Transport Phenomena of Non-Newtonian Fluids
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Lecture - 22
Flow of Non -Newtonian Fluids through Packed Beds

Welcome to the MOOCS course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Flow of Non-Newtonian Fluids through Packed Beds.

Till now what we have seen, we have seen different geometries where single phase flow taking place like geometries, like you know infinitely long cylinders or infinite parallel plates, inclined surfaces, concentric cylinders etcetera; those kind of geometries we have taken. Now, in this lecture and then coming couple of lectures we will be taking a geometry where, you know multiple particles are you know packed as a kind of bed or maybe those particles under fluidized conditions and then how to get the required frictional pressure drop for such kind of situations.

So, that is what we are going to see in this and then coming couple of lectures.

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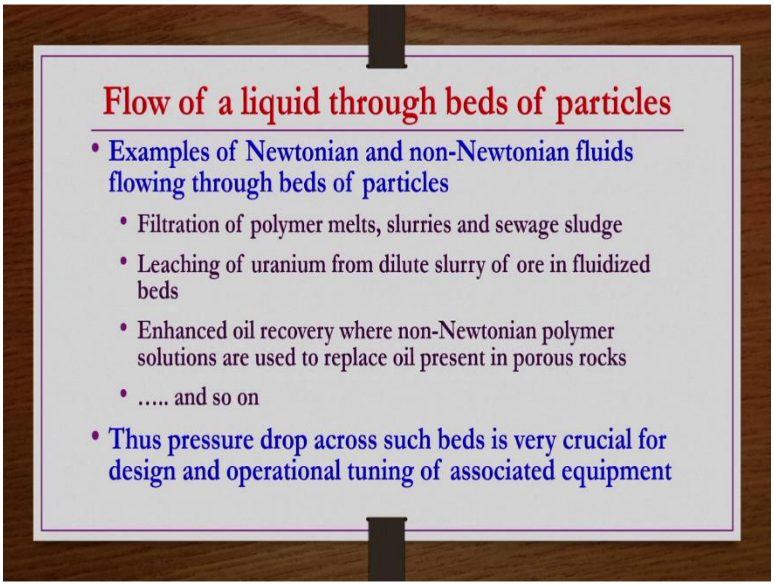

Before going into the details of pressure drop calculations for the case of flow of a liquid whether Newtonian or non-Newtonian flowing through beds of particles; what we do? We, will be having a kind of a list of applications where we can have such kind of applications, alright.

Examples of Newtonian and non-Newtonian fluids flowing through beds of particles. As chemical engineers we must be aware that you know in any of the chemical industries or almost all chemical industries we may be having a situations where, we may be doing kind of filtration. Filtration of slurries and then getting clear filtrate etcetera in general that is possible.

So, that may also be taken as a kind of a flow through you know bed of particles kind of thing. Like that you know sometimes polymer melts, slurries, sewage sludge etcetera are also being filtrated using this packed beds or bed of particles.

Then leaching of uranium from dilute slurry of ore in fluidized beds another example. Then enhanced oil recovery where non-Newtonian polymer solutions are used to replace oil present in porous rocks. In general what happens in you know petroleum rocks lot of oil is filled in the porous structure. So, in the primary and secondary extraction process or recovery of the oil, so most of the oil is taken out from the oil rocks, right.

But however, oil is very important energy resource, so then we cannot leave even a small amount or traces of you know oil in the rocks. So, then for that you know there is a tertiary method where, tertiary method which is also known as enhanced oil recovery method.

So, what we do in that such conditions where most of the oil is recovered, so only small amount is there that we cannot separate from separate by using primary recovery method. So, then we allow some liquids sometimes Newtonian, but most of the times non-Newtonian polymer solutions are allowed to replace the oil that is present in the porous structure.

So, because this solutions are higher density compared to the density of the oil that is a crude oil that is present inside the rock. So, then these polymer solutions that replaces the oil present in the rocks. So, that is another examples that is one more another example for the case where the flow of a non-Newtonian fluids flowing through beds of particles.

So, like that if you keep on listing there may be n number of applications especially associated with the chemical engineering applications we may find. So, having these many applications obviously it becomes very much essential to find out the pressure drop, because wherever there is a flow it is it becomes very essential to have a relation volumetric flow rate versus pressure drop or equations for the pressure drop calculations or friction pressure drop etcetera are very much essential.

So, that is what we have been doing for so many you know geometries, right. So, because that pressure drop is coming into the picture in velocity distribution or you know volumetric flow rate etcetera all those things so that is indirectly we are having relations. If you have the velocity you can get the other kind of thing, if you have known fixed value of a volumetric flow rate how much pressure drop you need to apply all those things we can calculate.

So, such kind of information is also required for the case of you know flow through packed beds or bed of particles especially when the fluid is non-Newtonian, right. So, pressure drop across such beds is very crucial for both design point of view as well as the operational tuning point of view of the associated equipment, where these where any of the systems are having this bed of particles kind of system, right.

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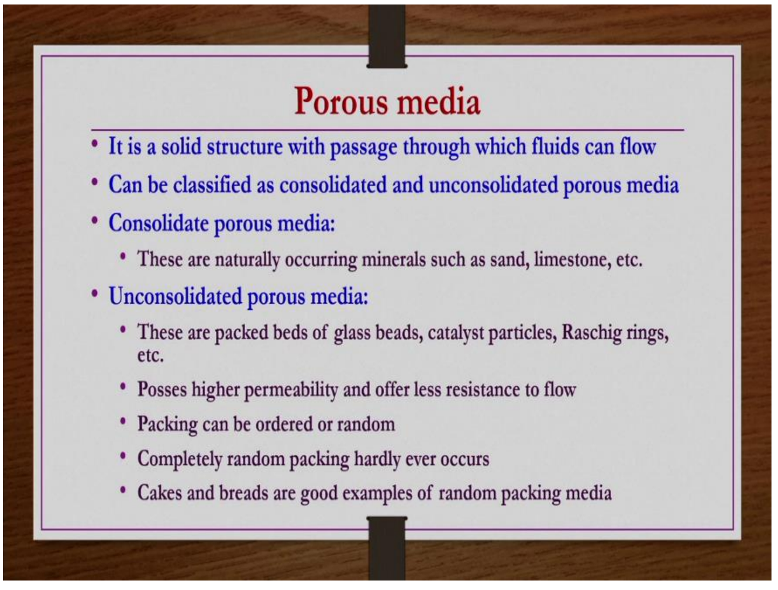

So now, we understand this bed of particle. So, porous media is the one immediately that comes to the mind, so then it is it becomes very essential to understand what this porous media is. So, though all these details basic details we know from our UG level transport phenomena or fluid mechanics course. However, it is important to have a kind of a recapitulation of these basic concepts before going into the frictional pressure drop calculations for the flow through packed beds.

Porous media is a solid structure with passage through which fluids can flow, alright. And then it can be classified as consolidated and unconsolidated porous media, because the this classification is one of the classification. Not necessarily that this is the ultimate classification, there may be classification based on the other properties also right.

So, consolidated porous media are in general naturally occurring minerals such as sand, limestone, etcetera, so these materials are often called as consolidated porous media. Whereas, the unconsolidated porous media in general what we have, we have like you know heat transfer taking place through a bed of particles right or mass transfer is taking place through bed of particles or a catalytic reaction is taking place in a packed bed reactor where the bed is packed with catalyst or sand etcetera.

So, all that we do in general for our chemical engineering applications. So there, so such kind of beds or the porous media whatever are there they are known as the unconsolidated porous media. These are packed beds of glass beads, catalyst particles, Raschig rings, etcetera those kind of materials you know; broken glasses whatever different types of packed packing materials are available.

So, this packing material are in general you know placed in a column. We have a column in that column some portion of the column is packed with these kind of materials so that you have a packed bed. And these packed beds are very much important in many of the transport phenomena problems.

Whether it is momentum transfer or mass transfer or heat transfer, not only in the transport phenomena without reaction, but with reaction also there are many cases. So, such kind of self made, such kind of beds that we make for our applications point of view are known as the unconsolidated porous media.

And then this possesses high permeability. So obviously, this unconsolidated porous media whatever we have, they have a high higher permeability. If the permeability is higher obviously, resistance to the flow would be low. Then this packing can be ordered or random, but rarely we find completely random packing.

However, some examples of a completely random packing are you know cakes and breads; cake material or bread material that we breads we in general consume we see. So, they are also having porous structures. So then, so that those whatever the porous structure is there so that you know it is very random and then inter-connection between one pore to the other pore is very very complex.

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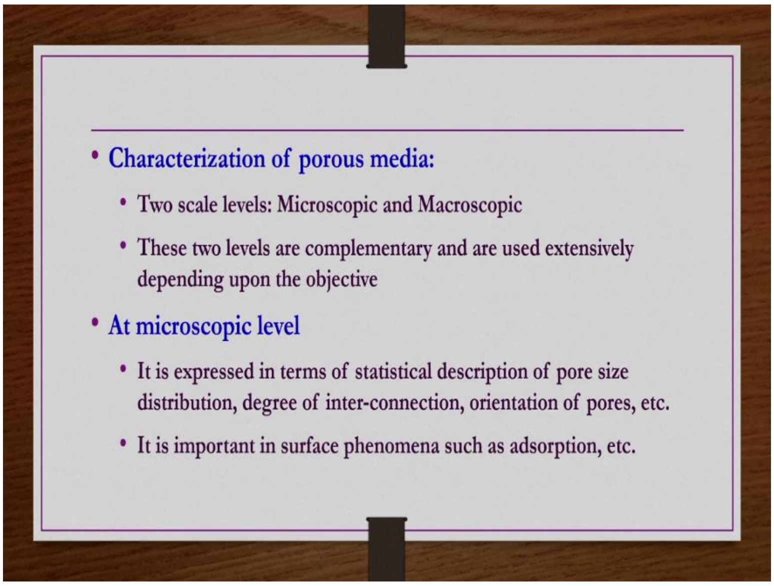

So, how to characterize this porous material? Obviously, when you have a material if you go into the engineering aspects first one of the thing what we do? We characterize the material, we find the properties of the material etcetera. So now, here this in this case we have to characterize the media.

So, how do we do that one? So, there are two possible ways are there. So, that depends on the scale at which are you using this porous media, if you are using this porous media at microscopic levels so then certain different characterizations are there. If you are using the same porous media at macroscopic level, like you know packed beds are you know fluid catalytic reactors etcetera so then we have different characteristics, right.

So, based on the scales at which we are using this porous media this can be characterized macroscopic level as well as the microscopic level. The characterization at these two scales in general are complementary to each other. We cannot say one is superior to the other one, like that it is not possible to say. Sometimes you know both of them are very useful in general or both the characterization may be used, but majority of the times they are characterized separately for different applications, right.

So, they are complementary to each other and they are extensively used depending upon the objective, right. So, objective in the sense let us say you have a column packed with activated charcoal right, and then you wanted to separate out the color from a color solution. So, color solution you pass through that activated charcoal, so then what happened that activated charcoal will adsorb the will adsorb the color that is present in the solution. And then almost like clear or colorless solution you can get as a kind of filtrate from the other end. That is one application.

So now, here the entire bed if you take you; entire bed if you take then you have to see the characteristics at the macroscopic level. But, if you take individual particles, if you take one single particle and then see how much it is adsorbing and all that those calculations if you have to do, then you have to do the microscopic characteristics or these or characterization of such individual particles at microscopic level. Those things we have to see.

So, at microscopic level we in generally what we do; we find statistical description of pore size, pore volume, degree of inter-connection, orientation of pores, surface area, like how many metres square per gram etcetera those things that we get from BET equipment. So, all those things comes under you know microscopic level, even the isotherms etcetera that we get. So, all these things comes under the microscopic level characterization of porous media. So, one example is like adsorption as just mentioned, right.

