Transport Phenomena of Non-Newtonian Fluids
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Lecture - 19
Transition and Turbulent Flow of GNF in Pipes – 2

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Transition and Turbulent Flow of Generalized Newtonian Fluids in Pipes, part 2. So, before going into the details of today's lecture, what we will have? We will have a kind of recapitulation of what we have seen in the last class, and then we see a couple of example problem today based on the things that we have discussed in a last class. Then, we start moving to the new topics of the day.

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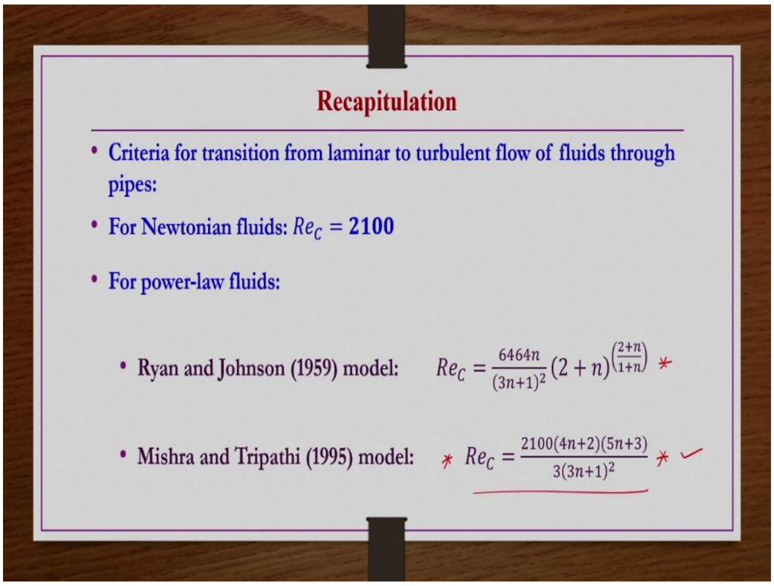

In the previous class, we have seen criteria for transition from laminar to turbulent flow of fluid through pipes. So, if the fluid is Newtonian, so then we already know that it is its critical Reynolds number for transmission of flow from laminar to turbulent is 2100. Whereas, what is that for the case of power-law fluids what is that critical Reynolds number for the case of viscoplastic fluids, etcetera, how to find out them, we have seen, right.

So, for Newtonian fluids we know that critical Reynolds number is 2100. For power-law fluids we have a several correlations out of which two we have discuss. So, one is by Ryan and Johnson model who has proposed the critical Reynolds number as function of n and then that empirical equation is given like this.

Then, Mishra and Tripathi have also developed empirical correlation based on their experimental results and then they have given this correlation for critical Reynolds number which is again function of n. However, this correlation by Mishra and Tripathi seems to be more reliable because as n decreases the critical Reynolds number gradually increasing that is that trend is followed.

But, however, in the case of Ryan and Johnson, we have seen that as n decreases critical Reynolds number increases up to certain value of n like 0.3 or 0.38. And then after that if you further decrease the n value critical Reynolds number was found to decrease, which is not reliable, which is not acceptable from the fundamental viewpoint. So, this seems to be more reliable, ok.

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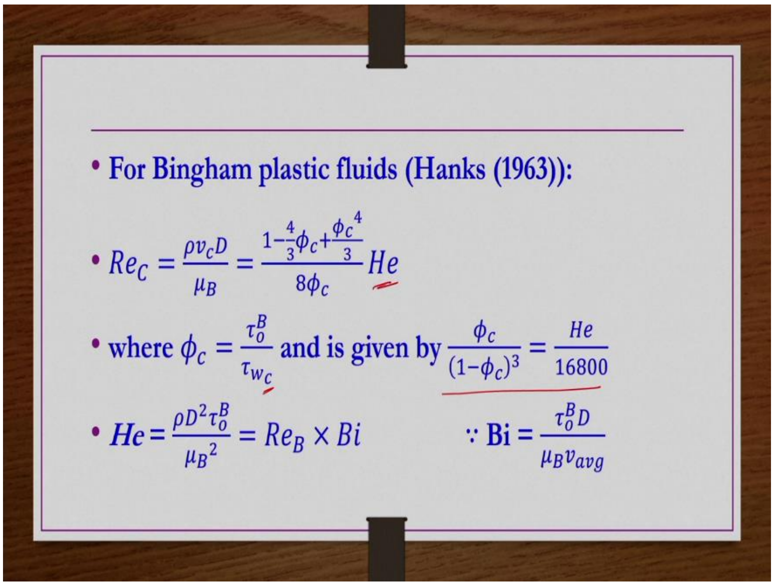

