Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 28
Temperature distribution for FDF of Newtonian fluids in tubes

Welcome to the MOOCs course, Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Temperature distribution for a Fully Developed Flow of Newtonian fluids in tubes.

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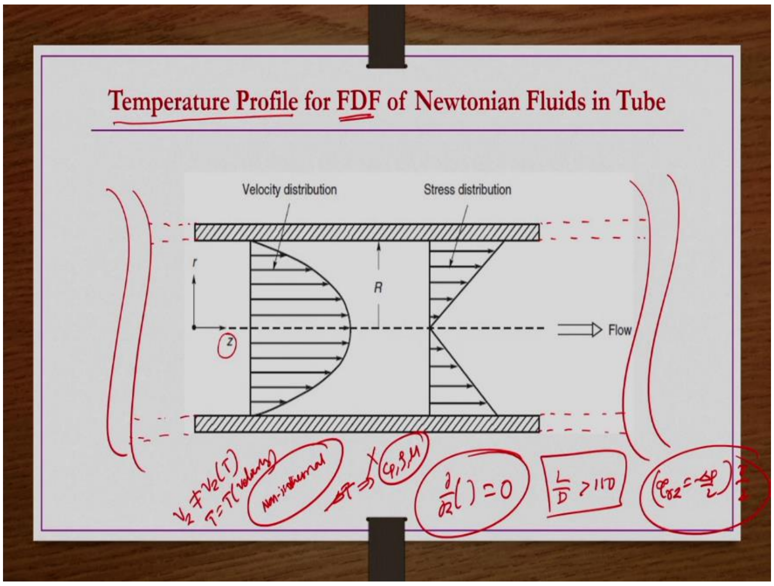

So, the major constraint of this problem is the flow is fully developed. So, whatever the temperature profile that we are going to obtain for the case of Newtonian fluids flowing in tube under non-isothermal conditions, this temperature profile is applied only for that region of the tube, where the flow is fully developed. Actually this problem we have taken previously under isothermal conditions.

So, where we tried to obtain the velocity profiles for the power law fluids right and then, we substitute n = 1 and then, got the velocity profile. So, that we know actually. So, this that we have to revisit here. Actually since we are finding out the temperature profile and then, energy equation there is a velocity term. So, then, we must know the velocity profile as well for that given profile, for that given geometry ok.

So, for fully developed flow conditions especially for circular tubes, what you have taken? L/D is very very large; in general more than 150 or something like that or 100 to 150 something like that we have taken right. That also depends on the nature of the fluid; sometimes even 100 L/D of 100 is sufficient, sometimes even L/D of 200 may also not be may not be sufficient, if it is a viscoelastic fluids. So, that is a different issue right.

So, what we see under such conditions? The flow is fully developed. We take only that region of the pipe in which the flow is fully developed. Fully developed in the sense, in the flow direction, let us say in the z direction flow is taking place; $\frac{\partial}{\partial z}$ of any you know velocity or vector components etcetera are going to be 0. That is what we have seen right.

So, now the same, we are revisiting again the same problem; but only thing that now we are taking non isothermal conditions right and then, that non-isothermal conditions are also such a way that the ΔT or temperature difference whatever is there, it is not affecting the C_p, ρ, μ etcetera, these kinds of physical properties. These things are not being affected by the temperature right.

So, that means, velocity profile is not function of temperature; but the temperature is function of velocity as well right. So, that is the reason for this case, we have to find out the velocity profile and then that velocity profile, we have to incorporate in solving the energy equation ok. So, now, here the coordinate system stand, the same problem in the flow direction, we are taking z; other direction, we are taking r right.

If it is fully developed flow, when we simplified the momentum equations etcetera what we understand? $\tau_{rz} = \left(\frac{-\Delta p}{L}\right)\frac{r}{2}$ that is what we have we obtain. So, that means, you know the shear stress is linear. So, that is what shown here and then, velocity profile, we have seen like in depends on the value of n because we have done it for power law fluids and then, being on plastic fluid separately. Individually for Newtonian fluids, we have not done.

So, but that velocity profile, if you substitute n = 1, you get the parabolic velocity profile that is given here right. So, now, we are bringing in this non-isothermality in this problem so that to find out the temperature profile also, but only when the only for the fully developed region of the geometry, not for the entire entry to exit.

Because when we are saying the fully developed flow, end effects that is entry effect and then, exit effects are negligible right. So, the whatever the solution whether the velocity distribution that we already got or the temperature distribution that we are going to get today, it is not valid; region close to the entry, region close to the exit, in between only it is valid and then, especially L/D has to be very large ok.

So, now, we give the conditions like you know what are the velocity conditions, boundary conditions are required? So, at wall, no slip condition. So, velocity is 0, we know at centre velocity is maximum. So, $v_z = v_{z\,max}$ that we know. This is that is sufficient for in order to get the velocity profile, this parabolic profile that we can get that is $v_z = v_{z\,max}\left(1-\frac{r^2}{R^2}\right)$ that we can get using these boundary conditions.

But temperature profile, you have to wanted to find out you have to have the conditions with respect to the temperature also. So, let us say at the inlet the temperature is T_1 and then, this wall whatever the pipe walls are there, they are maintained at constant heat flux q_0 right and the temperature at the centre, what it is? We do not know; whether it is maximum or minimum or whatever we do not know.

So, what we are saying that temperature is finite at the centre; T is finite at the centre that is at r = 0, it is finite that is what we are saying fine. So, now, three boundary three conditions are there for temperature, so we have to find out. Another thing that most important thing for the velocity profile in this fully developed region, we found that it is function of r only right.

And then, accordingly, we got you know velocity distribution and then, that is true also if you have a one dimensional laminar flow. So, definitely stands well. But under such conditions, even though velocity is function of r only, you cannot say the temperature is function of r only, it may also be function of z.

Especially because the temperature condition at the inlet is different from the wall conditions, you know centre temperature conditions. So, now here more than one variable is coming into the picture, so whose effects should also be taken into the consideration.

So, now, even though here in this problem, the velocity is function of only r; but temperature is function of both r and z. So, now, the problem becomes more complicated so to solve mathematically. So, however, there are certain approaches certain you know you know certain types of solutions are available approximate solutions which are you know almost close to the reality.

So, those things we are one of such kind of solution, we are going to discuss now. So, what we have to do here? First we have to list out the constraints of the problem, then solve the simplify the momentum equations, then get the velocity profile, then after that you know simplify the energy equation in that energy equation if the velocity terms are there, so then substitute the velocity distribution, then simplify the simplified energy equation to get the final temperature distribution. So, those are the steps ok.

