Transport Phenomena of Non-Newtonian Fluids
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Lecture - 26
Viscous Heat Generation

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of today's lecture is Viscous Heat Generation.

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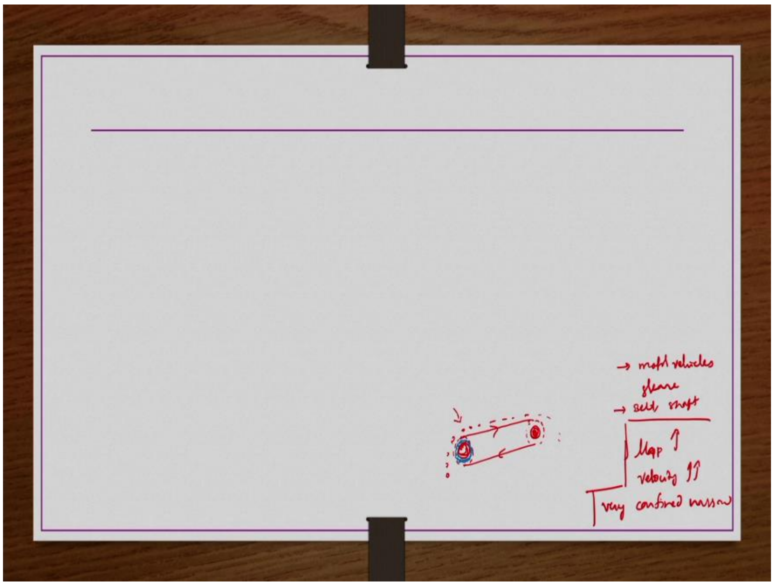

So, viscous heat generation in general is important where this apparent viscosity is very high, and then it is moving at a high velocity and then it is flowing in very confined narrow gaps. So, under such conditions what happens? Whatever the mechanical energy is there that converts into the thermal energy and then because of that one heat generation takes place. And then that is known as the viscous dissipation, ok.

So, now in the lecture what we will do? We will take a few example problems, where the viscous heat generation is very essential and then in order to know how much is the temperature variation or temperature increasing because of this viscous heat dissipation etcetera. Those things we are going to see by solving a few problems, ok.

Some examples of this viscous heat generation in general we see like you know motor vehicles; grease is actually you know confined between two cylinders and then one of them may be rotating based on the design it requirement. So, there it general this grease is high viscous, apparent viscosity of a grease is really very high. And then this motor vehicles obviously we know that they run at very high velocities.

So, and then these greases etcetera are confined between very narrow spaces, so then viscosity generation etcetera takes place. Then the conveyors like you know belt conveyors, shaft conveyors etcetera in general in chemical industries. Where they are being conveyed, because they have been you know confined along the two cylinders at different locations.

And then these cylinders may be rotating and then because of that one this shafts or belts may also be rotating right. So, this cylinder is rotating and but it is rotating, there is a inner confined inner cylinder which is confined and then that is not rotating, right.

So, let us say this is the inner cylinder and then there is an outer cylinder is there. So, like this there are two cylinders are there and then they are confined like this, right. And then this belt is moving like this. So, that the material whatever the solid materials etcetera are there so they can be taken poured here. So, they can be conveyed to the other locations that is what happens; that is what happens in general right.

So, now, this whatever this confined area is there here between these two cylinders. So, this is very narrow actually, this blue colour maybe you can take it as a confined region. So, that is very narrow and then that area is in general you know provided with some kind of a high viscous lubrication fluids.

So, in this phenomena what happens? This because of the high speed velocity and then high viscous fluid the viscous dissipation takes place. That mechanical energy converts into the thermal energy and there is a heating of the cylinders etcetera may takes place. So, that is also not good if it is too much heating.

So, accordingly based on the temperature variation for a given speed of the rotation and then for a given high apparent viscosity, how much temperature variation is there those calculation one has to do accordingly the coolant fluid etcetera may be supplied.

So, that is the purpose of you know considering these viscous heat generation problems in general in the majority of the material or automobile the kind of industry; material processing industries or automobile industries or wherever they are applicable, ok. So now, in this lecture we are going to take a couple of problems and then do the derivation to get the temperature distribution for these kind of geometries.

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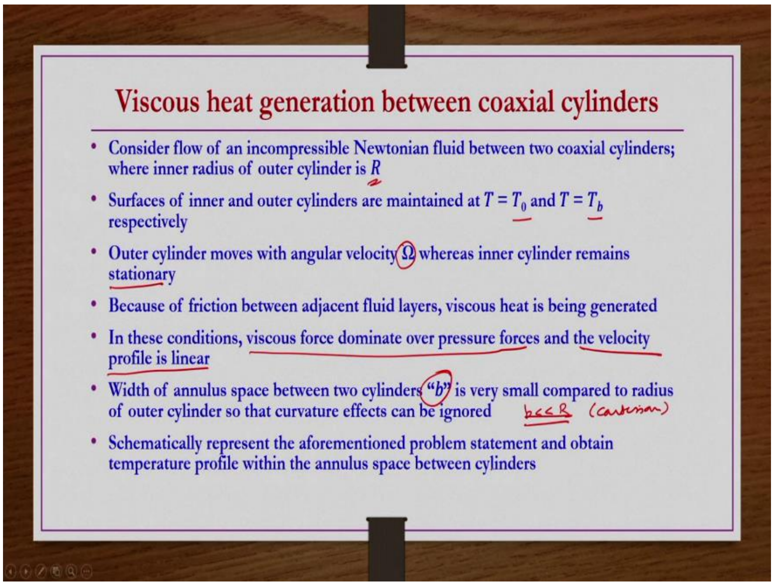

So, the first instant is viscous heat generation between coaxial cylinders. This we are taking as a first problem of today's lecture. First let us see the statement of the problem. Consider flow of an incompressible Newtonian fluid between two coaxial cylinders, where inner radius of outer cylinder is R. Surfaces of inner and outer cylinders are maintained at T_0 and T_b respectively.

Outer cylinder moves with angular velocity Ω whereas, the inner cylinder remains stationary. Because of friction between adjacent fluid layers viscous heating is being generated. In these conditions, viscous force dominate over the pressure force that is one condition given and then velocity profile is linear that is also another condition given.

Width of annulus space between two cylinders b is very small compared to the radius of outer cylinder R, so that curvature effects can be ignored. That is b is very very smaller than R, so that we can avoid the curvature effects. If you are not considering the curvature effects, so even the geometry cylindrical geometry we can do the problem solving in Cartesian coordinate system ok. That we see now.

Then the statement is a problem statement is schematically represent the aforementioned problem statement and obtain temperature profile within the annulus space between two cylinders. So, first schematically we represent this flow problem and then we get into the solution steps to get the temperature profile.

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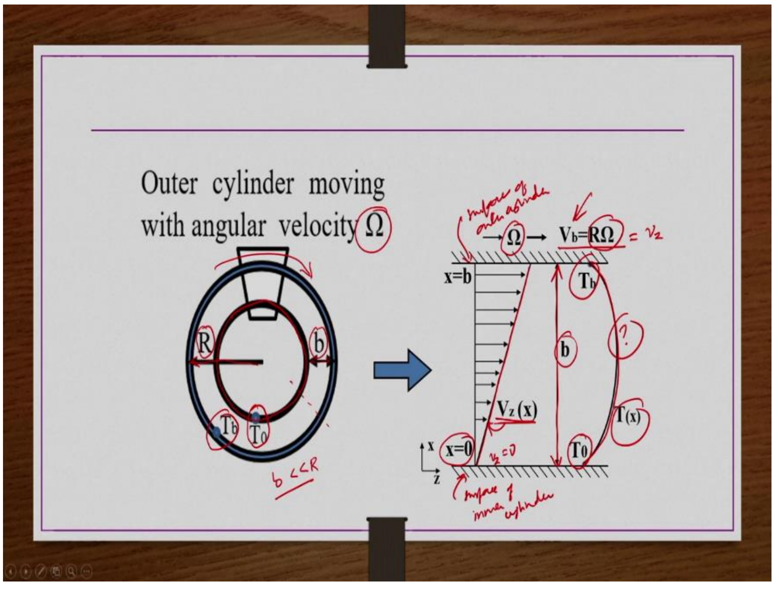

So, now schematically if you see this is the inner cylinder whose radius is not known, not required also for this problem is maintained a temperature T_0. And then this is the outer cylinder whose radius is capital R and then it is maintained a temperature T_b. The gap between these two cylinders is b. And then b is very very smaller than R so that you know we can avoid the curvature effects, and then this outer cylinder is rotating with angular velocity *Ω*, right.

So, now since b is very very small smaller than the radius of an outer cylinder capital R, what we can say? Curvature effect is negligible and then that is given in the problem statement as well. So now, this problem we can represent in Cartesian coordinate system. So, how we can do?

You just cut the cylinder at any location and then stretch it. Then what you get? You have a geometry like this, where the surface of inner cylinder is x_0; surface of inner cylinder which you can designate at x = 0 and then top layer is surface of outer cylinder right which is moving at velocity Ω.

So, if this velocity if you convert into the rectilinear coordinates velocity component then you get R Ω as that velocity. And then let us designate R Ω as V_b, right. The gap between these two cylinders is small b, inner cylinder is at T_0 temperature, outer cylinder is at T_b temperature, the velocity profile is linear, right. So, that is also given V_z and it is function of x only.