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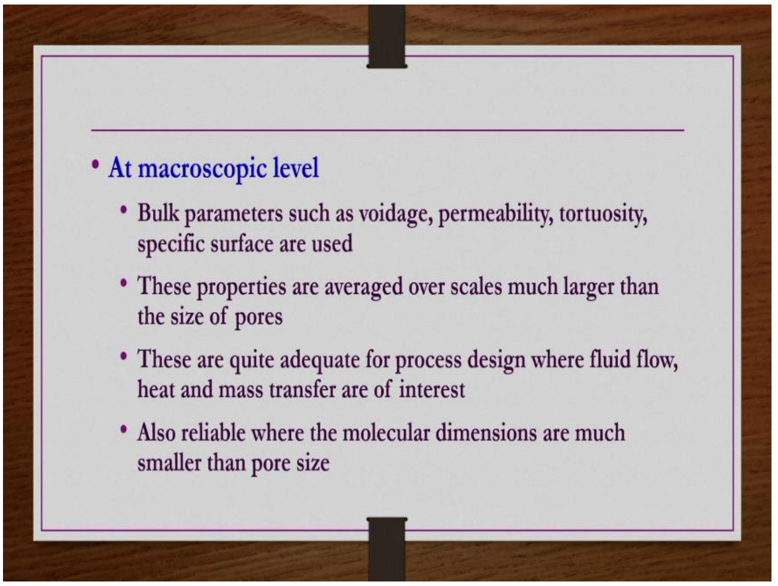

Other level is at macroscopic level. At macroscopic level it is very essential to do proper characterization of the porous material, because most of the transport phenomena problem that exist, in chemical engineering you know we use this macroscopic level characterization.

So, how do we do? Bulk parameters such as voidage, permeability, tortuosity, specific surface are used, right. So, those things are very essential right. So, and these properties are averaged over scales much larger than the size of pores, ok. And these quantities are adequate for the process design where the fluid flow, heat and mass transfer are of interest, ok.

So, these are the things that one has to be aware about the macroscopic level. And then these characteristics at macroscopic level are also reliable where the molecular dimensions are much smaller than the pore size. As I have mentioned like you know, if you take individual charge particle it is a microscopic level, but if you take the same charge particle but n number of particles you make as a bed, so then you know you can have a kind of a macroscopic level this thing. So, whether this macroscopic level is also reliable when the; where the molecular dimensions are much smaller than the pore size as well.

So now, this particular lecture what we are going to do we are going to obtain frictional pressure drop for a packed bed at using the characteristics of the bed at macroscopic level. So, what are those characteristics?

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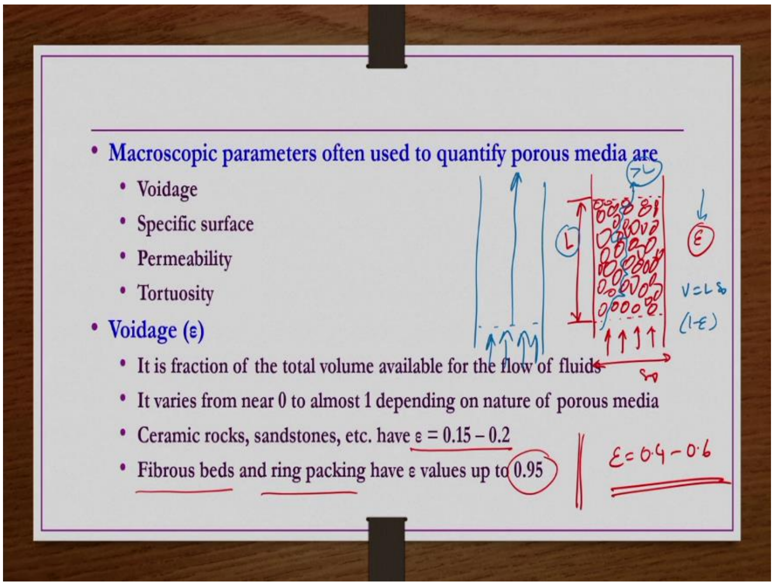

Just now we have mentioned the voidage, then specific surface, then permeability, then tortuosity. So, all these things are known to us. However, we have a kind of recapitulation.

Let us say you have a vertical column right, and there is a perforated plate at the bottom. And then we are packing this material with certain kind of particles be glass, beads or broken glasses or you know raschig rings, regular or irregular particles whatever possible we are having. So, we are packing them, right. And then at the top again we have a perforated plate ok.

So, that particles should not move out and then this bed is compactly packed in order to that one. So now, the fluid is passing through here, some momentum transfer is taking place or heat transfer mass transfer or whatever the reaction is taking place. So, let us not worry about those details.

So now, what happens? So, voidages if you wanted to understand, so then this height of the bed let us say L and then this cross section of the column is S_0 let us say. So, L S_0 would be the volume of the bed, right. Out of this volume of the bed only ε fraction is available as a free space, so that this particles these fluid particles to flow out.

So, out of this total volume $V = L S_0$ only some fraction is available that is nothing but this interstitial spaces between the particle whatever is so, that is only available. And then particle goes away. How much is that one? That is we call voidage. That is the volume free volume fraction out of the bed that is available for the fluid to flow through, right that we call voidage ε and then whatever these solid volume fraction is there that we call it 1 - ε.

So, that is void fraction is nothing but free volume that is available for the fluid to flow through divided by total volume of the bed. The total volume is L S_0 which is having two fractions; one is the volume fraction of the solid particles, another one is the void space or the free space of the volume fraction of the free space ok.

So, that free space whatever is that that we call it as voidage. And then these particles like if you have only column simple column alright, there is no packing so then it is possible that whatever the fluid element is coming it may pass through straight forward without any deviations, right.

So, but now here you have so many particles in between, so resistance is there from one particle to other particles so then fluid particle cannot go straight. So, the distance that any given fluid particle is travelling would be more than the L; would be more than the L because of this tortuous parts and then that that is characterized by the tortuosity right.

Specific surface is that is total surface area of the bed per unit volume. So, that is what you know another important factor. Permeability, how much you know volume; what is the velocity at what velocity per unit volume of that you know bed it is passing through those things we have to see. So, all these things are known to us, however we have a kind of a recapitulation here. So now, we see one-by-one.

So, voidage is ε, it is the fraction of the total volume available for the flow of fluids. It varies from nearly 0 to almost 1 depending on nature of porous media. If it is close to 1, that means the bed is very sparsely a packed and then lot of volume is available for the fluid to pass through. If it is close to 0 that bed is very compactly packed that there is no or very little space available for the fluid element to pass through. So, then under such conditions you know frictional pressure drop is going to be very high ok.

So, either of the extremes are possible. But, in general ceramic, rocks, sandstones, etcetera such kind of consolidated porous materials have the ε range 0.15 to 0.2 that is possible. So but, fibrous beds and then ring packing may have the ε value even up to 0.95. So, it is not like that you know only by a certain range of ε in general we have, but it is possible close to 0 it is possible close to 1 as well, right.

But however, most of the chemical engineering applications ε whatever that we have would be varying between 0.4 & 0.6 or maybe in between 0.45 & 0.55 something like that.

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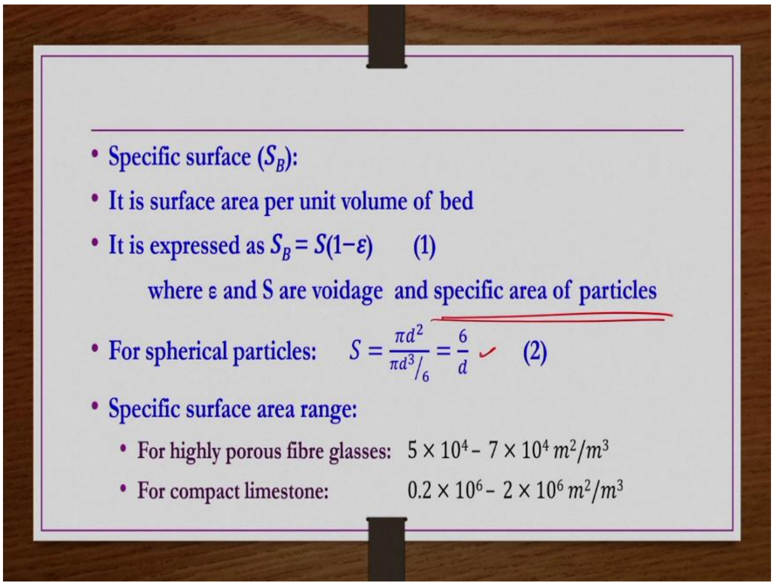

Specific surface: It is the surface area per unit volume of the bed. And then it is expressed S (1 – ε) right, because 1 - ε is nothing but the volume fraction of the particles. Now, we are getting the surface area per unit volume of the bed. So, then whatever the volume fraction of solids is there that should be multiplied by the specific area of the particle, so that to get the specific surface of the entire bed ok.

So, if you have this specific area of particles that depends on the nature of the particles; which type of particles are you using for the packing. If it is spherical particles then you know area of the particle is πd^2 divided by volume of the particles spherical particle is $\pi d^3/6$; 6/d is the specific area of the particles spherical particles.

If you have an irregular particle, so then it will be different. There are different ways of getting these things also that is also possible. We have seen in mechanical unit operations how to find out the specific area of a particles if the packing is or the pack or the particles are irregular, non-structured particles ok. So, we cannot go into all those this all those details we can you can go through those details.

Then specific surface area range, it depends. Again, it is not necessary that you know it may be very small or very large in general for highly porous fibrous material it may be order of 10^4. But, for compact limestone's it may be order of 10^6 m^2 per m^3.

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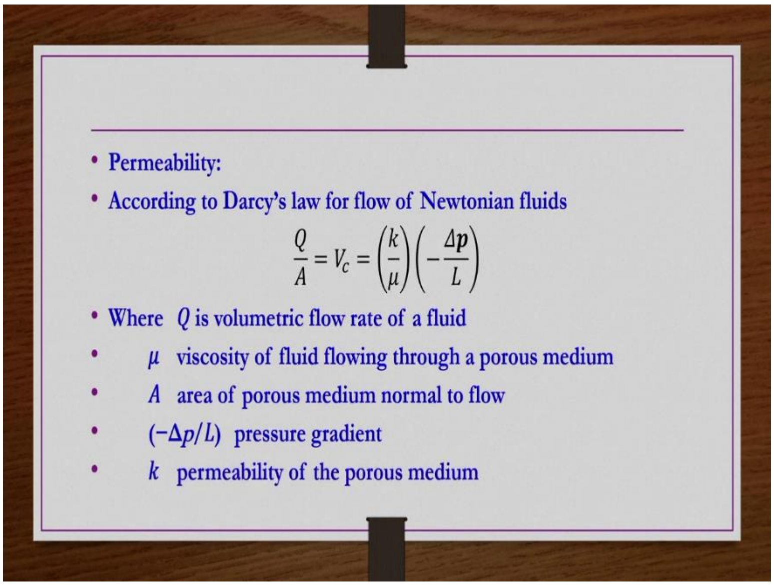

Then permeability: According to Darcy's law for a flow of a Newtonian fluids the velocity of the fluid that is flowing through or the volumetric flow rate per area of the bed through which the fluid is flowing is proportional to the pressure gradient and inversely proportional to the viscosity. Whatever the proportionality constant k is there that is known as the permeability ok.

So, Q is volumetric flow rate of fluid, μ is viscosity of fluid flowing through the porous media, and then area A is the area of porous media normal to the flow right, and then $\left(\frac{-\Delta p}{L}\right)$ is pressure gradient ok. So, then next is the k, k is the permeability of porous media.

