Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 03
Time Dependent Non-Newtonian Fluids and Viscoelastic Non-Newtonian Fluids

Welcome to the MOOCs course Transport Phenomena of Non Newtonian Fluids. The title of this lecture is Time Dependent Non Newtonian Fluids and Viscoelastic Non Newtonian Fluids. Now before going into the details of today's lecture we will be having a recapitulation of what we have studied till now. In previous couple of lectures what we have seen?

We have seen several characteristics of Newtonian fluids and then making those characteristics of Newtonian fluids as a kind of base to define non-Newtonian fluids. And then based on the different possibilities of non-Newtonian fluids we have classified them into three different categories.

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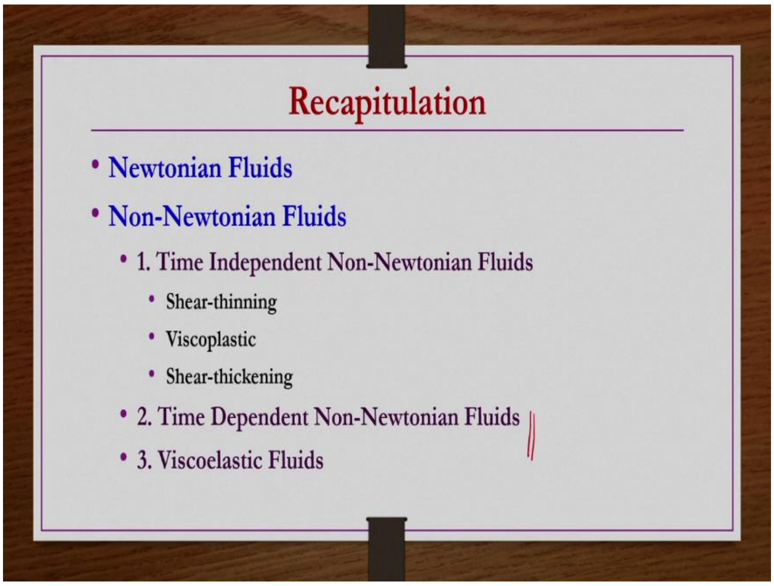

One category is time independent non-Newtonian fluids, another category is time dependent non-Newtonian fluids and then third category is the viscoelastic non-Newtonian fluids right. So, in the previous lecture what we have seen? We have seen in detail discussion we have we had in detailed discussion about time independent non-Newtonian fluids.

And then sub classification of those time independent non Newtonian fluids, models for those time independent non Newtonian fluids that is what we have seen. That is for a shear thinning, viscoplastic and shear thickening type of a time independent non Newtonian fluids we have seen almost all possible details in the previous lecture.

Now, in today's lecture we will be discussing a few details about time dependent non Newtonian fluids and then viscoelastic non Newtonian fluids. So, but before going into the details of time dependent non Newtonian fluids, what we will be doing? We will be having a few problems on time independent non Newtonian fluids, because in the previous lecture we could not have a time to discuss example problems.

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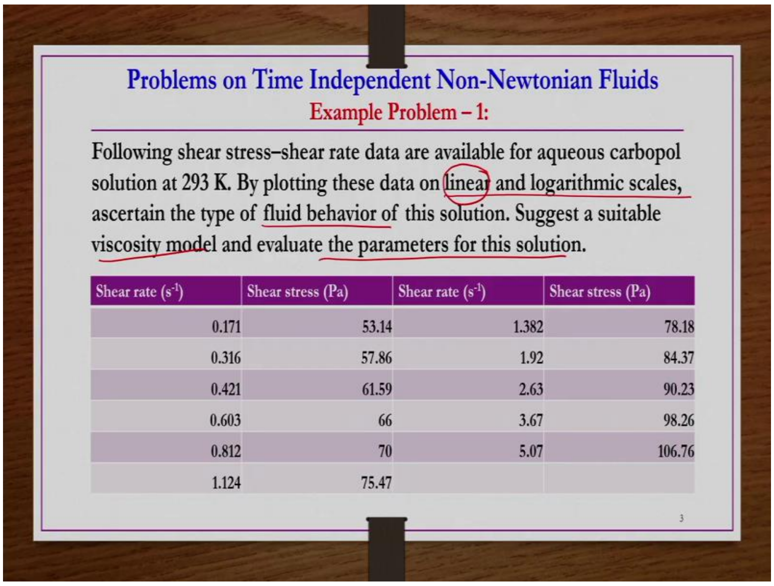

So, now we see a few example problems on time independent non Newtonian fluids before going into the time dependent non Newtonian fluids. So, the first problem on time independent non Newtonian fluids is as follows. Following shear stress versus shear rate data are available for aqueous carbopol solution at 293 Kelvin.

By plotting these data on linear and logarithmic scales ascertain the type of fluid behaviour of this solution suggest a suitable viscosity model and evaluate parameters of this solutions, rheological parameters of this solution that is the question.

And then data shear stress versus shear rate data is given here that is shear, shear rate is given in second inverse shear stress is given in pascals. So, what we have to do? This experimental data whatever is there that we have to first plot we have to plot on linear scale.

So, that when you have a this shear stress versus shear rate curve then you can come to know whether this data is linear or non-linear is it passing through the origin or is it not passing through the origin. If it is not passing through the origin then again is it linear or non-linear after crossing that yield stress point.

So, all those things we can come to know only when you draw this shear stress versus shear rate information right. So, first that we will do and then we are also asked to make a logarithmic scale. Logarithmic scale plotting also we have to do. That is we have to take log of shear stress and then we have to take log of shear rate and then we have to plot them log of shear stress versus log of shear rate. Or straight forward on a log-log graph sheet we can plot shear stress versus shear rate as a second part of the question.

Then from the first part of the question when we have plot this shear stress versus shear rate data on a linear scale then based on that one we have to ascertain which type of fluid is it is it a Newtonian or non Newtonian. If it is non Newtonian what type of non Newtonian fluid that we can know from the linear plot of shear stress versus shear rate on a linear scale, so that we that is what we do.

Once we realize that you know if it is non Newtonian fluid. So, then which type of non Newtonian fluid that we can realize from the plot of a shear stress versus shear rate then we can do accordingly simple calculations to obtain the model parameters ok.

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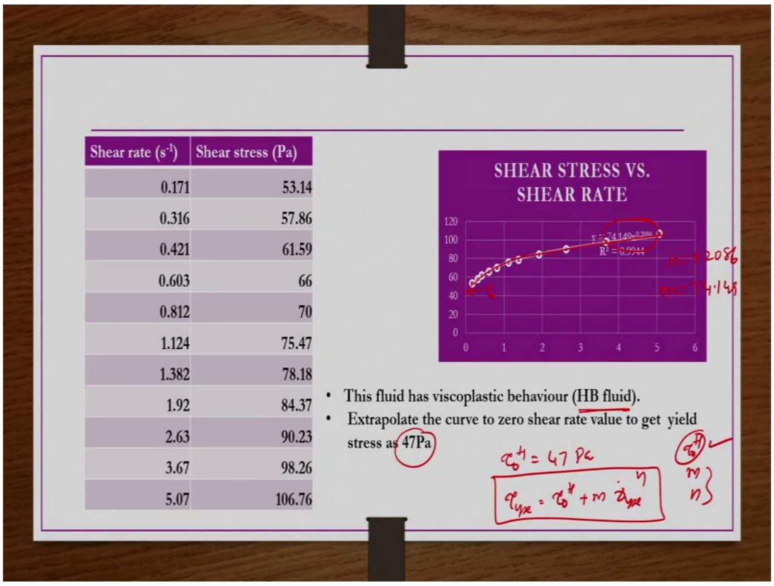

So, this is the given data once again re-drawn here. So, now, the shear stress versus shear rate data first we will have to plot on a linear scale. So, when you plot you know shear stress versus shear rate on a linear scale. So, then what you can see? It is not passing through origin; obviously, and then after that you know it is having a non-linear curve like this ok. So, that is what I have done on a excel sheet. We can do on a normal graph sheet using pen and pencil like this. So, then we get this curve like this.

So, then what is this curve? This curve is; obviously, what indicates if you extrapolate it to this x = 0 line that is you know $\dot{\gamma}$ = 0. So, then you get corresponding value of yield stress that clearly we can see that when $\dot{\gamma}$ = 0 shear stress is not 0.

So obviously, it is not a, it is not a kind of a Newtonian fluid and then it is not a kind of a power law fluid or shear thinning or shear thickening fluid. It is a viscoplastic fluid because it is having some yield stress right. So, that is what we understand.

So, now since we have done in excel sheet, so directly we can understand. So, it is a after crossing this yield stress point whatever the behaviour of these curve is there that can be well represented by you know power law model with n = 0.2086 and then m = 74.149 or something like this. But; however, in exams we may we may not be having the Microsoft excel to plot directly. So, then what we do? We have to have a kind of alternative approach of calculating them ok.

So, now from here what we understand? This fluid has a visco plastic behaviour and in that also Herschel Bulkley fluid behaviour. Why Herschel Bulkley fluid behaviour?

Because after crossing this yield stress it is having a non-linear curve like this shown it. After crossing this yield stress if it is linear then it is a Bingham plastic. Since it is not linear after crossing this yield stress it is Herschel Bulkley fluid or yield plastic of fluid right.

So, now extrapolate this curve to zero shear rate value to get yield stress value and then that is 47 Pascals that is 47 Pascals is your τ_0^H right. You know that $\tau_{yx} = \tau_0^H + m(\dot{\gamma}_{yx})^n$ for Herschel Bulkley fluid. So, now, at least by having this shear stress versus shear rate curve on a linear scale, so then we realize that it is Herschel Bulkley fluid.

So, then second part of the question is that we have to find out the model parameters and then there are three parameters out of three one you already got it τ_0^H right as a 47 Pascals. So now, m n and you have to find out right. We can find it out find out that one as well.

So now, what we do? Next part of the question is that we have to plot this data on a log log scale as well. So, what I have done rather taking the log-log graph sheet I have taken log of shear rate versus log of shear stress whatever the shear rate shear stress information is given.