Then, coming to the viscoplastic fluids, under the category of Bingham plastic and then Herschel-Bulkley fluids we have two different models. One is the Hanks model which is for the Bingham plastic fluids, where critical Reynolds number has been reported as function of Hedstrom number and then ϕ_c value. ϕ_c is nothing but the ϕ at critical Reynolds number, ok. ϕ is nothing but τ, $\frac{\tau_0}{\tau_w}$ that we know, ok.

So, now if you wanted to know critical Reynolds number for Bingham plastic fluids. So, then you have to know the Hedstrom number which is possible because the properties of the material and then dimensions of the geometry are in general available, so then you can find out. But ϕ_c is not available.

So, for that reason they have also provided a relation ϕ_c as function of Hedstrom number and then that is given by this one. Whereas, ϕ_c is as mentioned is nothing but $\frac{\tau_0^B}{\tau_{w_c}}$, c stands for at critical condition, so critical Reynolds number, right.

So, now, if you know the Hedstrom number you can find out the ϕ_c and then once you know both phi c and Hedstrom number you can find out the critical Reynolds number, right. So, what we have seen? We have seen an example problem as well based on this model in the previous lecture.

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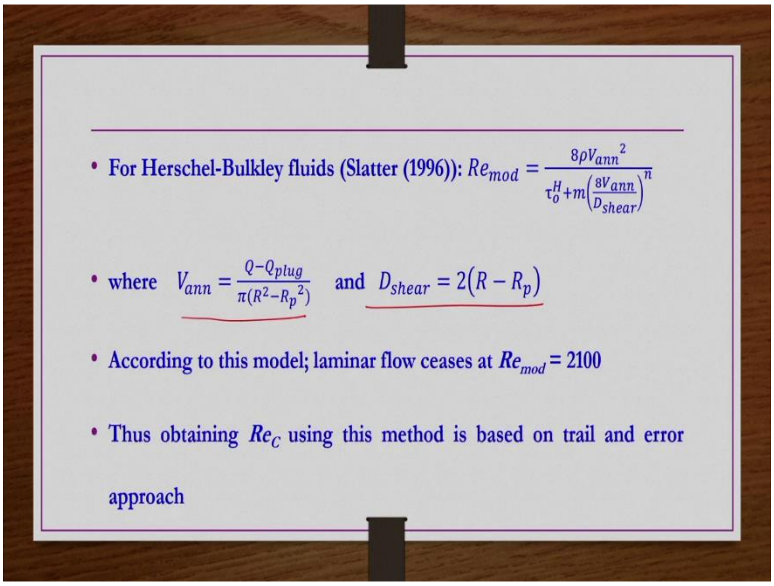

Then, Herschel-Bulkley fluids Slatter has defined a modified Reynolds number based on the angular velocity or the velocity of the fluid in the deforming region only because viscoplastic fluids, what we know? We there we know that the flow geometry is or the flow cross section is divided into the two fractions, one is the deforming fraction, another one is the non-deforming fraction.

Non-deforming fraction towards the center of the pipe where the shear stress or the applied shear stress is less than the yield stress. And then, when the applied stress becomes more than the yield stress then there will be a deforming region. So, two regions we know.

So, since the flow is taking place only certain region. So, then the velocity should be taken only for that region that is for their you know argument and then based on that one they have defined or Re modified or modified Reynolds number and they propose this Reynolds number has to be less than 2100 for a Herschel-Bulkley fluid to be in laminar flow conditions, right.

