Transport Phenomena of Non-Newtonian Fluids
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Lecture - 30
Transpiration Cooling

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of today’s lecture is Transpiration Cooling. We have been discussing several problems associated with the heat transfer with or without reactions, right. So, what is today’s lecture is all about is transpiration cooling.

If we have a system of transpiration cooling say then how to get the temperature distribution in that particular system and then how to get you know the heat removal that is that has been affected because of the particular system that we have taken. This transpiration cooling systems they may be spherical in nature or cylindrical in nature.

And the size of this transpiration cooling systems are very small in general very very small few mm size and then often they are used in a nuclear reactors etcetera as a kind of applications. So, that to avoid or remove the heat that is being evolved because of the associated nature of the radioactive materials, ok.

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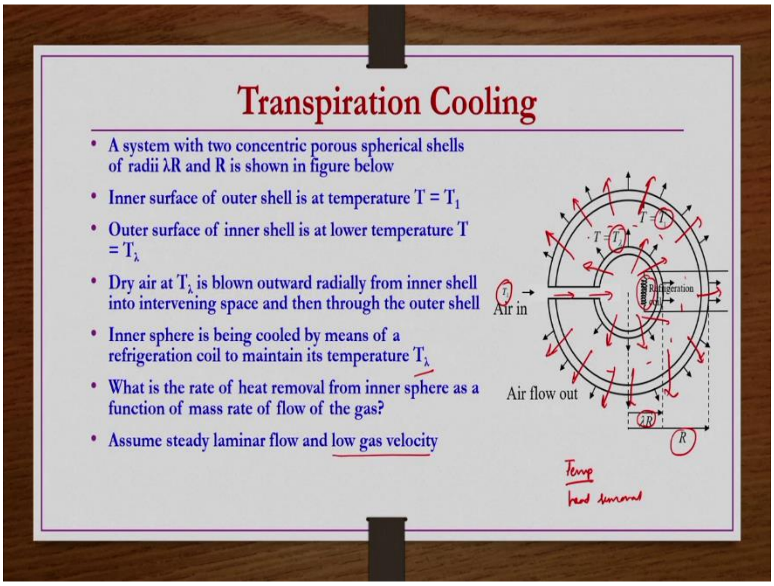

So, here we take a concentric spherical shells, which are porous, ok. So, now, we have a; that means, we have a two spherical shells are there both of them are porous. The radius of inner spherical shell is λR, the radius of outer spherical shell is R, right. So, now the temperature at the inner spherical shell is maintained at T_λ out outer spherical shell is maintained temperature at T_1, right.

So, now, what happens here? So, the air or some kind of gas in general, it not necessarily air, but some other gases is also possible. The gases at temperature T_λ is coming into the shell right and then thrown out through the porous structure of the shell like this and then while they are moving like this so, they are carrying the heat.

And then this air will be occupying the intervening space or the concentric annular space that is the available between two spherical shells and then from there from the outer shell surface there this gas is going out. So, this is the system we are having right. Sometimes you know in these kind of systems what happens? You need to maintain the temperature of required you know of specified temperature sometimes you need to maintain the specified temperature.

So, then spherical coil, the refrigeration coil is provided to the inner spherical shell here. So, now, what is the temperature distribution and then what is the heat removal at the inner spherical shell that is what we are going to derive now ok. The inner surface of outer shell is at temperature $T = T_1$.

The outer surface of an inner shell is at temperature T_λ. So, as shown here in the picture then dry air at T_λ is blown outward radially from inner shell into intervening space and then through the outer cell it is going out. Inner sphere is being cooled by means of a refrigeration coil to maintain the constant temperature of T_λ.

Then what is the rate of heat removal from inner sphere as a function of mass rate of flow of the gas mass rate in kg per second some gas you are allowing a let us say dry air you are allowing here. So, what is the rate of heat removal because of this air circulation ok? So, assume steady laminar flow and low gas velocity; gas velocity is very low; that means, only conduction may be dominating in this problem also, alright.

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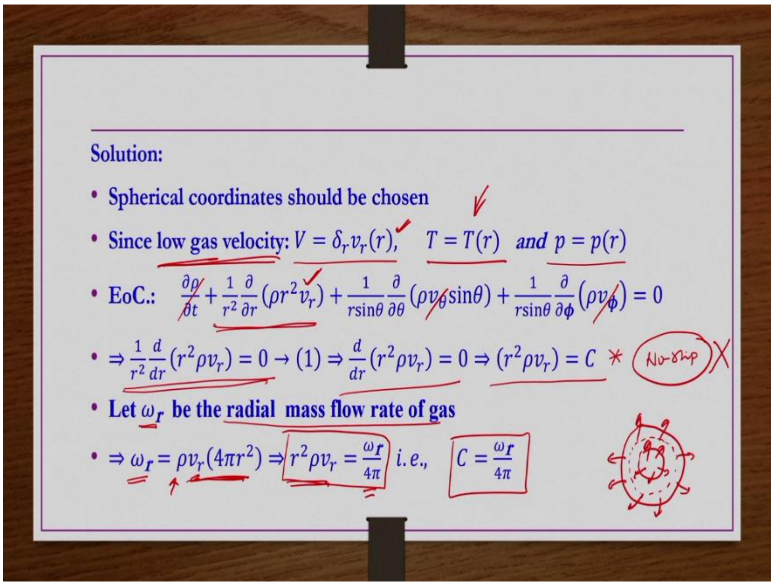

So, now what we do? We start the solution by you know by presenting the constraints or restrictions of the problem. So, here in this particular transpiration cooling system we have taken spherical shell. So, the spherical coordinates should be used and then since gas velocity is low and then that gas is radially blown right. So, radially blown from the inner spherical shell to the intervening space and then to the outer shell from the outer shell it is going out.

And then it is all taking place radially that is given in the problem even if it is not given as long as the velocity of whatever the air or gas you are taking if it is low, so, then this should this would be practically feasible. And then temperature would also be a function of r, pressure in general we do not know in general about the pressure.

So, it is function of r that also we have to check we will prove it and then what is that function that also we get, right. So, before getting this temperature because you get the temperature distribution then only you can get the heat removal. So, in order to simplify the energy equation you need to know what is the velocity right. So, though it is a low velocity, what is that low velocity that we have to find out, ok.

Then so that in order to find out the velocity, what we do in general? We simplify conservation of mass and momentum so that we get some information about the shear stress or the velocity as function of you know whatever the radial coordinates are you know in this case it is radial coordinates. Then similarly we get the pressure distribution information also from the conservation of a momentum equations right.

So, now, what we do? First as a convention we simplify the equation of continuity or conservation of the mass. Then this is the equation in spherical coordinates because of the steady state the first term is 0 and then only r velocity component is existing. So, that would be there and then it is also a function of r. So, then this entire term would be there.

And then v_θ v_ϕ velocity components are not existing, right. Remember we cannot deduce more number of assumptions from the schematic only that steady laminar flow we can take or incompressible fluid as a standard, but whether it is symmetry or not fully developed flow or not these things we cannot get any information from you know schematic.

So, we will be; we will be having less number of restrictions here. So, then we have to be very careful to get the required velocity distribution, pressure distribution and temperature distribution, right. So, now, here from the continuity equation, what we are understanding? We are getting an expression for the velocity. So, that is what $\frac{1}{r^2}\frac{d}{dr}(r^2\rho v_r) = 0$ this is the equation, now we need v_r right.

So, we can calculate we can obtain this one from here itself right, ok otherwise it is given also we will do different way also anyway. So, now, this equation if you integrate what you get? $r^2\rho v_r$ equals to constant, right. So, what is this constant you have to find out; no slip boundary condition will not be valid in this case, why? Because the spherical shells whatever we have taken they are porous in nature, if they are porous in nature. So, then fluid steam may be passing through you know porous structure.

So, then we cannot have the no slip boundary condition effectively in the entire surface because along the surface there are some porous you know holes are also porous structure is also there. So, we cannot use that boundary condition. So, we no other boundary condition we are having for a velocity, it is not given in the problem from the schematic also we cannot understand anything about the boundary condition for the velocity.

So, we cannot solve this equation further to get this v_r, but we have to get ok. So, what is given? Some mass rate air is entering at some mass rate. Basically whatever the dry air that is being circulated that velocity only we are finding out how it is being distributed in the radial direction that we are finding out we have to find out.

Because that is the fluid that is you know being circulated in intervene space right, so, but that is constant mass rate of dry air let us assume it is known, ok. So, let us take that one as ω_r be the radial mass flow rate of the gas, that is whatever the dry air is there so, that is radially blown that is what mentioned in the problem. So, that is a radial mass flow rate ok that is let us say assume we assume it is known.

So, then if you wanted to find out this one in terms of the velocity, density etcetera, so, if the velocity if you multiplying by the surface area of the spherical shell then you will get the volumetric flow rate v_r multiplied by $4\,\pi r^2$, r is any given location between λ_R to R.

So, that can be anything so, that is at any that like; that means, we have spherical shells like this one spherical shell and another spherical shell. So, these are porous in nature. So, they are not having you know no slip kind of boundary condition, right. So, now, what happens? You take one layer of element here at that element if you wanted to find out the mass flow rate. So, let us say the radius of this element is r.

