Transport Phenomena of Non-Newtonian Fluids

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Lecture - 23
Dispersion in Packed Beds: Non-Newtonian Effects

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Dispersion in Packed Beds: Non-Newtonian Effects. In the previous lecture we have seen if a power law or Bingham plastic fluid is flowing through a packed, bed how to obtain expression for the frictional pressure drop etcetera those things, the derivations etcetera those things we have seen, we have also seen an example problem.

Now, in the same packed bed let us say if you wanted to include the effect of dispersion as well in the overall transport phenomena, then how to proceed. So, what are the things that you have to worry about, so all those things that we are going to see right.

We are going to see an equation for a dispersion in packed bed as well. If there is a dispersion in packed bed, so they how the concentration is varying because of the dispersion in addition to the because of the bulk flow those things we are going to see and then derive an equation as well.

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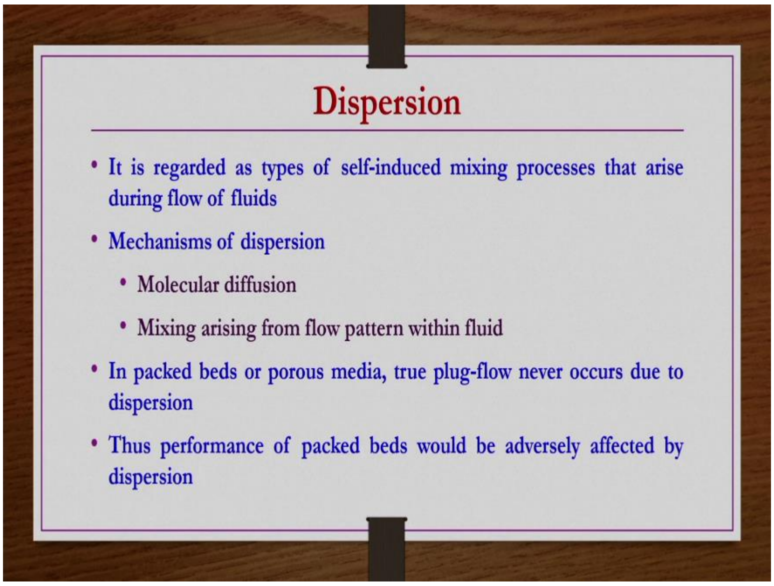

Dispersion

- It is regarded as types of self-induced mixing processes that arise during flow of fluids
- Mechanisms of dispersion
  - Molecular diffusion
  - Mixing arising from flow pattern within fluid
- In packed beds or porous media, true plug-flow never occurs due to dispersion
- Thus performance of packed beds would be adversely affected by dispersion

Often dispersion is regarded as types of self-induced mixing process that arise during flow of fluids. Mechanism of dispersion; there are two different possible mechanisms in general, one is the molecular diffusion another one is the mixing arising from flow pattern within the fluid. How does it appear, how does it occur, and that we see especially in the case of packed bed now.

In packed beds or porous media true plug flow never occurs due to the dispersion. Obviously, if there is a dispersion, we will never have a kind of a true plug flow. And then because of that one performance of packed beds would be adversely affected by the dispersion. So, why dispersion is very much essential in the case of packed bed that we see.

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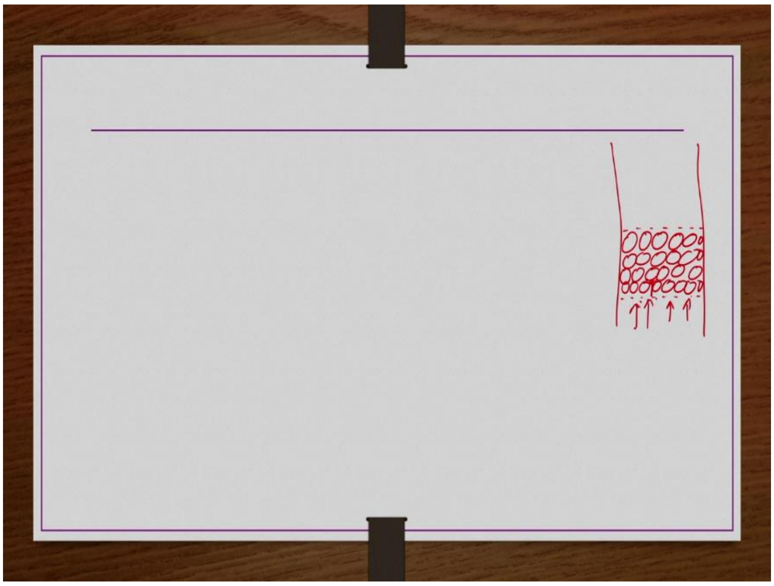

So, let us say if you have a column right, and then that is packed with certain particles regular or irregular whatever particles right. So, now, what happens when a fluid enters from the bottom let us say it is entering like this. So, whatever the fluid element is entering that is entering at certain high velocity. So, then it must be entering the column at certain high kinetic energy.

So, when this fluid element comes and then enters the perforated plate here. So, immediately it may be encountering you know resistance from the particle right. When this fluid elements comes and then hits the particle because of high kinetic energy that kinetic energy has to disperse other way. So, then what happens? A kind of rotational of the rotation of the fluid elements takes place in the interstitial spaces.

So, because of that one the rotation is there that causes a kind of self-inducing of a mixing in the process, mixing of the fluid within the bed. So, because of such kind of reasons what happens the dispersion is very much important in the case of a packed beds.

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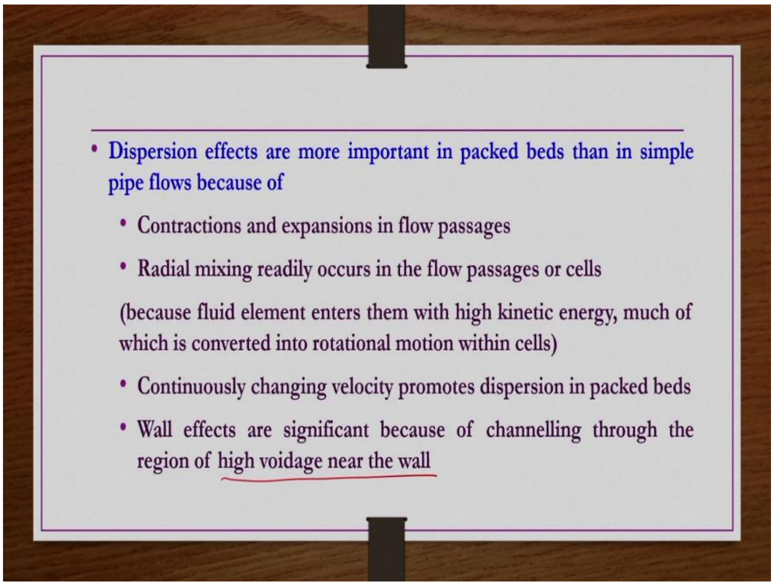

- Dispersion effects are more important in packed beds than in simple pipe flows because of
  - Contractions and expansions in flow passages
  - Radial mixing readily occurs in the flow passages or cells

  (because fluid element enters them with high kinetic energy, much of which is converted into rotational motion within cells)
  - Continuously changing velocity promotes dispersion in packed beds
  - Wall effects are significant because of channelling through the region of high voidage near the wall

So, dispersion effects are more important in packed beds than in simple pipe flows because of several reason, one of them is the contraction and expansion in flow passages in general takes place.

So, in that; because of that contraction and expansion of a flow passages interconnecting interstitial spaces there will be a kind of a rotational motion of the particles. And in that induces dispersion in the packed beds that is one region. Another region is this radial mixing readily occurs in the flow passages or cells because of the loss of kinetic energy that fluid element is carrying before hitting a particle. And then that loss of kinetic energy induces a rotational motion in the neighboring interstitial spaces.

Because fluid element enters with high kinetic energy much of which is converted into rotational motion within the cells, because of that one radial mixing readily occurs in the case of packed beds and then that induces additional dispersion. And a continuously changing velocity promotes dispersion in packed beds.

In packed beds we know that completely laminar flow or completely turbulent flow does not appear in the entire column. At some interstitial spaces there may be laminar flow other interstitial spaces maybe there turbulent kind of thing may be there.

So; that means, you know from one flow passage area to the other flow passage area the velocity is drastically changing, sometimes it is decreasing sometimes, it is increasing. And then because of this change in velocity that also promotes dispersion in packed beds.

Also wall effects are also significant because of a channeling through the region of high voidage near the wall. Near the walls usually what we have you know the voidage or volume fraction is more the actually this voidage is also not uniform in the entire packed bed.

If you take region wise if you do right, overall volume fraction we take for an engineering calculations. But if you are using especially regular non-spherical particles etcetera then what happens the voidage is varying from location to the location. So, that voidage is in general very high near the walls right compared to the voidage at the center of the bed right. So, in because of that one also wall effects near the wall you know that also induces dispersion in packed beds.

Because of these many reasons what happens? We have seeing that you know dispersion is very much essential in the case of packed bed if a column is packed with particles rather than if you are having only empty column. If there is an empty column dispersion may be very small or may be negligible in many of the applications.

But if the column, part of the column is packed with particles then dispersion is going to be very large. And then one cannot avoid the considering it in engineering calculation either for the design of the equipment or for tuning of the operational parameters. So, why does at; why does it happen in the case of packed beds because of a these many listed reasons.

