Transport Phenomena of Non-Newtonian Fluids
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Lecture - 02
Time Independent Non-Newtonian Fluids

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Time Independent Non-Newtonian Fluids. Time independent non-Newtonian fluids that is the apparent viscosity of these non-Newtonian fluids does not depend on the time, but they depend only on the shear rate. That is what it mean by.

So, however, before going into the details of this time independent non-Newtonian fluids, what we do? We have a kind of recapitulation of what we have discussed in the previous lecture. In the previous lecture, we have seen a several characteristics of a Newtonian fluids and then based on those characteristics we have a defined non-Newtonian fluids, and then we have categorized non-Newtonian fluids.

We classified these non-Newtonian fluids. Those classification of non-Newtonian fluids we once again see, and then we go into the details of time independent non-Newtonian fluids.

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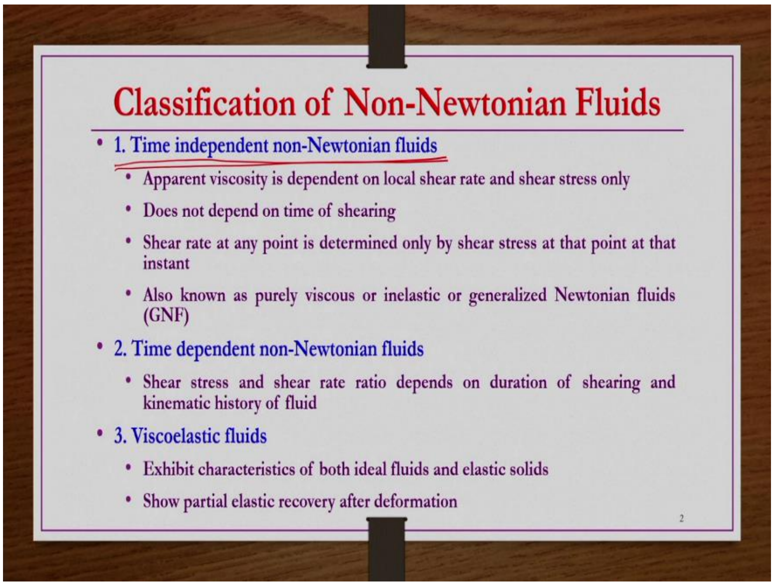

We have seen that this classification of non-Newtonian fluids may be done in 3 different ways. One is the time independent non-Newtonian fluids or shear dependent non-Newtonian fluids, in which the apparent viscosity of this non-Newtonian fluids depend only on the shear rate, but does not depend on the time of shearing.

The second category is time dependent non-Newtonian fluids, in which the apparent viscosity it also depends on the duration of shearing as well as the kinematic history of the fluid. And then third category of non-Newtonian fluids is a viscoelastic fluids, where the material is expected to display some amount of an elastic behaviour in addition to the viscous nature. These are the 3 primary categories of non-Newtonian fluids one can have.

Within the time independent non-Newtonian fluids, what we have seen? We have seen that the apparent viscosity is dependent on the local shear rate and then shear stress only. It does not depend on the time of shearing. And then shear rate at any point is determined by the shear stress at that point, at that instant only.

And these materials are also known as the purely viscous or inelastic or generalized Newtonian fluids. Because under certain limiting conditions, these fluids display Newtonian behaviour, that is the reason, these are also known as the generalized Newtonian fluids.

Whereas, the time dependent non-Newtonian fluids, the shear stress and shear rate ratio depends on duration of shearing and as well as the kinematic history of the fluid. Viscoelastic fluids exhibit, characteristics of both ideal fluids as well as the elastic solids that is they show partial elastic recovery after deformation. These are the few classification that we have already seen in the previous lecture.

Now, in this lecture, we will be discussing in detail about this time dependent non-Newtonian fluids, right. So, we see some details and then also we see if at all sub-category or sub-classification of this time independent non-Newtonian fluids are possible or not, that we see. And then we see, different models, and then a few example problems as well.

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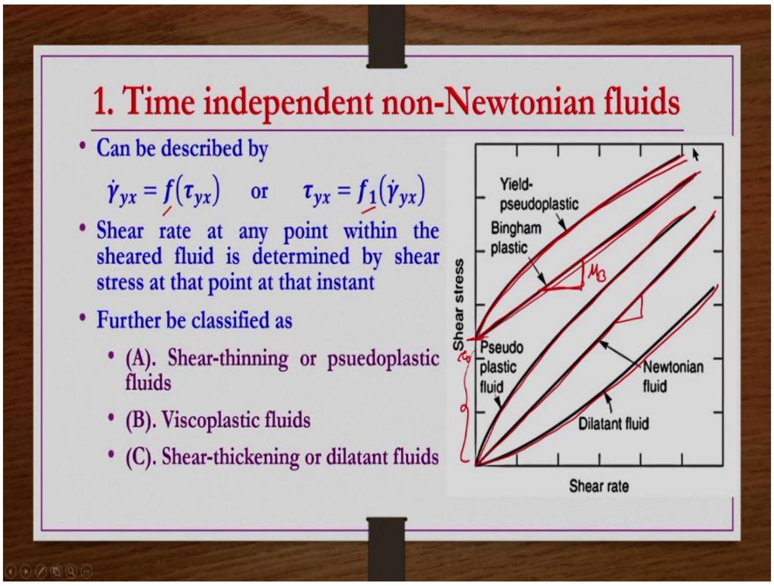

So, time independent non-Newtonian fluids can be described by a functional form like $\dot{\gamma}_{yx} = f(\tau_{yx})$ or $\tau_{yx} = f_1(\dot{\gamma}_{yx})$. And then, this function whatever f or f_1 that we are writing they are not linear, they are not linear. Even if they are linear, they will not pass through the origin, they will not pass through the origin. If they are linear and then pass through the origin, then they will be Newtonian fluids, ok.

So, shear rate at any point within the sheared fluid is determined by the shear stress at that point, at that instant only that is what we have already seen. And then further these are classified as 3 different ways, right because we are saying, what we are saying, this time independent non-Newtonian fluid? We are saying that these time independent non-Newtonian fluids, their apparent viscosity depends on the local shear rate.

So, that means, if the shear rate is changing, then the viscosity is changing. So, that change is increasing or decreasing. So, obviously, if it is increasing, we can have a one category, one classification. If it is decreasing, we can have another classification, right. So, that function we are saying f or f_1 function, whatever we have written here, we are calling them as a kind of non-linear function. If at all they are linear, they do not pass through the origin that is what also we are saying.

So, that means, if they are not passing through the origin; that means, they are not the deforming until and unless the applied stress crosses a certain characteristic yield stress of the material. So, that means, under certain category of you know applied stress, the material behaving as a kind of a plastic material. It does not deform at all, right. And then another, after that after crossing that characteristic yield stress, then material start flowing like a viscous material so, that is the, that is the other category is possible.

So, in all these 3 categories we are not having any time dependency. We are having only function of shear rate. How the apparent viscosity is changing with the shear rate that is what only we are having. So, that is the region 3 classifications are possible within this time independent non-Newtonian fluids category.

One is the shear-thinning or psuedoplastic fluids. Shear-thinning that the word it indicates that you know material becoming thinner by shearing. So, that is with increasing the shear rate, the material apparent viscosity decreases and then such materials are known as the shear-thinning materials or shear-thinning fluids.

Then, viscoplastic fluids, by the word viscoplastic, what we can understand? That material displaying both viscous and fluid like behaviour as well as the plastic solid like behaviour. That means, it may not be deforming at all. Like usually when you apply an external force to a fluid, it undergoes a certain kind of a continuous deformation depending on the nature of the material as long as that force is applied. But if it is not flowing; that means, that is behaving as a kind of plastic solids, right.

So, at under certain range of shear rate or under certain range of the applied stress, this materials do not deform they display solid plastic kind of behaviour and after certain value of applied stress this material start deforming and then flow like a viscous material. So, that is the reason that is the viscoplastic category. That is a second category.

And then third category, shear-thickening or dilatant fluids. Shear-thickening by word it indicates the material become thicker viscosity increases by shearing. So, the apparent viscosity increases with shearing, so those materials or those fluids are known as the shear-thickening fluids.

And then all these categories as I have already mentioned, there is no function of duration of shearing, ok. That is the reason these materials are these or these fluids are known as the time independent non-Newtonian fluids which are also known as the shear dependent non-Newtonian fluids.

So, now if you see the rheogram of this time independent non-Newtonian fluids, then we can have these characteristics here as shown here. That is shear rheogram in the sense, shear stress versus shear rate the information whatever is there, that we can call it as a rheogram.

So, for Newtonian fluids what happens? It passes through the origin and then we get a straight line like this, and then constant slope of viscosity μ whatever. Whereas, for the shear-thinning fluids, you can get the concave downward curves like this something like this, right. So, that is known as the shear-thinning fluids.

And then shear-thickening fluids, again they pass through the origin, but you know they will be having concave upward something slightly like this and then the apparent viscosity increases with the shear rate.

Whereas, the viscoplastic fluids, what we can see here? We can see that you know for an initial certain applied stress unless until and unless applied stress crosses a some certain initial value, it does not flow, it does not flow. It start flowing after that only, and then after that if it is having a linear curve then we can call it as a kind of Bingham plastic fluid.

And then after this characteristic yield stress, when the applied stress applied, when the applied stress is more than the characteristic yield stress of the material, the curve if it is non-linear like this where the apparent viscosity decreases, then we call them as a kind of yield pseudoplastics materials, right.

So, now here this plastic materials are having two range viscosity. You may be thinking that let us say for Bingham plastic material its remaining constant. So, then after crossing this τ_0 value, after crossing this yield stress τ_0 value it is remaining constant, that is what you may be thinking. But you know before that it is having as almost like an effectively infinite viscosity, behaving like a kind of a you know almost solid plastic material kind of thing.