So, then $4\,\pi r^2$ multiplied by v_r would be giving the volumetric flow rate of the gas at this particular r dotted line that is shown that can be anywhere between λ_R to R, ok. So, now that one that v_r multiplied by $4\,\pi r^2$ if you multiply by density you will get. What you will get? You will get the mass rate and that mass rate is nothing but ω_r is given it is given, ok.

So, now, we are having this one. So, from here $r^2\rho v_r$ is nothing but $\frac{\omega_r}{4\,\pi}$ so; that means, $C = \frac{\omega_r}{4\,\pi}$. So, this is the velocity distribution that we are looking for we got it, $v_r = \frac{\omega_r}{4\,\pi\rho r^2}$ is the velocity distribution ok, alright.

So, now it is a slightly different from what we have done previously, previously we tried to get the velocity distribution from the equations of motion by simplifying the equations of motion, but now directly we are getting from the continuity equation ok, right.

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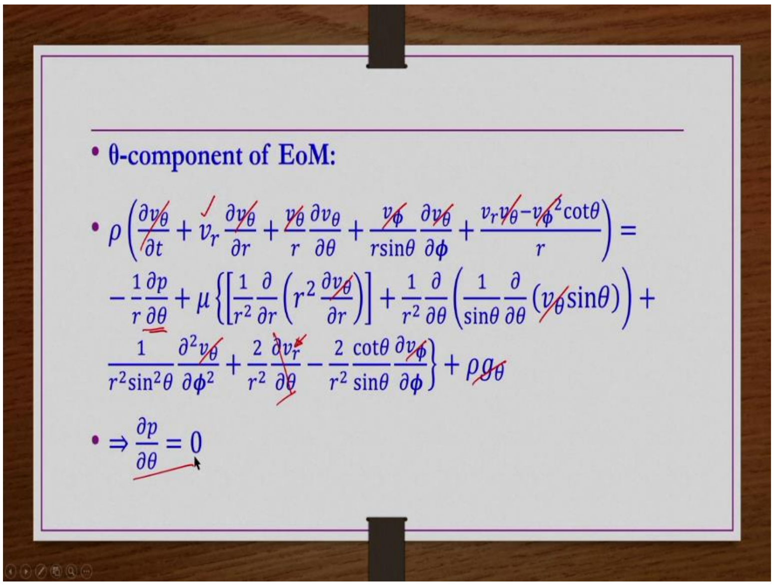

So, now however, what we do? We simplify the equation of motion also because we wanted to find out the pressure distribution is it really a function of r alone or is it function of θ and ϕ also that we have to find out. If it is function of r only what function of r that also we have to find out, ok.

So, for that what we have to do? We have to simplify all components of equations of motion r, θ, ϕ. So, let us start with θ component of equation of motion that is given here. So, let us say when you apply the constraints of the problem what we have? The steady state this term is 0, v_r is existing, but v_θ is not there. So, this term, this term, v_ϕ is also not there.

So, this term, this term is also cancel out v_r is there, but v_θ v_ϕ are not there. So, left hand side all terms are you know being cancelled out, pressure in general we do not know. So, in the viscous terms also since we are taking the Newtonian fluid an equations we have written Navier-Stokes equation we have written, ok

So, v_θ is not there. So, this term is 0, v_θ is not there. So, this term is also 0; v_θ is not there, v_r is existing, but it is not function of θ it is function of r that is what we have already found out from the continuity equation also as well it is given in the problem.

So, this is 0, v_ϕ is not existing. So, this term is also 0 gravity we are not taking. So, what we get? $\frac{\partial p}{\partial \theta} = 0$; that means, pressure is not function of θ, right. So, similarly ϕ component of equation of motion in spherical coordinates that is what I have given here.

So, steady state this term is 0, v_ϕ is not existing. So, this term is 0, v_θ is not existing. So, v_ϕ is not existing, v_ϕ is not existing. So, these terms or all these terms in the left hand side are cancelled out only v_r is existing. So, individual term where we are having only v_r is not there in the left hand side.

So, that here also altogether left hand side terms are cancelled out. Pressure in general we do not know what it is right. So, we keep it as it is. So, in the viscous terms also v_ϕ terms we can straight forward cancel out. So, all the terms are having v_ϕ. So, v_θ term is also cancelled out, gravity is not there. So, that is also cancelled out we can delete we can take off, here v_r term is there, but it is not function of ϕ it is function of r only. So, then this term is also gone.

So, what we understand from here? Pressure is not function of ϕ coordinate as well, ok. So, if it is not function of θ and ϕ direction; obviously, it will be a function of r direction alone because if there is no pressure variation, so, then obviously, flow will not take place because we are blowing the fluid dry air. So, that blowing the some pressure should be there then only it is possible. So, that pressure distribution probably we can get from r component of equation of motion is it really possible or not that we are going to see now.

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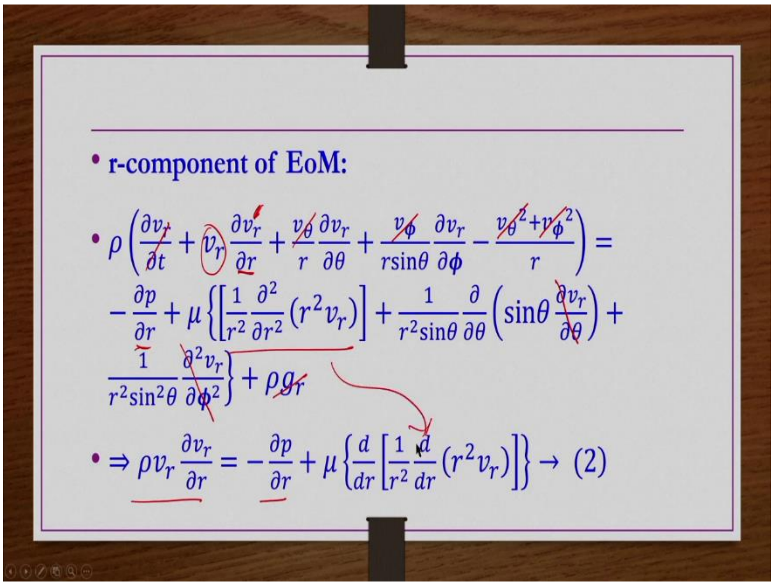

So, r component of an equation of motion given here. So, steady state this term is 0, v_r is existing; v_r is existing and then it is function of r. So, we cannot cancel out this term let it be like that. v_θ is not there, v_ϕ is not there; v_θ v_ϕ both of them are 0. So, left hand side only one term is existing that we should retain because v_r and then that is function of r so, those two.

So, that particular term should be there. So, pressure we do not know, v_r is there and then it is function of r only and it is function of r only. So, then it should be there v_r is there, but it is not function of θ. So, this term is gone v_r is there, but it is not function of ϕ. So, this term is also gone.

So, pressure we do not know anything. So, what we get? $\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu\left\{\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}(r^2 v_r)\right]\right\}$. So, this term we can expand like this. Whether you write partial derivative or total derivative it does not mean does not affect here because now whether it velocity or pressure they are function of only r.

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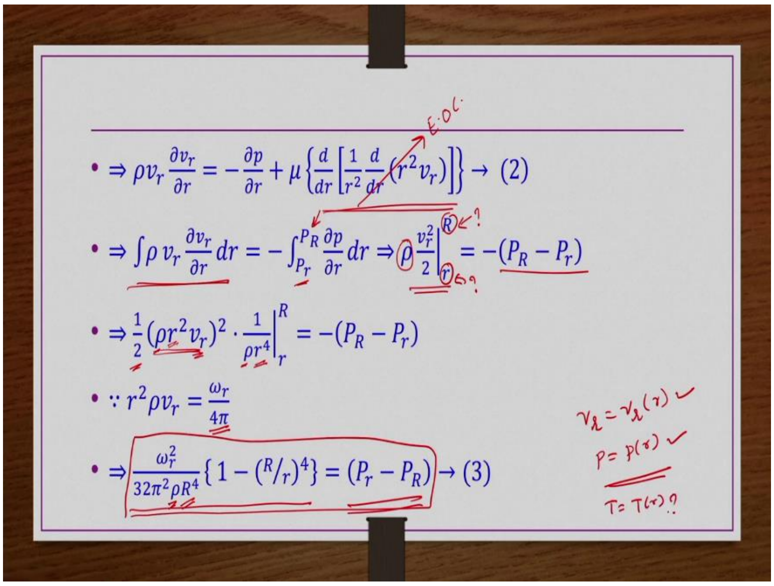

So, the same equation is written here also. So, now, what we do? We integrate this equation both sides with respect to r. So, $\int \rho v_r \frac{\partial v_r}{\partial r} dr = -\int_{P_r}^{P_R} \frac{\partial p}{\partial r} dr$. Let us say pressure at some r location is P_r pressure at outer shell r = R is let us say P_R, right.

