Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 27
Temperature distribution in fluids confined between co-axial cylinders

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of today's lecture is Temperature distribution in fluids confined between coaxial cylinders.

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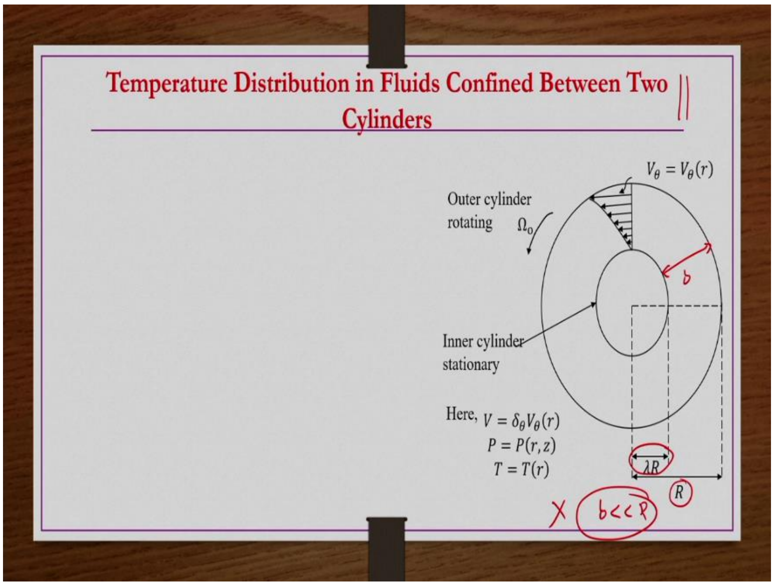

So, in the previous lecture we have taken a similar problem there also we have taken a coaxial-cylinders and then fluid is confined between them. So, we got the velocity profile and then temperature distribution.

But what we have done the case in the previous lecture what we have taken is you know the cylinders are very close to each other, the confinement gap whatever is there very very small almost like both the cylinders inner and outer cylinders are you know touching to each other, right.

So, under such conditions what we had taken, since the gap between these two cylinders is very very very very smaller compared to the radius of outer cylinder the curvature effect we have neglected and obtain the velocity profile. And then under such conditions we have seen that velocity profile is linear whether the fluid is Newtonian or power law or power law fluids that also we have taken we have seen right.

So, that is one thing right and then accordingly according to that velocity profile we have found the temperature distribution and then we got a Brinkman number for Newtonian fluid as well as the non-Newtonian power law fluid case as well, ok.

So, that Brinkman number we have defined for both Newtonian case as well as the power law fluids case right, but now the geometry is similar here in today's lecture, but difference major difference is that gap is not very small, the cylinders are not touching to each other almost it is not the case.

And then also the gap is not very large that the inner cylinder radius is very very small compared to the radius of outer cylinder that is also not the case the gap is moderate the gap is small, but not very small that you know you can avoid the viscous dissipation. It is small, but still viscous dissipation is there it is small, but you cannot, but you cannot avoid the curvature effects.

Under such kind of I mean geometry is same, but the confinement is in such a way. So, then; obviously, the velocity profile is going to be different if it is linear or non-linear that is also, we cannot say and then even if it is linear, you know we are going to get a, we supposed to get a different velocity compared to the previous case.

Because now the curvature effects are coming into the picture, we are going to see that in this case the velocity profile itself is going to be non-linear and then; obviously, the subsequent temperature distribution is also going to be very different from what we have seen.

So, that is one difference further from the problem-solving point of view also there are certain differences that are what are those things that we are going to see it. So, before getting into the solution of this problem of coaxial cylinder fluid confined between coaxial cylinders, but the gap is small, but not very small. So, before solving that problem we have to see the schematic right, schematic if you see the radius of inner cylinder, you are taking λ R and then radius of outer cylinder you are taking R right, the gap between these two is there whatever is there.

Let us say for the time being we calling b, but it is not very very small compared to the R as in the previous case we have seen right, if it is like this case is this. So, then curvature effect we can avoid and then we can do the problem in a Cartesian coordinate and then velocity profile you are going to get linear.

But in this case the gap is not such small; it is small, but it is not such small that you can avoid the curvature effects, ok. So, that is one thing so; obviously, the gap between these two cylinders is not very small.

So, then the velocity profile whatever is there between these I mean velocity profile of the fluid that is confined between these two fluids is going to be different compared to the previous case ok. So, now, coming to the other details of the geometry the inner cylinder is stationary.