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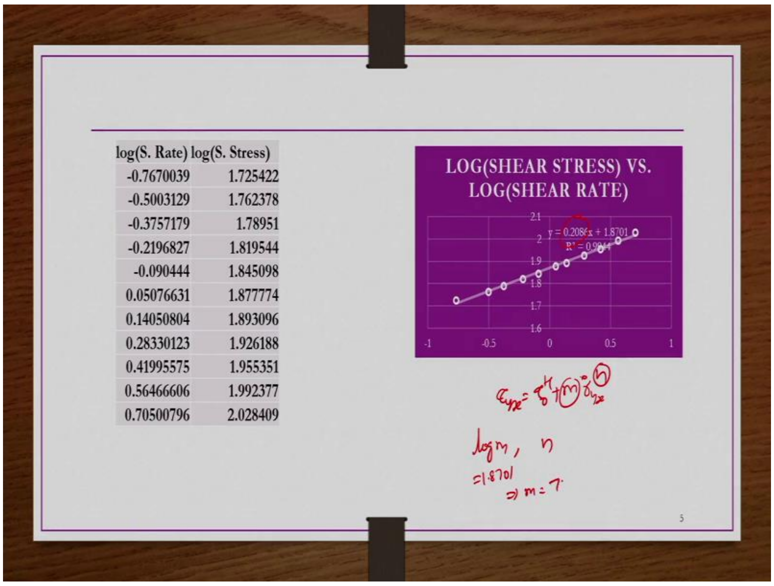

So, that I have taken into I have converted into the log of shear rate and then log of shear stress corresponding respectively and then I have plotted them. So, log of shear stress versus log of shear rate then if I plot this data what I get? I get a straight line. So, I get a straight line. So, that what I am here doing is that I am not taking that tau naught H value into the consideration remaining values only I am taking. So, remaining part is having you know power law behaviour kind of thing.

So, if you exclude the τ_0^H this is what you are having right. So, then from if you take the log of either side. So, then you will get whatever this m and n values. So, from here intercept is nothing but log of m and then slope is nothing but n ok. Slope is 0.2086 that you are getting intercept log of m that is 1.8701 you are getting from here m you will get around 74 point something like this, so that you can clearly get.

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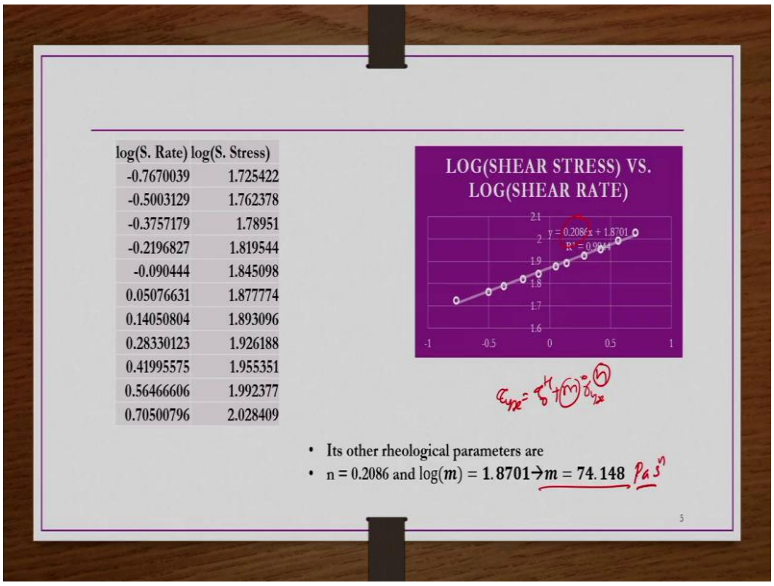

So, then rheological parameters we can find out n the slope of this log of shear stress versus shear rate curve that is nothing but 0.2086 and then intercept is nothing but log of m that is1.8701. So, that is m is equals to 74.148 Pas^n right. This is how we can do, so very simple straight forward.

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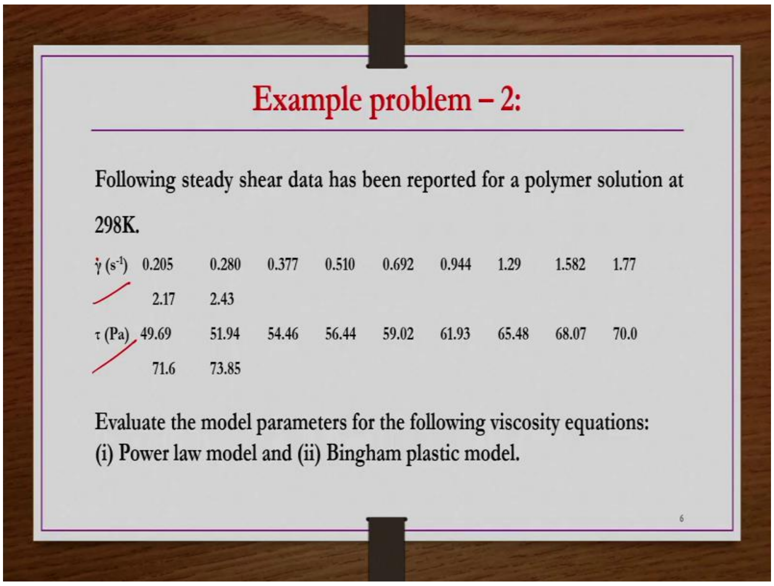

So, we take another example problem. Following study shear data has been reported for a polymer solution at 298 Kelvin. So, $\dot{\gamma}$ is given in s^{-1} τ is given in Pascal right. The question is evaluate the model parameters for the following viscosity equations power law model and then Bingham plastic model.

In the previous question it has not been mentioned which model you have to do the curve fitting for that given shear stress versus shear rate data. But now here it has been given like we have to fit this data using the power law model and Bingham plastic model ok.

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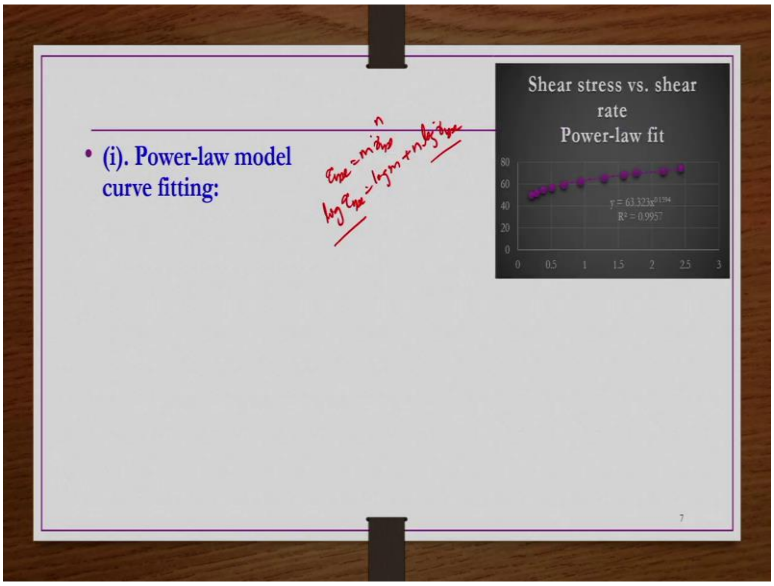

So, first we do the power law model curve fitting. So, this power law model what you get? $\tau_{yx} = m(\dot{\gamma}_{yx})^n$. So, if you take log either side log $log\,(\tau_{yx}) = log(m) + nlog(\dot{\gamma}_{yx})$ this is what you have. So, now, if you plot $log\,(\tau_{yx})$ versus $log(\dot{\gamma}_{yx})$ then you will get a straight line of slope n and then intercept log of m.

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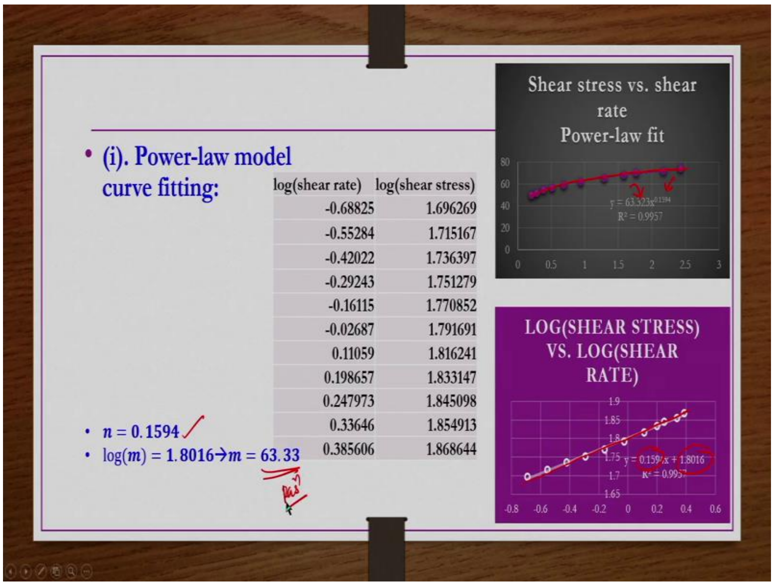

So, first we have to do the curve fitting using the power law model. So, whatever the data shear stress versus shear rate data is there. So, this is the given data we have to apply the trend line of a power law behaviour. And then is it really replicating or not that is what we have to see. So, it is anyway it is fine ok. So, what we get from directly if you are doing excel directly you are getting this m this n and then this m values directly you are getting right.

But if you wanted to do using graph sheet in pencil. So, then you have to do it you have to plot it on the log-log graph sheet or convert the data in a log of shear rate and then log of shear stress like that and then you plot them in a linear scales ok, linear graph sheet you can plot. So, then when you convert this data into the log terms like log of shear rate and then log of shear stress then you plot it.

So, then you have a straight line like this which is having a slope of 0.1594 and then which is having a intercept of 1.8016. So, n is equals to 0.1594 and then m is equals to 63.33 Pas^n right.