So, that is what we are having and then this second term in the right hand side is 0. How it is 0? From the continuity equation this particular term we have already found it as 0, from the continuity equation this is what we found. By simplifying the continuity equation what we got? $\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right]$ is 0 that is what we got.

So, then that term we are not considering here, right. So, now, here you know when you integrate this particular part we have $\int \rho v_r dv_r$, alright. So, then $\frac{\rho {v_r}^2}{2}$ should be the integration, after integration this is what we get. And then lower limit is r upper limit is R alright.

So, at these limits what is the v that we do not know, here also at v = R what is the v_r that we do not know. So, for the time being let us not worry about that one also and then pressure here. So, integral of dp is nothing but $(P_R - P_r)$ after substituting the limits, this is what we are having.

So, now, what I will be doing? I will be dividing by r^4 and then multiplying by r^4 and then dividing by ρ and then multiplying by the ρ, when I multiply the ρ already there is one ρ. So, ρ^2 I will get, right. So, one ${v_r}^2$ is already there. When I am multiplying with r^4, so, $(r^2)^2$ I can write.

So, $(\rho\, r^2\, {v_r}^2)^2$ I can write it here and then 1/2 is as it is and then $\frac{1}{\rho r^4}$ we are getting here, right. So, because see in terms of velocity we do not have the upper and lower limits above the integration. So, then we cannot substitute those limits. So, that is the reason we are rearranging like this. What is the benefit of rearranging like this? In place of $\rho\, r^2\, v_r$ we can write $\frac{\omega_r}{4\,\pi}$, right.

So, that this term would be $\frac{{\omega_r}^2}{32\,\pi}$ and then ρ is as it is and then after substituting the upper and lower limit you will be; you will be getting $\frac{1}{R^4} - \frac{1}{r^4}$. So, out of which $\frac{1}{R^4}$ if you take common you will get $1 - \left(\frac{R}{r}\right)^4$ in the left hand side that should be equals to this minus I if I take in inside the parenthesis $(P_r - P_R)$ that is what we get.

So, the pressure distribution whatever is there in the radial direction that is given by this expression. So, now what we did till now? We found what is this v_r as function of r that is what we have found and then what is this pressure function of r that also we have found. Now, what we have to do? We have to find out temperature as function of r what it is.

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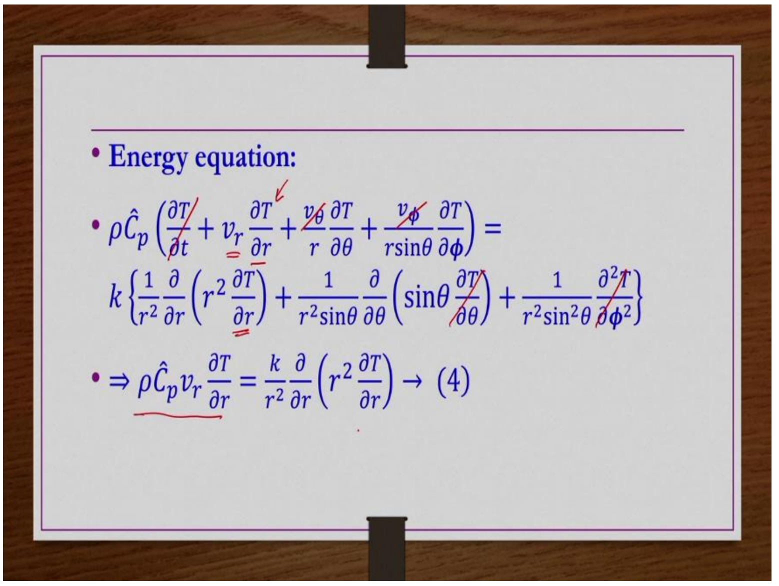

So, that, we can get by simplifying the energy equation in spherical coordinates. So, that is given here steady state. So, this term is 0, v_r is existing and then temperature is also a function of r. So, then this term would be there, v_θ is not there v_ϕ is not there. So, these two terms are cancelled out in the left hand side.

Temperature is function of r. So, this term should be there, but it is not function of θ or ϕ. So, these two terms are not there. Since the velocity is very low we will not have the viscous dissipation etcetera those kind of terms and then there is no reaction. So, no additional terms would be there. Now, so, what we have here? $\rho \hat{C}_p v_r \frac{\partial T}{\partial r}$ in the left hand side or right hand side we have $\frac{k}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right)$.

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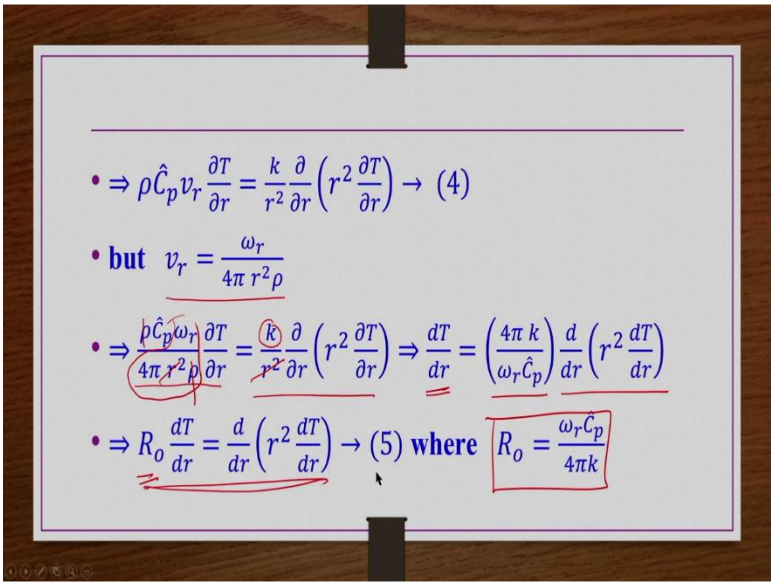

So, now this equation if you solve you will get temperature as function of r. This same equation is written here again. So, now, what we are taking? v_r we already know it as $\frac{\omega_r}{4\,\pi r^2 \rho}$ right. So, this we can substitute here. So, what we have here, left hand side $\frac{\rho \hat{C}_p \omega_r}{4\,\pi r^2 \rho}$ and then $\frac{\partial T}{\partial r}$. So, this entire term is nothing but v_r and then right hand side as it is.

So, now, what we do? We do some simplification this ρ this ρ is cancelled out and this r^2 this r^2 is cancelled out. So, left hand side we can have $\frac{\hat{C}_p \omega_r}{4\,\pi k}$ if this k also if you bring it here or what we do? We can take this $\frac{\hat{C}_p \omega_r}{4\,\pi}$ to the right hand side. So, we get $\left(\frac{4\,\pi k}{\hat{C}_p \omega_r}\right)\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$ in the right hand side left hand side we will be having only $\frac{dT}{dr}$.

So, now this $\left(\frac{4\,\pi k}{\hat{C}_p \omega_r}\right)$all are constant. So, instead of writing so many things so many times the same value we write in place of this one you know $\frac{1}{R_0}$ or $R_0 = \frac{\hat{C}_p \omega_r}{4\,\pi k}$. So, that in the left hand side we have $R_0 \frac{dT}{dr}$ is equals to and then right hand side we have $\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$ right then this equation we have to integrate.

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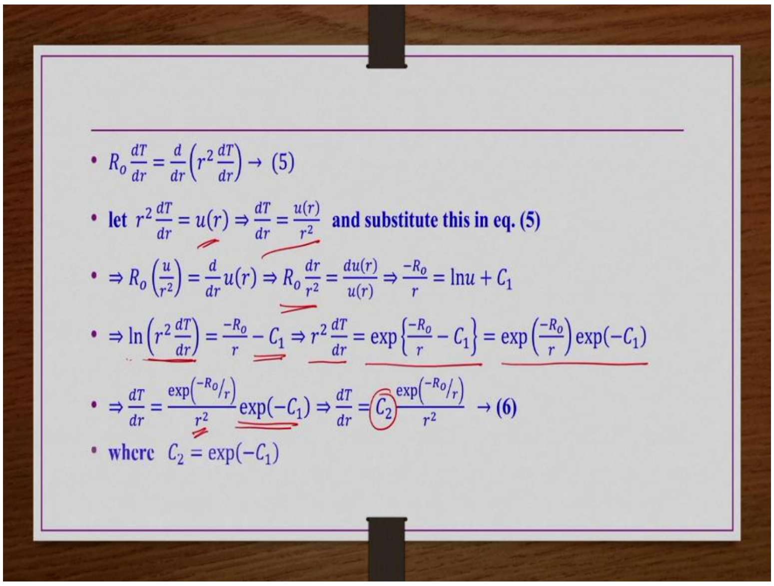

So, what we do here? You know in order to have the simple steps of integration $\left(r^2 \frac{dT}{dr}\right)$ we take as u which is also a function of r. So, that from here $\frac{dT}{dr}$ we can write u(r) or $\frac{\mathrm{u(r)}}{r^2}$ that is what we can write. So, these two quantities we if you make use in equation number 5 then we have $R_0\left(\frac{\mathrm{u}}{r^2}\right) = \frac{d}{dr}\mathrm{u(r)}$, right.

