Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 24
Liquid-Solid Fluidization by Power-law Liquids

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is, Liquid-Solid Fluidization by Power-law Liquids. We will have a comparison of a packed bed when the flow is downward and upward, then we enter into the fluidized bed conditions.

In the downward flow through packed beds of particles no relative moment occurs between the particles except in the case of an unstable initial packing.

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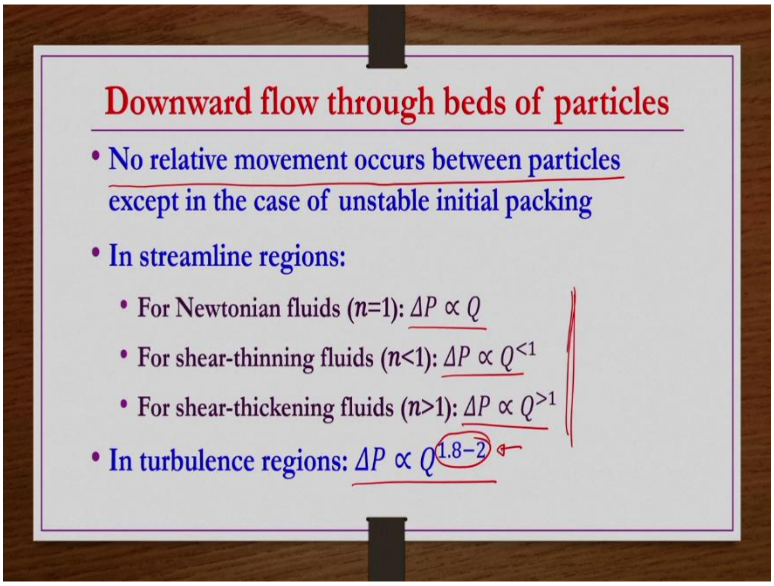

If the initial packing is not properly stable; not have not been you know properly vibrated and shaken before the packing then that means, the initial packing is unstable. So, under such conditions you know there will not be any relative movement of the particles if the flow is downward.

When there is no relative moment of particles you know; what does it mean? It does mean it means that the particle is stable it is like a fixed bed, it is like a packed bed.

So, for the packed beds we have seen you know frictional pressure drop equations both under the streamline conditions as well as the turbulent conditions.

So, what we understand from there? For Newtonian fluids ΔP is directly proportional to Q volumetric flow rate, but for shear thinning fluids ΔP is proportional to the Q to the power of less than 1. This you can you can get whatever the frictional pressure drop equation that we got F = some α constant by Re’ or Re^* + β constant so, that you expand and then you know you write in terms of you know pressure drop versus you know a volumetric flow rate.

Then what you can understand? Pressure drop is you know proportional to the Q to the power of less than 1 if it is a shear thinning fluid. If it is a shear thickening fluid, pressure drop is proportional to the Q to the power of greater than 1. This you can understand from the frictional pressure drop equation whatever we have a derived in previous two lectures for the case of packed beds, right.

So, directly that those equations are not in terms of you know volumetric flow rate, but they are in terms of a superficial velocity. So, both of them are similar, because superficial velocity if you multiply by the cross section area of the bed through which it is the fluid is flowing, then you get the volumetric flow rate, ok. So, from those equations this is what you understand right.

But if the flow is under turbulent region, then we understood that ΔP is proportional to the Q to the power of 1.8 to 2 irrespective of the nature of the fluid; that is whether Newtonian fluid or shear thinning fluid or shear thickening fluid. That indicates under turbulent conditions, the non-Newtonian effect is very less in the case of packed beds. Especially when you know flow is downward and then there is there is no relative movement between the particles.

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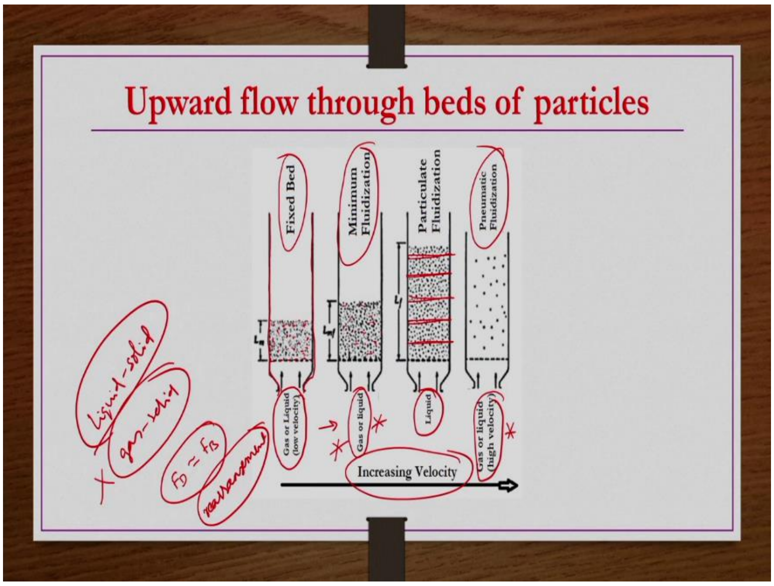

Now, what happens? Upward flow through beds of particles takes place ok. So, that is what we are going to see now. So, this is what pictorially we see now.

So, what we have here? We have a column right, at the bottom there is a perforated plate. And then particles have been poured to some extent to some portion of the column on to this perforated fluid; onto this perforated plate right. So, the perforated plate perforation is such a way that it is a the perforation is, size of the perforation is smaller than the particle size so that the particle should not fall down or fall down through the perforation of the plate.

Why the perforation is required? Because we have to allow a fluid to enter. For the required momentum, heat or mass transfer or reaction whatever reason is occurring right for this reason. So, now these different pictures are shown here, have been shown here. So, this is with increase in the velocity, right.

So now, the top of the packing is not covered or you know constrained by another perforated plate ok. So, what happens? If the velocity whether you use the gas or liquid you know, if the velocity is very very small or if the velocity is small so that there is no relative movement of the particles right.

There is no relative movement of the particles. Or the entering fluid is not having such a high velocity or such you know force that it is altering you know bed arrangement.

It is not making any rearrangement of the bed such a way the velocity is there. So, then whether it is downward flow, upward flow we can say that the bed is under fixed conditions. There is no relative movement of the particles, the velocity is maintained such a way.

Remember, so whatever that we are going to discuss in this lecture is primarily confined to liquid solid fluidized fluidization only, right. Why? Because to some extent whether the fluidizing medium is liquid or solid the behaviour is same, but after certain value of the velocity; fluidizing medium velocity, the behaviour changes the fluidization behaviour changes you know if it is a solid if it is a gas; if it is a gas solid fluidized bed.

So, but we are not going into the gas solid fluidized beds that is not required for us. Because we are talking about the liquid solid fluidization especially the liquid is a power law fluid or you know Bingham plastic fluid or such kind of non-Newtonian fluids ok.

Now next what happens? This is the; this is the case at low velocities. If gradually if you increase the velocity, still it is whether it is gas or liquid does not makes difference in the second picture also. The, when you gradually increase the velocity of the fluidizing medium then what happens? The drag force on the particles is balanced by the buoyant weight of the particles or the drag acting on the particle is balanced by the buoyant weight of the particles.

And then you know once this point reaches in terms of the velocity, because we are increasing the velocity; that means, you know we are increasing the force of the fluid that is entering into the bed, right.

So, then from that point onwards the rearrangement of the particles will start take place. Rearrangement of particles will start take place and then particles would be slightly rearranging from one point to the others. And then the initially they are compact you know; initially they are lying on to each other there is a stiff contact because of low velocities that is not being moved.

So, as a velocity increases slightly there is a rearrangement of the particles; slightly rearrangement like this, changes slightly their locations right. That occurs when the velocity such a way that the drag is balanced by the buoyant weight of the particles. Whatever the drag on the particles are there is balanced by the buoyant weight of those particles. So, then that is slight rearrangement takes place.

