

Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 29
Heat Transfer Combined with Chemical Reactions

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Heat Transfer Combined with Chemical Reactions. Till now we have seen an individual movement on transfer problems, we have seen individual heat transfer problems, today we will see a few heat transfer problems followed by heat transfer combined with chemical reactions ok.

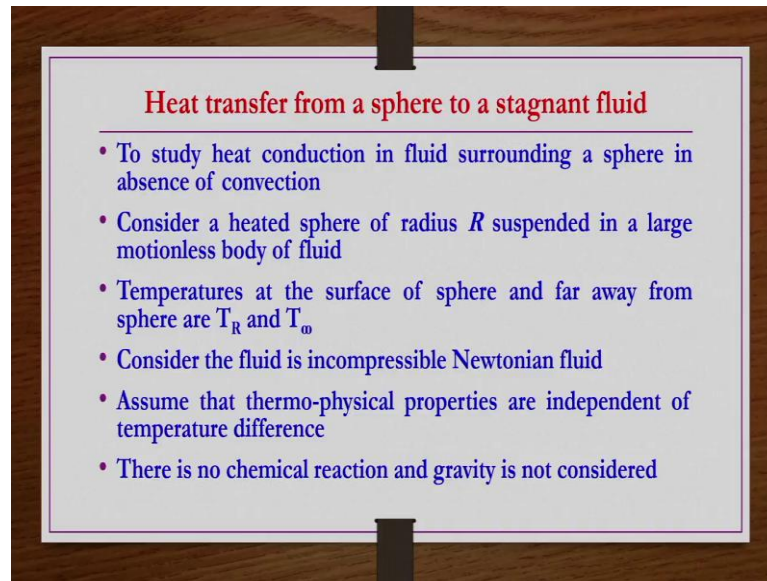
In today's lecture what we are going to see? We are going to discuss three different problems. All of them are based on a spherical geometry like even the reaction whatever we are going to take. So, that is also on a spherical catalyst pellet surface ok spherical catalysts we have. So, then there is a reaction is taking place and then because of that reaction you know the heat transfer I mean heat transfer is being affected ok.

So, then how the temperature distribution is going to change with the heat generation because of the reaction that is what we are going to see. Since we are taking you know spherical geometry spherical particles we are taking. So, what we do? Before going into the problem where the reaction is also involved, we take a spherical geometry and then see how the temperature distribution is a taking place.

Let us say if the same spherical particle is surrounded in an infinite fluid medium right. The infinite fluid medium is there so, but that fluid medium is a stagnant that is motionless and then in that fluid medium there is a heated sphere is there. So, now, that heated sphere from that sphere surface to the surrounding stagnant fluid the heat transfer is taking place. And then heat transfer is taking place only because of the conduction because the fluid is stagnant right.

So, then under such conditions we know the correlation for the Nusselt number is $Nu = 2$. So, that derivation also we see first and then we go into a porous spherical particle then we go into catalytic spherical particle where the reaction is also taking place in a step by step manner.

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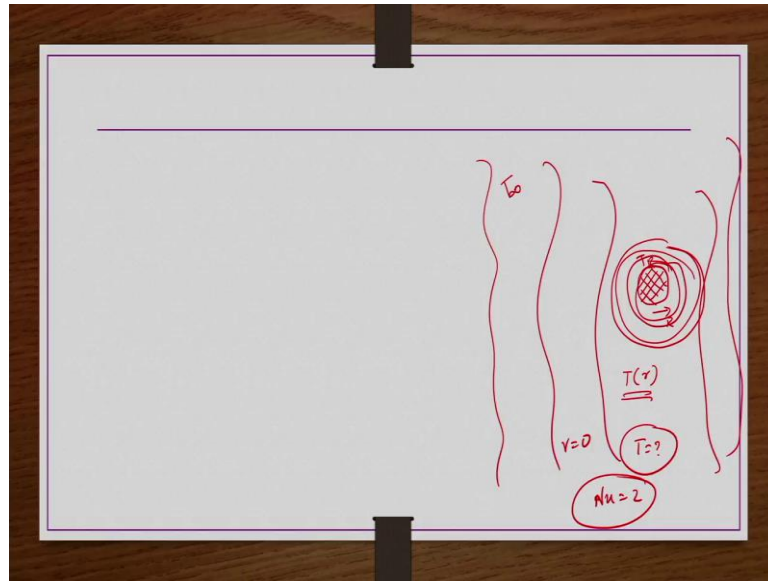
Heat transfer from a sphere to a stagnant fluid

- To study heat conduction in fluid surrounding a sphere in absence of convection
- Consider a heated sphere of radius R suspended in a large motionless body of fluid
- Temperatures at the surface of sphere and far away from sphere are T_R and T_∞
- Consider the fluid is incompressible Newtonian fluid
- Assume that thermo-physical properties are independent of temperature difference
- There is no chemical reaction and gravity is not considered

So, heat transfer from a sphere to a stagnant fluid. So, the purpose is to study the heat conduction in fluid surrounding a sphere in absence of convection because we are taking infinitely bounded stagnant Newtonian fluid. Since it is stagnant there will not be any convection. So, consider a heated sphere of radius R suspended in a large motionless body of fluid right. Temperatures at the surface of sphere and far away from the sphere are T_R and T_∞ respectively.

Consider the fluid is incompressible Newtonian liquid. And then assume that thermo-physical properties that is C_p , ρ , μ etcetera are independent of are independent of the temperature difference. So, they are not being affected by the difference $T_R - T_\infty$. And there is no chemical reaction and gravity is not considered in this problem ok.