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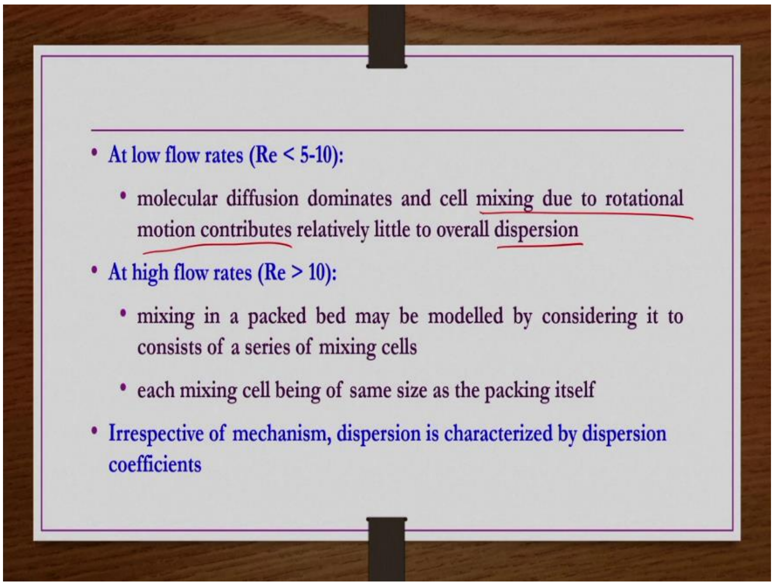

- At low flow rates (Re < 5-10):
  - molecular diffusion dominates and cell mixing due to rotational motion contributes relatively little to overall dispersion
- At high flow rates (Re > 10):
  - mixing in a packed bed may be modelled by considering it to consists of a series of mixing cells
  - each mixing cell being of same size as the packing itself
- Irrespective of mechanism, dispersion is characterized by dispersion coefficients

So, in general what happens at low flow rates or small Reynolds number molecular diffusion dominates in general that we know. And then cell mixing due to the rotational motion contributes relatively very little to overall dispersion in general right. So, this dispersion also it we have seen like you know because of the molecular diffusion as well as the because of you know flow right.

So, at low Reynolds number molecular diffusion dominates that we understand the convection or contribution because of the bulk flow is very less. So obviously, under such conditions if you have the flow rates small that is small Reynolds number reason, so molecular diffusion would be dominating. And cell mixing due to the rotational motion or cell mixing due to the rotational motion contributes very little to the overall dispersion if the Reynolds number is small.

However, if Reynolds number is high mixing in packed bed may be modelled by considering it consist of series of mixing cells, how to do that all we do. So that means, at high Reynolds number or high flow rates the dispersion is going to be significantly large. So, that one has to consider appropriately and then each mixing cell being of same size as the packing itself. So, next slide we are going to derive the equation for the dispersion in packed bed. So, for that region these assumptions are important.

So, we are assuming the packed bed is divided into several you know small cells in which dispersion or mixing is taking place. And those cells are also having the same shape like a column in which we have done the packing to get the packed beds right. So, each mixing cell being of same size and then all such cells have been assumed to be like equal size.

And then finally, irrespective of mechanism dispersion is characterized by the dispersion coefficient. So, like you know in the case of diffusion we have diffusivity or diffusion coefficients, when we are considering this dispersion then we have a dispersion coefficients.

And then diffusivity does not depend on the flow regime and then you know geometry in which the flow is taking place etcetera, it does not depend on that one. But dispersion strongly depends on the flow regime and then also depends on the flow geometry in which the flow is taking place.

For example, just said you know if you have an empty column in which fluid is flowing so then dispersion coefficients may be very small and negligible, but the same column part of the column if you packed with certain kind of materials.

So, what happens the dispersion significantly increases right so; that means, it depends on the flow geometry as well. And then; obviously, definitely it is if it is depending on a flow geometry definitely it will be depending on the flow conditions whether the low Reynolds number region or high Reynolds number region, that is you know flow of the fluid also having the effect on the dispersion.

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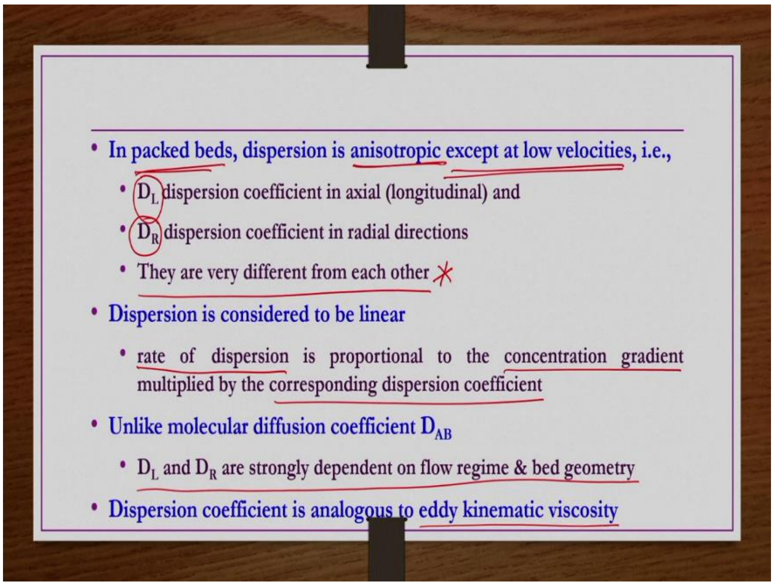

- In packed beds, dispersion is anisotropic except at low velocities, i.e.,
  - D_L dispersion coefficient in axial (longitudinal) and
  - D_R dispersion coefficient in radial directions
  - They are very different from each other *
- Dispersion is considered to be linear
  - rate of dispersion is proportional to the concentration gradient multiplied by the corresponding dispersion coefficient
- Unlike molecular diffusion coefficient D_{AB}
  - D_L and D_R are strongly dependent on flow regime & bed geometry
- Dispersion coefficient is analogous to eddy kinematic viscosity

So, in packed bed dispersion is anisotropic except at low velocities, because at low velocities usually you know we understand that molecular diffusion dominates. Whatever the mixing radial mixing that occurs in the cells that is very small you know because of the dispersion or the contribution to the dispersion. The contribution to the dispersion because of the radial mixing is very small compared to the molecular diffusion at low velocities or small Reynolds number that we know.

However, dispersion if it is not in the low Reynolds number region or low flow rate regions, it is not anisotropic it will be having different values in different directions right. So, molecular diffusion we have only one, for a given system diffusivity or diffusion coefficient we have only one irrespective of the flow region, flow regime, irrespective of the geometry we have only one.

But here in the case of you know dispersion it depends on the geometry. And then especially in the case of packed beds, what happens? You know this dispersion is anisotropic especially beyond the low Reynolds number region right. If it is low Reynolds number region in general dispersion which is very small. And then molecular diffusion is going to dominate.

So; that means, above the beyond the beyond certain low Reynolds number region, after crossing certain low Reynolds number region dispersion is going to be different in different direction. That is it is going to be different in radial direction, it is going to different in axial direction, if you are taking a cylindrical column for packing of the bed. So, that is what it means by it is anisotropic.

So, that is D_L is dispersion coefficient in axial or longitudinal direction and D_R is dispersion coefficient in radial directions ok. And these are very different from each other especially when the Reynolds number is beyond the low Reynolds number region or the flow rate is not in the low velocities region, but high velocities region right. So, they are different from each other when the velocity is more than the low velocity region are small Reynolds number region.

So, when the Reynolds number region is higher or when you go beyond the streamlines region then, what we have? We have this D_L D_R are very much different from each other. Dispersion is considered to be linear in general and then rate of dispersion is proportional to the concentration gradient multiplied by the corresponding dispersion coefficient. It is defined slightly similar way as we defined molecular diffusion right.

So, where here dispersion whatever is the rate of dispersion, rate of dispersion whatever is there that is proportional to the concentration gradient. Like in molecular diffusion the flux says a proportional to the concentration gradient, here also rate of dispersion is proportional to the concentration gradient. And the proportionality constant is known as the dispersion coefficient. It changes in the different direction, all three direction of the geometry it may be having different dispersion coefficients.

Let us say if you have a cylindrical packed bed then you know dispersion coefficient in longitudinal direction and dispersion coefficient in the radial direction are going to be different ok. So, unlike in a molecular diffusion coefficient D_{AB} as I already mentioned D_L D_R are strongly dependent on flow regime and bed geometry ok. So, dispersion coefficient is as analogous to eddy kinematic viscosity which is not same in all the direction, it varies from direction to direction.

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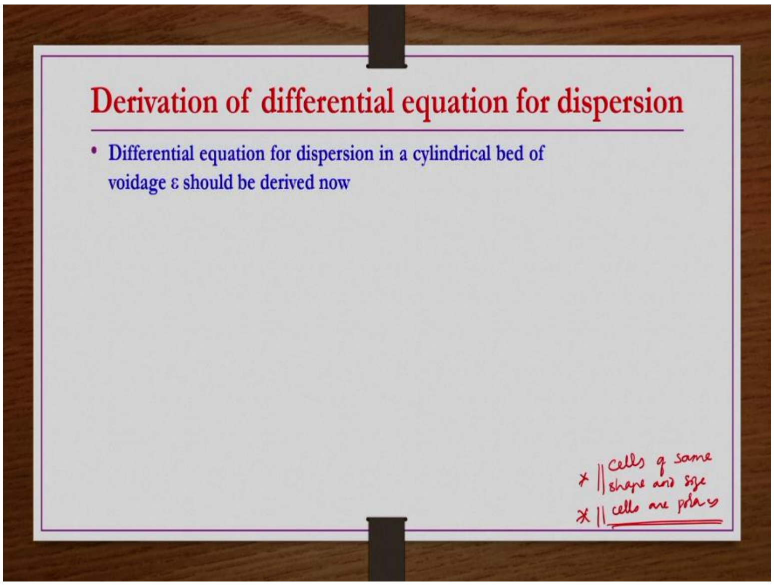

Derivation of differential equation for dispersion

- Differential equation for dispersion in a cylindrical bed of voidage ε should be derived now

* || cells of same shape and size
* || cells are [illegible]

So, now what we do? We do derivation of differential equation for dispersion. So, very first assumption in this case is that whatever the column is there, if the column is cylindrical column. Then whatever the packed bed is there that we are dividing into the small small cells which is which are also in cylindrical shape. And then all those cells are having same size ok.

So, cells of same shape and size this is one essential important assumption right. Another one is that these cells are porous, because our whatever the packed bed is there that is having you know that is a one type of porous media and then interstitial spaces are there. So, these entire bed is not completely packed some of the volume fraction is void space is there. So, because of that one you know we have to take these cells also porous.

In general, whenever we do the momentum or mass balance for the given geometry we take that walls or you know solid no slip walls kind of thing. But now this is packed bed which is like a porous media. So, then cells we have to take they are as porous right. So, that these two are the important essential assumptions in this derivation.

Other than these two after having these two and then it is simply what is the rate of you know dispersion at the inlet, what is the rate of dispersion at the outlet, what is the net rate etcetera those things we have to mathematically represent. And then do you know substitution in the overall balance equation.