So, though the change is there only two times, from effectively infinite viscosity to some constant mu b viscosity, but there is a change. That is the reason these material is also kept in the category of a non-Newtonian fluids, ok. Yield pseudoplastic anyway, after crossing this characteristic yield stress, they are showing a non-linear curve here between shear stress and then shear rate. Now, we see a few more characteristics of this shear-thinning, viscoplastic, and then shear-thickening materials, ok.

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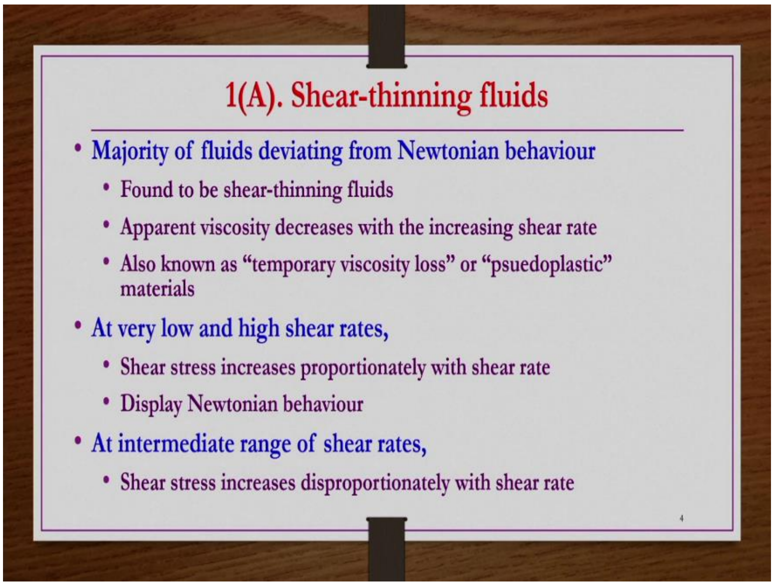

So, under shear-thinning fluids, what we understand? We realize that majority of the fluids which are deviating from non-Newtonian behaviour whether it is polymer industry or food industry or you know dairy industries or you know petroleum fluids or you know materials from the different types of material processing units, etcetera, they have been found that they are deviating if from the Newtonian behaviour and they are displaying the shear-thinning behaviour.

And they are displaying the shearing thinning behaviour. That is in other words majority of non-Newtonian fluids that are common in many of the polymer industries are found to display shear-thinning fluids. Of course, in addition to the shear-thinning behaviour, they may also display some other characteristic like an elastic nature. So, viscoelastic material behaviour it may be having, but the viscous part may be showing the shear-thinning behaviour.

Then, for this material apparent viscosity decreases with the increasing shear rate. When you increase the shear rate the apparent viscosity gradually decreases. They are also known as the temporary viscosity loss or psuedoplastic material. And then they have a two different behaviour.

What are those two different behaviours? At low and high shear rates, low shear rate high shear rate depends on the material and then depends on the application that we have. But in general, shear rate having order of $10^{-2} s^{-1}$ are referred to as a low shear rate range and then shear rate having 10^4 or higher s^{-1}values they are referred to as high shear rate range.

So, under those low and high shear rates, this shear-thinning fluids they display Newtonian behaviour, they display Newtonian behaviour, where shear stress increases proportionately with the shear rate, and then they display Newtonian behaviour with a constant viscosity.

Of course, that constant viscosity would be very different at low shear rate and then it will be very different at high shear rate as well. That is at two ends of shear rates or two extremes of shear rates, low and high extremes of shear rates they may be displaying Newtonian behaviour and they may be having a constant viscosity, but that constant viscosity would be very different from each other, would be very different from each other.

Then, at intermediate range of shear rates, the shear stress increases disproportionately with the shear rate. So, that the apparent viscosity decreases with increasing the shear rate, ok. These are the two characteristics the shear-thinning fluids in general display.

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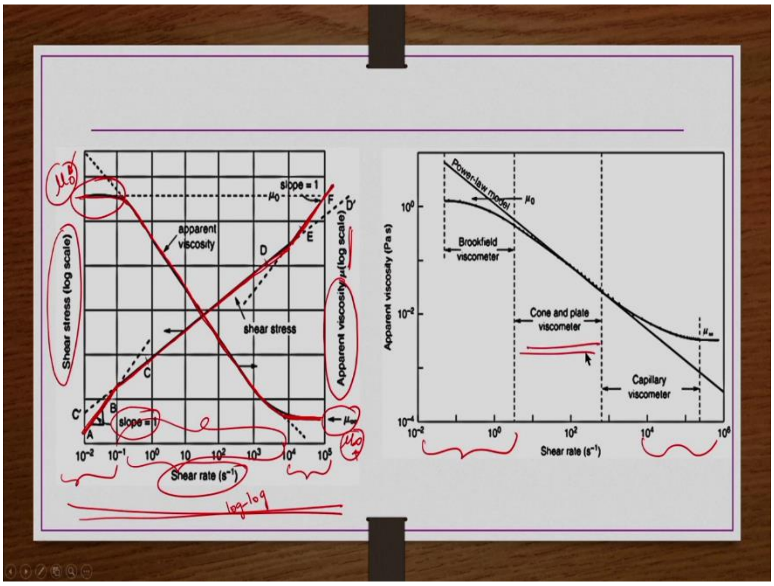

So, now if we see pictorially, the shear stress versus shear rate information, what we do? We have this information shear stress and then shear rate on a log-log scale, on a log-log scale. So, then what you can see here? This particular curve, this A, B, C, D, E and then F this curve is for this shear stress versus shear rate on a log-log scale. You can see at low shear rate range, at low shear rate range; you have a, you have a straight line. Since, it is a log-log plot, so then you are having a straight line, but the slope is 1, but the slope is 1, ok.

Whereas, at high shear rate range, at high shear rate range also, the shear stress increasing proportionately with the shear rate with a slope 1 again, with a slope again 1. But in between this intermediate range, in between these true ranges of shear rate whatever the intermediate range of shear rate is there, here we can see and the shear stress increasing disproportionately with the shear rate, right. That is what we are saying.

So, what does mean by here? So, at low shear rate it is having a kind of, if it is having slope 1; that means, we can say that is a it is having it is increasing proportionately, shear stress increasing proportionately with the shear rate, ok. So, it is having a Newtonian behaviour.

Same at high shear rate also, the shear stress increasing proportionately with the shear rate that is the reason slope is 1, because this shear stress and shear rate are plotted on a log scale, right. In between it is changing disproportionately, ok. So, now that is what we understand.

Now, exactly the same information you convert, whatever the shear stress, local shear stress divided by corresponding shear rate you do, and then you obtain apparent viscosity, and then you obtain apparent viscosity, ok. This information local shear stress divided by its shear rate then you get the apparent viscosity. And then it changes for non-Newtonian fluids that we already realized in the previous class.

This curve is displaying the apparent viscosity versus shear rate information on a log-log scale, right. You can see here again, at low shear rates, at low shear rates the apparent viscosity almost constant even though shear rate is increasing from 10^{-2} to $10^{-1}\ s^{-1}$.

And then this apparent viscosity value is very large, is very large, μ_0, ok. This value. Whereas, at low at high shear rates also, at very high shear rate range also, the curve is having you know horizontal line like you know almost like the viscosity does not change, even the shear rate increasing from 10^4 to $10^5\ s^{-1}$, right. So, it is again displaying a Newtonian behaviour, that if the viscosity does not change with the shear rate, that means, it is a Newtonian behaviour, right.

So, but this constant viscosity whatever is there, μ_∞ we are calling, this is very low value. Whereas, for intermediate range of shear rate what we can see that the apparent viscosity whatever is there that is decreasing from higher value of μ_0 to lower value of μ_∞. So, whatever non-Newtonian characteristic is been found in the shear-thinning fluids, they are in general found in the intermediate range of shear rate only.

Whereas, at low and high shear rate range, in general even those shear-thinning fluids are you know displaying a Newtonian behaviour with different constant viscosity. At low shear rate it is having μ_0, that naught indicates that low shear rate towards the 0, right, but its value is very high, μ_0 value is very high.

Whereas, the μ_∞, ∞ indicates that shear rate go going towards the infinite large values, all right. But, however, this mu infinity is having very low value, right. In between shear rate, this apparent viscosity is decreasing from μ_0 to μ_∞.

Obviously, these are given in a kind of a log scale. So, then they will be having the log values, ok, right. So, this is about the shear-thinning fluids in general.

But, however, we are going to see in the subsequent chapters also of the course that you know, not one single rheometer or viscometer is sufficient to cover entire range of shear rate. Even if they are able to cover the entire range of shear rate of from very low shear rate to the very high shear rate accuracy maybe doubtful.

So, that in general, what you do? You if you wanted to have the rheology of the fluid in the entire range of a low to high shear rate range, then you have to you have to select accordingly rheometers and then that information we can have here. For example, this is not true for all.

For example, if you wanted to have a kind of you know a rheology at low shear rate range, so then it is better to have a kind of a Brookfield viscometer. At high shear rate ranges if you wanted to have the rheology information, then it is better you go for a capillary. In between range, in general cone and plate viscometers are better. We are going to study this rheometers as well in the subsequent lecture, right.