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So, schematically if you see what we have? We have a spherical particle non-porous spherical particle is there and then it is bonded in infinite amount of fluid, that is there or no wall effects that is the purpose taking infinitely bounded large volume of the fluid ok. So, this spherical particle is suspended in this large volume of the fluid right. So, the surface temperature is T_R radius of the sphere is R and then far away temperature is T_∞ right.

So, now in this case the T_R is higher than the T_∞ . So, then heat transfer is taking place and then surrounding fluid whatever is there so, heat transfer may be taking place in the in this manner because of the conduction right. So, because why conduction only? This fluid is stagnant motionless. So, stagnant in the sense v is 0. If v is 0 all the convection terms in the left hand side whatever are there in energy equation they will be negligible ok.

So, for this case what is the T ? Right. How it is changing? And then here the conduction would be dominated in the radial direction only because there is no rotation. So, in θ direction there will not be anything any transfer of the momentum or there will not be any transfer of energy in θ direction because there is no rotation of the sphere right.

And then similarly in π direction also there is nothing is happening. So, whatever the transfer is there radially it is transferring to the surrounding fluid that is the region temperature is a function of r only. What is that function of r that is what we are going to find out now, ok. And then once we have the temperature profile. So, then you can find

out what is the Nusselt number. So, this you might have studied it is equals to 2. So, that also we do the derivation.

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Solution:

- Fluid is stagnant, thus no need to solve E.O.M.
- In energy equation consider only conduction as specified in the problem
- Energy equation for an incompressible fluid in spherical coordinates:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right]$$

$$\Rightarrow \frac{k}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = 0$$

$$\Rightarrow r^2 \frac{\partial T}{\partial r} = C_1 \Rightarrow \frac{\partial T}{\partial r} = \frac{C_1}{r^2} \Rightarrow T = -\frac{C_1}{r} + C_2$$

$r=R \Rightarrow T=T_R$
 $r=r \Rightarrow T=T_0$

So, fluid is stagnant that is given in the problem statement. So, there is no need to solve the equation of motion because when we solve the energy equation in the left hand side velocity terms are there. So, then in general before getting into the energy equation we have to get the velocity of the fluid or velocity distribution of the fluid we must get it. So, however, in this case that is not required because the fluid is motionless it is stagnant. So, v is 0. So, then left hand side terms of energy equation we do not need to worry.

And then all these problems we are doing for the steady state condition. So, $\frac{\partial T}{\partial t}$ will also be 0 because of the steady state ok. So, the energy equations we are we need to solve and then in energy equation also consider only conduction as specified in the problem because the fluid is stagnant. So, we will be taking only conduction terms.

However, we as a normal practice you know we will be simplifying the energy equation to get that expression for the conduction in the radial direction in spherical coordinates ok. So, that way we can do or you can do, what you can do? You can do simple heat balance because heat balanced because of the conduction.

You know then in heat balance that is heat transfer taking place because of the conduction in r direction. So, that heat balance you can do and then get the required energy equation

required simplified energy equation for the temperature distribution. But what we follow our normal you know convention whatever we have been following in this course, that is we simplify the energy equation.

So, energy equation in spherical coordinates is given here. So, steady state problems we are solving so, this term is 0, fluid is stagnant so, then all it all velocity terms should be 0 right. And then conduction is taking place in only in the r direction and then thus temperature is function of r . So, then this term would be there right. So, next is temperature is not function of θ , it is not function of ϕ also because only conduction in the radial direction that is what we are taking.

So, what we get from here? So, this is the equation. This equation we can integrate to get the required temperature distribution right. This same equation we may also get in a conventional way of a conventional way of a you know obtaining the governing equations, conservation equation that you take a spherical particle of radius R right.

And then this particle is suspended in infinite volume of a stagnant fluid. So, that also geometry also you what you take, you take R_∞ or $R = \infty$ also you can take. So, in general we write R_∞ or $R = \infty$ also you can take that is far away that. What does it mean by $R = \infty$? It is not ∞ , but compared to the R , R_∞ value is very large very very large that you can take ∞ ok.

So, now here the surface temperature is T_R , here the surface temperature is T_∞ right. So, now, what you can do? You can take a volume element like in between these two limits like this right. So, this is the volume element in the spherical geometry. So, whose thickness is dr whose thickness is dr .

So, let us say radius of this element inner radius of this element is small r and then radius of a outer element is r plus dr . So, that the thickness of this element is dr . So, now, what is the rate of heat entering in r direction because of the conduction minus what is the rate of heat leaving in the r direction at $r + dr$ because of the conduction that you can write down, right.

And then you take the balance is equals to 0 because we are not taking any accumulation term here steady state. So, then you get this equation that way also you can do it ok. So,

now, on integration what we get? $r^2 \frac{\partial T}{\partial r} = C_1$. So, the same thing we can write $\frac{\partial T}{\partial r} = \frac{C_1}{r^2}$. So, this is also going to be useful for our further calculations.

So, now, once again if you do the integration what you get? You get $T = -\frac{C_1}{r} + C_2$. So, now we need two boundary condition to get to these 2 constants, C_1, C_2 . One condition is at $r = R$ $T = T_R$ at $r = \infty, T = T_\infty$ this is what we know ok.