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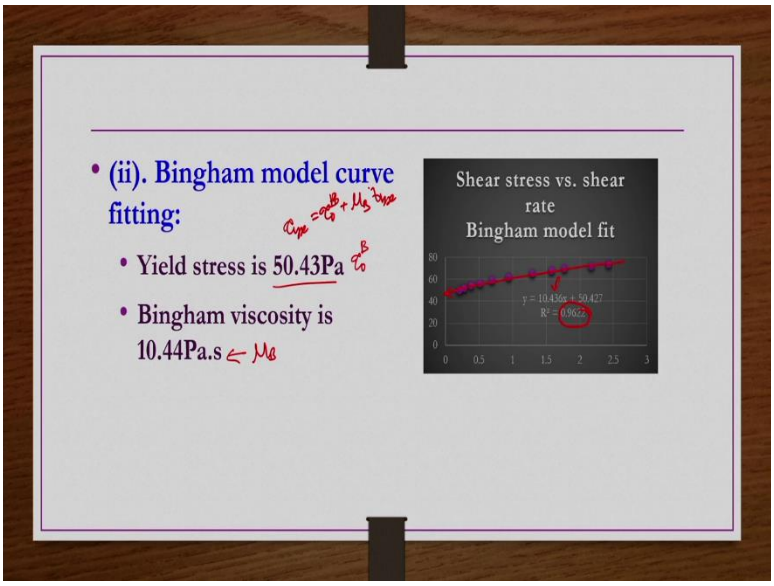

So, next part is that the same data we have to fit using Bingham model curve fitting. So, Bingham model curve fitting what we have? $\tau_{yx} = \tau_0^B + \mu_B(\dot{\gamma}_{yx})$ that is it. So, if you plot it linearly you get a straight line like this.

So, then τ_0^B this is not τ_0^H because we are doing for the Bingham plastic model. So, that you get approximately 50.427 and then viscosity μ_B you get 10.436. So, 50.43 pascal is τ_0^B yield stress and then Bingham viscosity that is $\mu_B$is equals to 10.44 pascal seconds.

So, you can see here the curve fitting is not very good 96 percent only R^2 is 0.96 only, but other case R^2 is almost like you know 0.99, but again there also we are not taking the yield stress point anyway.

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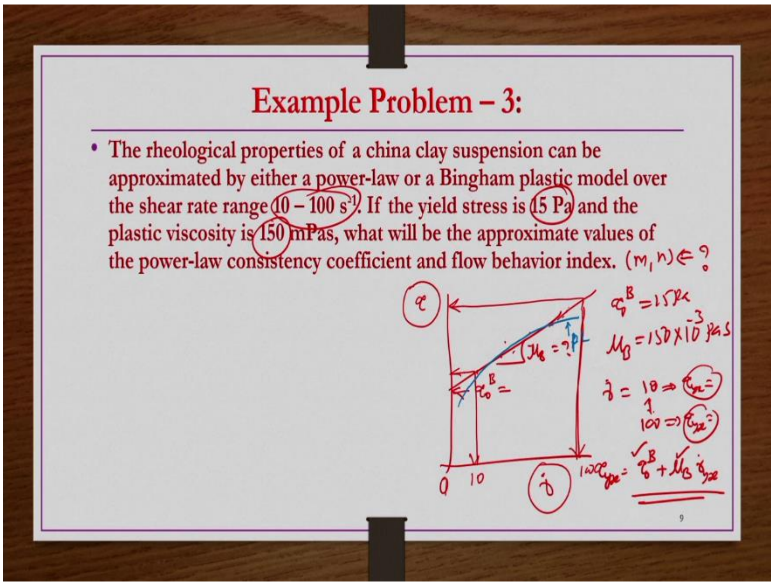

So, example problem number 3. The rheological properties of a china clay suspension can be approximated by either a power law or a Bingham plastic model over the shear rate range of 10 to 100 s^{-1}. If the yield stress is 15 pascals and a plastic viscosity is 150 millipascal second. What will be the approximate values of the power law consistency coefficient and then flow behaviour index?

So, now for you the data is given like τ_0^B is given and then μ_B is given 150 milli pascal second. So, 150 x 10^{-3} pascal second ok. And then $\dot{\gamma}$ range is given one is 10 another one is 100 between these two ranges only the this information is valid ok.

So, now we know this $\tau_{yx} = \tau_0^B + \mu_B(\dot{\gamma}_{yx})$. So, $\dot{\gamma}_{yx}$ is 10. So, what is the corresponding τ_{yx} we can find out from this equation, because τ_0^B is given and then μ_B is given.

Similarly when $\dot{\gamma}_{yx}$ is 100 what is the value of τ_{yx}. So, that is what you get. So, then so now, you get this information. So, then next model parameters you can find out ok. So, before going into the solution, one important thing that you have to understand is that this is the curve fitting right. So, for this whatever the china clay suspension is there already τ versus $\dot{\gamma}$ information has been obtained right.

Between 10 to 100 s^{-1} of shear rate of course, τ information is not given rather giving τ information it is given you know when that information τ versus $\dot{\gamma}_{yx}$ is plotted using Bingham plastic model right. So, whatever the τ versus $\dot{\gamma}$ information is there that has been plotted using the Bingham plastic model and then whatever this τ_0^B is there that is given right and then slope of this curve μ_B how much that is given right. So, now, what you have now this is let us say you know 10, 100 this is 0 anyway. So, this values of shear rate are given right.

So, corresponding this information you have to find out; that you can find out by using this equation. So, whether you are doing the curve fitting using the Bingham plastic model or power law model this τ versus $\dot{\gamma}$ information is not changing that is an experimental information. So, now, rather fitting like this curve what you are doing you are fitting like in a different way using a power law model you are fitting.

So, now same data if you are fitting using the power law behaviour. So, then let us say you may be getting data something like this for example, for a power law model right. So, whether you are using this red colour Bingham plastic curve fitting this blue colour power law curve fitting your data τ versus $\dot{\gamma}$ data is not changing it is going to be same, it is going to be same alright.

So, then so, that understanding is required. So, then once you have these thing then corresponding tau information you can find out from the Bingham model and in that same tau information you can use for the power low model to find out the m and n values ok.

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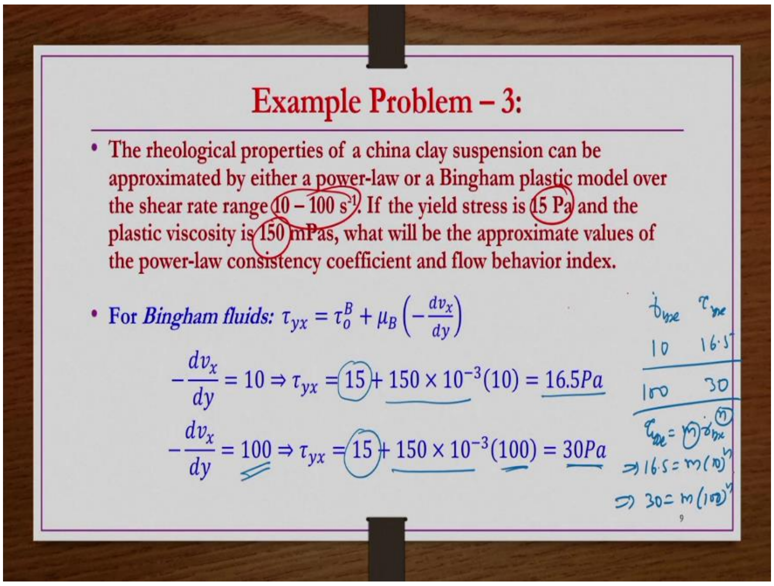

So, now that is how we will progress to solve this problem. So, for Bingham plastic model $\tau_{yx} = \tau_0^B + \mu_B\left(\frac{-dv_x}{dy}\right)$and then $\frac{-dv_x}{dy}$ lower limit is 10. So, when it is 10 τ_{yx} is equals to

τ_0^B is 15 and then μ_B is 150 millipascal second and then this $\dot{\gamma}_{yx}$shear rate is nothing but 10, 10 s^{-1}. So, when you substitute all these you will get 16.5 pascals.

Then similarly other limit of a $\dot{\gamma}_{yx}$is 100 second inverse. So, when it is 100 you know τ_{yx} is how much? Tau naught B is remaining same any way μ_B will also remain same only this in place of $\dot{\gamma}_{yx}$ you will be substituting 100. So, then you will get 30 pascals. So, now, when you have this information $\dot{\gamma}_{yx}\tau_{yx}$ when it is 10 $\tau_{yx}$is 16.5 when it is 100 this $\tau_{yx}$is 30. So, now, power law model τ_{yx}= m $\left(\dot{\gamma}_{yx}\right)^n$.

So, now you have two data points you need to you have a two unknowns. So, then you can get the two equations let us say 16.5 = m 10^n then 30 = m 100^n you have two equations two unknowns you solve and then you get the solution.

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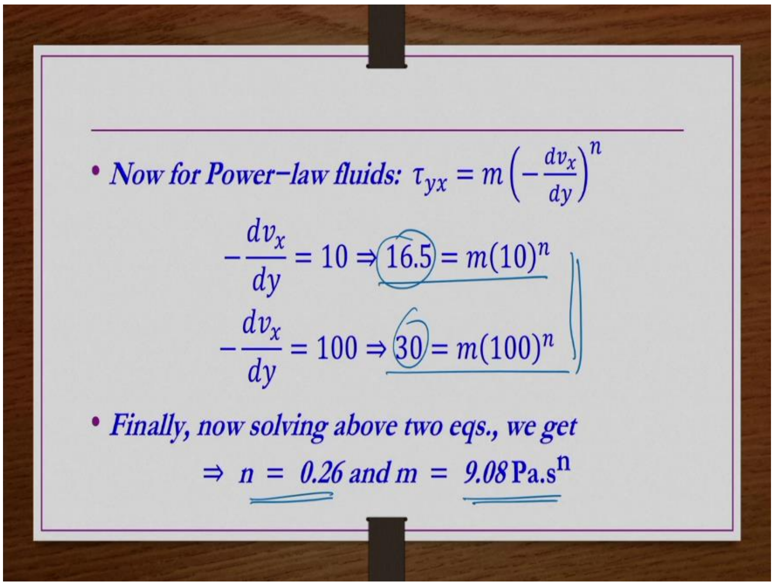

So, that is what we are going to do for power law model. Because same data if you are applying for the power law fluid or power law model then what are the corresponding model parameters m and n that we have to find out. So, when $\dot{\gamma}_{yx}$ is 10 you get this equation 16.5 = m $10^n$when $\dot{\gamma}_{yx}$is 100 then you get this equation 30 = m 100^n.

So, this τ_{yx} we already found using the parameters given for the other model Bingham plastic model right. Then finally, if you solve this two equations you will get n = 0.26 and then m = 9.08 $Pa.s^n$ right. This is how we have to solve the problems simple and straight forward.