So, that is all mathematical procedure right. So, now, this is these two are the very essential and then after that we can understand schematically what it what does it mean by exactly.

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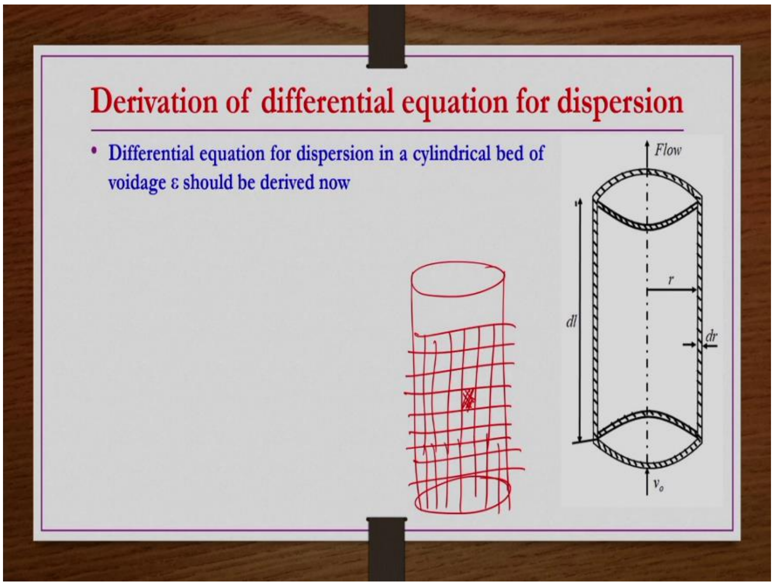

Derivation of differential equation for dispersion

- Differential equation for dispersion in a cylindrical bed of voidage ε should be derived now

So, differential equation for dispersion in a cylindrical bed of voidage ε should be derived now. So, this is the cell that I was mentioning, so we have taken porous packed bed ok or packed bed we have taken were interstitial spaces are there. And then the packed bed we have made in a cylindrical column by making some kind of packing material. So, that is the reason that packed bed whatever is there.

So, let us say this packed bed. So, now, this packed bed we have divided into several cylindrical object cylindrical cells. Because this bed itself is you know made in a kind of cylindrical column like this. So, what will happen? So, then we have to have a kind of a bed cells also cylindrical type. And then these cells you know they are also having the equal size and then same shape like a bed or like a column in which we have done you know packing. So, then we have this representation.

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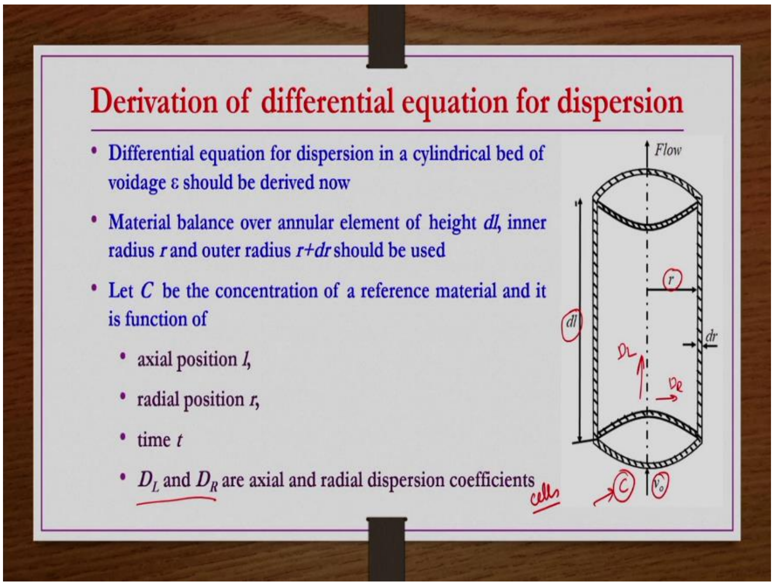

Derivation of differential equation for dispersion

- Differential equation for dispersion in a cylindrical bed of voidage ε should be derived now
- Material balance over annular element of height *dl*, inner radius *r* and outer radius *r+dr* should be used
- Let *C* be the concentration of a reference material and it is function of
  - axial position *l*,
  - radial position *r*,
  - time *t*
  - D_L and D_R are axial and radial dispersion coefficients

cells

So, now this is the representation of such cells of same size and shape and then they are porous in nature. Shape we have taken cylindrical cell, because there packing we have done in a cylindrical column right. The height of this cell is dl, the inner radius of the cell is r, and outer radius of cell is r + dr. So, that thickness of the cell we can take as dr the material the fluid is entering at v_0 velocity at the bottom and then leaving from the top ok.

The concentration of the material that is entering is C. So, how this C, variations in C are occurring because of the dispersion that is what we are going to derive mathematically right. So, material balance over annular element of height dl, inner radius r and outer radius r + dr should be used now.

Let C be the concentration of reference material and it is function of; obviously, the space, time and then dispersion coefficients right, as well as the voidage ok. So, it is a function of axial position l, radial position r that is what mean by it is function of space. And then it is function of time as well, in addition to that one it is also function of the dispersion coefficients D_L and D_R ok.

D_L is the dispersion coefficient in the axial direction, D_R is the dispersion coefficient in the radial direction. We understand that you know if the flow Reynolds, if the flow rate is higher, if you are not in the stream line flow region. Then this dispersion coefficients are anisotropic, we have more than one dispersion coefficient and then they may be different in the different directions, unlike a molecular diffusivity D_{AB}. D_{AB} same irrespective of the flow geometry and then flow regime. But here it is not like that ok.

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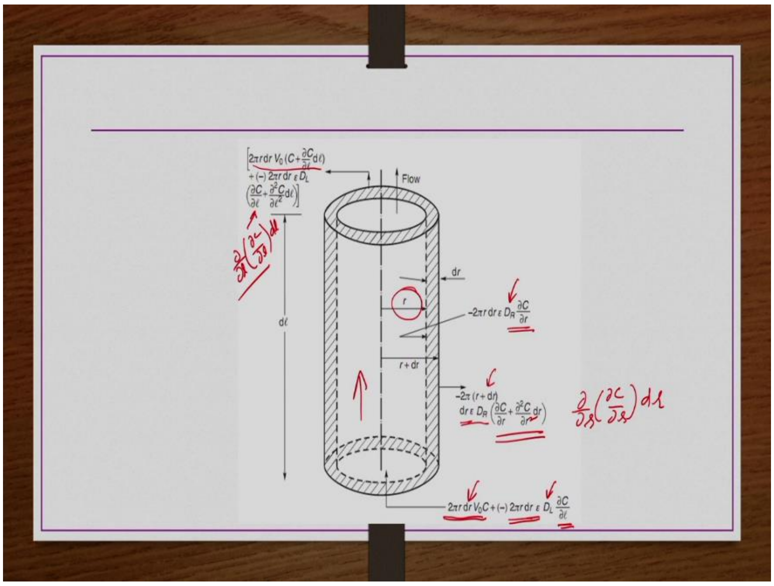

So, pictorially we write quantities that are entering here. So, the rate of material that is entering at the inlet because of the flow is this one $2\pi r\ dr\ V_0\ C$. $2\pi r\ dr$ is the cross section area of the cell through which the material is entering. And then V_0 is the velocity at which entering, C is the concentration, this is because of the motion bulk motion ok.

And then it may because the mechanism, two mechanisms are there here also, molecular diffusion mechanism or molecular mechanism as well as the bulk motion mechanism that we are having.

So, this is because of the bulk motion because of the molecular mechanism the rate at which material is entering is 2πr dr and then ε, because it is a porous. Multiplied by $D_L\frac{\partial C}{\partial l}$ actually $D_L \frac{-\partial C}{\partial l}$ that $(-)$ we have written here right ok. So, D_L, why D_L? Why not D_R, why not D_R? Why D_L here? Because it is an axial direction we are taking, axial direction components we are taking that is the region it is D_L.

So, similarly at the outlet what are the corresponding quantities they are leaving because of the convection and then because of the bulk motion and because of the diffusion. Because of bulk motion this is the component that is 2 π r dr V_0 C in C there is a change of Δ C the change in Δ C we are writing $\frac{\partial C}{\partial l} D_L$ ok.

And then similarly material leaving because of the diffusion is 2 π r dr ε dl is as it is, what we have? $\frac{\partial C}{\partial l}$ is as it is it is having the change in C is change in $\frac{\partial C}{\partial l}$ we have to write, that change in $\frac{\partial C}{\partial l}$ is nothing, but $\frac{\partial}{\partial l}\frac{\partial C}{\partial l} dl$.

So, that is $\frac{\partial^2 C}{\partial l^2} dl$ that is the addition the change in concentration at the outlet because of the diffusion is $\frac{\partial^2 C}{\partial l^2}$ that is what is this, so that we have written here right. Now this is in the axial direction; now in the radial direction there is no convection, whatever the bulk flow is there that is there only in the axial direction right.

So, bulk flow component or the rate of the material entering because of the bulk flow will not be there in the case of radial components. Because the bulk flow is there only in the axial direction, in the radial direction we have only diffusion or molecular mechanism is taking place.

So, what is the rate at which the material is entering at r is 2 π r dr ε and then because of the molecular diffusion $dr\frac{\partial C}{\partial r}$. Why dr here? Because now the now the dispersion we are considering in the radial direction.

And then the change in concentration also in the radial direction that is the region $\frac{\partial C}{\partial r}$ ok. So then, what is the rate at which it is leaving at r + dr? It should be 2 π in place of r we have to write r + dr and then d r ε dr as it is and then change in $\frac{\partial C}{\partial r}$. Change in $\frac{\partial C}{\partial r}$ is nothing, but $\frac{\partial}{\partial r}\frac{\partial C}{\partial r}dr$.

So, that is $\frac{\partial^2 C}{\partial r^2}dr$. So, that is the component that is the rate of the material that is living at r + dr because of the molecular diffusion in the radial direction ok. So, in the pictorially we have shown all the components here.

So, now, all this should be balanced by the total overall accumulation. So, inlet minus outlet in the radial direction + inlet minus outlet in the axial direction that you do, then whatever the entire all the summation is there of two components that should be equated to the overall accumulation ok. So, that is 2 π r dr dl $\frac{\partial C}{\partial t}$ ok. So, then when you do then you get the final equation. So, that we are representing here.