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$\Rightarrow T = -\frac{C_1}{r} + C_2$ ✓

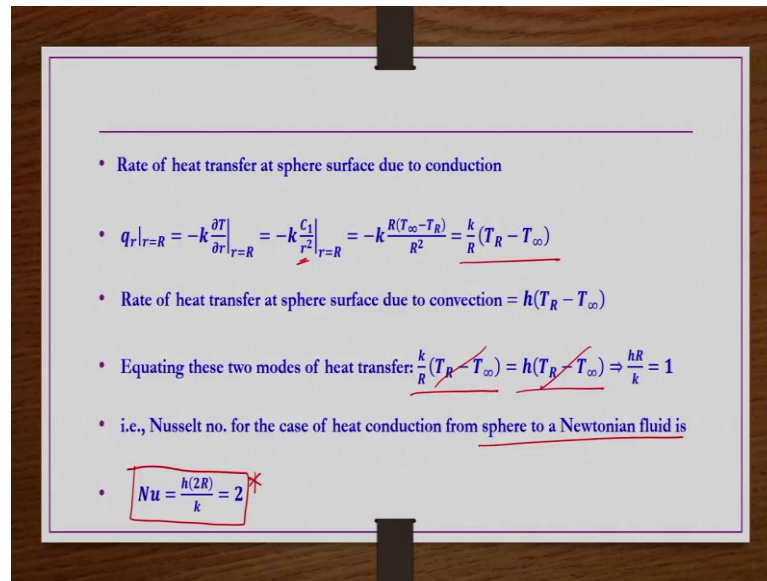
- BC-1: at $r = R \Rightarrow T = T_R = -\frac{C_1}{R} + C_2$
- BC-2: at $r = \infty \Rightarrow T = T_\infty = -C_1(0) + C_2 \Rightarrow \underline{C_2 = T_\infty}$
- From BC-1: $-\frac{C_1}{R} = (T_R - C_2) \Rightarrow \underline{C_1 = (T_\infty - T_R)R}$
- Thus temperature distribution: $\Rightarrow T = -(T_\infty - T_R) \left(\frac{R}{r}\right) + T_\infty \Rightarrow \left(\frac{T - T_\infty}{T_R - T_\infty}\right) = \frac{R}{r}$ ✗

So, this is the temperature profile that we got. Now, we will obtain these constant C_1, C_2 . At $r = R$ $T = T_R$. So, $-\frac{C_1}{R} + C_2 = T_R$. And then at $r = \infty, T = T_\infty$ that is $-\frac{C_1}{\infty}$ that is $0 + C_2$ so; that means, $C_2 = T_\infty$ we get.

Now, from this boundary condition if you substitute $C_2 = T_\infty, -\frac{C_1}{R} = T_R - C_2$ that is T_∞ . So, C_1 would be $(T_\infty - T_R)R$ alright. So, now, these two constants C_1 and C_2 if you substitute here you get the final temperature distribution like $T = -(T_\infty - T_R) \left(\frac{R}{r}\right) + T_\infty$.

The same thing we can write $\left(\frac{T - T_\infty}{T_R - T_\infty}\right) = \frac{R}{r}$. So, this is the temperature distribution that we get for this case of heat conduction from a sphere to a stagnant Newtonian fluid ok.

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So, now, we try to find out the Nusselt number expression for this case. So, rate of heat transfer at sphere surface due to conduction is nothing but $q_r|_{r=R} = -k \frac{\partial T}{\partial r}|_{r=R}$. So, $\frac{\partial T}{\partial r}$ we have seen is $\frac{C_1}{r^2}$ right at $r = R$. So, $-\frac{k}{R^2}$ and then C_1 is nothing but $R(T_\infty - T_R)$. So, that is $\frac{k}{R}(T_R - T_\infty)$ is nothing but the rate of heat transfer at a sphere surface due to the conduction.

Similarly, if at all there is some convection little convection whatever is there. So, that rate of heat transfer at sphere surface due to the convection is $h(T_R - T_\infty)$, h may be very very small in this case so, which can be negligible ok. So, equating these two modes of heat transfer that is $\frac{k}{R}(T_R - T_\infty) = h(T_R - T_\infty)$. So, $(T_R - T_\infty)$ both sides cancel out.

So, then we have $\frac{hR}{k} = 1$. So, both sides if you multiply by 2, then you have $\frac{h2R}{k} = 2$ that is nothing but $\frac{hd}{k}$ is Nusselt number. So, Nusselt number for the case of heat conduction from sphere to Newtonian fluid is 2. This you might have studied in your heat transfer courses right you may be remembering. So, this is how you get the derivation for this problem right.

Now, we take next problem heat conduction in spherical shell. So, now, what we have taken till now? We have taken a geometry spherical particle right and then from there the surrounding fluid the heat conduction is taking place. Now we take a spherical shell alright

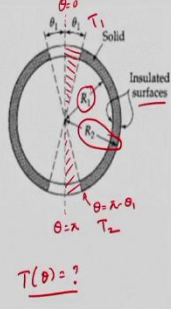
and then in that shell you know some kind of holes have been made and then remaining portion of the shell is insulated.

Then how to get the temperature profile? So, here also in this case also only heat conduction is taking place because there is no fluid involvement at all. At least in the previous case there was a fluid, but that was stagnant, but in this problem there is no fluid at all ok.

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Heat Conduction in Spherical Shell

- Consider a spherical shell with inner and outer radii R_1 and R_2
- Make a hole in the shell at north pole by cutting out conical segment in the region $0 \leq \theta \leq \theta_1$
- Similarly make a hole at the south pole by cutting out conical segment in the region $(\pi - \theta_1) \leq \theta \leq \pi$
- Surface at $\theta = \theta_1$ is maintained at constant temperature $T = T_1$
- Surface at $\theta = \pi - \theta_1$ is maintained at constant temperature $T = T_2$
- What is the temperature profile in the shell



So, consider a spherical shell with inner and outer radii R_1 R_2 . Make a hole in the shell at North Pole by cutting out conical segment in the region between $\theta = 0$ to $\theta = \theta_1$. Pictorially if you see here inner radius of the shell is R_1 outer radius of shell is R_2 right. And now, what we are having? Here this solid is insulated surface whatever the solid surface is this insulated. So, now, we have $\theta = 0$ location this one and $\theta = \pi$ location is this one.