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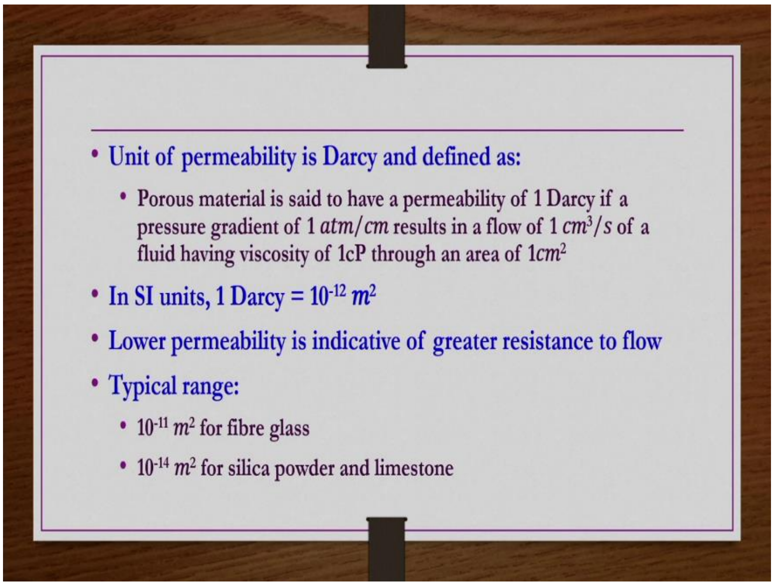

Units for the permeability is Darcy and then it is defined as Porous material is said to have a permeability of 1 Darcy if a pressure gradient of 1 atmosphere per centimeter results in a flow of 1 metre cube per second of a fluid having a fluid viscosity 1 centipoise through an area of 1 centimeter square.

So, that if you convert in metre square 1 Darcy you will be getting 10^{-12} metre square. Lower the permeability indicates the higher is the resistance. Obviously, it is inversely proportional to the resistance or it is otherwise. If the permeability is lower, then resistance would be high. If the permeability is higher, then resistance would be low for the flow to occur.

Typical range of permeability is order of 10^{-11} metre square for fiber glasses order of 10^{-14} metre square for silica powder and limestone etcetera.

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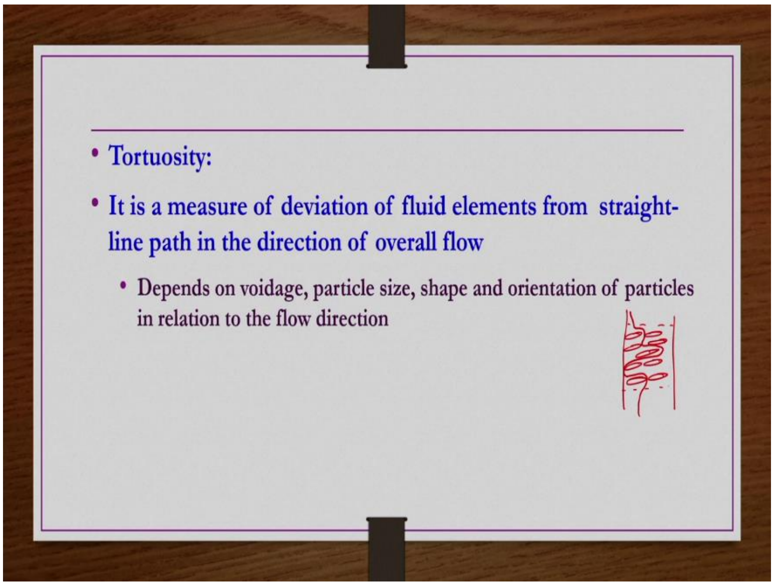

Tortuosity it is a measure of deviation of fluid elements from straight line path in the direction of overall flow, as I explained through the picture. If the column is not having any packing, so the fluid element that is entering and then leaving would be traveling the length of the L which is the length of the column. But, part of the column is packed with particles, so then fluid element would be traveling more than the column length right, so or height of the column because of the tortuous path that is to pass through, ok. It cannot go through straight because so many particles are hindering in between.

And then obviously, this tortuosity should be depending on the voidage, particle size, shape, and orientation of particles in relation to the flow direction. What do you mean by in relation to the flow directions? Let us say this is the column that you have, right. So, and then you are packing the material. If you have a needle like particles like this for packing, right.

So, then this fluid element that is entering that has to pass through different you know higher distance compared to you know spherical particles, right. So, that is the reason the particle orientation of particles relation to the flow direction is also essential.

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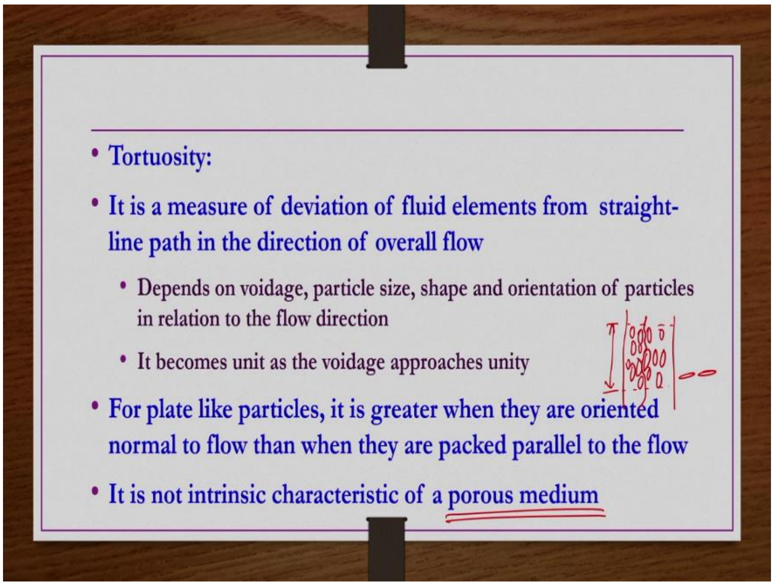

Let us say if the same way same needle like particles. If you packing like this parallel to the flow direction like this the same needle like particle, so then the distance that particle has to transfer it will be still higher than the height of the column, but it will be less than the case where these particles are oriented in the normal direction like this, ok.

It becomes unit as the voidage approaches unity. And then for a plate like particles it is greater when they are oriented normal to the flow than when they are oriented or packed parallel to the flow, ok. Just as explained here. And it is not intrinsic characteristics of porous medium, ok. Voidage specific surface are the intrinsic characteristics of you know packed bed, but tortuosity is not.

So now, that was the recapitulation of a characterization or characteristics how to obtain the characteristics of a packed bed at a macroscopic level through voidage, specific surface, tortuosity, permeability etcetera those things we have seen. So now, what we do for the same cases; for the same packed bed what we do? We try to obtain the pressure drop in flow through packed beds.

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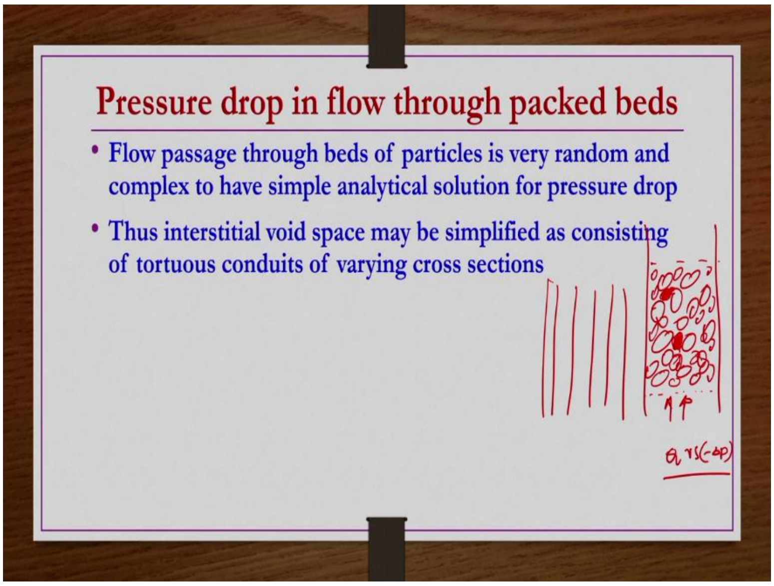

Flow passage through beds of particles is very random and complex to have simple analytical solution for a pressure drop. If you have a simple pipe you know we already have done the analysis, how to get you know pressure drop, velocity profile or volumetric flow rate velocity versus pressure drop calculations etcetera all those things we have already seen.

Now, we have particles right. So, the packing is like this. So, much of packing is there right, because of this packing what happens? The flow is not uniform or not same from one location to the other location even. So, that is fully developed flow or like simple streamline flow or laminar flow kind of thing; in the entire column is not possible.

Let us say in the packing if it is like this, so it may happen that in some portion it may be having in this region maybe it may be having like you know laminar region or low velocity region, whereas, in this region it may be having a very high velocity turbulent flow kind of thing. So, the flow is very complicated, you cannot have you cannot generalized as a kind of laminar flow, transition flow or turbulent flow like that.

Within the bed certain region one region you may be having streamline flow, another region you may not have streamline flow you may be having turbulent flow all that is possible. And then because these complications now how do you generalize. The problem with this one you know generalizing this one; how do you generalize in order to have a simple Q versus $(-\Delta P)$ relation as we have done for the flow through simple pipe or infinite parallel plates or inclined surfaces or concentric cylinder etcetera that is not possible here. Because of this random and then complex packing that we are having in the bed, and then because of that packing the flow is very complicated.

So, analytical solutions for pressure drop are not possible in general, but however we make some kind of assumptions or engineering simplifications in order to get a certain analytical solution for these cases as well. But, there may be some constant, those constants we may be fixing based on the experimental results. How to do all that? We are going to do anyway.

So, thus interstitial void space may be simplified as consisting of tortuous conduits of varying cross sections. So, what you do now this bed is there it is having certain you know void space it. So, right at certain given $(-\Delta P)$ values it may be having certain velocity. So now what you do? You can have a different conduits; conduits are you know of a different size like this, right.

So, that and then you join them together. So, that whatever the characteristics like you know here that we are having for the packed bed especially in terms of the voidages and then specific surface etcetera. So, the same you make a kind of you know n number of conduits so that those n number of conduits when joined together you will be having a kind of a voidage and specific surface same as the initial bed.

So, that way we do and then we make a simplification to get the required pressure drop versus volumetric flow rate relations. So, what we do thus interstitial void space may be simplified as consisting of tortuous conduits of varying cross sections.

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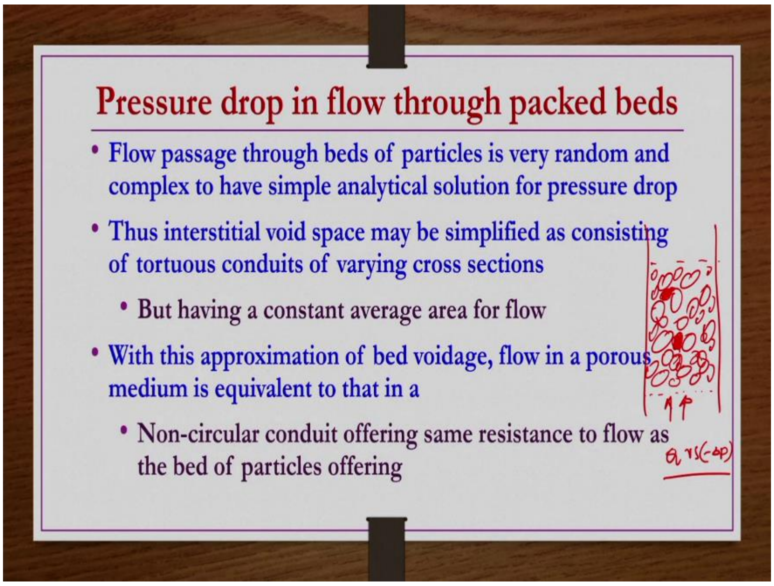

But, having a constant average area for flow which is as same as the case of you know real packed bed. With this approximation of bed voidage flow in porous media is equivalent to that in non-circular conduit offering same resistance to flow as a bed of particles offering.