V_{ann}, we can find out using this one, and D shear is nothing but diameter of the pipe minus diameter of the plug, ok. Then, we have also seen couple of example problems, how to use these things; and obviously, what we understand that V_{ann} is not known. If you wanted to know that one you should know R_p, so which is not known a priori. So, then for that region we have to go for a trial and error approach, that is what we have seen, and then we have seen couple of problems as well.

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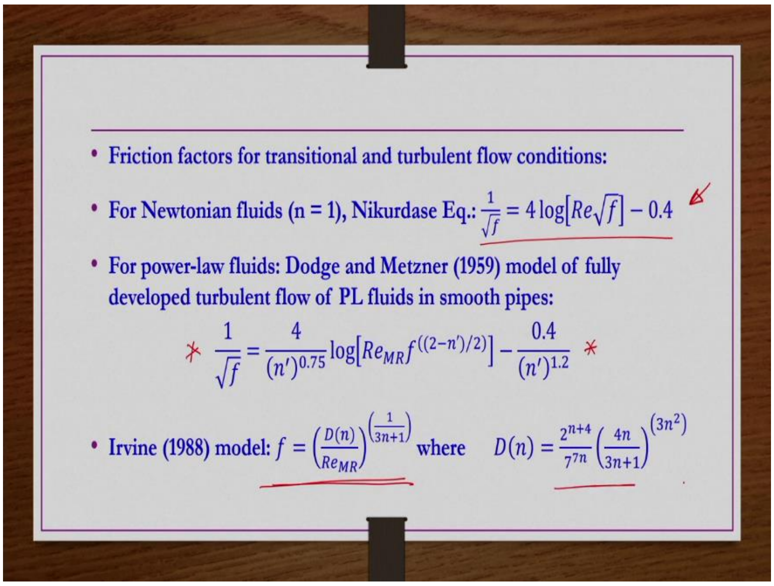

Then, friction factors also we have obtained. Friction factors for the transitional and turbulent flow conditions in smooth pipes, right. If it is Newtonian fluid then we know that Nikurdse equation for friction factor is this one, right. But what is if the fluid is power-law fluid? Then we have according to Dodge and Metzner. They have developed similar equation which is similar to the Newtonian case and then that is given by this one, right.

Now, in this equation if you substitute n’ which is nothing but power-law behavior index as equals to 1, so then you get back to Newtonian correlation whatever given here, right.

Then, Irvine have proposed altogether a very different expression for friction factor which is not having any relation with the form of a corresponding Newtonian friction factor correlation, ok. So, that correlation by Irvine is given by this equation. $\left(\frac{D(n)}{Re_{MR}}\right)^{\left(\frac{1}{3n+1}\right)}$, whereas, this D function of n is given by this function, right.

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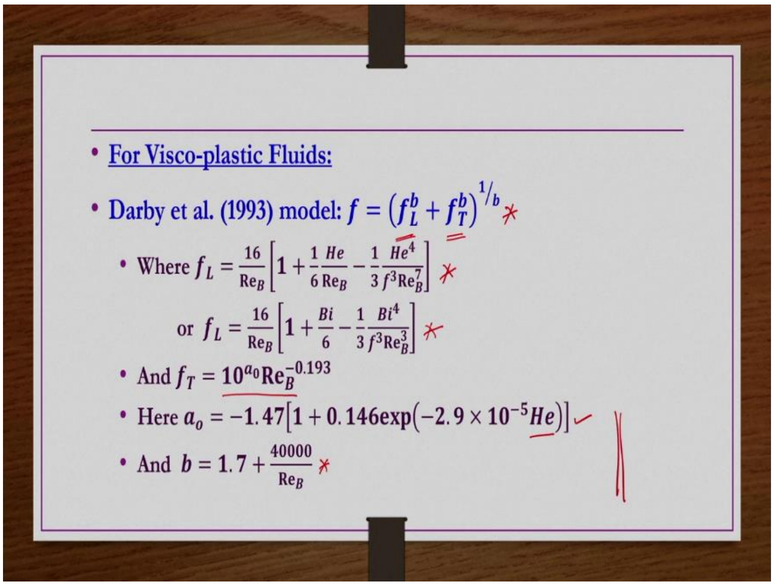

Then, we have also seen for a viscoplastic fluids how to find out the friction factor if the flow is under transitional or the turbulent flow conditions. So, Darby, what they have proposed? They have proposed friction factor, the overall friction factor is having two components that is the friction factor because of the laminar flow conditions and then because of the turbulent conditions.