The temperature is function of x, right. The nature of temperature profile we do not know what it is, right. So, that is what we are going to find out ok. But nature of the velocity profile is given it is linear. So, we have to find out what is the slope intercept of this linear profile that we have to do, ok.

So, since we have two boundary condition at x = 0, V_z = 0 and then at x = b, V_z = V_b or R Ω, right. So, we can find out those constants anyway, ok. So, this is the schematic representation of the problem. The temperature profile whatever is shown here it is just of a rather representation shown, it is not the true nature of the curve that we have to find out.

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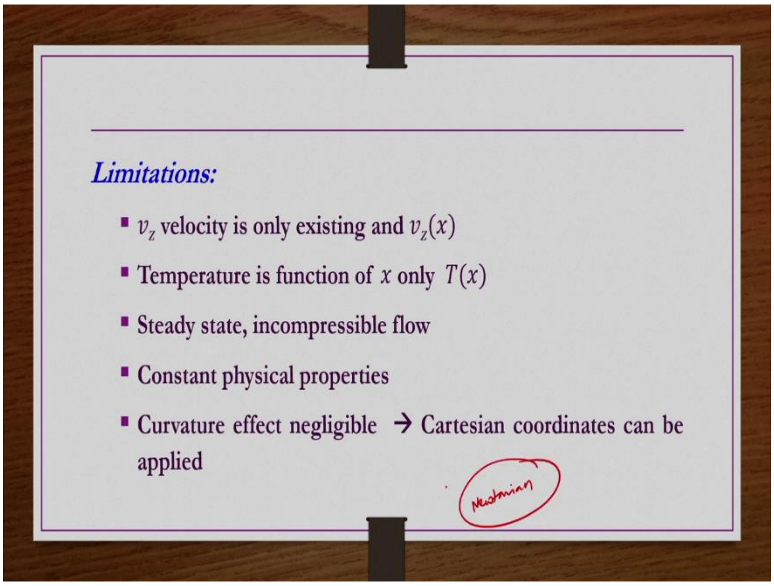

So, limitations of this problem if you list out; v_z velocity is only existing and then it is function of x. Temperature is function of x only, so that is T is function of x only.

Steady state and incompressible flow. Constant physical properties and then curvature effect negligible, so Cartesian coordinates can be applied ok. The fluid nature is Newtonian fluid, it is not given any fluid. So, then we can take a Newtonian fluid, ok.

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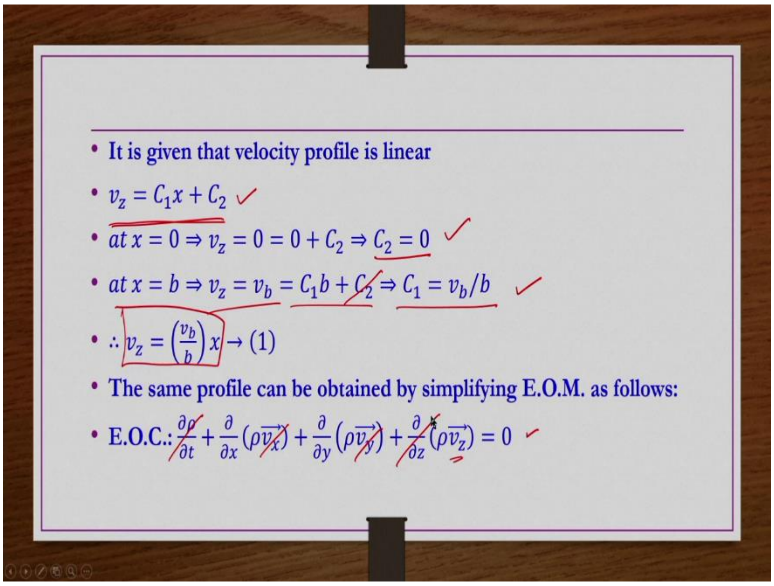

So, it is given that velocity profile is linear in the problem statement. That means, v_z is some constant x + another constant that is what we have, we can have that linear profile like this. So, what is C_1 C_2, we find out. At x = 0 v_z is 0, so $C_2 = 0$ you will get. At x = b $v_z = v_b$, so $v_b = C_1 b + C_2$. So, C_1 b is nothing but v_b/b because C_2 is 0, right.

Now, C_2 is known, C_1 is known, if you substitute them here in this velocity profile $v_z = v_b/b$ is C_1 multiplied by x + C_2 is 0. So, linear velocity profile that is given which is having this form the linearity and the constant v_b/b is this slope, right and then there is no intercept, ok. So, this is the velocity profile is given.

What if it is not given or what if you are not sure from the problem statement whether the velocity profile is linear or not, then what we have to do? We have to follow the conventional step of simplifying the continuity equation and then momentum equations getting an expression for the shear stress and then substituting corresponding expression for the shear stress as per the rheology of the fluid, then integrating it and getting the velocity profile. That is what we have to do, right.

So, as a practice we do that one also though it is given. So, continuity equation in Cartesian coordinates is given this one. Steady state this term is 0, v_x v_y are not existing, v_z is existing but it is not function of z it is function of x only. So, continuity is satisfied. So, the constraints are whatever the constraints that we have listed they are reliable ones.

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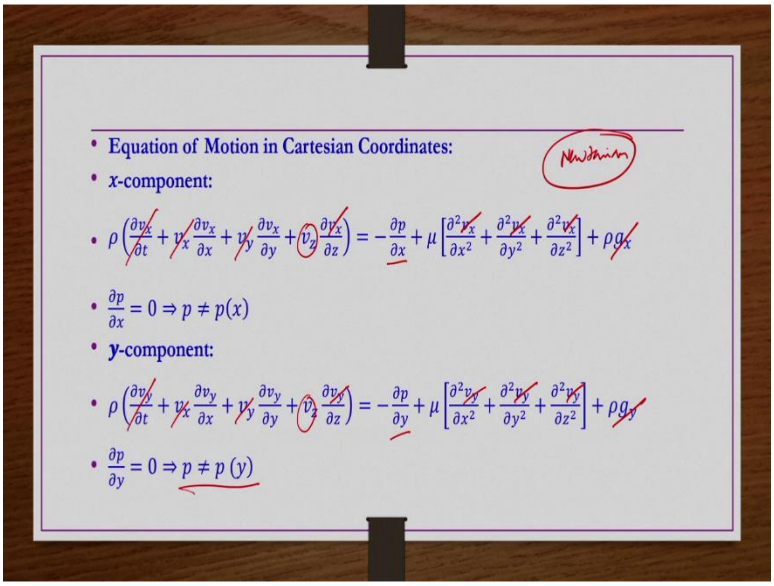

Then equation of motion in Cartesian coordinates, x-component of an equation of motion in Cartesian coordinates if you write. So, this is what we have; we have written for the Newtonian fluid, directly we have written for the Newtonian fluids. So, then it is a Navier-Stokes equation ok. So, Navier-Stokes equation x component is this is given here, steady state this term is 0, v_x is not existing, v_y is not existing, v_z is existing but v_x is not there. So, this term is also 0.

In general pressure we do not know what is the function; v_x is 0, so all these three terms v_x are there so then all that viscous force term is 0. Then gravity we are not taking horizontal configuration we are taking. So, what we get? $\frac{\partial p}{\partial x} = 0$; that means, pressure is not function of x.

Similarly, y-component equation of motion or Navier-Stokes for Newtonian fluid is given here. Steady state, so this term is 0, v_x is not there, v_y is not there, v_z is existing, but v_y is not there. So, here also left-hand side all terms are negligible. Pressure in general we do not have any information; v_y is 0, so all these three terms again here viscous force terms are 0.

Then gravity we are not taking here horizontal configuration. So, here also we get $\frac{\partial p}{\partial y} = 0$; that means, you know pressure is not function of y, right.

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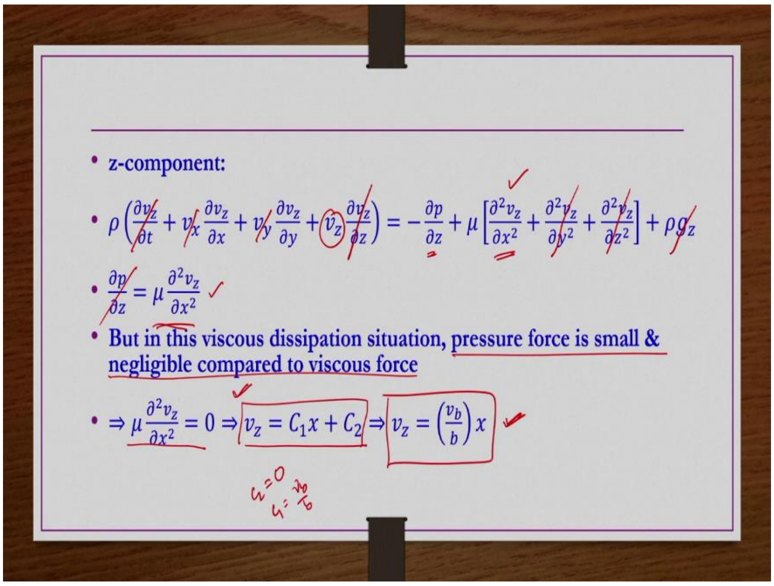

Next z-component of equation of motion in Cartesian coordinates for a Newtonian fluid if you write, so, that is this equation. So, steady state this is 0, v_x is 0, v_y is 0, v_z is existing but v_z is not function of z, it is function of x only. So, this term is 0. So, that way also now left-hand side term all the terms here also 0, negligible.