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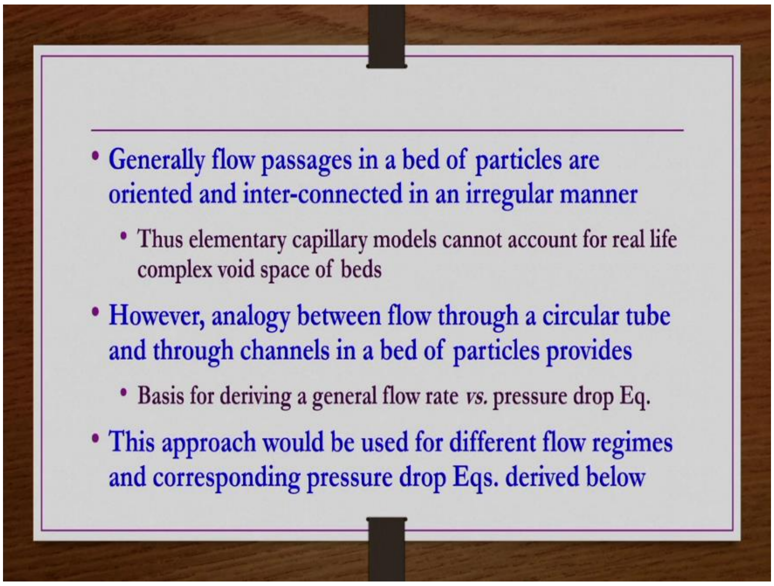

So, generally flow passages in a bed of particles are oriented or interconnected in an irregular manner that we know. Thus elementary capillary models cannot account for real life complex void space of the beds in general.

So however, analogy between flow through circular tube and through channels in a bed of particles provides, basis for deriving general flow rate versus pressure drop equations.

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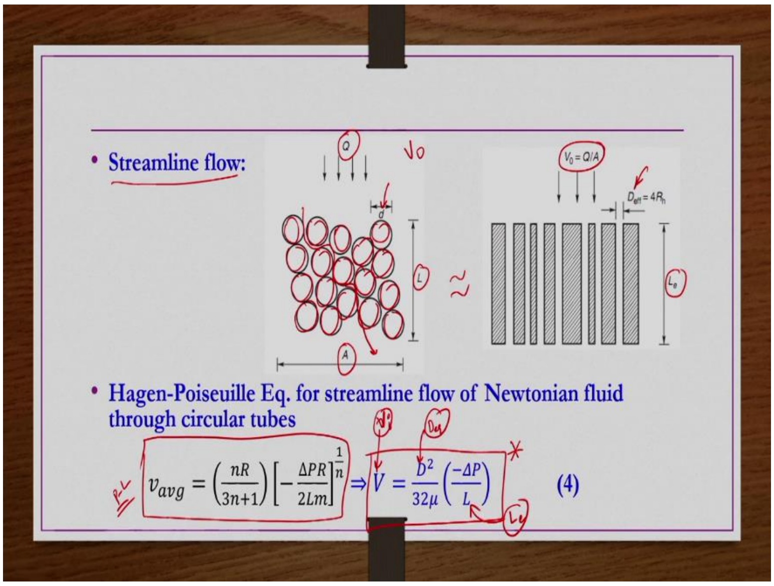

So, that we are going to do now here. How do we do?

So now, this is the case you know real life packed bed where the bed is packed with you know spherical particles like this. In real life problem we have a packed bed which is packed with the spherical particles like this, ok. So, the diameter of these particles here is d, the height of the packing is L the cross section area of the column or the bed through which the fluid is flowing is A, and then the fluid is flowing at a volumetric flow rate Q. So, which may be corresponding to some average velocity V_0 = Q/A.

So now, what we do? We replicate this system to have n number of conduits non- circular conduits like this ok. So, that here also we get average velocity V_0 = Q/A same as packed bed, pressure drop would also be same thing. And then this L is going to be L_e because L is the fixed one but fluid particles we have to take with respect to the fluid particles. Fluid particle may be traveling more than the L value as I mentioned because of the tortuosity. So, that is the reason we have L_e here.

And then whatever the void space because of this non-circular conduits that we are having that you take a D effective, right. So, because for this case let us say if these columns conduits if you have a kind of circular pipe; n number of circular pipe, so then if the flow is let us say in this one if you have the stream line flow in the packed bed, if you have stream line flow. So, such kind of streamline flow if you have using a capillary or if you having a circular tube, so then for circular tube streamline flow is taking place so then we know the Hagen-Poiseuille equation right.

So, that Hagen-Poiseuille equation now we use here whatever the Hagen-Poiseuille equation is there and then whatever the L, d or V etcetera that are there in Hagen-Poiseuille equation they will be replaced by the realistic true L, d, V of the packed bed. So, that kind of analysis we do, right.

Hagen-Poiseuille equation for a flow of Newtonian fluid through circular pipes is nothing but $\left(\frac{-\Delta p}{L}\right) = \frac{32\mu V}{D^2}$ this is what we know. The same thing for the power-law fluids we have derived in one of the previous lecture week 3 or 4 this is what we have derived. In this equation if you substitute n is equals to 1 you get this Newtonian case right.

So, in this case what you do now? In place of D you find out D equivalent right, which gives the same average velocity like in packed bed and then this V you replace by the velocity that is there in the interstitial spaces; in the interstitial spaces what is the average velocity. So, let us say V_i that we call interstitial spaces that you replace. And then this L you replace by effective length that is the particle is flowing through.

And then you substitute here, so then you get you know required a pressure drop relation for the streamline flow in packed bed that is what you can get by this analysis or by this analogy between packed beds and then n number of non-circular conduits or circular conduits whatever you take, ok. So, that is what we are going to do.

So now, the question is that what is this V_i, what is this D equivalent, what is this L_e? So, those things only we have to find out and then after that the problem is simply mathematical.

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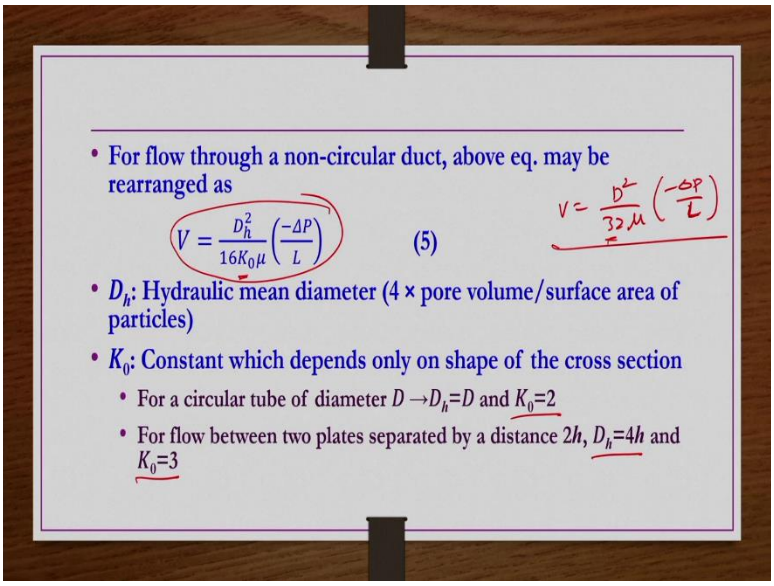

For flow through a non-circular duct above equation may be rearranged as this one ok. The previous equation whatever $V = \frac{D^2}{32\mu}\left(\frac{-\Delta p}{L}\right)$ was there. This is for the circular column, tubes kind of thing you know Hagen-Poiseuille equation. But, the same equation if you derive for non-circular conduits you get this one. Here D_h is nothing but hydraulic radius and then this constant K_0 depends on the shape; depends on the shape ok.

So, D_h is hydraulic mean diameter which is nothing but 4 multiplied by pore volume per surface area of particles. And then K_0 constant which depends only on shape of the cross section, right. Which shape you are taking, if you are taking circular cross section then K_0 should be 2, so that here you get 32 like this case ok.

So, that is for circular tube of diameter D = D_h K_0 should be 2. Likewise for flow between two plates separated by a distance 2 h D_h should be 4 h and K_0 should be 3. So, this flow between two plates that already we have derived in a couple of lectures before. So, then there also we have the average velocity expressions that equation you rearrange in this kind of form. So, then you can get that; when you rearrange that equation in the form of this one so then you get K_0 = 3, ok.

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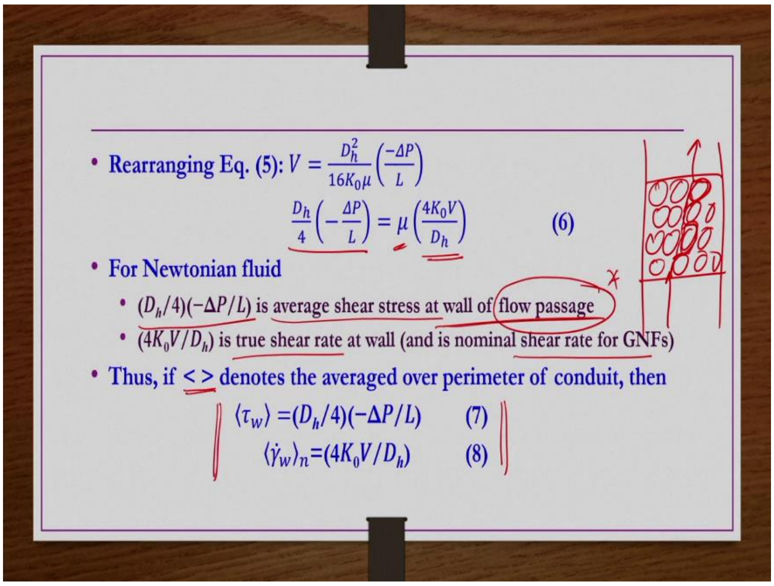

Now, that same equation 5 for non-circular conduits we are rearranging. How are we rearranging? We are writing $\left(\frac{-\Delta p}{L}\right)\frac{D_h}{4}$; one side we are writing other side whatever the remaining terms are writing we are writing ok.

So now, here what we have $\left(\frac{-\Delta p}{L}\right)\frac{D_h}{4}$ is nothing but what, it is nothing but shear stress right. So now, this shear stress is at the wall of flow passage right; at the wall of the flow passage that is along the particles whatever is there. So, that is changing from one particle to other particle, but we are taking average shear stress at wall of flow passage, it is not at the wall, right. Wall of the flow passages, it is not at the wall of the column but at the flow passage. That is you have these particles like you know the packing like this.

So, now when the fluid element is moving there is some shear stress here, some shear stress along this, some shear stress along this, some shear stress along this. So all that shear stress along the fluid passage areas whatever is there that is the average; that is $\left(\frac{-\Delta p}{L}\right)\frac{D_h}{4}$. It is similar to whatever the τ_{rz} that we got $\left(\frac{-\Delta p}{L}\right)\frac{r}{2}$ for the case of a circular conduit. So, analogy to that one this we can write as an average wall shear stress, right.

And then right-hand side whatever is there. So μ is what? μ is the viscosity and then shear stress is equals to viscosity multiplied by the shear rate. So, then whatever the parenthesis term here 4 K_0 V/D_h should be what; it should be shear rate it should be the true shear rate

$4 K_0 V/D_h$ is true shear rate for the case of a Newtonian fluids, but it is a nominal shear rate for the case of a time independent non-Newtonian fluids generalized Newtonian fluids. That we know already ok.

This is also at the wall of flow passages, not at the wall of the column ok. So, if you designate this symbol for average quantities. So, this equation number 6, how we can write? Average τ_w is nothing but $\frac{D_h}{4}\left(\frac{-\Delta p}{L}\right)$. And then average nominal shear rate is nothing but $4 K_0 V/D_h$ ok.

So, these two equations we are going to use later on when we do the frictional pressure drop calculations.