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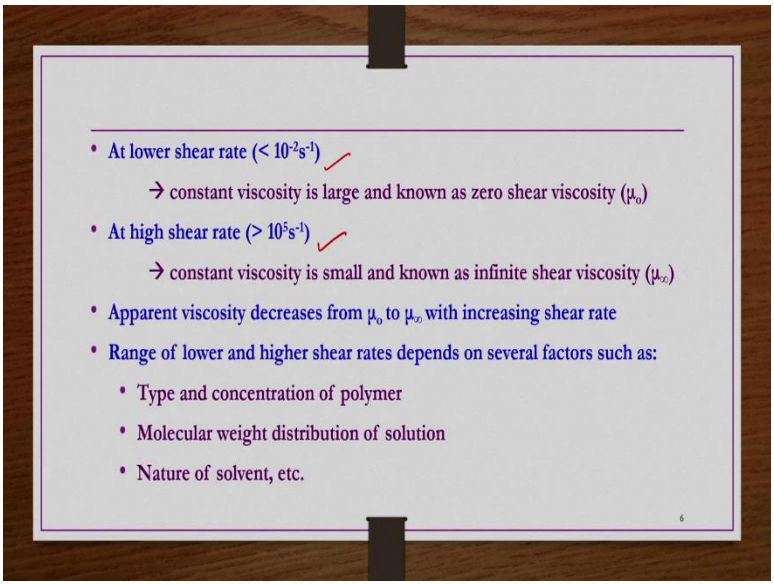

So, this is what, whatever we have seen in the previous slide that is a given here as a kind of notes here. At low shear rates that is less than $10^{-2}s^{-1}$, the constant viscosity is large and known as 0 shear viscosity, and then μ_0 is the kind of symbol is in general used.

At high shear rate again constant viscosity is found, and but it is small and known as the infinite shear viscosity $\mu_\infty$is the symbol that is used. Whereas, apparent viscosity decreases from μ_0 to $\mu_\infty$with increasing shear rate for this shear-thinning fluids.

And then, range of lower and higher shear rates depends on several factors, such as type and concentration of polymer solution, molecular weight distribution of the solution, nature of solvent, etcetera. These are given with respect to the polymeric shear-thinning fluids only, but they may also change depending on the type of fluid that you are handling.

So, obviously, the lower and higher range of shear rates depends on several factors. So, then in general, but, however, these are the kind of things that are often consider as a kind of acceptable ranges.

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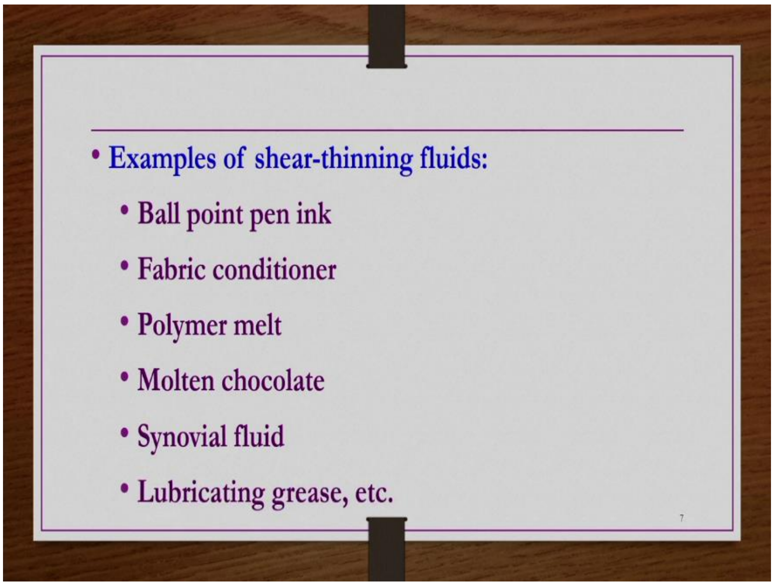

Some examples of shear-thinning fluids if you see, ball point pen ink, fabric conditioner, polymer melt, molten chocolate, synovial fluid and so on so, right. Now, we see a few mathematical models that are available for shear-thinning fluid behaviour, ok. So, most of the mathematical model that are available for, especially time independent non-Newtonian fluids are primarily based on a kind of curve fitting information. One or two are available which are based on the molecular kinetic information also.

So, now, we are going to see some of the models for the shear-thinning fluid behaviour. There may be n number of models are available. In fact, in fact for shear-thinning fluids a large number of models are available, but we are going to discuss which are famously known and then famously known because of their consistency with the experimental results, ok. So, those things are only we are going to discuss here.

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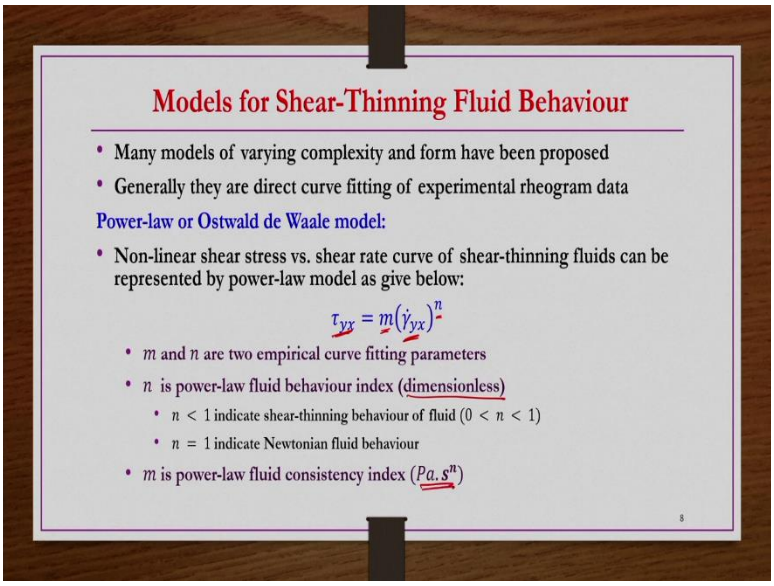

Many models of varying complexity and form have been proposed, and these are generally obtained by direct curve fitting of experimental rheogram data. Whatever the experimental τ versus $\dot{\gamma}$ information is there, you plot it and then do the curve fitting.

And then whatever the curve fitting trend equation is there that is nothing, but your, you know a model for that particular shear-thinning fluid. That is what the normally the models have been developed, except one Carreau model fluid that we are going to see anyway.

So, the first one is the power-law or Ostwald de Waale model. This is the model which have been used extensively to describe the shear-thinning behaviour of non-Newtonian fluids. One of the primary reason of using this model is that it is the simplest mathematical model. It is having simplest mathematical form.

Non-linear shear stress versus shear rate curve of shear-thinning fluids can be represented by power-law model as given by this, $\tau_{yx} = m(\dot{\gamma}_{yx})^n$. Power-law model it is having. So, now here what we see? τ_{yx}, experimental information $\dot{\gamma}_{yx}$, experimental information.

When you are doing curve fitting this experimental information you are getting a power-law curve trend line power type of trend line you can easily fix, if you are doing in excel, right. So, then we have a two parameters, model fitting parameters or two; m and n are the two model fitting parameters.

So, m and n are two empirical curve fitting parameters, n is power-law fluid behaviour index which is dimensionless, right. Behaviour index by the name it indicates, it tells the nature of the material, how strongly shear-thinning, or how weakly shear-thinning, how moderately shear-thinning a given material is that that information you can get from this n, right. So, if n is less than 1, it indicate shear-thinning behaviour of fluid, right. So, when n is less than 1, then only when you use this equation you will get apparent viscosity decreasing with increasing the shear rate, ok.

But, however, in this equation if you substitute n is equals to 1, what will happen? $\tau_{yx} \propto \dot{\gamma}_{yx}$ and then proportionality constant is m. So, that is nothing, but viscosity. So, that means, as we said already mentioned in the classification these time independent non-Newtonian fluids are also known as the generalized Newtonian fluid.

So, here in this case, in the case of shear-thinning fluid fluids described by power-law model. In this model, if you substitute n is equals to 1, you are getting the Newtonian fluid behaviour. That is n is equals to 1 indicate Newtonian fluid behaviour. Whereas, the m is power-law fluid consistency index which is having units pascal second power n.

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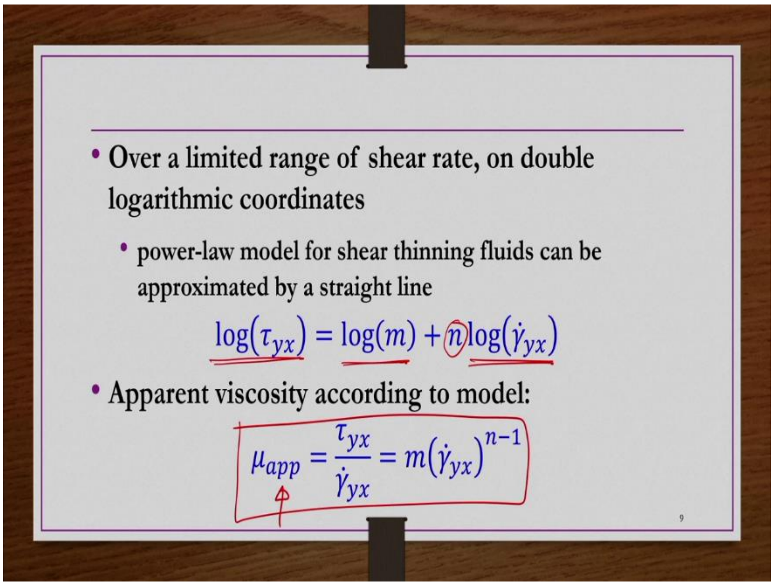

So, over a limited range of shear rate on double logarithmic coordinates, if you draw this information, then we will get a straight line, we will get a straight line. How? Because whatever τ_{yx}= $m\left(\dot{\gamma}_{yx}\right)^n$, and if you apply logarithm either side. So, then you get $log\ (\tau_{yx}) = log(m) + nlog(\dot{\gamma}_{yx})$.