Pressure in general we do not know what it is; v_z is existing and then it is function of x. So, we cannot cancel out this term; v_z is existing but it is not function of y and z, so these two terms are 0. Gravity we are not having any term here. So, what we get here? $\frac{\partial p}{\partial z} = \mu\frac{\partial^2 v_z}{\partial x^2}$.

But in the problem statement it is given, in this viscous dissipation problems in general the viscous forces are very strong compared to the pressure and in other forces. So, we can in comparison to these viscous forces the pressure forces are small so then we can strike out. So, that is given in the statement also, in the problem statement. Pressure is very small and negligible compared to the viscous forces.

That means we have $\mu \frac{\partial^2 v_z}{\partial x^2} = 0$; that means, $v_z = C_1 x + C_2$ and then same constant. Now, once after this the constants obtaining is same like you know whatever we have done by assuming this velocity profile directly.

So, then same constant we get $C_2 = 0$ and then $C_1 = \frac{v_b}{b}$ you get; when you apply the boundary conditions then you get $v_z = \left(\frac{v_b}{b}\right) x$. Same velocity profile we are getting, ok. So however, for this problem this step was not required because it is already given, but as a practice we are doing it.

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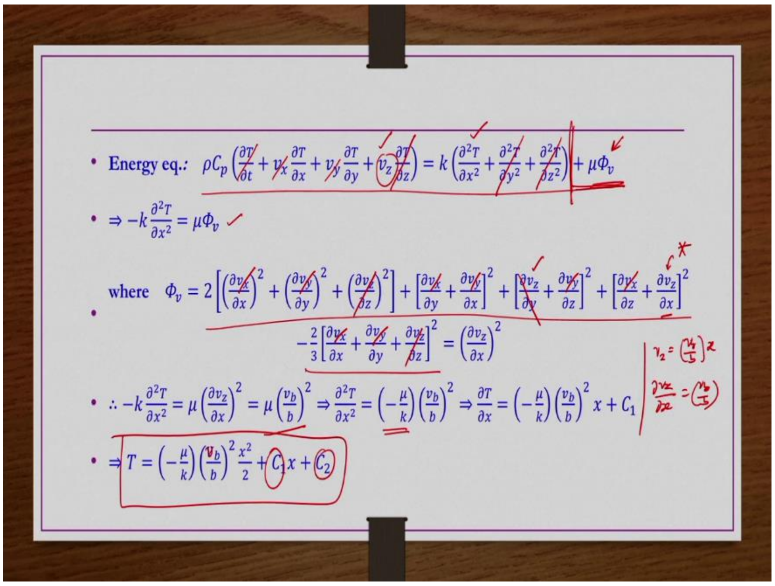

Now, energy equation we have to simplify. Energy equation in Cartesian coordinates is given this one, $\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \mu\phi_v$. This $\mu\phi_v$ is nothing but viscous dissipation term right, it is having 9 terms right. So, that we are going to write in the next step.

So, in general when you write the energy equation we take only this temporal term and then convection term and then conduction terms only, other terms as per the requirement of the problem reaction terms, viscous dissipation etcetera whatever are there; if the specific to the problem then only we write otherwise we do not write. So, now specific to this problem viscous dissipation term should be included. So, that is that term is this one.

So, we expand this one in the next step, before that we simplify this equation. Steady state so, this term is 0, v_x is 0, v_y is 0, v_z is not 0 it is there, temperature is not function of z it is function of x only it is given. So, left-hand side here also all the terms are 0. And then temperature is function of x only it is said. So, then this term should be there, it is not function of y and z so these two terms are negligible.

So, then what we get? $-k\frac{\partial^2 T}{\partial x^2} = \mu\phi_v$. And now we expand ϕ this; now we if we expand ϕ_v we get this expression, this entire expression from here to here this is the viscous dissipation term. Now, here also we have to apply the constraints.

So, limitations that is the reason we have written at the beginning of the solution. Limitations we have not only written just for the temperature purpose, we also written for the flow point of view also because the flow is also associated. And then that is being affected because of this temperature profile in general ok or temperature profile is being affected by the flow phenomena. So, that is the reason.

Now we are, we have that is the reason we have listed the constraints associated with the flow so that those constraints would be helpful now to simplify this ϕ_v term right. So, v_x is 0 this term is 0, v_y is 0, this is 0 v_z is there but v_z is not function of z, it is function of x only.

So, this term is also 0; v_x is 0, v_y is 0, v_z is there but v_z is function of x only it is not function of y. So, this term is 0; v_i v_y is not there so this is 0, v_x is not there this is 0, v_z is existing right and then it is function of x also. So, this term should be taken. So, this term would be there right.

So, from here v_x is 0, v_y is 0 and then v_z is function of x only, it is not function of z. So, what we get here? ϕ_v is nothing but just $\left(\frac{\partial v_z}{\partial x}\right)^2$. That means, $-k\frac{\partial^2 T}{\partial x^2} = \mu\left(\frac{\partial v_z}{\partial x}\right)^2$. But v_z what we have? $v_z = \left(\frac{v_b}{b}\right)x$, so $\frac{\partial v_z}{\partial x} = \left(\frac{v_b}{b}\right)$. So, $\mu\left(\frac{v_b}{b}\right)^2$ this is what we are having.

Now, this minus k also we take it to the right-hand side. So, $\frac{\partial^2 T}{\partial x^2} = -\frac{\mu}{k}\left(\frac{v_b}{b}\right)^2$. So, if you integrate it $\frac{\partial T}{\partial x} = -\frac{\mu}{k}\left(\frac{v_b}{b}\right)^2 x + C_1$. If you integrate it again $T = -\frac{\mu}{k}\left(\frac{v_b}{b}\right)^2\frac{x^2}{2} + C_1 x + C_2$, right.

So, this is the temperature profile, but it is having two unknowns C_1 C_2. So, those two unknowns now we will be finding out using the boundary conditions. What are the boundary conditions? At x = 0, T = T_0, at x = b, T = T_b; they are given.

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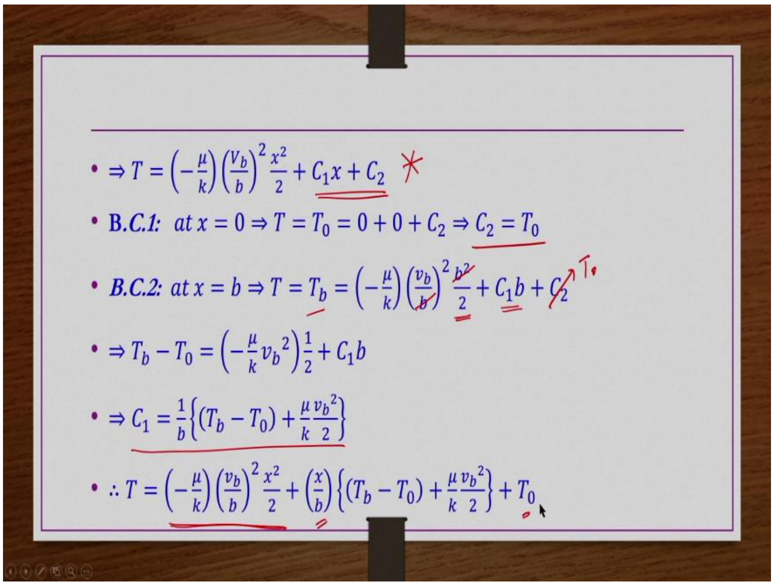

So, when you substitute s at x = 0, T = T_0 then we have right-hand side 0 + 0 + C_2. That is C_2 = T_0. Other boundary condition at x = b, T = T_b = $-\frac{\mu}{k}\left(\frac{v_b}{b}\right)^2$. In place of b in place of $\frac{x^2}{2}$, now you have $\frac{b^2}{2}$ + C in place of C_1 x you have C_1 b + C_2.

C_2 you already got it as T_0, so this is T_0. Now, this T_0 we take to the left-hand side, so that $T_b - T_0 = \left(-\frac{\mu}{k} v_b^2\right)\frac{1}{2}$ + C_1 b, because this b square this b^2 can be cancelled out. So, from here C_1 we get it as $\frac{1}{b}\left\{(T_b - T_0) + \frac{\mu}{k}\frac{v_b^2}{2}\right\}$. So, this is C_1. Now, this C_1 and this C_2 both we substitute here in this equation and then simplify to get the required temperature profile.

So, this is what we have substituted. On the first term is irrespective of the constant, so that is as it is. C_1 x is nothing but $\frac{1}{b}\left\{(T_b - T_0) + \frac{\mu}{k}\frac{v_b^2}{2}\right\}$ x. So, $\frac{x}{b}$ we have written + C_2, C_2 is nothing but T_0.