If you further increase gradually keep on increasing the velocity what happens? The particles more and particles becomes looser and then they move away from each other right. So now what happens? The same number of particles are occupying more space in the column they are occupying more space in the column.

So, then that means, what happens? The voidage is increasing; voidage is increasing. So, if the voidage is increasing what happens? The resistance to the flow decreases, right. So, then like this you know there would be a point of velocity at which is known as the minimum fluidization velocity at which you know where this pressure drop is balanced by the buoyant weight of the particle.

So, that point of velocity at that conditions you know we call it bed is started fluidizing under fluidized conditions. If you further increase the velocity the bed become looser and then particles you know move at a you know higher distance from each other; the gap between the particles increases and then voidage increases right.

So, like that you know the bed keep on expands. And then after this point of a minimum fluidized conditions you know what will be there? You know the behaviour of the bed is going to be different, depending on the nature of the fluidizing medium. If it is liquid, we have a almost like a homogeneous particulate fluidization will have. What does mean by homogeneous particulate fluidization? That means, at any cross section if you measure the voidage if you measure the voidage you are going to get approximately the same voidage right.

But if it is not a homogeneous fluidized bed, then what happens? At one location the voidage is different, its other location it may be different like that you know voidage is going to be very very different from one point to the other point. Even in this case also particulate fluidization also; particulate homogeneous fluidized bed also there may be slight variations voidage. You do not get exactly let us say at this location it is 0.4, so exactly 0.4 you get the other location. Slight variation 0.39, 0.41 like that you know on the average if you take on the average at any cross section you get the same voidage; approximately the same voidage ok.

That is what you; such kind of fluid behave; fluidized bed you get if you have a liquid fluidized bed right. So, but if you have a gas fluidized bed then you know bubbling fluidization, fast fluidization you know all those different kind of a behaviour are possible that we are not going into the details. Because that is not part; that is not the theme of the lecture we are taking only liquid solid fluidization right.

So now, but after this particulate fluidization condition also reached. So, it you can see almost like you know bed is expanded to the almost to the top of the column, right. So, the velocity is so high that you know the bed is expanding fluidizing and then it is almost like the particles are reached almost top of the column right.

Further, if you increase the velocity, then there would be you know condition where pneumatic fluidization takes place. Where the particles may be throwing particles are being thrown out of the column and then they are collected back by using a cyclone separator at the top. And then sent back to you know bed from the bottom again, that is all different things.

So now, at such high velocities the particles are very far from each other and then such conditions we call them part pneumatic fluidization. So, pneumatic fluidization behaviour is also approximately same for the gas or liquid fluidizing medium ok.

So, pictorially this is how you can say. You can say the how the fluidization of the particle behaviour look like with increase in the velocity. This is what we can understand.

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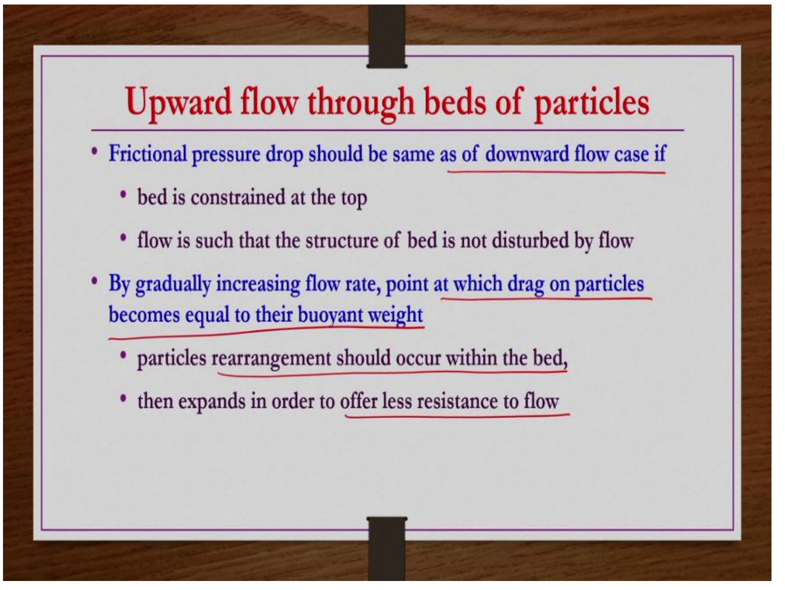

So, now we see the same details again here. So, upward flow through beds of particles the frictional pressure drops should be same as of the downward flow case if under two conditions; one is if the bed is constrained at the top also by a closing perforated plate or the velocity is low such a way that the rearrangement of particle is not taking place; or there will not be any relative movement of the particles if the bed is not constrained at the top. These two conditions.

Under two conditions we have whether upward flow or downward flow the frictional pressure drop is going to be same, ok. Then by gradually increasing the flow rate point at which drag on particles becomes equal to their buoyant weight. Particles rearrangement should occur within the bed as I shown pictorially. And then expands in order to offer less resistance to flow.

Because the rearrangement is taking place, now particles are having more space to occupy so they are you know slightly away from each other. So, then obviously, the voidage going to be higher than the initial voidage. So, if the voidage is higher so then it will be offering the less resistance.

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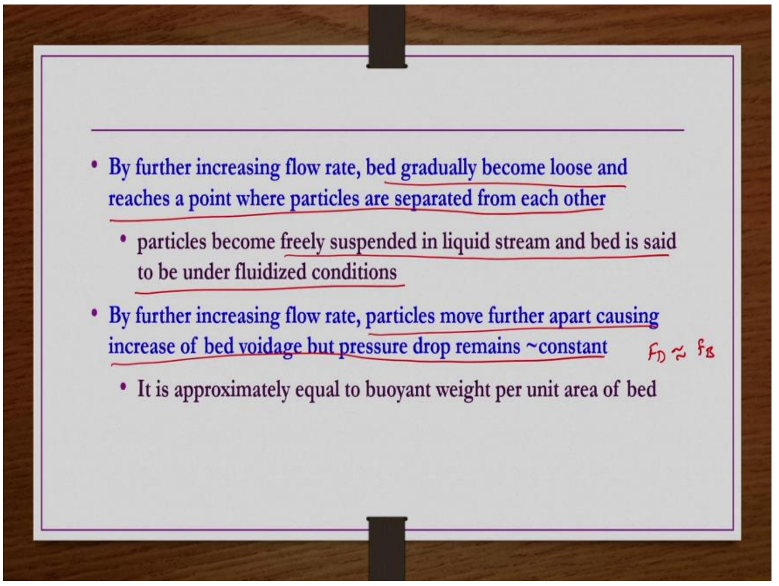

By further increasing the flow rate what happened? Bed gradually becomes loose and reaches a point where the particles are separated from each other and then particles become freely suspended in liquid stream and bed is said to be under fluidized conditions.

So, the particles, even if they are solid particles, they behaves like a fluid particles they are under fluidized condition, right. So, that is the reason these kind of beds are called as a fluidized beds. Even the solid particles behaves like a fluids, they are suspended like a fluid particles ok.

By further increasing the flow rate particles move further apart. And then increase bed of voidage but the pressure drop remains constant; pressure drop almost remains constant. Because after reaching certain minimum fluidization conditions you know whatever the velocity increase you know bed voidage increases, but the pressure drop does not change; pressure drop does not change ok.

So, the change in pressure drop whatever is there with increase or increase in the pressure drop with increase in the flow rate or superficial velocity is there, that is there only; that is there only for the case of a fixed bed. Or the conditions such a way that the there is no relative movement of the particles even if it is not fixed bed; if it is not constrained at the top.

But after that fluidization start occurring you know if you gradually increasing the flow rate, what happens you know the voidage increases, but the pressure drop remains constant. Because, from that point onwards the drag force is balanced by the buoyant weight of the particle or the drag force is balanced by you know buoyant force of the particles right.

So, then from that point onwards you know obviously, the pressure drop is going to remain constant. And it is approximately equal to the buoyant weight per unit area of the bed.

