ar}(2\pi(r+dr)^2)\big|_{r+dr} = 0$
- $\frac{d}{dr}\left(r^2 N_{Ar}\right) = 0$ (2)
- But $N_{Br} = 0$, so Eq. (1) becomes
- $N_{Ar} = x_A(N_{Ar}) - cD_{AB}\frac{dx_A}{dr} \Rightarrow N_{Ar} = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}$
- Substitute this eq. in eq. (2): $\frac{d}{dr}\left(r^2 \frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}\right) = 0$ (3)

So, first what we do the same problem. We solve for the isothermal case first, then we go for the non-isothermal case. Because it will be helpful to compare the results how they are changing when you are incorporating non-isothermality of diffusive mass transfer, ok. So, that is the region. First, we finished case 1 where we are taking isothermal case.

So, we know that the combined flux $N_{Ar} = x_A(N_{Ar} + N_{Br}) - cD_{AB}\frac{dx_A}{dr}$. Now, the mass transfer or diffusion is taking place in the radial direction. That is the region the flux are you know designated with r and then, a differentiation of concentration we are doing with respect to r, ok.

Now, if you do the steady state mass balance on a spherical shell; let us say if you have a droplet here so now you take a spherical shell right. So, whose radius is r and then the radius of the outer cell is r + dr, so that the thickness of the cell is dr, ok. And then this gas film that is r_2 is anyway is there. So, between r_1 & r_2 values we are taking a spherical thin film of thickness dr and for that one we are making the balance.

So, when we do that one; so mass rate at r = r is nothing but $N_{Ar}(2\pi r^2)$ and then, mass rate at r + dr is nothing but $N_{Ar}\ 2\pi\ (r+dr)^2$ and should be equals to 0 because there is no reaction involved here in this case. Now this one you simplify. You take dr tends to 0 and then apply the limiting conditions to get the differential form of this equation, then you get $\frac{d}{dr}(r^2 N_{Ar}) = 0$.

Now, you can know the concentration profile, if you know the N_{Ar}, but N_{Ar} you cannot know until and unless N_{Br} is known. N_{Ar} expression you can find out from equation 1; that is fine, but that you can do only when N_{Br} is known. But however in this problem it is mentioned that non-diffusing stagnant gas film.

So, that means B is not diffusing, only A is diffusing. So, that means if B is not diffusing, its flux would be 0 that is N_{Br} is 0. So, now this equation 1 would become when you take $N_{Br} = 0$, you get $N_{Ar} = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}$.

Now, here this you substitute in equation number 2, then you get $\frac{d}{dr}\left(r^2 \frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}\right) = 0$. Now whatever the changes occur in this equation, that comes through this D_{AB} and then c; if non-isothermality is there. So that we take as a case 2, but first case we are taking isothermal case.

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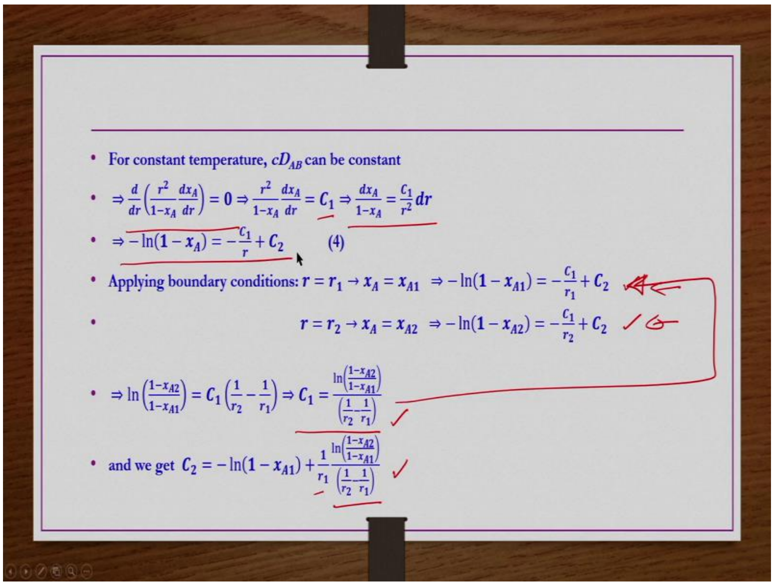

- For constant temperature, cD_{AB} can be constant
- $\Rightarrow \frac{d}{dr}\left(\frac{r^2}{1-x_A}\frac{dx_A}{dr}\right) = 0 \Rightarrow \frac{r^2}{1-x_A}\frac{dx_A}{dr} = C_1 \Rightarrow \frac{dx_A}{1-x_A} = \frac{C_1}{r^2}dr$
- $\Rightarrow -\ln(1-x_A) = -\frac{C_1}{r} + C_2$ (4)
- Applying boundary conditions: $r = r_1 \rightarrow x_A = x_{A1} \ \Rightarrow -\ln(1-x_{A1}) = -\frac{C_1}{r_1} + C_2$
- $r = r_2 \rightarrow x_A = x_{A2} \ \Rightarrow -\ln(1-x_{A2}) = -\frac{C_1}{r_2} + C_2$
- $\Rightarrow \ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right) = C_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \Rightarrow C_1 = \frac{\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}$ ✓
- and we get $C_2 = -\ln(1-x_{A1}) + \frac{1}{r_1}\frac{\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}$ ✓

So, under isothermal conditions c and D_{AB} both are constant. So, then if you take them as constant, then we have $\frac{d}{dr}\left(\frac{r^2}{1-x_A}\frac{dx_A}{dr}\right) = 0$ on integrating once we get $\frac{r^2}{1-x_A}\frac{dx_A}{dr} = C_1$. So, that is $\frac{dx_A}{1-x_A} = \frac{C_1}{r^2}dr$.

Now, once again if you integrate, you get $ln\frac{1-x_A}{-1}$. So, that is $-ln(1-x_A) = \int r^{-2}$ is $\frac{-1}{r}$. So, $\frac{-C_1}{r} +$ another integration constant C_2 you get. Now we have two boundary condition at r = r_1. We have $x_A = x_{A1}$ at r = r_2, $x_A = x_{A2}$.

So, when you apply these two boundary conditions, you get these two expressions. Simply replacing r = r_1 and then $x_A = x_{A1}$ and r = r_2 and then $x_A = x_{A2}$ in the second equation. In the first and second equations here respectively.

Then what you do? Subtract the equation; you subtract the below equation from the above equation, then you get $ln\frac{1-x_{A2}}{1-x_{A1}} = C_1\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ or C_1 is equals to this one. This C_1 you take and then substitute here in the first equation, so you get C_2 that is $C_2 = -ln(1-x_{A1}) + \frac{C_1}{r_1}$. So, $\frac{1}{r_1}$ and then this is nothing but C_1.

So, now you have both C_1 and C_2 expressions you are having. So, those you substitute in equation number 4 to get the concentration profile.

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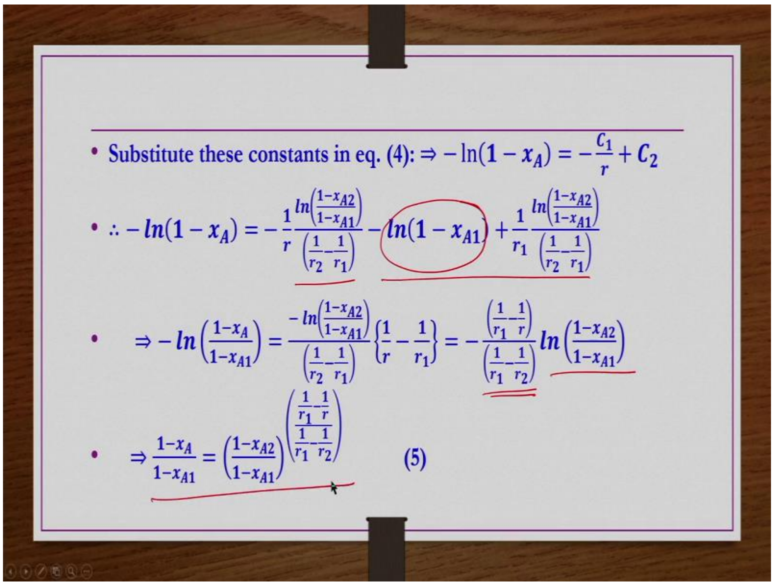

- Substitute these constants in eq. (4): $\Rightarrow -\ln(1-x_A) = -\frac{C_1}{r} + C_2$
- $\therefore -ln(1-x_A) = -\frac{1}{r}\underline{\frac{ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}} \underline{-ln(1-x_{A1}) + \frac{1}{r_1}\frac{ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}}$
- $\Rightarrow -ln\left(\frac{1-x_A}{1-x_{A1}}\right) = \frac{-ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\left\{\frac{1}{r}-\frac{1}{r_1}\right\} = -\underline{\underline{\frac{\left(\frac{1}{r_1}-\frac{1}{r}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}}}\,\underline{ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)}$
- $\Rightarrow \underline{\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\left(\frac{\frac{1}{r_1}-\frac{1}{r}}{\frac{1}{r_1}-\frac{1}{r_2}}\right)}}$ (5)

So, here you substituted C_1 and this is C_2. Then now you what you do, this particular component you take to the left-hand side, so that you have $-ln\frac{1-x_A}{1-x_{A1}}$ in the left-hand side and then right-hand side $-ln\frac{1-x_{A2}}{1-x_{A1}}\frac{1}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}$ is common if you take common. So, multiplied by $\left(\frac{1}{r}-\frac{1}{r_1}\right)$you are having, right.

So, this expression what we can write? Here numerator also what I do? I write $\frac{1}{r_1}-\frac{1}{r_2}$ and then, this one also I write $\frac{1}{r_1}-\frac{1}{r}$, so that I have this expression and then remaining term is as it is. So, concentration profile what you have this is the expression you have given by equation number 5, right.

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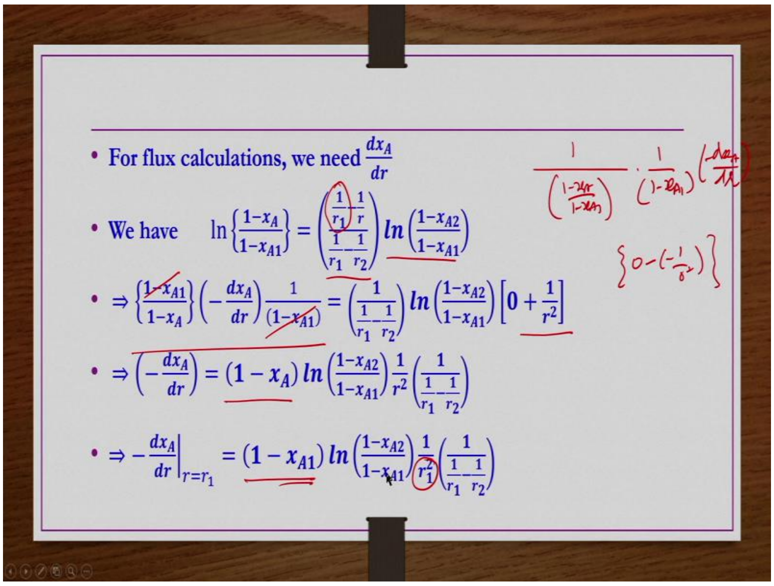

- For flux calculations, we need $\frac{dx_A}{dr}$
- We have $\ln\left\{\frac{1-x_A}{1-x_{A1}}\right\} = \left(\frac{\frac{1}{r_1}-\frac{1}{r}}{\frac{1}{r_1}-\frac{1}{r_2}}\right) ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$
- $\Rightarrow \left\{\frac{\cancel{1-x_{A1}}}{1-x_A}\right\}\left(-\frac{dx_A}{dr}\right)\frac{1}{\cancel{(1-x_{A1})}} = \left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right) ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\left[0+\frac{1}{r^2}\right]$
- $\Rightarrow \left(-\frac{dx_A}{dr}\right) = (1-x_A)\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{r^2}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right)$
- $\Rightarrow -\frac{dx_A}{dr}\bigg|_{r=r_1} = (1-x_{A1})\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{r_1^2}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right)$

$\frac{1}{\left(\frac{1-x_{Ar}}{1-x_{A1}}\right)} \cdot \frac{1}{(1-x_{A1})}\left(-\frac{dx_A}{dr}\right)$

$\left\{0-\left(-\frac{1}{r^2}\right)\right\}$

Now, if you wanted to know the flux what you have to do, you have to obtain the $\frac{dx_A}{dr}$ because N_{Ar} expression we have $\frac{dx_A}{dr}$, right. So, for that what we do? We take a concentration profile in this ln form, then differentiate with respect to r both sides. So, first left-hand side if you differentiate with r, you get $\frac{1}{\left(\frac{1-x_A}{1-x_{A1}}\right)}$ multiplied by $\frac{1}{1-x_A}$.

And then $-\frac{dx_A}{dr}$, you are getting; right-hand side this and these are constant. So, that and then remaining term if you do the differentiation, you get this is also constants. So, 0 −

$\left(-\frac{1}{r^2}\right)$ as differentiation. So, then you have this $1 + \frac{1}{r^2}$ in the right-hand side and then left-hand side this is what you are having right.

So, this $1 - x_{A1}$, this $1 - x_{A1}$ you can cancel out and then this $1 - x_A$ you can take it to the right-hand side or you keep it here itself. There is no issue right. So, if you take to the right-hand side, so this is what you are having number. This $\frac{dx_A}{dr}$ we need at r = r_1. Flux, we wanted to calculate at r = r_1 because from that surface on only; from that position, only droplet is evaporating surface.

You know evaporation is taking place from the droplet surface which is designated as r = r_1. So, that is the region this flux we have to calculate at r = r_1. So, this $\frac{dx_A}{dr}$ also we should calculate at r = r_1. So, when you do; at r = r_1 x_A is nothing but x_{A1} and then r is nothing but r_1. So, rest of the terms are as it is.