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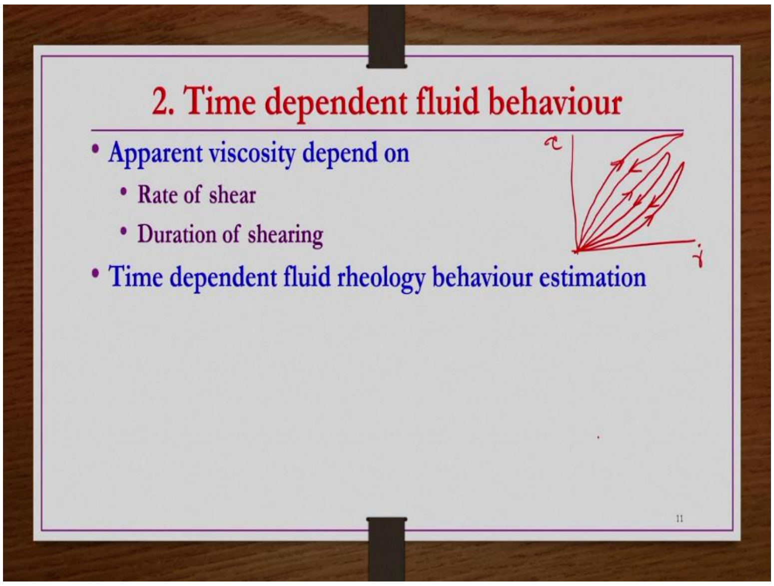

Now, we see the details of time dependent fluid behaviour. While categorizing or while classifying the non Newtonian fluids we have already seen this time dependent non Newtonian fluids the shear stress versus shear rate information whatever you say that is also function of time. That is duration of shearing is also going to affect the apparent viscosity of these fluids.

So, if the duration with the duration of shearing if the apparent viscosity is changing then we call such kind of fluids are a time dependent fluid behaviour, because with time it is changing the apparent viscosity is changing. So, the apparent viscosity depends on the rate of shear plus duration of shearing as well ok.

So, then time dependent fluid rheology behaviour estimation, how to do it, how do you find out whether the material is having time dependent rheological behaviour or not? So, for that simply what you do?

You do the shearing experiment ok get the τ versus $\dot{\gamma}$ information. How you do it? At a constant rate you increase the shear rate and then let us say then τ versus $\dot{\gamma}$ information you are getting like this. Now, to after reaching certain higher shear rate what you do? You should not change the rate at which you are changing the shear rate you decreased shear rate and then get the τ versus $\dot{\gamma}$ information again.

So, upper curve this one is while increasing the shear rate at a constant rate, lower curve is the same τ versus $\dot{\gamma}$ information for the same fluid. But decreasing the shear rate at the same rate for the same fluid using the same equipment at the same rate we are increasing the shear rate and then we are decreasing the shear rate. But both the thing we are doing you know at the constant rate at the same constant rate we are not changing the rate at which you are changing the shear rate information right.

So, then when you plot these two curves if you are, so if you both the curves if these are super-imposing then you can say that there is no time dependent behaviour, but if they are forming some hysteresis like this. So, then; that means, it must be having some time dependent behaviour. That is the reason when increasing and then decreasing you know the time is time of shearing is changing right, you are at a constant rate you are increasing the shearing.

And then you reach you reach a maximum shear rate at certain value and then you are again at the same constant rate you are decreasing the shearing. So, the time parameter has come into the picture so, but if the both the times if it is super-imposing τ versus $\dot{\gamma}$ information is superimposing then the material is time independent fluid rheologically fluid behaviour.

Rheological fluid behaviour of such material is time independent if they are superimposing, but if they are forming hysteresis like this then it is having some time dependent behaviour.

You also may have something like this ok. You also may have something like this initially in you know you may have a kind of almost steady straight line then again it may be having something like this that is also possible. Whatever the nature of the curve while increasing the shear rate and decreasing the shear rate if they are forming a hysteresis then time dependent fluid behaviour is existing right.

If they are not forming any hysteresis then such materials rheologically time independent ok right. So, rheogram is measured by increasing shear rate steadily from zero to some maximum value at a constant rate that is very much important.

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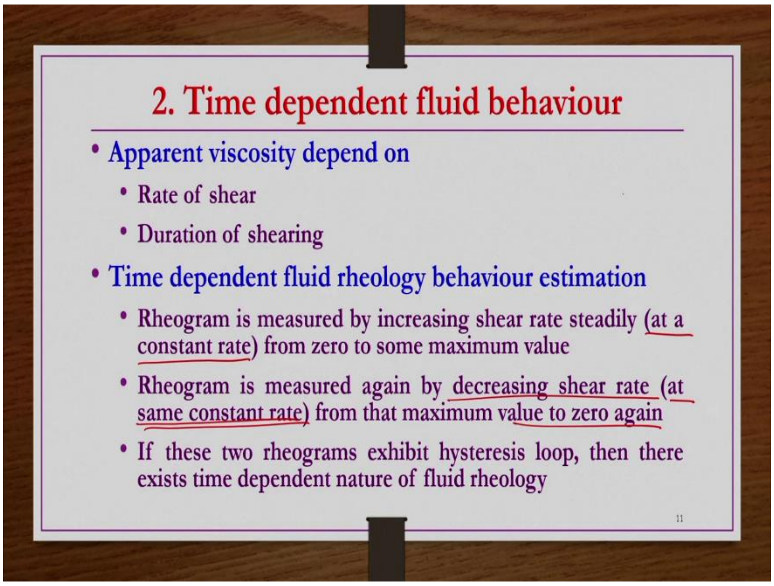

Now, after reaching some maximum value of shear rate rheogram is again measured by decreasing shear rate from that maximum value to zero value, but at the same constant rate at the same constant rate. Whatever rate you increase the shear rate to reach the maximum shear rate information or maximum shear rate value at the same rate you have to decrease the shear rate to come back to zero shear rate value.

Then this rheograms whatever the corresponding τ versus $\dot{\gamma}$ information is that you have to plot right. If these two rheograms exhibit hysteresis loop then there exist time dependent nature of fluid behaviour right.

If there exist a kind of hysteresis loop between these two rheograms while increasing the shear rate and then while decreasing the shear rate then; obviously, the material is having time dependent rheological behaviour. If they are super-imposing then the material is not having any time dependent rheological behaviour.

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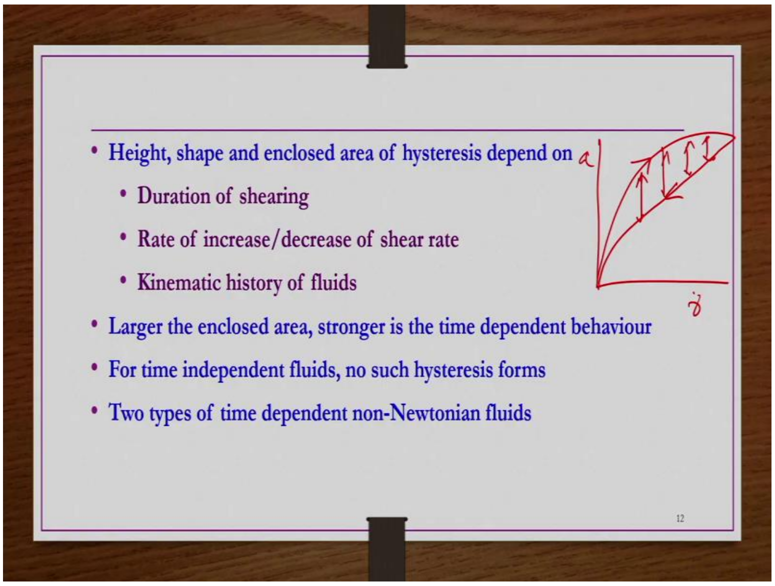

So, the height shape and enclosed area of hysteresis depends on several parameters. Some of them are duration of shearing right; how long are you doing the shearing. You know you are doing for 10 minutes or 10 hours or you know 100 minutes that also makes a difference and then rate at which increasing or decreasing of shear rate is taken place. At what rate you are increasing the shear rate or rate what rate you are decreasing or both going to have an influence and the height shape and enclosed area of the hysteresis.

Then, kinematic history of the fluids as well playing a role on the height shape and enclosed area of the hysteresis in the case of time dependent non Newtonian fluids. So; obviously, the larger the enclosed area stronger is the time dependent behaviour right. Whatever, this τ versus $\dot{\gamma}$ information is there if the hysteresis having the larger gap.

So, then; obviously, you know the time dependent behaviour is more if the gap is more here like ok. If this the hysteresis is narrower than we can say that a time dependent nature is weak it is not very strong. For time independent fluids no such hysteresis will form and then two types of time dependent non Newtonian fluids are possible; obviously, what are the two? Is the apparent viscosity is increasing or decreasing with the duration of shearing right.

So, these are the two possibilities are existing these are the two possibilities are existing. One is the if the apparent viscosity is decreasing with the duration shearing then we call them as thixotropic material if the apparent viscosity is increasing with the duration of shearing then we call them rheopectic or negative thixotropic materials. Now we see a few details of these two as well now.

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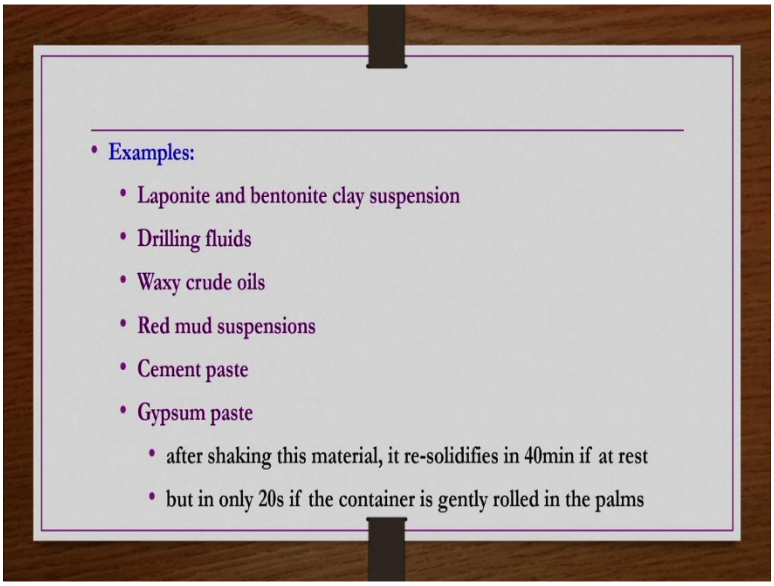

But examples for this time dependent non Newtonian fluids are laponite and bentonite clay suspensions, drilling fluids, waxy crude oils, red mud suspensions, cement paste, gypsum. Gypsum paste you know you make a gypsum paste and then shake it well and you make a gypsum paste and then keep it on a table on a container. So, then what happens? It re solidifies in larger time in larger time

Let us say it is a same container whatever in which you are taken the gypsum paste and then you what you do? You take in your palms and then rub like this what happens you know that may be re solidifying very quickly; that means, you know with the duration of shearing the gypsum paste apparent viscosity is sync increasing.