Rather, laminar or turbulent conditions, whatever the contribution is there from the f_L expression that should also be included and then the that f_L is nothing but whatever the friction factor that we have developed previously for viscoplastic fluids flowing through pipe and then that is given by this one, right. So, this is for the Bingham plastic fluid, right. And this can also be written in terms of Bingham numbers like this. This is also we have derived.

Whereas, f_T is nothing but the friction factor because of the contribution from the turbulence and then that is given by this expression $10^{a_0} Re_B^{-0.193}$. Whereas, a_0 is function of Hedstrom number and then given by this expression, right. And then this here the power b, whatever is there in this equation, right, so that b is function of Re_B and then given by this expression.

So, till this point we have seen in last lectures. So, now, what we do? We take a couple of problems based on these expressions and then solve couple of problems then we go into the details of next lecture, today's lecture.

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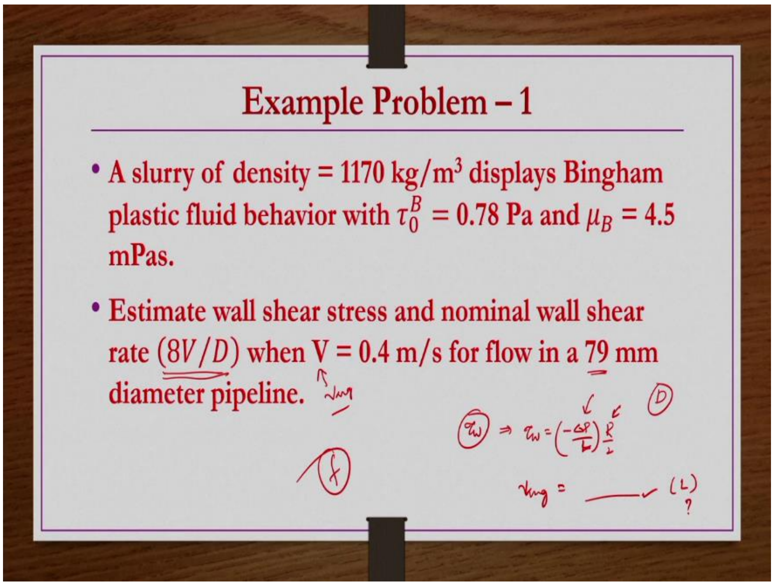

Example 1, a slurry of density 1170 kg per meter cube, this displays Bingham plastic fluid behavior with $\tau_0^B = 0.78$ pascal and μ_B = 4.5 millipascal seconds. Estimate wall shear stress and nominal wall shear rate when V = 0.4 meter per second for flow in a 79 mm diameter pipeline.

So, D is given. This V is nothing but the V_{avg} in general whatever we are taken. So, if you wanted to know nominal or apparent wall shear rate that is nothing but $\frac{8V_{avg}}{D}$, so that you can directly find out because V is also given, D is also given as 79 x 10^{-3} meters. So, you can substitute, you can get it directly.

But how to get the wall shear stress τ_w? If you wanted to know wall shear stress, what is the expression that we have developed for pipe flow? $\left(\frac{-\Delta p}{L}\right)\frac{R}{2}$, right. So, now here R is given, D is given, so R is known, but L is not given. So, if L is not known, so even if you know Δp you cannot know all shear stress because V_{avg} is given. So, V_{avg} expression for you know Bingham plastic fluids etcetera that we have already developed, so that we can use here. So, but that also is having values of L, so which is not known.

So, calculating $-\Delta p$ from either of expression V_{avg} or anywhere else and then finding out τ_w is not possible. So, what we can do? Why it is not possible? Because in those corresponding equations L value is required which is not given. So, what we can do now here? So, we have to find out friction factor; because if friction factor is known, friction factor $f = \frac{\tau_w}{\frac{1}{2}\rho v^2}$. So, f is known, V is given and then τ_w you can find out.