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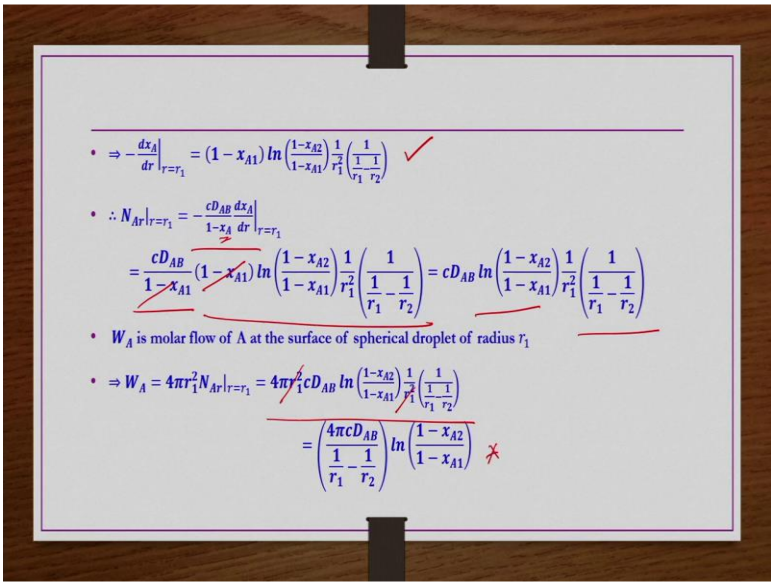

- $\Rightarrow -\left.\frac{dx_A}{dr}\right|_{r=r_1} = (1 - x_{A1})\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{r_1^2}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right)$ ✓
- $\therefore N_{Ar}|_{r=r_1} = -\frac{cD_{AB}}{1-x_A}\left.\frac{dx_A}{dr}\right|_{r=r_1}$

$$= \frac{cD_{AB}}{\cancel{1-x_{A1}}}\cancel{(1-x_{A1})}\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{r_1^2}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right) = cD_{AB}\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{r_1^2}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right)$$

- W_A is molar flow of A at the surface of spherical droplet of radius r_1
- $\Rightarrow W_A = 4\pi r_1^2 N_{Ar}|_{r=r_1} = 4\pi \cancel{r_1^2} cD_{AB}\, ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\frac{1}{\cancel{r_1^2}}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}}\right)$

$$= \left(\frac{4\pi c D_{AB}}{\frac{1}{r_1}-\frac{1}{r_2}}\right) ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right) \quad *$$

So, this is what we get $-\frac{dx_A}{dr}$ at r = r_1. Now flux N_{Ar} at r = r_1 is nothing but $\frac{-cD_{AB}}{1-x_{A1}}\frac{dx_A}{dr}$ at r = r_1 x_{A1} because we are calculating this one flux at r = r_1. So, that is $\frac{cD_{AB}}{1-x_A}$ and then $-\frac{dx_A}{dr}$ is this one. So, $(1 - x_A)$ $(1 - x_A)$cancelled out, cD_{AB}ln of this term and then $\frac{1}{r_1^2}$ and then this term, this is the flux.

Now, if you wanted to know the rate of mass transfer in terms of moles per time that is W_A is molar flow of A at the surface of spherical droplet of radius r_1. So, what you have to do? This flux whatever is there that you have to multiply by surface area through which the mass transfer is taking place. Surface area of the droplet that is at r_1 location that is 4 $\pi\, r_1^2$ is the surface area of the droplet.

So, flux if you multiplied by this multiply by this surface area, you get the mass transfer rate in moles per seconds; that expression is given this. So, now 4 $\pi\, r_1^2$. So, this r_1^2, this r_1^2 is cancelled out. So, the remaining term is this one. So, this is the isothermal case.

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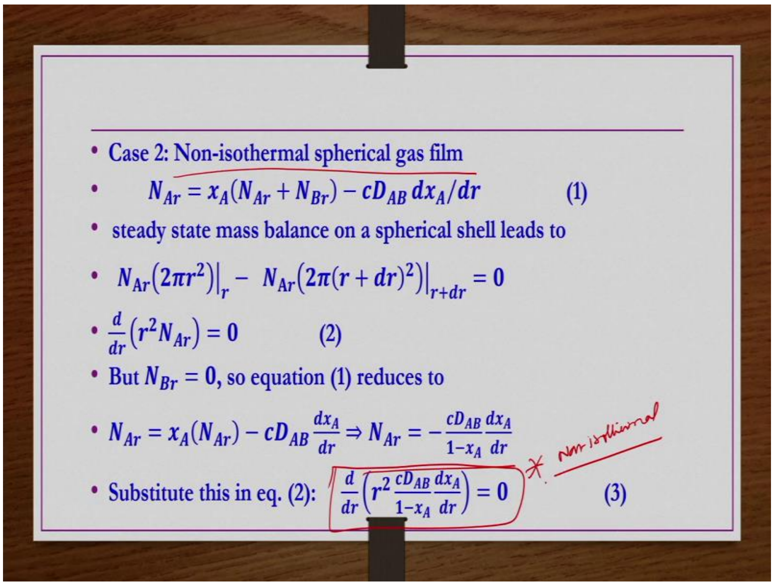

- Case 2: Non-isothermal spherical gas film
- $N_{Ar} = x_A(N_{Ar} + N_{Br}) - cD_{AB}\, dx_A/dr$ (1)
- steady state mass balance on a spherical shell leads to
- $N_{Ar}(2\pi r^2)\big|_r - N_{Ar}(2\pi(r+dr)^2)\big|_{r+dr} = 0$
- $\frac{d}{dr}(r^2 N_{Ar}) = 0$ (2)
- But $N_{Br} = 0$, so equation (1) reduces to
- $N_{Ar} = x_A(N_{Ar}) - cD_{AB}\frac{dx_A}{dr} \Rightarrow N_{Ar} = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}$
- Substitute this in eq. (2): $\boxed{\frac{d}{dr}\left(r^2 \frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}\right) = 0}$ * Non isothermal (3)

Now, we take non-isothermal case of the same problem. So, this non-isothermal case up to derivation of $N_{Ar} = \frac{-cD_{AB}}{1-x_A}\frac{dx_A}{dr}$ up to that point it is quite same, because till that point temperature issue has not come into the equations or any of the balances, ok.

So, combined flux is this one. So, steady state balance is the same like isothermal case. Then by applying the limiting conditions you get the differential form of this equation, this balance equation given by equation number 2. But N_{Br} is 0 because stagnant non-isothermal stagnant gas film. So, gas film is not diffusing; non-diffusing stagnant gas film that is the statement is given. So, B is not diffusing; gas film is of component B. So, N_{Br} is 0, so that again you substitute in equation number 1, you get N_{Ar} is equals to this one.

So, this one you substitute in equation number 2. So, then this equation you get. So, now this equation you have to solve for non-isothermal conditions. Till now the non-isothermality has not brought into the picture. There is no term which required to take consider this non-isothermality here. So, till this point it is solution is same as case 1. So, from here it will change.

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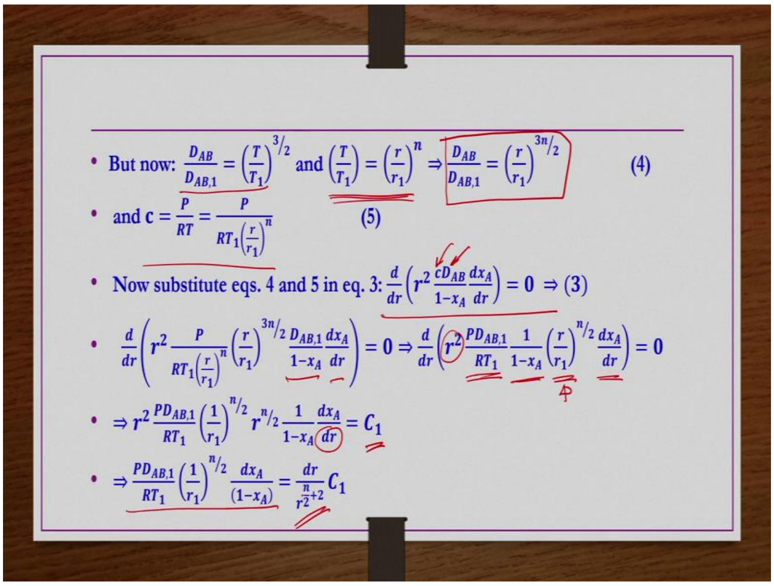

- But now: $\frac{D_{AB}}{D_{AB,1}} = \left(\frac{T}{T_1}\right)^{3/2}$ and $\left(\frac{T}{T_1}\right) = \left(\frac{r}{r_1}\right)^n \Rightarrow \boxed{\frac{D_{AB}}{D_{AB,1}} = \left(\frac{r}{r_1}\right)^{3n/2}}$ (4)
- and $c = \frac{P}{RT} = \frac{P}{RT_1\left(\frac{r}{r_1}\right)^n}$ (5)
- Now substitute eqs. 4 and 5 in eq. 3: $\frac{d}{dr}\left(r^2 \frac{cD_{AB}}{1-x_A}\frac{dx_A}{dr}\right) = 0 \Rightarrow (3)$
- $\frac{d}{dr}\left(r^2 \frac{P}{RT_1\left(\frac{r}{r_1}\right)^n}\left(\frac{r}{r_1}\right)^{3n/2}\frac{D_{AB,1}}{1-x_A}\frac{dx_A}{dr}\right) = 0 \Rightarrow \frac{d}{dr}\left(r^2 \frac{PD_{AB,1}}{RT_1}\frac{1}{1-x_A}\left(\frac{r}{r_1}\right)^{n/2}\frac{dx_A}{dr}\right) = 0$
- $\Rightarrow r^2 \frac{PD_{AB,1}}{RT_1}\left(\frac{1}{r_1}\right)^{n/2} r^{n/2}\frac{1}{1-x_A}\frac{dx_A}{dr} = C_1$
- $\Rightarrow \frac{PD_{AB,1}}{RT_1}\left(\frac{1}{r_1}\right)^{n/2}\frac{dx_A}{(1-x_A)} = \frac{dr}{r^{\frac{n}{2}+2}}C_1$

So, now this relation is given $\frac{D_{AB}}{D_{AB,1}}$ is nothing but $\left(\frac{T}{T_1}\right)^{3/2}$ and then this $\frac{T}{T_1}$ is given by$\left(\frac{r}{r_1}\right)^n$. So, now what we can write $\frac{D_{AB}}{D_{AB,1}} = \left(\frac{r}{r_1}\right)^{3n/2}$. You may be thinking that in place of D_{AB} we can why cannot we write $D_{AB,1}$ multiplied by $\left(\frac{T}{T_1}\right)^{3/2}$. So, that way we can write and then do the differentiation, but T is related to the r; T is related to the r that is given in this statement.

So, the T is also a function of r, so then we have the differentiations and then integration subsequent in the calculation. So, until and unless this T function of r if you do not include, so the solution will not be reliable; because T function of r that statement is given in the problem that should also be incorporated.

Had that not been given, so then we should have taken only $D_{AB} = D_{AB,1}\left(\frac{T}{T_1}\right)^{3/2}$. That is it. So, but now $\frac{T}{T_1}$ is given by this expression in the problem statement. So, that information should also be brought into the picture.

Further see we can write as $\frac{P}{RT}$ and then T is nothing but $T_1\left(\frac{r}{r_1}\right)^n$, right. So, now in this equation number 3 in place of c in place of D, we have to use these two expressions ok. That if you do $\frac{d}{dr}r^2$, c is nothing but $\frac{P}{RT_1\left(\frac{r}{r_1}\right)^n}\left(\frac{r}{r_1}\right)^{3n/2} D_{AB,1}$ that is nothing but D_{AB} and then $\frac{1}{x_A}$ and then $\frac{dx_A}{dr} = 0$.

So, that you rearrange. Now this r^2 is as it is $\frac{PD_{AB,1}}{RT_1}$ are constant now. $\frac{1}{1-x_A}$ as it is $\frac{dx_A}{dr}$ is as it is. So, remaining terms whatever this $\left(\frac{r}{r_1}\right)^{3n/2}$ and then $\left(\frac{r}{r_1}\right)^n$, we can write $\left(\frac{r}{r_1}\right)^{n/2}$, right. So, that is what this additional thing.

Now, you can integrate. So, then left-hand side differentiation will go off and then we have C_1 constant in the right-hand side. So, this one r^2 and then $\frac{r^{n/2}}{{r_1}^{n/2}}$, we are separating out. So, that we can combine this $r^{n/2}$ and then r^2 as one term.

So, that you know it will be useful in integration subsequently. So, that $r^{\frac{n}{2}+2}$ that we are taking to the right-hand side. Even this dr also we are taking to the right-hand side. So, we are getting $\frac{dr}{r^{\frac{n}{2}+2}}C_1$ in the right-hand side, all other terms in the left-hand side.

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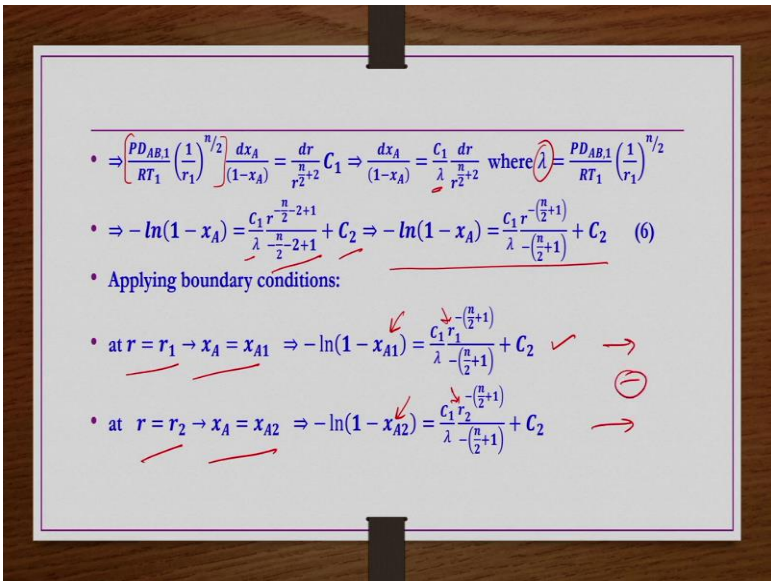

- $\Rightarrow \left[\frac{PD_{AB,1}}{RT_1}\left(\frac{1}{r_1}\right)^{n/2}\right]\frac{dx_A}{(1-x_A)} = \frac{dr}{r^{\frac{n}{2}+2}}C_1 \Rightarrow \frac{dx_A}{(1-x_A)} = \frac{C_1}{\lambda}\frac{dr}{r^{\frac{n}{2}+2}}$ where $\lambda = \frac{PD_{AB,1}}{RT_1}\left(\frac{1}{r_1}\right)^{n/2}$
- $\Rightarrow -ln(1-x_A) = \frac{C_1}{\lambda}\frac{r^{-\frac{n}{2}-2+1}}{-\frac{n}{2}-2+1} + C_2 \Rightarrow -ln(1-x_A) = \frac{C_1}{\lambda}\frac{r^{-\left(\frac{n}{2}+1\right)}}{-\left(\frac{n}{2}+1\right)} + C_2$ (6)
- Applying boundary conditions:
- at $r = r_1 \rightarrow x_A = x_{A1}$ $\Rightarrow -\ln(1-x_{A1}) = \frac{C_1}{\lambda}\frac{r_1^{-\left(\frac{n}{2}+1\right)}}{-\left(\frac{n}{2}+1\right)} + C_2$ ✓ →

$\ominus$

- at $r = r_2 \rightarrow x_A = x_{A2}$ $\Rightarrow -\ln(1-x_{A2}) = \frac{C_1}{\lambda}\frac{r_2^{-\left(\frac{n}{2}+1\right)}}{-\left(\frac{n}{2}+1\right)} + C_2$ →

So, in the left-hand side except $\frac{dx_A}{1-x_A}$ everything is constant, right. So, the same equation is written here. So, what we do for simplicity? We take this expression which is a constant, we represent by λ. So that that λ also we take into the right-hand side. So, that we can write $\frac{C_1}{\lambda}$ as 1 constant and then remaining terms are as it is, ok.