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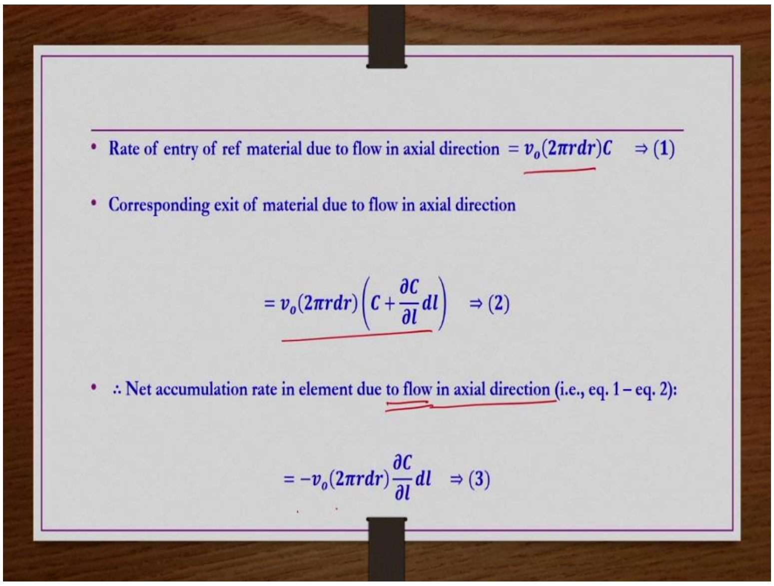

- Rate of entry of ref material due to flow in axial direction $= v_o(2\pi r dr)C \Rightarrow (1)$
- Corresponding exit of material due to flow in axial direction

$$= v_o(2\pi r dr)\left(C+\frac{\partial C}{\partial l}dl\right) \Rightarrow (2)$$

- ∴ Net accumulation rate in element due to flow in axial direction (i.e., eq. 1 – eq. 2):

$$= -v_o(2\pi r dr)\frac{\partial C}{\partial l}dl \Rightarrow (3)$$

So, rate of entry of reference material due to flow in axial direction is this one same because of the, then corresponding exit of material due to the flow in axial direction is this one that we have seen pictorial in the picture. So, net accumulation rate in element due to flow in flow in axial direction due to flow only. So, this 1 - 2 if you do this is what you get that is what we have written in picture, figure as well.

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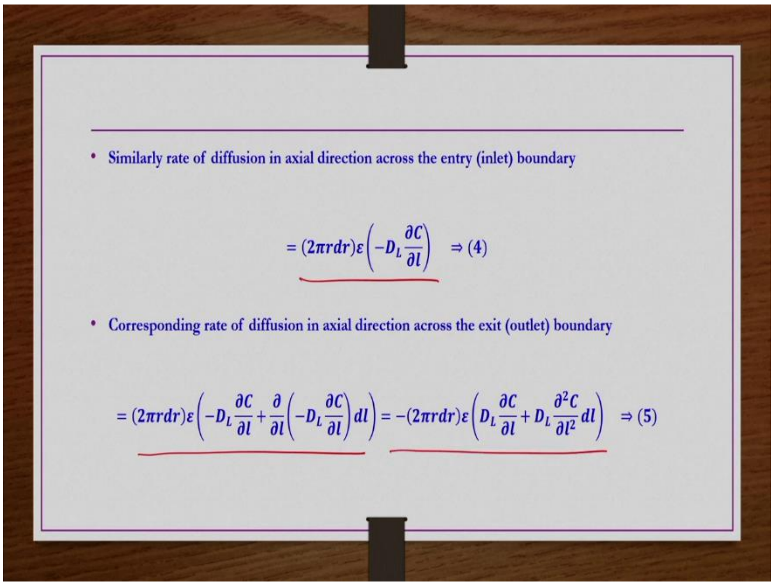

- Similarly rate of diffusion in axial direction across the entry (inlet) boundary

$$= \underline{(2\pi r dr)\varepsilon\left(-D_L \frac{\partial C}{\partial l}\right)} \quad \Rightarrow (4)$$

- Corresponding rate of diffusion in axial direction across the exit (outlet) boundary

$$= \underline{(2\pi r dr)\varepsilon\left(-D_L \frac{\partial C}{\partial l} + \frac{\partial}{\partial l}\left(-D_L \frac{\partial C}{\partial l}\right) dl\right)} = \underline{-(2\pi r dr)\varepsilon\left(D_L \frac{\partial C}{\partial l} + D_L \frac{\partial^2 C}{\partial l^2} dl\right)} \quad \Rightarrow (5)$$

Similarly, rate of diffusion in axial direction across the entry inlet this is what we have written. And then corresponding rate of diffusion in axial direction across the exit or outlet boundary is this one that is what we have written after simplifying this one we got.

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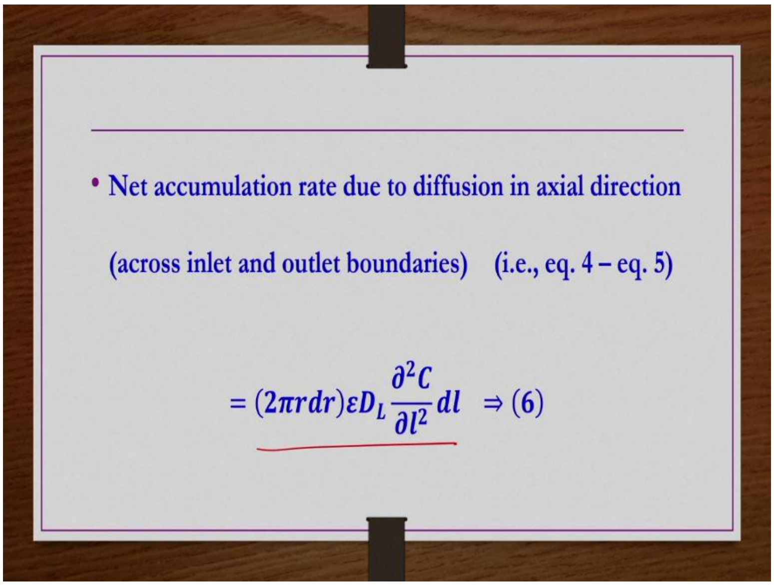

- Net accumulation rate due to diffusion in axial direction

  (across inlet and outlet boundaries) (i.e., eq. 4 – eq. 5)

$$= \underline{(2\pi r dr)\varepsilon D_L \frac{\partial^2 C}{\partial l^2} dl} \quad \Rightarrow (6)$$

Then net accumulation rate due to diffusion in axial direction should be across, that is across inlet and outlet boundaries that is equation number 4 and 5 when you do you get this equation right. All these pictorially I have shown these quantities we are again writing here.

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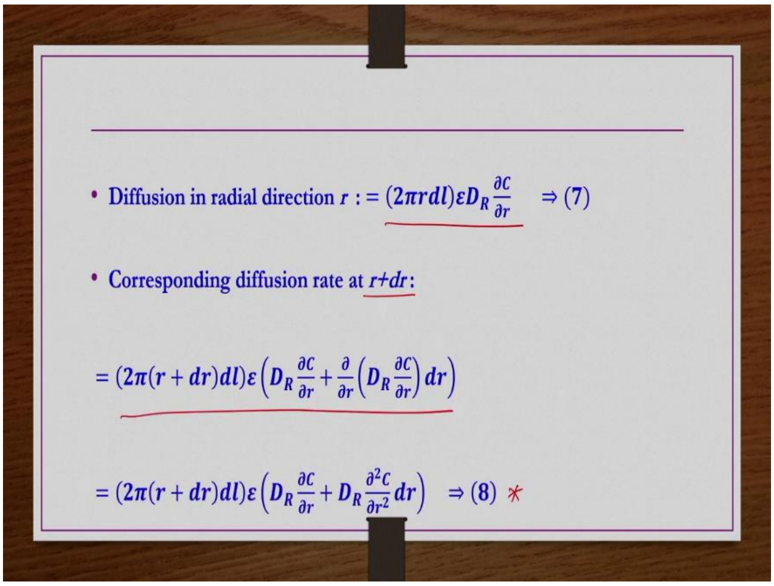

- Diffusion in radial direction $r: = \underline{(2\pi r dl)\varepsilon D_R \frac{\partial C}{\partial r}} \Rightarrow (7)$
- Corresponding diffusion rate at $\underline{r+dr}$:

$$= \underline{(2\pi(r+dr)dl)\varepsilon\left(D_R\frac{\partial C}{\partial r}+\frac{\partial}{\partial r}\left(D_R\frac{\partial C}{\partial r}\right)dr\right)}$$

$$= (2\pi(r+dr)dl)\varepsilon\left(D_R\frac{\partial C}{\partial r}+D_R\frac{\partial^2 C}{\partial r^2}dr\right) \Rightarrow (8) \; *$$

So, now diffusion in radial direction at r we have written this one. And then corresponding diffusion rate at r + dr the other side of the boundary we have written this one, after simplifying we get this equation.