It is a negative thixotropic material or rheopectic material. So, after shaking this gypsum paste it resolidifies in 40 minutes if you keep it at rest, but it resolidifies within 20 seconds if the container is gently rolled in the palms ok.

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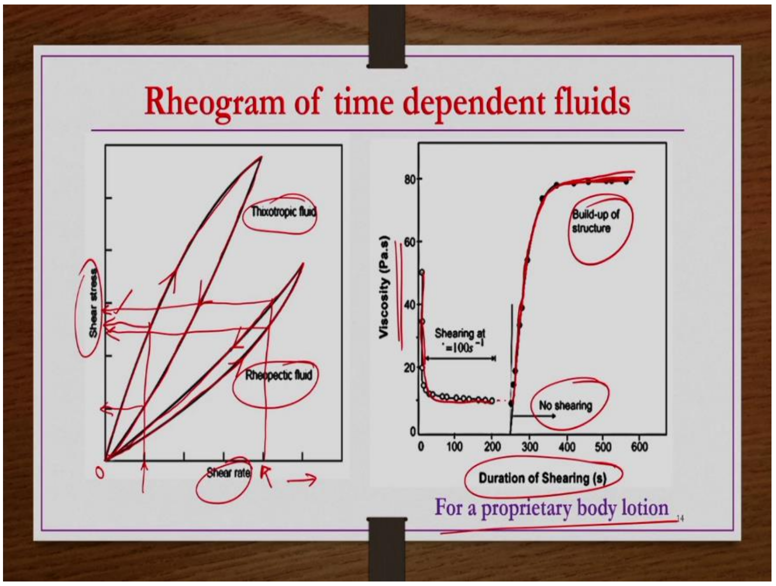

Now, rheogram of time dependent fluids as I already mentioned shear stress versus shear rate information you have to get for a you know for an increasing shear rate. Let us say increasing shear rate from 0 to some maximum value of shear rate you are increasing right.

But you are increasing at a constant rate and then reaching some maximum value of the shear rate. And then what you are doing next you are decreasing the shear rate from that maximum value of shear rate to 0 value, but at the same rate as we increase the shear rate, but at the same rate.

So, then now you can see you take this at this you know $\dot{\gamma}$ value you can see this shear stress is more while increasing shear rate and then shear stress is less while decreasing the a shear rate from maximum value to the 0 value. But both of them both of them have been done increasing and decreasing have been done at the same constant rate. So, the shear stress you can see they are very different from each other.

So, while increasing the shear rate this shear stress is having the larger value while decreasing the shear rate this shear stress is having the lower value for the same constant value of the shear rate. So, then apparent viscosity which is; obviously, shear stress divided by the shear rate right. So, that is decreasing initially it is having larger shear stress by same constant shear rate, then it is lower shear stress divided by the same constant that shear rate ok.

So, apparent viscosity has decreased. So, there thixotropic fluids may be having these kind of nature. Whereas, the other one you know rheopactic material when you are increasing the shear rate at a constant rate you may have curve like this while reaching the maximum value of shear rate. Once reaching the maximum value of shear rate as per your requirement, decrease the shear rate again at the same rate to reach the zero shear value then you have this curve.

So, now you can see here in this case at this constant shear rate information while increasing you have the lower value of shear stress while decreasing you have the higher value of shear stress for the same shear rate value, for the same shear rate value. That means, apparent viscosity has increased for this material and these are known as the rheopactic or negative thixotropic materials.

So, one example we are taking here proprietary body lotion. So, now, here if you plot the apparent viscosity versus duration of shearing you can see initially the viscosity has drops suddenly to some lower value from approximately 50 pascal second to approximately 10 pascal second right, as long as the duration of shearing is maintained up to something like you know 250 to 100 to 250 seconds.

And then you stop shearing. After stopping the shearing you know you can see a kind of a buildup of the structure of the material is taking place. So, then whatever the body lotion is there its viscosity is increasing when there is no shearing. This is one example of a time dependent non Newtonian fluids. Similarly we have more examples will be seeing, but we see in different categories within the thixotropy and then within the rheopectic categories will be seeing these examples.

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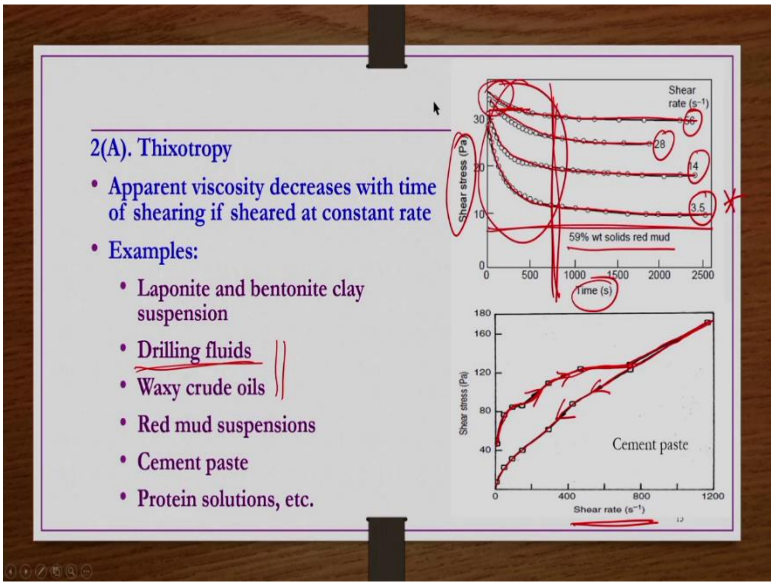

So, thixotropy apparent viscosity decreases with the time of shearing if sheared at constant rate. And in some examples are laponite and bentonite clay suspension drilling fluids, waxy crude oils, red mud suspensions and then cement paste. And then remember some of these examples like drilling fluids, waxy crude oils etcetera you might have also seen in the other type of non Newtonian fluids also in the previous lecture.

But again I said that mentioned I already mentioned in previous couple of lectures that this nature of the non Newtonian behaviour, nature of non Newtonian behaviour may be different under different type different range of shear rate. So, this thixotropy you may be finding under certain different range of shear rate whereas, the same material may be displaying power law behaviour under different range of shear rate ok.

So, shear rate is very essential; however, these examples are generalized way they are provided. So, that is the reason shear rates are not provided here ok. So; that means, the same fluid drilling fluids may also show some viscoplastic behaviour and then may also show thixotropic behaviour as well.

So, this cement paste we can take an example see now cement paste what we are doing, you know we are increasing the shear rate. So, when you are increasing the shear rate so the curve is like this. Shear stress versus shear rate curve is like this alright. We increase at certain constant rate.

Now, at the same rate we are decreasing the shear rate of this material. So, the corresponding shear stress versus shear rate information has been noted down. So, the curve the data points are like this and then we have drawn this curve. Now you can see you know whatever the previous slide the rheological behaviour whatever τ versus $\dot{\gamma}_{yx}$ are given they are theoretically plotted. But true data you can see a different forms may be you may not have a smooth curve in general like this in general.

So, you may be having curve like this ok, but; however, what you can see here? Here there is a hysteresis only within this only up to let us say how much it is approximately up to shear rate you know approximately 700 s^{-1} you know there is a time dependent behaviour after that there is no time dependent behaviour. You can see both of them are you know almost constant value after 800 $s^{-1}$are something like that.

Whereas now you take a 59 weight percent solids red mud, then shear stress versus time. Now this is shear stress versus time it is showing; that means, you know apparent viscosity also shear stress versus shears, apparent viscosity shear stress divided by the shear rate local shear stress divided by the corresponding shear rate. So, if the shear stress is changing with time; that means, apparent viscosity is also changing right.

So, now this experiment we have done at different shear rates right. 3.5 s^{-1}, s^{-1}, 28 $s^{-1}$and 56 s^{-1}. So, then what we can see when the shear rate is low the shear stress is sharply dropping down and then remaining almost constant right.

As you gradually increase the shear rate, what happens? Shear stress versus time curve these sharp drop whatever is there in the shear stress value in at initial time of shearing that is decreasing.

So; that means, at the higher shear rates whatever the time dependence behaviour is there that is also there only for the initial period of time only that is very small. So, for this material what we understand here that shear rate you know the time dependent behaviour is more significant if you are shearing at a low shear rates if you are shearing at low shear rates that is what you can see.

If the material is not having any time dependent behaviour when you are shearing it a constant rate you will be having a straight line you will be having a straight line you know for the shear stress versus time curve. But now you are not having straight line that straight line you are going to get after certain time only, but before that you can see there is a time dependence.

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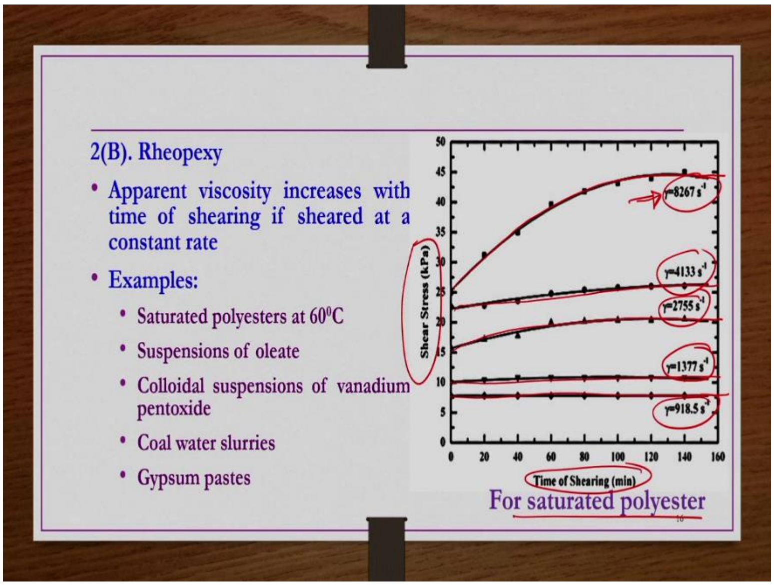

The next type is rheopexy. So, where apparent viscosity increases with the time of shearing if sheared at constant rate and then examples are saturated polyesters, suspension of oleate, colloidal suspension of vanadium pentoxide, coal water slurries, gypsum paste etcetera. So, now if you take this saturated polyesters now again shear stress versus time of shearing is a data is provided, but these data has been conducted a different shear rate.