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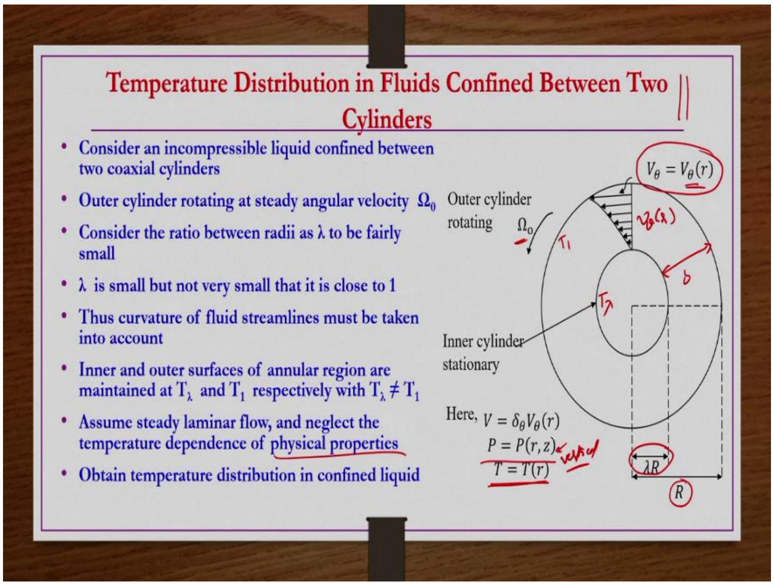

And then it is maintained at temperature T_λ, outer cylinder is rotating with the rotational velocity Ω_0 and then temperature is maintained at T_1, right. So, now this outer cylinder is rotating in angular direction so; obviously, we are going to have a component velocity component in angular direction is going to be dominating that is v_Θ component is going to be dominating compared to the v_r and v_z components of the velocity right.

And then further the variation since the inner cylinder is stationary and then outer cylinder is rotating so; obviously, the variations in the velocity are going to be you know more significant in the radial direction that is from $R = \lambda R$ to $R = R$.

So, V_Θ is predominating component of the velocity and it is function of r, right. For the temperature distribution in general we cannot say whether it is function of r only or function of Θ also, but for the time being we take it is function of r only, ok.

So, if you take function of Θ also problem will become more complicated, but anyway such kind of problem we are taking in the next lecture right and then pressure is also function of not only function of r, but function of z also because we are taking vertical cylinder. So, hydrostatic pressure will also come into the picture ok.

So, these two coaxial cylinders are arranged in a vertical configuration and then whatever the fluid between these you know two cylinders is there is rotating in a Θ direction because outer cylinder is rotating in Θ direction at the angular velocity Ω_0.

So, obviously, this velocity profile v_Θ as function of r is going to be non-linear in this case that we are going to find out how what exact form and then temperature profile anyway when you include the velocity also in the temperature distribution. So, then definitely the final temperature profile is going to be non-linear.

Because in the temperature distribution whatever the non-linearity is coming to the picture that is coming through the convection terms and then convection terms in general velocity components would also be there right indeed and in convection terms velocity components would be there. So, they will be bringing the more non-linearity in the temperature distribution that finally, we are going to get, right.

So, this is the basic about the problem statement right. Now, we see all these things written here consider an incompressible liquid confined between two coaxial cylinders, nature of the fluid is not given. So, for the time being we take a Newtonian fluid if it is power law fluid also then exactly similar way we have to do.

Outer cylinder is rotating a steady angular velocity Ω_0, the ratio between radii is a λ it is fairly small it is not very very small that the curvature effect can be avoided it is not such small it is small, but fairly small not very small, right.

So, the curvature effect must be taken into the account while obtaining the velocity profile then inner and outer surfaces of angular region are maintained at T_λ and T_1 respectively and both of them are not equal to each other then only we are going to have a temperature distribution right. Assume steady laminar flow and neglect the temperature dependence of the physical property. So, then ρ, μ, C_p etcetera are independent of the temperature.

If ρ, μ are independent of the temperature; that means, the velocity profile is not going to be affected by the temperature difference right; however, the temperature difference is going to be affected by the velocity distribution. Obtain temperature distribution in confined liquid for this case. So, now what we do in general? We first you know list out the assumptions and then constraints of the problem.

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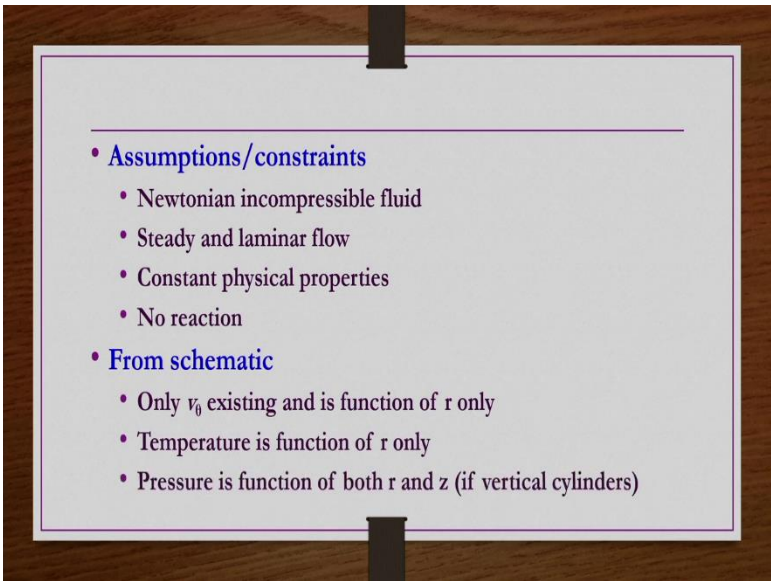

So, Newtonian incompressible fluid steady and laminar flow, constant physical properties, no reactions and then from schematic, v_Θ is only existing it is function of r temperature is function of r only and then pressure is function of both r and z because we are taking vertical cylinders.