Now, you can see, if you plot this information $log\ (\tau_{yx})$ versus $log(\dot{\gamma}_{yx})$, what are you going to get? You are going to get a straight line as we have seen in the one of the graph just a few slides before that is what we have seen. We have seen that a τ versus $\dot{\gamma}$, shear stress versus shear rate, when we plot on a log-log scale for this shear-thinning material, then you get a straight line, straight line slope is n, slope is n and then intercept is log of m.

And then if this slope is equals to 1; that means, n is equals to 1; that means, it is a Newtonian behaviour, ok. That is also we have seen for a both low and then high range of shear rates in the previous slides, one of the previous slides. And then apparent viscosity according to this model is simply $\frac{\tau_{yx}}{\dot{\gamma}_{yx}}$ which is nothing, but m $\left(\dot{\gamma}_{yx}\right)^{n-1}$. This is what we have. So, the apparent viscosity according to power-law model is this one.

Remember, henceforth sometimes we may skip the apparent term, whenever we are talking about the non-Newtonian fluids, because now we realize that it is already very clear to us that for non-Newtonian fluids, the viscosity is not going to remain same by changing the shear rate. So, that is the reason we are calling it apparent viscosity.

So, then sometimes we may skip this writing apparent, or we may, if you are clearly clear about it you do not need to write apparent. It is clearly understood that for non-Newtonian fluids the apparent viscosity is the viscosity that is not remaining same, that is changing with the shear rate, ok.

So, sometimes we may be explicitly write μ_{app}, sometimes we may skip this subscript apparent as well, ok. Because it is now clear to us that non-Newtonian fluids are having more than 1 viscosity value and then this viscosity value is changing with changing the shear rate.

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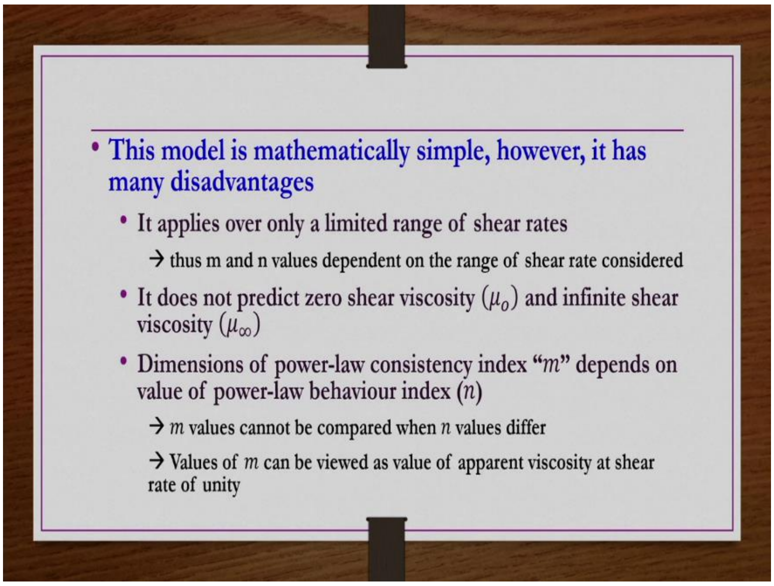

Though this model is mathematically simple as we have seen, but it has a few disadvantages. Some of them, it applies over only a limited range of shear rates. We have already seen $log\ (\tau_{yx})$ versus $log(\dot{\gamma}_{yx})$ plot.

So, there, what we have seen? You know whatever the μ_0, μ_∞information is there that is at low and high shear rate we are having the constant viscosity Newtonian behaviour, but those μ_0, μ_∞ informations are not incorporated in this power-law model. We have only $\tau_{yx}, m, \dot{\gamma}_{yx}\ and\ n$ only in this power-law model. We do not have any mu naught mu infinity values there, right. And then, also we realize that it is valid only for a limited range of shear rate, thus whatever the m and n values dependent on the range of shear rate considered.

Similarly, it does not predict 0 shear viscosity and infinite shear viscosity. Obviously, this power-law model, for shear-thinning fluids, what we have? We have $\tau_{yx}= m(\dot{\gamma}_{yx})^n$. So, there is no term like μ_0, μ_∞, so obviously, it cannot predict them.

And then dimensions of power-law consistency index m whatever is there that depends on the power-law behaviour index n also; m is having units of you know Pascal second power n. So, if a material, if a shear-thinning fluid having different n values, so you cannot compare their corresponding m values that is the problem with this mode. So, obviously, m values cannot be compared when n values are differing. So, values of m can be viewed as value of apparent viscosity at shear rate of unity.

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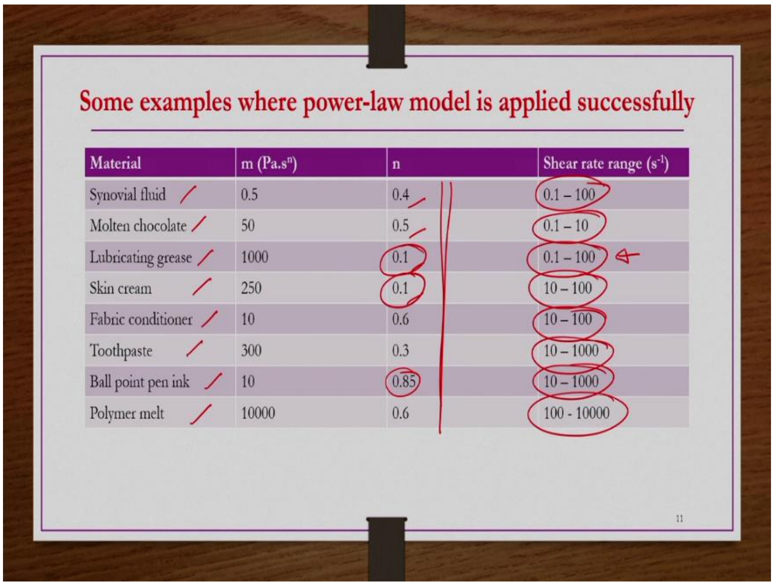

So, some examples where power-law models are applied successfully we can see, this material like synovial fluid, molten chocolate, lubricating grease, skin cream, fabric conditioner, toothpaste, ball point pen ink, polymer melt, etcetera. You can see n values 0.4, 0.5, 0.1, 0.1.

See lubricating greases, we have seen that in examples you know highly shear-thinning. Smaller the n value higher is the shear-thinning behaviour that is what we should understand from this n value now, right. From that n that is power-law behaviour index that indicates you know how strong or how weaker is the shear-thinning nature that material or that fluid is having.

So, if n value is smaller, then it is highly shear-thinning, right. If n value is close to 1, this that is weakly shear-thinning. If n is equals to 1, then it is a Newtonian behaviour anyway, right. Skin cream etcetera, so we can see they are highly shear-thinning, ok. So, whereas, ball point pen ink etcetera you can see that is 0.85, so that is weakly shear-thinning behaviour. It is close to n is equals to 1 range, right.

And then corresponding shear rate, these values are again you know as we have been discussing, whenever we talk about the non-Newtonian behaviour we should talk about the range of shear rate. So, these values of n whatever are provided, you know they are obtained when experiments are conducted in this range of shear rate.

Probably for let us say lubricating grease if you change the shear rate from 0.1 to $100s^{-1}$, so then you may not get n value 0.1, right. If you go to like shear rate of order of 10^3 or 10^4, it may be having even smaller value of 0.1 or if you go to the order of 10^{-2} or 10^{-1} between 10^{-2} to 10^{-1} in that range, you may have this n value 0.2 or 0.3 that also possible, ok.

So, it is very essential to report the range of shear rate as well, when you are reporting a non-Newtonian nature of a given non-Newtonian fluid.

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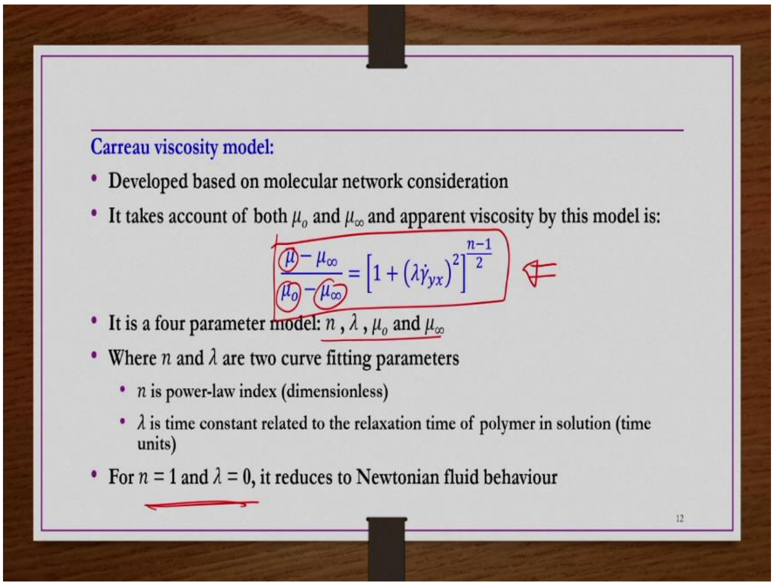

Next, Carreau viscosity model, it is developed based on the molecular network consideration; it is not based on the experimental data. It is not based on the experimental data and curve fitting. It is based on the molecular network consideration and then have been done by some molecular simulations, etcetera, ok.

So, it takes account of both μ_0 and μ_∞, and then apparent viscosity by this model whatever it is given. So, this is the Carreau model, Carreau viscosity model, right. So, it is the improvement over the power-law model because, why it is improvement over power-law model? Because in the power-law model we do not have anything information about μ_0

and μ_∞, but now we are having such information again here. So, obviously, that is a kind of improvement, ok.

Here, but it is the 4 parameter model that is n, γ, μ_0 and μ_∞ are there. So, where this n and lambda are two curve fitting parameters, n is power-law index dimensionless which is same as the power-law model that we have discussed already, and then γ is a time constant related to the relaxation of time of polymers in solution which is having the time units.