So, now, $R_0 \frac{dr}{r^2}$ I keep one side another side $\frac{\mathrm{du(r)}}{\mathrm{u(r)}}$, this u is function of r that is ok, ok. So, now, we can if you integrate left hand side dr terms are there right hand side du terms are there. So, integration could be straightforward. So, left hand side we get $\frac{-R_0}{r}$, right hand side ln u function of r you get plus constant C_1, right.

So, now, ln u in place of u I can write $\left(r^2 \frac{dT}{dr}\right)$ and then $\frac{-R_0}{r}$ is as it is that constant I am taking to the other side. So, that becomes $-C_1$, right. So, now, what I do? You, I take an exponential either side. So, then left hand side $\left(r^2 \frac{dT}{dr}\right) = exp\left\{\frac{-R_0}{r} - C_1\right\}$ that I can write $exp\left(\frac{-R_0}{r}\right) exp(-C_1)$, right.

So, $\frac{dT}{dr}$ if I wanted to, write so, that whatever the r square in the left hand side is there that I take to the right hand side. So, $\frac{exp\left(\frac{-R_0}{r}\right)}{r^2} exp(-C_1)$. So, this $exp(-C_1)$ I call it C_2 just for simplicity or easiness of writing, ok.

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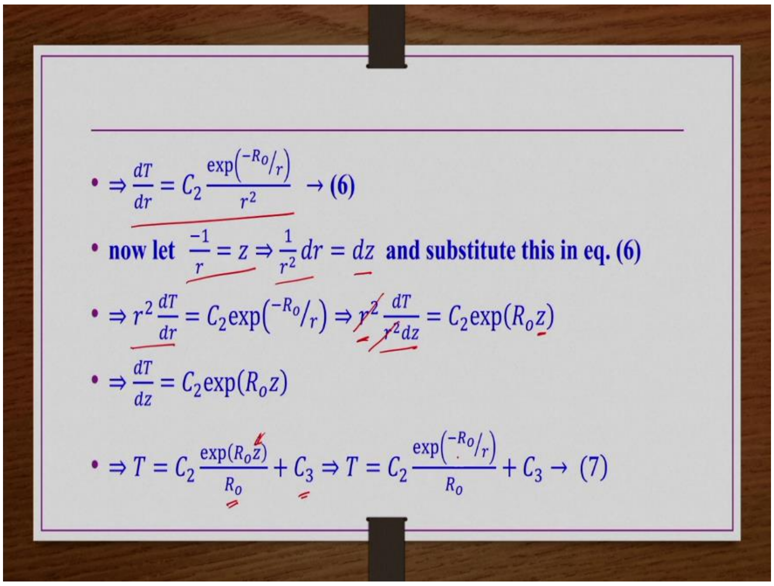

So, now the same equation is given here again. So, final equation that we have $\frac{dT}{dr} = C_2 \frac{exp\left(\frac{-R_0}{r}\right)}{r^2}$. So, now, this equation again we can integrate to get the temperature profile ok. So, what we do here again? In place of ,$-\frac{1}{r}$ I take it as z. So, that wherever I have $\frac{1}{r^2} dr$ I can write dz in this above equation number 6, when I do this one what I get?

In the left hand side $r^2 \frac{dT}{dr}$is there. So, r^2 as it is dT in place of dr r^2 dz I am writing. Right hand side $C_2\ exp\left(\frac{-R_0}{r}\right)$ is there. So,$-\frac{1}{r}$ z. So, $exp(R_0 z)$ I am getting here and then left hand side this r^2 this r^2 is cancelled out. So, what I have? $\frac{dT}{dz} = C_2 exp(R_0 z)$, right.

So, now, this integration even simpler now. So, when you integrate this one $T = C_2 \frac{exp(R_0 z)}{R_0} +$ another constant C_3 right and then this z is nothing but$\frac{-R_0}{r}$. So, this is the final temperature distribution.

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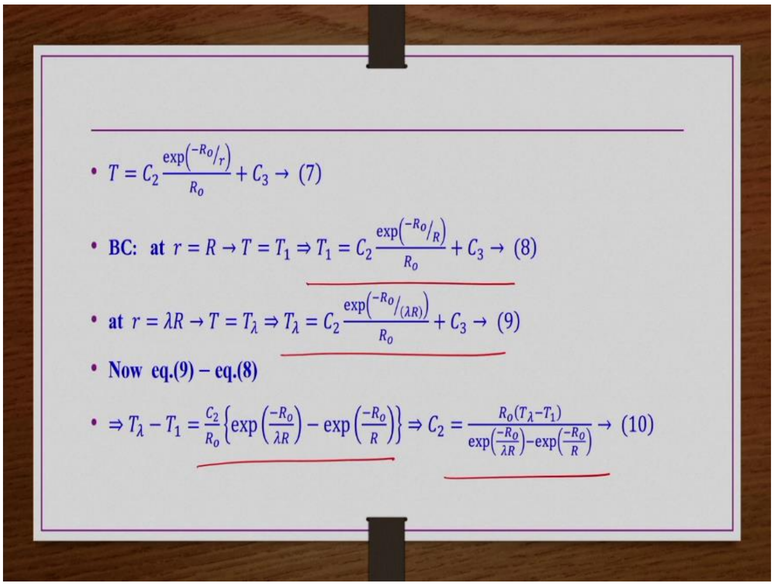

Now, this final temperature distribution is useful only when we find out what are these C_2 C_3 constants, at least luckily for temperature we have the boundary conditions given in the problem. So, we can find out these constants whereas, the velocity there no boundary conditions was given. So, we had a slightly difficult difficulty.

So, then we had to do some other you know constant mass rate statement we have to use in order to get the you know constant in the case of velocity distribution, but here we do not need to have worry such kind of simplifications because fixed constant temperature conditions are given.

That is at r = R, T = T_1 right so; that means, $T_1 = C_2 \frac{exp(-R_0/R)}{R_0} + C_3$, whereas, at r = λ that is inner spherical shell T = T_λ so; that means, $T_\lambda = C_2 \frac{exp(-R_0/\lambda R)}{R_0} + C_3$.

Now, if I do 9 minus 8 equation, so, $T_\lambda - T_1 =$ exponential of this one, this C_3 would be cancelled out; that means, $C_2 = \frac{R_0(T_\lambda - T_1)}{exp\left(\frac{-R_0}{\lambda R}\right) - exp\left(\frac{-R_0}{R}\right)}$ alright. So, this is what we are having now. So, now, C_2 constant is known this C_2 constant we can substitute either of these equations 8 or 9 and then find out this C_3 constant.

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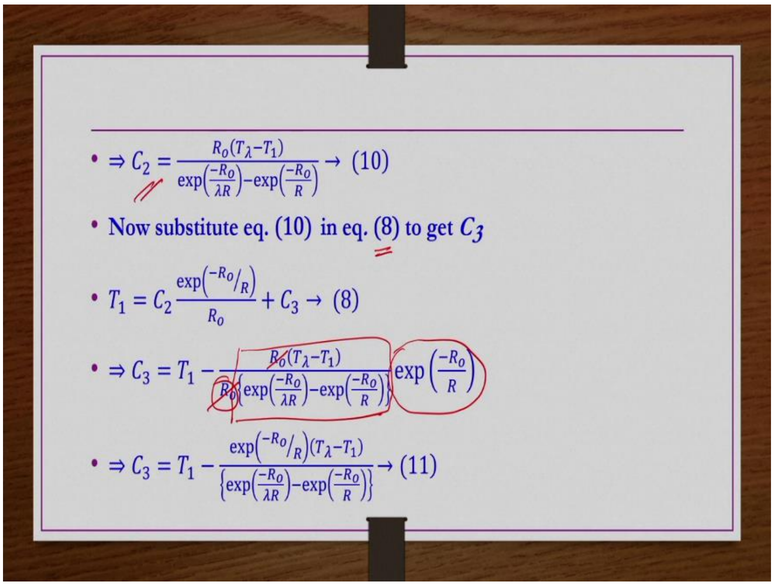

So, but; however, what we do? We substitute in equation number 8. So, equation number 8 is nothing but $T_1 = C_2 \frac{exp(-R_0/R)}{R_0} + C_3$. So, in place of C_3 now we substitute equation number 10. So, that is then we get C_3, $C_3 = T_1 - C_2 exp(-R_0/R)$.

So, then $exp(-R_0/R)$ and then $\frac{1}{R_0}$ are already there remaining terms are nothing but the C_2 constant. This is nothing but C_2 right. So, we do not need to do any further simplification; however, if we want we can do it right.

So, $C_3 = T_1$ and then after canceling out this $R_0 R_0$ $\frac{exp(-R_0/R)(T_\lambda - T_1)}{exp\left(\frac{-R_0}{\lambda R}\right) - exp\left(\frac{-R_0}{R}\right)}$ this is what C_3. Now, we have both C_2 and C_3, this we can substitute in the final temperature distribution that we already got equation number 7. So, that is this equation, right.