Now, if you integrate this equation again, you get left-hand side $ln\frac{1-x_A}{-1}$. So, that is $-ln(1-x_A)\frac{C_1}{\lambda}$ is constant and then integration of $r^{-\frac{n}{2}-2}$ is nothing but this one, + C_2 constant. So, when you simplify, this is what you have this equation right.

Now, we have two boundary conditions at $r = r_1$ $x_A = x_{A1}$. So, we have this equation in place of x_A, we are writing x_{A1} in place of r, we are writing r_1. Other boundary condition at $r = r_2$, $x_A = x_{A2}$. So, same thing here in place of x_A we are writing x_{A2} in place of r, we are writing r_2. Rest all other terms are same.

So, now this equation minus this equation if you do you get an expression for C_1 because $C_2 - C_2$ is 0.

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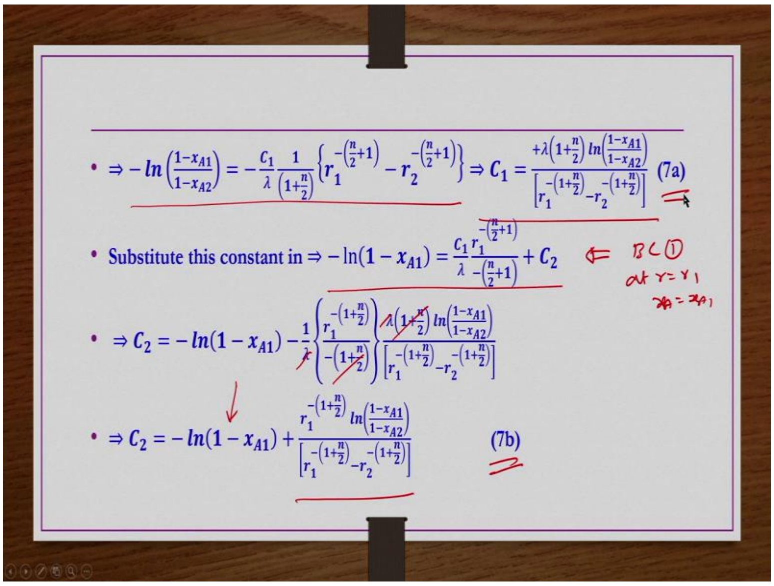

- $\Rightarrow -ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) = -\frac{C_1}{\lambda}\frac{1}{\left(1+\frac{n}{2}\right)}\left\{r_1^{-\left(\frac{n}{2}+1\right)} - r_2^{-\left(\frac{n}{2}+1\right)}\right\} \Rightarrow C_1 = \frac{+\lambda\left(1+\frac{n}{2}\right) ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]}$ (7a)
- Substitute this constant in $\Rightarrow -\ln(1-x_{A1}) = \frac{C_1}{\lambda}\frac{r_1^{-\left(\frac{n}{2}+1\right)}}{-\left(\frac{n}{2}+1\right)} + C_2$ ⇐ BC ① at r = r1 xA = xA1
- $\Rightarrow C_2 = -ln(1-x_{A1}) - \frac{1}{\cancel{\lambda}}\left\{\frac{r_1^{-\left(1+\frac{n}{2}\right)}}{-\cancel{\left(1+\frac{n}{2}\right)}}\right\}\frac{\cancel{\lambda\left(1+\frac{n}{2}\right)} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]}$
- $\Rightarrow C_2 = -ln(1-x_{A1}) + \frac{r_1^{-\left(1+\frac{n}{2}\right)} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]}$ (7b)

So, this is what you get and then from here C$_1$ you get this expression. Now, this C$_1$ if you substitute in this equation, you get C$_2$. This is B C 1. That is at r = r$_1$, x$_A$ = x$_{A1}$ whatever we have written that equation is this one, right. So, now here $C_2 = -ln(1 - x_{A1}) - \frac{1}{\lambda}$ and then, this is of C$_1$ from 7 a equation number 7a.

So, now we have C$_1$ C$_2$, here C$_2$ what we can do we can simplify slightly this λ, this λ this 1 + $\frac{n}{2}$, this 1 + $\frac{n}{2}$ cancel out. So, remaining terms are this and then this term is as it is. So, 7 b C$_2$, 7 a C$_1$.

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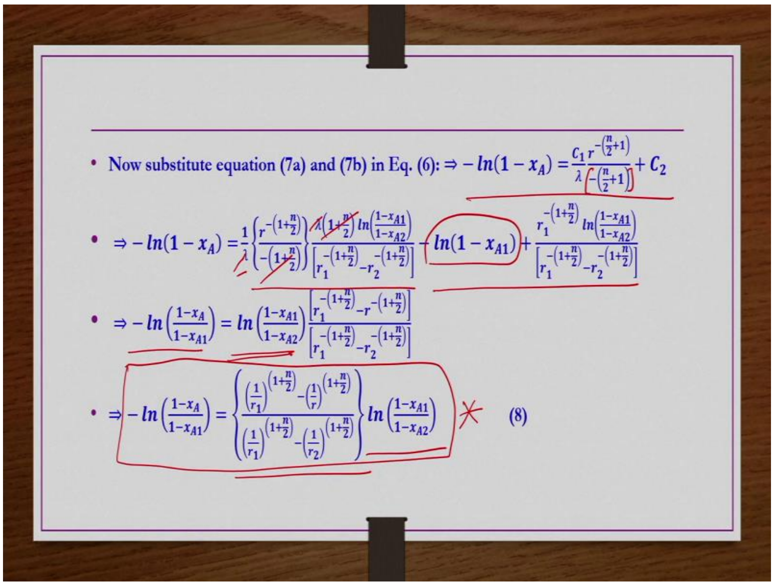

- Now substitute equation (7a) and (7b) in Eq. (6): $\Rightarrow -ln(1-x_A) = \frac{C_1}{\lambda}\frac{r^{-\left(\frac{n}{2}+1\right)}}{\left[-\left(\frac{n}{2}+1\right)\right]} + C_2$
- $\Rightarrow -ln(1-x_A) = \frac{1}{\cancel{\lambda}}\left\{\frac{r^{-\left(1+\frac{n}{2}\right)}}{-\cancel{\left(1+\frac{n}{2}\right)}}\right\}\frac{\cancel{\lambda\left(1+\frac{n}{2}\right)} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]} - ln(1-x_{A1}) + \frac{r_1^{-\left(1+\frac{n}{2}\right)} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]}$
- $\Rightarrow -ln\left(\frac{1-x_A}{1-x_{A1}}\right) = ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\frac{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r^{-\left(1+\frac{n}{2}\right)}\right]}{\left[r_1^{-\left(1+\frac{n}{2}\right)} - r_2^{-\left(1+\frac{n}{2}\right)}\right]}$
- $\Rightarrow \boxed{-ln\left(\frac{1-x_A}{1-x_{A1}}\right) = \left\{\frac{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r}\right)^{\left(1+\frac{n}{2}\right)}}{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}}\right\} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}$ ✱ (8)

So, this constant C₁ C₂ if you substitute in the equation 6; that is $-ln(1-x_A) = \frac{C_1}{\lambda} r^{\frac{-\frac{n}{2}+1}{-(\frac{n}{2}+1)}} + C_2$, right. So, now C₁ C₂ you substitute $\frac{1}{\lambda}$ this is C_1, this entire thing is C_2. Now what we do? This one this $ln(1-x_{A1})$ term that we take to the left-hand side, so that we can have left-hand side $-ln\left(\frac{1-x_A}{1-x_{A1}}\right)$.

And then remaining terms the first term here λ, λ you can cancel out and 1 + $\frac{n}{2}$, 1 + $\frac{n}{2}$ also you can cancel out. So, the remaining terms; from the remaining two terms you can take $ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$ common, then we have $\frac{{r_1}^{-\left(1+\frac{n}{2}\right)} - r^{-\left(1+\frac{n}{2}\right)}}{{r_1}^{-\left(1+\frac{n}{2}\right)} - {r_2}^{-\left(1+\frac{n}{2}\right)}}$.

So, then all of these r's are having power minus, so then we can write the reciprocal of them. So, when you write. So, then this is what you are having multiplied by $ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$. So, under the non-isothermal conditions, the concentration profile for this spherical gas film problem diffusion of droplet to a spherical gas film is provided by this expression, right.

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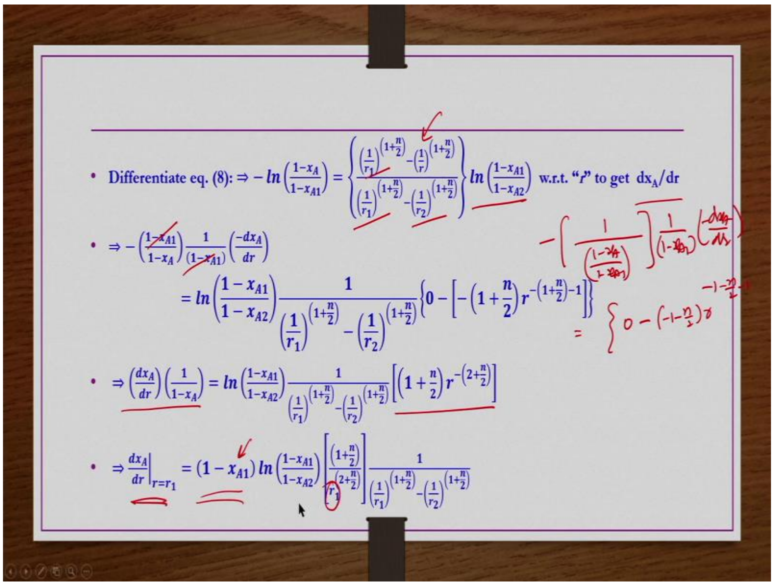

- Differentiate eq. (8): $\Rightarrow -ln\left(\frac{1-x_A}{1-x_{A1}}\right) = \left\{\frac{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r}\right)^{\left(1+\frac{n}{2}\right)}}{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}}\right\} ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$ w.r.t. "r" to get dx_A/dr
- $\Rightarrow -\left(\frac{1-x_{A1}}{1-x_A}\right)\frac{1}{(1-x_{A1})}\left(\frac{-dx_A}{dr}\right)$

$$= ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\frac{1}{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}}\left\{0-\left[-\left(1+\frac{n}{2}\right)r^{-\left(1+\frac{n}{2}\right)-1}\right]\right\}$$

$-\left[\frac{1}{\left(\frac{1-x_A}{1-x_{A1}}\right)}\right]\frac{1}{(1-x_{A1})}\left(\frac{-dx_A}{dr}\right)$

$= \left\{0-\left(-1-\frac{n}{2}\right)r^{-1-\frac{n}{2}-1}\right.$

- $\Rightarrow \left(\frac{dx_A}{dr}\right)\left(\frac{1}{1-x_A}\right) = ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\frac{1}{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}}\left[\left(1+\frac{n}{2}\right)r^{-\left(2+\frac{n}{2}\right)}\right]$
- $\Rightarrow \left.\frac{dx_A}{dr}\right|_{r=r_1} = (1-x_{A1})\, ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\left[\frac{\left(1+\frac{n}{2}\right)}{{r_1}^{\left(2+\frac{n}{2}\right)}}\right]\frac{1}{\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)} - \left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}}$

Now, if you wanted to know the flux, what you have to do is, you have to find out $\frac{dx_A}{dr}$. So, for that what we do this equation whatever is there that we differentiate with respect to r.

So, left-hand side what we will be having; $-\left[\frac{1}{\left(\frac{1-x_A}{1-x_{A1}}\right)}\right]\left(\frac{1}{1-x_{A1}}\right)\left(\frac{-dx_A}{dr}\right)$; you get in the left-hand side.

Right-hand side this is constant, this is constant, this is constant, this is constant. So, only this one we have to differentiate. So, in the numerator we will be having $\left\{0-\left[-\left(1+\frac{n}{2}\right)r^{-\left(1+\frac{n}{2}\right)-1}\right]\right\}$. This is what we have. Rest all other terms are constants in the right-hand side. So, then we have right-hand side this particular term as written here.

So, $\frac{1}{\left(\frac{1-x_A}{1-x_{A1}}\right)}$, I can write $\frac{1-x_{A1}}{1-x_A}$ and then remaining two terms as it is. Right-hand side, we have this term is constant and then denominator is constant, numerator differentiation you get this expression, right. So, this if you write $\frac{dx_A}{dr}$ one side and then $\frac{1}{1-x_{A1}}$ side. Rest all other terms to the other side if you take. So, here this $(1-x_{A1})$ $(1-x_{A1})$ is cancelled out. So, the remaining terms are as it is right-hand side term. This is what we got, ok.

So, now we need the flux at the droplets surface which is evaporating. So, at r = r_1, we have to get this expression. So, at r = r_1, x_A = x_{A1}. So, whatever $\frac{1}{1-x_{A1}}$ in the left-hand side that also be brought to the right-hand side. And then in this expression whatever these $r^{-2+\frac{n}{2}}$ is there that I am writing $\frac{1}{r^{2+\frac{n}{2}}}$ and then r we are replacing by r_1.