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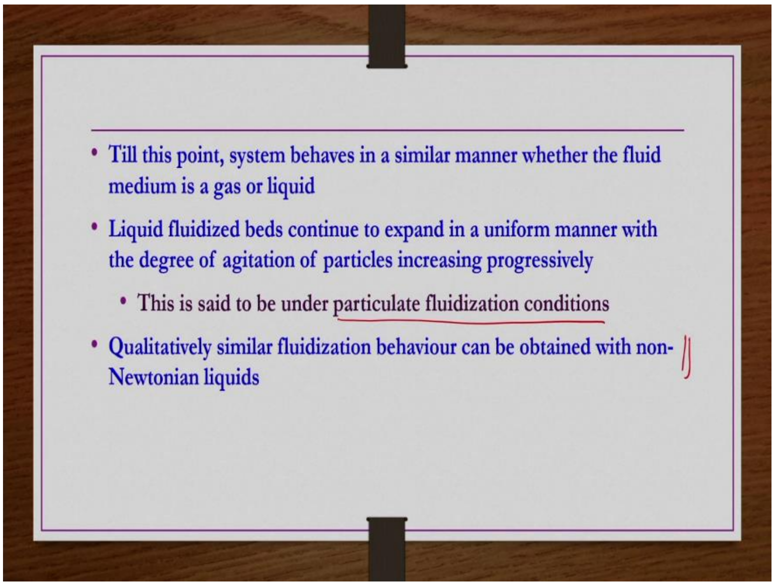

Till this point, system behaves in a similar manner whether the fluidizing medium is gas or liquid. So, the till that point where the pressure drop start becoming constants. So, that point we called as a minimum fluidized conditions. The point at which the pressure drop start becoming constant even if you increasing the velocity volumetric flow rate. So, that point we call it as a point of incipient fluidization right.

So, that incipient fluidization or the fluidization starts taking place, till that point whether the fluid is you know gas or liquid the same equations are going to be used in order to get the frictional pressure drop, ok. So that means, the gas or liquid does not make any difference ok.

Liquid fluidized bed continue to expand in a uniform manner with the degree of agitation of particles increasing progressively. And this is said to be under particulate fluidized conditions where the average voidage at any cross section of the bed if you take, it is going to be approximately the same. Then such fluidized beds we call particulate fluidized beds.

And then qualitatively similar behaviour for a non-Newtonian liquids also possible. Like if your fluidizing medium is non-Newtonian liquid, then obviously, then also you are going to get the similar behaviour; qualitatively similar behaviour you may be getting right.

Now in terms of a pressure drop versus volumetric flow rate or pressure drop versus superficial velocity, if you see the liquid solid fluidization behaviour how does it look like, right?

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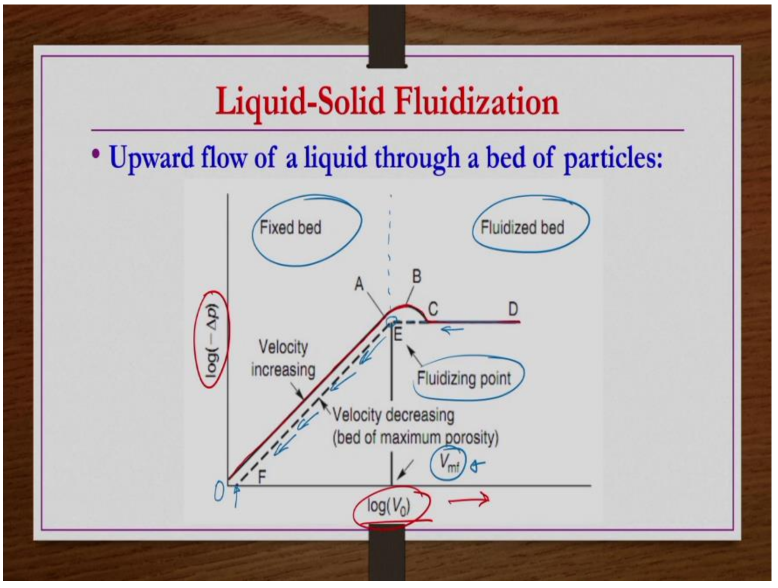

So, then obviously, it has to be upward flow, because upward flow only then it is possible that the fluidization of the particle is taking place right.

So now, you take a column and then you have a perforation at the bed at the bottom of the column, then you pour in some particles ok; crushed you know glass or whatever the packing material you use. And then you do not constraint at the top you do not use any other perforated plate at the top for the constraining this bed.

Then what you do? You allow some fluid to flow through it; fluid to flow through it especially liquids you take because we are talking about the liquid solid fluidized beds right. Then for a given average volumetric flow rate or superficial velocity what is the corresponding pressure drop that you measure. Experimentally that you measure. And then that whatever the Δ p versus V_0 information is there that you plot on a log-log coordinates that you plot on a log-log coordinates.

Then, how do you know what kind of course, you expect is this one. So, initially you know when the velocity is low when gradually velocity is increasing, the pressure drop increases pressure drop increases because till this point; till this point A what happens? The velocity is such a low that it is not causing any rearrangement of the bed it is not causing any rearrangement of the bed.

And then there is no relative moment between the particles even it is not constrained at the top. But if you further increase the velocity what happens? The slope of this curve gradually diminishes and then reaches a point B at which the pressure drop is maximum right ok.

Then further if you increase the velocity then what happens? This pressure drop slightly decreases from A to B there is an increase in pressure drop, but there is a slight increase. From B to C there is a decrease in pressure drop with increase in the velocity right. So, but that decrease is also very small right.

After this point whatever the value you go to the; whatever the higher superficial velocity you take, the pressure drop is not going to be changing. Because from this point onwards whatever the pressure drop is there that is balanced by the buoyant weight of the bed ok. So, that is the region. So, from that point onwards the pressure drop is going to be constant.

Initially you increase the superficial velocity and then you reach this, you trace this A B C D curve ok. Right; now after reaching D because after that point you realize that you know whatever increasing the after this C point; after this point what you realize that? Whatever the increase in the fluidized medium velocity pressure drop is not changing because of the; because of the balanced pressure drop and then buoyant weight of the beds right.

So, then what you try to do? You try to decrease the velocity gradually you decrease gradually velocity ok. Then what happens? To certain point C you are going to get back your curve C D curve. Now, you will be getting D C curve you are decreasing the velocity. Then further decreasing the velocity what happens? You are not going to get the pressure drop same as pressure drop at B and then A.

But rather what happens? You know further decreasing the velocity, the pressure drop still remains constant up to the point E; up to the point E. Why it is? Because initially the any when you do the initial packing the bed was compact, right.

So, it was giving slightly higher pressure drop at this corresponding velocity right. But when you once it has been fluidized and then you are gradually decreasing the velocity, gradually decreasing the velocity what happens? The particles initially at the you know at the D point they are you know fluidized conditions of free from each other; they are suspended like fluid particles right.

As decreasing the velocity particles you know the still up to C points still they are you know free from each other and then they are suspend like a part fluid particles like under fluidized conditions. But you know further decreasing the velocity what happens? The particle starts settling starts settling and then distance between this particle decreases.

And then there would be a point E at which the particles are lying on to each other. They are just lying on to on to each other. But there is no vibration or mechanical agitation kind of thing additional external force is not there so that this particle become compact.

Since it is initially fluidized the bed is initially disturbed then again by decreasing the velocity the particles are settled down and then resting onto each other. So, then at that conditions voidage would be the slightly higher than the initial packing voidage. That is the reason the pressure drop is going to be slightly less up to this E point.

Now after this E point further if you decreasing the velocity what happens? You know the pressure drop gradually decreases because, now the after this reaching this E point when you decreasing the velocity the particles are at this E point, at this E point what happens? Particles already resting on to each other.

They are like you know confined they are now like a packed bed fixed bed kind of thing. Only thing that the slight you know the packing is not very compact as initial packing, because they are disturbed the packing is disturbed and then they are just resting onto each other.