And then for n is equals to 1 and then γ is equals to 0, it reduces to Newtonian fluid behaviour. If you substitute n is equals to 1, right hand side everything would be become you know something power of 0. So, it is a constant 1. So, then $\mu - \mu_\infty$ is equals to μ_0 by μ_∞; that means, it will be showing a kind of a Newtonian fluid behaviour with constant viscosity μ_0, ok, yeah.

Now, as I mentioned previously, since we already realize that viscosity of non-Newtonian fluid is not constant, it changes with shear rate, so we call it apparent viscosity. So, sometimes we may not try this mu apparent specifically because we already realized it. So, otherwise this μ is nothing, but μ_{app}.

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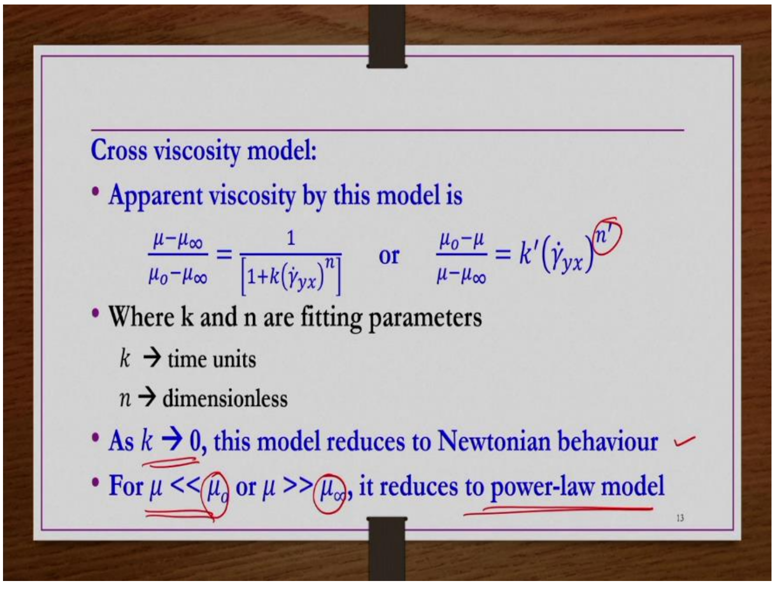

Now, cross viscosity model, apparent viscosity by this model is given by this particular form or it can also be written in this particular form, ok. So, here k and n are nothing, but curve fitting parameters, where k is having time units, n is dimensionless, same as in power-law behaviour power-law model. As k tends to 0, here again if you apply k tends to 0, then this model reduces to Newtonian fluid behaviour of having constant viscosity μ_0 only.

However, but case when you have a μ, the apparent viscosity whatever we are taking if that range is very very small that that is value is much smaller than μ_0 or when the apparent viscosity value is much larger than the μ_∞, then it reduces the power-law model, then it reduces the power-law model. So, you can apply this limiting conditions in this equation simply. So, then you can understand that final model is having the power-law model, power-law form.

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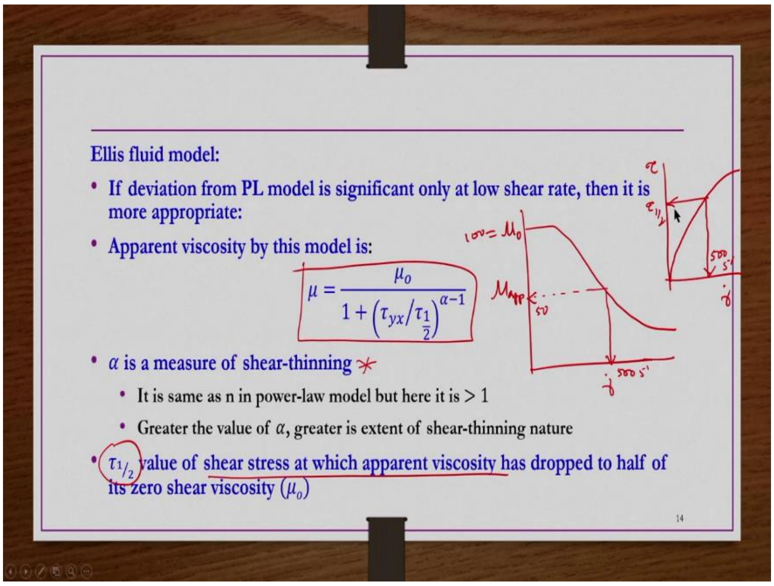

Then is the Ellis fluid model. If deviation from power-law model is significant only in the low shear rate range, then it is better to go for Ellis fluid model, right. And then according to this Ellis fluid model we have this apparent viscosity μ_{app} is given by this expression, right.

Here alpha is a measure of shear-thinning, ok, but it is having greater than 1 value. It is similar like n in the previous models like power-law and then Carreau model, but it is having greater than 1 value, ok. But qualitatively, its functioning is that is same as it is, it gives a measure of shear-thinning, larger the α value higher is the shear-thinning nature of that material. That is what we indicates. It is same as n in power-law model, but here it is greater than 1 and then greater the value of α greater is the extent of shear-thinning nature.

Tau half is nothing, but whatever the shear stress at which the apparent viscosity dropped to half of the 0 shear viscosity, ok. So, this way we can write let us say μ_{app} versus $\dot{\gamma}$ you are plotting. And then since we know that for power-law behaviour, we have you know μ_{app} decreasing with increasing the $\dot{\gamma}$, but initial at low shear rates you are having μ_0 constant viscosity, right.

So, let us say if it is 100 millipascal seconds, so wherever this 50 is there, wherever this mu naught drops to 50 pascal second. So, then corresponding shear rate you find out, right, let us say this is coming out to be 500 s^{-1}.

So, then you can go back to your initial τ versus $\dot{\gamma}$ information for this shear-thinning material and then wherever this $\dot{\gamma}$ and then wherever this $\dot{\gamma}$ corresponding to $500 s^{-1}$ is there, right, so corresponding to that one whatever the shear stress is there that is nothing, but tau half, ok.

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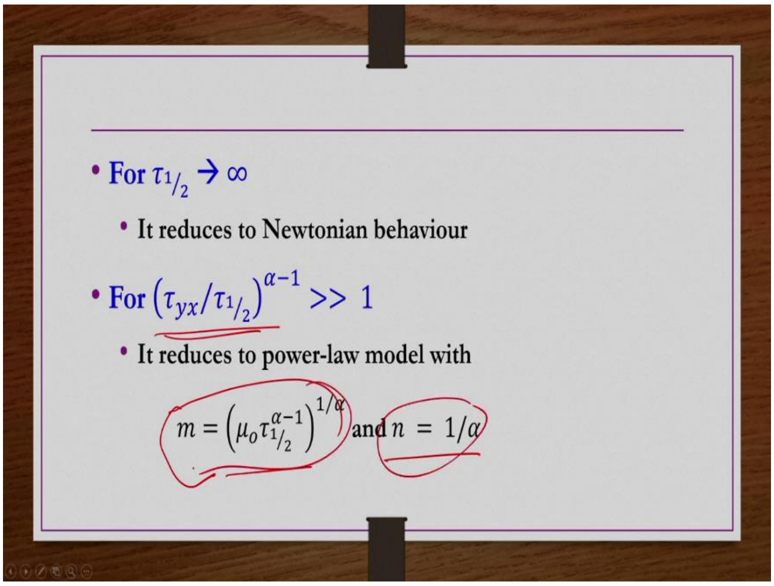

So, for tau half tending to infinity, it reduces to Newtonian behaviour, and then for $\left(\tau_{yx}/\tau_{1/2}\right)^{\alpha-1}$, if this value is very very larger than 1 very very larger than 1, then it reduces to power-law model with $m = \left(\mu_0 {\tau_{1/2}}^{\alpha-1}\right)^{1/\alpha}$ and $n = 1/\alpha$.

You can do this. You can take it as a kind of a take home problem and then you apply these conditions in the previous Ellis fluid model expression, and then you try to find out this expression what are you getting this m, and then this n or not that you can find out, ok.

So, that is about shear-thinning fluids. Shear-thinning fluids that is what we have seen. Now, what we see? We see viscoplastic fluids the second category within the time independent non-Newtonian fluids is nothing, but viscoplastic fluids. So, that is 1 b is viscoplastic fluid behaviour. Within the first category of time independent non-Newtonian fluids, second nature of non-Newtonian behaviour is viscoplastic fluid behaviour.

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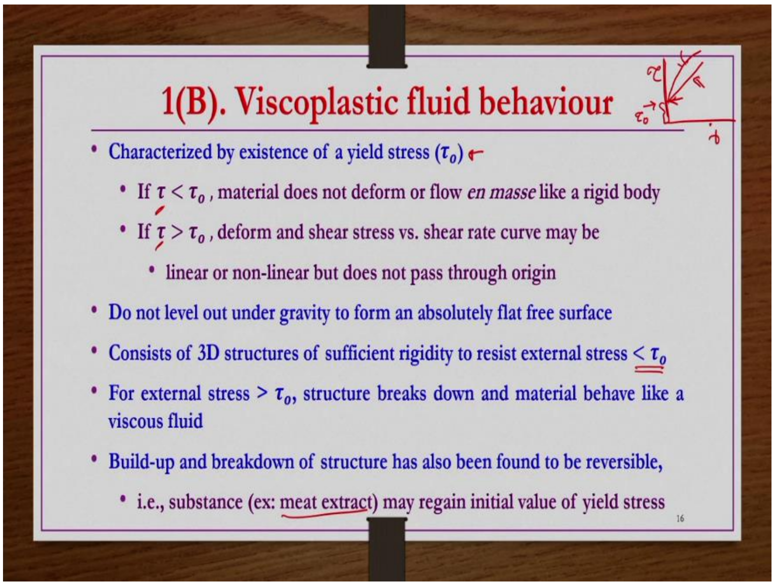

Here what we have seen? τ versus $\dot{\gamma}$ information in one of the previous slides that we have seen that it is having either this form or it is having this form like this, right. So, that means, the material is not deforming, material is not deforming until and unless applied stress.