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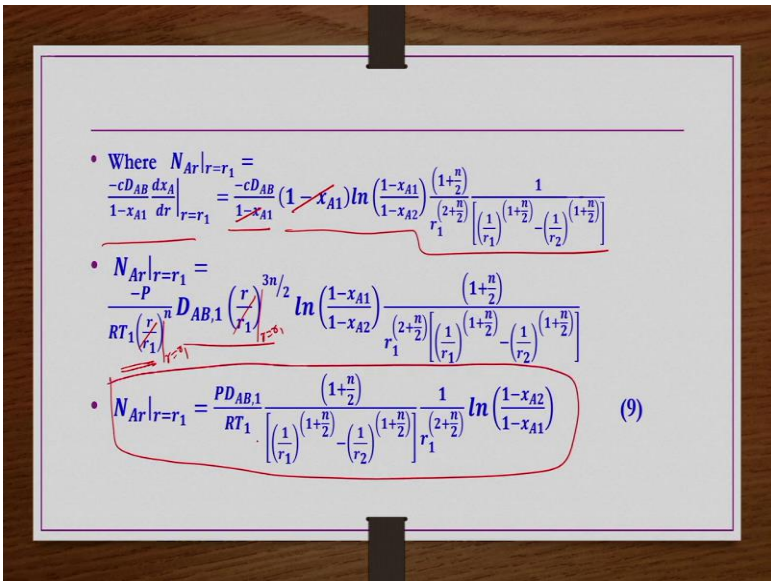

- Where $N_{Ar}|_{r=r_1}=$
$$\frac{-cD_{AB}}{1-x_{A1}}\frac{dx_A}{dr}\Big|_{r=r_1}=\frac{-cD_{AB}}{\cancel{1-x_{A1}}}(\cancel{1-x_{A1}})ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\frac{\left(1+\frac{n}{2}\right)}{r_1^{\left(2+\frac{n}{2}\right)}}\frac{1}{\left[\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)}-\left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}\right]}$$
- $N_{Ar}|_{r=r_1}=$
$$\frac{-P}{RT_1\left(\frac{\cancel{r}}{\cancel{r_1}}\right)^n\Big|_{r=r_1}}D_{AB,1}\left(\frac{\cancel{r}}{\cancel{r_1}}\right)^{3n/2}\Big|_{r=r_1}ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)\frac{\left(1+\frac{n}{2}\right)}{r_1^{\left(2+\frac{n}{2}\right)}\left[\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)}-\left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}\right]}$$
$\Rightarrow$
- $$\boxed{N_{Ar}|_{r=r_1}=\frac{PD_{AB,1}}{RT_1}\frac{\left(1+\frac{n}{2}\right)}{\left[\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)}-\left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}\right]}\frac{1}{r_1^{\left(2+\frac{n}{2}\right)}}ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)} \qquad (9)$$

So, now this flux N$_{Ar}$ at r = r$_1$ is nothing but $\frac{-cD_{AB}}{1-x_{A1}}\frac{dx_A}{dr}$ at r = r$_1$. So, this part is as it is; $\frac{dx_A}{dr}$ is given by this expression. Just previous slide we got it, ok. So, this $1-x_{A1}$, this $1-x_{A1}$ is cancelled out. And now the c is nothing but $\frac{P}{RT}$ and then D_{AB} is nothing but $D_{AB,1}\left(\frac{r}{r_1}\right)^{3n/2}$. So, those things we are writing. And then T is nothing but $T_1\left(\frac{r}{r_1}\right)^n$, right.

So now this, we are actually evaluating at r = r$_1$. So, $\frac{r}{r_1}$ is 1. Similarly this also we are evaluating at r = r$_1$. So, $\frac{r}{r_1}$ is 1. So, whatever the power does not matter, 1 power anything is 1, right. So, now here so this is what we get. So, finally the flux is given by this expression $\frac{PD_{AB,1}}{RT_1}$ multiplied by this particular term, right.

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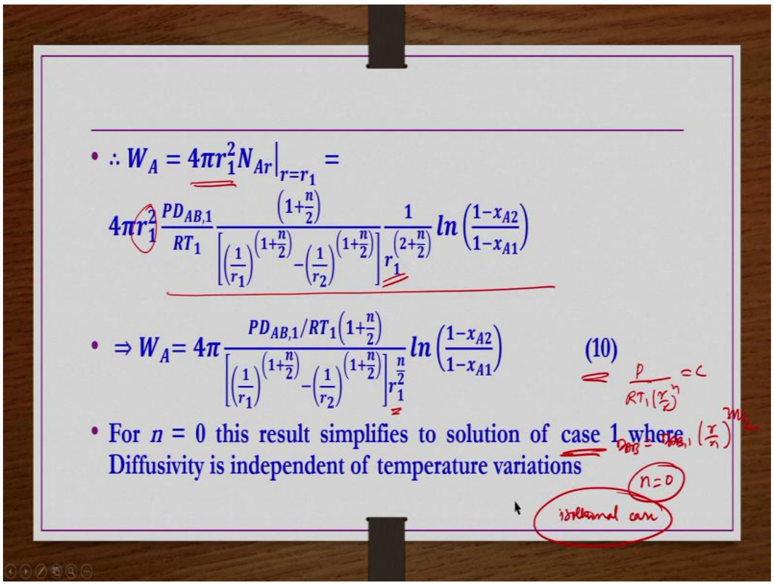

- $\therefore W_A = 4\pi r_1^2 N_{Ar}\big|_{r=r_1} =$

$$4\pi r_1^2 \frac{PD_{AB,1}}{RT_1}\frac{\left(1+\frac{n}{2}\right)}{\left[\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)}-\left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}\right]}\frac{1}{r_1^{\left(2+\frac{n}{2}\right)}}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$$

- $\Rightarrow W_A = 4\pi\frac{PD_{AB,1}/RT_1\left(1+\frac{n}{2}\right)}{\left[\left(\frac{1}{r_1}\right)^{\left(1+\frac{n}{2}\right)}-\left(\frac{1}{r_2}\right)^{\left(1+\frac{n}{2}\right)}\right]r_1^{\frac{n}{2}}}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$ (10)
- For $n = 0$ this result simplifies to solution of case 1 where Diffusivity is independent of temperature variations

$\frac{P}{RT_1\left(\frac{r}{r_1}\right)^n} = c$

$D_{AB} = D_{AB,1}\left(\frac{r}{r_1}\right)^{3n/2}$

$n=0$

isothermal case

Now, if you want to know the mass transfer rate in moles per second here, so whatever N$_{Ar}$ at r = r$_1$ is that we have to multiplied by surface area of the droplet that is 4 π r_1^2. Because from the droplet surface, the transfer of the mass is taking place right.

So, N$_{Ar}$ at r = r$_1$ is nothing but this one. So, now this r_1^2 this r_1^2 can be cancelled out. So, then we have only r$^{n/2}$ here. So, that r is also at r = r$_1$, ok. So, remaining r terms are same.

So, now in this equation if you write $\frac{P}{RT_1\left(\frac{r}{r}\right)^n}$ as c and then $D_{AB} = D_{AB,1}\left(\frac{r}{r_1}\right)^{3n/2}$ if you write and then, you take n = 0, then this expression will reduce to the solution of so called isothermal case. You can cross check it is, right.

So, this is about the non-isothermal diffusive mass transfer. Now, we take another case where we are considering forced convective mass transfer, ok. For this situation of forced convective mass transfer, we are considering diffusion into a falling liquid film. There is a liquid film. So, that is falling with certain velocities. So, there is a velocity distribution, that velocity distribution is affecting the diffusive mass transfer. How it is affecting that we have to find out, right.

So, it is also possible that diffusion is taking place into the liquid film. So, then liquid composition may change and then viscosity may change; that is also possible. So, can we consider can we handle that situation? If not, how we; how to make approximation simplification and all those things we have to see.

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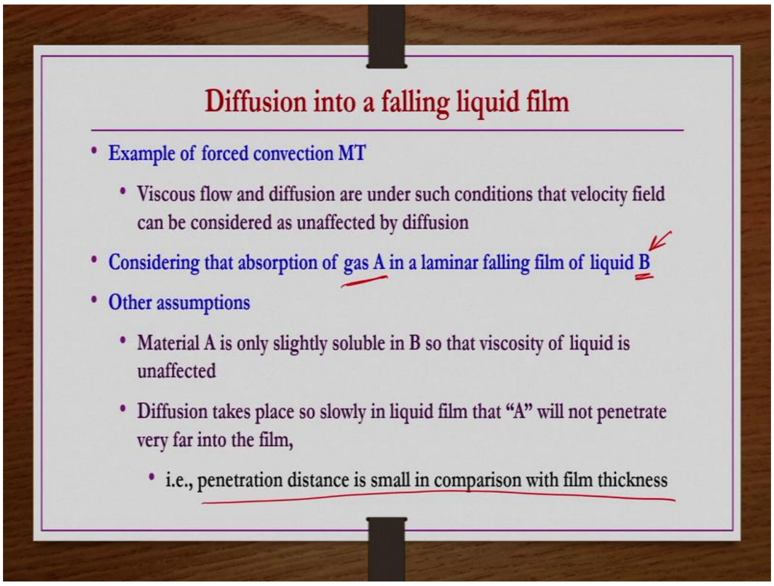

Diffusion into a falling liquid film

- Example of forced convection MT
  - Viscous flow and diffusion are under such conditions that velocity field can be considered as unaffected by diffusion
- Considering that absorption of gas A in a laminar falling film of liquid B
- Other assumptions
  - Material A is only slightly soluble in B so that viscosity of liquid is unaffected
  - Diffusion takes place so slowly in liquid film that "A" will not penetrate very far into the film,
    - i.e., penetration distance is small in comparison with film thickness

So, this is an example of forced convective mass transfer as explained. Where viscous flow and diffusion are under such condition that velocity field can be considered as an affected by the diffusion. That is momentum transfer is not affected by the mass transfer, but mass transfer is affected by the momentum transfer, ok.

So, this mass transfer is not affecting the momentum transfer, but momentum transfer is affecting the mass transfer. So, this is the situation. This can be possible when diffusion is only taking place slightly. If it is taking place only slightly, then what happens? You know the composition of liquid composition does not change much.

So, liquid viscosity does not change. So, then obviously its velocity distribution will not be affected, because here it is a thin film; it is a thin film only ok. So, that is considering that absorption of gas A in a laminar falling film of liquid B. So, B is liquid component, A is the gas film which is diffusing. So, this A gas is diffusing into B liquid.

Other assumptions material A is only slightly soluble in B. So, that viscosity of liquid is unaffected, then only we can say that mass transfer is not affecting the momentum transfer. Diffusion takes place so slowly in liquid film that A will not penetrate very far into the film. So, into the film you know the transfer is taking place and then that is penetration distance is small in comparison with film thickness, ok.

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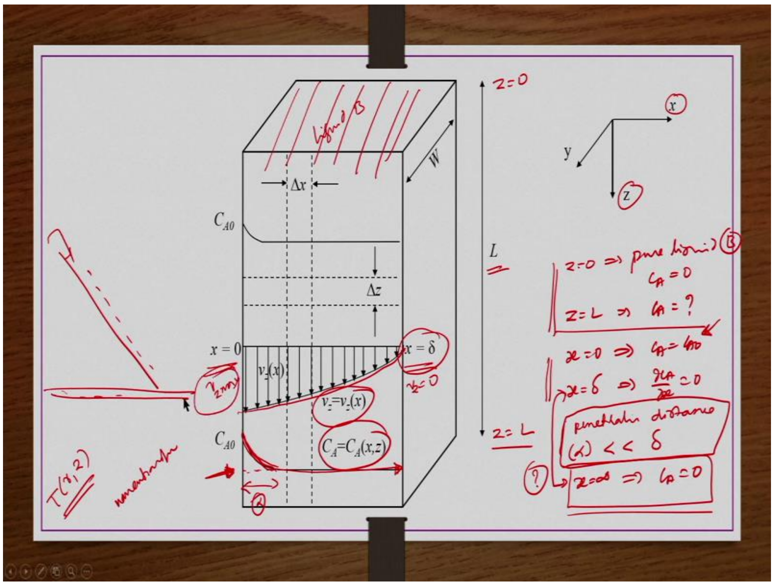

So, now pictorially if you see three-dimensional projection is shown here, so the horizontal axis is x, vertical axis is z right. So, the height of the column is L, width of this column is W through which along which the material is flowing down. So, there is A, this material whatever is there that B liquid is there. So, that is falling like a film here right. And now into this film the diffusion of component A is gas component A is taking place.

So, how do you visualise this location? So, what you have to see; this z = 0 is this top location of the column z = l is the bottom location of the column. Now left-hand side x = 0 we are taking, right-hand side x = δ we are taking. If in 2 d diagram, it is shown here if visualised in 3d diagram, x = δ is nothing but the your screen on which you are studying right.

Or if you are; if you are discussing this problem on a board, x = δ is a board location and then from the top of the board the material is falling, right. x = 0 is location is the outermost layer of the liquid which is exposed to the gas; surrounding gas. So, x = 0 is the surrounding gas.

The same thing if you visualise like this. So, this is the surface let us say this form is a surface. Now from the top the liquid is flowing down like a film, right. The from the top liquid is flowing down like this. So, x = δ location is the palm surface; let us say x = 0 is the film is film thickness is δ. So, that is coming here. So, the tip of the pen whatever is there, so that is x = 0, right.

So, now this pen entire thing whatever is the that is the gas film and then that area let us as you can say. So, x = 0. Here the gas is coming and then diffusing into the liquid film like this, right. So, that is what the problem. So, when this gas film molecules are diffusing into the liquid film how the concentration profile of this gas film whatever the gas component A is there, so that we have to find out.

So, when these molecules of gas A are diffusing into the liquid film B, so how the concentration of this gas A is changing in the thickness of this film; that is from x = 0 to x = δ that you have to find out, right. So, that is one thing. Other thing is that it is mentioned in the problem that this is penetrating only to a smaller distance.

So, only that let us say from this is x = 0, it is penetrating only to small distance only. After that it is not able to penetrate completely to reach the wall surface; that is the limitation is given. So, why that limitation is given when it is only slightly soluble or penetrate to small penetration thickness? You can say that viscosity of this film liquid film is not affected. So, velocity the film thickness velocity distribution within the film will not be affected.

So, this understanding is important, once you have this understanding. Problem solving is very simple. So, now we see the boundary conditions also here, this all these details are given in the subsequent slides also, but from picture if you understand it will be easy. So, $z = 0$ is the location at the top where the pure liquid is there, pure liquid is there and then that is falling as a film.

So, concentration of A should be 0 because pure liquid B is there; if the liquid B is pure. So, then there is no A that that is what it mean by and then at other location $z = L$. What is C_A? That is not given to us. So, let us not worry about it. We will think of it. And then $x = 0$. What we have; that is the free surface that is of the liquid film; free surface of the liquid film which is exposed to the gas. That is the very first layer of the liquid film at which gas is coming and then interacting and then further into penetrating into the film.

So, that should be its initial concentration C_{A0} which is usually taken as the solubility of that particular component A and B. So, we are taking C_{A0} and $x = \delta$. What we have it has been mentioned that this is this component is penetrating to only smaller distance only, A is able to penetrate into the smaller distance only. That is the reason its concentration is varying. Variations in concentration is shown in pictorially only for small distance. After that it is almost remaining same. That means we can see $\frac{\partial c_A}{\partial x}$ is 0, right.

So, now from the you know this penetration distance, this penetration distance let us say if I call it you know some α let us say. So, penetration distance α is very very smaller than the film thickness δ. So, whatever the molecules at the surface are there at the free surface, they can never reach to the this wall of the plane which is at $x = \delta$.