So, further increase decreasing the velocity, there would be no compacting of the bed will not take place. So, then what happens? The voidage is going to remain constant, but what happens? The pressure drop is going to decrease and then you are not going to get this let us say origin O. O to A curve whatever is there that you are not going to get, you are going to get E F curve. And then the pressure drop at smaller velocity is you know slightly lesser than the pressure drop while increasing the velocity.

Why? Because while decreasing the velocity the bed rearrangement has been taking substantially and then even when you decrease the velocity to almost very small 0 close to 0 velocity, the voidage is slightly higher than the initial packing voidage; initial packing voidage. That is the region you know the pressure drop curve your O A curve whatever is there that is not being traced rather you get E F curve.

So, this point E whatever is there we call fluidizing point or a point of incipient fluidization ok and then corresponding velocity is whatever is there minimum fluidization velocity. That is what we call. So now, we have to find out how to find out this minimum fluidization velocity, because in the fluidized bed conditions we know that the pressure drop is constant, right.

So, now this bed is we can divide into two parts up to this point E whatever you say that behaviour is same like a fixed bed condition. So, then whatever the equations that we have a developed in the previous lectures that those things we can use. But after that point the bed is under fluidized conditions.

So, here we in this conditions we understand this pressure drop is constant; is constant and it is approximately equals to the buoyant weight of the bed. So, that we can calculate if you know the minimum fluidization.

So, how to find out this minimum fluidization that is what we are going to see. And then we are also going to see how to find out the friction factors etcetera for this minimum fluidized conditions.

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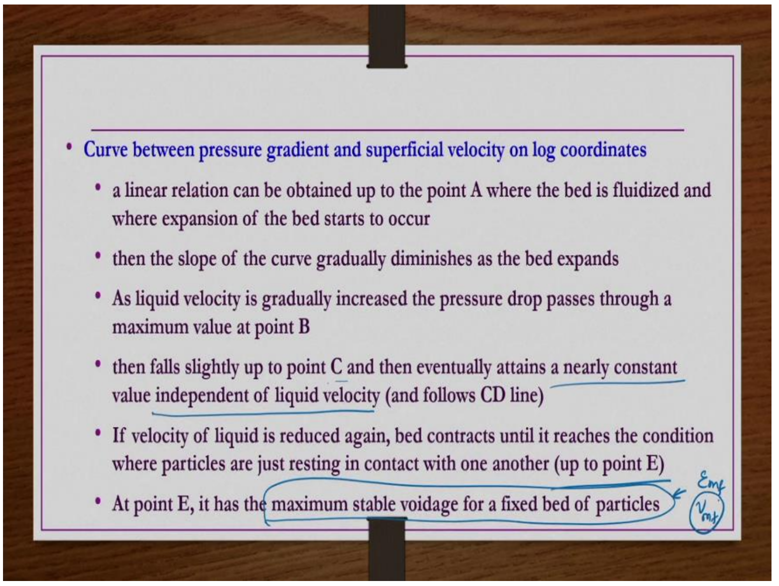

So, whatever the details that we have discussed here in this figure the same thing is presented here in the text form as a notes. The curve between pressure gradient and superficial velocity when you plot on log coordinates for a bed which is not constrained at the top and then flow is taking from up on from bottom to top as an upward motion. Then a linear relation can be obtained up to point A where the bed is fluidized and where expansion of the bed starts to occur. Till that point there is no expansion start, but at from A point onwards it starts occur.

Then the slope of curve gradually diminishes. As the bed expands as liquid velocity is gradually increased the pressure drop passes through maximum value at point B. Then falls slightly up to point C and then eventually attains a nearly constant value independent of the liquid velocity; independent of the liquid velocity and follows CD line.

Then if velocity of liquid is reduced again bed contracts until it reaches the condition where the particles are just resting in contact with another up to point E. At point E it has the maximum stable voidage for a fixed bed conditions. For a fixed bed conditions this voidage whatever is there that is maximum for a fixed bed condition and then minimum for a fluidized bed condition ok.

Further, if you increase the velocity you can increase the voidage right. But this voidage at the point E whatever is there that is the maximum for the fixed bed and then minimum for the fluidized bed right. Corresponding velocity is also minimum fluidization velocity. Below that velocity there will not be any fluidization.

That is the reason you know this voltage whatever that is there is ε_{mf} is written which is the minimum voidage for the fluidized conditions and then maximum voidage for the fixed bed conditions. And then corresponding velocity whatever is there we call it minimum fluidization velocity.

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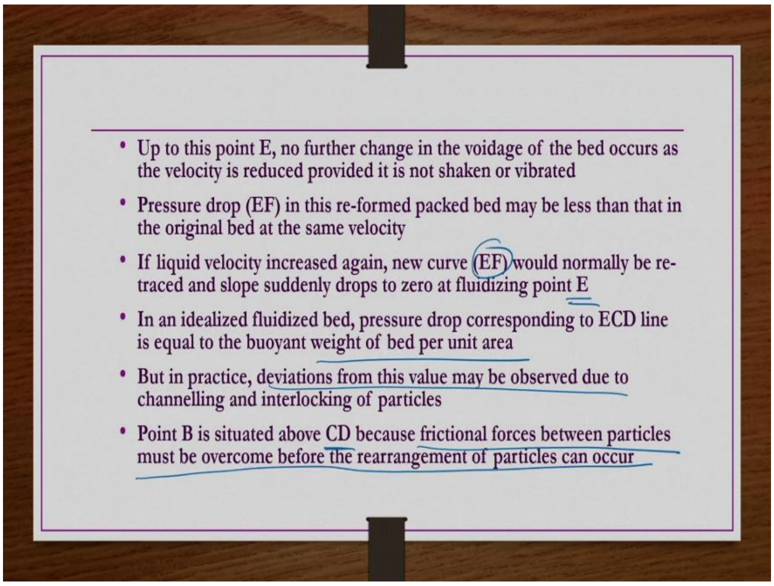

Up to this point E, no further changes in the voidage of the bed occurs as the velocity is reduced provided it is not shaken or vibrated. Then a pressure drop EF in this re-formed packed bed may be less than that in the original bed at the same velocity, because of the rearrangement of the bed. If the liquid velocity increased again new curve EF would normally be retraced and slope suddenly drops to zero at fluidizing point E.

So, in the figure we have increasing the velocity. So, then O A B C D curve that is what we have, while decreasing the velocity we have D C E F curve that we are having. So now, again if you increase the velocity to see the how the fluidization will take place. So, then under such conditions we are going to get this EF and then CD curve rather than O A B C D curve ok.

So, once it, once the bed is disturbed you are not going to get the same O A A B C D curve whatever the figure that log of pressure drop versus log of you know fluidization velocity that we have drawn ok. Once bed is disturbed you are not going to get the same O A curve again ok; you are going to get EF curve only.

In an idealized fluidized bed pressure drop corresponding to ECD line is equal to the buoyant weight of the bed per unit area. But in proxy; obviously, in reality small deviations from this value may be observed due to several regions like channeling, interlocking of the particles, wall effects, etcetera.

Point B situated above the CD because frictional pressure of frictional forces between the particles must be overcome before the rearrangement of a particles can occur.

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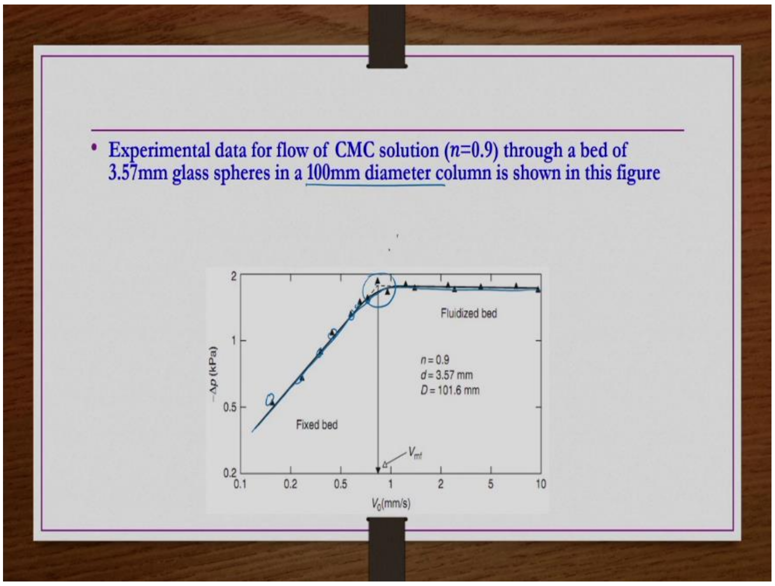

So, one example we take a CMC solution 0.9 and then it is pass through a bed of 3.57 mm glass spheres and then packing A has been done in 100 mm diameter column. Then the experiments have been carried out whatever the $-\Delta P$ versus V_0 information is there that has been plotted on a log-log scale.