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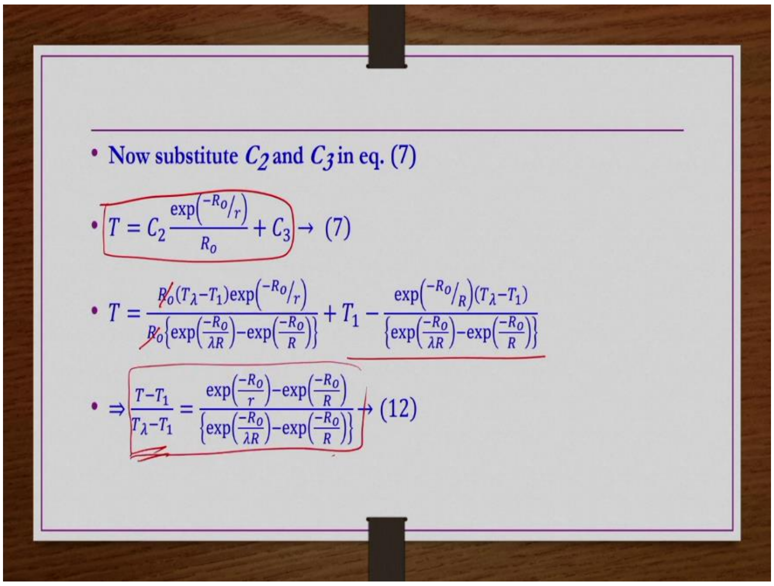

So, then we have this one. So, C_2 is nothing but this $\frac{R_0(T_\lambda - T_1)}{exp\left(\frac{-R_0}{\lambda R}\right) - exp\left(\frac{-R_0}{R}\right)}$ C_3 is nothing but this remaining term (Refer Time: 29:40) right. So, now, T_1 I take to the left hand side. So, that I can write $(T - T_1)$ and then I divide both sides by $(T_\lambda - T_1)$, ok. So, then we have this expression because here this R_0 this R_0 is cancelled out.

So, what we have? $\frac{exp\left(\frac{-R_0}{r}\right) - exp\left(\frac{-R_0}{R}\right)}{exp\left(\frac{-R_0}{\lambda R}\right) - exp\left(\frac{-R_0}{R}\right)}$ this is the temperature distribution final temperature distribution, alright.

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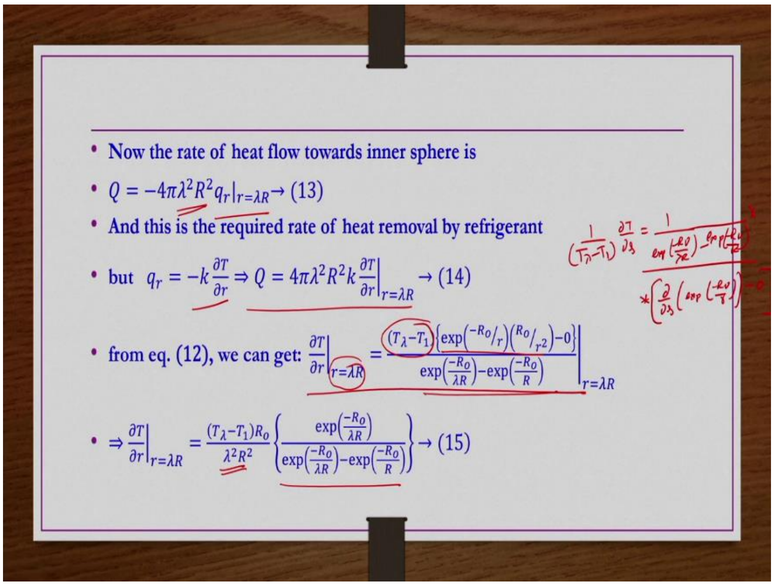

So, but we need to find out the heat removal as well. So, that is the main purpose. The rate of heat flow towards inner sphere is $Q = -4\pi R^2$. Now, R is nothing but λ R because at the inner sphere we are finding out and then multiplying by the heat transfer rate $q_r|_{r=\lambda\,R}$, but q_r is nothing but $-k\frac{\partial T}{\partial r}$. So, this Q would become $4\pi\lambda^2 R^2 k\frac{\partial T}{\partial r}|_{r=\lambda\,R}$.

So, now, we have to find out what is this $\frac{\partial T}{\partial r}$ from equation number 12. So, from equation number 12 that is temperature distribution $\frac{T-T_1}{T_\lambda-T_1}$ if you do the differentiation with respect to R, so, left hand side only; left hand side only temperature T is varying with r. T_1 T_λ are constant right hand side also only $exp(-R_0/r)$ is only function of r all other terms are independent of r. So, they remain constant.

So, left hand side $\frac{1}{T_\lambda-T_1}$ would be there. So, that we get to the right hand side that is in the left hand side we have $\frac{1}{T_\lambda-T_1}\frac{\partial T}{\partial r}$ right hand side whatever 1 by denominator term of $exp\left(\frac{-R_0}{\lambda r}\right) - exp\left(\frac{-R_0}{R}\right)$ that would be remain constant. And then this entire thing is multiplied by the differentiation of the $exp\left(\frac{-R_0}{r}\right) - 0$. So, this entire thing divided by this thing we are having.

So, that is divided by is as it is and then whatever $\frac{1}{T_\lambda-T_1}$ in the left hand side was there that we have taken to the right hand side and then differentiation of $exp\left(\frac{-R_0}{r}\right)$ we can have $exp\left(\frac{-R_0}{r}\right)$ multiplied by $\frac{R_0}{r^2}$ that is what we will be having minus 0, right so; that means, this equation actually we are evaluating at r = λR.

So, in this equation if you substitute λ r, so, this we get $exp\left(\frac{-R_0}{\lambda r}\right)$ and then denominator as it is ok. So, this $\frac{R_0}{r^2}$ is nothing but $\lambda^2 R^2$ now. So, this is $\frac{\partial T}{\partial r}$ at r = λ r. This we can substitute in equation number 14 to get the rate of heat removal.

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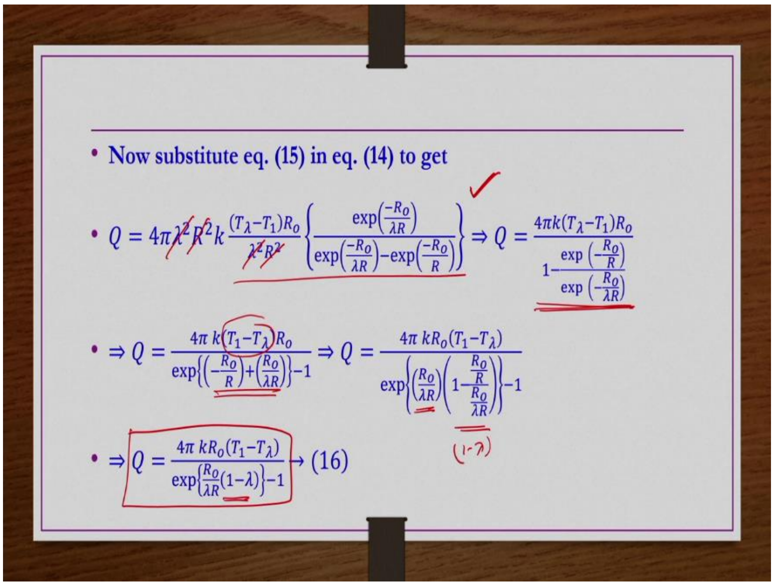

So, this is what $\frac{\partial T}{\partial r}$ at r = λ R right. Now this same equation I can write $\frac{4\pi k(T_\lambda - T_1)R_0}{1-\frac{exp\left(\frac{-R_0}{R}\right)}{exp\left(\frac{-R_0}{\lambda R}\right)}}$. Simply what I am doing? In the denominator I am taking $exp\left(\frac{-R_0}{\lambda R}\right)$ common factor and then that cancelling out with the numerator thing.

So, then I have 1 minus of this one in the denominator now whereas, rest of the terms after cancelling these all $\lambda^2 R^2$ remaining the same that is $4\pi k(T_\lambda - T_1)R_0$ is remaining the same right. So, now, further this, what we can do here? We can write it as an exponential of this whatever the exponential of a divided by exponential of b is there that I can write exponential of a − b right.

So, that if I write $exp\left(\frac{-R_0}{R} - \left(\frac{-R_0}{\lambda R}\right)\right)$ is nothing but $\frac{R_0}{\lambda R}$. And then −1 because −1 + 1 is becoming − 1 because in the numerator whatever the $(T_\lambda - T_1)$ was there I have written as $(T_1 - T_\lambda)$, ok. So, now, this is what we are having. So, after this step here again I am taking common as from these two terms I am taking common $\frac{R_0}{\lambda R}$.

So, that I have $1 - \frac{\frac{R}{R_0}}{\frac{R_0}{\lambda R}}$. So, this R_0, R_0 is cancelled out R, R is cancelled out. So, that I can have 1 −λ here that is 1 −λ rest all other terms are same, right. So, in order to have a simpler form only we have done all this simplification otherwise this step itself this is final step ok.