So, when the penetration distance α is very very small compared to the δ, what we can say from the molecules of the gas film viewpoint, this δ location is far away from their positions. So, they cannot go there. It is so far away that we it can say; it can say that x is equals to infinity from the molecules gas molecules from your point of view.

So, at x is equals to infinity, the molecules are not able to come up to this location. They are penetrating only at the smaller distance. So, after that you know a presence of C_A is 0, it is not there. So, C_A should be 0. So, the conversion of this boundary condition like this is very essential in solving this problem, otherwise we were not able to solve this problem.

It is similar like you know previous heat transfer case where the flow through; a non-isothermal flow through cylindrical pipe were taking place, where T was taken as function of both r and z. So, similar kind of situation here also here concentration is function of both x and z.

We cannot say C_A is function of only x, it is function of z also. So, we need to know two boundary conditions in the z direction, two boundary condition in the x direction. So, in the two; in the z direction we have only one boundary condition we can specify other boundary condition at the bottom we cannot specify, we cannot even say $\frac{\partial c_A}{\partial z}$ also 0; we cannot say that one physically.

Other two boundary condition in these directions. So, these are the things, but this boundary condition has been simplified like this. From the molecules of gas, gas molecules A viewpoint we can say that x is equals to infinity, $C_A = 0$ which is same as x is at $x = \delta$ ∂c_A at $x = \delta$ $\frac{\partial c_A}{\partial x}$ is 0 because the penetration distance or penetration depth whatever is there that is very very small compared to the film thickness.

Actually film itself is very very thin. It is a few mm thickness film usually most of the polymer industries we have this kind of problems. Remember in one of the course momentum transfer, we have seen that you know; we have a inclined plate and then the fluid is flowing down. So, then we found the velocity profile etcetera.

So, now here the same situation is there, but only thing that we have taken the vertical plate. And then in addition to the momentum transfer, there is a mass transfer is also occurring. So, it is a combined mass transfer and momentum transfer that is what we are calling forced convective mass transfer, because the flow whatever the velocity distribution is there that is affecting the mass transfer also.

Now, this velocity distribution here we see; at $x = 0$ is the outermost layer of the fluid which is having the maximum velocity. And $x = \delta$ is nothing but the surface of the plane on which the material is flowing. So, because of the no slip condition, the velocity would be 0, v_z would be 0 here and then v_z would be maximum here at $x = 0$. And then, between these two locations the velocity profile changes like this which is function of x. So, that also we have to find out, ok.

So, this understanding is very much essential. So, after that everything is mathematical only. Since here the velocity also needed to be find out. So, first what we do, we simplify the conservation of mass and momentum in order to get the velocity profile.

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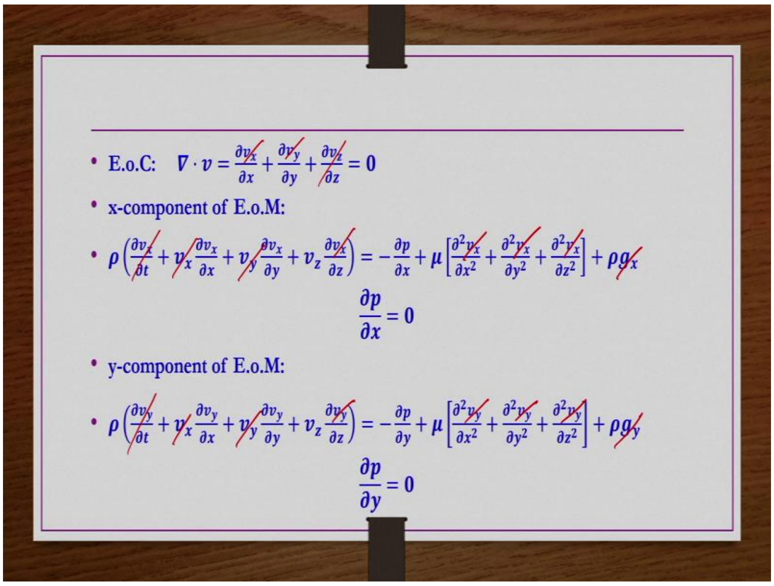

- E.o.C: $\nabla \cdot v = \frac{\partial \cancel{v_x}}{\partial x} + \frac{\partial \cancel{v_y}}{\partial y} + \cancel{\frac{\partial v_z}{\partial z}} = 0$
- x-component of E.o.M:
- $\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + \cancel{v_x}\frac{\partial v_x}{\partial x} + \cancel{v_y}\frac{\partial v_x}{\partial y} + v_z\frac{\partial \cancel{v_x}}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 \cancel{v_x}}{\partial x^2} + \frac{\partial^2 \cancel{v_x}}{\partial y^2} + \frac{\partial^2 \cancel{v_x}}{\partial z^2}\right] + \rho \cancel{g_x}$

$$\frac{\partial p}{\partial x} = 0$$

- y-component of E.o.M:
- $\rho\left(\cancel{\frac{\partial v_y}{\partial t}} + \cancel{v_x}\frac{\partial v_y}{\partial x} + \cancel{v_y}\frac{\partial v_y}{\partial y} + v_z\frac{\partial \cancel{v_y}}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 \cancel{v_y}}{\partial x^2} + \frac{\partial^2 \cancel{v_y}}{\partial y^2} + \frac{\partial^2 \cancel{v_y}}{\partial z^2}\right] + \rho \cancel{g_y}$

$$\frac{\partial p}{\partial y} = 0$$

So, continuity equation in Cartesian coordinates is given here. Only v_z is there and then it is not function of z, it is function of x only. So, this is 0 v by v_x are anyway 0, so then continuity is satisfied; x component of equation of motion is given here. So, here v_x steady state. So, this term is 0, v_x is 0, v_y is 0, v_x is 0, right v_x is 0, so all these three terms are 0. And gravity we are taking only in the z direction as per the schematic, so g_x is 0. So, $\frac{\partial p}{\partial x}$ is 0.

Similarly, y component of equation of motion is given here steady state, this term is 0, v_x is 0, v_y is 0, v_y is 0, v_y is 0. So, all these three terms are 0. Gravity is only in this z direction so this term is also 0. So, we have $\frac{\partial p}{\partial y} = 0$.

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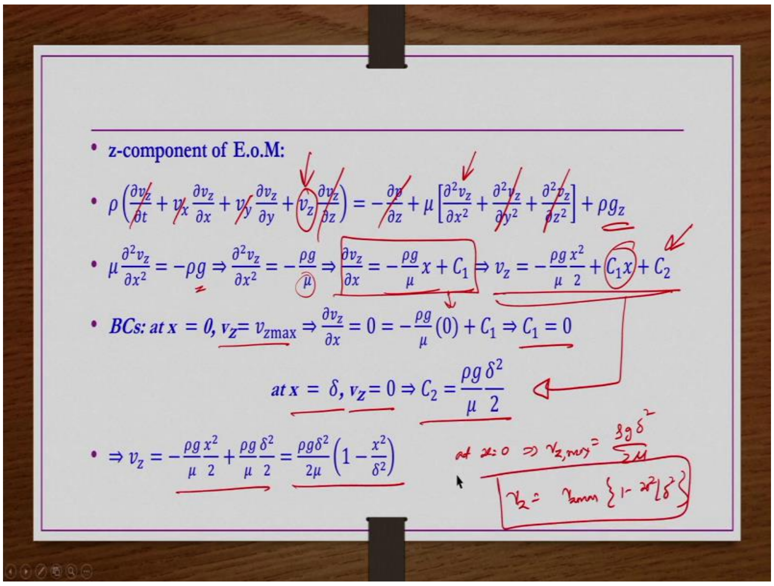

- z-component of E.o.M:
- $\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_x}\frac{\partial v_z}{\partial x} + \cancel{v_y}\frac{\partial v_z}{\partial y} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\cancel{\frac{\partial p}{\partial z}} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$
- $\mu\frac{\partial^2 v_z}{\partial x^2} = -\rho g \Rightarrow \frac{\partial^2 v_z}{\partial x^2} = -\frac{\rho g}{\mu} \Rightarrow \boxed{\frac{\partial v_z}{\partial x} = -\frac{\rho g}{\mu}x + C_1} \Rightarrow v_z = -\frac{\rho g}{\mu}\frac{x^2}{2} + C_1 x + C_2$
- **BCs:** *at x = 0,* $v_z = v_{z\max} \Rightarrow \frac{\partial v_z}{\partial x} = 0 = -\frac{\rho g}{\mu}(0) + C_1 \Rightarrow C_1 = 0$

  at $x = \delta,\ v_z = 0 \Rightarrow C_2 = \frac{\rho g}{\mu}\frac{\delta^2}{2}$
- $\Rightarrow v_z = -\frac{\rho g}{\mu}\frac{x^2}{2} + \frac{\rho g}{\mu}\frac{\delta^2}{2} = \frac{\rho g\delta^2}{2\mu}\left(1 - \frac{x^2}{\delta^2}\right)$

at $x = 0 \Rightarrow v_{z,max} = \frac{\rho g\delta^2}{2\mu}$

$\boxed{v_z = v_{zmax}\{1 - x^2/\delta^2\}}$

Then z component of equation of motion is given here steady state. This term is 0, v_x is 0, v_y is 0, v_z is existing it is function of x but it is not function of z. It is function of x only it is not function of z, so this term is also 0. And then the film whatever the film that is falling, it is falling because of the gravity only or compared to the gravity pressure forces are 0 here, because the film is falling because of the gravity in the z direction, right.

So, v_z is function of x. So, this term would be there, but v_z is not function of y and z. So, these two terms are these are 0. So, then we get $\mu\frac{\partial^2 v_z}{\partial x^2} = -\rho g_z$ and g_z is nothing but g. So, you take the μ also to the right-hand side, then integrate first time.

So, $-\frac{\rho g}{\mu}x + C_1$ left-hand side $\frac{\partial v_z}{\partial x}$. If you integrate again, $v_z = -\frac{\rho g}{\mu}\frac{x^2}{2} + C_1 x + C_2$. Then BCs at x = 0, v_z is maximum, so $\frac{\partial v_z}{\partial x}$ is 0. If $\frac{\partial v_z}{\partial x}$ is 0 at x = 0 from this part of the equation, we get $C_1 = 0$, right. And then at x = δ $v_z = 0$.

So, then v_z is if v_z; from this part of the equation, you get $v_0 = -\frac{\rho g}{\mu}\frac{\delta^2}{2} + C_1\delta + C_2$. C1 is 0. So, $C_2 = \frac{\rho g}{\mu}\frac{\delta^2}{2}$ you get. So, this C_2 you substitute here in this equation C_1 is anyway 0, so this term is gone. So, then you have this thing and then out of these two, if you take $\frac{\rho g}{\mu}\frac{\delta^2}{2}$ common, you get $1 - \frac{x^2}{\delta^2}$.

And then maximum velocity where are we having it x =0, v_z is maximum. So, $v_z = v_{z\,max} = \frac{\rho g}{\mu}\frac{\delta^2}{2}$ because x is 0 here. So, this is m equation we can write $v_z = v_{z\,max}$ multiplied by $1 - \frac{x^2}{\delta^2}$ we can write.

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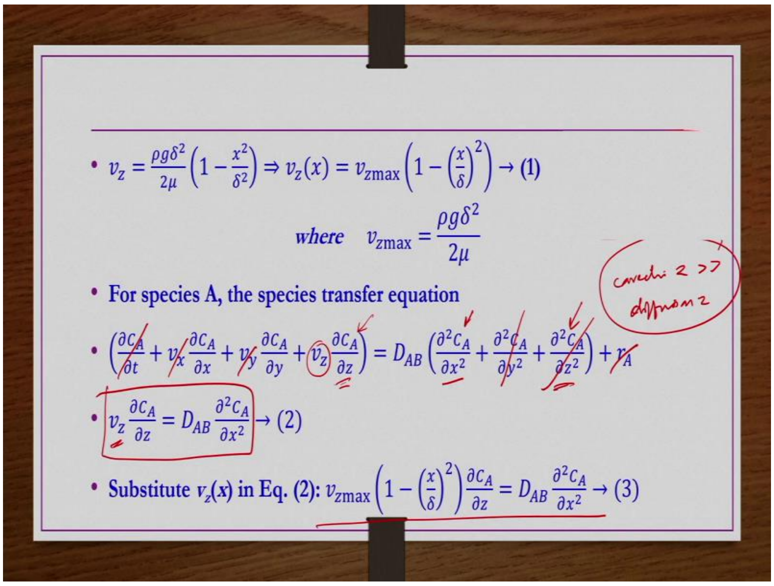

- $v_z = \frac{\rho g \delta^2}{2\mu}\left(1 - \frac{x^2}{\delta^2}\right) \Rightarrow v_z(x) = v_{zmax}\left(1 - \left(\frac{x}{\delta}\right)^2\right) \rightarrow (1)$

$$\textit{where} \quad v_{zmax} = \frac{\rho g \delta^2}{2\mu}$$

- For species A, the species transfer equation

convectn z >>
diffusn z

- $\left(\cancel{\frac{\partial C_A}{\partial t}} + \cancel{v_x}\frac{\partial C_A}{\partial x} + \cancel{v_y}\frac{\partial C_A}{\partial y} + v_z\frac{\partial C_A}{\partial z}\right) = D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \cancel{\frac{\partial^2 C_A}{\partial y^2}} + \cancel{\frac{\partial^2 C_A}{\partial z^2}}\right) + \cancel{r_A}$
- $\boxed{v_z\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}} \rightarrow (2)$
- Substitute $v_z(x)$ in Eq. (2): $v_{zmax}\left(1 - \left(\frac{x}{\delta}\right)^2\right)\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2} \rightarrow (3)$

So, that is given here. Now a velocity distribution you got it. Now what we have to do? You have to find out the concentration profile for (Refer Time: 43:54). The species transfer equation is this one like heat transfer equation we have like momentum transfer equation, we have the species transfer equation is provided by this one. We can find out any books, we can derive this one also.

So, the steady state this term is 0, v_x is not existing, v_y is not existing, v_z is existing and then C is concentration of which is function of z and then x also. So, this you can you cannot cancel out. Now D_{AB} this C_A is function of x. So, this term should be there, but it is not function of y, so it is 0. C is function of z also, but what we have the convection in z direction is much higher compared to the diffusion in z direction.

So, pictorially what we have the diffusion is taking place in the x direction. So, diffusion is dominating in the x direction convection, the flow is taking place in the z direction. So, convection terms are dominating in the z direction, diffusive terms are dominating in the x direction. So, diffusive terms whatever in the z direction is there, so that would be small compared to either of these two terms that is diffusion in x direction and then convection in z direction. So, accordingly this term can be cancelled out. There is no reaction finally. So, what we have is, we have $v_z \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$.