So, one is the 918.5 s^{-1}like that gradually shear rate has been increased. Now what you can see here? When the shear rate is less than 1000 or even up to 1400 something like that shear stress versus time of shearing curve they are almost straight line, they are almost straight line.

That means, you know up to 1500 or up to 1400 s^{-1}shear rate. Roughly the material is showing time independent behaviour the material is showing time independent behaviour.

But if you are doing shearing it further higher shear rate then you can see you can see the shear stress is increasing with increasing the duration of shearing; that means, apparent viscosity is increasing, that you can clearly see. And then this increase is very stiff very significant especially when you are doing the shearing at a very large shear rate values of order of 10^4or something like that.

So, now, in the previous slide one example we have seen here in the case of thixotropy what you have seen for red mud suspensions the effect of time dependency behaviour on rheology of the material is higher at lower shear rate; is very higher at lower shear rate.

Whereas, in rheopexy whatever the material that now we have taken here the effect of time dependence behaviour on rheology of this material is higher at higher shear rate opposite trend we can say and then it not necessarily be you know true for all the cases.

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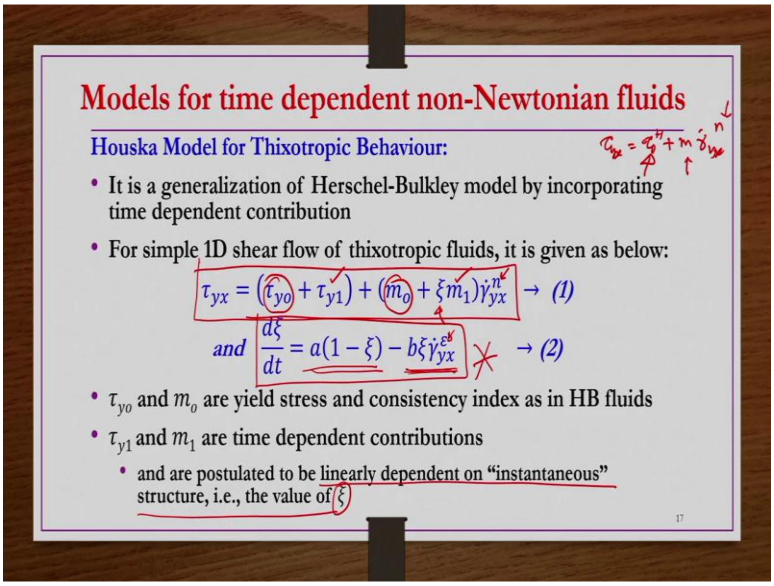

So, now we see models for time dependent non Newtonian fluids actually there are not many models for the time dependent non Newtonian fluids because of the dynamic nature of the material rheology. So, getting such information and incorporating them in a proper model form it is very difficult.

So, we have taken only one model Houska model for thixotropic behaviour which is having similar form like Herschel Bulkley fluid model. Herschel Bulkley fluid model what we have we have $\tau_{yx} = \tau_0^H + m(\dot{\gamma}_{yx})^n$ in this form.

So, now when we talk about Houska model for a thixotropic material, so then we have some addition here, we have some addition here and then we have some addition here like that you know the additions are you know corresponding to the time parameters. This τ_0^H m n are with respect to the you know you know just without considering without having any time behaviour when you have the time behaviour some additional parameters will come and join them.

So, that is what this model it is a generalization of Herschel Bulkley model by incorporating time dependent contribution for a 1 D simple shear flow of thixotropic fluids it is given as $\tau_{yx} = \left(\tau_{y0} + \tau_{y1}\right) + (m_0 + \xi m_1)\dot{\gamma}_{yx}^{\,n}$. So, this τ_{y0} and then this m and this n are same as you know model parameter as in Herschel Bulkley fluids whereas, this $\tau_{y1}$and then and this m_1 are corresponding to time dependent contribution.

And then ξ whatever is there ξ whatever is there that gives the dynamic equilibrium between the for the material because of the time dependent nature ok. How it is? Like you know we have $\frac{d\xi}{dt} = a(1-\xi) - b\xi\dot{\gamma}_{yx}^{\,\epsilon}$ right.

This is the buildup of the structure and then this is the breakup of the structure. These two terms indicates buildup of the structure because of the time dependent contribution and then this gives the breakup of the structure because of the time dependent contribution ok.

So, now τ_{y0} and m_0 are yield stress and consistency index as in Herschel Bulkley fluids τ_{y1} and m_1 are time dependent contributions. And these are postulated to be linearly dependent on instantaneous structure that is represented by the value of ξ. Instantaneous structure of the material is represented by this value ξ alright.

So, that is how it is you know incorporating how the buildup of the structure or breaking of the structure is has been brought into the equation number 2; that has been brought into the mathematical form by using this equation number 2.

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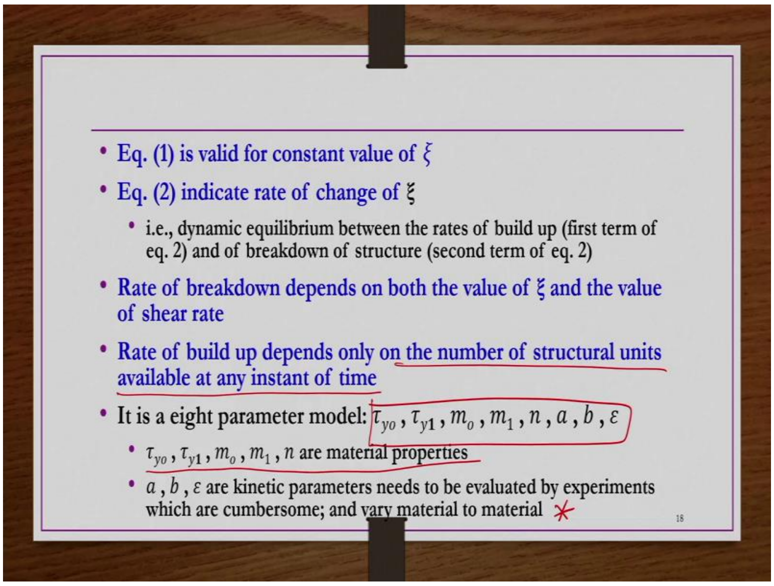

So, equation is valid for constant value of ξ; obviously. So, the equation number 2 indicate rate of change of ξ that is dynamic equilibrium between the rates of build up and then rates of breakdown of structure correspondingly represented by the first term and then second term on this equation number 2 as I shown in the previous slide.

So, rate of breakdown depends on both the values of ξ and the value of shear rate as we have seen. Whereas, because that is having the term $b\dot{\gamma}_{yx}^{\epsilon}$ these terms are there.

So, it depends on ξ as well as the $\dot{\gamma}_{yx}$that is shear rate also whereas, the rate of build up is having the term $a(1-\xi)$. So, it is not dependent on the shear rate, but rate of build up depends only on the number of structural units available at any instant of time that is the value of ξ ok. And then this model is very complicated because there are 8 parameters are existing τ_{y0}, τ_{y1}, m_0, m_1, n, a, b,ϵ.

So, this τ_{y0}, τ_{y1}, m_0, m_1, n are the material properties whereas, the a, b, ϵ are the kinetic parameters needs to be evaluated by experiments which are very cumbersome and very material to material; that is the primary drawback of this material ok. So, this is all about the time dependent non Newtonian fluids.

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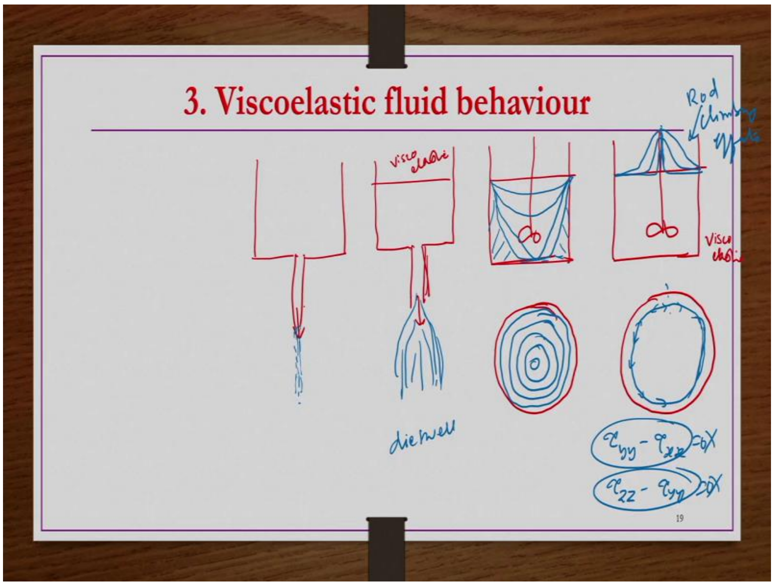

Now, we see the details of viscoelastic fluid behaviour. Viscoelastic fluids examples it is better to see, how we see? So, we take two materials or two solutions one which is not having any elastic behaviour and then one which is having elastic behaviour.

So, the second one rights right hand side is viscoelastic material, we are steering them now. When we are steering them, so let us say in the left hand side when there is a there is no elastic behaviour the surface of the liquid you know that falls something like this.

If you are rotating at higher speeds then this may falls like this. If you are even rotating very high speed something like this. So, then you may even have the surface like this or you know something like this right, all the fluid is being thrown towards the wall like this that is the pointer also wall of the container. Whereas if the material having elastic behaviour.