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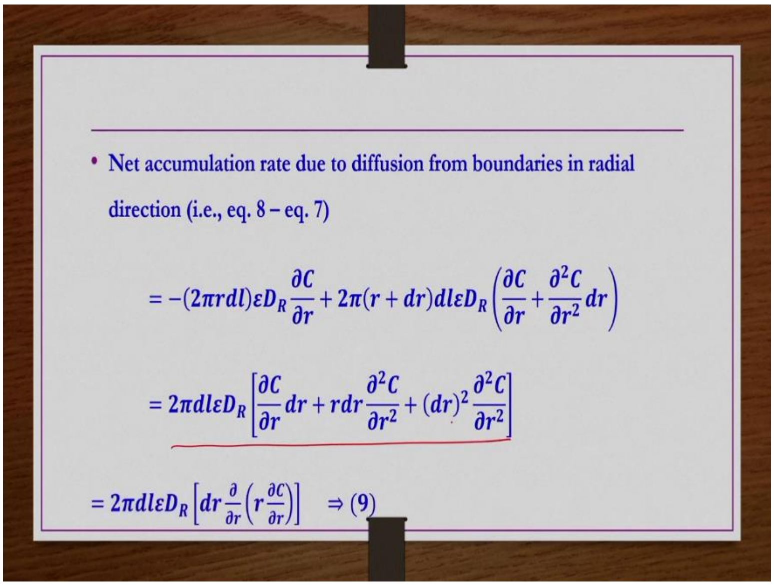

- Net accumulation rate due to diffusion from boundaries in radial direction (i.e., eq. 8 – eq. 7)

$$= -(2\pi r dl)\varepsilon D_R\frac{\partial C}{\partial r} + 2\pi(r+dr)dl\varepsilon D_R\left(\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial r^2}dr\right)$$

$$= \underline{2\pi dl\varepsilon D_R\left[\frac{\partial C}{\partial r}dr + rdr\frac{\partial^2 C}{\partial r^2} + (dr)^2\frac{\partial^2 C}{\partial r^2}\right]}$$

$$= 2\pi dl\varepsilon D_R\left[dr\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right] \Rightarrow (9)$$

So, net accumulation rate due to diffusion from boundaries in radial direction from r to r + dr is equation 8 minus equation 7 that we you do simply you get this equation right. So, now dr we are taking very small, so then this term may be cancelled out ok. And then these two terms we can combined and write like this term right.

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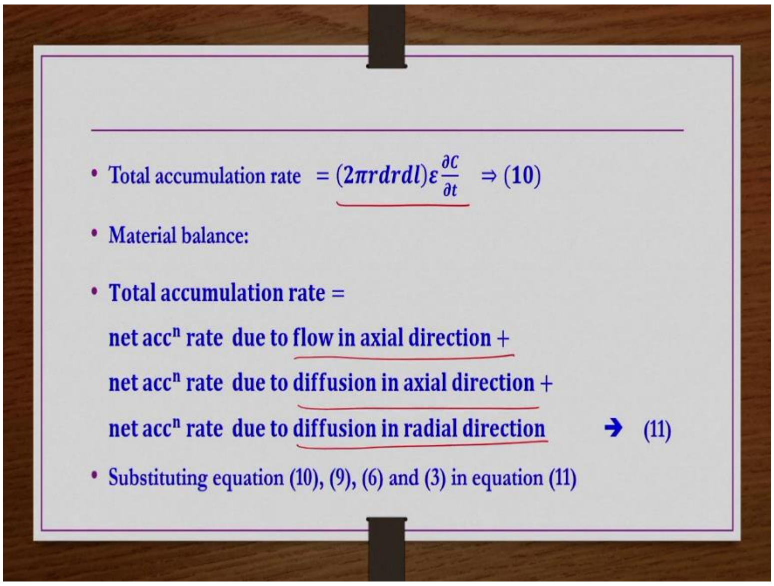

- Total accumulation rate $= (2\pi r dr dl)\varepsilon\frac{\partial C}{\partial t}$ ⇒ (10)
- Material balance:
- **Total accumulation rate =**

net acc^n rate due to flow in axial direction +

net acc^n rate due to diffusion in axial direction +

net acc^n rate due to diffusion in radial direction → (11)

- Substituting equation (10), (9), (6) and (3) in equation (11)

Total accumulation rate in the overall cell is (2 π r dr dl) ε $\frac{\partial C}{\partial t}$. Total volume of the cell multiplied by ε because whatever the transfer or of the material is taking place because of the dispersion that is taking only in the ε void space fraction only.

So, that the total volume of the cell multiplied by ε and then that multiplied by $\frac{\partial C}{\partial t}$, $\frac{\partial C}{\partial t}$ that will give total accumulation rate. So, now, this total accumulation rate should be balanced by net accumulation rate due to flow in axial direction + net accumulation rate due to diffusion in axial direction + net accumulation rate due to diffusion in radial direction that all the quantities we have written in previous slides ok.

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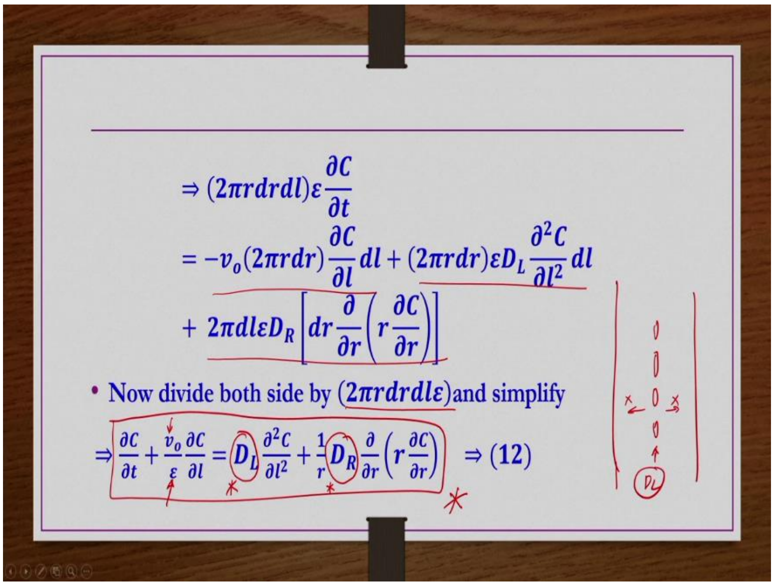

$$\Rightarrow (2\pi rdrdl)\varepsilon\frac{\partial C}{\partial t} = -v_o(2\pi rdr)\frac{\partial C}{\partial l}dl + (2\pi rdr)\varepsilon D_L\frac{\partial^2 C}{\partial l^2}dl + 2\pi dl\varepsilon D_R\left[dr\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right]$$

- Now divide both side by $(2\pi rdrdl\varepsilon)$ and simplify

$$\Rightarrow \frac{\partial C}{\partial t} + \frac{v_o}{\varepsilon}\frac{\partial C}{\partial l} = D_L\frac{\partial^2 C}{\partial l^2} + \frac{1}{r}D_R\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) \Rightarrow (12)$$

When you do this material balance (2πrdrdl)ε $\frac{\partial C}{\partial t}$ is this one. This is the net accumulation in the axial direction because of the bulk flow and then this is a net accumulation in the again axial direction but because of the molecular mechanism.

And then this is net accumulation in the radial direction because of the molecular mechanism. In the radial direction there is no bulk flow; bulk flow is dominating in only axial direction.

So, now, this entire equation you divide by (2πrdrdl) ε then you get simply $\frac{\partial C}{\partial t} + \frac{v_0}{\varepsilon}\frac{\partial C}{\partial l} = D_L\frac{\partial^2 C}{\partial l^2} + \frac{1}{r}D_R\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)$ this is what you get. And this the, this equation is nothing, but change in concentration of reference material in a packed bed because of the dispersion.

It is similar like you know our species concentration equation that we have derived the for a mass transfer part ok. So, but only thing that here the whatever the change in concentration are there that because of the dispersion and then bulk motion only we are taking here taking into the consideration.

So, now, here at what velocity you are allowing the material to flow that in general you know. You also know what is the ε voidage of a bed that you have made for your real life situation, right.

So, if you wanted to find out the change in concentration with respect to the space n time that is C as function of r, l and t then what you should have? You should also have the information of D_L and D_R then only you can use this equation in order to know the change in concentration because of the dispersion.

Otherwise, this equation is as useless as nothing has been done. So, the importance of this equation depends on if how effectively you going to measure this D_L and D_R for your system in which you are going to apply this equation ok.

So, that we see now this is in general done let us say if you wanted to find out you know D_L dispersion coefficient in the axial direction. So, you take a tracer element and then you release in the axial direction only like this without it any shape variations in the radial direction.

Shape variations of this tracer element should not be there in the radial direction, it should go in the axial direction such a way that such a way you have to measure this. You have to release this tracer element and then measure the change in shape you know along the axial direction as well as the change in shape with respect to the time.

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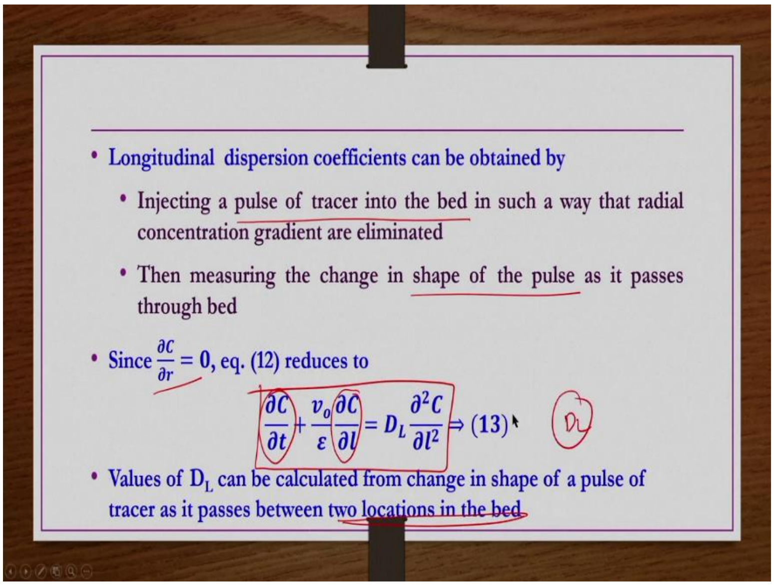

- Longitudinal dispersion coefficients can be obtained by
  - Injecting a pulse of tracer into the bed in such a way that radial concentration gradient are eliminated
  - Then measuring the change in shape of the pulse as it passes through bed
- Since $\frac{\partial C}{\partial r} = 0$, eq. (12) reduces to

$$\frac{\partial C}{\partial t} + \frac{v_o}{\varepsilon}\frac{\partial C}{\partial l} = D_L \frac{\partial^2 C}{\partial l^2} \Rightarrow (13) \quad D_L$$

- Values of D_L can be calculated from change in shape of a pulse of tracer as it passes between two locations in the bed

So, longitudinal dispersion coefficient if you wanted to obtain inject a pulse of tracer into the bed in such a way that radial concentration gradient are eliminated, these are the pulse of tracer. So, then the concentration gradient in the sense that shape in change in shapes, the shapes you know there should not be change in shape in the radial direction, whatever the change in shape is there that should be there only in the axial direction.