So, now, between $\theta = 0$ to $\theta = \theta_1$, what you do? Make a hole that hole should be conical segment like this right. So, the angle of this conical segment is θ_1 that is what we understand. Now, similarly make a hole at South Pole by cutting out conical segment in the region $\theta = \pi - \theta_1$ to π .

So, now, this is this location is nothing but $\theta = \pi - \theta_1$. So, from $\theta = \pi$, $\theta = \pi - \theta_1$ whatever the angle it is making from that angle you make another hole at the South Pole like this right. And then remaining surfaces insulated here.

The temperature at $\theta = \theta_1$ is T_1 at $\theta = \pi - \theta_1$ temperature is T_2 right. Since now this the spherical shell this holes have been made. So, the remaining portion though it is insulated the heat conduction is taking place right. And then heat conduction is taking place in the θ direction; obviously, here, so, that temperature as function of θ direction that we have to find out now, ok. What is the temperature profile in the shell?

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- In this case there is no involvement of fluid, hence only conduction is the mode of heat transfer
- Conduction in θ -direction only
- Energy equation in spherical coordinates:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right]$$

$$\Rightarrow \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) = 0 \Rightarrow \sin \theta \frac{\partial T}{\partial \theta} = C_1 \Rightarrow \frac{\partial T}{\partial \theta} = \frac{C_1}{\sin \theta}$$

$$\Rightarrow T = C_1 \ln \left| \tan \frac{\theta}{2} \right| + C_2$$

$$\Rightarrow T = C_1 \ln \left(\tan \frac{\theta}{2} \right) + C_2$$

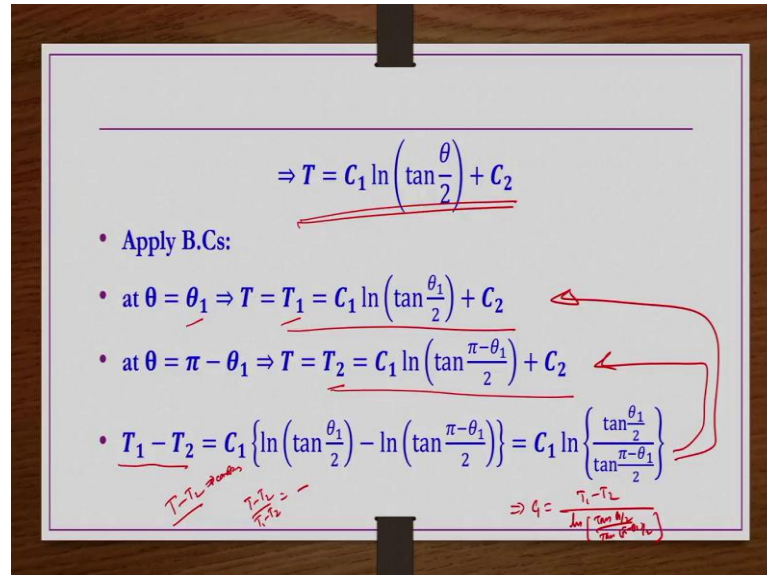
In this case there is no involvement of fluid, hence, only conduction is the mode of heat transfer. Conduction would also be there only in θ direction, it will not be there in r and ϕ direction according to schematic given. And then energy equation in spherical coordinates we have to simplify since steady state.

So, this term is 0 there are no velocity terms at all since there is no fluid involvement. So, left hand side all terms are 0. Temperature is function of θ only it is not function of r and ϕ . So, these two terms are gone. So, then what we get? We get $\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) = 0$ right.

So, that means, $\left(\sin \theta \frac{\partial T}{\partial \theta} \right) = C_1, \frac{\partial T}{\partial \theta} = \frac{C_1}{\sin \theta}$. This further if you integrate say you get $T = C_1 \ln \left(\tan \frac{\theta}{2} \right) + C_2$, but now here the range of θ is between 0 to π only. So, then whatever the tan value should be there of that part θ region. So, you will be getting only positive

values. So, then because of that one you can remove this modulus. So, that is $T = C_1 \ln \left(\tan \frac{\theta}{2} \right) + C_2$.

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Now this even C_2 we have to find out using the boundary conditions that we are having. The same temperature, expression is written again here. At $\theta = \theta_1$, $T = T_1$ that is given. So, $T_1 = C_1 \ln \left(\tan \frac{\theta_1}{2} \right) + C_2$ right. At $\theta = \pi - \theta_1$ $T = T_2$ that is also given.

So, that should be equal to $C_1 \ln \left(\tan \frac{\pi - \theta_1}{2} \right) + C_2$. So, $T_1 - T_2$ that is this equation minus this equation if you do, you get $C_1 \ln \left(\tan \frac{\theta_1}{2} \right) - \ln \left(\tan \frac{\pi - \theta_1}{2} \right)$, that is $C_1 \ln \left\{ \frac{\tan \frac{\theta_1}{2}}{\tan \frac{\pi - \theta_1}{2}} \right\}$ you can get.

So, now this expression you can substitute either in this here this equation or you substitute here in this equation and then you can find out C_2 right. Then you will get some kind of expression because from here you can get you are getting $C_1 = \frac{T_1 - T_2}{\ln \left(\frac{\tan \frac{\theta_1}{2}}{\tan \frac{\pi - \theta_1}{2}} \right)}$ this is what you are getting, but not necessary to do that one again it becomes lengthier calculations.