So, that now you can see; you can see the points are slightly scattered slightly scattered right. But what you can see here? You can see whatever the details that we have seen in the theoretical thing that we are getting back here. While decreasing this you know there is a slight change here. So, the dotted one is increasing one the solid one is decreasing the velocity.

So, even not only the nature of the curve even that whatever A B C D and then D E F curve are there. So, that also we can clearly see here.

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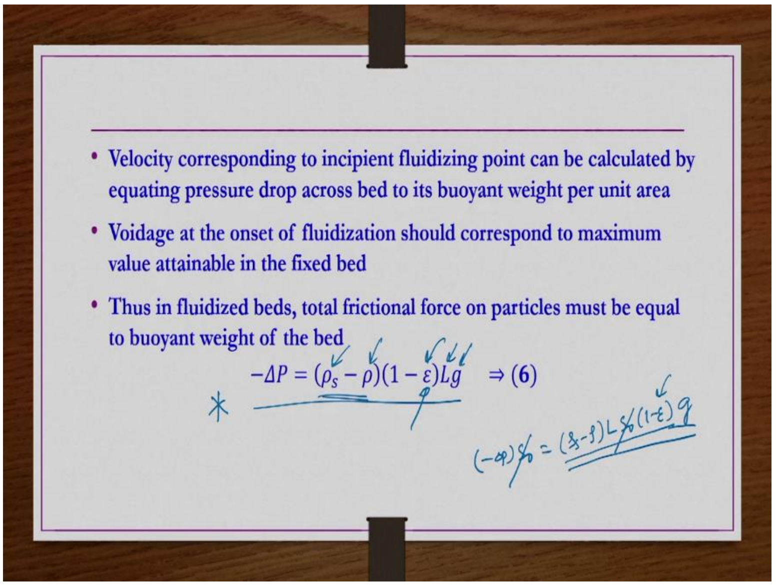

So, velocity corresponding to incipient fluidization point can be calculated by equating the pressure drop across the bed to its buoyant weight per unit area. So, voidage at that; at the onset of fluidization should be correspond to the maximum value attainable in the fixed bed, but minimum value in the case of a fluidized bed.

Thus, fluidized beds total frictional force in particles must be equal to the buoyant weight of the bed. So, let us say frictional force whatever is there. So, $-\Delta$ P multiplied by say let us say cross section area is S_0, then the volume of the fluid that has been replaced is $(\rho_S - \rho)$ right.

If you wanted to know the mass of the fluid that has been replaced by the particle movement. So, it has to be multiplied by the volume of the bed. So, L S_0. But in a in the case of packed bed what we have? We have only $(1 - \varepsilon)$ fraction of the solid particles; not the entire column is filled by the particles. So, white space is ε is there. So, $(1 - \varepsilon)$ is the fraction of the particles that are present and then.

So, this is, this gives the mass of the particles right and then multiplied by g. So, that gives this entire expression will give you the band force of the bed the; of the entire bed ok. We are not talking about one particle for the entire bed because we are multiplying by this $1 - \varepsilon$ right.

So, now this S_0 S_0 is cancelled out. So, $-\Delta P$ is $(\rho_s - \rho)(1 - \varepsilon) Lg$ that is what we get right. So, this is the constant pressure drop in the case of fluidized bed. That means, in the case of fluidized bed if you wanted to know what the constant pressure drop is.

So, only thing that you need to know what is the voidage that is it. And because fluid density, particle density are known ρ_s; ρ, ρ_s are known ρ_s is the particle density and then ρ is the fluid density those things are known, g is known. The height of the packing of fluidized height is known. So, only thing that you need to know is ε, once it is known you can know what that constant pressure drop is ok.

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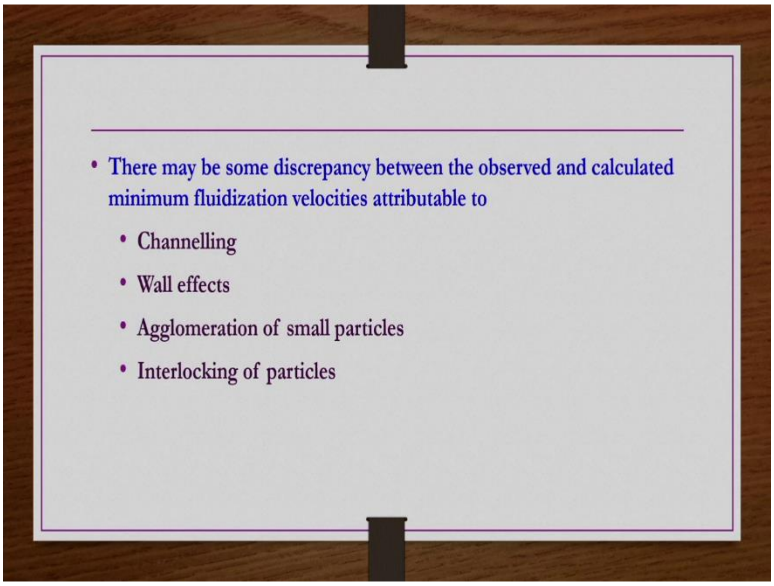

So, there may be some discrepancies obviously, between the observed and calculated minimum fluidization velocities, because the same regions are there for $-\Delta P$ versus V_0 curves also. These are channeling wall effects, agglomeration of small particles, and then interlocking of particles if the particles are having like needle like shape etcetera.

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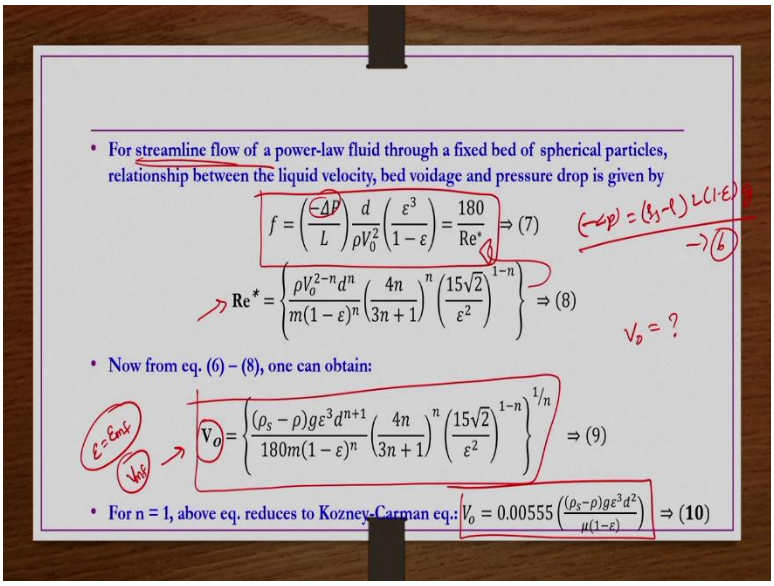

Then for streamline of flow of power law fluid through a fixed bed of spherical particles relation between the liquid velocities, bed voidage and pressure drop; we have already calculated in the previous lecture. And then that is given by this expression right.

So, in this expression in place of $-\Delta P$. If you substitute $(\rho_s - \rho)(1 - \varepsilon) Lg$ then you can get you know whatever the velocity V_0 at which the fluidization is taking place. So, here in this equation Re^* this is also we have derived in previous lectures.