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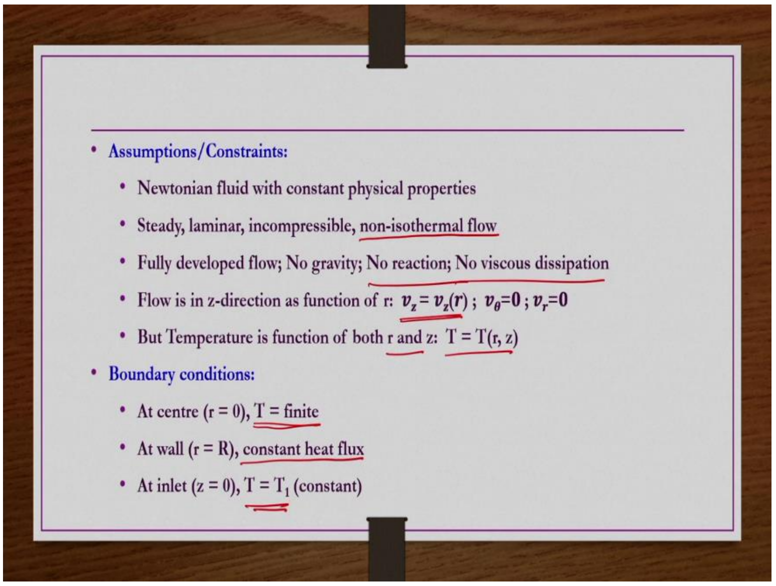

So, first constraints are assumptions of the problem, Newtonian fluid with constant physical properties; ρ, μ etcetera are not changing even though non-isothermal conditions are there. Then, steady, laminar, incompressible, non-isothermal flow; previously, what we have taken you know when we are taking only momentum transfer, we have taken isothermal flow.

But it is non-isothermal flow in this case. So, temperature profile one has to find out. Flow is fully developed, no gravity, no reaction, no viscous dissipation, no reaction kind of things standard and then, only z component of velocity is existing that is function of r; whereas, v_θ, v_r are 0.

But temperature is function of both r and z that is T is function of both r and z. Then, boundary conditions for the temperature, we are writing because for the fluid we already know, very simple ones. At the centre, at r = 0, we do not know whether it is maximum minimum or negative, positive, we do not know anything right.

So, then, what we are saying, that it is a finite value; just finite and then, walls are maintained at constant heat flux ok. At the inlet that is z = 0, T = T_1. Since T is function of z also, we need to have a boundary condition as function of z also, for different values of z also.

So, for the time being, it is given only at the inlet that is at z = 0, only the temperature condition is given that is T = T_1 right. Now, we solve the problem. First we write the continuity equation in cylindrical coordinates.

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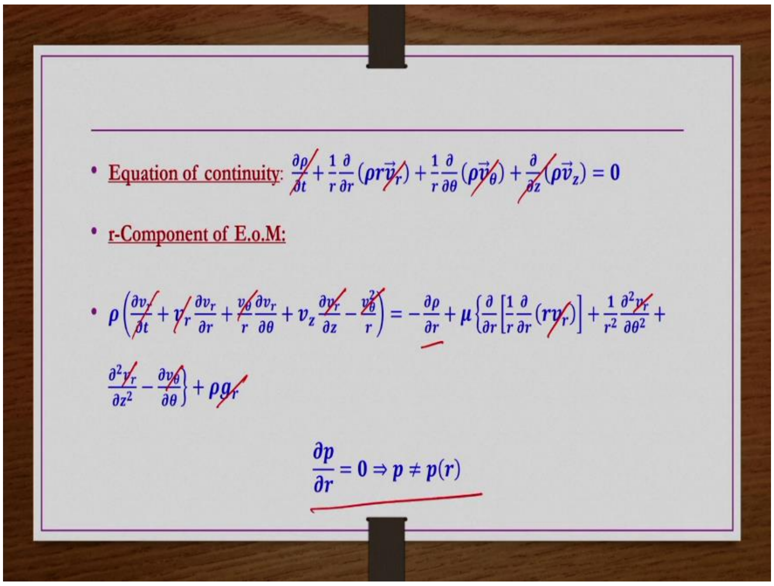

Then, we have this one. Steady state, so this term is 0, v_r is not existing, v_θ is not existing and then, fully developed flow, so $\frac{\partial}{\partial z}$ of anything is 0. So, continuity is satisfied. r-component of equation of motion is given here; v_r is not existing and then, steady state term, so it is 0; v_r is not existing, v_θ is not existing; v_z it is there, but v_r is not there, v_θ is not there.

Pressure in general, we do not know; v_r is not existing. So, all these three terms are not there; v_θ is also not existing. The gravity, we are not taking; horizontal pipes, we are taking. So, pressure is not function of r.

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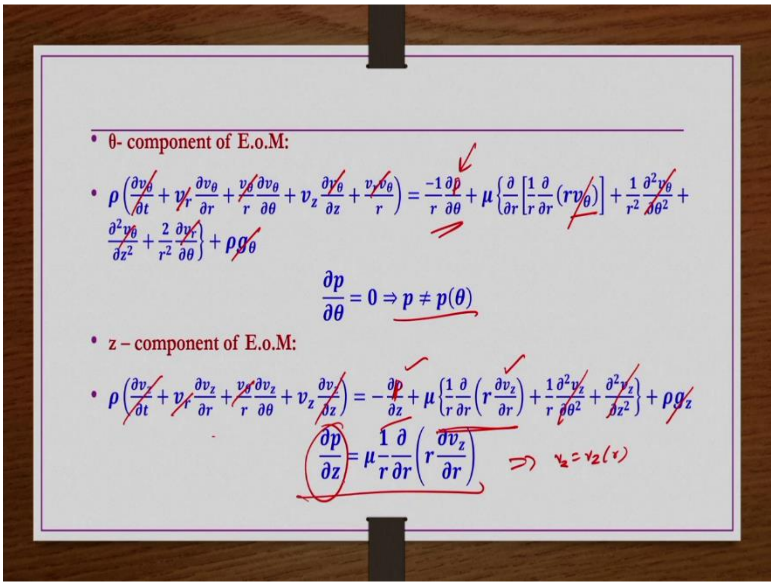

Then, θ component of equation of motion is given here. So, steady state this term is 0, v_r is not there, v_θ is not there; v_z is there, but v_θ is not there, v_r v_θ both are not there. So, left hand side all terms are canceled out. The pressure $\frac{\partial p}{\partial \theta}$ 1 by the pressure term, we do not know in general.

So, we just keep it as it is. v_θ is not there, so this term is 0, this term is also 0, this is also 0; v_r is not there, so this is also 0; gravity, we are not taking. So, what we understand pressure is not function of θ. So, pressure should be function of z only because in the z direction, flow is taking place and then that flow is taking place because of the pressure gradient. It is a same problem we are revisiting for a Newtonian case.

This exactly the same problem, we have done for the power law and being in plastic fluids previously. So, z component of equation of motion is given here. So, steady state, this is 0; v_r is not there, v_θ is not there, v_z is not function of z or fully developed flow, this is 0.

The pressure in general, $\frac{\partial p}{\partial z}$ now we do not know, so let us keep it as it is. v_z is there and it is function also and it is function of r, so this term would be there. v_z is not function of θ and z, so these two terms are 0, there is no gravity. So, $\frac{\partial p}{\partial z} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$, this is what we are getting.

So, this equation if you solve, then you get v_z as function of r. But now, already we understand that pressure is not function of r and θ. So, when we integrate this equation, left hand side is only z pressure and then, z terms are there. So, velocity is not you know function of z. So, when you integrate this one, you can take the left hand side term as a constant, when you integrate the right hand side term.