Applied stress crosses certain value let us say τ_0 I am calling and then it is a characteristics of the material, until and unless the applied stress is more than this one, the material is not deforming it is behaving as a plastic solid. And then, after crossing this applied stress after when the applied stress crosses this characteristics yield stress of τ_0 then it is deforming and then after that it may be showing a kind of a Newtonian behavior or it may be showing a kind of a shear-thinning behavior, right.

So, the change in viscosity is there, but only at two points, two viscosities. Initially, when it is not deforming it is having effectively infinite viscosity like solids. After that it may be having the viscosity according to nature of the behaviour, either decreasing apparent viscosity with increasing gamma dot or constant viscosity like in a Newtonian behaviour.

Both of them are possible, but there is a change, ok. At least two times change is there. If it is Bingham plastic, then two time two viscosity values are there. If it is you know yield pseudoplastic kind of material then it is having you know large number of variations as gamma dot increases.

So, that is these materials are characterized by existence of a yield stress tau naught which is the characteristics of the material. It is like a property of the material like viscosity density, thermal conductivity etcetera that material properties that are you are having in general.

So, similarly, τ_0 yield stress is also a material property for this viscoplastic fluids. And then what is the nature of what is the importance of this characteristic of the material, that if the applied stress, τ is applied stress, if it is less than this yield stress then material does not deform or flow en masse like a rigid body like a plastic solid material without deforming.

However, if applied stress τ is greater than this yield stress, τ_0, then material will deform and then having a shear stress versus shear rate curve, maybe linear like this one or may be non-linear like this one. Linear or non-linear, but does not pass through the origin, right.

And then, they do not level out under gravity to form an absolute flat free surface. Let us say you take water, water is a kind of Newtonian fluid and then you just drop it on a flat surface it will spread out. But if you take a viscoplastic material and then drop it on the same flat surface, it will not you know it will not level out, it will not level out like a paste. You take paste and then drop it on a flat surface it will not level out, until and unless you are applying certain higher stress.

Now, dropping on a flat surface, that means, you are applying gravity, you are applying gravity and then making that to drop on a flat surface. So, the accordingly there is a stress there, right. So, whereas, the other material which is not having plastic material that will spread out, that is spread out on the surface, ok. So, this is one of the other characteristic. And then this consist of 3D structure of sufficient rigidity to resist external stress which is less than τ_0.

So, when the external stress is less than τ_0, this material is resisting such external stress or applied stress. It is this material opposing that that applied stress to deform and then it is remaining like a solid.

For external stress greater than τ_0, that is when you applied stress is more than this yield stress the structure whatever the 3D structure of this material is there that breaks down and material behave like a viscous fluid. And then, buildup and breakdown of structure has also been found to be reversible, sometimes it is reversible, sometimes it may not be reversible, but there are a few cases where there reversible is there.

So, for example, like meat extract, may regain initial value of yield stress, ok. Let us say paste you are having like you know shaving cream or paste, it is like a viscoplastic material when you are applying with a brush; that means, you are applying external force and then there is a stress, right.

When the applied stress is more than the yield stress of that paste or shaving cream, whatever you have taken, that material start becoming thinner and thinner. It start flowing like a like a viscous fluid, though foams etcetera you can see, but it viscosity is decreased that is what you can see, ok.

So, but, however, when you stop this applied stress, that paste or you know shaving cream will not get its initial yield stress. So, there whatever the build up and then breakdown structure of this material is there, that is irreversible in such kind of material. But if you have a meat extract such kind of material, then it is possible that you know it may regain initial value of yield stress.

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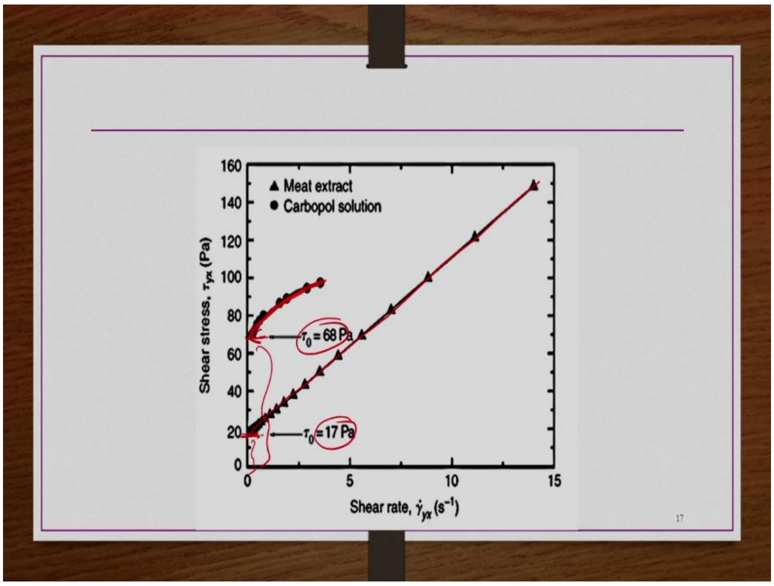

Some examples, we see two examples here meat extract and Carbopol solution. Carbopol solution if you obtain τ versus $\dot{\gamma}$information experimentally and you plots so then you can have a curve like this. And then you know that is it does not deform until and unless applied stress is more than 70 or 68 Pascal. It does not deform. It stayed as a kind of solid, ok.

But after this after apply after this crossing this yield stress, when the applied stress is more than this yield stress then material started deforming and then it has shown a shear-thinning behaviour like this, right. So, here tau naught is 68 Pascal. Up to 17 Pascal even this meat extract solution whatever is there that regained, that was not deforming, that was not deforming, and that that was not flowing.

Once the applied stress is more than 17 Pascal, it start deforming and then start flowing and then flowing displaying a kind of Newtonian behaviour of constant viscosity. So, the so, below the 17 Pascal, the apparent viscosity of this meat extract is almost effectively infinite.

Whereas, similarly for the Carbopol solution below the yield stress, below the 68 Pascal the material is having effectively infinite viscosity. Apparent viscosity is almost like infinite like in solids. So, now, what we do? We see a different possibilities of a viscoplastic behaviour.

Two possibilities we have already seen, after crossing that yield stress material is deforming and then showing a kind of a fluid like behaviour. So, that fluid like behaviour is having two behaviours that we have seen two natures it is showing. It is a linear curve and then another one is the concave downward curve like that we have seen.

So, whichever is the linear curve is there, after crossing the yield stress that curve, that kind of material you know we call them as a Bingham plastic material. So, that is one category. Another category, after crossing this yield stress if τ versus $\dot{\gamma}$ information is having non-linear like shear-thinning fluid concave downward kind of curve, then we call them yield pseudoplastic materials, right.

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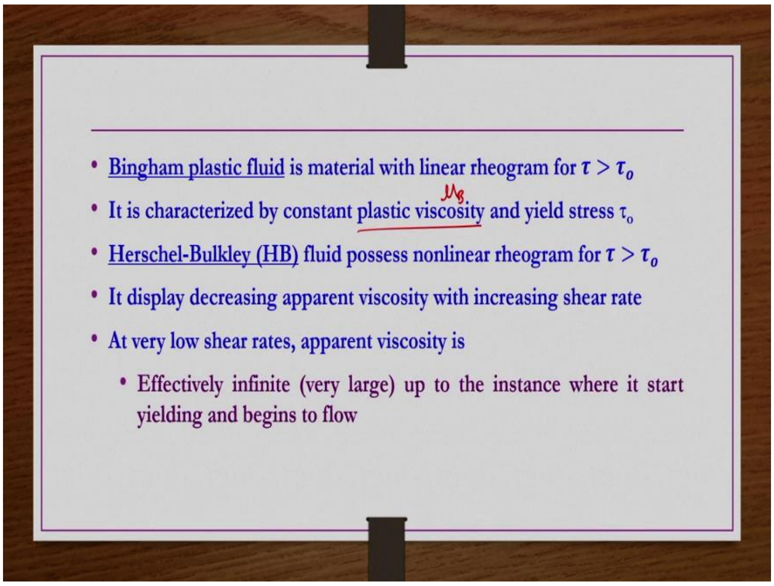

So, Bingham plastic fluid is material with linear rheogram for $\tau > \tau_0$. It is characterized by constant plastic viscosity and yield stress tau naught. Obviously, after deformation, when it start deforming it is having linear curve, so then it is the viscosity apparent viscosity is constant. That apparent viscosity we call it plastic viscosity which is in general represented by μ_B.

Then, Herschel-Bulkley fluid processes non-linear rheogram for $\tau > \tau_0$, and then it display decreasing apparent viscosity with increasing shear rate. Obviously, we have seen concaved downward kind of curve with increasing $\dot{\gamma}$ after crossing this yield stress, so that means, obviously, displaying decreasing apparent viscosity with increasing shear rate.

And at very low shear rates apparent viscosity is effectively infinity that is what we have seen. Effectively infinity very large up to the instance, where it start yielding and begins to flow from the point where the applied stress is more than the yield stress. Below that one it is having effective effectively you know infinite viscosity.