Now, first we have to get the velocity profile before getting the temperature distribution, without knowing the velocity profile you cannot simplify or solve you know convection part of an energy equation. So, that is the reason you know we have to first get the velocity profile, right.

So, how do we, how do we get the velocity profile? As we have, as we have seen in previous lectures you know first we have to simplify the you know continuity and then momentum equation and then one of the momentum equations one component of momentum equation would be giving a relation for the shear stress and in that we have to solve to get the velocity profile, right.

However, now we are doing the case for a Newtonian fluid. So, momentum equations also we are going to write for the Newtonian fluids directly that is Navier-Stokes equations and cylindrical coordinates that we are going to use. So, now, what we do?

We simplify the continuity equation, before simplifying the continuity equation v_Θ is function of r that we know, but we do not know whether the flow is fully developed or the flow is like you know symmetric those things we do not know, but some kind of information we can get by simplifying the continuity equation also.

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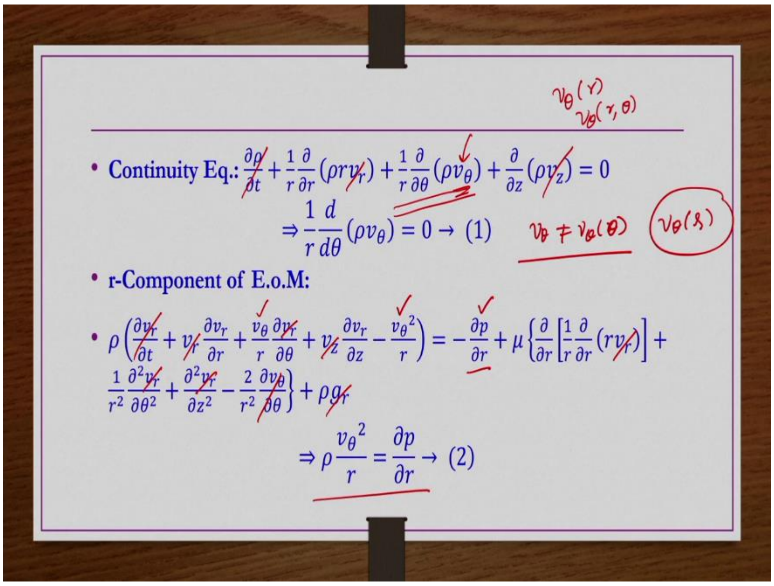

So, continuity equation in cylindrical coordinates is given here. So, steady state term. So, this term is 0 v_r is not existing v_Θ is existing, but it is not function of Θ that is given. So, v_Θ is function of r only it is not function of Θ. So, then all together we can cancel out this term v_z is not existing.

So, this term is 0. So, altogether $\frac{\partial}{\partial \theta}(\rho v_\theta)$ is 0 because v_Θ is function of r. So, that way you can cancel out and then say the continuity is satisfied otherwise from the schematic if you are not sure whether v_Θ is function of r only or is it function of Θ also. So, that information you can get it from here.

Let us say; let us say you are confused whether the v_Θ is function of r only or is it function of both r and Θ if you are not sure from this schematic. So, then you know from this continuity equation that information you can get right. So, now, from here what we get? $\frac{\partial}{\partial\theta}(v_\theta) = 0$.

Because incompressible fluid we are taking; that means, v_Θ is not function of Θ, right. So, v_Θ is only function of r. So, that conformation also we can get, if you are not sure whether it is function of Θ or not then we have to obtain the final conclusion like this.

So, now equation of motion r, Θ, z components we simplify and see what we get. So, r component of equation of motion is given here. So, we apply the constraints of the problem. Steady state so, this term is 0 v_r is not existing v_Θ is existing, but v_r is not there, v_z is not there v_Θ is there, pressure in general we do not know.

So, let us keep it as it is and then then the viscous force terms v_r is not there. So, v_r is not there, v_r is not there, v_Θ is not function of Θ it is function of r only. So, this term is also 0. And then gravity we are taking in the z direction, the configure you know vertical cylinders we are taking and then such a way that the gravity is only in the z direction. So, this term is also 0.

So, what we get from here? $\rho\frac{{v_\theta}^2}{r} = \frac{\partial p}{\partial r}$; that means, pressure is function of r that is; obviously, because the flow is taking the velocity is variation is taking place in the r direction. So, then in the direction definitely pressure variation should be there ok.

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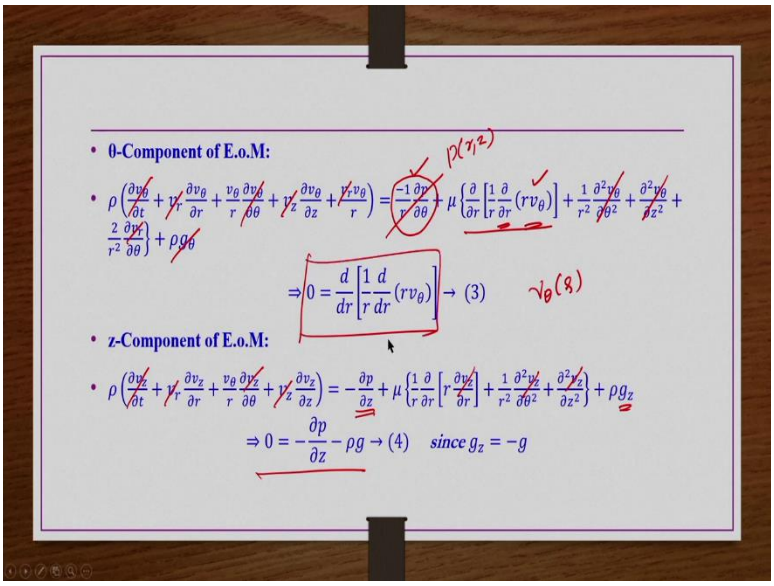