Similarly, right hand side only function of r is there ok; but pressure is not function of r. So, when you integrate the left hand side term, what you can do? You can take a right hand side as a constant and then, integrate it ok. Since pressure is not function of r and θ, but it is function of z only and then, $\frac{\partial p}{\partial z}$ whatever is there, that you can treat it as a constant.

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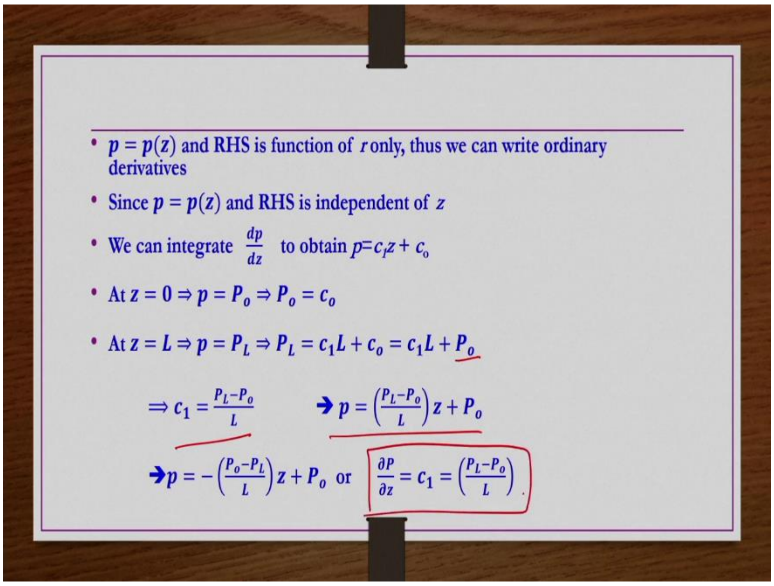

And then, when you integrate, you get the pressure $p = c_1 z + c_0$. At z = 0, $p = P_0$. So, $c_0 = P_0$. At z = L, $p = P_L$, so that means, $P_L = c_1 L + c_0$ and then, c_0 is nothing but P_0. So, that means, $c_1 = \frac{P_L - P_0}{L}$; that means, $P = \frac{-\Delta P}{L}z + P_0$ that is what we can write or $\frac{\partial p}{\partial z} = c_1$ and then that $c_1 = \frac{-\Delta P}{L}$.

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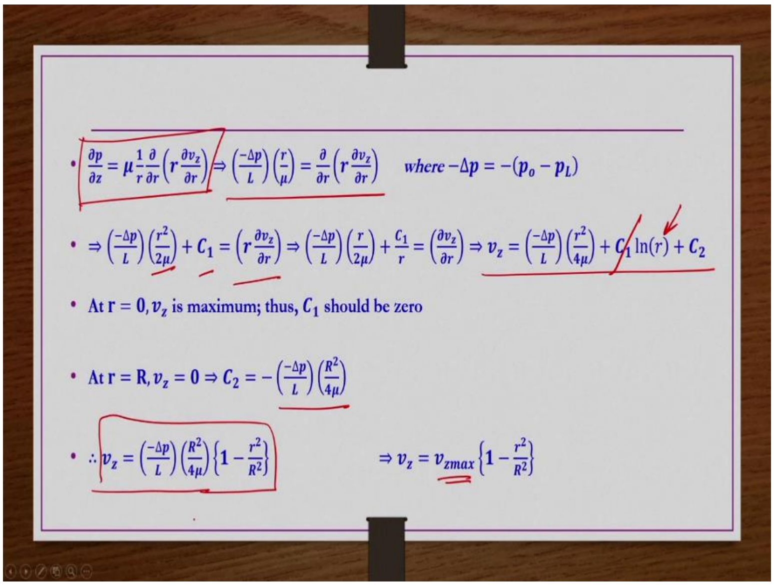

So, now this equation in place of $\frac{\partial p}{\partial z}$, this is the equation that you know when we simplifies when we simplified the z component of an equation of motion, this is what we get in the previous slide, one of the previously slide. Now, $\frac{\partial p}{\partial z}$ in the previous slide, we got it as a constant; that constant is $\frac{-\Delta P}{L}$.

So, $\frac{-\Delta P}{L} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$ so that we can write $\left(\frac{-\Delta P}{L}\right)\frac{r}{\mu} = \frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$ and then, when you integrate $\left(\frac{-\Delta P}{L}\right)\left(\frac{r^2}{2\mu}\right) + C_1 = r\frac{\partial v_z}{\partial r}$.

This r you take to the left hand side, so $\left(\frac{-\Delta P}{L}\right)\frac{r}{2\mu} + \frac{C_1}{r} = \frac{\partial v_z}{\partial r}$ and then, once again if you integrate, $v_z = \left(\frac{-\Delta P}{L}\right)\left(\frac{r^2}{4\mu}\right) + C_1 lnr + C_2$. Now, when you apply the boundary condition at r = 0, v_z has to be maximum; but ln of 0 you cannot define.

So, then, this C_1 constant has to be 0 and then, at r = R, $v_z = 0$, then C_2, you will be getting $-\left(\frac{-\Delta P}{L}\right)\left(\frac{R^2}{4\mu}\right)$ and then this C_2, you substitute here, C_1 is 0 anyway, then $\left(\frac{-\Delta P}{L}\right)\left(\frac{R^2}{4\mu}\right)$, if you take common, you get $1 - \frac{r^2}{R^2}$ that is the velocity profile that we have the parabolic nature for Newtonian fluid right.

This $\left(\frac{-\Delta P}{L}\right)\left(\frac{R^2}{4\mu}\right)$, you can write it as $v_{z\,max}$ because in this equation if you substitute r = 0, it will be you know $v_{z\,max}$ because at r = 0 that is at the centre of the pipe velocity is maximum. So, at r = 0, $v_z = v_{z\,max}$. So, and the in this equation, if you substitute r = 0, you will be having only $\left(\frac{-\Delta P}{L}\right)\left(\frac{R^2}{4\mu}\right)$ that I am writing $v_{z\,max}$.

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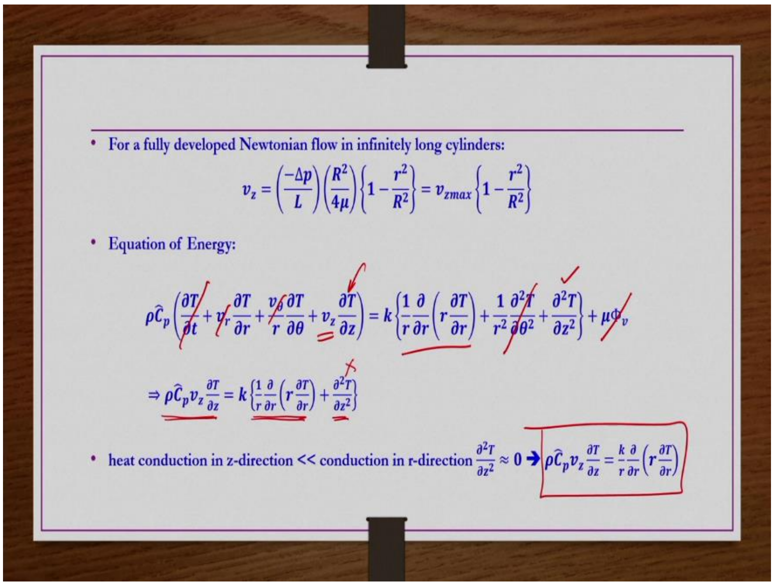

So, this is what we have. Now, equation of energy, we are going to simplify ok. In cylindrical coordinates, equation of energy is given here. A steady state, this term is 0; v_r is 0; v_θ is 0; v_z is not 0 and then, temperature is also function of z in addition to function of r. So, this term should be there. Temperature is function of r, so this term would be there, but it is not function of θ.