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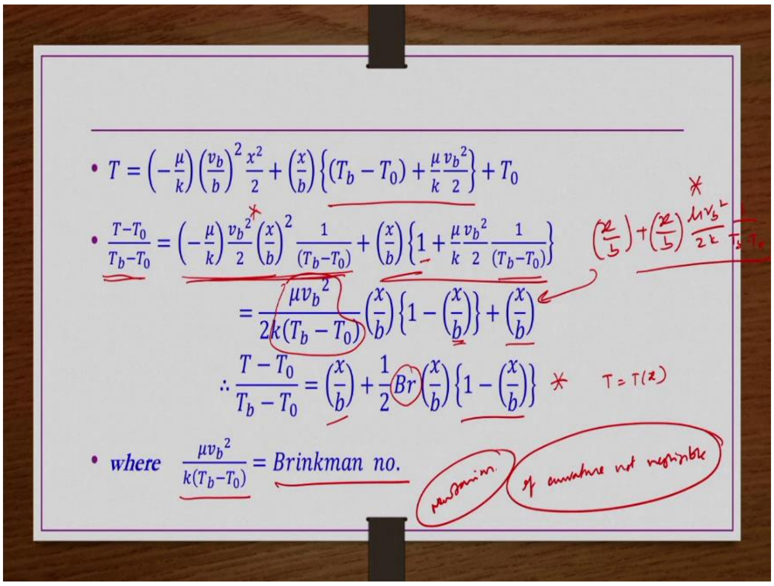

So now, this is the final temperature profile, but however what we do? We do some more simplification so, that we can have this equation in simpler form. So, now this T_0 will take to the left-hand side. So, we have $(T - T_0)$ and then both the sides we are dividing by $(T_b - T_0)$.

So, in the first term as it is and then divided by $(T_b - T_0)$, second term x by b; this term if you divide by $(T_b - T_0)$, so here 1 and then here you get second term you get $(T_b - T_0)$; T_0 we already taken to the left-hand side before dividing by $(T_b - T_0)$, ok.

So, now what we do? This term if you expand, this is $\left(\frac{x}{b}\right) + \left(\frac{x}{b}\right)\frac{\mu v_b{}^2}{2k}\frac{1}{T_b - T_0}$ this is what we are having. So now, what we do? This term and then this term, from these two terms we will be taking this $\frac{\mu v_b{}^2}{2k(T_b - T_0)}$ as common. So, $\left(\frac{x}{b}\right)$ also we are taking common. So now, this is that is this term we are taking common. So, this multiplied by $1 - \left(\frac{x}{b}\right)$ from this term, the same thing if you take common $\left(\frac{x}{b}\right)$ another $\left(\frac{x}{b}\right)$ would be remaining from this term right. And then this $\left(\frac{x}{b}\right)$ is as it is, right.

So now, this term $\frac{(T - T_0)}{(T_b - T_0)} = \left(\frac{x}{b}\right) + \frac{1}{2}$, this entire term whatever is there that I am calling it Br $\left(\frac{x}{b}\right)\left\{1 - \left(\frac{x}{b}\right)\right\}$. This is the final temperature profile in a simplified manner because we know that T is function of x only all other constants whatever are there.

So, they are grouped as a $\frac{\mu v_b{}^2}{k(T_b - T_0)}$ which is nothing but Brinkman number, right. So, this is how we can obtain the temperature profile in the case of a viscous dissipation problem. Now, the same problem becomes complicated if curvature effects are not negligible.

This problem we are going to take in subsequent lectures so, but the same problem exactly same thing, but only additional thing that if you say that if the curvature effect is not negligible. So, then mathematically simplifying these equations, the process steps are same right but only thing that you will be solving in cylindrical coordinates when the curvature effects are not negligible, then under such conditions mathematical simplifications becomes little tough.

So, those problems we take in subsequent lectures anyway. Now, this we have taken if the fluid is Newtonian fluid, but primarily our purpose is to see such kind of problems especially when the fluid is obeying some kind of non-Newtonian rheology, right. So now, the same problem we will revisit, but now the fluid we take a power-law fluid, ok.

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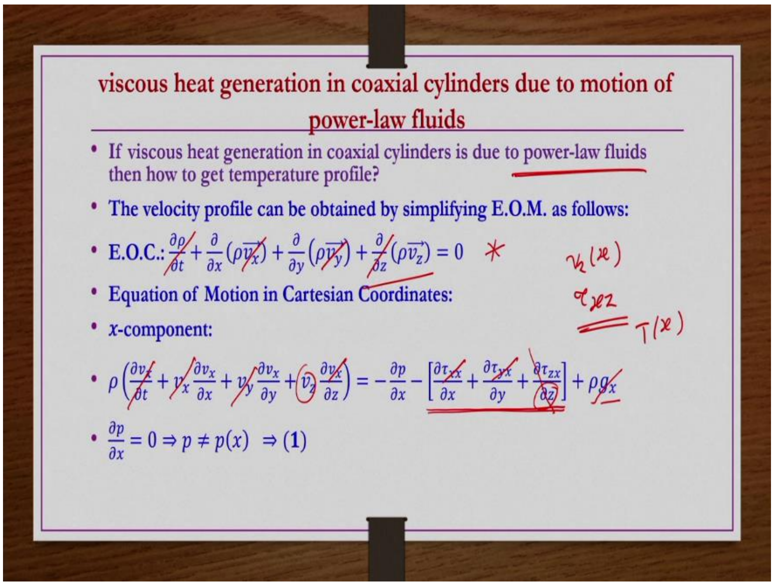

So, viscous heat generation in coaxial cylinders due to motion of power-law fluids, all conditions are same, everything is same, the only thing that you know the fluid is power-law fluid. The velocity profile can be obtained by simplifying the equation of motion and all those steps we can follow, because let us assume in this case we are not sure whether the profile is linear or not, because the fluid rheology is power-law behaviour is non-linear.

So, we are not sure even if the confinement the gap between two cylinders is very small, whether the velocity profile is going to be linear or not we do we are not sure if the fluid is obeying non-linear non-Newtonian behaviour such like power-law fluids or something else like that. So, that is the reason we follow the standard steps to obtain the velocity profile as well, ok.

So now, what we do? See the same equations of a continuity and then motion we will be writing in Cartesian coordinates and then applying the constraints to get the velocity profile, ok. So, continuity equation is given here, steady state this term is 0, v_x is 0, v_y is 0, v_z is not function of z it is function of x only. So, this term is also 0; that means, continuity is satisfied.

Then equation of motion in Cartesian coordinates x component is given here. So now, here the difference you can see the viscous terms have been provided in terms of the extra stress component τ_{xx}, τ_{yx} and τ_{zx} etcetera. It is not given directly in terms of velocity, because τ we will be substituting later on when simplifying this equation based on the nature of the rheology, right.

The nature of the rheology is simpler in the case of Newtonian fluid. So, that directly you substitute here and then simplify. So, then directly you get $+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$ that is what directly you get on this term.

But here now in this case of power-law fluid this τ expression is complicated non-linear. So, when you substitute here it becomes very complicated. So, that is the reason we substitute it later. And then here v_z is function of x and then only τ_{xz} component is only existing which is function of z, temperature is anyway function of x.

So, these constraints we apply and then simplify now. Steady state this is 0, v_x is 0, v_y is 0, v_z is existing, but v_x is not there. So, left-hand side altogether again 0, we do not know. So, this is having; all this is having like you know v x terms. So, this is 0 here v_x v_y terms only be there.

So, this is also 0. Here v_z term will also be there, but v_z is not function of z, it is function of x so then this term is also 0. Then there is no gravity, so this term is also 0. So, we get $\frac{\partial p}{\partial x} = 0$; that means, pressure is not function of x, here again.

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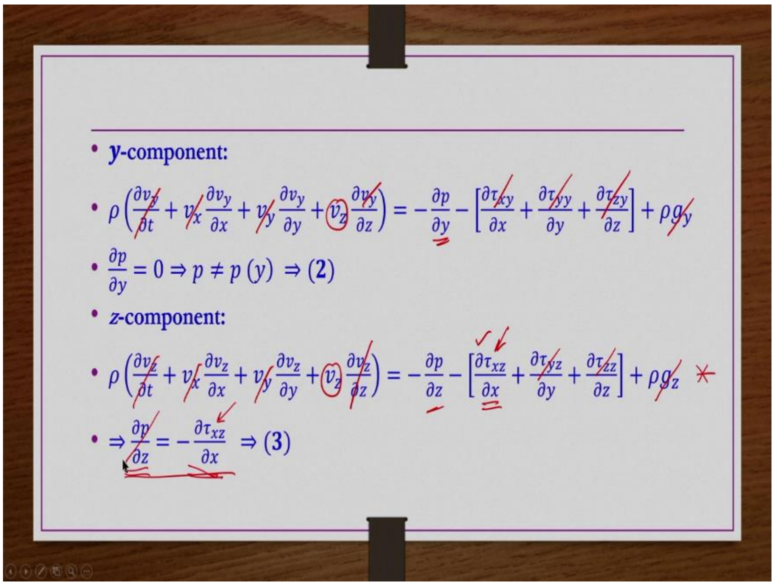

Now, y-component of equation of motion for a non-Newtonian fluid if you write in Cartesian coordinates this is that equation. So, steady state this term is 0, v_x is not existing, v_y is not existing, v_z is existing but v_y is not existing. Pressure terms we do not know, so let us leave them as it is.

Here from τ_{xy} terms if you expand you will have v_x v_y term, so then this is 0; only τ_{zx} or τ_{xz} is existing, τ_{yy} is not existing, τ_{zy} is also not existing ok. So, that is 0 and then gravity is not there. So, here also we get $\frac{\partial p}{\partial y} = 0$. So, the pressure is not function of y. Again z-component of equation of motion in Cartesian coordinates for a non-Newtonian fluid if you write this is what you have.

So, steady state this is 0, v_x is not there, v_y is not there, v_z is there, but v_z is not function of z. So, this term is also 0, pressure we do not know right. τ_{xz} is existing and then it is function of x, so we cannot cancel out it should be there. τ_{yz} is not existing, τ_{zz} is not existing, the gravity is not existing because of horizontal configuration we are taking.