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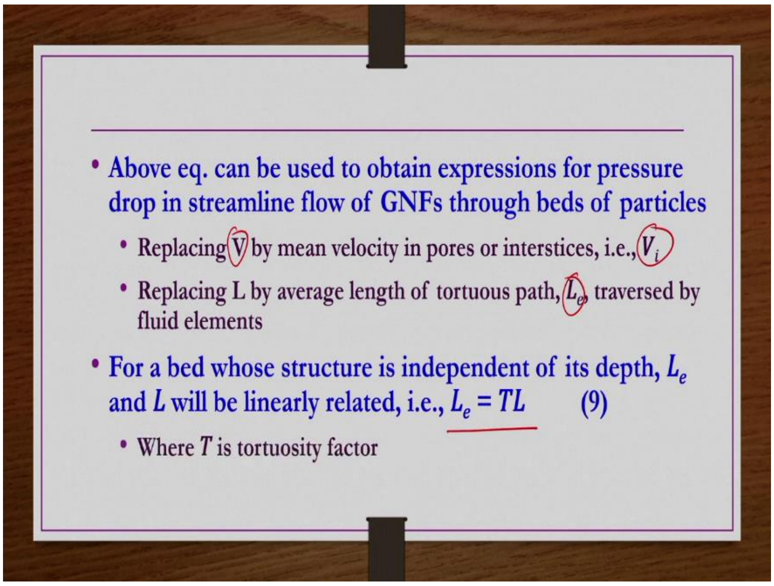

So, now this equation can be used to obtain expression for pressure drop in streamline flow of a generalized Newtonian fluids through beds of particles by replacing V by interstitial or the pore velocity or mean velocity in pores or interstitial spaces V_i. And then replacing L by average length of tortuous tortures path L_e. And L_e is nothing but T L; T is nothing but the tortuosity factor, ok

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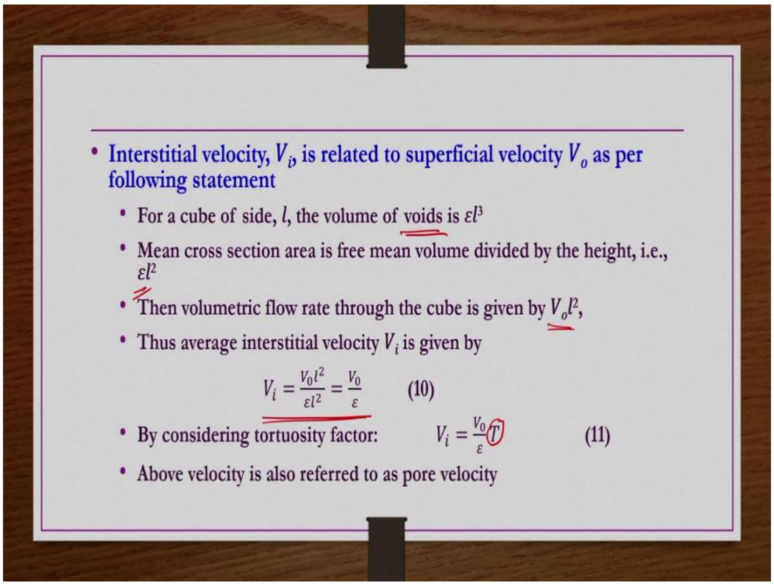

So, interstitial velocity V_i is related to the superficial velocity V_0, how it is? It is related like $V_i = V_0/\varepsilon$, how it is that we see now. For a cube of side l the volume of voids is $\varepsilon\, l^3$ voids or cube if you have a particle is a cube cubical shaped let us say the particle that you are taking it is a cubical shape, then its volume is l^3 if the side of cube is l.

So, out of these n number of cubes are there, so then void space is or the volume of the voids is the $\varepsilon\, l^3$ that we understand. Then mean cross section area is a free mean volume divided by the height that is $\varepsilon\, l^3/l$. So, that is $\varepsilon\, l^2$ right.

Then the volumetric flow rate through the cube is given by $V_0\, l^2$, through the cube whatever the volumetric flow rate is that is nothing but whatever the average velocity V_0 multiplied by the area that is l^2, ok. So now, average interstitial velocity V_i should be obtained by V_0 this volumetric flow rate divided by the mean cross section area that is $V_0\, l^2/\varepsilon\, l^2$. So, that is V_0/ε is nothing but V_i ok.

So, this is how we get this you know interstitial velocity or the pore velocity or the average velocity in the pores. Now, if you consider tortuosity factor also, so this V_0/ε should be multiplied by T; some books it is multiplied some books it is not multiplied. Because there are so many constants are there all the all together they are clubbed at the end when you have the final frictional pressure drop relation, and then compared with the experimental results. And then this combination of whatever n number of constants are there replaced by one or two constants α or β like that, ok. Those things we are going to see.

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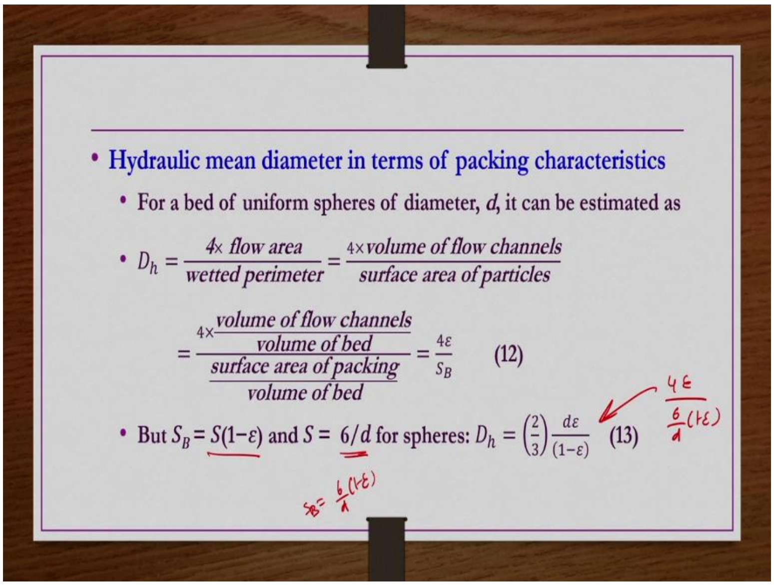

So, hydraulic mean diameter in terms of packing characteristics we have to find out. For a bed of uniform spheres of diameter d it can be like D_h = 4 multiplied by flow area divided by the wetted parameter. So, flow area is nothing but the volume of flow channels ok that is available and then wetted perimeter is nothing but the surface area of particles.

So now, what we do? We divide a numerator and denominator by total volume of the bed. So, that volume of flow channels divided by the volume of bed we can write as ε; void fraction ε and then surface area of packing divided by the volume of bed we can write it as specific surface S_B.

S_B for spherical particles we know S (1 – ε) and then S for spherical particle is 6/d. So, S_B is nothing but 6/d (1 – ε) right. So now, D_h is here 4 ε/S_B is 6/d (1 – ε), when you do simplification you get $D_h = \left(\frac{2}{3}\right)\frac{d\varepsilon}{(1-\varepsilon)}$ ok.

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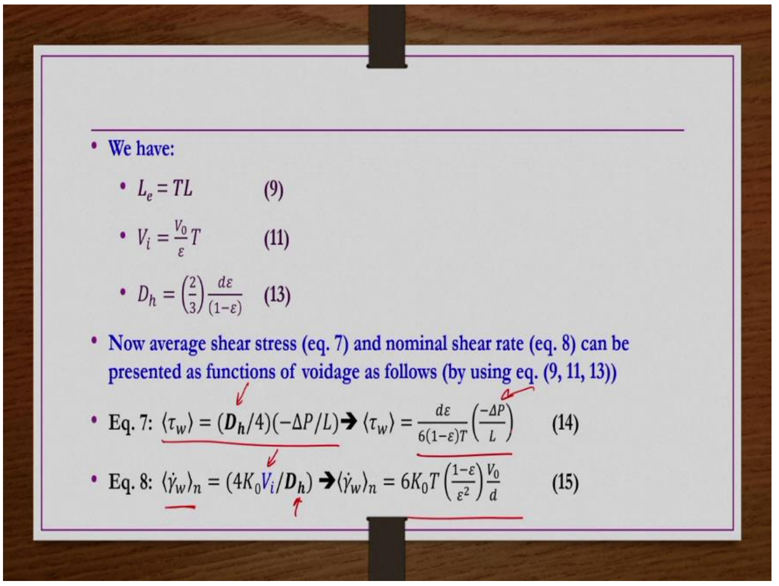

So now, L_e effective length we already got as TL that is height of the packing. V_i interstitial velocity we got it as $\frac{V_0}{\varepsilon}T$. And then D_h we got it as $\left(\frac{2}{3}\right)\frac{d\varepsilon}{(1-\varepsilon)}$.

So now, these three quantities we are going to substitute in τ_w and then $(\dot{\gamma}_w)_n$ average values right. So, when you substitute here τ_w is $D_h/4$ $(-\Delta P/L)$; D h you substitute from equation number 13 here and then you simplify, so average τ_w you get this one.

Now, similarly average $\langle\dot{\gamma}_w\rangle_n$; nominal average nominal shear rate is this one 4 K_0 V_i/D_h. V_i you substitute $\frac{V_0}{\varepsilon}T$, D_h you substitute $\left(\frac{2}{3}\right)\frac{d\varepsilon}{(1-\varepsilon)}$ and then simplify so then you get this one, right. So, but still here if you wanted to know the τ_w you need to know the pressure drop, right apriority. But, we are doing all these calculations in order to get that pressure drop or frictional pressure drop in terms of f right.

So, these equations are not final directly we cannot use because, though all other terms are like you know ε D etcetera are available from the experimental results right. So, you cannot a priori cannot calculate the shear stress or shear rate using these expressions because this $(-\Delta P/L)$ is not available.

So, for that reason what we do? We do further simplification and then obtain frictional pressure drop expression rather than pressure drop.

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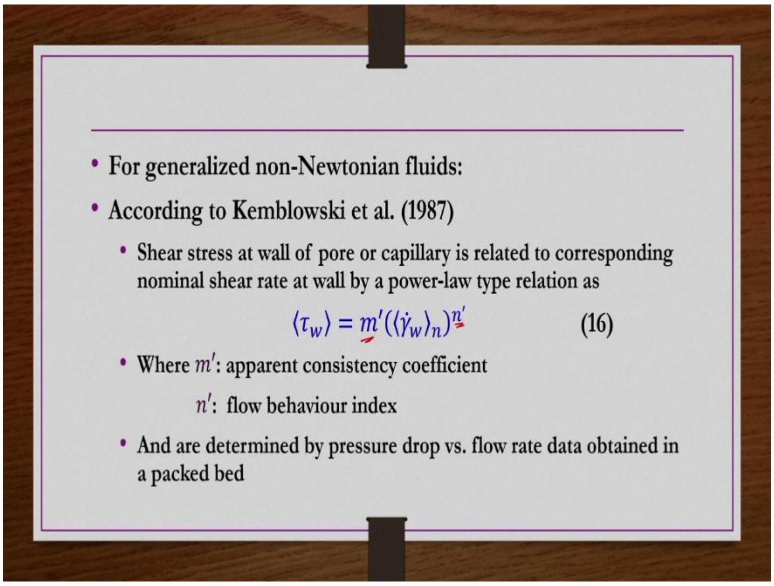

So, for generalized non-Newtonian fluids: Kemblowski have reported that the shear stress at wall of pore or capillary is related to corresponding nominal shear rate at wall by a power-law type of relation and then that relation is given by this one. $\langle\tau_w\rangle = m'(\langle\dot{\gamma}_w\rangle_n)^{n'}$. And these m' and then n' are not the rheological parameter of the fluid, they are not obtained they are not the rheological parameter of the fluid but rather they are obtained by the pressure drop versus volumetric information from the experimental results, ok.

So, from the experimental results what people found this $n' = n$.