So, it this term is canceled out. This term is there because temperature is function of z. What function it is? We do not know that we are going to obtain now right and then, we are not taking any viscous dissipation in this problem. So, what we have $\rho\hat{C}_p v_z\frac{\partial T}{\partial z} = k\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right\}$.

Now, can we further simplify this equation that is what we see. So, this term indicates the conduction in the radial direction; this term indicates the conduction in the z direction; this term indicates the convection in the z direction. So, flow is taking place in the z direction. So, in that direction, convection is going to be dominating compared to the conduction.

So, what we can do? We can cancel out these term $\frac{\partial^2 T}{\partial z^2}$ with that comparison to $\rho \hat{C}_p v_z \frac{\partial T}{\partial z}$ or otherwise, also if you take only conduction terms, conduction is going to be dominating in the direction normal to the flow that is in the r direction. So, in compare to so conduction is going to be more in the r direction than in the z direction.

So, by that analysis as well, $\frac{\partial^2 T}{\partial z^2}$, we can cancel out. So, by either of the region, when you cancel out $\frac{\partial^2 T}{\partial z^2}$, you will be having $\rho \hat{C}_p v_z \frac{\partial T}{\partial z} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$.

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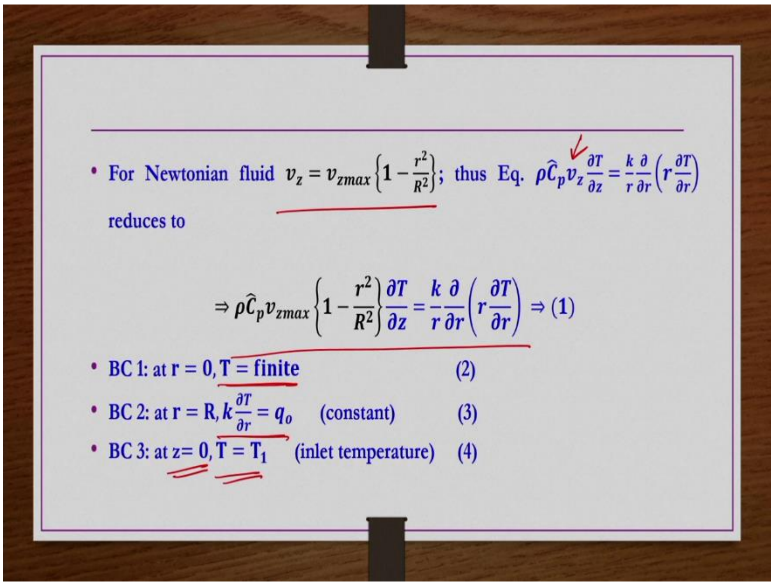

So, now here in place of v_z, what we write? We write $v_z = v_{z\,max}\left\{1 - \frac{r^2}{R^2}\right\}$. So, then this is what we are having. This equation, we have to solve. So, obviously, from this equation what we understand? We need two boundary condition for r with respect to the r and then, we need one boundary condition with respect to z right. So, boundary condition, what we have? At the centre, temperature is finite, we do not know how much it is. We are saying finite ok.

Obviously, it cannot be 0; it can be maximum and cannot be maximum that also we cannot say. So, but we are saying finite ok and then at wall, we are maintaining the constant heat flux that is $q_0 = k\frac{\partial T}{\partial r}$ right and then, at z = 0, at the inlet, we know the fixed temperature $T = T_1$; constant inlet temperature right. So, now equation 1, using the boundary conditions given in equations 2, 3 and 4, we have to solve.

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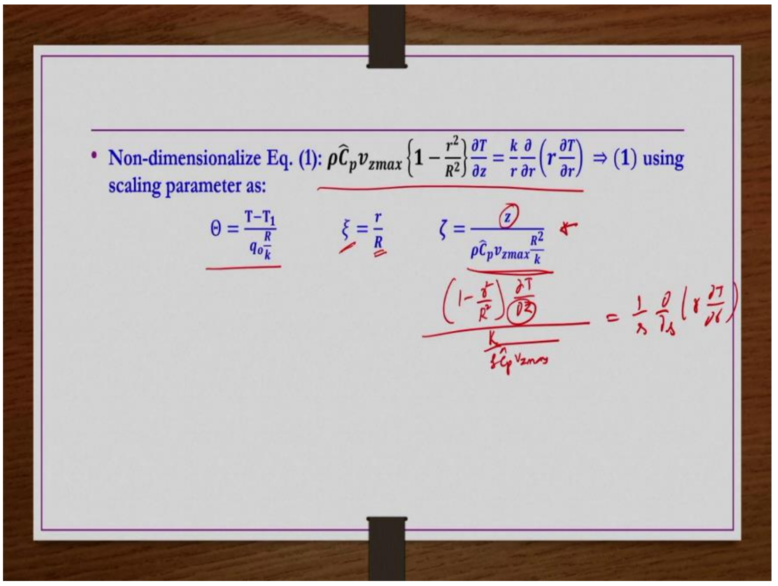

So, equation number 1, now what we do rather solving in this form, we make it non-dimensionalized right. So, now here, scaling parameters are given here. So, θ is nothing but dimensionless temperature right. In the previous problem, you may be thinking that ΔT is non-dimensionalized using the $(T_1 - T_\lambda)$; $(T_\lambda - T_1)$whatever. But now here, why are we taking $q_0\frac{r}{k}$ to non-dimensionalize the temperature difference?

Because we know only one fixed temperature; only T = T_1 at z = 0 that much only we know. We need at least two temperatures, if you wanted to non-dimensionalized the temperature difference using the temperature quantities. So, that is not possible here because we are we know only one temperature quantity T_1. So, that is not possible.

So, that is the region q_0, we are using that is constant heat flux right. So, θ is the dimensionless temperature which is $(T - T_1)q_0\frac{R}{k}$ and then, dimensionless radial coordinate dimensionless radial coordinate whatever is there $\frac{r}{R}$, we call it ξ right.

Now, this z dimensionless z coordinate also we have to define; how to define? Let us say this equation number 1, if I write $\frac{\left\{1-\frac{r^2}{R^2}\right\}\frac{\partial T}{\partial z}}{\frac{1}{\rho\hat{C}_p v_{z\,max}}}$ this k also I am bringing it here to the left hand side.