That means, only concentration gradient should be there only in the axial direction such a way you have to inject. So, then measuring the change in shape of the pulse as it passes through the bed you can find out you can measure that one. So; that means, $\frac{\partial C}{\partial r} = 0$ in equation 12 previous equation so; that means $\frac{\partial C}{\partial t} + \frac{v_0}{\varepsilon}\frac{\partial C}{\partial l} = D_L\frac{\partial^2 C}{\partial l^2}$.

So, C you are measuring that is what measuring the change in shape of the pulse as it passes through the bed that; that means, $\frac{\partial C}{\partial l}$ you are measuring. And then you can get the $\frac{\partial^2 C}{\partial l^2}$ as well if you have $\frac{\partial C}{\partial l}$ right.

So, then similarly $\frac{\partial C}{\partial t}$ also with respect to time you can measure. So, then D_L you can find out because v_0 ε are known ok. So, values of D_L can be calculated from change in shape of pulse tracer at as it passes between two locations in the bed ok. So, then you can get dl by solving this above equation 13 right.

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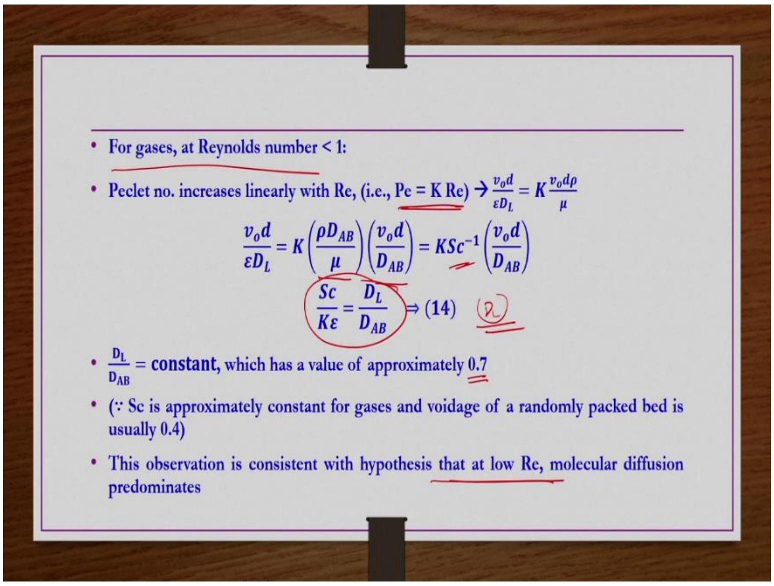

- For gases, at Reynolds number < 1:
- Peclet no. increases linearly with Re, (i.e., Pe = K Re) → $\frac{v_o d}{\varepsilon D_L} = K\frac{v_o d\rho}{\mu}$

$$\frac{v_o d}{\varepsilon D_L} = K\left(\frac{\rho D_{AB}}{\mu}\right)\left(\frac{v_o d}{D_{AB}}\right) = KSc^{-1}\left(\frac{v_o d}{D_{AB}}\right)$$

$$\boxed{\frac{Sc}{K\varepsilon} = \frac{D_L}{D_{AB}}} \Rightarrow (14) \quad ②$$

- $\frac{D_L}{D_{AB}}$ = **constant,** which has a value of approximately 0.7
- (∵ Sc is approximately constant for gases and voidage of a randomly packed bed is usually 0.4)
- This observation is consistent with hypothesis that at low Re, molecular diffusion predominates

So, for gases in general its small Reynolds numbers. Peclet number increases linearly with Reynolds number so that Pe = K Re, K is the proportionality constants for the time being ok.

So, usually this is the reality from the experimental results when the Reynolds number is very small for gases Pe increases proportionally with Reynolds number. So, this is the relation Pe = K Re. So, Pe for the packed bed is nothing, but $\frac{v_0 d}{\varepsilon D_L} = KRe$ is nothing, but $v_0 \frac{d\rho}{\mu}$.

So, now right side you divide by D_{AB} and multiply by D_{AB} and then rearrange, so that $\rho \frac{D_{AB}}{\mu}$ and then $v_0 \frac{d}{D_{AB}}$ you have two terms. This $\frac{\mu}{\rho D_{AB}}$ is nothing, but Schmidt number. So, then $\rho \frac{D_{AB}}{\mu}$ we can write Schmidt number inverse right.

So, from this equation Schmidt number divided by K ε = $\frac{D_L}{D_{AB}}$. So; that means, for gases system usually Schmidt number is known because μ, ρ, D_{AB} etcetera are known. From experimental results this K is known in general, for any gases system D_{AB} is known or you can measure right. So, then from this equation you can find out what is D_L especially, for gases at small Reynolds number regions right.

So, $\frac{D_L}{D_{AB}}$ is in general constant which has a value of approximately 0.7. So, Schmidt number is approximately constant for gases in general it does not vary much from points you know it is within 0.1, 0.5 to 0.7, 0.8 region in that region only.

And then voidage of randomly packed bed is also usually 0.4 or between 0.4 & 0.5. So; obviously, this $\frac{D_L}{D_{AB}}$ from this equation 14 that is also going to be constant. And then for most of the gases at low Reynolds number this $\frac{D_L}{D_{AB}}$ is found to be 0.7.

So; that means, for any gases system if you know D_{AB}, D_L you can find out by multiplying D_{AB} by 0.7. And then it is consistent with the assumption, what is the assumption? That its small Reynolds number molecular diffusivity dominates compared to the radial mixing etcetera or dispersion. So; that means, if D_L is 0.7 times the D_{AB}; that means, D_{AB} is having higher contribution compared to the D_L.

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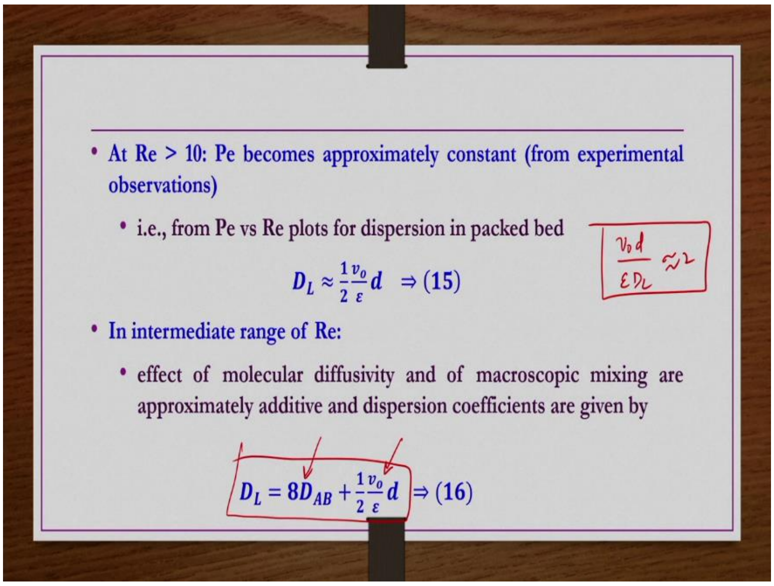

- At Re > 10: Pe becomes approximately constant (from experimental observations)
  - i.e., from Pe vs Re plots for dispersion in packed bed

$$D_L \approx \frac{1}{2}\frac{v_0}{\varepsilon} d \quad \Rightarrow (15)$$

$$\boxed{\frac{v_0 d}{\varepsilon D_L} \approx 2}$$

- In intermediate range of Re:
  - effect of molecular diffusivity and of macroscopic mixing are approximately additive and dispersion coefficients are given by

$$\boxed{D_L = 8D_{AB} + \frac{1}{2}\frac{v_0}{\varepsilon} d} \Rightarrow (16)$$

At high Reynolds number; obviously, in packed beds as we have seen the critical Reynolds number for transition from laminar to turbulent flow is in general 5 to 10. So, beyond 10 it is consider as the high Reynolds number or turbulent flow region in the packed beds.

So, at high Reynolds number Re greater than to 10, Pe becomes approximately constant as per the mania experimental observation especially for the gases. So, from Pe versus Re plots for dispersion in packed beds we can have this you know whatever the $\frac{v_0 d}{\varepsilon D_L}$ is approximately 2 that Peclet number is approximately 2 at high Reynolds number that is constant in general. So, from here we can get $D_L = \frac{1}{2}\frac{v_0 d}{\varepsilon}$ right.

So, you do not even need to know the diffusivity if the Reynolds number is high in order to know the D_L. If in order to know the D_L at high Reynolds number high Reynolds number region you do not need anything including the diffusivity is also not required. Only thing that you need to know what is the velocity at which you are releasing the fluid material, what is the voidage and then what is the size of the particle that you have used for the packing.

In the intermediate range of Re people found these two on these two contributions at low Reynolds number and then high Reynolds number whatever the contributions are there they are additive. And then people have written $D_L = 8\ D_{AB} + \frac{1}{2}\frac{v_0}{\varepsilon} d$So, this is the contribution because of the high Reynolds number, this is the contribution because of the low Reynolds number and then intermediate range these two are you know added together.

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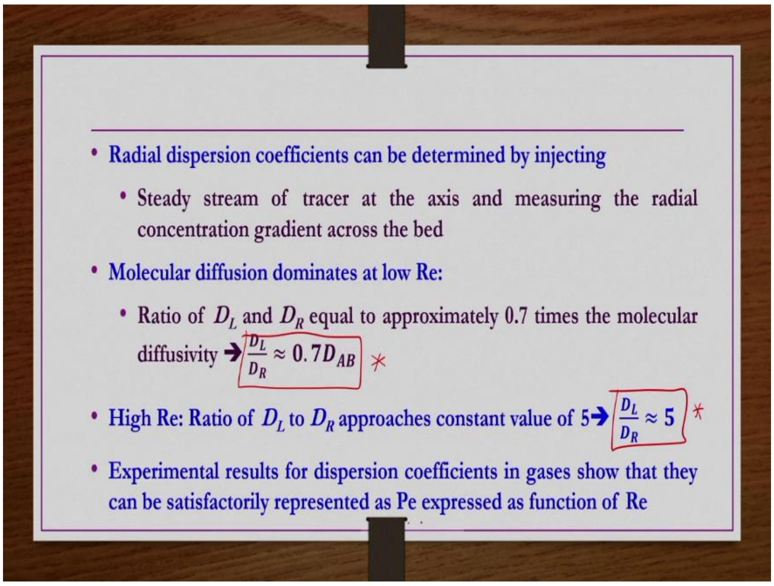

- Radial dispersion coefficients can be determined by injecting
  - Steady stream of tracer at the axis and measuring the radial concentration gradient across the bed
- Molecular diffusion dominates at low Re:
  - Ratio of D_L and D_R equal to approximately 0.7 times the molecular diffusivity → $\boxed{\frac{D_L}{D_R} \approx 0.7 D_{AB}}$ *
- High Re: Ratio of D_L to D_R approaches constant value of 5 → $\boxed{\frac{D_L}{D_R} \approx 5}$ *
- Experimental results for dispersion coefficients in gases show that they can be satisfactorily represented as Pe expressed as function of Re

So, similarly radial dispersion coefficients if you wanted to obtain what you have to do? You have to do inject you have to inject the steady stream of tracer at the axis and measure the radial concentration gradient across the bed. Now, at certain middle portion at the axis at the central axis, what you have to do?