So, here $\frac{\partial p}{\partial z} = -\frac{\partial \tau_{xz}}{\partial x}$ you are getting, right. Now, here in place of τ_{xz} you substitute $-m\left(\frac{\partial v_z}{\partial x}\right)^n$ and then substitute it and then simplify to get the required velocity profile. But before doing that one so, this is a viscous force term right and this is pressure force term right.

So, in this problem viscous heat generation problem in general viscous forces are very strong compared to the other one. And then pressure force is very small and negligible compared to the viscous forces. So, this term can be cancelled out.

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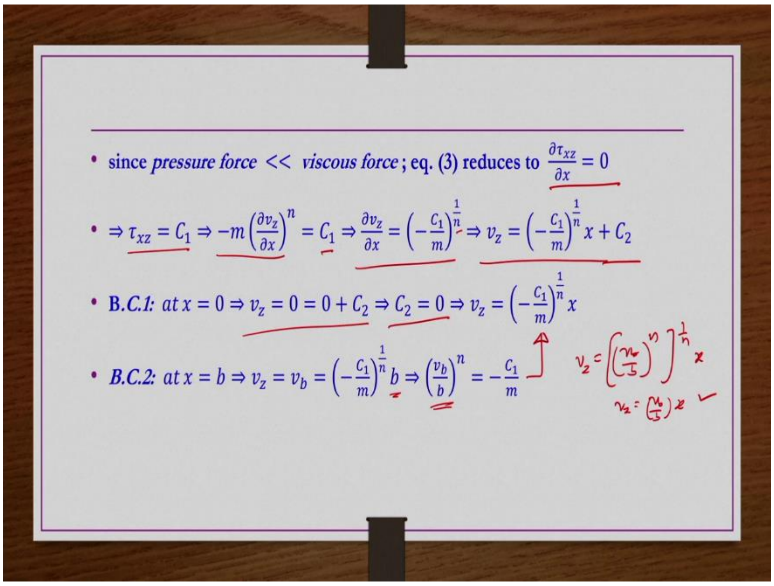

So, we have only $\frac{\partial \tau_{xz}}{\partial x} = 0$, right. Now, $\tau_{xz} = C_1$ on integration and then τ_{xz} we can have $-m\left(\frac{\partial v_z}{\partial x}\right)^n = C_1$. Now, you take you take $-m$ to the right-hand side, so that $\frac{C_1}{m}$ and then both sides you do whole power 1/n. So, then you get $\frac{\partial v_z}{\partial x} = \left(-\frac{C_1}{m}\right)^{1/n}$.

Now, you integrate this one $v_z = \left(-\frac{C_1}{m}\right)^{1/n} x + C_2$. So, here also before obtaining the this constant C_1 C_2 we can understand in the case of power-law fluids also the velocity profile is linear, only thing that constants we have to obtain. Let us see if the constants are also same or not. After obtaining the constants are we getting the same velocity profile as in the case of Newtonian fluid or not that we check?

So, at x = 0, v_z is 0 so, C_2 you are getting 0; that means, $v_z = \left(-\frac{C_1}{m}\right)^{1/n} x$ right C_2 is 0 anyway. Now apply the second boundary condition at x = b, $v_z = v_b$ and then that should be equals; that should be $= \left(-\frac{C_1}{m}\right)^{1/n}$ b in place of x, right.

So, now you can rearrange this one as $C_1 - \frac{C_1}{m}$ is nothing but $\left(\frac{v_b}{b}\right)^n$. So now, in place of this one in place of $-\frac{C_1}{m}$ you write $\left(\frac{v_b}{b}\right)^n$ and then this whole power 1/n is there x. So, $v_z = \left(\frac{v_b}{b}\right) x$ here also you are getting linear profile and then profile is exactly same as in the case of Newtonian fluid.

What does it mean by? It means like you know if the fluid is Newtonian or power-law fluid that confinement is very narrow then the velocity profile is going to be linear in either of the case. But, it does not mean that the temperature profile is also going to be same. Definitely it is going to be different, and then how different it is that we are going to derive now.

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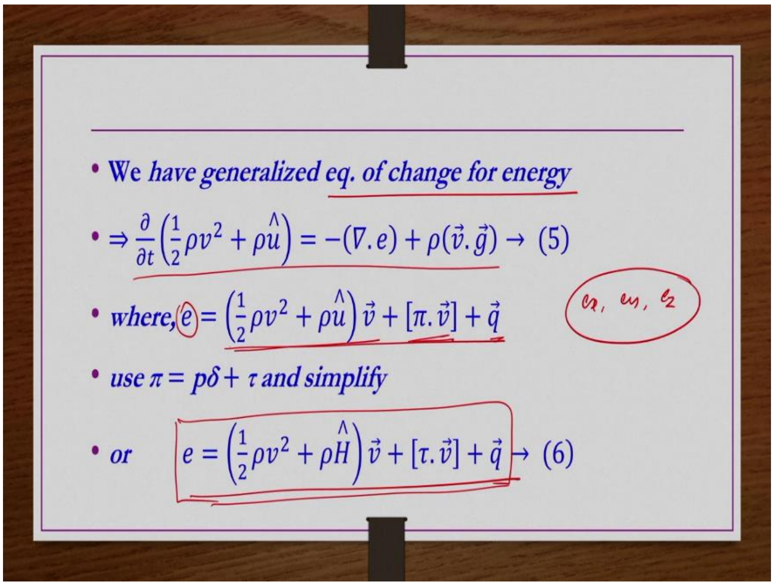

So, generalized equation of change for energy that we have derived in week number 3 or 4 previously. There what we derived? Generalized energy equation is this one; $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{u}\right) = -(\nabla.e) + \rho(\vec{v}.g)$, right.

So, this is the generalized one, then after that we have done several simplification and then the same energy equation we obtain in 4-5 different forms. Different forms of energy and finally we got energy equation in the form of temperature, ok. So now, this equation we are going to use here right.

Here e is nothing but this one. So, this term indicates the convection term, this is indicates the both pressure and then molecular force systems. Molecular force systems are also including the viscous dissipation terms etcetera, and then this term indicates the conduction term, right.

And then this e how many components are there? It is having e_x, e_y, e_z; 3 components we are having. So, if you expand this one you get the 9 such components, ok. This ϕ we know as p δ+ τ. And then finally this is also we have seen, e $=\left(\frac{1}{2}\rho v^2 + \rho\widehat{H}\right)\vec{v} + [\tau.\vec{v}] + \vec{q}$. This is what we had seen, right.

So, this we are going to simplify for given problem. Because see energy equation now directly in the temperature form we cannot write; why we cannot write? Because in that form you know the viscous dissipation term whatever is there that you know is not for power-law fluids it may be different way, for non-Newtonian fluid it is incorporated in a different way.

So, how different, that we can understand from here only, because this term is going to play leading role now because of non-Newtonian rheology. And this is directly not available in energy equation in the form of temperature. So, the same equation is written here also.

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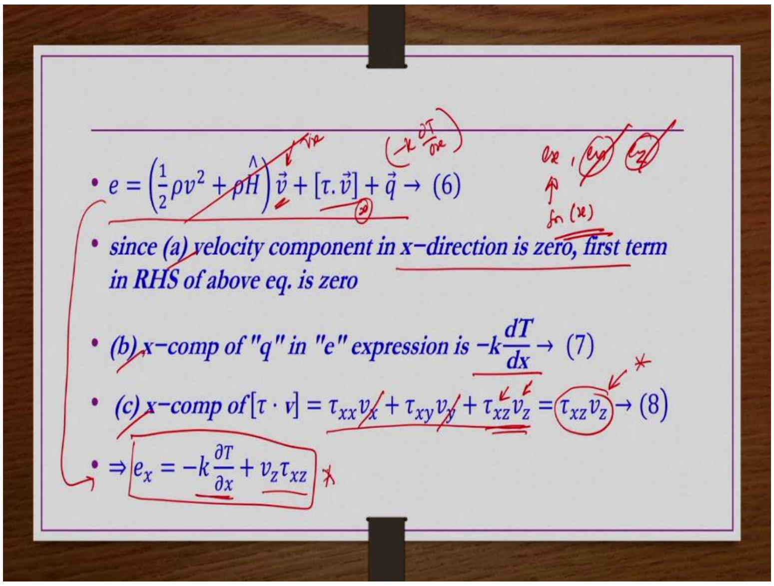

So, now what we understand? We are taking only see this e is having e_x, e_y, e_z; that means, energy changes it is there as function of x only. So, whatever the e_y term e_z terms are there, so, that are all having function of y z something like that. So, then altogether we can cancel out, we do not need to expand that, we do not need to worry about e_y e_z terms at all, right.

So, we take only x component of this e; x component of e if you see. So, this is going to be v_x and then v_x component is 0, so that means, this term is 0 altogether right. And then this q only conduction $-k\frac{\partial T}{\partial x}$ term would be there in the q also you have q_x q_y q_z terms, only q_x term is there and then that is $-k\frac{\partial T}{\partial x}$ only that term is required.

And then $[\tau.v_x]$ its x component if you see it is $\tau_{xx}v_x + \tau_{xy}v_y + \tau_{xz}v_z$ this is also we have seen previously, right. So, now v_x is 0, v_y is 0, v_z is there and then τ_{xz} is also there, so $\tau_{xz}v_z$. So, this term is going to give you the contribution of viscous dissipation in the change of temperatures. So, that is the reason this is how we are doing for the power-law fluid right.