Because in this of type of heat conduction problem the temperature distribution we are getting in the form of a ratio between differences two temperature differences this is one

temperature difference. Similarly other temperature difference expression if you get for $T - T_2$ then you can have an expression $\frac{T-T_2}{T_1-T_2}$ function of θ .

Because this $T - T_2$ will also be having some constant we will not evaluate and then see how we can get rid of this constant by taking these two ratios.

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$$\begin{aligned} \bullet \quad T_1 - T_2 &= C_1 \left\{ \ln \left(\tan \frac{\theta_1}{2} \right) - \ln \left(\tan \frac{\pi - \theta_1}{2} \right) \right\} = C_1 \ln \left\{ \frac{\tan \frac{\theta_1}{2}}{\tan \frac{\pi - \theta_1}{2}} \right\} \\ \bullet \quad \text{Similarly: } T - T_2 &= C_1 \left\{ \ln \left(\tan \frac{\theta}{2} \right) - \ln \left(\tan \frac{\pi - \theta}{2} \right) \right\} = C_1 \ln \left\{ \frac{\tan \frac{\theta}{2}}{\tan \frac{\pi - \theta}{2}} \right\} \\ \Rightarrow \frac{T - T_2}{T_1 - T_2} &= \frac{\ln \left\{ \frac{\tan \frac{\theta}{2}}{\tan \frac{\pi - \theta}{2}} \right\}}{\ln \left\{ \frac{\tan \frac{\theta_1}{2}}{\tan \frac{\pi - \theta_1}{2}} \right\}} = \frac{\ln \left(\tan \frac{\theta}{2} \right) - \ln \left(\tan \frac{\pi - \theta}{2} \right)}{\ln \left(\tan \frac{\theta_1}{2} \right) - \ln \left(\tan \frac{\pi - \theta_1}{2} \right)} \end{aligned}$$

So, this $T_1 - T_2$ is written once again here. Now, similarly if you wanted to write $T - T_2$, what you have to do? Θ_1 you have to replace by θ right. T is nothing but temperature at unknown θ T_1 is nothing but temperature at some known θ_1 right, T_2 is nothing but temperature at some known $\pi - \theta_1$ angle fine.

So, now, similarly if you wanted to know unknown temperature at some unknown θ value you can write you can replace the θ_1/θ by writing $T - T_2 = C_1 \ln \left\{ \ln \left(\tan \frac{\theta}{2} \right) - \ln \left(\tan \frac{\pi - \theta}{2} \right) \right\}$ that is this one $C_1 \ln \left\{ \frac{\tan \frac{\theta}{2}}{\tan \frac{\pi - \theta}{2}} \right\}$. So, now you can do this divided by this expression so, then C_1 C_1 would be cancelled out.

So, that will simplify our life not requiring to calculate the C_1 C_2 separately ok. So, that is $\frac{T-T_2}{T_1-T_2}$ if you write you get $\frac{\ln \left(\tan \frac{\theta}{2} \right) - \ln \left(\tan \frac{\pi - \theta}{2} \right)}{\ln \left(\tan \frac{\theta_1}{2} \right) - \ln \left(\tan \frac{\pi - \theta_1}{2} \right)}$. So, this at this unknown θ value what is unknown temperature that you can find out using this expression right.

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Combined Heat Conduction and Reaction Energy in Spherical Catalyst Pellet

- Consider a spherical catalyst pellet of size R
- Thermal conductivity of the pellet is k
- Inside the pellet, chemical reaction is occurring due to which the rate of heat generation is S_C
- Heat is lost to outer surface of pellet and then to a gas stream by convective heat transfer
- Gas stream at temperature T_g
- Convective heat transfer coefficient is h
- Assume rate of heat generation is constant throughout*
- Develop steady state temperature profile and obtain expression for the maximum temperature in the system

Handwritten notes: A small diagram of a sphere with a red circle around it. Next to it, the text reads: $T_R = T_R(S_C)$ and $T(r) = ?$. There are also some scribbles and the words "Handwritten" and "my own" written in red.

So, now, combined heat conduction and reaction energy in spherical catalyst pellet. So, that is the last problem that we are taking in this class ok. So, let us say this statement. Consider a spherical catalyst pellet of size R . Thermal conductivity of the pellet is k . Inside the pellet chemical reaction is occurring due to which the rate of heat regeneration is S_C .

Then heat is lost to outer surface of pellet and then to a gas stream by convective heat transfer so; that means, you have a spherical catalyst pellet some pores may be connected or broken or whatever. So, like this you have a catalyst particle. So, now, inside this one a reaction taking reaction is taking place in this inside this you know catalyst pellet.

So, because of the reaction the heat is being generated and then that is S_C . So, then obviously, the temperature distribution would be there here in this case we will not have isothermal conditions. So, at this surface you know, what is the temperature? We do not know.

Because now the reaction is taking place inside the catalyst surface so, then depending on that S_C whatever heat generation is there, so, this temperature on the surface let us say you designate is T_R . So, the T_R is not a constant, it is a function of whatever that heat generation because of the chemical reaction ok.

So, now, whatever the heat is this since the reaction is taking place inside the catalyst porous catalyst pellet the temperature would be maximum at the centre right at centre yeah

maximum temperature ok because reaction is taking place inside the catalyst pellet. So, now, whatever the heat is there, so, at the centre maximum that is being transferred towards the outer surfaces outer layers of the catalyst pellets right.