So, $\frac{\left\{1-\frac{r^2}{R^2}\right\}\frac{\partial T}{\partial z}}{\frac{k}{\rho \hat{C}_p v_{z\,max}}} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$ right. So, now, whatever this things you know being you know multiplied by this z that we are taking as a kind of you know scaling parameter to non-dimensionalize z. So, dz whatever z things is there that is multiplied by $\frac{r^2 \rho \hat{C}_p v_{z\,max}}{k}$ from this equation.

So, that is the reason dimensionless z that is $\zeta = \frac{z}{\rho \hat{C}_p v_{z\,max}\frac{r^2}{k}}$ we are taking. Sometimes you know this comes with the experience also. So, that is what we are doing now here ok. So, otherwise, you may be confusing because non-dimensionalizing the radial coordinate, you are simply dividing that one with the radius of the tube ok.

Temperature difference, you are simply taking the constant heat flux terms and then, doing it; but in this case of z it is not so simple because there is no another L coordinate kind, L kind of thing in this equation; otherwise z/L, we could have taken as ζ right. So, that is the reason, we have written like this.

How to select? Now, you can that that you know you have to rearrange this way and then, whatever the terms you know being multiplied by this z terms whatever are there. So, that you take as a kind of you know scaling parameter to non-dimensionalize the z coordinate fine.

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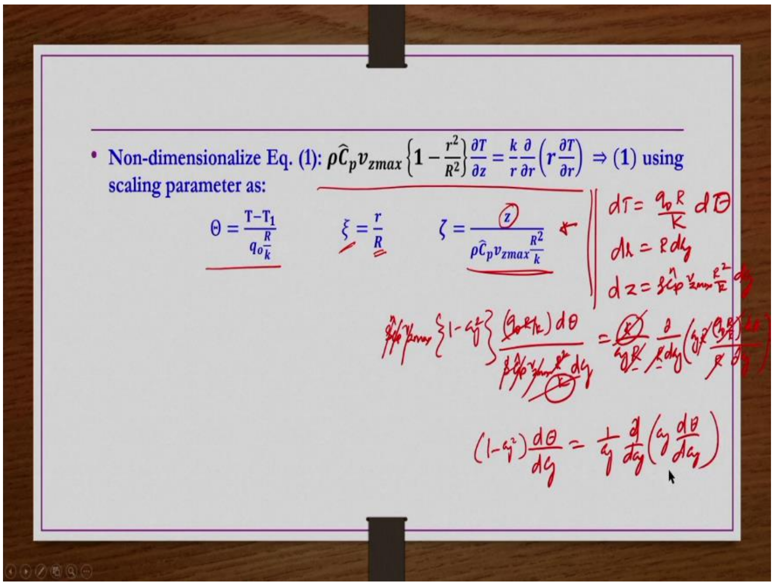

So, now, these scaling parameters, we are going to use to non-dimensionalize this equation number 1. So, in the equation number 1, we are having dr, dz, dT terms are there. So, now, from this θ, what we can understand? $dT = q_0 \frac{R}{k}\,\mathrm{d}\theta$ right. From this definition dr is nothing but R dξ and then, from this ζ information, dz is nothing but $\rho\hat{C}_p v_{z\,max}\frac{R^2}{k}\,\mathrm{d}\zeta$. So, now, we apply these things in this equation number 1.

So, what we will have? $\rho\hat{C}_p v_{z\,max}\left\{1-\frac{r^2}{R^2}\right\}$ is nothing but ξ^2, ΔT is nothing but $q_0\frac{R}{k}\frac{d\theta}{dz}$ is nothing but $\rho\hat{C}_p v_{z\,max}\frac{R^2}{k}\,\mathrm{d}\zeta = k$, r is nothing but ξR; r is nothing but ξR, $\frac{d}{dr}$ is nothing but R dξ of r is nothing but ξR and then, dθ is nothing but $q_0\frac{R}{k}\frac{d\theta}{dr}$ is nothing but R dξ.

So, now, this is the equation $\rho\hat{C}_p v_{z\,max}\left\{1-\frac{r^2}{R^2}\right\}\frac{\left(q_0\frac{R}{k}\right)d\theta}{\rho\hat{C}_p v_{z\,max}\frac{R^2}{k}\mathrm{d}\zeta} = \frac{k}{\xi R}\frac{\partial}{R\partial\xi}\left(\xi R\frac{\left(q_0\frac{R}{k}\right)d\theta}{R\partial\xi}\right)$. So, this $\rho\hat{C}_p v_{z\,max}$ this $\rho\hat{C}_p v_{z\,max}$ cancelled out right. So, this $q_0\frac{R}{k}$ and this $q_0\frac{R}{k}$ is cancelled out; this R, this R is cancelled out right. So, now, here ask R and R, R^2 is this. So, this R^2, this R^2 is cancelled out and then, this k, this k cancelled out. So, what do you get from this equation? $\{1-\xi^2\}\frac{d\theta}{d\zeta} = \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right)$. This is what you get.

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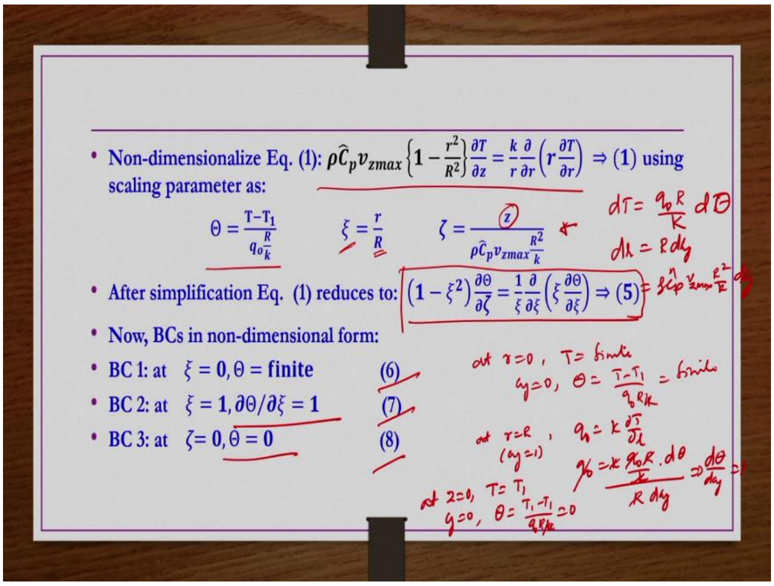

So, the same equation simplified final equation is written as an equation number 5 here right. So, now, boundary conditions also we have to define in non-dimensionalized quantities right. At r = 0, T is equals to finite that is the boundary condition. If r = 0; that means, ξ is also 0 and then, if T is T is finite, so θ that is $\frac{T-T_1}{q_0\frac{R}{k}}$ is this, so that should also be finite.

We do not know how it how much it is, we just write it as. Then, at r = R; that means, ξ = 1 and then what is the boundary condition? The constant heat flux that is $q_0 = k\frac{\partial T}{\partial r}$ right. So, at r = R, constant heat flux q_0 is maintain that is nothing but $k\frac{\partial T}{\partial r}$; forget about the minus + and all, that is not required ok. So, now q_0 = k, ΔT is nothing but $q_0 \frac{R}{k}$ dθ.