So, this is one. Now, Θ component of equation of motion is given here. A steady state so, this term is 0 v_r is not existing v_Θ is existing, but v_Θ is not function of Θ. So, this is 0, v_z is not existing, v_r is not existing. So, altogether all the terms in the left-hand side are you know cancelled out.

$\frac{\partial p}{\partial \theta}$ pressure how it is varying in general we do not have any information about the pressure. So, let it be like that then v_Θ is existing and it is function of r. So, then this term should be there v_Θ is not function of Θ, v_Θ is not function of z, v_r is not existing.

And then gravity is only in the z direction, right. So, now, about the pressure it is mentioned in the problem that pressure is function of r and z only it is mentioned, it is not function of Θ that is given in the problem statement. So, at least now it is given. So, then we can strike off this term as well.

So, then what we have? We have only one term remaining and then all other terms are cancelled out then we have $\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rv_\theta)\right] = 0$ this equation if you solve you get the velocity profile v_Θ as function of r right. Now z component of equation of motion given here.

So, steady state this term is 0, v_r is not existing v_z is not existing, v_z is not existing. So, all the terms in the left-hand side are negligible. Pressure it is function of z because we are taking vertical cylinder and then gravity in the negative z direction. So, it would be there. So, v_z is not existing.

So, all these three terms are 0. So, g_z is nothing but $-$ g. So, $-\frac{\partial p}{\partial z} - \rho g = 0$, right. So, that is what we have now ok. So, now we need this equation for getting the velocity profile. So, this equation we are going to solve it.

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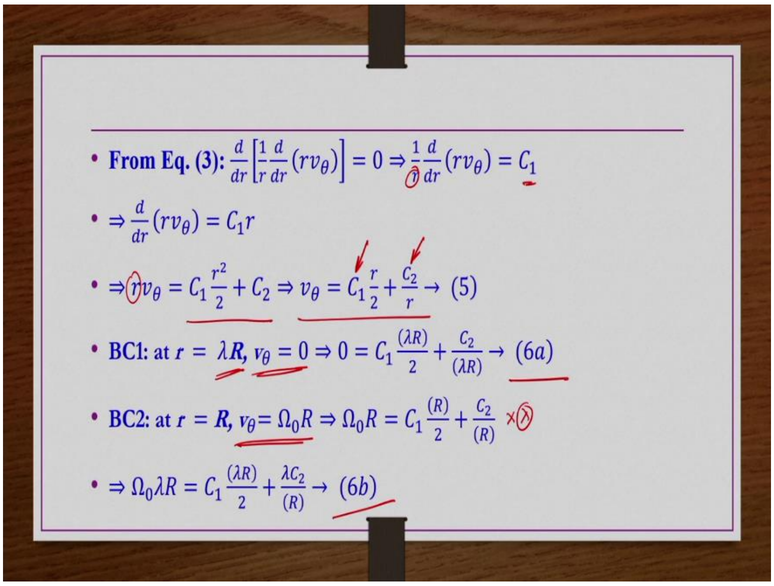

So, when you do first time integration you get C_1 constant right-hand side now this C and this r you take to the right-hand side. So, $C_1 r = \frac{d}{dr}(rv_\theta)$ when you integrate next step you get $rv_\theta = C_1\frac{r^2}{2} + C_2$. Now, this r if you bring it to the right-hand side, you have $v_\theta = C_1\frac{r}{2} + \frac{C_2}{r}$.

So, now from this step itself is clear that you know whatever the values of C_1 C_2 the velocity profile is not linear, it is non-linear ok whereas, when we have the confinement gap very narrow be very smaller than the R then we got a velocity profile a linear velocity profile ok, but it is not true here if the gap is fairly small, it is not very small, but only fairly small then it is non-linear velocity profile.

Now, boundary condition inner cylinder is stationary, inner cylinder location is at r = R λ. So, $0 = C_1 \frac{(\lambda\, R)}{2} + \frac{C_2}{(\lambda\, R)}$ whereas, the outer cylinder is rotating with Ω_0 angular velocity. So, $v_\Theta = \Omega_0$ R at r = R. So, that is Ω R = C_1 R + C_2/R.

So, now this equation both sides we multiply by λ. So, that we get $\Omega_0 \lambda\, R = C_1 \frac{(\lambda\, R)}{2} + \frac{C_2}{(\lambda\, R)}$ you are going to have. Why because this now 6a minus 6b you do $C_1 \frac{(\lambda\, R)}{2}$would be cancel out and then you will have an equation for C_2, right.