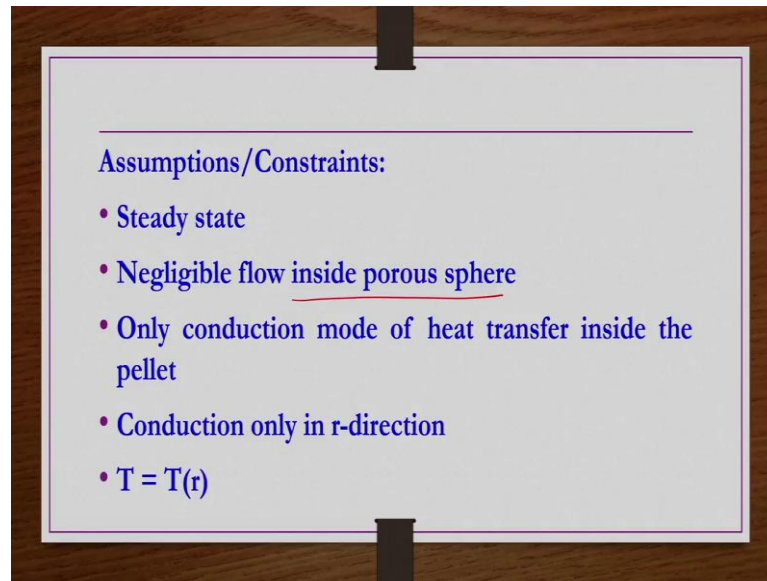
So, it reaches the surface or the catalyst surface then it moves to the surrounding gases stream because they convective heat transfer. So, there is some amount of loss of temperature is there. So, that temperature of gas stream is T_g that is known. So, because usually these catalytic reactions you know heterogeneous reactions and then one of the phase is either gas or liquid in general mostly gases reaction. So, that there is a gas stream flowing around this catalyst surface.

So, that it is forming a kind a film layer. So, that temperature of that gas stream is T_g that is known ok. So, now, gas stream at temperature T_g that is known. Convective heat transfer coefficient is h that is also known, thermal conductivity of the pellet k that is also known. Assume rate of heat generation is constant throughout, this is very important right. Develop steady state expression for the temperature profile and obtain expression for the maximum temperature in the system. So, this is what we are going to have.

So, now, some temperature now here, what we understand? The temperature is function of r that is what only you are taking, so, this you to find out ok. So, the conduction equation you to use here also because mostly it is conduction dominated within the because the the changes in the temperature are there within the catalyst pellet are only there that is what we are taking place.

Whatever the T_g temperature is the surrounding gas stream temperature, but that is forming as a film only very thin film only ok.

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So, assumptions constraints, steady state problem we are taking, negligible flow inside porous sphere so, that we can take only conduction in general. Because inside the porous structure whatever the catalyst pellet you take inside the porous structure in general these reactants whether the gases or liquid reactants whatever are there they move they are moved from one location to the other location primarily because of the diffusion.

Primarily because of the diffusion and then under such conditions the bulk motion is very very small negligible so, that we can take only conduction heat transfer here. So, only conduction mode of heat transfer inside the pellet, conduction is only in the r direction. So, T is function of r that is what we have to find out right.

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• Energy equation in spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right)$$

$$= k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + S_c$$

$$\Rightarrow \frac{k}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + S_c = 0 \Rightarrow (1)$$

$$\Rightarrow \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -\frac{S_c r^2}{k} \Rightarrow r^2 \frac{\partial T}{\partial r} = -\frac{S_c r^3}{k} + C_1 \Rightarrow \frac{\partial T}{\partial r} = -\frac{S_c r}{k} + \frac{C_1}{r^2}$$

$$\Rightarrow T = -\frac{S_c r^2}{k} - \frac{C_1}{r} + C_2 \Rightarrow (2)$$

Handwritten notes on the slide:
 $\frac{1}{r} = \frac{1}{r} \times \frac{r}{r}$
 $\frac{1}{r} = \frac{1}{r} \times \frac{r}{r}$

Now, energy equation in spherical coordinates is given here, $\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + S_c$ this is nothing but heat generation because of the chemical reaction inside the catalyst pellet.

So, steady state this term is 0 and then there is no fluid moment at all or conduction only we are taking. So, velocity components are 0 temperature is function of r only it is not a function of θ or ϕ . So, these two terms are 0 and then this heat generation because of the you know chemical reaction should be there. How much it is we do not know actually, it is given let us assume it is given ok.

But it in general changes during the progress of the reaction, but however, in our problem it has been mentioned, it is being maintained throughout the reaction, constant S_c has been maintained throughout the reaction. So, finally, we have $\frac{k}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + S_c = 0$ right.

Now, this equation we can rewrite $\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -S_c \frac{r^2}{k}$. Now, when you integrate $\left(r^2 \frac{\partial T}{\partial r} \right) = -S_c \frac{r^3}{k} + C_1$, now you take that r^2 to the right hand side. So, $\frac{\partial T}{\partial r} = -S_c \frac{r}{k(3)} + \frac{C_1}{r^2}$.

Now, once again if you integrate $T = -S_c \frac{r^2}{k(6)} - \frac{C_1}{r} + C_2$. So, now, you have to find out this C_1, C_2 right then only you can find out the temperature. So, it is not a straightforward

because only T_g one temperature is known right. Boundary conditions you how to apply such a way that its known value is there if you use certain kind of boundary condition which is again dependent on some other variable or parameter it is of no use

For example, here T_R is the temperature at the surface, but it is function of this S_c . So, this is not known as of now, this is not known, so, we have to find out. Then we can get this constant and then get this final temperature directly we cannot assume T_R is constant, though S_c is constant you know we do not know what that is S_c , but T_R . Once S_c is known T_R can be found out what is that constant T_R can be found ok. So, that first we have to do ok.