This entire divided by dr is nothing but R dξ. So, this q_0, q_0; this k, this k; this r, this r is cancelled out. So, $\frac{d\theta}{d\xi} = 1$. So, that is what is this one and then, at z = 0, T = T_1. If z = 0; that means, ζ = 0 ,right and then, if T = T_1, so θ is going to be $\frac{T_1-T_1}{q_0\frac{R}{k}}$. So, this is going to be 0 anyway. So, that is this one ok.

So, now, we are going to solve this equation number 5 using the boundary conditions given in equation 6, 7, 8. So obviously, it is not possible without many without making certain kind of approximations ok. Because now, temperature is function of r and z, so θ is also function of ζ and ξ here ok. So, what are the approximations we do in general, that we see.

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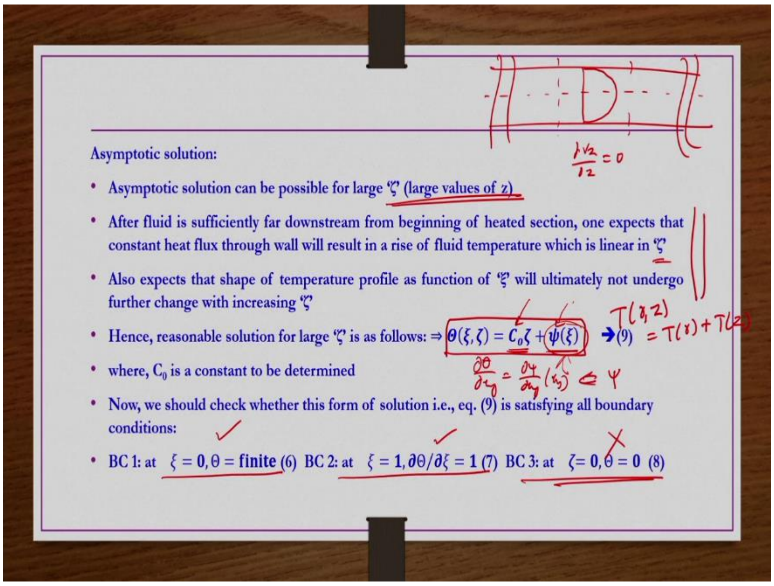

We are taking asymptotic solution. First one is it is possible for large values of z or ζ because it has been mentioned that you know temperature profile for a fully developed flow region. So, that is true only for large values of z ok. Then, after fluid is sufficiently far downstream from beginning of heated section, one expects that constant heat flux through wall will result in a rise of fluid temperature which is linear in z direction of ζ.

So, let us say this is the pipe ok. So, now, we are far away from the inlet and outlet, so where the velocity profile is fully developed alright? And then, we are having some parabolic profile like this. So, in this region, what happens? So, whatever the variation in the velocity is not there. So, variation in the fully in the fully developed region only, in the fully developed region only, variation in the velocity in the flow direction are 0; isn't it?

So, that means, you know the with respect to the wall, the fluid relative motion between wall and then fluid regions is going to be same. So, then obviously, that velocity is not going to affect much the temperature profile here in the fully developed region and then, that also in which direction, it will not affect? It will not affect much in the z direction.

The convection whatever is there that is not going to affect you know; the velocity profile is not going to affect the heat transfer because of the convection much. So, only whatever the variations are there in the z direction, they are expected to be linear ok.

So, temperature profile whatever is there, T function of z are T function of ζ is there that is a linear that is what we are saying because velocity profile is not changing in the flow direction that is in the z direction. Then, also expect that the shape of temperature profile as function of ζ will ultimately not undergo further change with increasing ζ or z value.

So, whatever the variation in the temperature profile is there, so T as function of r is there, so that is not affected by the temperature variations in the z direction for the further longer you know you know increasing ζ values. So, basically, what we are trying to say?

What we are trying to do here now from these you know two assumptions? From these two assumptions, we are saying that whatever the T function of r and z is there, so that we are trying to write T is separate function of r and then, T is separate function of z and then, we are taking you know their affects as an additive and then, that is quite possible, if the flow is fully developed.

So, then we have this temperature, temperature non-dimensionalized temperature would be having this form. This is the form of the solution now. This is not the problem equation. This is solution equation right. So, what we understand from these two assumptions?

The temperature is a linear function of ζ, so C_0 ζ and then whatever the temperature as function of r, or you know θ as function of ψ is there that is not affected by the you know temperature variations in the z direction. So, in this part, it is only function of ξ; it is not function of ζ ok that is what we are trying to do.

So, now, if you find out what is the C_0 constant and then, what is this ψ function of ξ whatever is there that you find out you got the solution ok and then, this solution should also obey the boundary conditions that we have you know three boundary conditions equation 6, 7 and 8. Then only, we can say that whatever the assumed the form of the solution, this is assumed form of the solution ok.

So, then we can say that is approvable or otherwise, do we need to make any amendments that we have to see. So, we are will check; equation first boundary condition is you know at ξ = 0, θ is finite. So, so if let us say this second term psi function of ξ whatever is there, if it is having all the terms ξ term.

So, if you substitute ξ, so the second term is going to be 0. But despite of that one, this term is going to have some value. So, then θ is equals to finite is possible, even when ξ = 0. Then, when ξ = 1, $\frac{\partial\theta}{\partial\xi} = 1$; is it possible or not?

When you do the $\frac{\partial\theta}{\partial\xi}$, you will get $\frac{\partial\psi}{\partial\xi}$ right. So, this is function of ξ. So, then you know we substitute ξ = 1 here, so then it is possible that you may get 1 right. So, it is possible that kind of you know ψ function, you can find out right.

So, now this; so second boundary condition is also satisfied. So, the solution 9 is satisfying boundary condition 6 and 7. Then boundary condition 8, is it satisfying or not that we have to check. When ζ = 0, then θ = 0. So, let us say if ζ = 0, then θ is 0; that means, you know these all that is possible when altogether this function is 0.

But that is not possible. So, that means, this boundary condition is not being satisfied by this form. So, then what we do? We have to obtain one conditions for the z direction or so. So, that we try to do it. How we try to do?

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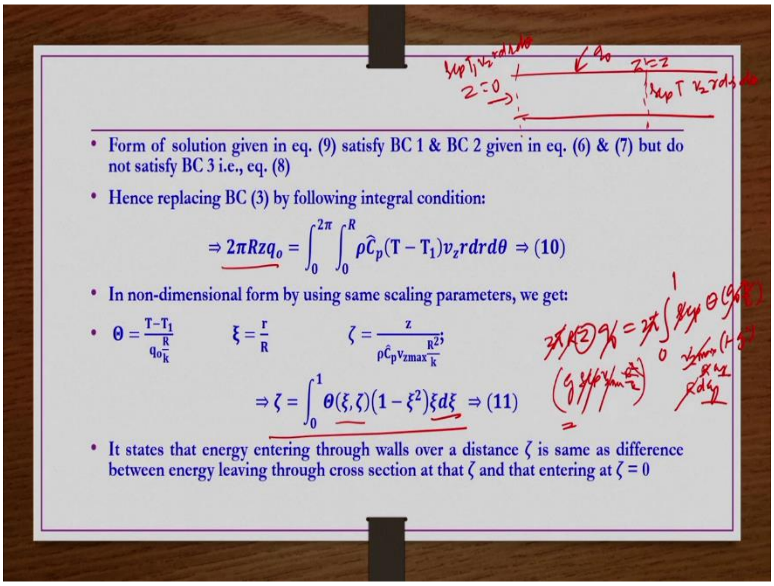

Let us say this is the pipe. So, at certain locations z. So, from the inlet, z = 0 to certain location z = z whatever is there. So, heat is coming in because of the constant heat flux. So, whatever the 2 π R v_z q_0 is there, so that should be you know balanced by the whatever the ρ $\hat{C}_p$ now here ρ $\hat{C}_p$ T v_z r dr dθ integration whatever the quantity is this, so that is at z = z.