Now, so, finally e_x. So, the same equation when you simplify by using this 3 constraints e_x component only we if we take. So, e_x = q_x + $\tau_{xz}v_z$, q_x is nothing but $-k\frac{\partial T}{\partial x}$. So this is the one, right.

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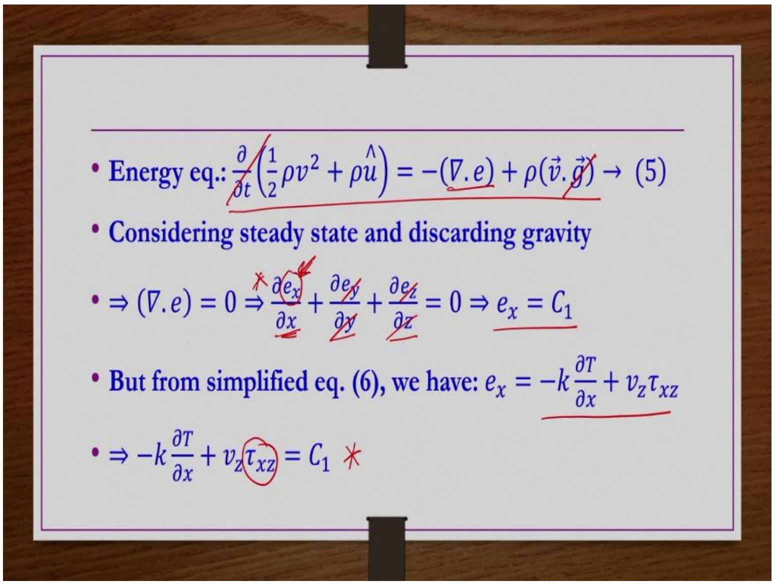

Now energy equation, equation number 5 in the generalized form this is what we had right. So, this now we have to simplify as per the constraints of the problem. So, steady state. So, this term is 0 altogether and then we are not taking gravity. So, this term all the terms would be having gravity. So, all together we can cancelled out, even after expansion each and every term associated with this one is having gravity g term. So, we can cancel out altogether.

So, what we get? $(\nabla . e) = 0$ and then $\nabla . e$ if you expand it is $\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z} = 0$ and then e_x is there only in the x direction temperature variations are there. So, e_x is existing and it is function of x, so then we cannot cancel out it would be there.

But e_y is not there and then it is not function of y also e_z is not there and it is not function of z also. So, $e_x \frac{\partial e_x}{\partial x} = 0$ that means, $e_x = C_1$. And then what is this e_x? In the previous slide we derived it as $-k\frac{\partial T}{\partial x} + \tau_{xz} v_z$. So, this should be $= C_1$.

Now this is the final expression; final simplified expression for our viscous dissipation due to the power-law fluid. So, this equation now you can solve, right only thing that here in place of τ_{xz} you have to substitute $-\left(\frac{\partial v_z}{\partial x}\right)^n$. So, that when you write this equation you have, right.

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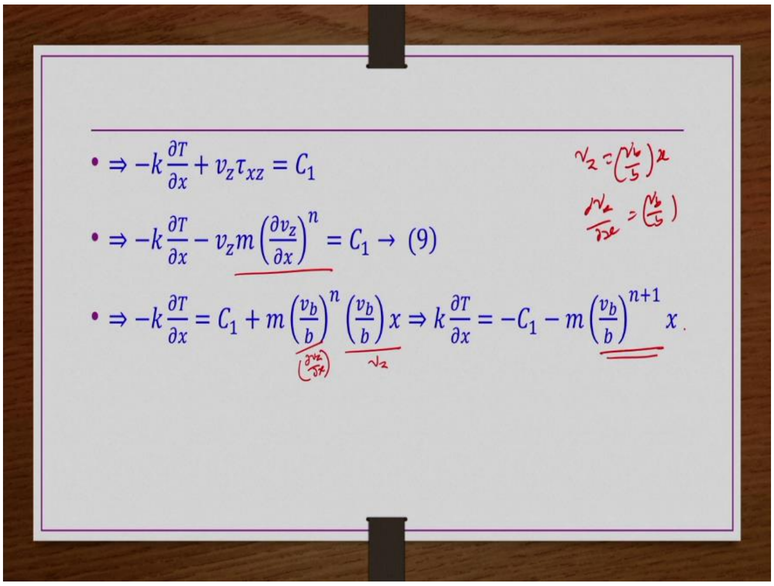

Now, $\frac{\partial \mathrm{v}_z}{\partial x}$ is nothing but $\frac{v_b}{b}$, because $v_z = \frac{v_b}{b}x$. So, $\frac{\partial \mathrm{v}_z}{\partial x}$ is going to be $\frac{v_b}{b}$. So, here in place of $\frac{\partial \mathrm{v}_z}{\partial x}$ we can write $\left(\frac{v_b}{b}\right)^n$. So, $-k\frac{\partial T}{\partial x} = C_1 + m\left(\frac{v_b}{b}\right)^n\left(\frac{v_b}{b}\right)x$. This $\left(\frac{v_b}{b}\right)x$ is coming because of v_z term and then this is coming because of $\frac{\partial \mathrm{v}_z}{\partial x}$ term.

So, $mx\left(\frac{v_b}{b}\right)^{n+1}$ we should have. So, that is when you take this $-k$ of left-hand side also in the right-hand side $k\frac{\partial T}{\partial x} = -C_1 - m\left(\frac{v_b}{b}\right)^{n+1}x$. Now, you integrate this equation both sides then you get $kT = -C_1 x - \frac{mx^2}{2}\left(\frac{v_b}{b}\right)^{n+1} + C_2$. This is what you get, right.

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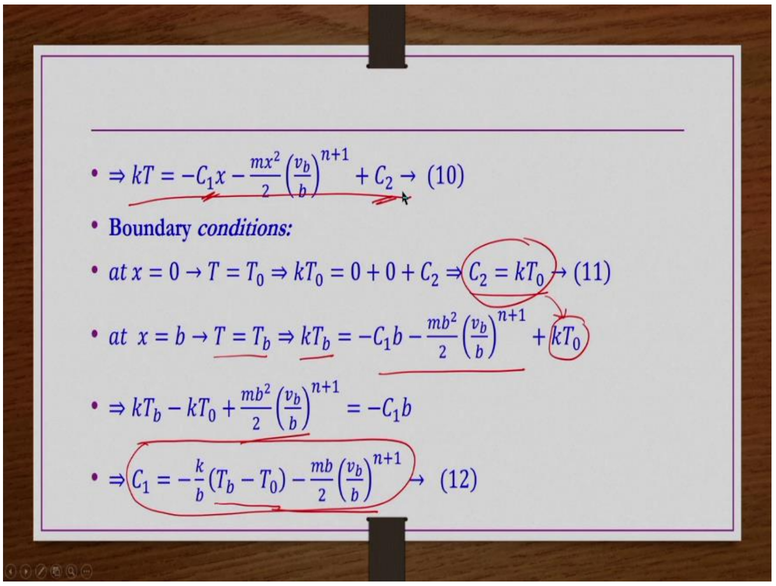

Now, this equation we are going to obtain the constant C_1 C_2. So now, this equation we have to obtain constant C_1 C_2 by applying the boundary conditions for the temperature, ok. So, at x = 0, T = T_0 so that means, $kT_0 = 0 + 0 + C_2$. That means, $C_2 = kT_0$ and then x = b, T = T_b.

That means, $kT_b = -C_1 b - \frac{mb^2}{2}\left(\frac{v_b}{b}\right)^{n+1} + C_2$; C_2 is kT_0, right. So, now what we do? This kT_0 we take to the left-hand side and then this $\frac{mb^2}{2}\left(\frac{v_b}{b}\right)^{n+1}$ also we are taking to the left-hand side.

So, here it is, we are not expanded expanding or cancelling b terms etcetera that not required. So, $-C_1 b$ is this one. So, C_1 is nothing but $-\frac{k}{b}(T_b - T_0) - \frac{mb}{2}\left(\frac{v_b}{b}\right)^{n+1}$, right. So, now this C_2 and this C_1 we are going to substitute in this equation number 10 to get the final temperature profile.

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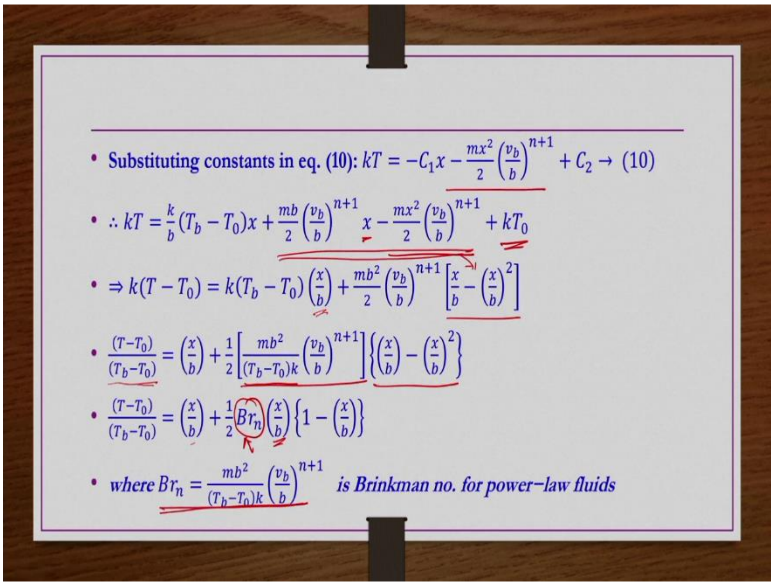

So, this is C_2, kT_0 and this is the $-\frac{mx^2}{2}\left(\frac{v_b}{b}\right)^{n+1}$ term is as it is. And then $-C_1x$ is nothing but $\frac{k}{b}(T_b - T_0)x + \frac{mb}{2}\left(\frac{v_b}{b}\right)^{n+1}x$. Now, this kT_0 we take to the left-hand side.