So, rather surface forming like this it you know it climbs the rod like this you can see the material is climbing the rod steering rod like whatever the steering rod is there that you know the material is steering that rod like this kind of thing that you can see in the viscoelastic material. Why it is happening, what are the reason that we are going to see anyway.

Now, we will do one more experiment. So, here you take a solution which is not having elastic behaviour ok. I am drawing the top view and then you take a solution which is having an elastic behaviour right like this. So, now, what we do now? We try to rotate right. So, now, when you rotating like this you can see the wrinkles let us say if you have some colour solutions etcetera. So, then you can see the wrinkles clearly something like this from the top right.

So, when you moment when you stop the rotations right, so you can see all the wrinkles are stopped, when there are no when there is no you know elastic behaviour in the material. But if the material is having elastic behaviour, so let us say you are rotating in an anticlockwise direction like this right. Now you rotate some for some period of time like few seconds or something like or a few minutes then you stop rotation.

So, then what you can see? This wrinkles whatever that you are you know moving in the anticlockwise direction even you stop rotating they will be further moving forward further slightly moving forward. Then they will stop they then they will start you know moving in the clockwise direction you rotated in the anticlockwise direction. Now, you can see that the some wrinkles are something like they are moving like you know anticlockwise direction. Why is it happening? Why is it happening?

Because the material whatever the viscoelastic material is there the elastic nature it is having the partial recover it is having some memory it is having some memory. So, it is trying to go back to its initial position right. So, that is one experiment. Then another experiment that you can do.

So, you take a container in which you have a solution which is not having any elastic behaviour you take another container in which you take a solution which is having elastic behaviour right. So, at the bottom there is an opening for both of the containers right.

So, when the material you release from the container you know in which you are taking a material which is not having any elastic nature. So, then you can see that material is flowing something like this, but the same thing if you do you open the container from the bottom the container in which you have taken the viscoelastic material.

So, then you can see the material flows out something like this a kind of swelling of this swelling behaviour you can see something like this. So, this is because of elastic nature because the material is having elastic nature.

So, why these are things happening? So, die swell these thing this is known as the die swell effect and this is known as the rod climbing effect. So, these are primarily found because of the normal stress differences this normal stress differences are in general found to be 0 for a Newtonian fluids. But for the viscoelastic materials they are not 0 they are not 0 for the viscoelastic material.

Because of this normal stress differences the materials are you know you know showing different kind of behaviour and then these kind of normal stress difference which are which are not 0 that you find primarily in viscoelastic fluids right. So, we see now some of these details right. Right now we see some of these details. So, this is about the viscoelastic fluid behaviour that is the material is having both viscous as well as the elastic nature.

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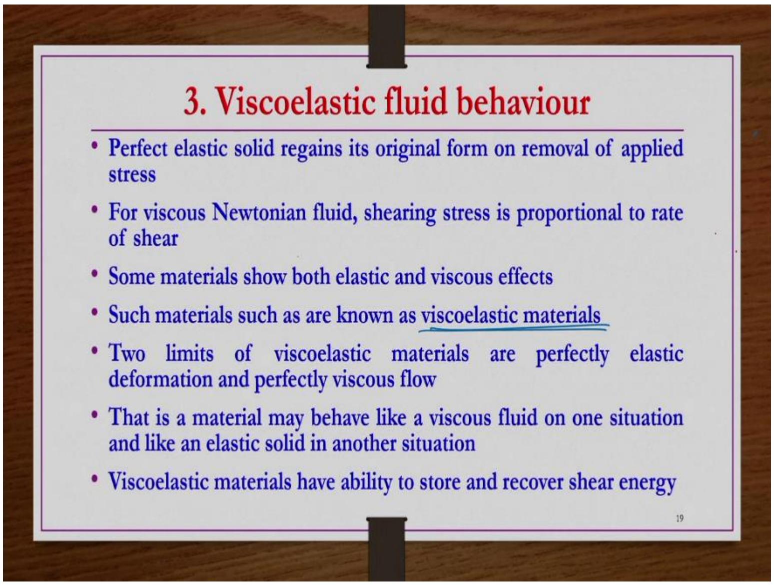

So, perfect electric solids try to regain its initial original form on removal of applied stress. When you apply a stress the external stress to the material which is elastic, so then it will undergo deformation. But when you remove that is applied external stress the material will try to come back to its initial position if it is coming completely to its initial position then we call them perfectly elastic solid material right.

But in the case of viscous Newtonian fluid shear stress is proportional to shear rate right. Once deform it will not come back to its initial position whatever is the reason ok. Whatever may be the range of shear rate or shear stress applied whatever the duration you applied and then when you stop the external force external stress you know nothing is going to have a kind of influence.

So, for viscous Newtonian fluid shear stress is proportional to the shear rate and then for such Newtonian fluids coming back to their initial position even partially is not possible. So, but there are some material which display both elastic nature and then viscous nature as well viscous flow nature as well. So, those material we call them viscoelastic materials, we call them viscoelastic materials.

Two limits of viscoeleastic materials are perfectly elastic deformation and perfectly viscous flow so; obviously, toward the two limits are possible. And then that is material may behave like a viscous fluid on one situation and like an elastic solid in other situation that is quite possible that is what it mean by. Then viscoelastic materials have ability to store and recover the shear energy.

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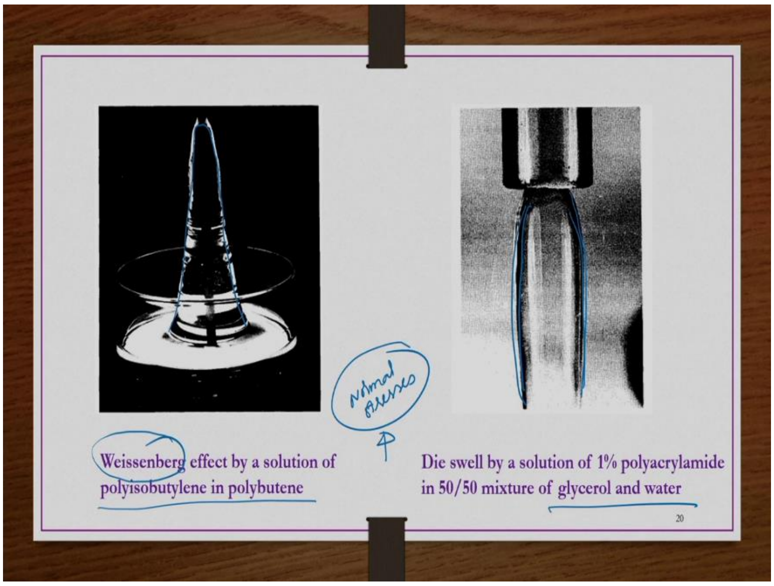

The material is climbing along this rod of the steerer. So, this is known as the Weissenberg effect of a solution of a polyisobutylene propylene. This is first reported by the scientist Weissenberg. Similarly die swell so, you can see the material when it is release so, and then it is swelling slightly right.

So, this is die swell by a solution of 1 percent polyacrylamide in 50 by 50 mixture of glycerol and water. So, both of these effects whatever we have seen into these two pictures are there because of because of the normal stresses which are not 0 and their difference is also not 0 ok.

So, this is the first instance where you know people realized about you know non zero normal stresses or their differences ok. So, that is the reason because of an investigation by Weissenberg this rod climbing effect is also known as the Weissenberg effect ok.

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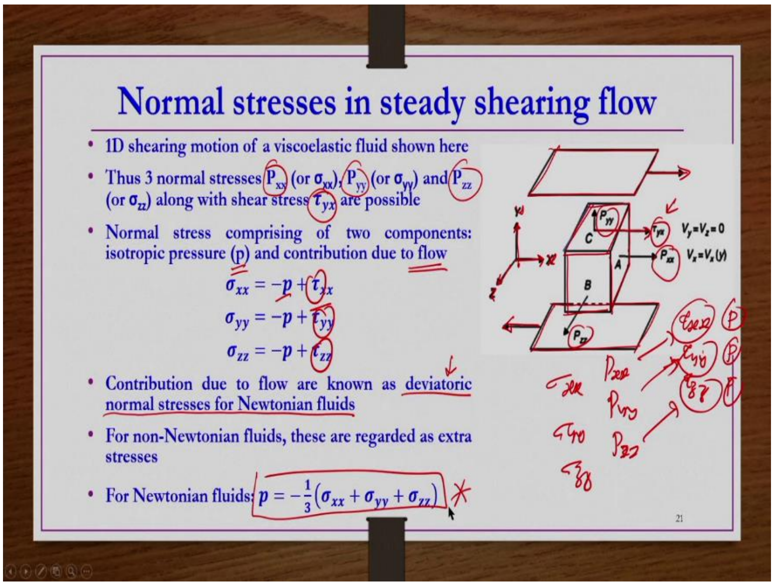

So, normal stress in steady shearing flow is important in the case of viscoelastic material that is what we realized right. So, in simple 1 D shearing motion you know what you get in general you get shear stress versus shear rate information only. But now if the material is having elastic behaviour even in simple 1 D shear motion. In addition to the shear stress you will also have the normal stresses and then their differences are going to play a role on the rheology of the material ok.

Let us take 1 D shearing motion of a viscoelastic fluid as shown here. So, now there are two plates are there. So, this is the one plate and then this is another plate and then we are doing the shearing experiment either side like this.

Whatever the fluid element that we have taken, let us say this fluid element we have taken between these two plates right. So, if it is Newtonian fluid you will be having only this shear stress τ_{yx} in general because this is our x direction this is our y direction and then this is our z direction.

So, we are taking 1 D shearing motion. So, in case of a material which is not having any elastic behaviour you will be having only τ_{yx} in the case of 1 D shearing motion. But if the material is having elastic nature as well, so then you will be having this $\tau_{xx}\tau_{yy}$and τ_{zz}also ok. So, that difference whatever the normal stresses; this normal stresses again they are having two components isotropic pressure and then normal stress because of the viscous flow.

Whatever the τ_{xx} τ_{yy} τ_{zz} are there. So, they are the normal stresses because of the viscous flow. Whatever because of the pressure isotropic pressure normal stress is there that is P ok. So, this P_{xx} is having these two contributions. Similarly P_{yy} is having these two contributions, P_{zz} is having these two contributions right fine. So, some books people write the $\sigma_{xx}\sigma_{yy}$ and then σ_{zz} also right ok.