So now, equation number 6 is this one and then 7 to 8 what does it mean by? Now you are trying to find out V_0 actually at the minimum fluidized conditions or the flow is under streamline conditions, but still the fluidization is taking place then what is the velocity at which the fluidization is taking place that you wanted to know. So, that you can get from this expression.

Because in this equation 7 ΔP is known from this equation number 6 right; Re^* is given by this one. So, now you substitute in place of $-\Delta P$ you substitute this equation 6 here, in place of Re^* you substitute equation number 8 and then simplify then you get V_0 is this one. Simple rearrangement you can do it. So, this equation if you substitute n = 1 you get this equation which is nothing but Kozney-Carman equation.

Now, this equation we are doing when we are we; when we are saying $-\Delta p$ is constant by this equation; that means, it is under fluidized conditions right. So, now in this equation if you substitute $\varepsilon = \varepsilon_{mf}$ what is ε_{mf}? ε_{mf} is nothing but the voidage at the minimum fluidized conditions or incipient fluidization conditions whatever the voidage of the bed is there that we are calling ε_{mf}.

And it is the maximum voidage for the fixed bed condition because after that condition; after that corresponding velocity the bed is no longer going to be under fluid under a fixed bed condition it is going to be under fluidized conditions. So, it is minimum for the fluidized bed condition. So, that minimum at minimum fluidized conditions whatever the ε is there that you substitute in this equation. Then whatever this V_0 is there that we can call it minimum fluidization velocity ok.

So, this is how we can get the minimum fluidization velocity, but it is only for the streamline flow of power law liquids. But always it is not possible that you have only streamlined flow, then we have to have the contribution from the turbulent flow also. So, whatever plus 1.75 is required in equation number 7 that should be taken into the consideration. Then how this equations will change?

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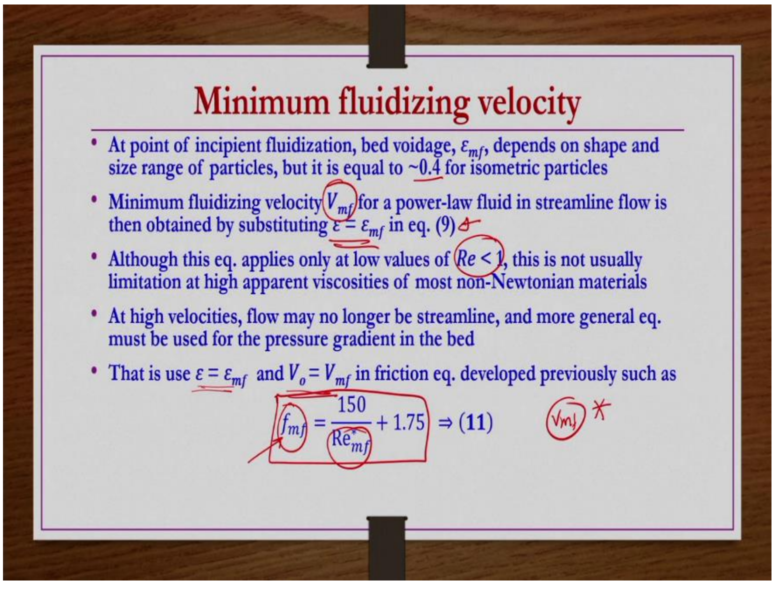

So, minimum fluidization velocity at the point of incipient fluidization bed voidage ε mf depends on shape and size and; shape and size range of particles, but it is equals to approximately 0.4 for isometric particles in general. Minimum fluidization velocity

V_{mf} for a power law fluid in streamline flow can be obtained by substituting $\varepsilon = \varepsilon_{mf}$ in the previous equation number 9 in the previous slide that we have seen.

Although, this equation applies only at low values of Reynolds number, this is not usually limitations at high apparent viscosities of most of the non-Newtonian fluids. We understand that apparent viscosity though the viscosity changes for our non-Newtonian fluids with respect to the shear rate and all that. So, but the apparent viscosity whatever is there that is local value of shear stress divided by the shear rate whatever is there that is apparent viscosity. That is usually very high for non-Newtonian fluids. So, then in general we whatever this limitation that we have seen that is fine ok.

So, at high velocities flow no; flow may not be under streamline conditions and more generalized equation must be used for pressure gradient in the bed. So, that is $\varepsilon = \varepsilon_{mf}$ and $V_0 = V_{mf}$ in friction factor equation whatever $f = 180/Re^* + 1.75$ is there. So, there we have to substitute.

So, when you take both the contributions from the stream line as well as the turbulent flow region. So, this is what we have this equation right. So now, this equation here f_{mf} is related to $\frac{-\Delta P}{l}$ right. So, $\frac{-\Delta P}{l}$ we know is $(\rho_s - \rho)(1 - \varepsilon)g$ from the fluidized conditions so, that we can substitute here, right.

Re_{mf} is having you know V_{mf} f_{mf} is also having V_{mf} values. So, solving these equations becomes very difficult. So, then what we have? Will have like you know this without V_{mf} value you cannot calculate Re^*_{mf} or f^*_{mf} or f_{mf} as well right. So, but we are doing all this analysis in order to calculate this what is V_{mf}.

So, in order to avoid such kind of complications what we do? We do some mathematical simplifications here; rearrangement of equation something like that.

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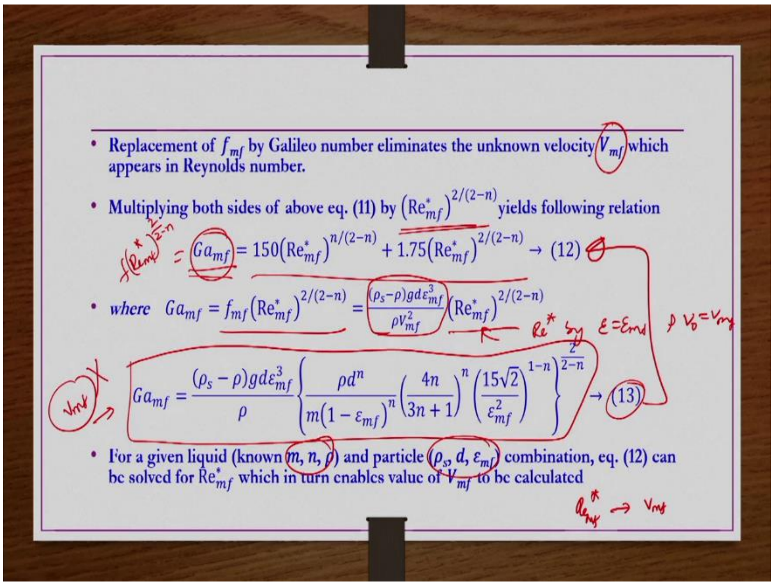

So, what we do? A replacement of f_{mf} by Galileo number eliminates the unknown V_{mf} which appears in the Reynolds number for that what we do? In the equation number previous equation number f_{mf} = $150/Re^*_{mf}$ + 1.75 you are multiplying both sides by $\left(Re^*_{mf}\right)^{2/(2-n)}$.

Here Re^*_{mf} is same definition is same as Re^* that we have seen previous lecture for packed bed. Only that you know wherever ε is there, ε should be replaced by ε_{mf} and then V_0 should be replaced by V_{mf}. That is the only difference.

So, when you do this whatever the left hand side f multiplied by $\left(Re^*_{mf}\right)^{2/(2-n)}$ is there. So, that we are calling it Galileo number and then right hand side. So, this is the things that we have ok. So, Galileo number at minimum fluidized conditions Ga_{mf} is nothing but this one and then f_{mf} is nothing but this one; f $\frac{(\rho_s-\rho)gd\varepsilon^3_{mf}}{\rho}$ V_0^2 this is what we have seen in previous lecture.