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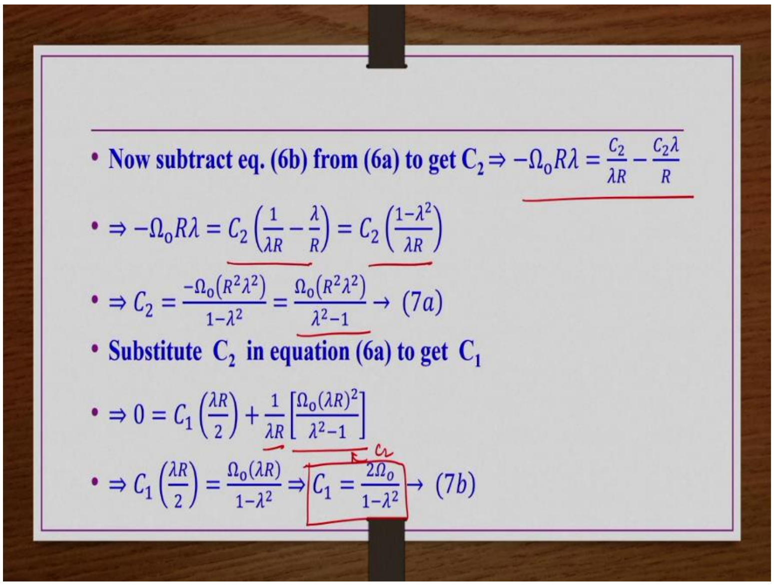

Then $-\Omega_0 \lambda\, R = \frac{C_2}{\lambda\, R} - \frac{C_2 \lambda}{R}$, this you first do you know C_2 you take common here then you are doing the LCM. So, then next step what you do? You take C_2 one side and then all other terms to the other side then you get $C_2 = \frac{\Omega_0\, R^2 \lambda^2}{\lambda^2 - 1}$.

So, now, this we substitute in equation number 6a which is nothing, but $0 = C_1 \left(\frac{\lambda\, R}{2}\right) + \frac{1}{\lambda\, R}\left[\frac{\Omega_0 (\lambda\, R)^2}{\lambda^2 - 1}\right]$. So, now C_2 is this one, right. So, you substituted here. So, now, what you do? Your C_1 you take this term one side and then simplify then you get $C_1 = \frac{2\Omega_0}{1-\lambda^2}$.

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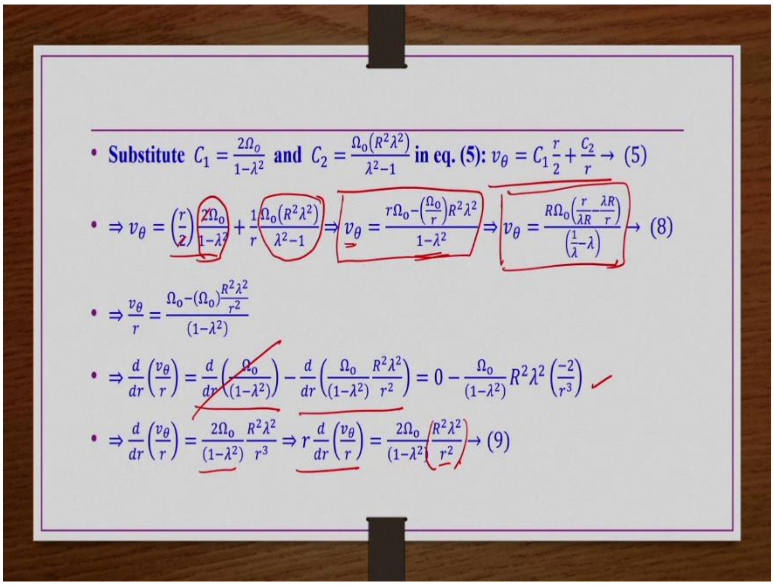

So, now, you have both C_1 C_2 this C_1 C_2 you substitute in $v_\theta = C_1\frac{r}{2} + \frac{C_2}{r}$ when you get. So, this is $\frac{r}{2}C_1$. So, this is C_1 and then $\frac{1}{r}C_2$ this is C_2 right. So, now, what you do? From both the terms $1-\lambda^2$ you take common in the denominator. So, then you have $r\Omega_0 - \left(\frac{\Omega_0}{r}\right)R^2\lambda^2$ because these two is cancelled out.

Now, this velocity profile you can see it is non-linear ok so, but further what we do? We write outer cylinder in terms of the velocity because outer cylinder is rotating with Ω_0 angular velocity. So, velocity profile we will write such a way that R Ω_0 multiplied by some whatever the additional factor or correction factors, right.

So, that is this one. So, this is the final velocity profile fine. So, now, what we do? This term we take and then do some more simplification, what are those simplification? We divide by r both sides. So, $\frac{v_\theta}{r} = \frac{\Omega_0-(\Omega_0)\frac{R^2\lambda^2}{r^2}}{1-\lambda^2}$.

Because we are taking viscous dissipation also under consideration here, the gap is small. So, viscous dissipation should be there, only thing that it is not very small that we velocity profile can be taken as linear ok that is the major difference. So, here also viscous dissipation is there and then in the viscous dissipation terms you are going to have $\frac{\partial v_\theta}{\partial r}$ kind of terms.