This is the rate of heat removal because of the dry air that is blown radially from the inner spherical shell to the intervene space and to the outer spherical shell and then out of the concentric spherical shells completely out of their system, fine.

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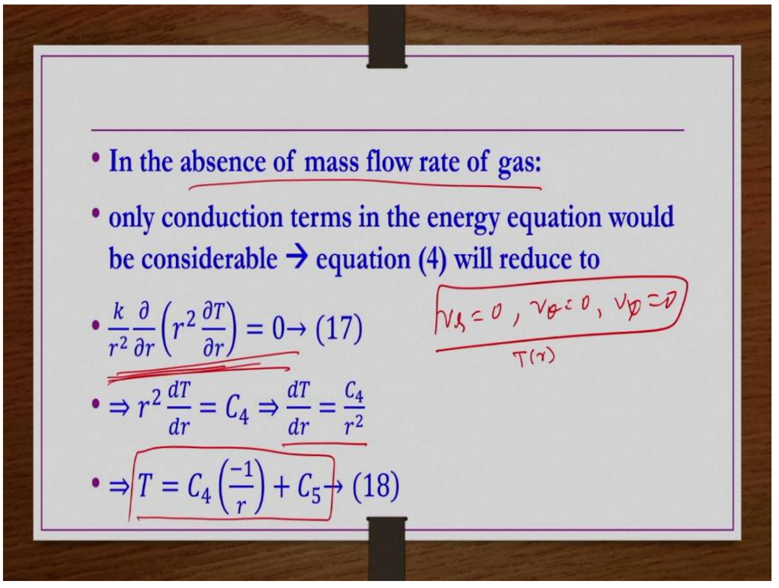

So, now, if there is no mass flow rate at all then what should we do? Do we have the same equation everything? Simply can we take that $\omega_r = 0$ and do it. So, that we have to see, blindly we cannot substitute $\omega_r = 0$ in final expressions otherwise we may not get the correct answers.

So, in the absence of mass flow rate of gas we can do the similar way whatever we have done. Now, v_r is 0 in the absence of mass flow rate; that means, v_r is completely 0 there is no flow at all, only thing that we have to find out what is the T_r ok. So, in the energy equations also since there is no flow at all only a mode of heat transfer would be the conduction and then that conduction is also predominating in the r direction that we know.

So, the when we simplify the energy equation only this $\frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$ would be there in the right hand side of energy equation that term only will be remaining and then the rest all other terms are 0. So, then this term is equals to 0, this is what we get.

So, I hope we now you are familiar at least this one how are we getting directly from the energy equation by simplifying it ok. So, because now there is no mass flow, so, then v_r is 0 v_θ is 0 v_ϕ is also 0 already. So, left hand side all the terms of energy equation are cancelled out and then temperature is function of r only.

So, in the right hand side whatever the temperature as function of θ and ϕ are there they have been cancelled out. So, only this term is remaining that is $\frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$. This when you integrate what you get? $\left(r^2\frac{\partial T}{\partial r}\right) = C_4$ constant again we can take this r^2 to the right hand side.

So, then $\frac{dT}{dr} = \frac{C_4}{r^2}$ you get further integration T = $C_4\left(\frac{-1}{r}\right)$ + C_5 that is what you get, this C_4 C_5 we can find out using the boundary conditions.

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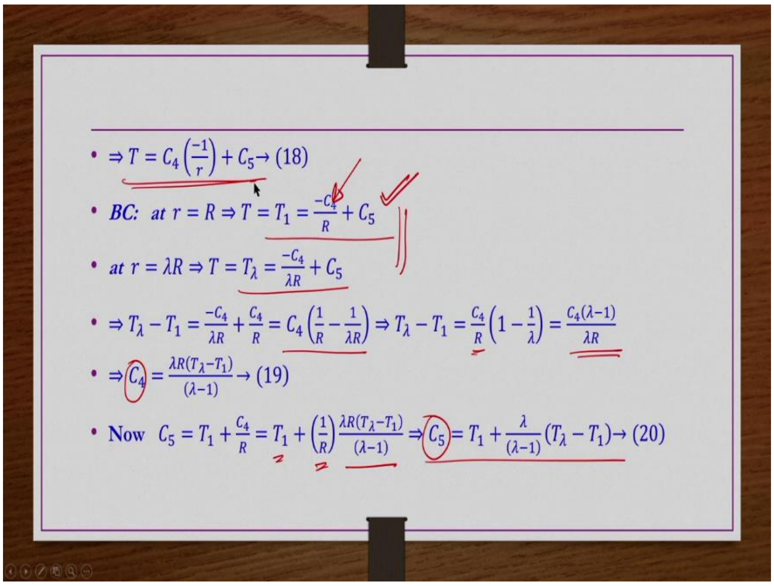

At r = λ r, T = T_λ at r = R, T = T_1. So, T_1 = $C_4\left(\frac{-1}{r}\right)$ + C_5 and then $T_\lambda = \frac{-C_4}{\lambda R}$ + C_5. So, if you substitute this one equation from the other from these two C_5 would be cancelled out. So, $(T_\lambda - T_1)$ I am doing. So, I get $\frac{C_4}{\lambda R} + \frac{C_4}{R}$ C_5 is cancelled out by subtraction.

Now, from the right hand side C_4 if I take common I get $\frac{1}{R} - \frac{1}{\lambda R}$. So, that R also I take common. So, then I get $(T_\lambda - T_1) = \frac{C_4}{R}\left(1 - \frac{1}{\lambda}\right)$ or we can write it as $\frac{C_4(\lambda-1)}{\lambda R}$. So, $C_4 = \frac{\lambda R\,(T_\lambda - T_1)}{(\lambda-1)}$, right.

So, C_5 now we are substituting this one in equation and this equation so, in this equation you are substituting C_4. So, $C_5 = T_1 + \frac{C_4}{R}$ T_1 is as it is $\frac{1}{R}$ is as it is C_4 is this one when you simplify C_5 you get this expression. So, now, you have both C_5 and then C_4, so, which we can substitute in equation number 18 to get the final temperature distribution.

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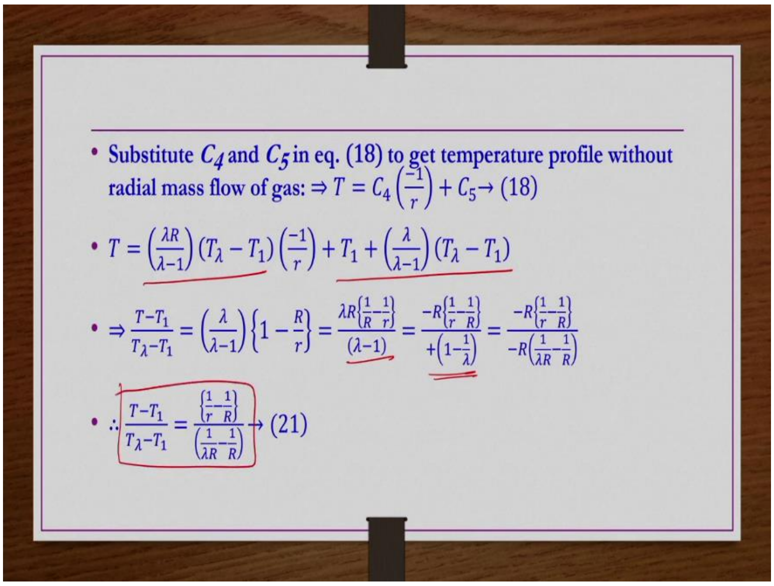

So, this is what we have. This entire thing is C_4, this entire thing is C_5. So, this equation I can write $\frac{T-T_1}{T_\lambda - T_1} = \left(\frac{\lambda}{\lambda-1}\right)\left\{1 - \frac{R}{r}\right\}$ I can write right. The same thing I can further simplify you know by multiplying by R and then dividing by R in the numerator. So, then $\frac{\lambda R\left\{\frac{1}{R}-\frac{1}{r}\right\}}{(\lambda-1)}$.

So, then further I am writing you know this in the denominator I am taking λ common and then cancelling with that λ with the λ in the numerator. So, then I can have $\frac{-R\left\{\frac{1}{r}-\frac{1}{R}\right\}}{\left\{1-\frac{1}{\lambda R}\right\}}$. So, this is what we have. So, finally, temperature profile what we can have? We can have this as the temperature profile fine.

Now, the same thing you can get same temperature profile you can get from the temperature profile that we got for the case of a dry air mass flow rate.