This v_z is what; $v_{z\,max} \left(1 - \left(\frac{x}{\delta}\right)^2\right) \frac{\partial C_A}{\partial z} = D_{AB} \frac{\partial^2 C_A}{\partial x^2}$, fine.

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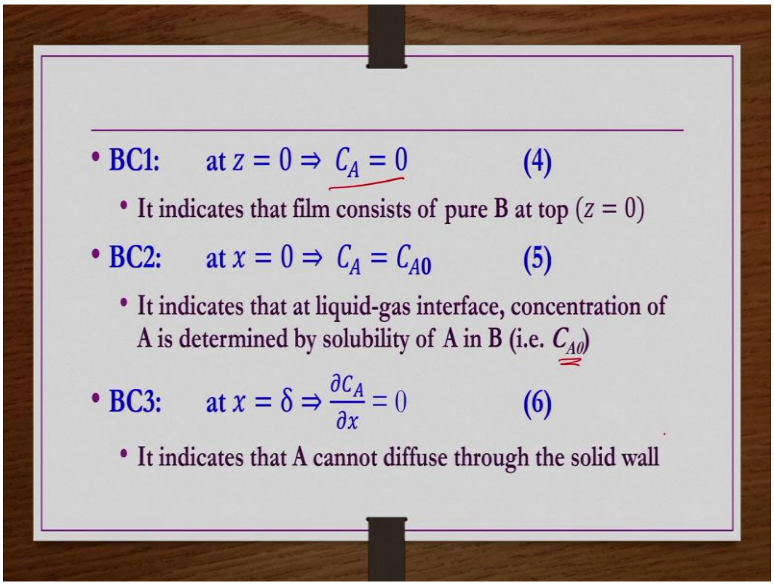

- **BC1:** at $z = 0 \Rightarrow C_A = 0$ (4)
  - It indicates that film consists of pure B at top ($z = 0$)
- **BC2:** at $x = 0 \Rightarrow C_A = C_{A0}$ (5)
  - It indicates that at liquid-gas interface, concentration of A is determined by solubility of A in B (i.e. C_{A0})
- **BC3:** at $x = \delta \Rightarrow \frac{\partial C_A}{\partial x} = 0$ (6)
  - It indicates that A cannot diffuse through the solid wall

Now, boundary conditions we have already seen. At z = 0 at the top of the column pure liquid B is there. So, gas A concentration is 0. It indicates that the film consists of pure B at tau that is z = 0 location. And x = 0, that is the free surface indicates that the liquid gas interface and then, concentration of A is determined by the solubility of A in B.

That is C_{A0} maximum concentration of A, that is possible at the interface that is C_{A0} right. And then at x = δ $\frac{\partial C_A}{\partial x} = 0$. This is also we have seen. That indicates that A cannot diffuse through the solid wall.

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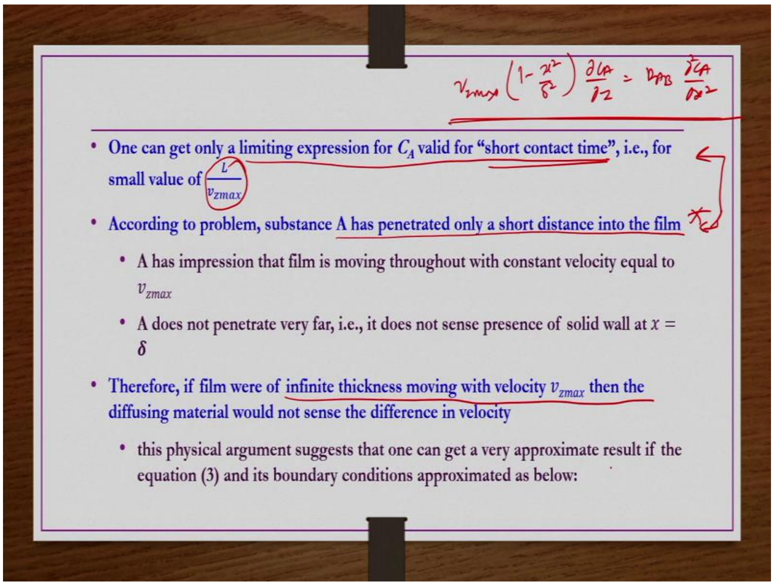

$$v_{zmax}\left(1-\frac{x^2}{\delta^2}\right)\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

- One can get only a limiting expression for C_A valid for "short contact time", i.e., for small value of $\frac{L}{v_{zmax}}$
- According to problem, substance A has penetrated only a short distance into the film
    - A has impression that film is moving throughout with constant velocity equal to v_{zmax}
    - A does not penetrate very far, i.e., it does not sense presence of solid wall at $x = \delta$
- Therefore, if film were of infinite thickness moving with velocity v_{zmax} then the diffusing material would not sense the difference in velocity
    - this physical argument suggests that one can get a very approximate result if the equation (3) and its boundary conditions approximated as below:

Now, this whatever the equation we had $1-\left(\frac{x}{\delta}\right)^2$, of course $v_{z\,max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$. So, this equation if you wanted to solve, you have to have certain limitations. Like in a heat transfer case, non-isothermal flow in pipe.

What we have taken? We have taken asymptotic approximation. So, like that you know some kind of limitations is required. So, then in this case one can get only a limiting expression for C_A valid for short contact time. That is when $\frac{L}{v_{z\,max}}$ is very small, then only one can get the solution easily, right.

So, why short contact time? Because within that small time only C_A; the A is able to penetrate to smaller distance into the liquid film. Only a smaller distance that is the reason. So, accordingly substance A has penetrated only a short distance into the film. So, this is the given statements, both of them are now interconnected right.

So, A has impression that film is moving throughout with A constant velocity equal to v_z. Now, here also $v_{z\,max}$ because that you know it is only able to penetrate. Now this is the entire film, but the gas molecules are here. So, the molecules are penetrating only to small distance into the liquid film; that gas molecules are in an impression that the film is moving with a constant velocity of maximum velocity that is possible at the gas liquid interface, because it is the penetrating only to a smaller distance A.

It is this gas molecules are not aware of the surface of the wall which they assume that they are the surface of the wall it is at infinite distance from their location of x = 0. This is with respect to the gas molecules viewpoint especially they are penetrating to a small distance.

So, A does not penetrate very far that it does not sense presence of solid wall at x = δ. Therefore, a film were of infinite thickness moving with velocity $v_{z\,max}$, then the diffusing material would not sense the difference in the velocity. And then this physical argument suggests that one can get a very approximate result. If the equation 3, this whatever and then boundary condition whatever you mentioned as approximated like this.

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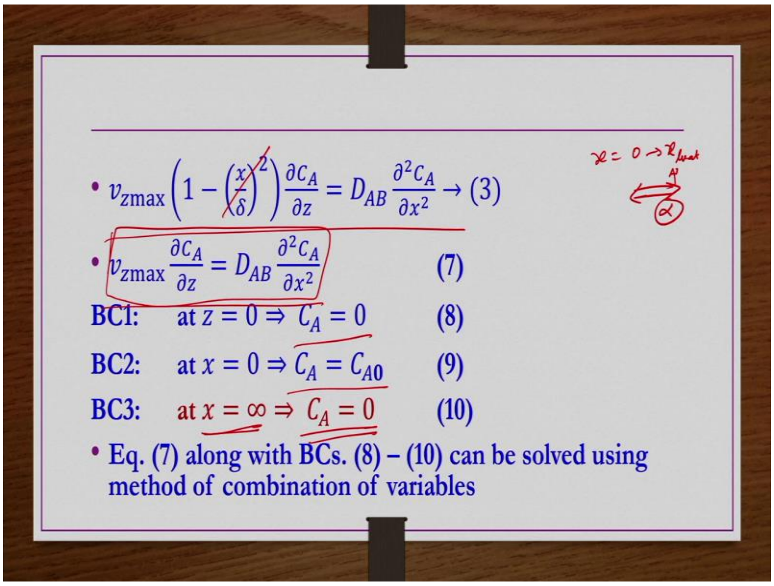

- $v_{z\max}\left(1-\cancel{\left(\frac{x}{\delta}\right)^2}\right)\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2} \rightarrow (3)$
- $\boxed{v_{z\max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}}$ (7)

BC1: at $z = 0 \Rightarrow C_A = 0$ (8)

BC2: at $x = 0 \Rightarrow C_A = C_{A0}$ (9)

BC3: at $x = \infty \Rightarrow C_A = 0$ (10)

- Eq. (7) along with BCs. (8) – (10) can be solved using method of combination of variables

$x = 0 \to x_{local}$

A?

α

So, this equation now x; $\frac{x}{\delta}$ is almost like 0, $\frac{x}{\delta}$ x is the distance from the view molecules point. From the gas molecules point of view, x = 0 to certain x value is there that is the penetration depth right from x = 0 to some x location which is these differences, it is the α penetration depth we are calling.

So, this α is very very small compared to the δ. So, $\frac{x}{\delta}$ is almost like you know 0. So, then this we can take off. So, that left-hand side we have $v_{z\,max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$. This is about the governing; simplified governing equations after couple of assumptions which are physically reliable of course.

Then boundary conditions at z = 0, $C_A = 0$ that will be as it is. At x = 0, $C_A = C_{A0}$ that would also be as it is. But at x = δ location is almost like infinite location very far for the gas molecules, because they are penetrating only smaller distance into the film and they are not able to sense the presence of wall.

So, they are thinking that those molecules are; thinking that x = δ is virtually x is equals to infinity and they cannot go up to that location. If they cannot go up to that location that their concentration at that location is 0, $C_A = 0$. So, this also pictorially we have explained.

So, now here still the problem is you know very difficult to solve because now C_A is function of both z and x, right. So, we cannot say that one side is independent of the other side like we say like we did in order to get the you know shear stress expressions etcetera here. So, that is not possible. So, for that what we do?

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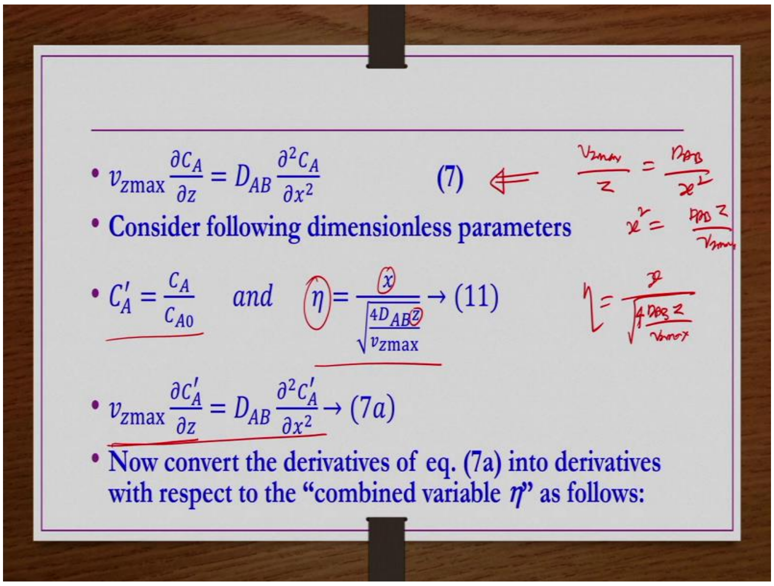

- $v_{z\max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$ (7) $\Leftarrow$ $\frac{v_{z\max}}{z} = \frac{D_{AB}}{x^2}$
- Consider following dimensionless parameters $x^2 = \frac{D_{AB}z}{v_{z\max}}$
- $C_A' = \frac{C_A}{C_{A0}}$ and $\eta = \frac{x}{\sqrt{\frac{4D_{AB}z}{v_{z\max}}}} \rightarrow (11)$ $\eta = \frac{x}{\sqrt{\frac{4D_{AB}z}{v_{z\max}}}}$
- $v_{z\max}\frac{\partial C_A'}{\partial z} = D_{AB}\frac{\partial^2 C_A'}{\partial x^2} \rightarrow (7a)$
- Now convert the derivatives of eq. (7a) into derivatives with respect to the "combined variable η" as follows:

We do combined variable approach. For that, we are taking dimensionless parameters $\frac{C_A}{C_{A0}}$ as C_A' and then, this one what do you can write from here? So, this dimensionless parameter which is you know having the both x and z together, that as a combined variable we can call this η; we can call combined variable now, ok.

So, how you get this expression particularly? Let us say from this equation what we can write $\frac{v_{z\,max}}{z} = \frac{D_{AB}}{x^2}$. So, if I write this one $x^2 = \frac{D_{AB}z}{v_{z\,max}}$. So, if I take square root of both sides, so $\frac{x}{\sqrt{\frac{D_{AB}z}{v_{z\,max}}}}$, this kind of form we can have. This four we are getting because of the experience of solving the problem, we can have this constant. So, this we are writing as η. This is how we can decide how this you know combined variable should be defined this is how we can do it, ok.

So, now this equation $v_{z\,max}$. In place of $\frac{\partial C_A}{\partial z}$, I can write C_{A0} $\frac{\partial C_A'}{\partial z}$. And then right-hand side in place of $\frac{\partial^2 C_A}{\partial x^2}$, I can write C_{A0} $\frac{\partial^2 C_A'}{\partial x^2}$. So, the C_{A0} C_{A0} is cancelled out and then this is what we are having.