So, now, what we do? We use the Darby's equation $f = \left(f_L^b + f_T^b\right)^{1/b}$. And then, we find out all those values, f we find out, then we get τ_w value that is what we are going to do here.

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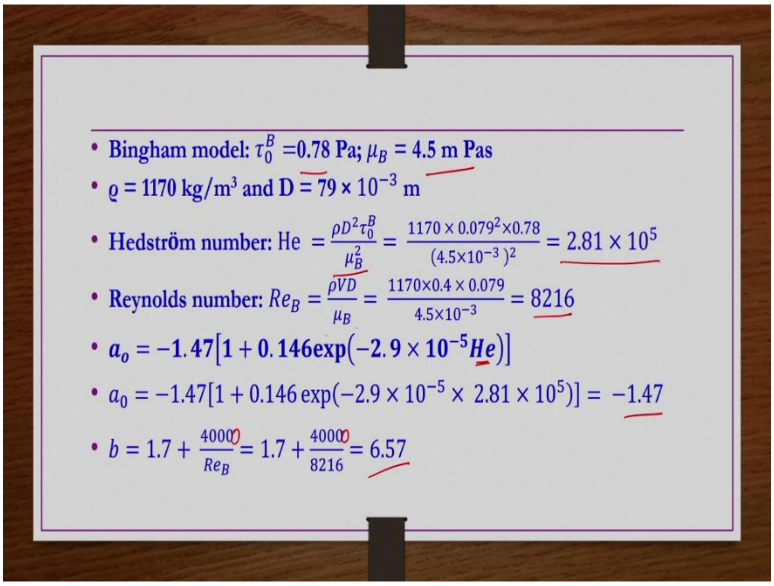

So, for Bingham model τ_0^B, μ_B values are given like this. These values are given ρ is given and then D is also given. So, according to Darby model we have to find out f, right. So, f you wanted to find out, so then what you have to find out? You have to find out Hedstrom number, you have to find out the Reynolds number, you have to find out a naught, you have to find out B value etcetera those things you have to find out.

So, first what we do? First, we found we, first we will find Hedstrom number which is given by this value. So, now, here ρ, D, τ_0^B, μ_B everything is given. So, substitute simply here and then get the Hedstrom number value as 2.8 x 10^5.

So, once it is known, so the Reynolds number can also be found easily because Reynolds number is nothing but $\frac{\rho v_{avg} D}{\mu_B}$, V_{avg} is given, D is given, ρ is given, μ_B is given. So, substitute, you get 8216 as the Reynolds number for Bingham plastic fluid in this case, alright

Then, a_0 we have to find out a_0 expression is given like this according to Darby's model. So, here also Hedstrom number we found. So, then simply substitute that Hedstrom number here find out a_0, so then you get -1.47 roughly, right. So, then B, you have to find out B is 1.7 + $\frac{40000}{Re_B}$, so 40000, here, here also, one more 0. So, 1.7 + $\frac{40000}{8216}$ you get 6.57. So, if you know a_0, B, etcetera, so then you can find out all other things.

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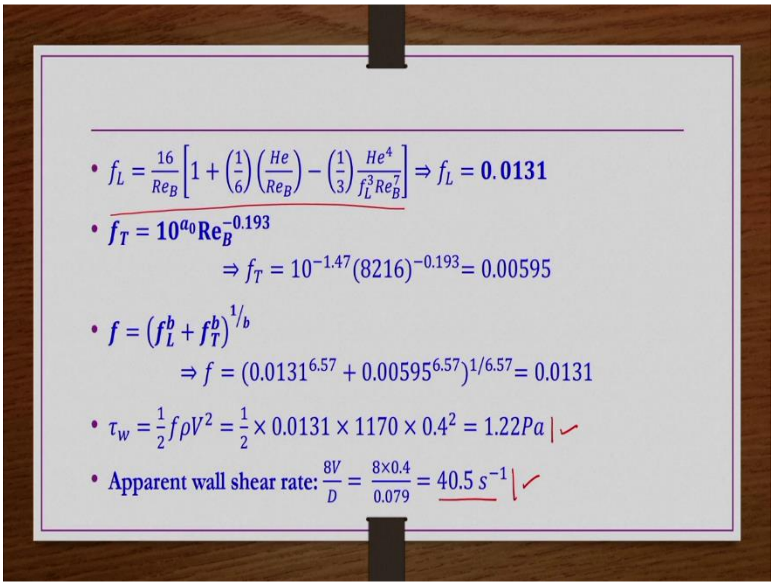