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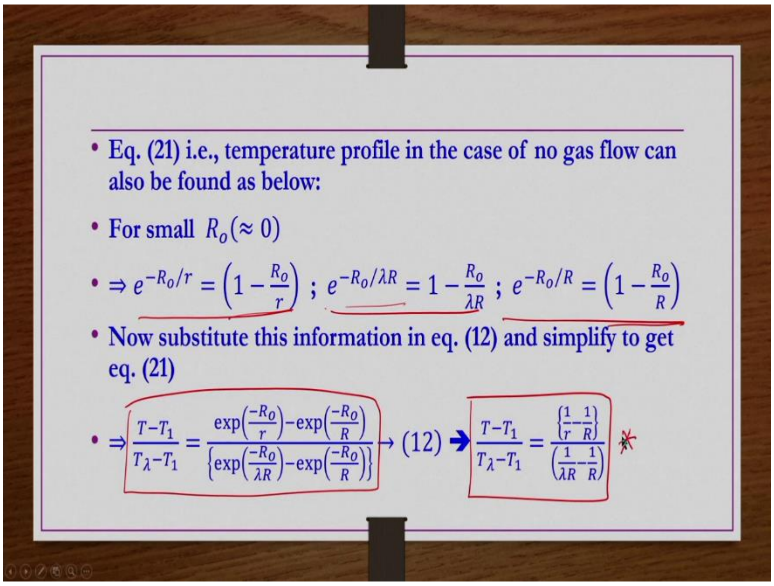

When the mass flow rate is there so, under such that expression we can use and then get the same thing by taking R_0 is approximately 0 because R_0 is nothing but what is R_0? Your $\frac{\omega_r \hat{C}_p}{4\,\pi k}$. So, $\omega_r$is nothing but mass rate if that is 0 R_0 is also 0 or if you take R_0 very very small. So, wherever $e^{-R_0/r}$ is there that I can write $\left(1-\frac{R_0}{r}\right)$.

Similarly, $e^{-R_0/\lambda R}$ I can write $1-\frac{R_0}{\lambda R}$ by expansion and then $e^{-R_0/R}$ I can write $1-\frac{R_0}{R}$ by neglecting the higher order terms. And these we can substitute in equation number 12 that we got this one.

When you substitute these expressions 3 expressions in this equation number 12 then simplify you get this expression simply $\frac{\left\{\frac{1}{r}-\frac{1}{R}\right\}}{\left\{\frac{1}{\lambda R}-\frac{1}{R}\right\}}$. So, the same expression we got it here, right either way you can do it.

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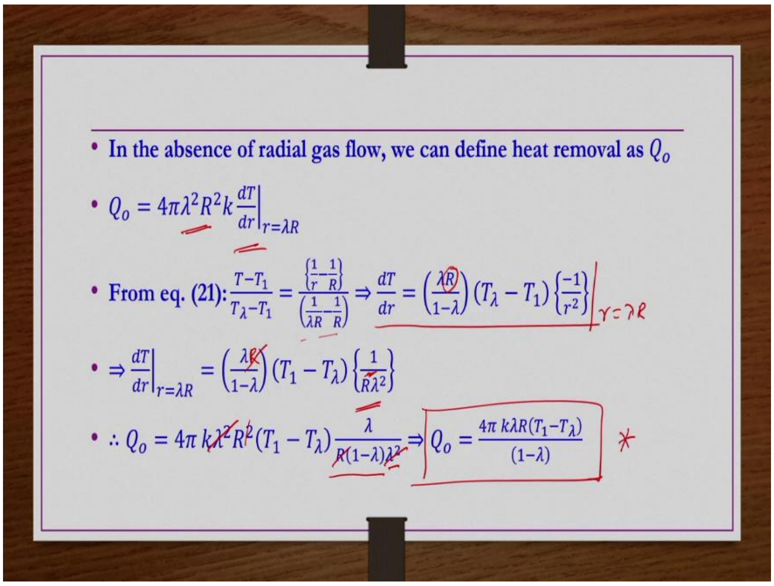

Now, in the absence of radial gas flow, we can define the heat removal as Q_0. So, that is $4\pi\lambda^2R^2k\frac{\partial T}{\partial r}|_{r=\lambda R}$ because r is nothing but R multiplied by λ at the inner sphere surface. So, that is the region λ^2R^2 we are having here.

So, from equation number 1 $\frac{T-T_1}{T_\lambda-T_1}$ is given by this one. So, this if you differentiate with respect to R. So, $\frac{dT}{dr} = \left(\frac{\lambda R}{1-\lambda}\right)(T_\lambda - T_1)\left\{\frac{-1}{r^2}\right\}$ you are getting here, fine.

This if you substitute here you know after you know substituting here r = λ R. So, then here you are having λ^2R^2, but one R we are cancelling with the other R. So, then this square is not here ok. So, that actually we have this λ R here.

So, R square we should get this square and then this R is cancelled out then final expression we are substituting here then we have $4\pi k\lambda^2R^2$ this is $(T_1 - T_\lambda)\left(\frac{\lambda}{R(1-\lambda)}\right)$ and then $\left\{\frac{1}{\lambda^2}\right\}$. So, now what we have this λ^2 this λ^2 we can cancel out and then this R and then square of this R is cancelled out $4\pi k\lambda R\frac{(T_1-T_\lambda)}{(1-\lambda)}$ is the heat removal in the absence of radial mass flow rate when there is no air or dry gas anything is not circulated then heat removal is given by this expression, right.

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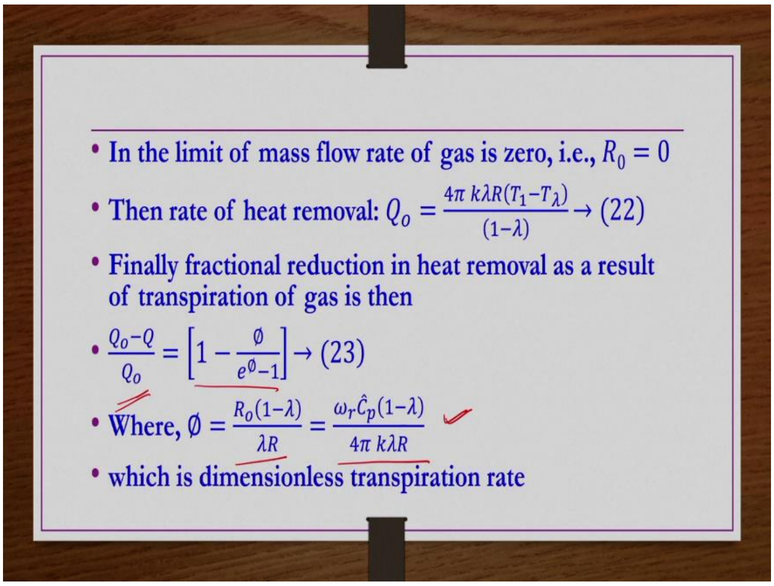

So, in the limit of mass flow rate of gas is zero, R_0 is 0 then rate of heat removal $Q_0 = \frac{4\pi k\lambda R(T_1 - T_\lambda)}{(1-\lambda)}$ that is what we got just now. Then finally, fractional reduction in heat removal as a result of transpiration of gas is nothing but $\frac{Q_0 - Q}{Q_0}$. Q we already got in the case of mass flow rate whatever the rate of heat removal is there that is Q, in the case of no mass flow rate then heat removal is Q_0.

So, by allowing the mass flow rate how much improved you know heat removal has been taken, how it has affected the heat removal process that you can get by this ratio. And then that Q_0 Q if you substitute and then simplify you get $1 - \phi$ by you get $\left[1 - \frac{\phi}{e^{\phi} - 1}\right]$ where ϕ is nothing but $\frac{R_0(1-\lambda)}{\lambda R}$ and that is nothing but $\frac{\omega_r \hat{C}_p (1-\lambda)}{4\pi k\lambda R}$, alright.

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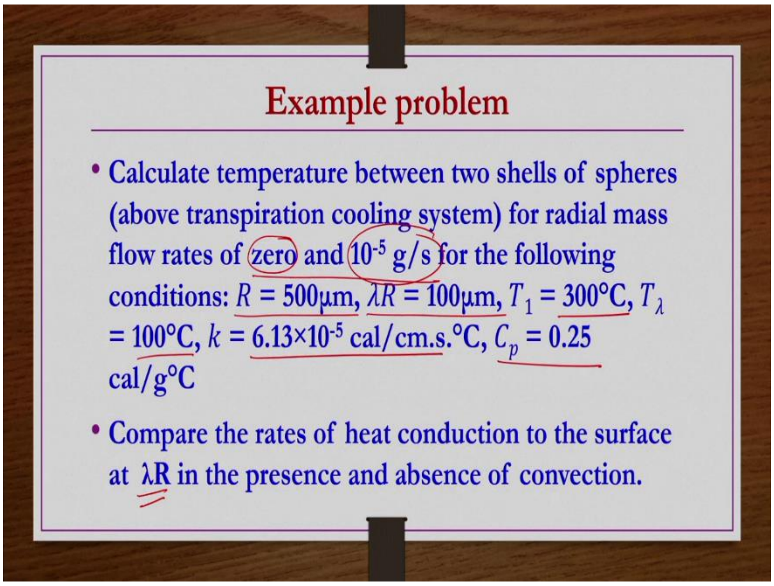

Now, we take an example problem: calculate temperature between two shells of spheres for radial mass flow rates of zero and 10 power of minus 5 gram per second. So, both the cases zero; that means, there is no mass flow rate and then when there is a mass flow rate 10 power minus 5 gram per second for the following conditions, right.