So, that for that I am making calculation in advance. So, this is $\frac{v_\theta}{r}$ you can write like this now differentiate with respect to r. So, $\frac{d}{dr}\left(\frac{\Omega_0}{1-\lambda^2}\right) - \frac{d}{dr}\left(\frac{\Omega_0}{1-\lambda^2}\frac{R^2\lambda^2}{r^2}\right)$.

So, this is constant. So, this $\frac{d}{dr}$ of first term is $0 - \frac{\Omega_0}{1-\lambda^2}R^2\lambda^2$ and then $\frac{d}{dr}\left(\frac{1}{r^2}\right)$ is $\left(\frac{-2}{r^3}\right)$, right. So, now, it is nothing, but $\frac{2\Omega_0}{1-\lambda^2}\frac{R^2\lambda^2}{r^3}$.

Now, what you do? You multiply both sides by r. So, $r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)$ is nothing, but $\frac{2\Omega_0}{1-\lambda^2}\frac{R^2\lambda^2}{r^2}$, right.

So, this term we are going to use in energy equation after simplifying, that is the reason we have done this part. Energy equation in cylindrical coordinates is given here.

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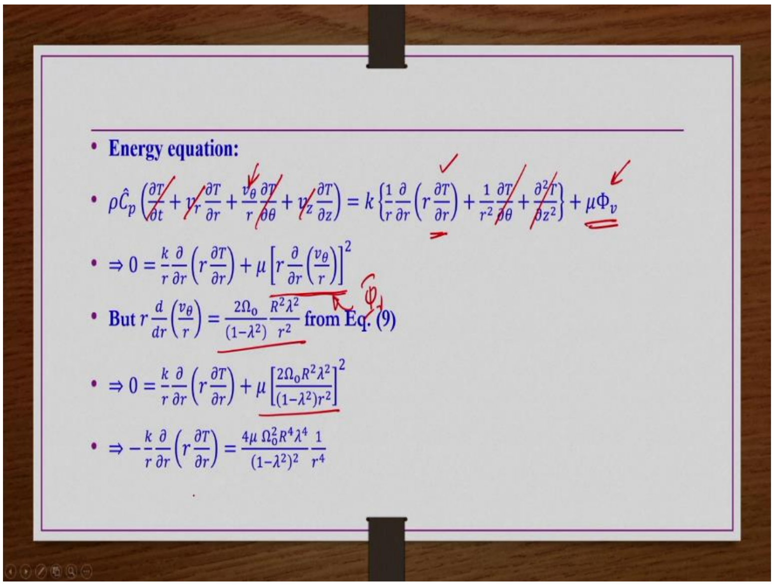

So, now here also you apply the constraints of the problem steady state. So, this term is 0 v_r is not existing, v_Θ is existing right, but the temperature is not function of Θ, it is function of r only is given. So, this term is 0 v_z is not existing. So, this term is also 0. So, right hand side temperature is function of r. So, this term would be there, but it is not function of Θ and z. So, second third terms of the parenthesis are cancelled out.

And then this term indicates the viscous dissipation term. So, viscous dissipation these 9 terms should be there. So, when you expand and then apply the constraints of the problem you get only $\left[r\frac{d}{dr}\left(\frac{v_\theta}{\mathrm{r}}\right)\right]^2$ this particular term you get from phi v after simplifying Φ_v this is what we get. So, now, $r\frac{\partial}{\partial r}\left(\frac{v_\theta}{\mathrm{r}}\right)$ we have already obtained it.

So, that is this one from equation number 9. So, that you can substitute here. So, you get this one and then further simplifying you get $\frac{4\mu\,\Omega_0{}^2R^4\lambda^4}{(1-\lambda^2)^2}\frac{1}{r^4}$ and then that whole $=$ $-\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$.

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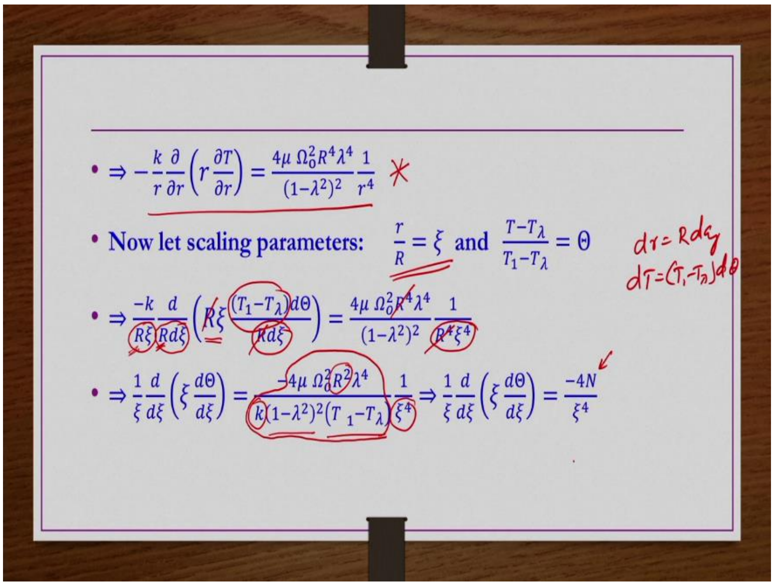

So, the same equation written here. So, now, what we do? We take a scaling parameters. So, $\xi = \frac{r}{R}$ and then Θ is dimensionless temperature, ok. So, which is defined as $\frac{T-T_\lambda}{T_1-T_\lambda}$, ok.