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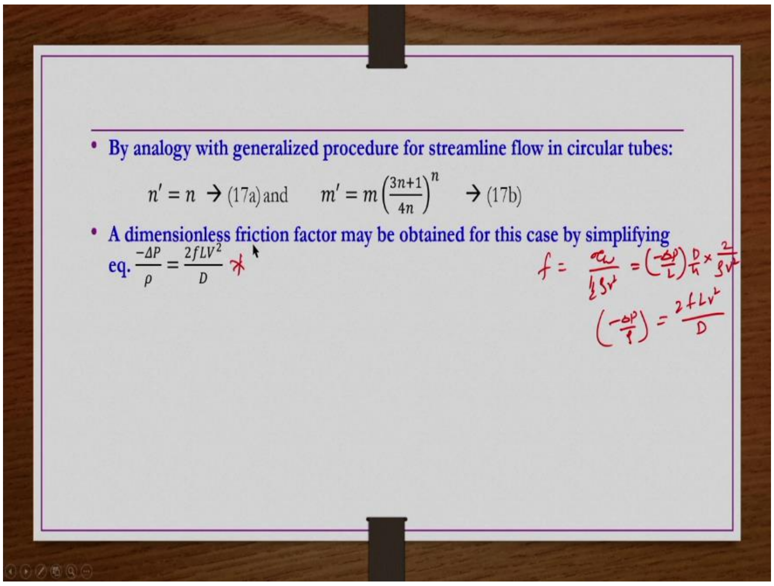

But, $m'$is $m\left(\frac{3n+1}{4n}\right)^n$. So, this m and n without primes they are the rheological parameters of the fluid whereas, n' and then m' are the characteristic parameter that are obtained from the packed bed by doing experiments and then getting the results volumetric flow rate versus pressure drop.

So now, what we do? We define the friction factor. Friction factor how we define? $f = \frac{\tau_W}{\frac{1}{2}\rho V^2}$, so τ_W is $\left(\frac{-\Delta p}{L}\right)\frac{D}{4}$ and then divided by $\frac{1}{2}\rho V^2$, this is what we have. So, then from here this same thing if you write $\left(\frac{-\Delta p}{\rho}\right)$ you can write it as $\frac{2fLV^2}{D}$ this is what you can have. So, that equation is written here, right.

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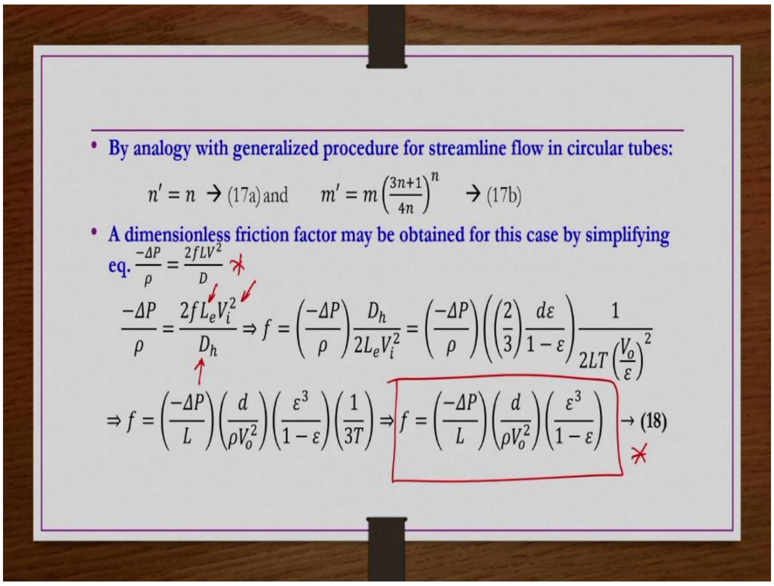

So, now in place of L we have to write L_e, in place of V we have to write V_i, in place of D we have to write D_h for the packed bed. And then corresponding equations that we have already derived in previous slide we substitute here so then you get expression for f as this one; $f = \left(\frac{-\Delta p}{\rho}\right)\frac{D_h}{2L_eV_i^2}$.

So, this is nothing but your frictional pressure drop; this is nothing but the frictional pressure drop expression. So now, this $\frac{1}{3T}$ constant that is we are not taking because we are writing as a kind of generalized expression. So, those constants are taken care by the final friction factor in terms of Reynolds number that we do.

So, this frictional pressure drop we can get from this expression ok, if you know the friction factor. If you know the friction factor $(-\Delta P/L)$ you can find out ok. So, how to find out the friction factor for a packed bed? That we do.

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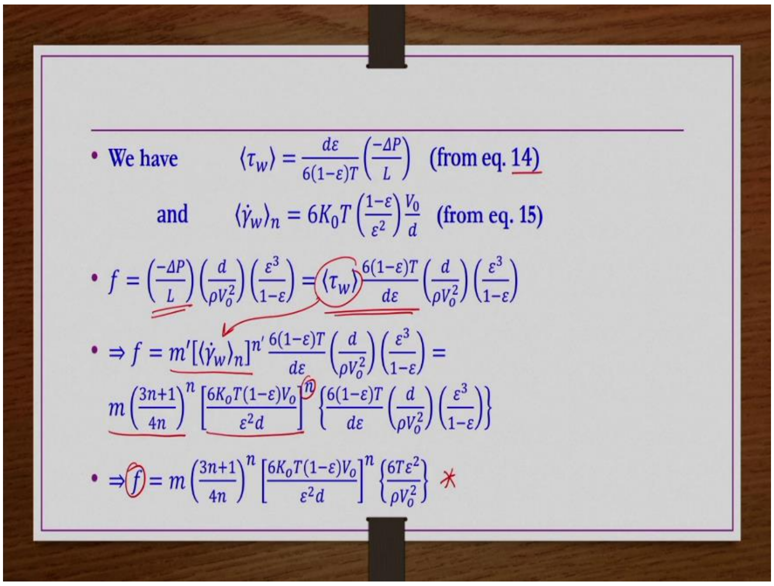

So, τ_w we have this one, $\langle\dot{\gamma}_w\rangle_n$ we have this one already from equation number 14 and 15 we have derived. So, f we are having this expression. So, in place of $(-\Delta P/L)$ what you write $\langle\tau_w\rangle\frac{6(1-\varepsilon)T}{d\varepsilon}$ from equation number 14 and do the simplification.

Before doing the simplification, now this τ_w is nothing but it is = $m'(\langle\dot{\gamma}_w\rangle_n)^{n'}$ right. So, that we write now here. So, m in place of τ_w we write $m'(\langle\dot{\gamma}_w\rangle_n)^{n'}$ this is what we write in place of τ_w. Then remaining terms are as it is.

Next step in place of $m'(\langle\dot{\gamma}_w\rangle_n)^{n'}$ we write $m\left(\frac{3n+1}{4n}\right)^n$ right. And then in place of $\langle\dot{\gamma}_w\rangle_n$ we write whatever $\frac{6K_0T(1-\varepsilon)V_0}{\varepsilon^2 d}$, this is what we write and then whole power n' n' is n, right. So, this is coming from equation number 15, here ok; in place of $\langle\dot{\gamma}_w\rangle_n$.

So now, after doing this one if you simplification simplify. So, this is the equation that you get. So now, here in this equation you do not have any unknown kind of thing, right. Usually this K_0 T ε for a given packed bed they are known from the experimental conditions. Average velocity how much average velocity you wanted to have through the bed, so all those things are known. Fluid rheology is also known a priori. So, then friction factor you can calculate using these relations. So, if the friction factor is there, so pressure drop you can find out, ok.

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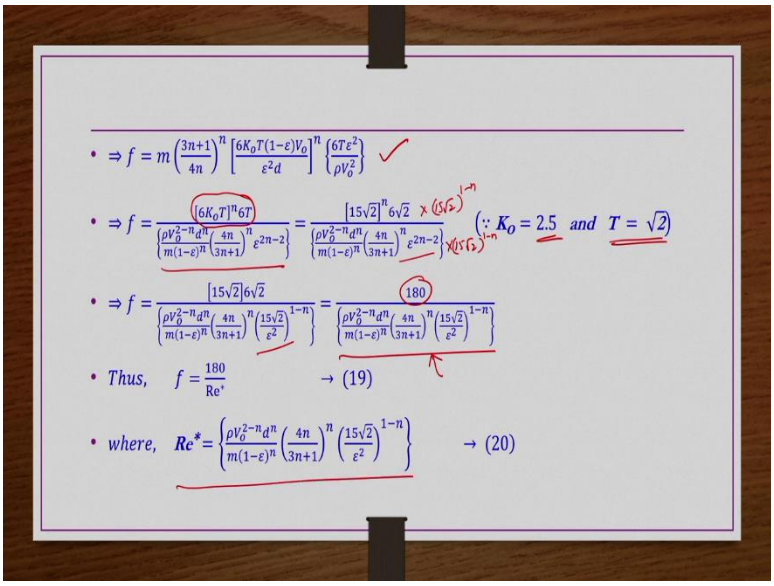

So, the same equation rewritten here. So now, this equation what are we trying to do? We are putting these constants $[6K_0T]^n$ 6 T these are the constants. So, these constants we are keeping in the numerator and rest all the terms we are bringing to the denominator.

Now K_0 in general for most of the case can be taken as on average 2.5, and then T in general for most of the case is $\sqrt{2}$. So, if you substitute K_0 T here in place of 6 K_0T you will get $(15\sqrt{2})^n$ is as it is and then 6 $\sqrt{2}$, right.

So now, next step what we are doing? We are just take multiplying this term here what we do $(15\sqrt{2})^{1-n}$ we are doing multiplying numerator and then with the same quantity we are multiplying the denominator as well.

So, that in the numerator we have only $15\sqrt{2}$ and the denominator whatever is there $15\sqrt{2}$ that is $\left(\frac{15\sqrt{2}}{\varepsilon^2}\right)^{1-n}$ we can write, and rest all the terms are as it is.

So, this $15\sqrt{2}$ $6\sqrt{2}$ is nothing but 180 and then divided by all these things as it is. So, this is in the form 180/Re right, this all this is nothing but Reynolds number for a power-law fluid flowing through a packed bed. Like $Re_{m\,r}$ that we have defined for a case of power-law fluid flowing through a circular pipe. Now, same the power same power-law fluid is flowing through packed bed then Re is defined by this whatever the quantity in the parenthesis. That is rewritten here, ok.

So, for the streamline flow of a power-law fluid flowing through a packed bed the friction factor f is 180/Re* that is what we got, ok.

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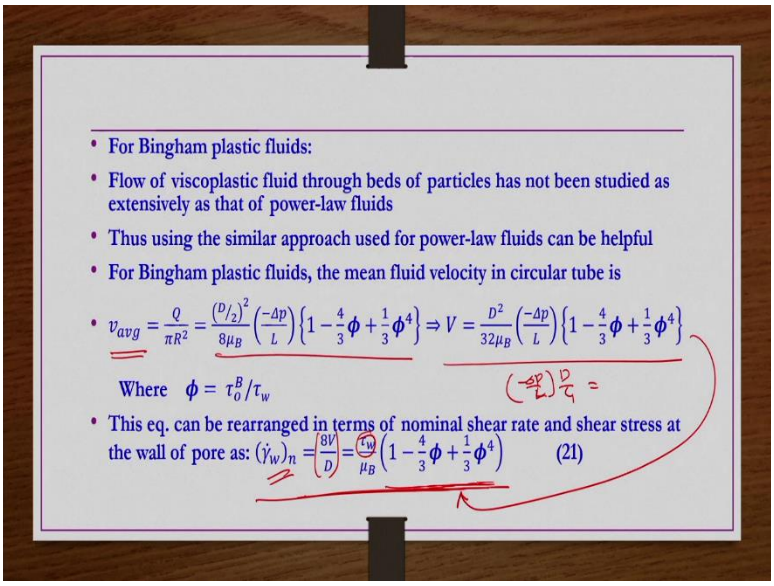

So, for Bingham plastic fluids also we can do similar analysis, but not much literature is available. But, if you follow the similar analysis for the Bingham plastic fluids, Bingham plastic fluids v_{avg} we found this expression. This is also we have done in the week number 3 or 4 sometimes previously. So, this is what we have, we adopted it here.