We have to find out the temperature right temperature distribution we have to find out. R is only 500 microns whereas, λ R is only 100 microns, T_1 is 300 degree centigrade, T_λ is 100 degree centigrade, k is given this one, C_p is given this one. And then compare the rates of heat conduction to the surface at λ R that is in this inner sphere shell ok, in the presence and in absence of convection.

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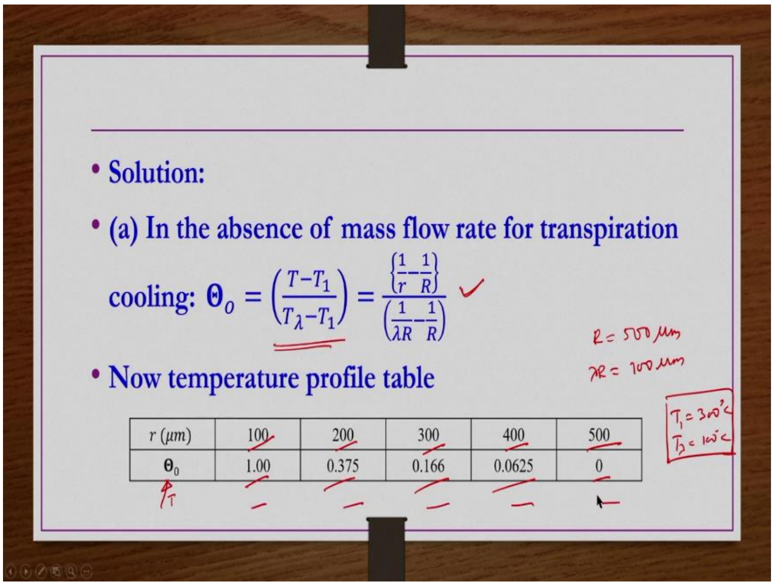

When there is no mass rate what is the heat removal, when there is a mass rate what is the heat rate that we have to find out. So, we take in the absence of mass flow rate that we take. So, in that case whatever $\left(\frac{T-T_1}{T_\lambda-T_1}\right)$ is this is what we got this equation number 21 in the lecture slides. And now R is given as 500 microns and λ R is given as 100 microns. So, in between you take a limits of 200, 300, 400 microns.

So, then 100, 500 are 2 limits in between 200, 300, 400 microns when you take. So, corresponding θ naught that is this ratio temperature difference ratio that is $\left(\frac{T-T_1}{T_\lambda-T_1}\right)$ is 1, 0.375 0.166 0.0625 and 0 this is dimensionless temperature difference ok dimensionless. Now this if you have then $T_\lambda - T_1$ let us say in terms of T if you wanted to find out. So, T_1 is 300, T_λ is 100. So, now this T_λ, T_1 are provided here. So, then you can find out what is the corresponding T from this in terms of degree centigrade's, ok.

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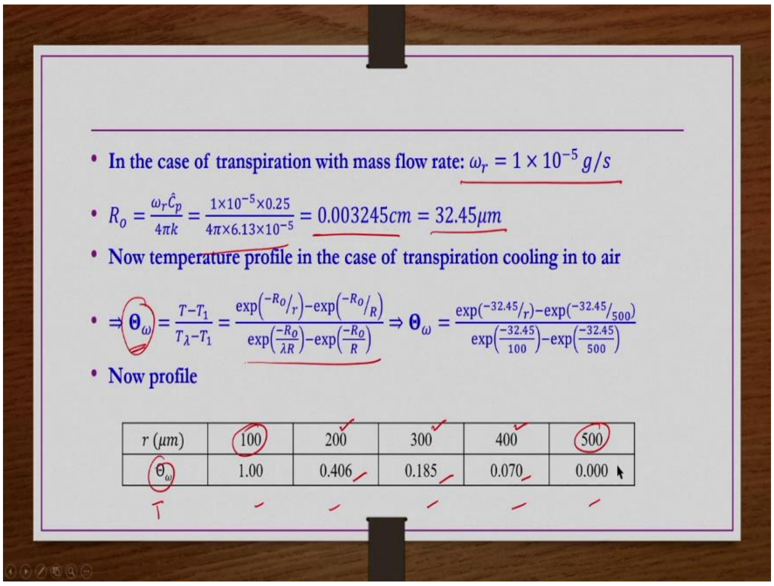

Now, in the case of transpiration with mass flow rate that is ω_r is given 10 power minus 5 gram per second. Luckily all the units are given in CGS unit. So, $R_0 = \frac{\omega_r \hat{C}_p}{4\,\pi\,k}$ we can calculate by directly substituting in CGS unit. So, then you get 0.003245 centimeters that is 32.45 microns, alright.

So, now, temperature profile in the case of transpiration cooling in to air. So, that let us say $\left(\frac{T-T_1}{T_\lambda-T_1}\right)$ whatever is there in the case of air flow rate that I am designating as a capital θ ω, right. So, this is the dimensionless temperature distribution. So, now, here right hand side this is the equation number 12 that we have derived. So, now, here R_0 you found it as you know 32.45 and then R is 500, λ R is 100. So, well all substituted here.

So, different R values between 100 microns and then 500 microns let us take 200, 300, 400 different R values. So, then corresponding θ ω you can find out and then once you have this one, so, directly you can find out T in temperature, T temperature in degree centigrade that you can find, right.

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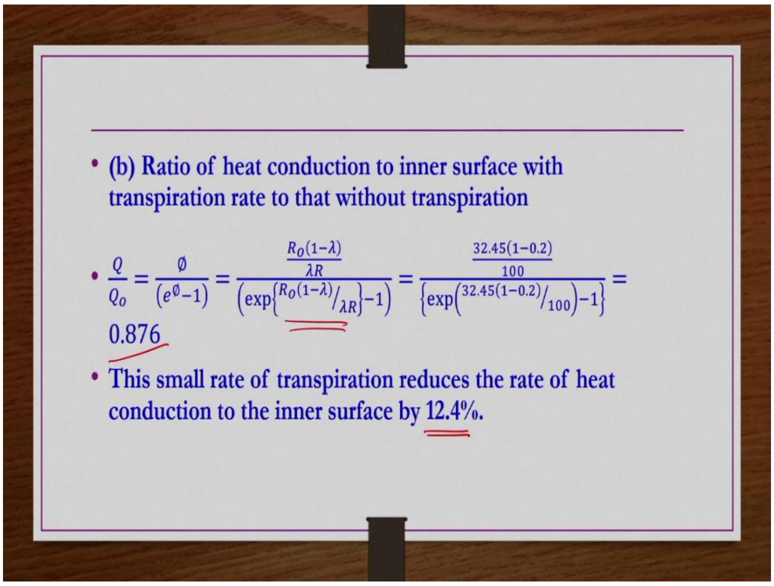

So, now ratio of heat conduction to inner surface with transpiration rate to that without transpiration also we have to find out that is the second part of the problem. So, that is $\frac{Q}{Q_0}$ is $\frac{\phi}{(e^{\phi}-1)}$. So, ϕ is nothing but $\frac{R_0(1-\lambda)}{\lambda R}$. So, that we know, so, derived in the equations. So, when you substitute R_0 λ R and then λ etcetera in this equation and then simplify. So, then you get 0.876 this ratio as 0.876.

That means this small rate of transpiration reduces the heat conduction to the inner surface by 12.4 percentages. Just maintaining 10 power minus 5 gram per second, such a small flow rate when you are allowing the heat conduction to the inner surface is reduced by 12.4 percent. So, that is the importance of transpiration cooling.

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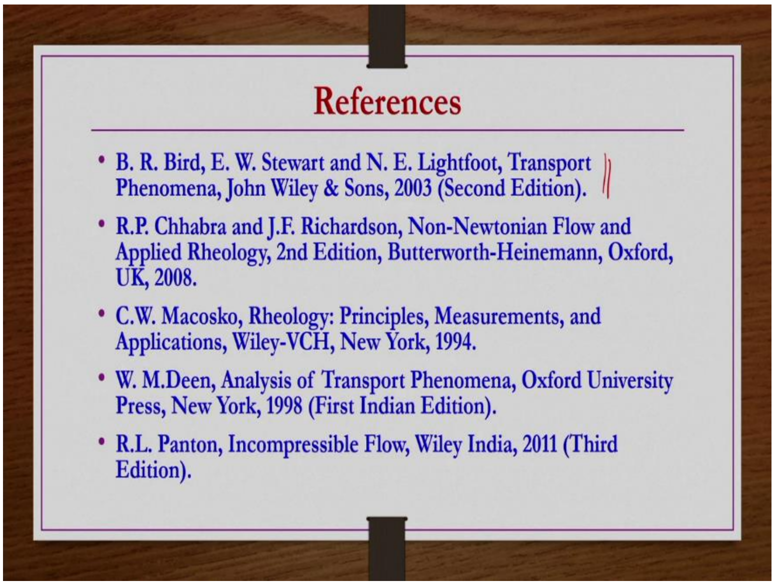

References are provided here. So, this problem can also be found in this book Transport Phenomena by Bird, Stewart and Lightfoot. However, as a exercise problem you can take this transpiration cooling problem in cylindrical geometry as well and then try to obtain the similar expressions as a take home problem for yourself.

Thank you.