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$$\Rightarrow T = -\frac{S_c r^2}{k \cdot 6} - \frac{C_1}{r} + C_2 \Rightarrow (2)$$

• C_1 must be zero because T cannot be infinite for any value of r

$$\Rightarrow T = -\frac{S_c r^2}{k \cdot 6} + C_2 \Rightarrow (3)$$

$T_b \checkmark \Rightarrow (2)$
 at $r=0$ $r=?$ $T_b \times$
 $T_R \leftarrow r=R$
 $\uparrow T_c (S_c)$

So, the same equation is written here again. So, the C must be 0 in this case because this temperature cannot be infinite for any value of r . And then if you substitute $r = 0$ that is at the centre you this equation is going to produce the infinite value that is not acceptable. So, C_1 must be 0. So, the final simplified temperature is this one right.

So, now this C_2 we have to find out right. So, you may be thinking again here we have only one T_g value. So, we can find out. We can find out this C_2 that you may be thinking right, but that is also not possible here. Why is not possible? T_g is known, but at what r value this T_g is there that we do not know then only we can apply these boundary conditions.

In the case of a T_R we know the location at $r = R$, but this T_R is function of S_c . So, then even though location we know T_R exact value of T_R we do not know every early. So, we cannot use this boundary condition also. Whereas, the other value of temperature the other large other side wise you know the reverse way is true. T_g is known, but at what location that T_g is existing that we do not know.

What is the film thickness that we do not know, film of gas stream that is surrounding catalyst pellet in which the heat loss is taking place because of the convection that thickness of the film is not known? So, either of the boundary conditions directly we cannot use. So, what we have to do? We have to do a few simplifications now that we are going to do now.

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$$\Rightarrow T = -\frac{S_c r^2}{k \cdot 6} - \frac{C_1}{r} + C_2 \Rightarrow (2)$$

- C_1 must be zero because T cannot be infinite for any value of r

$$\Rightarrow T = -\frac{S_c r^2}{k \cdot 6} + C_2 \Rightarrow (3)$$

- Heat losses to surroundings would provide required B.C. to obtain C_2
- At the surface of sphere: rate of heat transfer due to conduction and convection must be equal
- $\Rightarrow -k \frac{\partial T}{\partial r} \Big|_{r=R} = h(T_R - T_g) \quad (4)$
- where T_R is temperature on the surface of spherical catalyst pellet

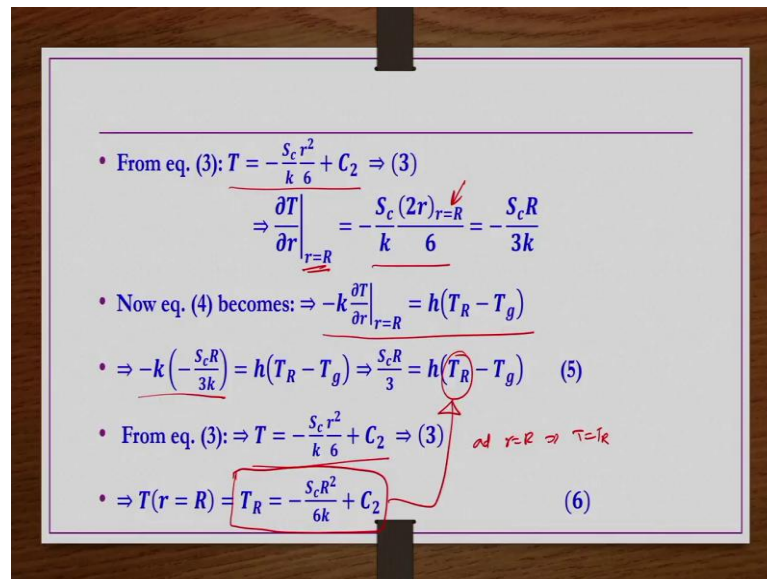
So, heat losses to surroundings would provide required boundary condition to obtain C_2 . So, what are the heat losses that we have to find out? At the surface of sphere rate of heat transfer due to conduction and convection must be equal in order to find out the new boundary condition to get this C_2 . So, we have to find out rate of conduction at the surface as well as the rate of convection at the surface to get a final boundary condition.

So, rate of conduction at surface that is $-k \frac{\partial T}{\partial r} \Big|_{r=R}$ should be equals to the rate of convection at the surface is $h(T_R - T_g)$ right. So, where T_R is temperature on surface of

spherical catalyst pellet that is function of S_c , it is not one constant value. T_R is also constant, but it is not a priori known ok. Why T_R is constant?

Because S_c is constant throughout the reaction that log in that way it is constant, we know that what is that constant we do not know. So, then that is the reason we cannot use directly that one as a boundary condition.

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So, from equation number 3 we are finding out $\frac{\partial T}{\partial r}$ because we have to find out $-k \frac{\partial T}{\partial r}$. So, $\frac{\partial T}{\partial r}$ from equation number 3 is $-\frac{S_c 2r}{6k}$ and then this $\frac{\partial T}{\partial r}$ we are finding out at $r = R$ that is at this surface. So, you substitute $r = R$ then you get $-\frac{S_c R}{3k}$.