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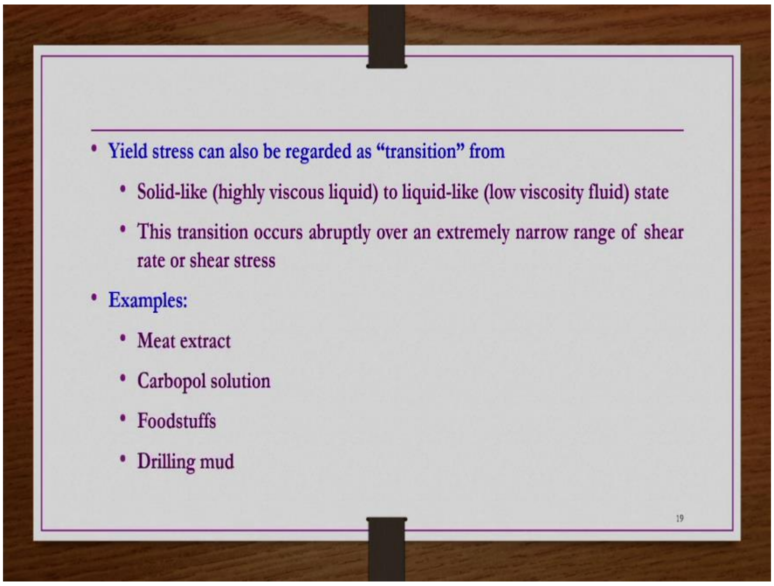

Yield stress can also be regarded as a transition from a solid like state to the fluid like state. Obviously, we can that way also we can say, right. So, yield stress can also be regarded as a transition from solid like highly viscous liquid to liquid like low viscosity fluid state. And this transition occurs abruptly over an extremely narrow range of shear rate. Some examples are meat extract, Carbopol solution, foodstuffs, drilling mud, etcetera.

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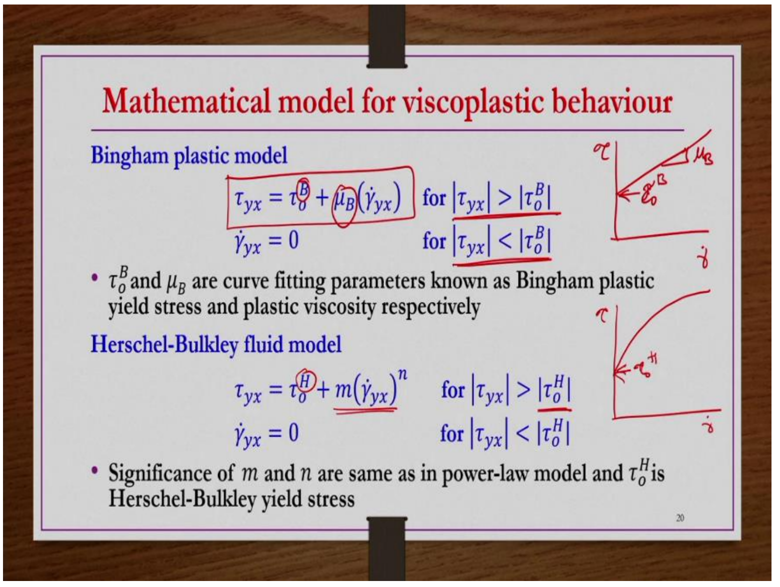

Now, coming to the mathematical model for viscoplastic behaviour. So, we start with Bingham plastic model which is very easy because for $\tau > \tau_0$, for $\tau > \tau_0$ it is having a kind of a linear behaviour that is $\tau_{yx} = \tau_0^B + \mu_B(\dot{\gamma}_{yx})$. So, that is this behaviour.

So, this here we are calling μ τ_0, the superscript B is for a indication of you know Bingham plastic nature, right, B, B stands for Bingham. It is not a superscript, it is not having any constant value like that, ok. And then after crossing this it is deforming and then the viscosity is constant μ_B.

However, for applied stress less than this yield stress τ_0^B, the material does not deform. If the material is does not deforming; that means, there is no shear rate that is $\dot{\gamma}_{yx} = 0$.

So, now, what you understand compared to the shear-thinning fluids here? You understand in order to represent viscoplastic materials, you have to write two equations, one equation for tau less than tau naught, another equation $\tau > \tau_0$, ok. $\tau < \tau_0$is always $\dot{\gamma}_{yx} = 0$ whichever model you take.

But $\tau > \tau_0$onwards, it depends on the model. If we are taking Bingham plastic model then we have this expression. This τ_0^B and μ_B are curve fitting parameters, known as Bingham plastic yield stress, and then plastic viscosity, respectively.

Next one is the Herschel-Bulkley fluid model. So, here again we need two equations. Herschel-Bulkley fluid model is like this, for a $\tau > \tau_0^H$, right, we have $\tau_{yx} = \tau_0^H + m(\dot{\gamma}_{yx})^n$.

So, pictorially once again if you draw this τ versus $\dot{\gamma}$. So, you have a information like this. So, this is τ_0^H. So, this superscript H again here indicates Herschel-Bulkley. It is not a constant, it is not having any value, ok. It indication, it is a kind of indication for this. So, that to differentiate one from other one.

And then after this, crossing this τ when applied stress is more than τ_0 it is having non-linear like a shear-thinning fluid, like a shear-thinning fluids. So, then that part is represented by the power-law model here.

So, Herschel-Bulkley fluid $\tau_{yx} = \tau_0^H + m(\dot{\gamma}_{yx})^n$, when $\tau_{yx} > \tau_0^H$. And then obviously, when $\tau_{yx} < \tau_0^H$, then there is no deformation. So, in obviously, this $\dot{\gamma}_{yx}$ is 0. Significance of m and n are same as in power-law model, and τ_0^H is the Herschel-Bulkley yield stress.

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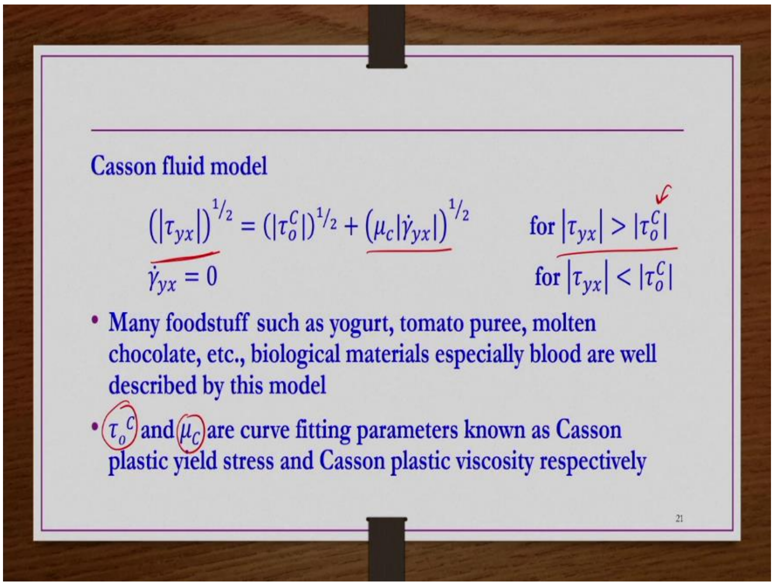

So, next model is the Casson fluid model. So, here we have exactly same like a Bingham plastic model, but each term in the model is having a power of 0.5 or square root. That is $\left(|\tau_{yx}|\right)^{1/2} = (|\tau_0{}^C|)^{1/2} + \left(\mu_C|\dot{\gamma}_{yx}|\right)^{1/2}$, when this $\tau_0{}^C$ is when the $\tau_{yx} > \tau_0{}^C$, right.

Again this C, superscript C, whatever is there that C stands for Casson, yeah, right. It does not have any value kind of thing. It is a kind of indication symbol. And then, obviously, the other condition is that when $\tau_{yx} < {\tau_0}^C$ $\dot{\gamma}_{yx}$ is 0 because there is no deformation, ok. So, many foodstuff such as yogurt, tomato puree, molten chocolate, etcetera and then some biological materials you know they especially like blood are well described by this Casson fluid model, ok.

This ${\tau_0}^C$ and then μ_C or curve fitting parameters known as the Casson plastic yield stress and then Casson plastic viscosity respectively, right.

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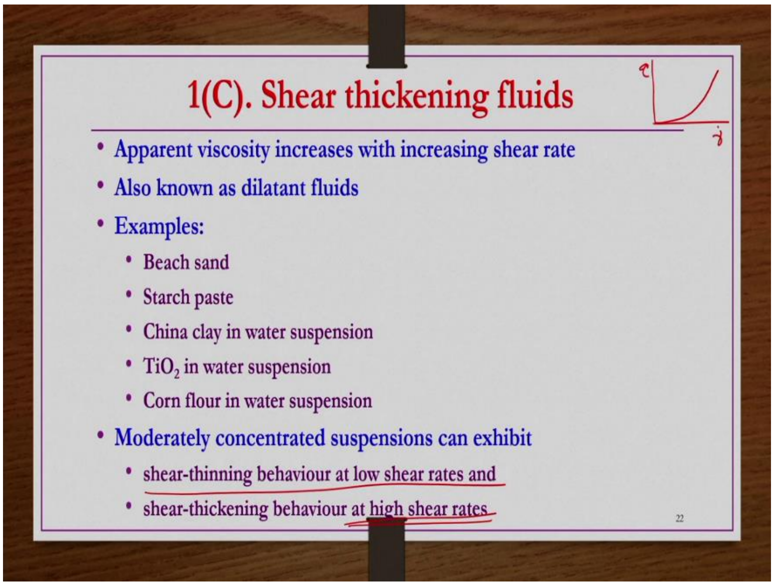

This is about a viscoplastic materials. The last category within the time independent non-Newtonian fluids is shear-thickening fluid. So, that shear-thickening fluids that we are going to see now. Shear-thickening fluid that we are going to see.

What we have seen for shear-thickening fluids? If you remember that τ versus $\dot{\gamma}$ you know it passes through origin, but you will be having concave upward curve like this, where the apparent viscosity increases with increasing the $\dot{\gamma}$. Apparent viscosity increases with increase in the gamma dot or shear rate.