So, because what we are trying to do? This equation we are trying to write in dimensionless form by using this scaling parameters, directly also if you can integrate and get the solution there is no difficult there is no problem, but this is going to be a kind of exercise for us for the next problem that we are going to solve in the next lecture.

So, now if $\frac{r}{R}$ if you are taking as ξ then $dr = Rd\xi$ and then $\frac{T-T_\lambda}{T_1-T_\lambda}$ if you are taking as Θ then $dT = (T_1 - T_\lambda)\ d\Theta$ this Θ is nothing but dimensionless temperature, ξ is nothing but dimensionless radial coordinate that is what we understand now from here, ok.

So, now these things we are going to use here in this equation when you use it you get − k in place of r you are having R ξ from this definition d by again in place of dr you are writing $Rd\xi$. And then parenthesis then r is again nothing but R ξ and dT is nothing but $(T_1 - T_\lambda)$ dΘ.

So, this one and dr is nothing but R d ξ ok and then right-hand side only r term is only this one R^4 in place of R^4 we can write R^4 ξ^4 this is what we have, right.

Now, this equation if you simplify what happens this R this R is cancelled out right now here you have R^2 R and R. So, R^2 that will come here, but here R^4 and then this R^4 is cancelled out. So, that you have $\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\theta}{d\xi}\right)$ and then whatever $(T_1 - T_\lambda)$ is there that also we are taking to the right-hand side.

So, then $-4\,\Omega_0{}^2 R^2$. So, this R^2 is coming from the left-hand side R and then R here is this. So, R^2 that brought it here.

And then λ^4 divided by this k also coming from the left-hand side $(1-\lambda^2)^2$ is already there and then this $(T_1 - T_\lambda)$ is also coming from the left-hand side and then $\frac{1}{\xi^4}$. So, now, what we do?

In the right-hand side except this ξ^4 rest of the terms whatever are there other than negative symbol we write it as N, N which is similar like you know Brinkman number. So, then this equation would be $\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\theta}{d\xi}\right) = \frac{-4N}{\xi^4}$ right.

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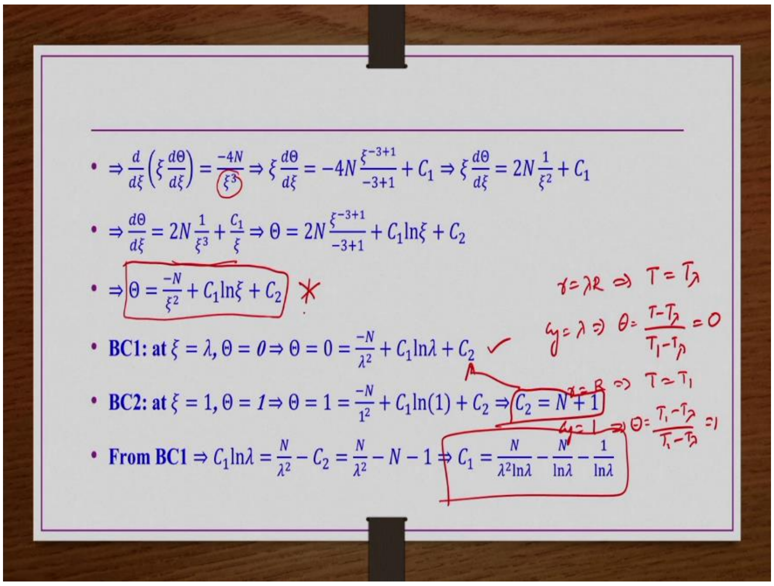

So, now $\frac{1}{\xi}$ whatever in the left-hand side is there if you take to the right-hand side that would become $\frac{-4N}{\xi^3}$ it will become earlier it was ξ^4. So, then left hand side whatever ξ is there that we brought it the brought it to the right-hand side.

So, then ξ^3 it has become. So, now, you do the integration. So, $\left(\xi \frac{d\theta}{d\xi}\right) = -4N\frac{\xi^{-3+1}}{-3+1} + C_1$ that is $\left(\xi \frac{d\theta}{d\xi}\right) = \frac{2N}{\xi^2} + C_1$.

Now, this ξ you take it to the right-hand side. So, then here in the right-hand side you have $\frac{2N}{\xi^3} + \frac{C_1}{\xi}$. So, now, once again if you do the integration $\theta = 2N\frac{\xi^{-3+1}}{-3+1} + C_1 ln\xi + C_2$ that is $\Theta = -\frac{N}{\xi^2} + C_1 ln\xi + C_2$.

So, this is the temperature distribution in the non dimensional form that is non dimensionalized temperature as function of non dimensionalized radial coordinate is this one only thing that C_1 C_2 constants we have to find out right.

So, r = λ r is nothing but the inner cylinder surface and then at inner cylinder surface T = T_λ, right. So, now, Θ is nothing but $\frac{T-T_\lambda}{T_1-T_\lambda}$ this is what we have. So, now, here if you substitute T = T λ then Θ = 0 at r = λ r. So, Θ = 0$= -\frac{N}{\lambda^2} + C_1 ln\lambda + C_2$, because at r = λ R; that means, ξ is nothing but λ, ok.