So, now, $-k \frac{\partial T}{\partial r} = h(T_R - T_g)$ that is what we know. So, now, here in place of $\frac{\partial T}{\partial r} \Big|_{r=R}$ we substitute $-\frac{S_c R}{3k}$. So, $-k \left(-\frac{S_c R}{3k} \right) = h(T_R - T_g)$. So, that is we get $\frac{S_c R}{3} = h(T_R - T_g)$.

Now, from equation number 3 that equation number 3 is $T = -\frac{S_c R^2}{6k} + C_2$. Now, here you what you can do you need to know this T_R . So, what you do? At $r = R$, $T = T_R$ you substitute here. So, then you get $T_R = -\frac{S_c R^2}{6k} + C_2$ right. So, now, this you can use here in this expression equation number 5.

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• Now substitute $T_R = -\frac{S_c R^2}{6k} + C_2$ in eq. 5:

• $\Rightarrow \frac{S_c R}{3} = h(T_R - T_g)$ (5)

• $\Rightarrow \frac{S_c R}{3} = h\left(-\frac{S_c R^2}{6k} + C_2 - T_g\right)$

• $\Rightarrow \frac{S_c R}{3h} = -\frac{S_c R^2}{6k} + C_2 - T_g$

• $\Rightarrow C_2 = \frac{S_c R^2}{6k} + \frac{S_c R}{3h} + T_g$ (7)

Now substitute $T_R = -\frac{S_c R^2}{6k} + C_2$ in equation number 5, equation number 5 is this one. $\frac{S_c R}{3} = h(T_R - T_g)$. So, now, in place of T_R we are writing this one. So, now, we can write $\frac{S_c R}{3h} = \frac{S_c R^2}{k \cdot 6} + C_2 - T_g$ that is $C_2 = \frac{S_c R^2}{k \cdot 6} + \frac{S_c R}{3h} + T_g$ this is what we get. So, this is the C_2 , C_2 is strongly dependent on S_c and then T_g as well in addition to h and k values right.

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• Final temperature profile can be obtained by using C_2 in eq. (3):

$$T = -\frac{S_c r^2}{k \cdot 6} + C_2$$

• $\Rightarrow T = -\frac{S_c r^2}{6k} + \frac{S_c R^2}{6k} + \frac{S_c R}{3h} + T_g$

• $\Rightarrow T - T_g = \frac{S_c R^2}{6k} \left(1 - \frac{r^2}{R^2}\right) + \frac{S_c R}{3h}$ *

• Maximum temperature is at centre, i.e., at $r = 0$

• $\Rightarrow T_{max} - T_g = \frac{S_c R^2}{6k} + \frac{S_c R}{3h} = \frac{S_c R^2}{6k} \left(1 + \frac{2k}{hR}\right)$

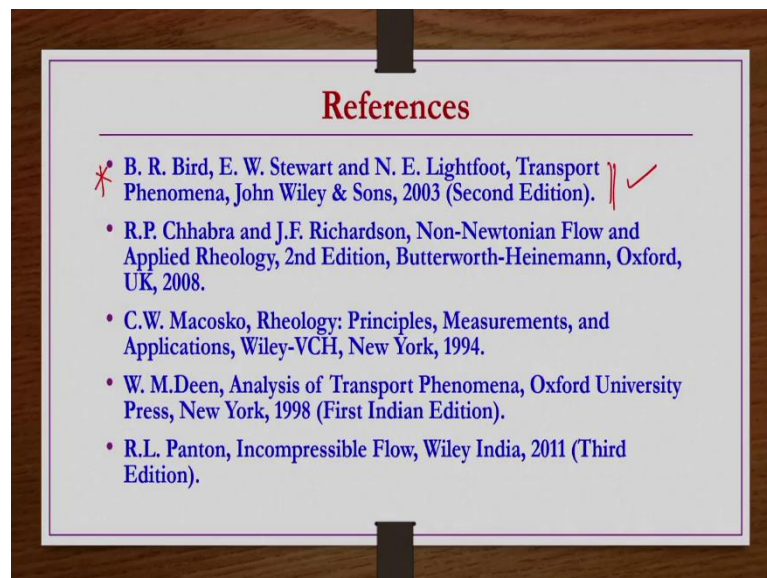
• $\Rightarrow T_{max} - T_g = \frac{S_c R^2}{6k} \left(1 + 4 \left(\frac{k}{h(2R)}\right)\right)$ *

So, now, this C_2 you can substitute in equation number 3, which is the final temperature distribution $T = -\frac{S_c r^2}{k \cdot 6} + C_2$, in place of C_2 we can write $\frac{S_c R^2}{k \cdot 6} + \frac{S_c R}{3h} + T_g$. So, $T - T_g =$

$\frac{s_c R^2}{k} \left(1 - \frac{r^2}{R^2}\right) + \frac{s_c R}{3h}$. This is the final temperature distribution right. So, now, we have to find out the maximum temperature which is nothing but at $r = 0$.

So, in this equation if you substitute $r = 0$ you get $T_{max} - T_g = \frac{s_c R^2}{k} \frac{1}{6} + \frac{s_c R}{3h}$. So, if you take $\frac{s_c R^2}{k} \frac{1}{6}$ common you will get multiplied by $1 + \frac{2k}{hR}$, this is in the form of you know reverse of Nusselt numbers. So, that way if you write you can write $\frac{4k}{2R}$. So, this is nothing but reverse of the r inverse of Nusselt number. So, this is the maximum temperature at the centre of the catalyst surface ok.

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The references for this lecture are provided here. Primarily the all 3 problems you can find out in this book Bird, Stewart and Lightfoot, Transport Phenomena. In this book Transport Phenomena by Birds, Stewart and Lightfoot you can find out all these problems.

Thank you.