So, now, three normal stresses that is P_{xx} or σ_{xx} P_{yy} or σ_{yy} P_{zz} or σ_{zz} along with the shear stress τ_{yx} are possible if the material is having you know elastic nature even in 1 D shearing motion like this alright. So, normal stress comprising of two components that is isotropic pressure p and then contribution due to the flow. So, they are $-p + \tau_{xx}$ is nothing but σ_{xx} similarly σ_{yy} is nothing but$-p + \tau_{yy}$ and then σ_{zz} is nothing but $-p + \tau_{zz}$.

So, contribution due to the flow are known as the deviatoric normal stresses for the case of Newtonian fluids. They are known as deviatoric normal stresses for the Newtonian fluids where are they are known as the extra stresses for the case of non Newtonian fluids. For the case of non Newtonian fluids whatever this τ_{xx} τ_{yy} τ_{zz} are there they are regarded as extra stresses. Whereas, for the Newtonian fluids they are regarded as a normal stresses deviatoric normal stresses.

So, for Newtonian fluids actually this τ_{xx} τ_{yy} $\tau_{zz}$are 0 they are not having they are individually 0 the difference is also 0. So, then p how you can define you can when you add this three $\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = -3p$ or $p = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$; like this you can relate. But you cannot relate you cannot have such kind of relation for this isotropic pressure in the case of a viscoelastic materials right.

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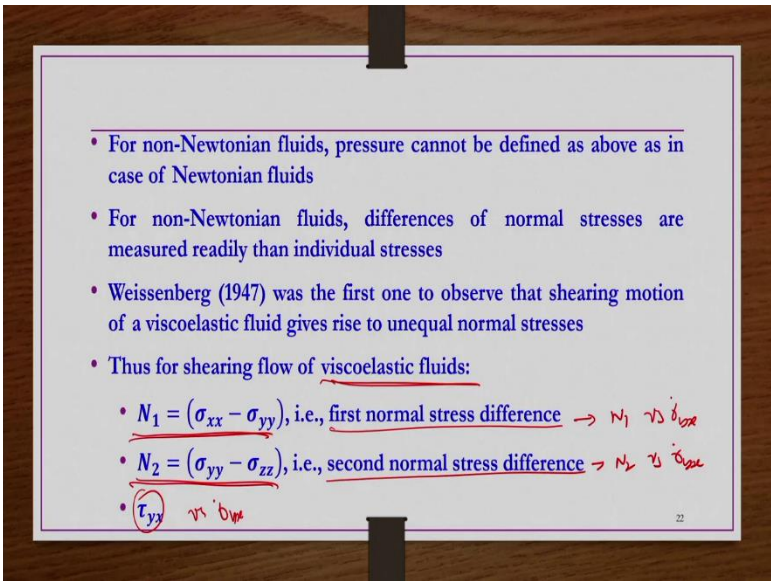

For non Newtonian fluids pressure cannot be defined as above as in the case of Newtonian fluids. For non Newtonian fluids differences of normal stresses are measured readily than individual stresses we are going to see in the subsequent lectures how to measure the rheology of the material.

So, there we can realize that measuring normal stress difference is quite easier compared to measuring individual normal stresses right. $\tau_{yy} - \tau_{xx}$ you can easily measure rather than measuring individually tau y y and then individually measuring τ_{xx}.

So, Weissenberg was the first one to observe that shearing motion of viscoelastic fluid gives rise to unequal normal stresses and then that is causing the material to climb the rod while stating this viscoelastic material viscoelastic solution when you are stating using a steerer. So, then the material is climbing the rod because of this normal stress differences are not equal for those materials.

So; obviously, for you know shearing flow of viscoelastic fluids, what are the information that you need to have? In general if it is not having you know normal stress are not important. So, then simply τ_{yx}versus $\dot{\gamma}_{yx}$ information is sufficient that is what we have seen for the time independent as well as time dependent fluids also.

In time dependent fluid additional term because of the dynamic equilibrium $\frac{d\xi}{dt}$ kind of terms have come otherwise, only τ versus $\dot{\gamma}$ information only we are using till now.

But now here in addition to the τ versus $\dot{\gamma}$ information you need to have the first normal stress difference information and second normal stress difference N_2 in addition to shear stress τ_{yx}. So, in order to define a viscoelastic fluid just having τ_{yx} versus $\dot{\gamma}_{yx}$ information is not sufficient you should also have N_1 versus $\dot{\gamma}_{yx}$ and then N_2 versus $\dot{\gamma}_{yx}$ information is also required.

N_1 is nothing but first normal stress difference that is $\sigma_{xx} - \sigma_{yy}$ and then N_2 is nothing but second normal stress difference that is $\sigma_{yy} - \sigma_{zz}$ right. In the case of you know material which are not having elastic behaviour viscous material.

So, then we have the viscosity or apparent viscosity. So, similarly you know can we have something additional parameter here in the case of a material which are having elastic behaviour or not. So, then we have this normal stress coefficients which are defined similar way alright.

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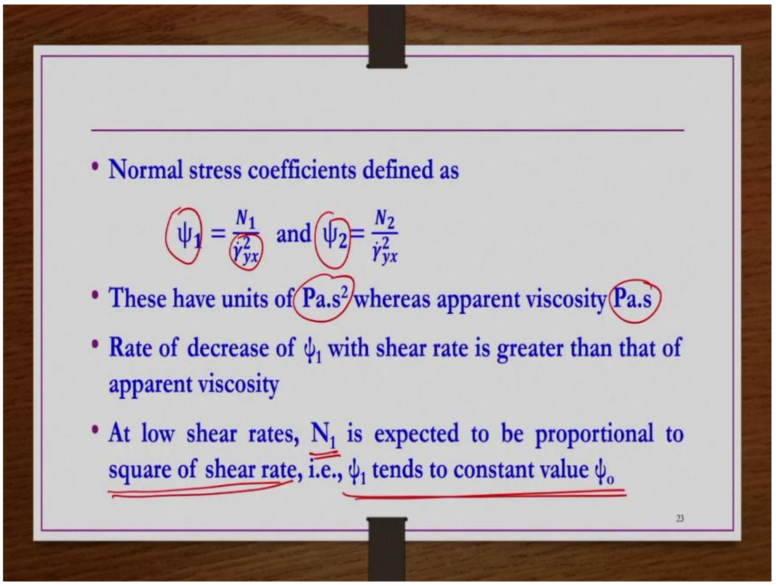

So, normal stress coefficient this $\frac{N_1}{\dot{\gamma}_{yx}^2}$ is ψ_1 and then $\frac{N_2}{\dot{\gamma}_{yx}^2}$ is ψ_2. This ψ_1 and then ψ_2 are nothing but the normal stress coefficients right which are like you know related to the first normal stress difference and then second normal stress difference respectively right.

So, these are the having these are having units of $Pa.s^2$ whereas, the apparent viscosity is having Pa.s. Because now here this stress difference whether it is shear stress or the normal stress the units are Pascals only.

But it is being in apparent viscosity shear stress is divided by the shear rate. So, pascal per pascal divided by second inverse you are having so that pascal second. But now here the normal stress differences we are dividing by the square of the shear rate. So, that is the reason $Pa.s^2$ we are having. Why are we doing this? Because you know when you plot this shear stress versus shear rate information you can have a kind of a then you do not have any kind of limiting kind of conditions.

But here N_1 versus shear rate when you plot. So, you can have a kind of limiting conditions which is having you know constant values. So, that we will see in the next slides anyway, so let us not worry about that one. Rate of decrease of ψ_1 which shear rate is greater than that of apparent viscosity at what rate it is decreasing because of this square term here.

And then at low shear rate at low shear rates N_1 is expected to be proportional to the square of shear rate that is the reason you know we are dividing N_1 by square of the shear rate. So, that to define this normal stress coefficients or the ψ_1 tends to have the constant value $\psi_0$ok. At low shear rates this N_1 is expected to be proportional to the square of shear rate that is the reasons $\psi_1$tends to constant value ψ_0.

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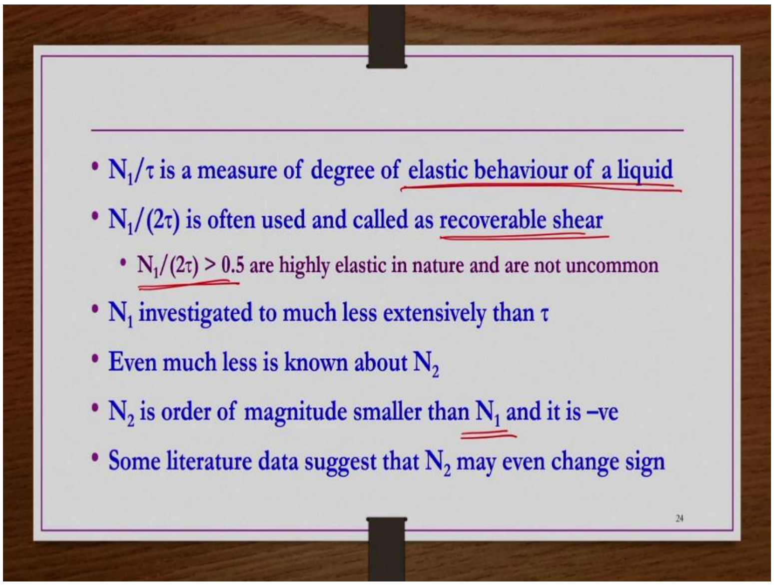

And then N_1/τ is a measure of degree of elastic behaviour of a liquid right. N_1/τ is a measure of degree of elastic behaviour of the liquid. $N_1/2\tau$ is often used and called as the recoverable shear in order to know whether the material is having any elastic behaviour or not ok. $N_1/2\tau > 0.5$ is also possible and it is not at all uncommon.

If it is $N_1/2\tau$ value is very much larger than 0.5 then we can say that material is very highly elastic in the nature. And then N_1 versus $\dot{\gamma}$ information is much less compared to the τ versus $\dot{\gamma}$ information available experimental data.

So, N_2 versus $\dot{\gamma}$ information even much less compared to N_1 versus $\dot{\gamma}$ and then tau versus gamma dot information that is available right. So, N_2 is also found is order of magnitude smaller than N_1 and then it is negative, sometimes it also changes the sign ok.

So, in the next class we will be discussing a few more details of the viscoelastic fluids and then few problems on viscoelastic fluids. References for this particular lecture are provided here.

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Thank you.