So, this is we are getting. Other boundary condition is at r = R, T = T_1. So, r = R in the sense $\xi = 1$ and then $\Theta = \frac{T_1 - T_\lambda}{T_1 - T_\lambda}$ that is 1. So, that you substitute here. So, $\Theta = 1 = -\frac{N}{1^2} + C_1 \ln(1) + C_2 \ln(1)$ is 0.

So, C_2 = you get N + 1 from this equation this C_2 you substitute here right and then you get $C_1 \ln \lambda = \frac{N}{\lambda^2} - C_2$. So, that is $\frac{N}{\lambda^2} - N - 1$ and then C_1 on expansion you get this one. So, this C_1 C_2 both now you can substitute here in this equation to get the final temperature distribution.

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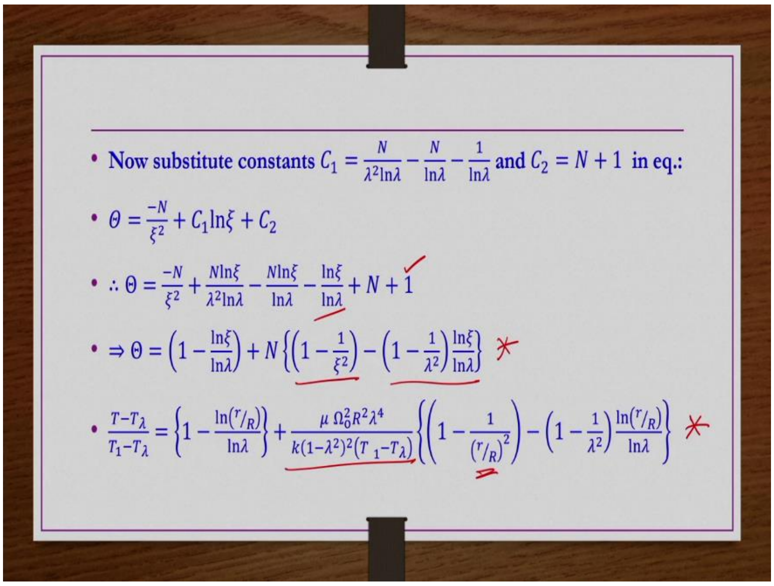

So, when you do it this is what you get and out of these two what you do? This 1 and then $-\frac{ln\xi}{ln\lambda}$ these are the two terms which are not having N. So, you combine these two terms as 1 term and then from other terms what you do?

You take the N common and then after that also $1 - \frac{1}{\xi^2}$ as 1 term and then $-\left(1 - \frac{1}{\lambda^2}\right)\frac{ln\xi}{ln\lambda}$ other term you can write it. So, this is the final temperature distribution in dimensionless form, ok.

Dimensionless temperature as function of dimensionless radial coordinate is this one right, but in dimensional form we have this T $-$ Θ is nothing but $\frac{T - T_\lambda}{T_1 - T_\lambda}$, right. So, this

$= 1 - \ln\xi$ is nothing, but $\frac{ln\frac{r}{R}}{ln\lambda}$ is as it is and then this N is nothing, but $\frac{\mu{\Omega_0}^2 R^2 \lambda^4}{k(1-\lambda^2)^2(T_1 - T_\lambda)}$ and then multiplied by wherever ξ is there we are writing $\frac{r}{R}$. So, this is the final temperature distribution.

Now, we can see how different it is compared to the case of a previous lecture where we avoided the curvature effects if you are considering curvature effects on the velocity distribution. So, the velocity distribution is not linear as we got unlike the previous problem also the temperature profile is also very different. So, we can realize how much important is the curvature effect especially when you are solving the viscous dissipation problem for coaxial cylinders like this.

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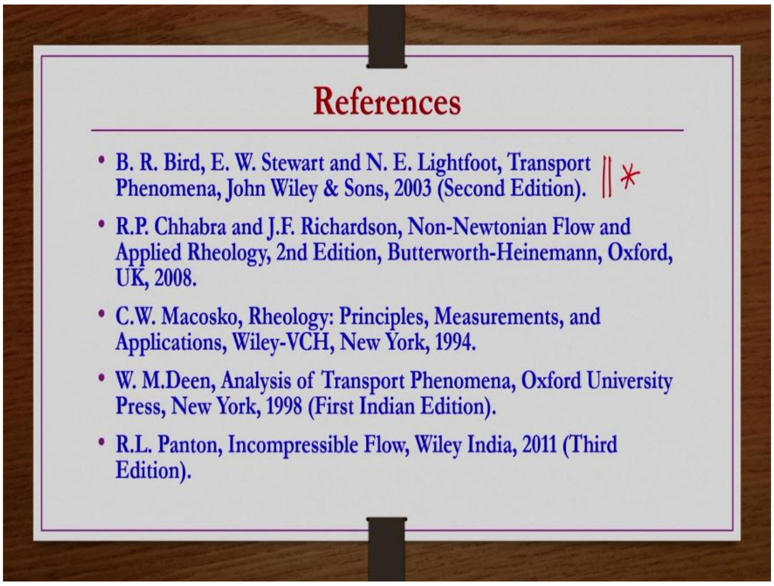

References: this problem is you know taken from this reference book ok, other useful references are provided here.

Thank you.