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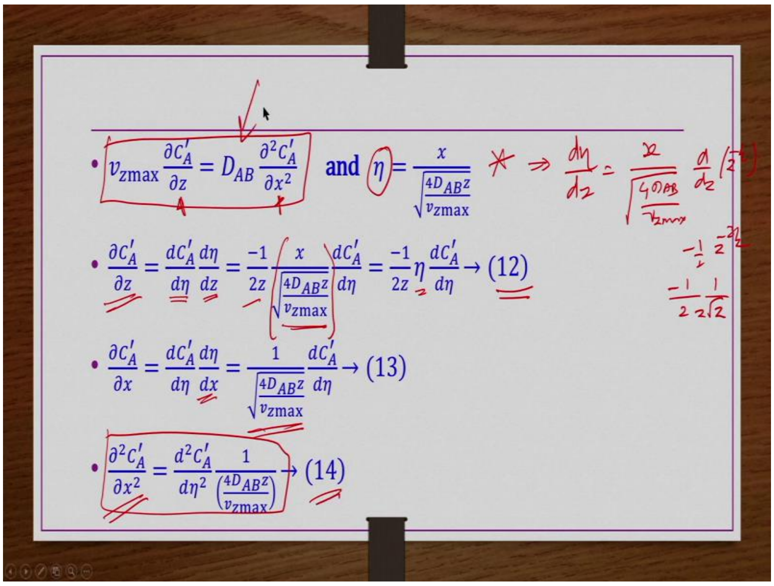

- $\boxed{v_{zmax}\frac{\partial C_A'}{\partial z} = D_{AB}\frac{\partial^2 C_A'}{\partial x^2}}$ and $\eta = \frac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}$ $* \Rightarrow \frac{d\eta}{dz} = \frac{x}{\sqrt{\frac{4D_{AB}}{v_{zmax}}}}\frac{d}{dz}\left(z^{-1/2}\right)$
- $\frac{\partial C_A'}{\partial z} = \frac{dC_A'}{d\eta}\frac{d\eta}{dz} = \frac{-1}{2z}\left(\frac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}\right)\frac{dC_A'}{d\eta} = \frac{-1}{2z}\eta\frac{dC_A'}{d\eta} \rightarrow (12)$
- $\frac{\partial C_A'}{\partial x} = \frac{dC_A'}{d\eta}\frac{d\eta}{dx} = \frac{1}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}\frac{dC_A'}{d\eta} \rightarrow (13)$
- $\boxed{\frac{\partial^2 C_A'}{\partial x^2} = \frac{d^2 C_A'}{d\eta^2}\frac{1}{\left(\frac{4D_{AB}z}{v_{zmax}}\right)}} \rightarrow (14)$

$-\frac{1}{2}z^{-3/2}$

$\frac{-1}{2}\frac{1}{z\sqrt{z}}$

Now, this equation we; this equation now here it is function of z and then x both. So, now we convert this as a function of one combined variable η using this definition of a dimensionless parameter. So, how we do? So, let us say $\frac{\partial C_A'}{\partial z} = \frac{\partial C_A'}{\partial \eta}$ multiplied by $\frac{\partial \eta}{\partial z}$. Since now the C_A' we are writing as a function of only one variable in place of ∂, I am writing d directly ok.

So, now this expression from here what you get $\frac{d\eta}{dz}$ is nothing but $\frac{x}{\sqrt{\frac{4\,D_{AB}z}{v_{z\,max}}}}$ and then $\frac{d}{dz}z^{-1/2}$. So, that is $-1/2\ z^{-3/2}$. So, $z^{-3/2}$. What I am writing $-1/2$ and then, $\frac{1}{z\sqrt{z}}$ I am writing and then $\sqrt{z}$ I am combining with these remaining terms. So, that I can write $\frac{4\,D_{AB}z}{v_{z\,max}}$ and then $\frac{-1}{2z}$ as it is. This $\frac{\partial C_A'}{\partial \eta}$ as it is.

So, now this again what is this? This is nothing but our η as per the definition of combined variables. So, this is what we are having. Similarly, $\frac{\partial C_A'}{\partial x}$ if you do that is $\frac{\partial C_A'}{\partial \eta}\frac{d\eta}{dx}.\frac{d\eta}{dx}$ is nothing but $\frac{1}{\sqrt{\frac{4\,D_{AB}z}{v_{z\,max}}}}$, right.

So, now when you are doing the differentiation of η with respect to x z has to be treated as constant. Similarly, when you are doing the differentiation of η with respect to z x has to be treated as constant; like in partial derivatives, then this is we have. So, once again if you differentiate with respect to x, this equation you get this form.

So, now we have $\frac{\partial C_A'}{\partial z}$ and then, $\frac{\partial^2 C_A'}{\partial x^2}$ from equation number 12 and 14, this we substitute here in this equation.

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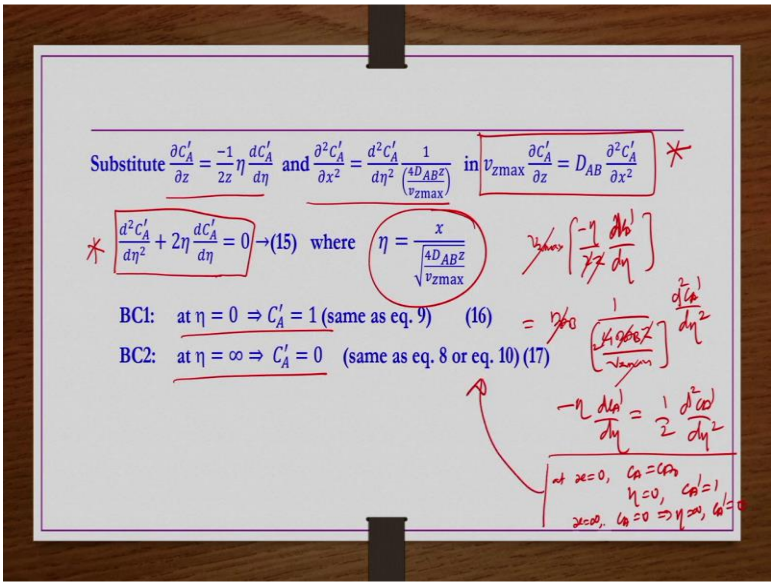

Substitute $\frac{\partial C_A'}{\partial z} = \frac{-1}{2z}\eta\frac{dC_A'}{d\eta}$ and $\frac{\partial^2 C_A'}{\partial x^2} = \frac{d^2 C_A'}{d\eta^2}\frac{1}{\left(\frac{4D_{AB}z}{v_{zmax}}\right)}$ in $\boxed{v_{zmax}\frac{\partial C_A'}{\partial z} = D_{AB}\frac{\partial^2 C_A'}{\partial x^2}}$ *

* $\boxed{\frac{d^2 C_A'}{d\eta^2} + 2\eta\frac{dC_A'}{d\eta} = 0}$ →(15) where $\eta = \frac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}$

BC1: at η = 0 ⇒ $C_A' = 1$ (same as eq. 9) (16)

BC2: at η = ∞ ⇒ $C_A' = 0$ (same as eq. 8 or eq. 10) (17)

$\cancel{v_{zmax}}\left[\frac{-\eta}{\cancel{2z}}\frac{dC_A'}{d\eta}\right] = \cancel{D_{AB}}\frac{1}{\left[\frac{4\cancel{D_{AB}}z}{v_{zmax}}\right]}\frac{d^2C_A'}{d\eta^2}$

$-\eta\frac{dC_A'}{d\eta} = \frac{1}{2}\frac{d^2C_A'}{d\eta^2}$

at $x=0$, $C_A = C_{A0}$
$\eta = 0$, $C_A' = 1$
$x = \infty$, $C_A = 0 \Rightarrow \eta = \infty$, $C_A' = 0$

So, that is $\frac{\partial C_A'}{\partial z}$ and $\frac{\partial^2 C_A'}{\partial x^2}$. We are substituting in this equation. Then what we have; $v_{z\,max}$, we can do this one.

$\frac{\partial C'_A}{\partial z}$ is $-\frac{\eta}{2z}\frac{\partial C'_A}{\partial \eta} = D_{AB}$ and then this part is $\frac{1}{\frac{4\,D_{AB}z}{v_{z\,max}}}\frac{d^2 C'_A}{d\eta^2}$. So, now this D_{AB}, this D_{AB} is cancelled out left-hand side $\frac{1}{z}$, right-hand side $\frac{1}{z}$ cancelled out left-hand side $v_{z\,max}$ right-hand side $v_{z\,max}$ cancelled out.

So, what we have? We have $-\frac{\eta}{2}\frac{dC'_A}{d\eta}$ and then these two also we can take because 2 times one is 2; 2 times 2. So, we have $1/2\frac{d^2C'_A}{d\eta^2}$. So, that is these two you take to the left-hand side and then you add them together by taking $\frac{d^2C'_A}{d\eta^2} + 2\eta\frac{dC'_A}{d\eta} = 0$. So, now this equation which is function of both x and z has been converted into the function of one single variable η, right. So, this is what we have.

So, coming to the boundary conditions. At x = 0, what we have $C_A = C_{A0}$. If x = 0, that means from this definition of combined variable η is 0 and then η is 0 that means, $\frac{C_A}{C_{A0}} = 1$ $\frac{C_A}{C_{A0}}$ is nothing but C'_A. So, that is equals to 1.

And then other equation at x is equals to infinity that is $C_A = 0$ that we understand. So, if x is equals to infinity, that means η is equals to infinity and then if C_A is 0, C'_A is also 0 because C'_A is nothing but $\frac{C_A}{C_{A0}}$. So, these are the two boundary condition in terms of η which are getting from these two expressions.

So, now this equation number 15 we have to solve using to this boundary condition given the equation number 16 and 17.

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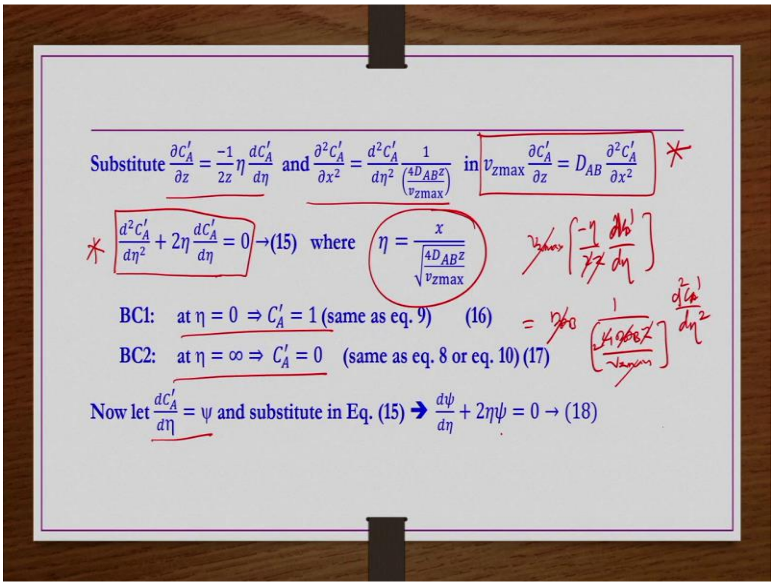

Substitute $\frac{\partial C_A'}{\partial z} = \frac{-1}{2z}\eta\frac{dC_A'}{d\eta}$ and $\frac{\partial^2 C_A'}{\partial x^2} = \frac{d^2 C_A'}{d\eta^2}\frac{1}{\left(\frac{4D_{AB}z}{v_{zmax}}\right)}$ in $\boxed{v_{zmax}\frac{\partial C_A'}{\partial z} = D_{AB}\frac{\partial^2 C_A'}{\partial x^2}}$ *

* $\boxed{\frac{d^2 C_A'}{d\eta^2} + 2\eta\frac{dC_A'}{d\eta} = 0}$ →(15) where $\eta = \frac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}$

$\cancel{v_{zmax}}\left[\frac{-\eta}{\cancel{2z}}\frac{dC_A'}{d\eta}\right] = \cancel{D_{AB}}\frac{1}{\left[\frac{\cancel{4D_{AB}z}}{v_{zmax}}\right]}\frac{d^2C_A'}{d\eta^2}$

BC1: at η = 0 ⇒ $C_A' = 1$ (same as eq. 9) (16)

BC2: at η = ∞ ⇒ $C_A' = 0$ (same as eq. 8 or eq. 10) (17)

Now let $\frac{dC_A'}{d\eta} = \psi$ and substitute in Eq. (15) ➔ $\frac{d\psi}{d\eta} + 2\eta\psi = 0 \rightarrow (18)$

So, let us say you take $\frac{\partial C_A'}{\partial \eta} = \psi$, then we have$\frac{d\psi}{d\eta} + 2\eta\psi = 0$, right.

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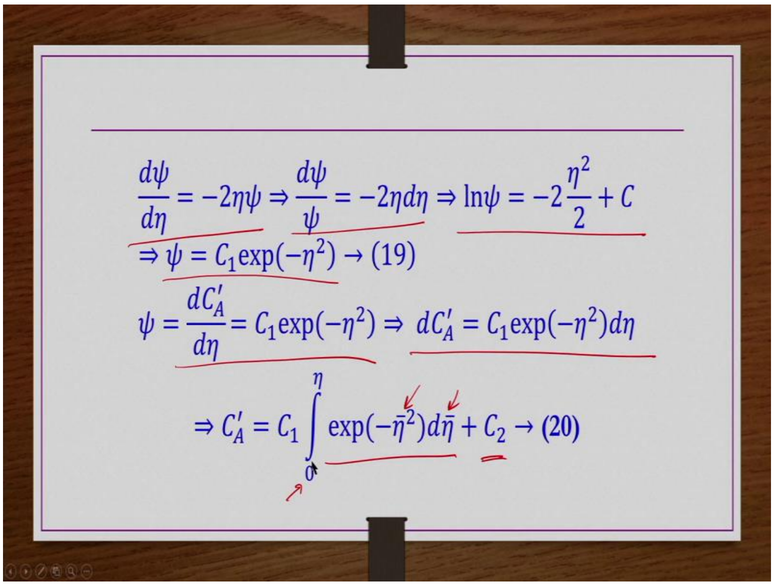

$$\frac{d\psi}{d\eta} = -2\eta\psi \Rightarrow \frac{d\psi}{\psi} = -2\eta d\eta \Rightarrow \ln\psi = -2\frac{\eta^2}{2} + C$$

$$\Rightarrow \psi = C_1\exp(-\eta^2) \rightarrow (19)$$

$$\psi = \frac{dC_A'}{d\eta} = C_1\exp(-\eta^2) \Rightarrow dC_A' = C_1\exp(-\eta^2)d\eta$$

$$\Rightarrow C_A' = C_1\int_0^\eta \exp(-\bar{\eta}^2)d\bar{\eta} + C_2 \rightarrow (20)$$

So, the same equation you have written like this by taking this to the right-hand side, then you are taking η terms ψ terms separately, two different sets. So, then integrating, so you get this one. Now, after removing the logarithmic you get this one. Now ψ is nothing but $\frac{\partial C_A'}{\partial \eta}$ that is $C_1 exp(-\eta^2)$. So, $dC_A' = C_1 exp(-\eta^2)d\eta$ that is $C_A' = C_1\int_0^\eta exp(-\bar{\eta}^2)d\bar{\eta} + C_2$. We are writing in a in terms of dummy variable same like that and then integration from 0 to η, right. So, this is what we have. We are not evaluating these constants directly, but we apply and then see what we get.