So, $k(T - T_0) = k(T_b - T_0)\left(\frac{x}{b}\right) + \frac{mb^2}{2}\left(\frac{v_b}{b}\right)^{n+1}$; what we are doing from these two terms? We are taking $\frac{mb^2}{2}\left(\frac{v_b}{b}\right)^{n+1}$. So, that the first term we will be having $\left(\frac{x}{b}\right)$ and then from this second term we have we will be having $\left(-\frac{x^2}{b^2}\right)$. So, this is what we are having.

So, in the next step what we do? We divide by $(T_b - T_0)$ and k both sides. So, that left-hand side we have $\frac{(T-T_0)}{(T_b-T_0)} = \left(\frac{x}{b}\right) + \frac{1}{2}\left[\frac{mb^2}{(T_b-T_0)k}\left(\frac{v_b}{b}\right)^{n+1}\right]\left\{\frac{x}{b} - \left(\frac{x}{b}\right)^2\right\}$ as it is.

So, now here this equation we can write $(T - T_b)$; $\frac{(T-T_0)}{(T_b-T_0)} = \left(\frac{x}{b}\right) + \frac{1}{2}$. Whatever the term that is there within the parenthesis this we are calling Br_n. And then from these two terms we are taking $\left(\frac{x}{b}\right)$ common, so multiplied by $1 - \left(\frac{x}{b}\right)$ we are having. So now, here this Br_n is nothing but Brinkman number for the case of power-law fluids fine.

So, if the fluid is obeying non-Newtonian behaviour; so simplifying the generalized energy equation and then getting things done is more appropriate especially if the molecular forces are contributing some changes in the temperature profile, like in viscous dissipation ok. Now, before winding up the class you know we take a one more similar case ok, that is viscous heat generation in slit flow.

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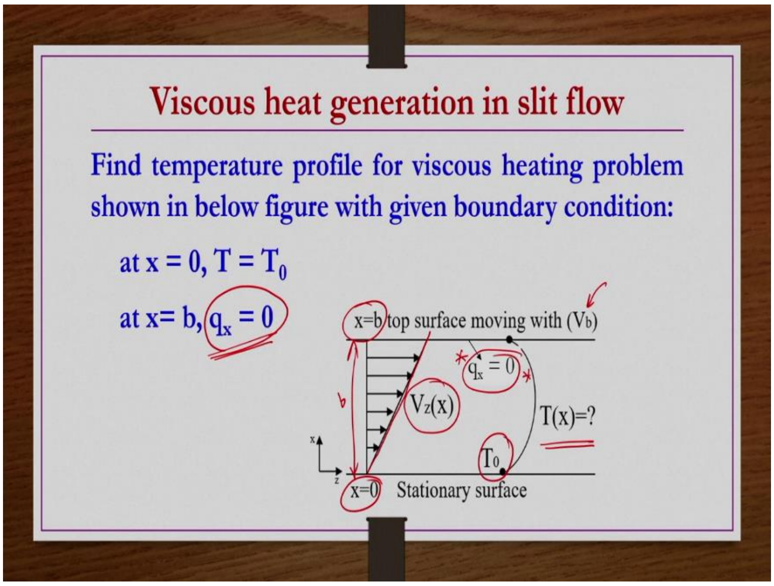

So, here the statement is find temperature profile for viscous heating problem shown in below figure with given boundary conditions. At x = 0, T = T_0, at x = b, q_x = 0. So now, the figure is given like this.

So, only thing that the boundary condition is same compared to the problem 1; this is let us say, if you take problem 3 of the class, problem 1 where we have taken like you know viscous heat generation between coaxial cylinders by avoiding the curvature effects.

So, then we had T = T_b at x = b now, in place of that one we are having constant flux, 0 flux case we are having that is q_x = 0 that boundary condition we are having. So, then how the temperature profile will change; that we will see now.

Here again bottom surface is stationary, top surface is moving with constant velocity v_b and then velocity profile between these two surfaces is linear and then that is function of x, right. The gap between these two is b, temperature of bottom stationary surface is T_0 and then boundary condition for the top moving surface is q_x = 0.

So, what is T as function of x; that is what we have to find. Only difference is that this boundary condition is only difference compared to the problem that we have seen previously.

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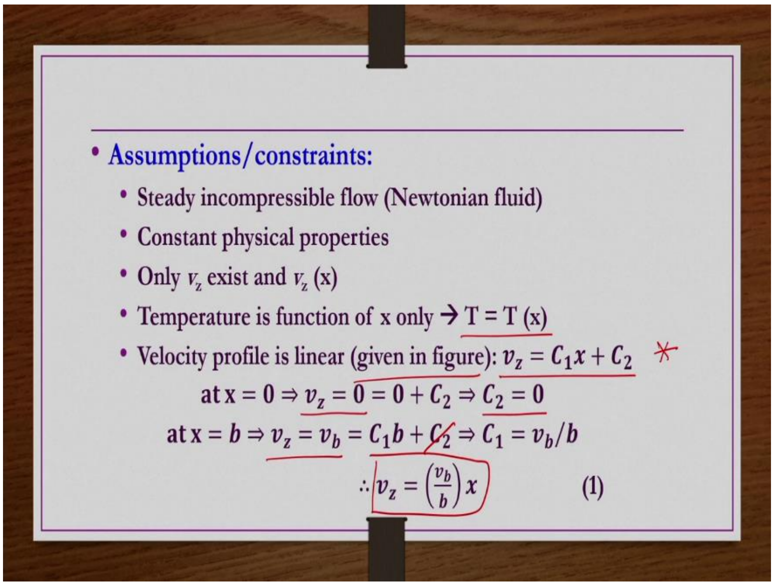

So assumptions: Constraints of the problem steady incompressible flow; nature of the fluid is not given so by default we should take Newtonian fluid. Constant physical properties, C_p, ρ, μ etcetera are constant. Only v_z is existing and it is function of x.

Temperature is function of x only that is T is function of x. Velocity profile is linear given in the figure, so we can have $v_z = C_1 x + C_2$. If it is not given then we have to simplify the equation of motion and get the required velocity profile applying the boundary conditions available for the velocity, right. So now, it is given in the profile in the picture it is given that velocity profile is linear, so then we can take this one. Now, the constant C_1 C_2 we have to obtain.

So, at x = 0, $v_z = 0$ so, $C_2 = 0$. At x = b $v_z = v_b$, so C_1 b + C_2, C_2 is 0 so $C_1 = \frac{v_b}{b}$. So that means, here also we are getting same velocity profile $v_z = \left(\frac{v_b}{b}\right) x$. So, even if the boundary condition of the moving surface has been changed for temperature the velocity profile is not being affected.

So that means, what does it mean by these problems? The velocity profile is affecting the temperature profile, but temperature profile change in temperature nonisothermality in the system that is not affecting the velocity profile, neither the rheology of the fluid is affecting the velocity profile because the narrow gap it is having and then all that, right. So, because of the flow temperature profile is changing, that we are measuring now.

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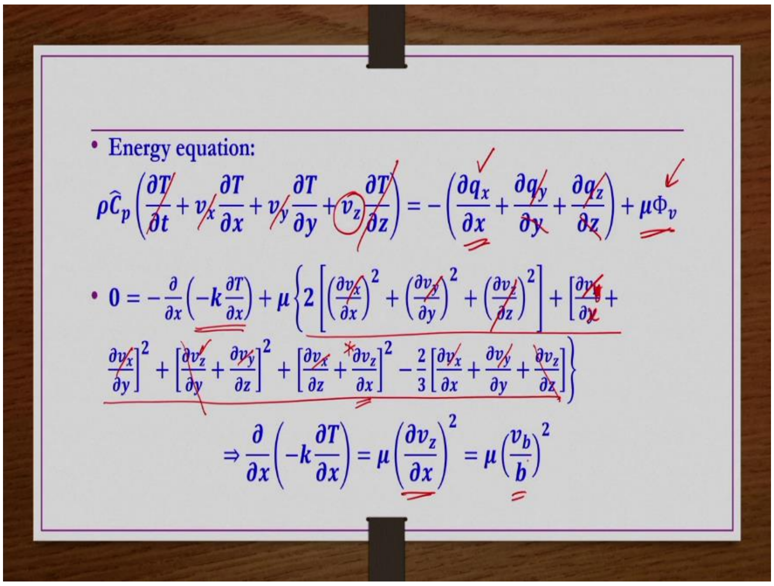

So, energy equation this is what we have in Cartesian coordinates, steady state this term is 0, v_x is 0, v_y is 0, v_z is existing but temperature is not function of z, it is function of x that is also given in the problem statement. Conduction or the heat changes are existing in the x direction. So, this would be there these two q_y q_z are not existing and they are not function of z. And then viscous dissipation this term should be there.

So, what we get? $0 = -\frac{\partial}{\partial x}$ of in place of q_x we are writing $-k\frac{\partial T}{\partial x}$ μ multiplied by in place of ϕ_v viscous dissipation term this is what this we have. So, now here again we apply the constraints of the flow; constraints of the flows to further simplify this equation.