So, apparent viscosity increases with increasing shear rate, also known as the dilatant fluids, and then some examples are beach sand. Beach sand, you know you might be having experience walking along the side of beach, right. When you are walking on a beach you can walk comfortably, right.

Rather, when you start running along the beach side then you will feel like you know you need to give more force and then you know higher energy is being consumed rather than when you are walking on a flat surface somewhere else, right. So, that is because this beach sand whatever is there when you apply the stress, when the more and more, when you apply more and more force what happens this particles are you know sand particles are coming closer to each other and then the viscosity apparent viscosity increases.

That is the reason you know to move your foot from one point to the other point in a faster manner then you have to give more force because the apparent viscosity of that beach sand increasing with increasing the shear rate. Starch paste is another example. China clay in water suspension, and then TiO_2 in water suspension, etcetera like that corn flour in water suspension.

Moderately concentrated suspensions can exhibit shear-thinning behaviour at low shear rates and shear-thickening behaviour at high shear rates which is also possible. As I was mentioning in a previous class that a given material, same material may be displaying more than 1 characteristics.

It may be displaying one Newtonian characteristic, one non-Newtonian characteristics under different ranges of a gamma dot. The same material may be displaying two different types of non-Newtonian behaviour under two different ranges of a shear rate that is also possible. And then some of the moderately concentrated suspensions, they show you know shear-thinning behaviour at low shear rates whereas, the shear-thickening behaviour at high shear rates, and then we see one example here, right.

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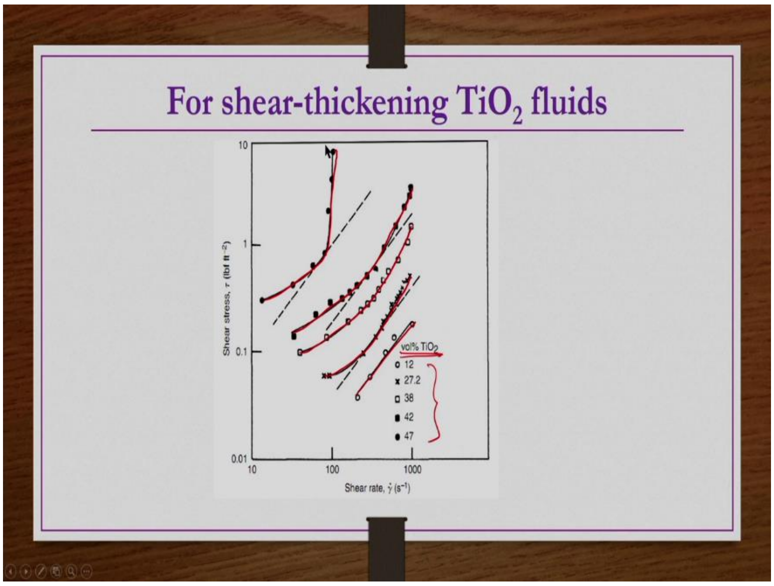

Now, here what you can see? The TiO_2 fluids like you know the suspension of different concentration have been taken, and then whatever the shear stress was a shear rate information is there that plotted here. And then the, when the concentration the volume percent of this TiO_2 increases different rheological behaviour you got and then you can see here.

When concentration is a small, so it is having a kind of a straight line like this behaviour. And then concentration increases gradually, so then you can see this shear-thickening behaviour like this, like this, and then here again like this it is increasing. But here again you will see, it is increasing like this and abruptly it is increasing like this, ok.

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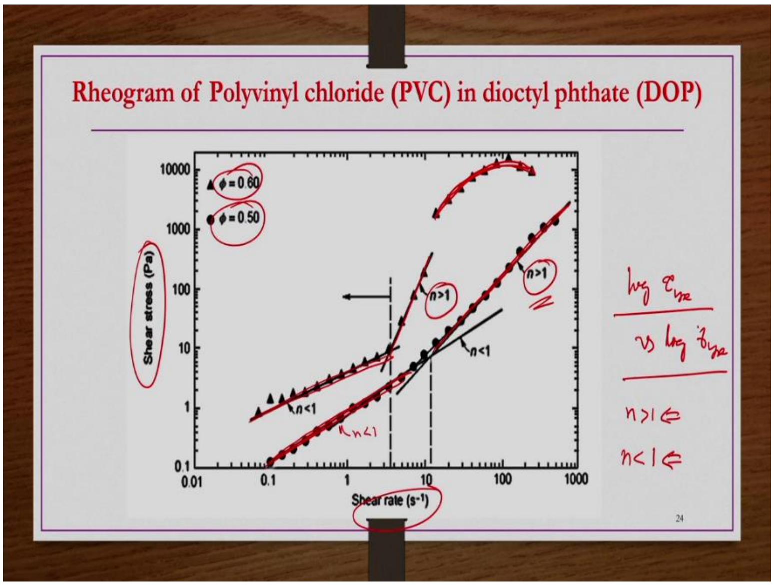

Now, this is the example. PVC in dioctyl phthalate where the material is showing a Newtonian, the material is showing a shear-thinning behaviour in low shear rate range and then shear-thickening behaviour in high shear rate range. So, this is the example.

When you take the 50 percent concentration of PVC in dioctyl phthalate, then we have this curve. You can see this curve this curve actually it is again plotted on a log-log scale, right. So, then you are getting straight line. So, then this curve you can see, some range up to certain range you know you are having low shear range, you are having the you know Newtonian behaviour n < 1.

After you know shear rate of 10 onwards you what you are having you are having again straight line you are having again straight line, but n > 1. That is shear-thickening behaviour. Slope greater than 1, for a log τ_{yx} versus log $\dot{\gamma}_{yx}$ when you are plotting, so then, obviously, for this materials you will get straight line. But the slope is greater than 1, then it is a shear-thickening fluid it is a. Slope is less than 1 then it is shear-thinning fluid.

So, you can see at small shear rate range it is having shear-thinning behaviour and then higher ranges, higher range of shear rate it is having shear-thickening behaviour. Whereas, the same material, but if you increase the PVC concentration to the 60 percent, then what you have? Here you have at low shear rates you are having shear-thinning behaviour up to shear rate of approximately 3 or 4.

And then after that it is having a shear-thickening behaviour because slope is greater than 1, but again after 10 second inverse we can see altogether different curves, different nature it is showing here. It is not even straight line. So, that means, again that is altogether showing a different non-Newtonian behaviour.

So, it is possible that the same material may be displaying more than 1 type of a non-Newtonian behaviour, in addition to Newtonian behaviour is also possible, like one of the example is shown here. Finally, mathematical model for shear-thickening fluids, right.

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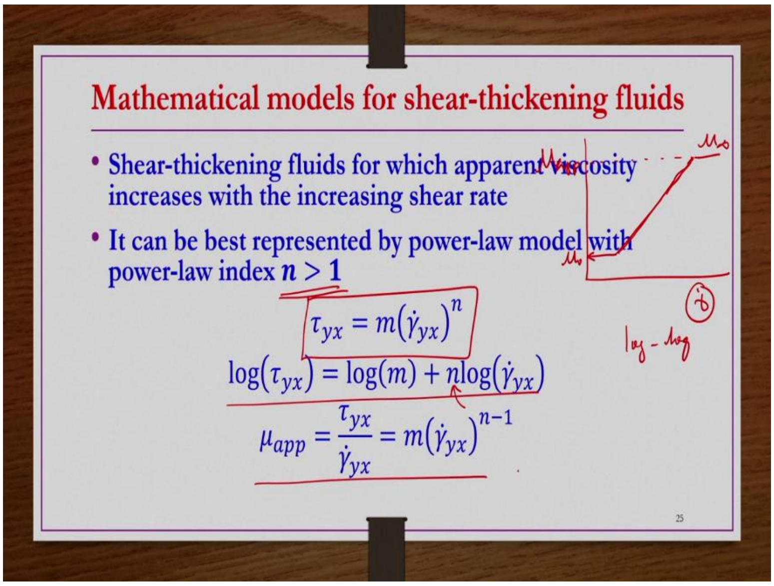

So, let us say for shear-thickening fluids, if you plot this μ_{app}versus $\dot{\gamma}$, right, then what you have, for like you know we are doing on log-log scale anyway, like we have done for you know shear-thinning material. So, then at low shear rate it is having it may be showing some constant viscosity mu naught, like a Newtonian behaviour.

After that you know disproportionately it increases, it the apparent viscosity increases with increasing the shear rate. And then at high shear rate again it may be displaying some constant viscosity μ_∞. So, now, what happens here? As $\dot{\gamma}$ increasing the apparent viscosity is increasing from μ_0 to μ_∞ in the case of shear-thickening fluids.

Whereas, in shear-thinning fluids, what happened? As $\dot{\gamma}$ increasing the apparent viscosity was decreasing from $\mu_0$to μ_∞. It is a reverse trend here, ok. For the case of shear-thickening, it is reverse trend, reverse opposite trend like opposite trend to the shear-thinning that we can see. Exactly opposite, rest everything is same.

So, shear-thickening fluids for which apparent viscosity increases with the increasing shear rate. It can be best represented by power-law model, but with n greater than 1, slope greater than 1, ok

So, the best model is power-law model for shear-thickening fluids that is $\tau_{yx} = m(\dot{\gamma}_{yx})^n$. And then on the log-log scale if you plot them, then you can get a straight line, but this slope will be greater than 1. But this, but this slope n would be greater than 1 for shear-thickening fluids, whereas, for shear-thinning fluids the slope would be less than 1.

Apparent viscosity is again defined exactly same way like in shear-thinning fluids that is $\frac{\tau_{yx}}{\dot{\gamma}_{yx}} = m(\dot{\gamma}_{yx})^{n-1}$.

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The references for this lecture are provided here.

Thank you.