You have to release this same tracer pulse of tracer, but how do you need to release? You have to release such a way that the change in shape should be there only in the radial direction it should not be there in the axial direction.

So, that you know $\frac{\partial c}{\partial l}$ etcetera can be taken 0 and then you can find out what is D_R from the dispersion coefficient equation that we have derived. So, we know we understand that molecular diffusion dominates at low Re. So, ratio of D_L and D_R equal to approximately 0.7 times the molecular diffusivity in general if you combined both, the previous expressions we have done only considering D_L.

Now, you have to considering both D_L and D_R it is found that $\frac{D_L}{D_R}$ is approximately 0.7 times the D_{AB} from the experimental results. At high Reynolds number people found this ratio

$\frac{D_L}{D_R}$ is approximately 5. So, in the case of high Reynolds number region the ratio is constant whereas, in the case of low Reynolds number region the ratio is not constant.

It is some constant multiplied by the diffusivity of the system that is 0.7 D_{AB} right. Experimental results for dispersion coefficients in gases show that they can be satisfactorily represented as Peclet number expressed as function of Re.

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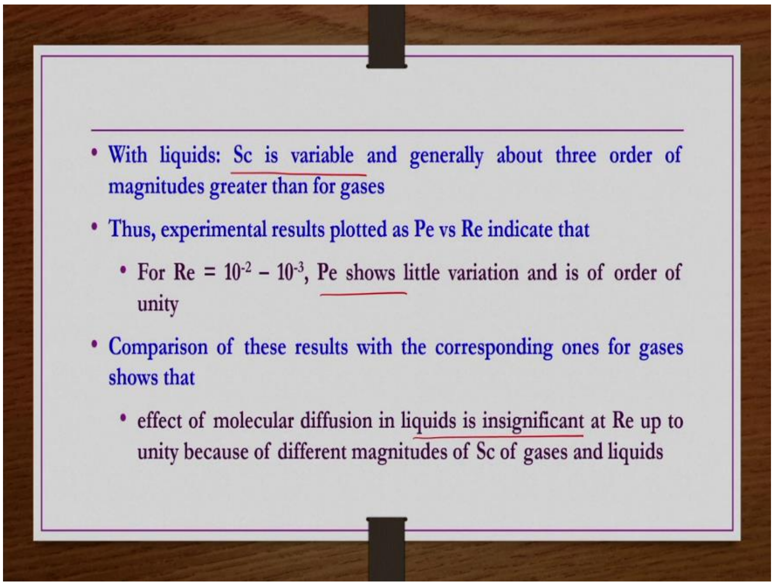

- With liquids: Sc is variable and generally about three order of magnitudes greater than for gases
- Thus, experimental results plotted as Pe vs Re indicate that
    - For Re = $10^{-2} - 10^{-3}$, Pe shows little variation and is of order of unity
- Comparison of these results with the corresponding ones for gases shows that
    - effect of molecular diffusion in liquids is insignificant at Re up to unity because of different magnitudes of Sc of gases and liquids

But in the liquid case in the case, whatever that whatever previous slides we have seen that is for the gases. In the case of liquids usually Schmidt number varies because the viscosity, density of the liquids are you know from one liquid to the other liquid variations may be large right. For the gases variations in the viscosity and the density values are very small from one gas to the other gas. That is the reason usually Schmidt number range for gases is very narrow, so then we can take the constant.

But in the case of liquids this ρ D_{AB} μ etcetera you know they are not they are changing by higher magnitude from one liquid to the other liquid. So, because of that one this Schmidt number is in general variable and generally about three order of magnitudes greater than that for gases.

For the liquid case when the experimental results are plotted Peclet number versus Reynolds number that indicated that its small Reynolds number, small in the sense Re less than or equals to order of 10 power minus 2 or 10 power minus 3 such small Reynolds number Peclet number does not show any variation, does not show any variation or only little variation and it is order of unity.

Comparison of these results with the corresponding ones for the gases shows that effect of molecular diffusion in liquids is insignificant at Re up to order of 10 power minus 2, 10 power minus 3 or less than Re is equals to 1.

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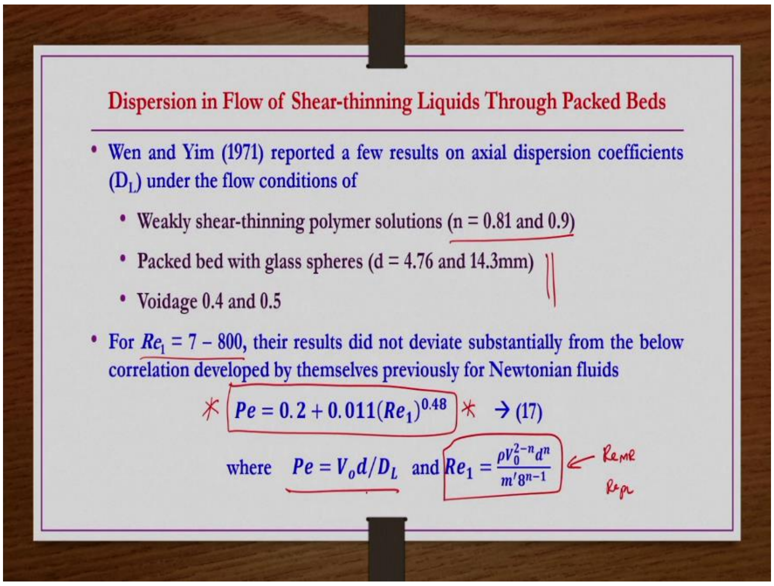

Dispersion in Flow of Shear-thinning Liquids Through Packed Beds

- Wen and Yim (1971) reported a few results on axial dispersion coefficients (D_L) under the flow conditions of
  - Weakly shear-thinning polymer solutions (n = 0.81 and 0.9)
  - Packed bed with glass spheres (d = 4.76 and 14.3mm) ||
  - Voidage 0.4 and 0.5
- For $Re_1 = 7 - 800$, their results did not deviate substantially from the below correlation developed by themselves previously for Newtonian fluids

$$* \boxed{Pe = 0.2 + 0.011(Re_1)^{0.48}} * \rightarrow (17)$$

where $\underline{Pe = V_o d/D_L}$ and $\boxed{Re_1 = \frac{\rho V_0^{2-n} d^n}{m' 8^{n-1}}} \leftarrow Re_{MR}$

Re_{PL}

Now, finally, before ending this lecture what we do little literature information for dispersion in flow of shear thinning liquids through packed beds. So, Wen and Yim reported few results on axial dispersion coefficient D_L under the following flow conditions weekly shear thinning polymer solutions, packed bed with a glass spheres of such and diameter and then such diameters, voidage of 0.4 and 0.5. And then experiments they conducted in the Reynolds number range 7 to 800 right they are all experimental results we are discussing now.

Because we wanted to see the effect of non-Newtonian fluids in the dispersion in packed beds ok. So, in these people they have developed this correlation for the case of Newtonian fluids say the voidage diameter of particles used for packing keeping same. They have taken Newtonian fluids and then have done the experiments in order to find out the dispersion coefficients etcetera. So, they found this correlation is suitable right.

However, when they change the fluid to weekly shear thinning fluids they found the same correlation is still valid, but with Reynolds number defined like Re_{MR} this is similar like Re_{MR} or Re_{PL} that we have seen previously. And then Peclet number is $V_0 \frac{d}{D_L}$ they have not taken ε in the definition of a Peclet number or Reynolds number.

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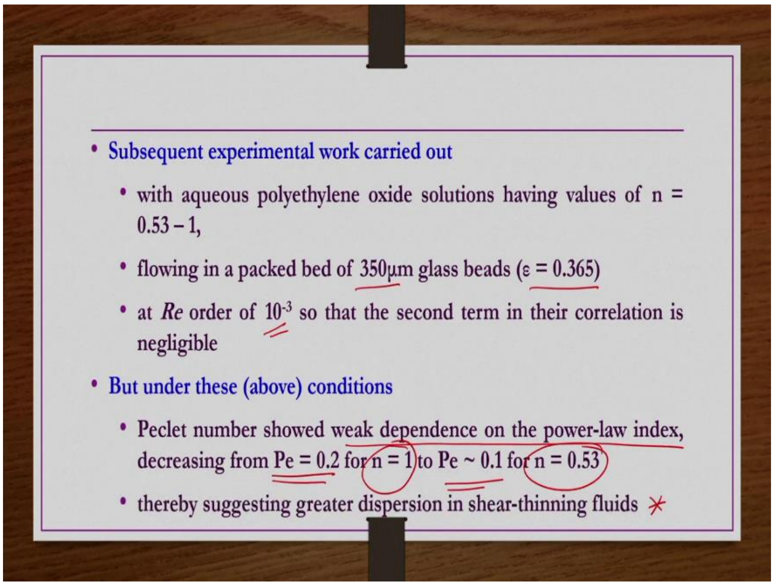

- Subsequent experimental work carried out
  - with aqueous polyethylene oxide solutions having values of n = 0.53 – 1,
  - flowing in a packed bed of 350μm glass beads (ε = 0.365)
  - at *Re* order of 10^{-3} so that the second term in their correlation is negligible
- But under these (above) conditions
  - Peclet number showed weak dependence on the power-law index, decreasing from Pe = 0.2 for n = 1 to Pe ~ 0.1 for n = 0.53
  - thereby suggesting greater dispersion in shear-thinning fluids

Subsequent experimental work carried out with wider range of n between 0.53 & 1. And then, but smaller voidage range and then very small particles of a glass beads, but at Reynolds number order of 10^{-3}. So, that the second term in their correlation is negligible, so that is what they found right.