But ε is replaced by ε_{mf} and then V is replaced by V_{mf} for the case of f_{mf} right. Re^* this thing we know right. So, Re^* we have already seen in previous lecture as well as the previous slide we have also seen. So, Re^* expression whatever is there in that Re^* expression what you do? $Re^*/\varepsilon = \varepsilon_{mf}$ and $V_0 = V_{mf}$ if you do that you substitute here and then simplify.

So, then whatever this V_{mf}^2 is here and then here also you will be getting V_{mf}^2 by after substituting Re_{mf}^* here, then that will be cancelled out and then you will be getting Galileo number is like this. Now here this Galileo number there is no V_{mf} requirement; unknown V_{mf} is not required here. In this Galileo number there is no unknown V_{mf} value right.

So, for a given fluid that is m, n, ρ are known, particles ρ s, d, ε_{mf} also known for a given conditions. So, Re_{mf}^* you can you cannot obtain directly now what you do? You Galileo number you first obtain right, from this Galileo number what you do? You Galileo this whatever the Galileo number you get from this equation 13 because you only need the ε_{mf} particle properties and then fluid properties all that you substitute.

So, then you get the Galileo number. This Galileo number you substitute in equation number 12, then you find this Re_{mf}^* once Re_{mf}^* is known. So, from there you can get V_{mf} ok. We see an example problem now.

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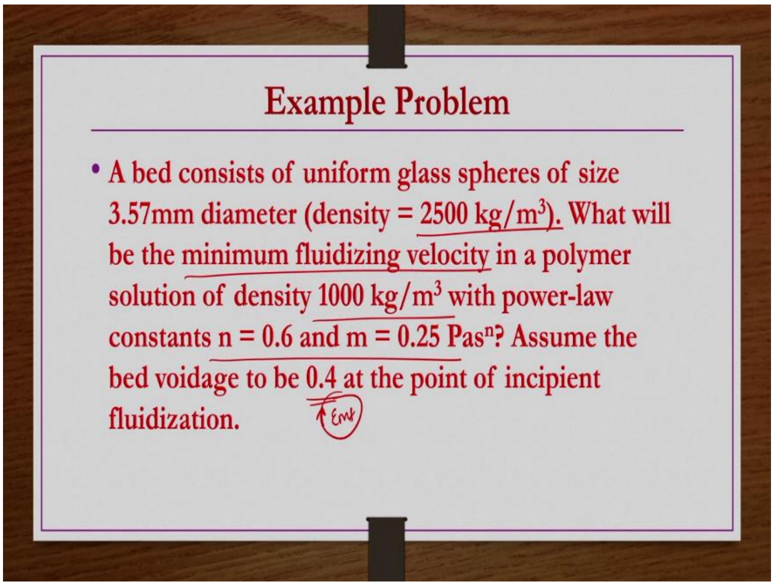

So, a bed consists of uniform glass spheres of size 3.57 mm diameter density of particle is given this one. What will be the minimum fluidizing velocity in a polymer solution of density? Fluid density is given this one, ρ is given this one, and then fluid is a power law fluid with these constants. Assume the bed voidage is 0.4 at incipient fluidization velocity. So that means, this is ε_{mf} not ε it is ε_{mf} is given. So, what you need to find out? You need to find out minimum fluidization velocity.

So, in the problem it has not been mentioned whether the flow is under the streamline conditions or turbulent conditions, that it is not mentioned. And then we cannot find out as well, because in order to know whether the flow is under streamline conditions or not we have to find out V_{mf} that we do not know. Indeed we are solving this problem to get that V_{mf}. So, for that reason what we are doing now? We first calculate the Galileo number ok.

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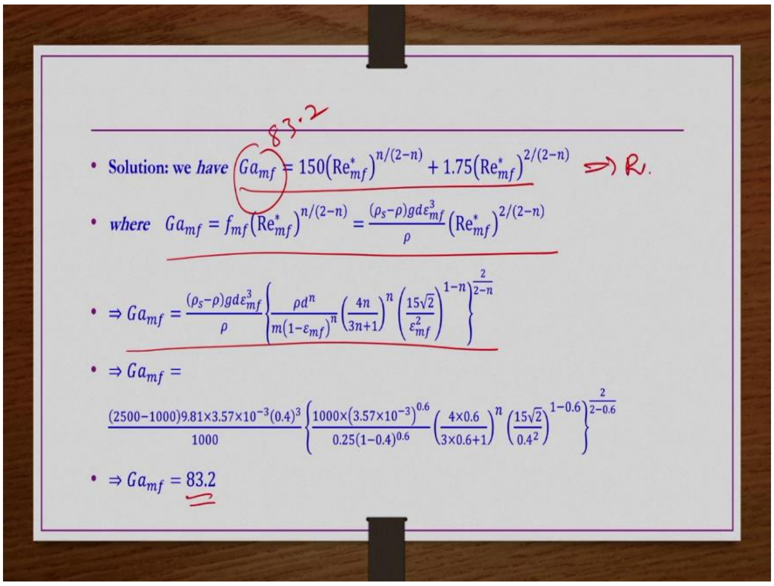

So, this is the equation that we have here in this equation Galileo number is given by this one. After simplifying substituting Re^*_{mf} and then simplifying what we get? Galileo number we got this expression. Now here in this equation when you substitute all these numbers. So, then you get Galileo number is 83.2; this 80 through; 83.2 you substitute here and then solve this equation for Re^*_{mf}.

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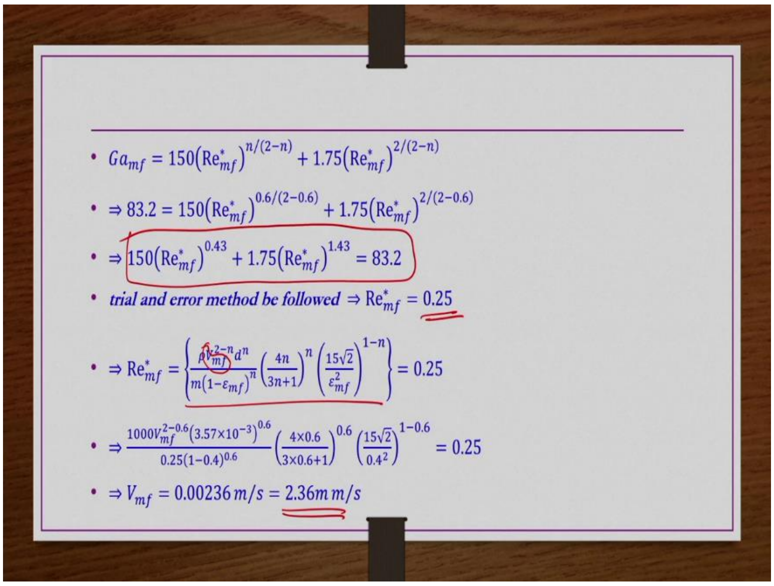

When you do that one you get $Re^*_{mf} = 0.25$. That is the flow under streamline conditions only. And then one E once and then once Re^*_{mf} is known then this is the Re^*_{mf}. So, then here everything is known except V_{mf}. So, then you can get the V_{mf} and then that comes out to be 2.36 mm per second straight forward simple. Only thing these equations slightly you know difficult to solve directly. So, then you have to do trial and error approach ok.

Now before ending today's class what we see? A few details of dispersion in fluidized beds and then we wind up the discussion on the flow through multiple particle system.