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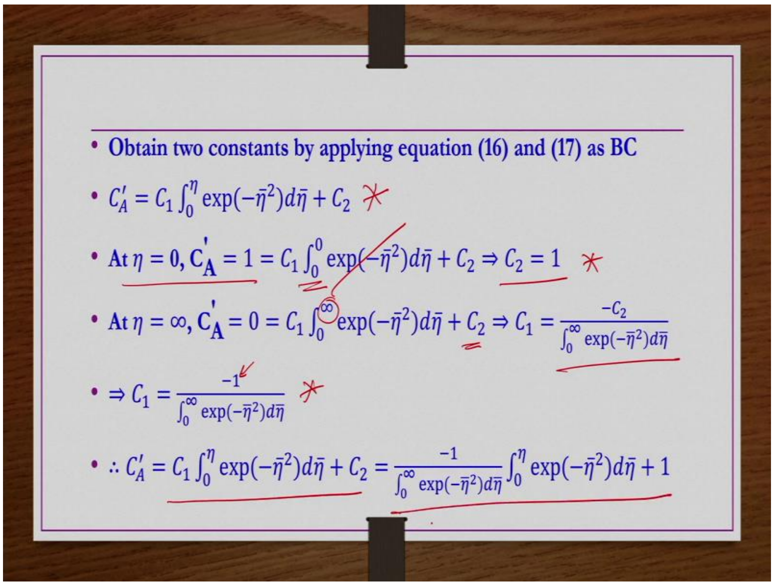

- Obtain two constants by applying equation (16) and (17) as BC
- $C_A' = C_1 \int_0^{\eta} \exp(-\bar{\eta}^2) d\bar{\eta} + C_2$ *
- At $\eta = 0, C_A' = 1 = C_1 \int_0^{0} \exp(-\bar{\eta}^2) d\bar{\eta} + C_2 \Rightarrow C_2 = 1$ *
- At $\eta = \infty, C_A' = 0 = C_1 \int_0^{\infty} \exp(-\bar{\eta}^2) d\bar{\eta} + C_2 \Rightarrow C_1 = \frac{-C_2}{\int_0^{\infty} \exp(-\bar{\eta}^2) d\bar{\eta}}$
- $\Rightarrow C_1 = \frac{-1}{\int_0^{\infty} \exp(-\bar{\eta}^2) d\bar{\eta}}$ *
- $\therefore C_A' = C_1 \int_0^{\eta} \exp(-\bar{\eta}^2) d\bar{\eta} + C_2 = \frac{-1}{\int_0^{\infty} \exp(-\bar{\eta}^2) d\bar{\eta}} \int_0^{\eta} \exp(-\bar{\eta}^2) d\bar{\eta} + 1$

So, in the first bon this is the equation. First boundary condition is that $\eta = 0$ $C_A' = 1$. So, 1 = $C_1 \int_0^0 exp(-\bar{\eta}^2) d\bar{\eta} + C_2$ is there. Whatever the integration when you substitute, both limits are same so then that will be 0. So, then $C_2 = 1$, right

Other one is at η is equals to infinity $C_A' = 0$. So, that is $C_1 \int_0^{\infty} exp(-\bar{\eta}^2) d\bar{\eta} + C_2$. C_2 is nothing but 1, it is given. So, C_1 you can get $-C_2$ by this one and then C_2 is nothing but 1. So, this is C_1 this is C_2 if you substitute these two in this equation, you get the expression for C_A' as this one.

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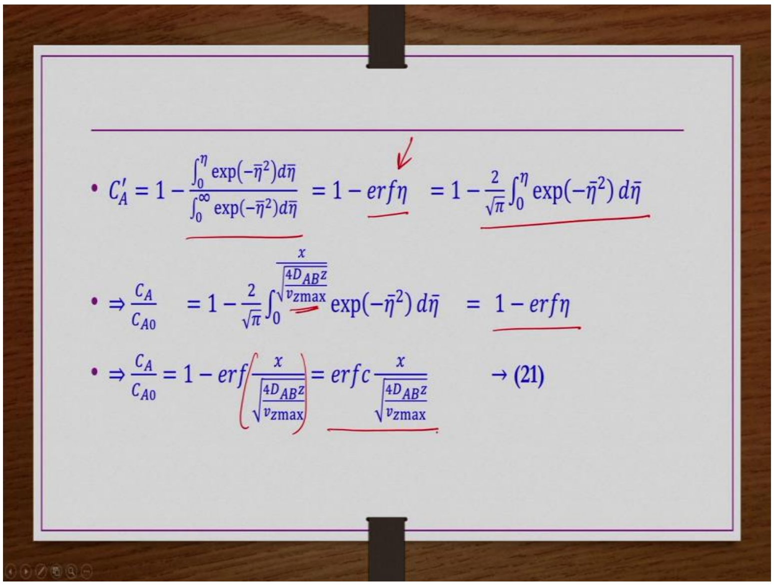

- $C_A' = 1 - \dfrac{\int_0^{\eta} \exp(-\bar{\eta}^2)d\bar{\eta}}{\int_0^{\infty} \exp(-\bar{\eta}^2)d\bar{\eta}} = 1 - erf\eta = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} \exp(-\bar{\eta}^2)\, d\bar{\eta}$
- $\Rightarrow \dfrac{C_A}{C_{A0}} = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}} \exp(-\bar{\eta}^2)\, d\bar{\eta} = 1 - erf\eta$
- $\Rightarrow \dfrac{C_A}{C_{A0}} = 1 - erf\left(\dfrac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}\right) = erfc\dfrac{x}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}} \rightarrow (21)$

This we can write $C_A' = 1 -$ of; 1 minus of this one and this is nothing but defined as error function of η. This is by the definition of error functions which is given by this expression, ok. Then the C_A' is nothing but $\frac{C_A}{C_{A0}} = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} exp(-\bar{\eta}^2)d\bar{\eta} = 1 - erf\eta$.

So, this whatever is there, so this entire thing we can write this error function of η. That is $1 - erf\left(\frac{x}{\sqrt{\frac{4\,D_{AB}z}{v_{z\,max}}}}\right)$ 1 minus error function of x is nothing but complimentary error function of x. So, that is given as $erfc\frac{x}{\sqrt{\frac{4\,D_{AB}z}{v_{z\,max}}}}$, ok.

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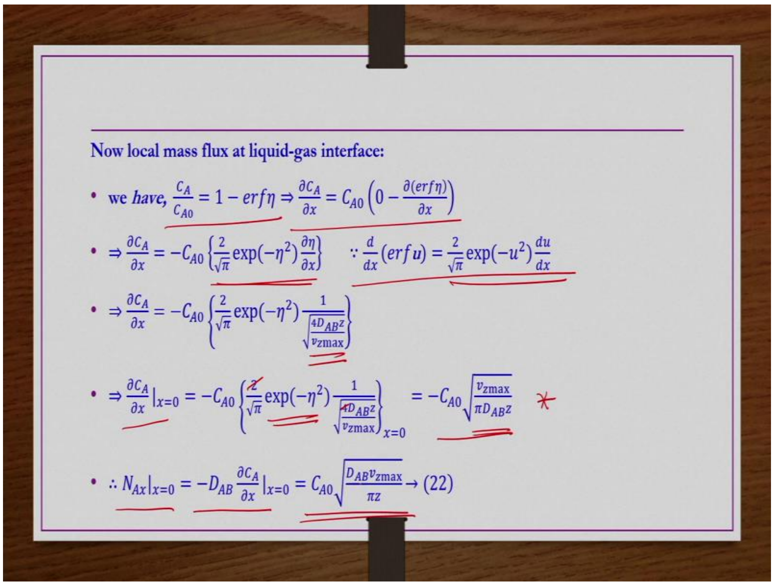

Now local mass flux at liquid-gas interface:

- we *have*, $\frac{C_A}{C_{A0}} = 1 - erf\eta \Rightarrow \frac{\partial C_A}{\partial x} = C_{A0}\left(0 - \frac{\partial(erf\eta)}{\partial x}\right)$
- $\Rightarrow \frac{\partial C_A}{\partial x} = -C_{A0}\left\{\frac{2}{\sqrt{\pi}}\exp(-\eta^2)\frac{\partial\eta}{\partial x}\right\}$ $\quad\because \frac{d}{dx}(erf\,u) = \frac{2}{\sqrt{\pi}}\exp(-u^2)\frac{du}{dx}$
- $\Rightarrow \frac{\partial C_A}{\partial x} = -C_{A0}\left\{\frac{2}{\sqrt{\pi}}\exp(-\eta^2)\frac{1}{\sqrt{\frac{4D_{AB}z}{v_{zmax}}}}\right\}$
- $\Rightarrow \frac{\partial C_A}{\partial x}\Big|_{x=0} = -C_{A0}\left\{\frac{\cancel{2}}{\sqrt{\pi}}\exp(-\eta^2)\frac{1}{\sqrt{\frac{\cancel{4}D_{AB}z}{v_{zmax}}}}\right\}_{x=0} = -C_{A0}\sqrt{\frac{v_{zmax}}{\pi D_{AB}z}}$ *
- $\therefore N_{Ax}|_{x=0} = -D_{AB}\frac{\partial C_A}{\partial x}\Big|_{x=0} = C_{A0}\sqrt{\frac{D_{AB}v_{zmax}}{\pi z}} \rightarrow (22)$

So, that is the concentration profile. If you wanted to know the flux, you have to find out the $\frac{\partial C_A}{\partial x}$. So, $\frac{C_A}{C_{A0}}$ is this one. So, $\frac{\partial C_A}{\partial 0}$ is nothing but this one. And then from the definitions $\frac{d}{dx}$ of error function of u is equals to this one, that if you apply, you get for our case this one, ok.

Now, here you substitute whatever $\frac{d\eta}{dx}$ is nothing but $\frac{1}{\sqrt{\frac{4\,D_{AB}z}{v_{z\,max}}}}$, right. So, $\frac{dC_A}{dx}$ at x = 0. That means, here if you substitute x = 0, what happens? η is 0 if x = 0 that means η is 0. So, exponential of 0 is 1. So, the these remaining terms would be there. The $\sqrt{4}$ is nothing but 2. So, that is cancelled out right and then, this π also we are taking into the square root of term. So, $-C_{A0}\sqrt{\frac{v_{z\,max}}{\pi D_{AB}z}}$ that is what we get.

So, this N_{Ax} that x = 0 is nothing but $-D_{AB}\frac{\partial C_A}{\partial x}$ at x = 0. So, that $\frac{\partial C_A}{\partial x}$ at x = 0 is nothing but minus of this one. So, + of this one we are having. And then multiplied by D_{AB} was there. So, $\frac{D_{AB}}{\sqrt{D_{AB}}}$, we have $\sqrt{D_{AB}}$. So, that is $\sqrt{\frac{D_{AB}v_{z\,max}}{\pi z}}$ we are having.

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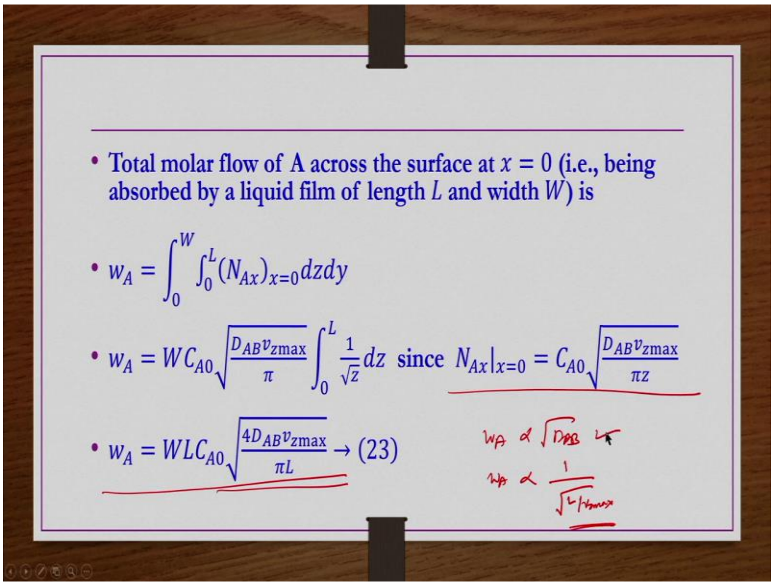

- Total molar flow of A across the surface at $x = 0$ (i.e., being absorbed by a liquid film of length L and width W) is
- $w_A = \int_0^W \int_0^L (N_{Ax})_{x=0} dzdy$
- $w_A = WC_{A0}\sqrt{\frac{D_{AB}v_{z\max}}{\pi}} \int_0^L \frac{1}{\sqrt{z}} dz$ since $N_{Ax}|_{x=0} = C_{A0}\sqrt{\frac{D_{AB}v_{z\max}}{\pi z}}$
- $w_A = WLC_{A0}\sqrt{\frac{4D_{AB}v_{z\max}}{\pi L}} \rightarrow (23)$

$w_A \propto \sqrt{D_{AB}}$

$w_A \propto \frac{1}{\sqrt{L/v_{z\max}}}$

Now, mass rate if you wanted to find out you have to multiply, you have to get $W_A = \int_0^W \int_0^L (N_{Ax})_{x=0} dzdy$. So, that is $WC_{A0}\sqrt{\frac{D_{AB}v_{z\,max}}{\pi z}} \int_0^L \frac{1}{\sqrt{z}} dz$ because N_{Ax} at x = 0 is nothing but this one.

Now, integration of $\frac{1}{\sqrt{z}} dz$ if you do you get simplified expression, this one here after substituting limits here. So, now what we understand? W_A is proportional to the square root of diffusivity and then W_A is inversely proportional to the $\frac{1}{\sqrt{L/v_{z\,max}}}$. $L/v_{z\,max}$ is nothing but the time contact time; inversely proportional to the contact time, but directly proportional to the diffusivity. So, this is how we have to solve the convective mass transfer problems.

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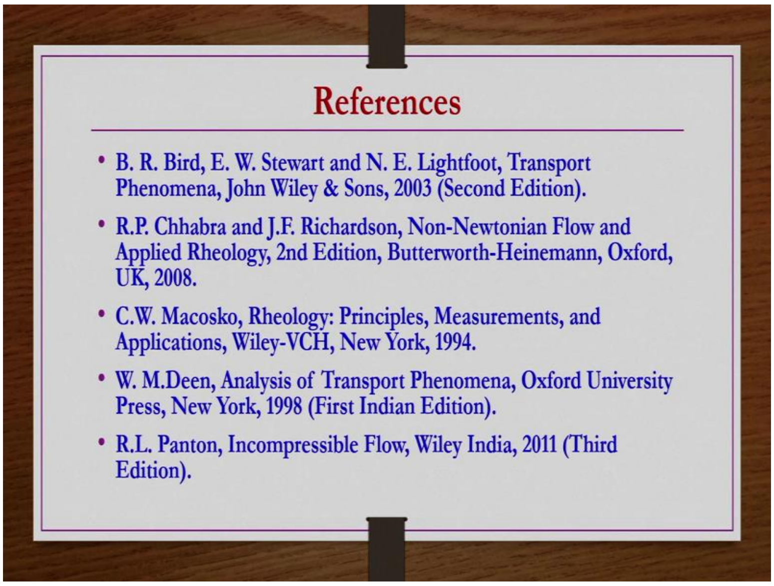

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The references for this lectures are provided here.

Thank you.