The same quantity at z = 0 is nothing but ρ $\hat{C}_p$ T_1 because at z = 0, T = T_1 v_z r dr dθ, this also you integrate whatever the quantity is there. So, that is the rate of heat in at z = 0. So, rate of heat out at z = z minus rate of heat in at z = 0 that should be balanced by the whatever the heat being supplied through the wall from z = 0 to z = L.

So, that balance when you do, you get these things that that is $2\pi Rzq_0 = \int_0^{2\pi}\int_0^R \rho\,\hat{C}_p(T-T_1)\,v_z r dr d\theta$. Now, the temperature is not function of θ. So, you can write 2 π $\int_0^R \rho\,\hat{C}_p(T-T_1)v_z r dr$, that is it. Then, 2 π 2 π will be cancelled out. Now, here also you apply this same scaling parameter exactly, then, you get this boundary condition. How? Later you can check it is.

Let us say 2πR z left hand side is same, q_0 is same, right hand side 2 π integral R = 0 means ξ = 0, R = R means ξ = 1. Then ρ $\hat{C}_p$ T, what we are using $(T-T_1)$ is nothing but θ multiplied by $q_0\frac{R}{k}$ multiplied by v_z; v_z we are writing $v_{z\,max}$ $\left\{1-\frac{r^2}{R^2}\right\}$ is $\xi^2$and then dr is nothing but R dξ, r is nothing but r ξ ok and then, left hand side in place of this v_z, what we have to write?

You know ζ ρ C_p $v_{z\,max}$ $\frac{R^2}{k}$ in place of z. So, this 2 π, this 2 π is canceled out; this q_0, this q_0 is canceled out. So, here one R and then another R here is canceled out; this ρ C_p, ρ C_p here it is canceled out; $v_{z\,max}$, $v_{z\,max}$ is canceled out. So, this R^2 would be getting from here only one R is there; so, then from this one R, then square of this R is gone. So, then this R, this R is gone. So, then what we have?

Left hand side, we have only $\zeta = \int_0^1 \Theta(\xi,\zeta)(1-\xi^2)\xi\, d\xi$; this is what you get. You can simply do it. Practice it and then, you can get to this equation. So, this is the third boundary condition that we got. Now, the solution whatever is there, after this part very mathematical. So, this is your solution this is your governing equation.

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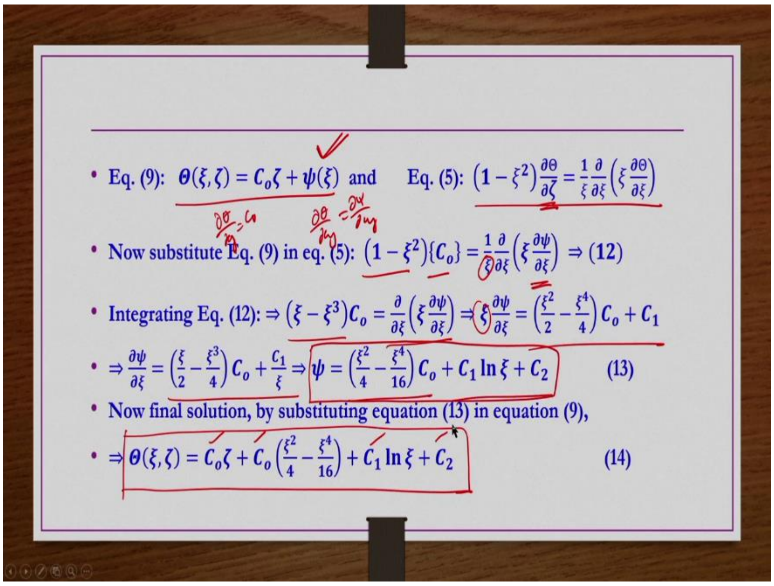

So, now, this equation you can substitute here. So, this equation should be satisfied. So, let us say from this equation $\frac{\partial\theta}{\partial\zeta}$ is nothing but C_1. So, I can substitute here in place of this one is C_1. So, $(1-\xi^2)$ C_1 = $\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\psi}{\partial\xi}\right)$ I get. Because $\frac{\partial\theta}{\partial\xi}$ is nothing but $\frac{\partial\psi}{\partial\xi}$; is not it?

So, that when you do $(1-\xi^2)$ in place of this one is C_1; in place of $\frac{\partial\theta}{\partial\xi}$, I have to write $\frac{\partial\psi}{\partial\xi}$ right. So, this is what we are having when you expand left hand side, this also you take to the left hand side. So, then you have $(\xi-\xi^3)$ C_0 and then this one right.

So, next what do you do? Integrate it; so, $\left(\frac{\xi^2}{2}-\frac{\xi^4}{4}\right)$ C_0 + C_1. So, this ψ you bring it to the right hand side again. So, then this is what you are having. Once again if you integrate, you get $\left(\frac{\xi^2}{4}-\frac{\xi^4}{16}\right)C_0$ + C_1 ln ξ + C_2. So, this ψ function whatever is this, so is this one. So, this you this unknown function, you got it now by given 30 number equation number 13.

So, now, over all final solution, we can have $\Theta(\xi,\zeta)=C_0\,\zeta+C_0\left(\frac{\xi^2}{4}-\frac{\xi^4}{16}\right)+C_1 ln\xi+C_2$. This is the final solution; only thing that what is this C_1? What is the C_0, C_1, C_2 that you have to find out. So, that we can find out simply by using boundary conditions. So, this is the solution.

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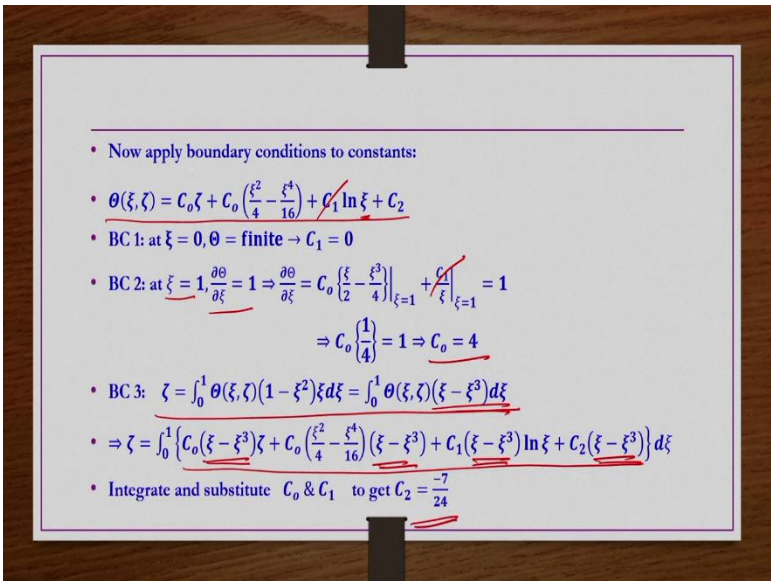

First boundary condition is at ξ = 0, θ is equals to finite, that means, if you substitute 0 here, ln 0 is not possible, so C_1 has to be 0. Then, second boundary condition at ξ = 1, $\frac{\partial\theta}{\partial\xi}=1$. So, from this equation, $\frac{\partial\theta}{\partial\xi}$ is nothing but $C_0\left\{\frac{\xi}{2}-\frac{\xi^3}{4}\right\}+\frac{C_1}{\xi}$; C_1 is any way 0 from boundary condition 1 after applying the boundary condition 1.