So, v_x is 0, v_y is 0, v_z is there but it is not function of z. So, first three terms are 0; v_z is there, but it is not function of y. This term should be $\frac{\partial v_y}{\partial x}$; right $\frac{\partial v_y}{\partial x}$ so v_y. So, there is a typo here, so it should be $\frac{\partial v_y}{\partial x}$.

So, here this v_y is 0, v_x is also 0, v_z is existing, but it is not function of y. So, it is 0; v_y is 0, v_x is not existing v_z is existing and it is function of x so this term would be there; v_x is 0, v_y is 0 but v_z is existing, but it is not function of z, so this term is also 0.

So, here also we get $\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) = \mu$ out of these terms only one term $\frac{\partial v_z}{\partial x}$ is remaining. So, $\frac{\partial v_z}{\partial x}$ is nothing but $\frac{v_b}{b}$ from the velocity profile. So, $\mu\left(\frac{v_b}{b}\right)^2$.

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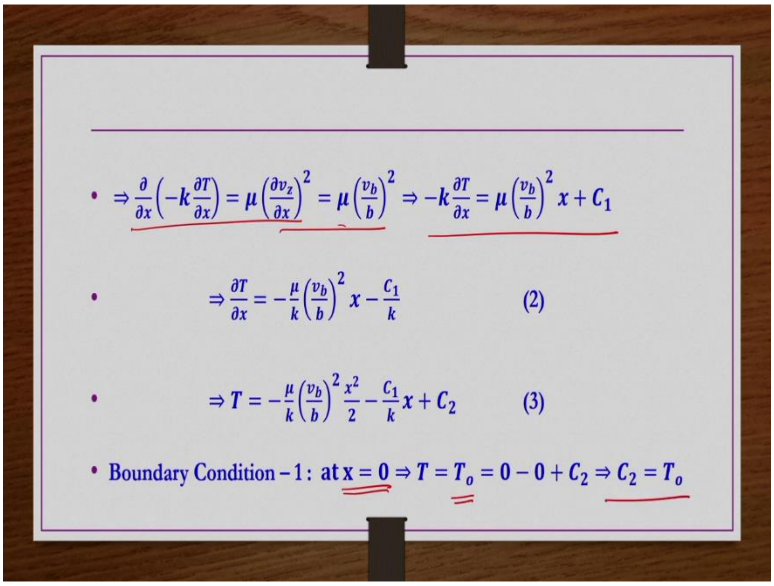

So, this equation now what we do? We integrate it. When you integrate first time $-k\frac{\partial T}{\partial x} = \mu\left(\frac{v_b}{b}\right)^2 x + C_1$. Then what we do? We take this $-k$ to the right-hand side then we have $\frac{\partial T}{\partial x} = \left(-\frac{\mu}{k}\right)\left(\frac{v_b}{b}\right)^2 x - \frac{C_1}{k}$.

On integration T = $\left(-\frac{\mu}{k}\right)\left(\frac{v_b}{b}\right)^2 \frac{x^2}{2} - \frac{C_1}{k}x + C_2$. Now the first boundary condition at x = 0, T = T_0. So, 0 – 0 + C_2 that is $C_2 = T_0$.

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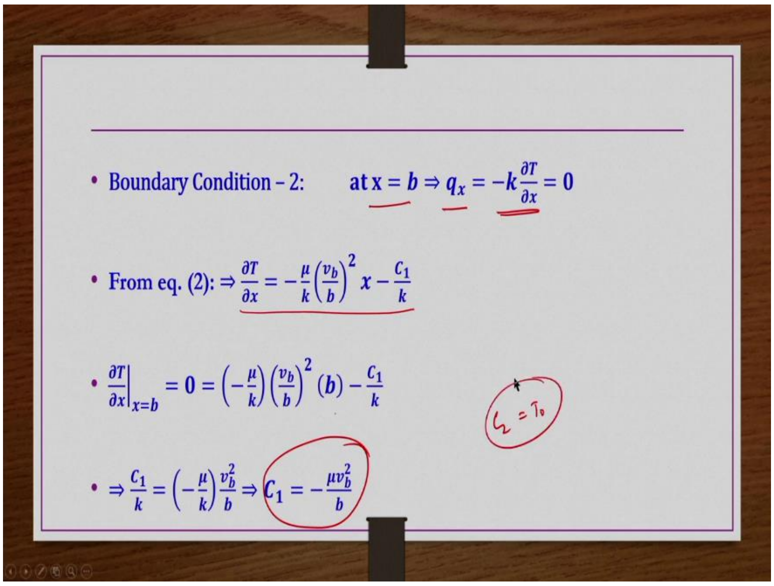

Another boundary condition at x = b, $q_x = 0$, q_x is nothing but $-k\frac{\partial T}{\partial x}$. So that means, $\frac{\partial T}{\partial x} = 0$. And then from equation number 2 we had $\frac{\partial T}{\partial x} = -\left(-\frac{\mu}{k}\right)\left(\frac{v_b}{b}\right)^2 x - \frac{C_1}{k}$. So here now, what we substitute? x = b we substitute and then equate the left-hand side term to 0, then we have $0 = \left(-\frac{\mu}{k}\right)\left(\frac{v_b}{b}\right)^2 - \frac{C_1}{k}$.

So, $\frac{C_1}{k} = \left(-\frac{\mu}{k}\right)\frac{v_b^2}{b}$. So, $C_1 = -\frac{\mu v_b^2}{b}$. So, this C_1 and then $C_2 = T_0$ these two we are going to substitute in this equation number 3 which we have derived previously.

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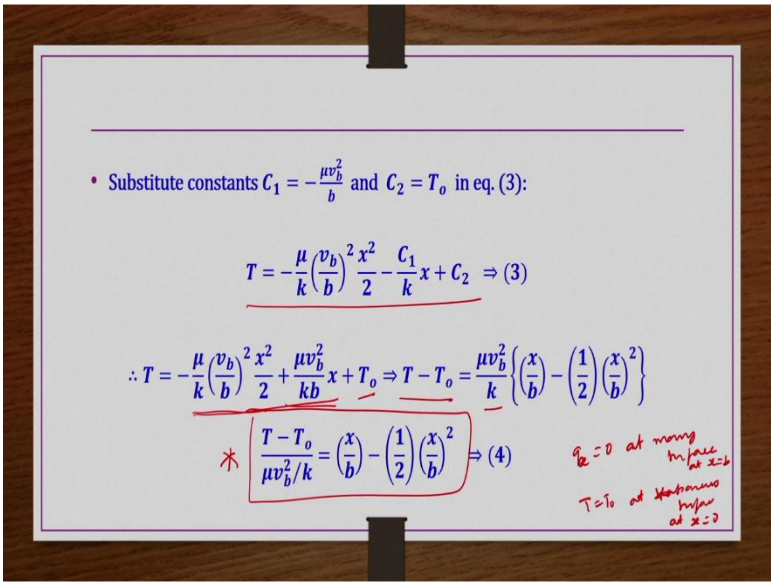

So, in place of C_1 you can write $-\frac{\mu v_b^2}{b}$ then we have this is the second term that is $+\frac{\mu v_b^2}{kb}x$ and in place of C_2 you can write T_0, the first term is as it is. So, this T_0 you take to the left-hand side and then right-hand side terms you take common $\frac{\mu v_b^2}{k}$, then you have $\frac{x}{b} - \frac{1}{2}\left(\frac{x}{b}\right)^2$.

So, that is $\frac{T - T_0}{\mu v_b^2/k} = \frac{x}{b} - \frac{1}{2}\left(\frac{x}{b}\right)^2$. This is the temperature profile for a viscous heat generation in a slit flow where $q_x = 0$ at moving surface. And $T = T_0$ at stationary surface; at $x = 0$ moving surface is at $x = b$. The form wise is looks similar, but the profile is different compared to the case where $T = T_b$ at moving surface that is at $x = b$.

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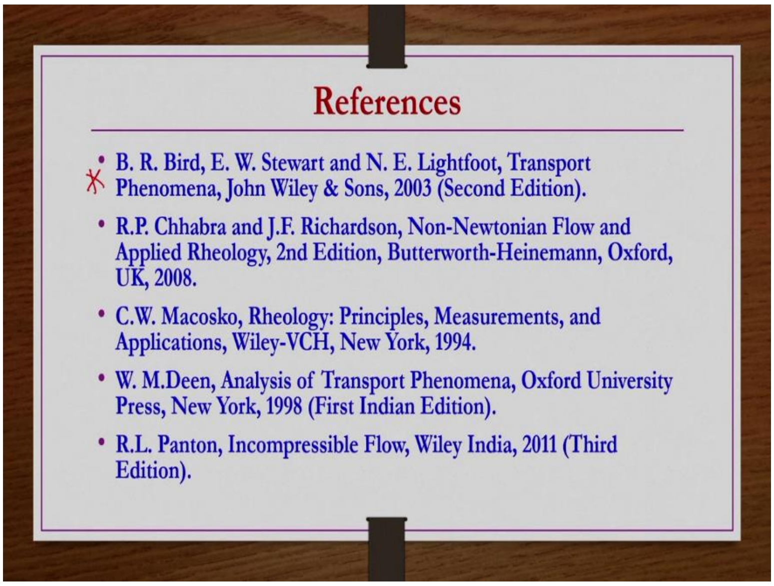

The reference: These problems can be found from this standard book, other references are also provided here.

Thank you.