So, this equation we can write like this; v_{avg} right. Next here ϕ is nothing but τ_0^B/τ_w that we have been discussing whenever we are having this viscoplastic fluids, right. So now, you apply this similar analysis like you have done for the power-law case or what you do you rewrite this equation like $\left(\frac{-\Delta p}{L}\right)\frac{D}{4}$ one side and then remaining terms other sides if you take you know you get these things.

Or you can do $\frac{8V}{D}$ multiplied by whatever these remaining constants. So, then this is what you are getting; this expression you get here $\frac{8V}{D}$ is nothing but nominal shear rate that = $\frac{\tau_w}{\mu_B}$ multiplied by this factor whatever is there.

It is just a rearrangement of this equation; simply you write $\frac{8V}{D}$ one side and then all other terms you keep other side. So, $\frac{8V}{D}$ is nothing but the nominal shear rate for a generalized Newtonian fluids and then right side wherever $\left(\frac{-\Delta p}{L}\right)\frac{D}{4}$ was there that you write τ_w and then remaining terms are as it is.

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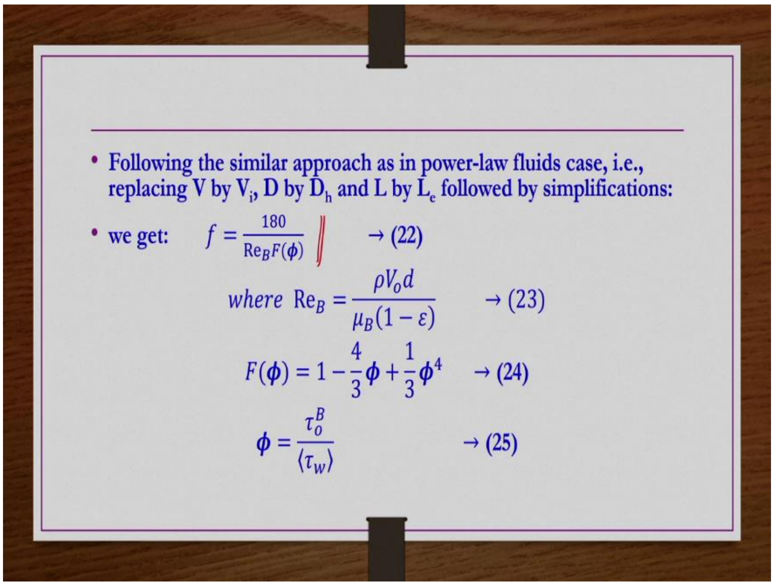

So, like that if you do and then follow the same analysis by replacing V by V_i and D by D_h and L by L_e then do the similar calculation what we have done till now for the case of power-law fluid, then you get $f = \frac{180}{Re_B F(\phi)}$ as a kind of friction factor for a Bingham plastic fluid flowing through packed beds.

Where, Re_B is nothing but $\frac{\rho V_0 d}{\mu_B (1-\varepsilon)}$, and an f_B is nothing but that ϕ function $1 - \frac{4}{3}\phi + \frac{4}{3}\phi^4$ ok. ϕ is nothing but $\frac{\tau_0^B}{\tau_w}$ as we already seen.

So, that is all about the streamline flow of a power-law or Bingham plastic fluid flowing through packed bed. What if, if the flow is transition or turbulent region? If it is in transition or turbulent region, so then what should we do? Similar analysis one can do or from the experimental analysis you can take some empirical correlation that is possible, right.

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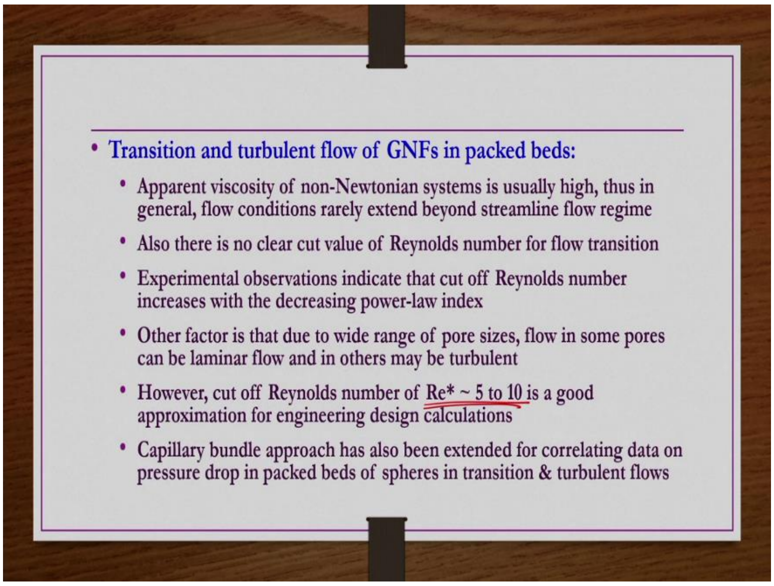

However, for non-Newtonian fluids usually apparent viscosity is higher. So, then in general you do not get very large Reynolds number flows, ok. Also there is no clear cut value of Reynolds number for flow transition in case of packed bed even for the Newtonian case. So, same is valid for non-Newtonian case.

So, experimental observation have found that this critical Reynolds number, where the flow is changing from stream line to transition or turbulent flow that increases with decreasing the power-law index. It is completely experimental observation. And then other factors due to a wide range of pore sizes flow in some pores can be laminar and in some other pores can be turbulent. These kind of other factors are also there as I have already mentioned.

So however, the cutoff Reynolds number from streamline to transition flow is taken between 5 to 10 value of Re^* for many of the applications. And then people found approximation is good for many of the engineering design calculations, ok.

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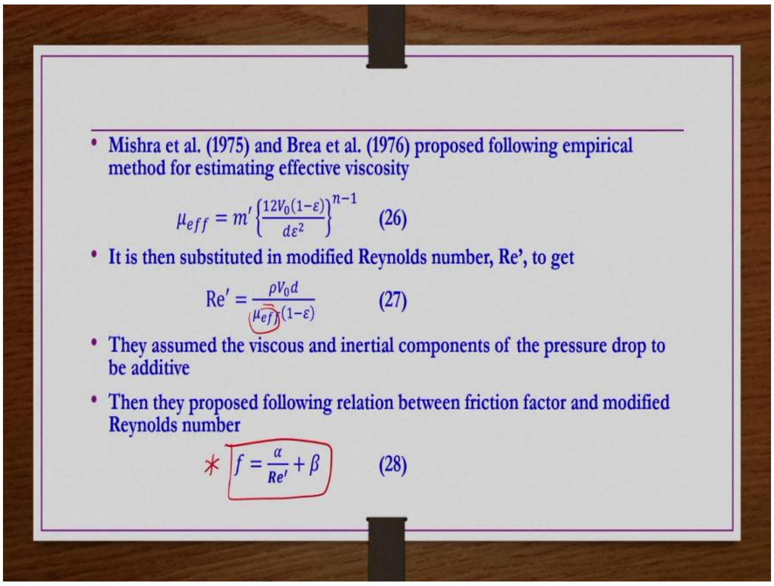

So, what we see? We see a few literature. So, Mishra et al and Brea et al proposed following empirical method for estimating the effective viscosity for a fluid flowing through packed beds that viscosity is given by this one. This is for the case of power-law fluids,

And now using this one they have defined the Reynolds number Re prime. So, like you know $\frac{\rho V_0 d}{\mu_{eff}(1-\varepsilon)}$ is the Reynolds number for a Newtonian fluid flowing through a packed bed. So, in place of mu we have μ_{eff}. So, now, you substitute μ_{eff} here and then get the Re' for the case of a power-law fluids you know flowing through packed beds.

Further these researchers assumed the viscous and inertial components of the pressure drop are additive. And then they proposed following correlation between friction factor and modified Reynolds number like this. This is for the entire range, not only for the streamline but also it is covered transition range also, right. This is the form they proposed.

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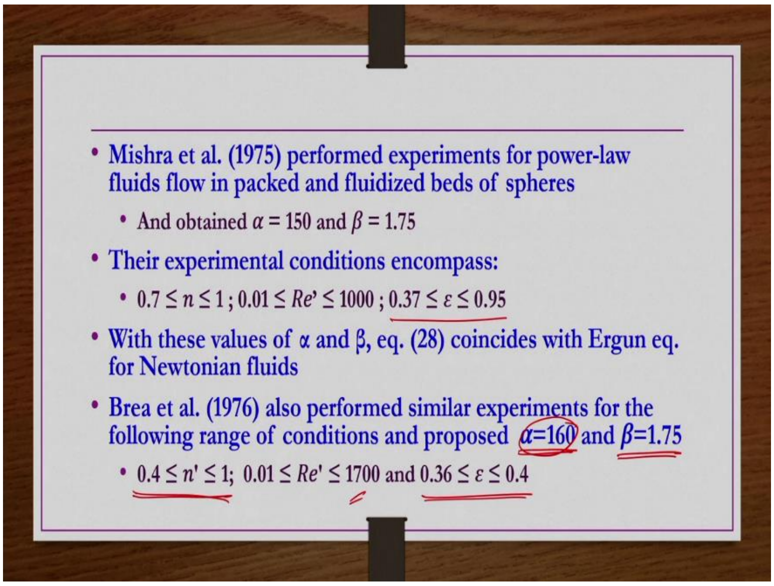

And they found this α = 150 and then β = 1.75 as per the experimental results of Mishra et al. Their experimental conditions are very narrow like n is 0.7 to 1 and then Re' 0.01 to 1000 only. However, ε range they have taken very wide 0.37 to 0.95 right.

Now, with these values of α and β whatever the f = α/Re + β is stay that is coinciding with the Newtonian results, because for the Newtonian results Ergun's equation we know that f = 150/Re + 1.75. So, same constant they also got here, only thing that Reynolds number is modified Reynolds number they have.

And then that modified Reynolds number is equals to the Reynolds number case of the Newtonian case if the fluid is Newtonian right. But, however, Brea et al also performed several similar kind of experiments and they proposed α should be 160 and then β can be 1.75 same like Mishra et al; only α is changing. However, their range of conditions are also very different, right. So, they have taken wide range of n values, but voidage is very narrow. And then Reynolds number is slightly higher up to 1700 they have taken, ok.

So, like that many correlations are there with slight different you know constants etcetera, right. However, one has to be careful using which correlation so for that Reynolds number we have to find out.

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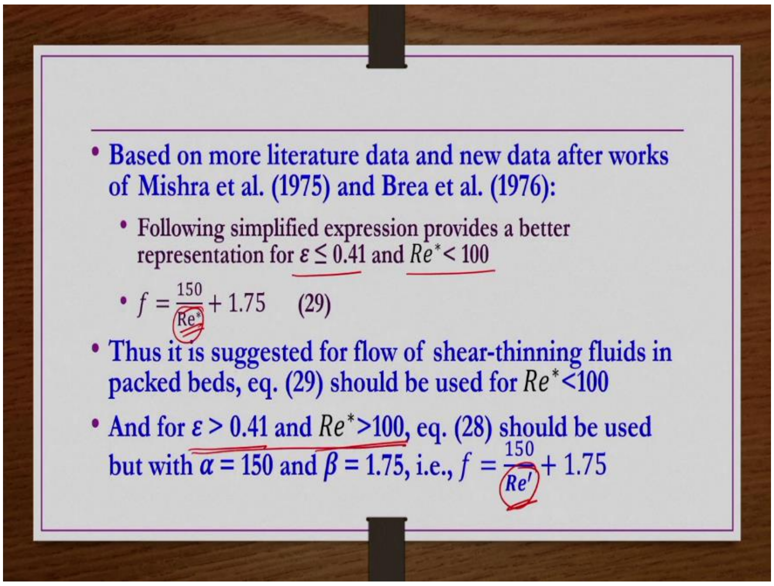

So, based on the more literature and new data after the works of Mishra et al and then Brea et al people found when ε is less than 0.41 and then when $Re^* < 100$ it is better to use $f = 150/Re^* + 1.75$. Whereas, if $Re^* > 100$ and then $\varepsilon > 0.41$ it is better to use $f = 150/Re' + 1.75$.