Then, ξ if you substitute 1 here, $\left\{\frac{1}{2}-\frac{1}{4}\right\}$ is $\frac{1}{4}$. So, that means, C_0 = 4. Then, third boundary condition is this one right $\zeta\int_0^1 \theta(1-\xi^2)\xi d\xi$. So, $(\xi-\xi^3\ )d\xi$, you can write; θ you can substitute, θ is this one. So, all the terms are being multiplied by $(\xi-\xi^3\ )$.

Now, you looks like lengthier, but simple you integrate this equation and then, substitute C_0, C_1 and then, get C_2. Then, C_2 you get $-\frac{7}{24}$. So, all the constants C_0, C_1, C_2 you got it.

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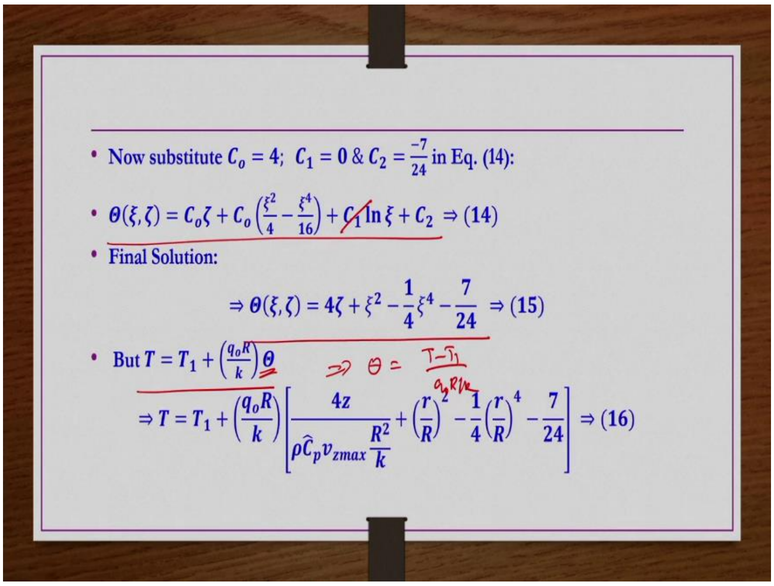

So, then final solution you got it. So, that is θ is equals to in this form that a final solution C_0, you substitute 4; C_0 its 4, C_1 is 0. So, then this term is anyway 0. Then after substituting, you get this final solution for the θ; θ as function of ζ and ξ you got it ok. But finally, what you have? What you need to have? You need to have temperature in dimensional form.

So, dimensional form T = T_1 + $q_0\frac{R}{k}$ θ. This is coming from your θ scaling parameter definition that is $\frac{T-T_1}{q_0\frac{R}{k}}$. So, now, here in this equation, in case of θ, you substitute equation number 15. So, then you have this final solution for the temperature distribution of a Newtonian fluid under fully developed flow conditions alright.

So, this temperature distribution for a Newtonian fluid flowing through pipe; but the flow is under fully developed flow conditions, temperature profile is also valid for only fully developed region.

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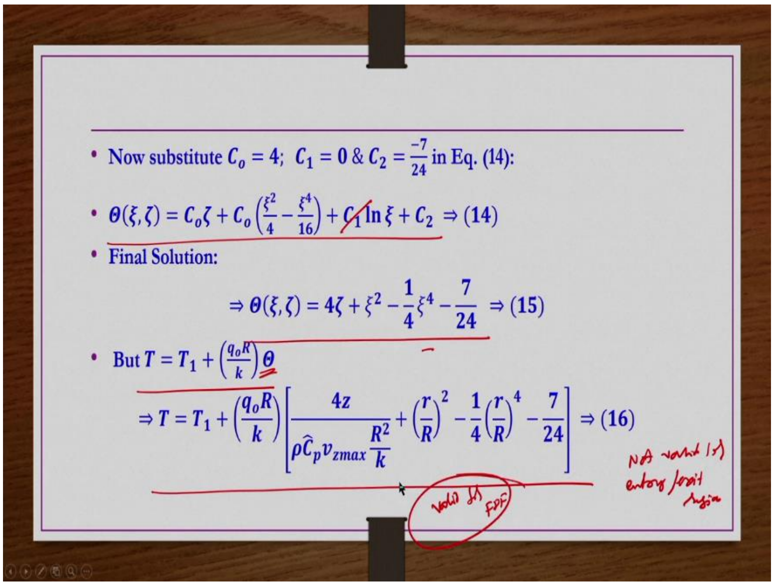

Not valid for entry and then exit regions; valid for only fully developed flow region only. If you have to include the entry exit regions also, then you have to do using the numerical solution; otherwise, it is not possible to get approximate solutions like this.

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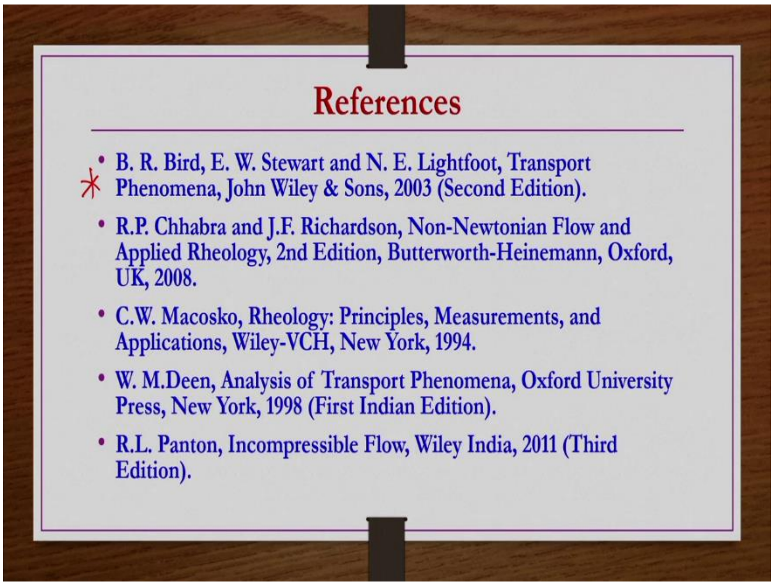

This problem is taken from this reference book. Other useful references are provided here.

Thank you.