So, in this correlation later on they have done the experiments by taking very small values of Reynolds number. If the Reynolds number is small, so Reynolds power 0.48 and then that is again multiplied by 0.011. So, if Reynolds number is small, so then; obviously, Peclet number is going to be constant for the case of a small Reynolds number case that also they have done.

But under these conditions Peclet number showed weak dependence on the power law index. That is Peclet number is decreasing from 0.2 to 0.1 when n is decreasing from 1 to 0.53 right. Thereby, suggesting greater dispersion in shear thinning fluids ok.

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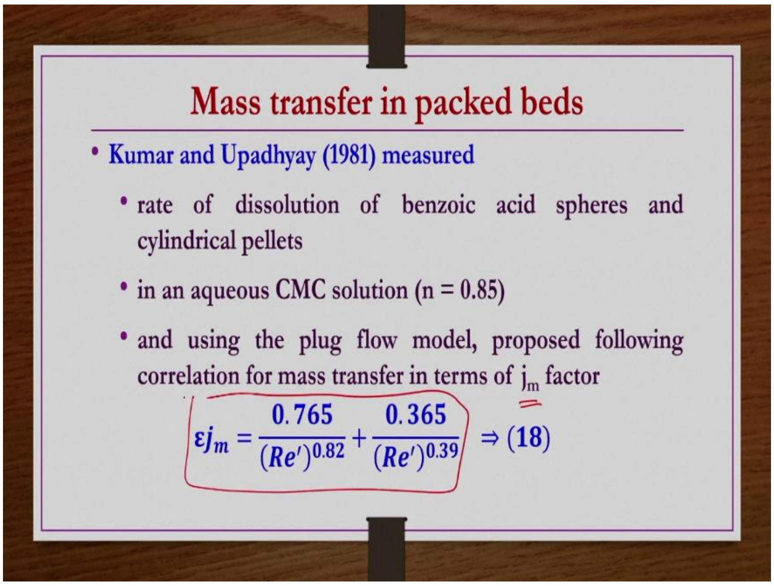

Mass transfer in packed beds

- Kumar and Upadhyay (1981) measured
  - rate of dissolution of benzoic acid spheres and cylindrical pellets
  - in an aqueous CMC solution (n = 0.85)
  - and using the plug flow model, proposed following correlation for mass transfer in terms of j_m factor

$$\boxed{\varepsilon j_m = \frac{0.765}{(Re')^{0.82}} + \frac{0.365}{(Re')^{0.39}}} \Rightarrow (18)$$

So, finally, mass transfer in packed beds Kumar and Upadhyay measured rate of dissolution of benzoic acid spheres and cylindrical pellets. So, they have taken benzoic acid the particles in two shapes spherical and cylindrical shapes.

And then they have done the experiments in CMC solution, dissolution or dispersion of these pellets in CMC solutions having n = 0.85. And then they have used to plug flow model and proposed a correlation for ε correlation for j_m factor and then that is given by this equation ok.

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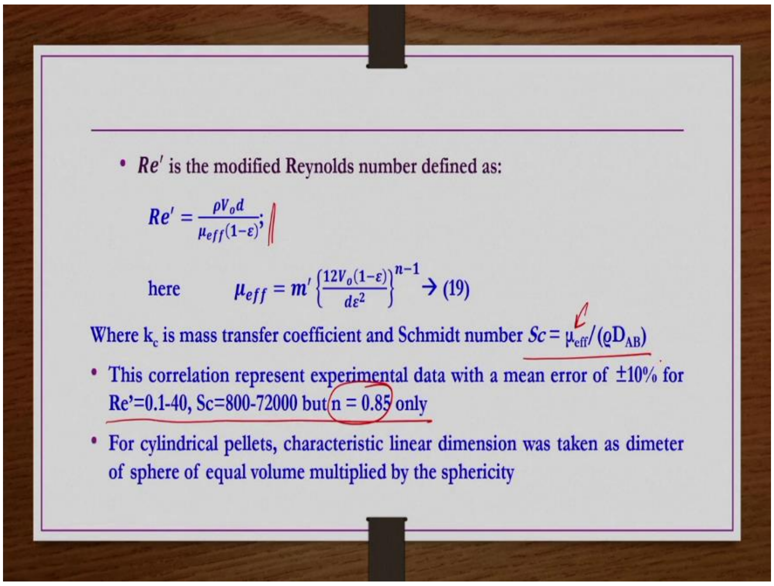

- Re' is the modified Reynolds number defined as:

$$Re' = \frac{\rho V_0 d}{\mu_{eff}(1-\varepsilon)};$$

here $$\mu_{eff} = m'\left\{\frac{12V_0(1-\varepsilon)}{d\varepsilon^2}\right\}^{n-1} \rightarrow (19)$$

Where k_c is mass transfer coefficient and Schmidt number $Sc = \mu_{eff}/(\varrho D_{AB})$

- This correlation represent experimental data with a mean error of ±10% for Re'=0.1-40, Sc=800-72000 but n = 0.85 only
- For cylindrical pellets, characteristic linear dimension was taken as dimeter of sphere of equal volume multiplied by the sphericity

Where Re’ is the modified Reynolds number that is defined as this one. Same like a packed bed case yesterday that we have seen where $\mu_{eff} = m' \left\{\frac{12V_0(1-\varepsilon)}{d\varepsilon^2}\right\}^{n-1}$.

This definition is same like yesterday’s class whatever we have studied for the frictional pressure drop calculations by Kumar and Upadhyaya. Here k c is mass transfer coefficient and Schmidt number is defined as a $\frac{\mu_{eff}}{\rho D_{AB}}$. Effective viscosity they have taken in order to define the Schmidt number that have been used for the j_m correlation.

And the range of condition of their experiments are given here they have taken only one n value. And they found that their correlation having within + or − 10 percent of mean error.

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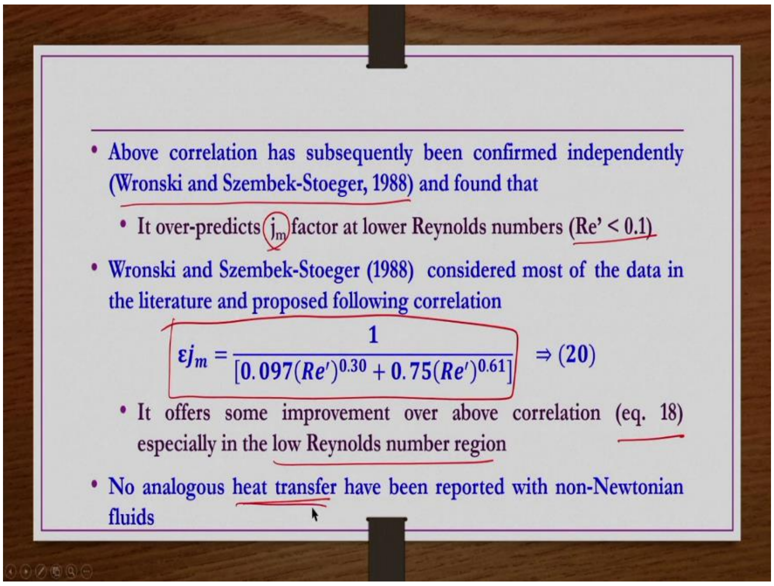

- Above correlation has subsequently been confirmed independently (Wronski and Szembek-Stoeger, 1988) and found that
  - It over-predicts j_m factor at lower Reynolds numbers (Re’ < 0.1)
- Wronski and Szembek-Stoeger (1988) considered most of the data in the literature and proposed following correlation

$$\varepsilon j_m = \frac{1}{[0.097(Re')^{0.30} + 0.75(Re')^{0.61}]} \Rightarrow (20)$$

  - It offers some improvement over above correlation (eq. 18) especially in the low Reynolds number region
- No analogous heat transfer have been reported with non-Newtonian fluids

So, above correlation has subsequently been confirmed independently by Wronski and Stoeger and they found that j_m factor over predicts at lower Reynolds number regions ok. And these people considered most of the data in the literature and proposed the following modified correlation for the j_m factor which is different from the Kumar and Upadhyaya.

It offers more improvement over the correlation of Kumar and Upadhyay that we have shown in the previous slides especially, in the low Reynolds number region. However, no analogous heat transfer results have been found in the literature especially, when you use non-Newtonian fluids flowing through packed beds.

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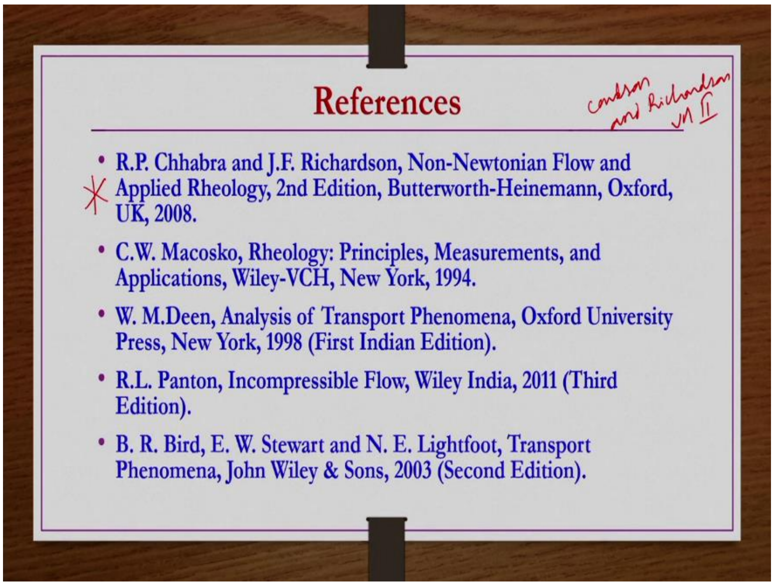

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References the entire lecture is prepared from this book Chhabra and Richardson. However, the derivation of the equation that we have done that has been taken from Carlson and Richardson book volume II, other useful references are provided here.

Thank you.