The constants are same, but only these Reynolds number different definitions of Reynolds numbers we have to use depending on the values of ε and then Re^*, ok.

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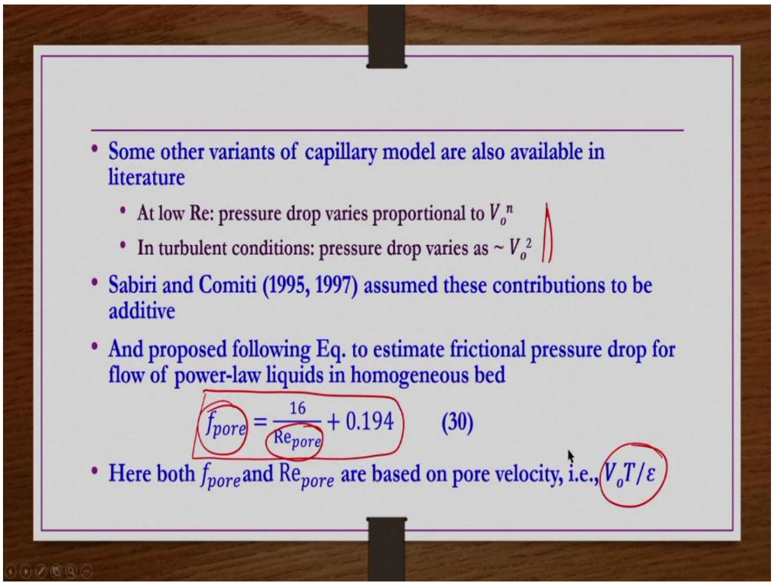

So, some other variants of capillary models are also available in the literature. At low Reynolds number pressure drop varies proportional to ${V_0}^n$. So, from the momentum equation itself we can understand. So the pressure drop, the viscous terms are having power n in the case of power-law fluids. Whereas the inertial terms that is at the high Reynolds number case pressure drop varies you know that is power of V naught square, because inertial terms whatever are there they are having V^2 terms. So, this is standard basic understanding from the momentum equations also we can get, right.

Sabiri and Comiti assumed these contributions low and then high Reynolds number cases are additive and then proposed the following correlation. They proposed $f_{pore} = 16/Re_{pore} + 0.194$; the constants are different, the altogether Reynolds number definition is different and then f definition is also very different according to them. How are they different that we are going to see, but they are defined based on the interstitial or pore velocity.

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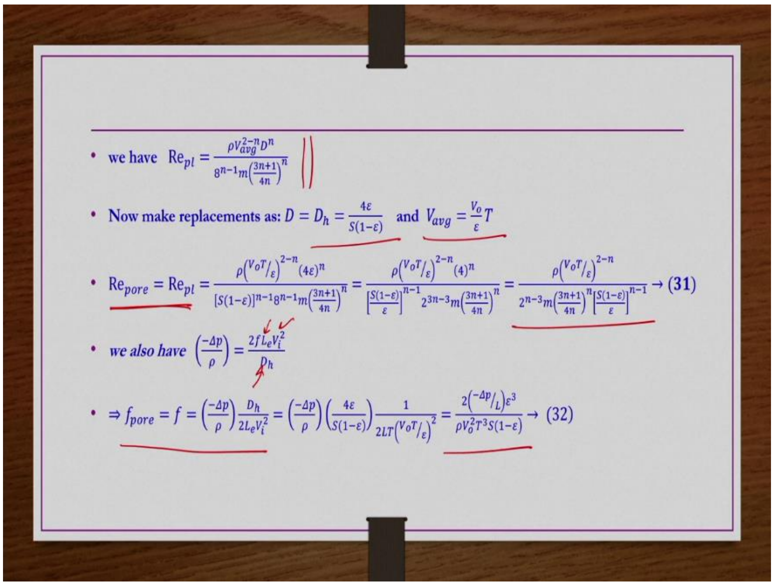

So Re pl, that we know. We are now trying to define what is that f pore and Re pore right. For that what we do? We take the Re_{pl} definition that we have $\frac{\rho V_{avg}^{2-n} D^n}{8^{n-1} m \left(\frac{3n+1}{4n}\right)^n}$, this is what we have seen. $Re_{m\,r}$ or Re_{pl} in week number 3 or week number 4 when we are studying flow through pipes.

So, now in this equations in place of D we write D_h that is $\frac{4\varepsilon}{S(1-\varepsilon)}$ and then V average we write $\frac{V_0}{\varepsilon}T$ and then do the simplification Re; then if you do this replacement whatever the Re_{pl} is there that would be Re_{pore} and then after doing certain simplification this is what you get very complicated expression.

Similarly, f also we have $\left(\frac{-\Delta p}{\rho}\right) = \frac{fLV^2}{D}$. So, there also in place of L you write L_e in place of V you write V_i in place of D you write D_h and then do the simplification. Then whatever the f is there that would be f_{pore}, right. So, that expression is given by this one, after some simplification you can get this.

So, now before winding up today's class we take an example problem right. So, that to understand which expression we should use especially when we have a flow of a non-Newtonian power-law fluid flowing through packed bed, ok.

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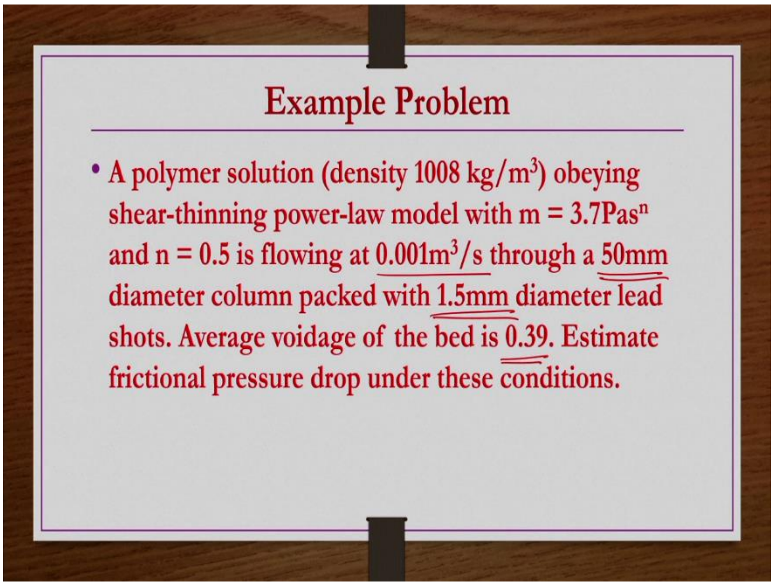

A polymer solution of density 1008 kg per metre cube obeying shear-thinning power-law model with m is equals to 3.7 pascal second power n and n = 0.5 is flowing at a 10 power minus 3 metre cube per second, through a 50 mm diameter column with 1.5 mm diameter lead shots. So, d is 1.5 mm ok, D is 50 mm ok n and m values are given Q is given as 10 power minus 3 metre cube per second, ρ is given, average voidage also ε is given as 0.39. Estimate the frictional pressure drop under these conditions.

So, what we have to do? We have to find out the Reynolds number. As we have seen Re^* we have to find out, if $Re^* < 100$ because $\varepsilon < 0.39$, if Re^* is also less than 100 then we can use $f = 150/Re^* + 1.75$ expression. If Re^* comes out to be more than 100 then we have to use $f = 150/Re' + 1.75$. So, then again Re' we have to find out.

So, in order to select the equation we have to first calculate the Re^*.

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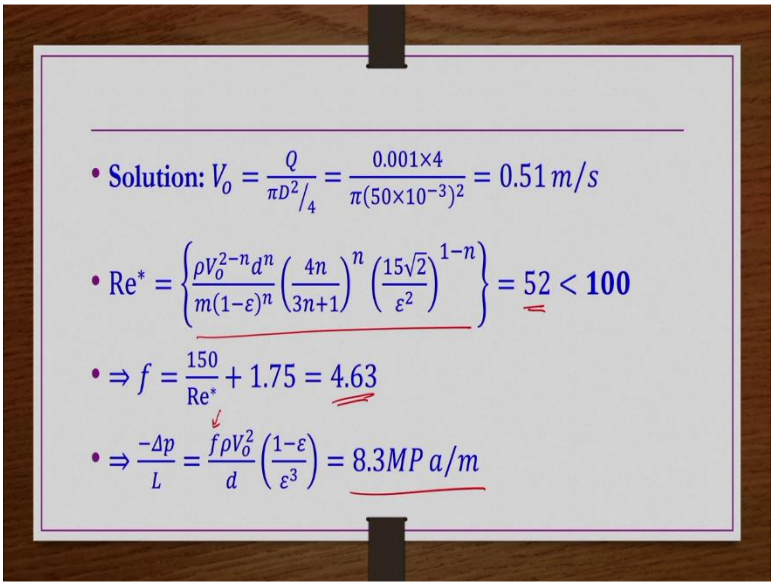

So, if you want to calculate Re^* you need V_0 that is you can get by $\frac{Q}{\pi D^2/4}$; Q is given, D is given, column diameter is given. So, V_0 you get 0.51 metre per second, Re^* expression is this one. So now, here in this equation ρ, V_0, D, n, m, ε everything is known, so when you substitute you get 52, so it is less than 100.

So, then you can use $f = 150/Re^* + 1.75$ this equation you can use, so then you get 4.63 as friction factor for this fluid flowing through the packed bed of a given characteristics. So, then if you have this one, pressure drop you can find it out as $\frac{-\Delta p}{L} = \frac{f\rho V_0^2}{d}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$. Now, here f you substitute ρ V_0 d ε etcetera all are known substitutes, so then you get 8.3 mega pascals per metre.

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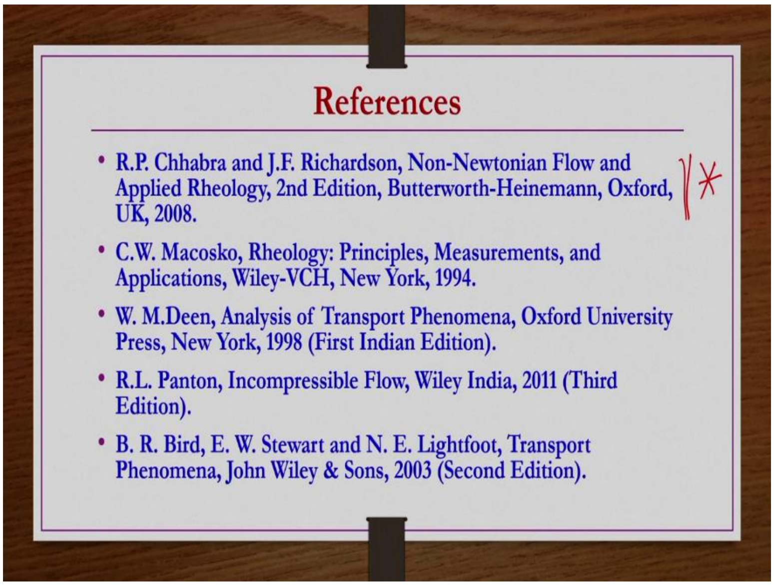

The references for today's lecture: The entire lecture is prepared from this reference book by Chhabra and Richardson, other references are provided here.

Thank you.