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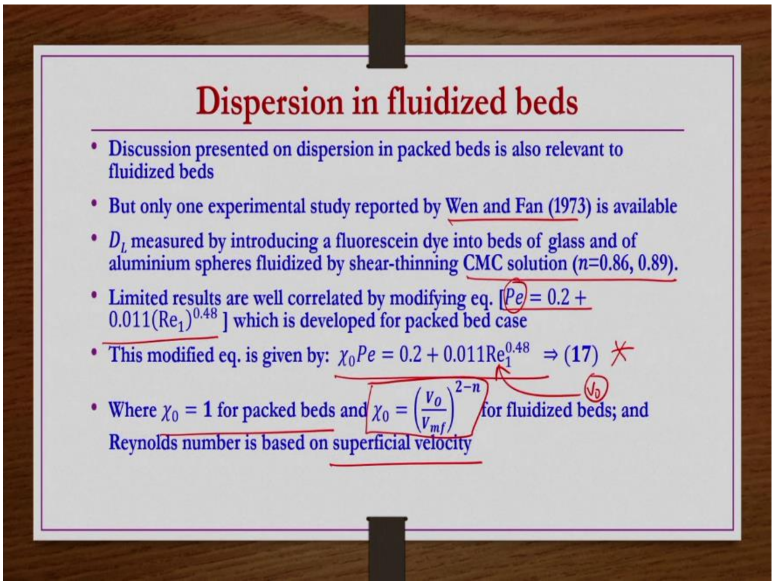

So, discussion presented on dispersion in packed bed is also relevant to the fluidized beds in general including that you know differential equation for you know dispersion in packed beds and all that. But only one experimental study is reported by these people.

They have measured D_L by introducing a fluorescence dye into beds of glass and of aluminium spheres fluidized by CMC solutions of limited n values. And they found that whatever this correlation Pe = 0.2 + 0.011 $Re_1^{0.48}$ is there the same equation can be used with a slight modification. Where this Peclet number is multiplied by a parameter χ_0.

That is χ_0 Pe = 0.2 + 0.011 $Re_1^{0.48}$ should be you know equations for the dispersion fluidized bed according to these people. What is this χ_0? It is a ratio between $\left(\frac{V_0}{V_{mf}}\right)^{2-n}$. If it is 1 then obviously, it is packed bed conditions and then whatever this Re_1 is there that is based on the V_0. It is not based on the V_{mf} ok; Re_1 definition whatever is that it is based on the superficial velocity not the minimum fluidization velocity.

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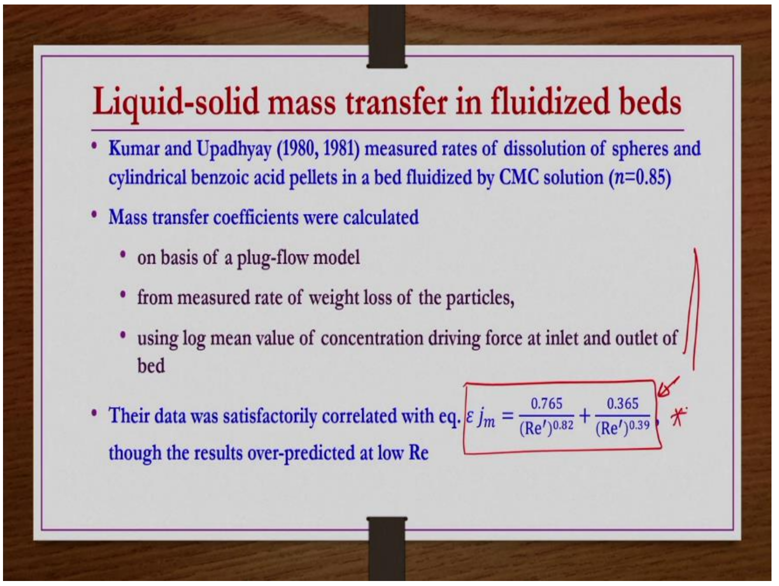

So, liquid-solid mass transfer in fluidized beds, here also very limited you know studies are available. Kumar and Upadhyay measured rates of dissolution of spheres and cylindrical benzoic acid pellets in a fluidized bed by CMC solution of only n value 0.85.

Then they measured the mass transfer coefficients on the basis of plug flow model, from measured rate of weight loss of the particles, using log mean value of concentration driving force at inlet and outlet of the bed. So, under such conditions, they measured the mass transfer coefficients. And they found whatever their packed bed correlation is there.

So, this is the correlation they have developed for the packed bed mass transfer in packed bed conditions, the same correlation maybe you know used; same correlation may be used without any problem. But however, this over predicts at low Re.

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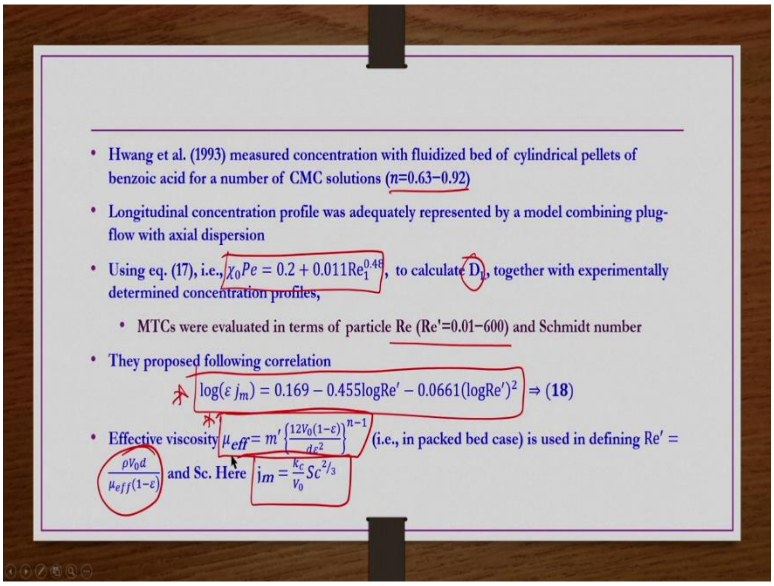

Hwang et al measured concentration with fluidized beds of cylindrical pellets of benzoic acid for a number of CMC solutions point n is equals to 0.63 to 0.92. And then they measure longitudinal concentration profile by combining plug flow model with axial dispersion. And they found this equation whatever is there is; they have used this equation to calculate D_L together with experimentally determined concentration profiles. And then mass transfer coefficients were evaluated in terms of Re prime between 0.01 to 600 and Schmidt numbers varying and they propose this correlation.

So, all these correlation Kumar and Upadhyay or Hwang et al whatever are there. So, all these studies are experimental studies and then their experimental results are there, they are empirically curve fitted and then these are the results.

In this equation effective viscosity is required for defining the Reynolds number. So, they have used this expression ok. And then Re’ they have used $\frac{\rho V_0 d}{\mu_{eff}(1-\varepsilon)}$ here also Re’ is defined based on the superficial velocity not the minimum fluidization velocity. And then whatever j_m is there that is given by $\frac{k_c}{V_0}Sc^{2/3}$.

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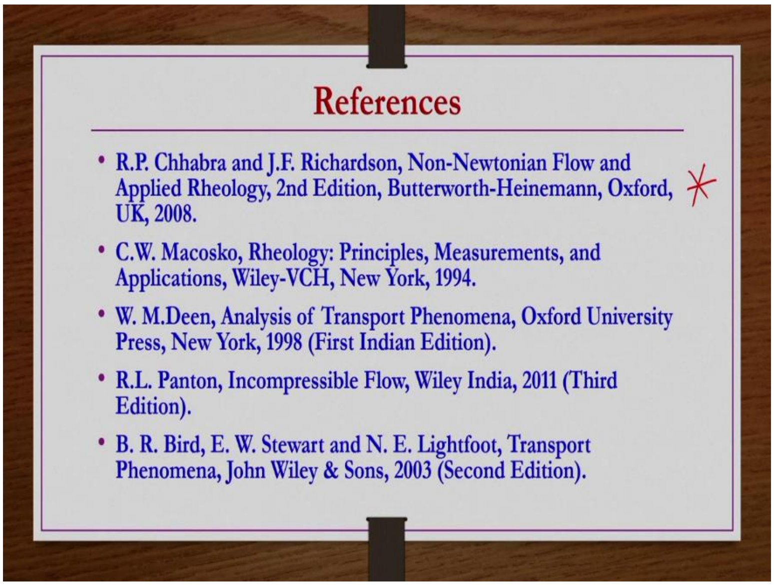

References: The entire lecture is prepared from this excellent book by Chhabra and Richardson which is a very excellent book especially for a Non-Newtonian Flow and Applied Rheology. Other reference books are provided here.

Thank you.