f_L you can find out. f_L we have derived this expression for Bingham plastic fluids. The suffix L stands for nothing but laminar flow. For the laminar conditions in week 3 or week

4, we have developed this expression for the friction factor of a Bingham plastic fluid flowing through pipe.

This is what we already found. And then we found that this expression is implicit because right-hand side also f_L terms are there. So, there while deriving this expression we did not use suffix L because specifically we are using a laminar flow condition. But now in this in this case, you know we are also having the turbulent contribution. So, in order to distinguish them we have f_L here in order to distinguish it from f_T that is because of the turbulence.

So, now, this equation when you solve by trial and error approach you get f_L = 0.0131, right. And then f_T is nothing but $10^{a_0} Re_B^{-0.193}$. So, a_0 we found already, Re_B we got 8216. So, then f_T you will be getting 0.00595. So, overall f is nothing but $\left(f_L^b + f_T^b\right)^{1/b}$. You substitute f_L f_T here along with the B values and then simplify f you get 0.0131.

So, once f is known, τ_w you can find out, $\frac{f\rho v^2}{2}$, so that you get 1.22 pascals, that is the wall shear stress, right. Then apparent wall shear rate $\frac{8V}{D}$ that you get. 8 V is given 0.4 and D is 79 mm, so you get 40.5 second inverse apparent wall shear rates, right.

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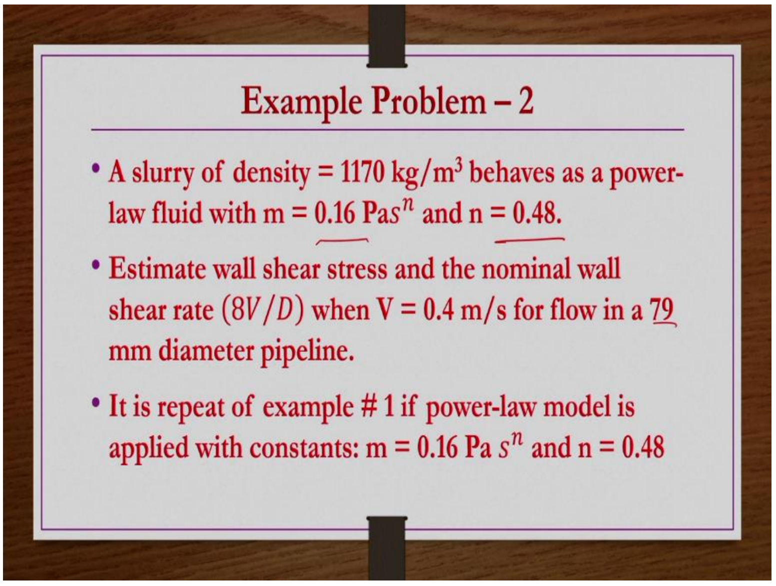

So, now we take another example problem. So, a slurry of density 1170 kg per meter cube behaves as a power-law fluid with m = 0.16 pascal second power n and n = 0.48 as its constants, right.

Then, estimate wall shear stress and the nominal wall shear rate $\frac{8V}{D}$ when V = 0.4 meter per second for the flow in a 79 mm diameter pipeline. So, basically it is the repetition of the example 1, but now the fluid is assumed to behave as a power-law fluid with the m and values are given like this.

So, now, here also what we have to do? We have to find out the friction factor. If you wanted to find out the friction factor, what you have to do? You have to first find out the Reynolds number.

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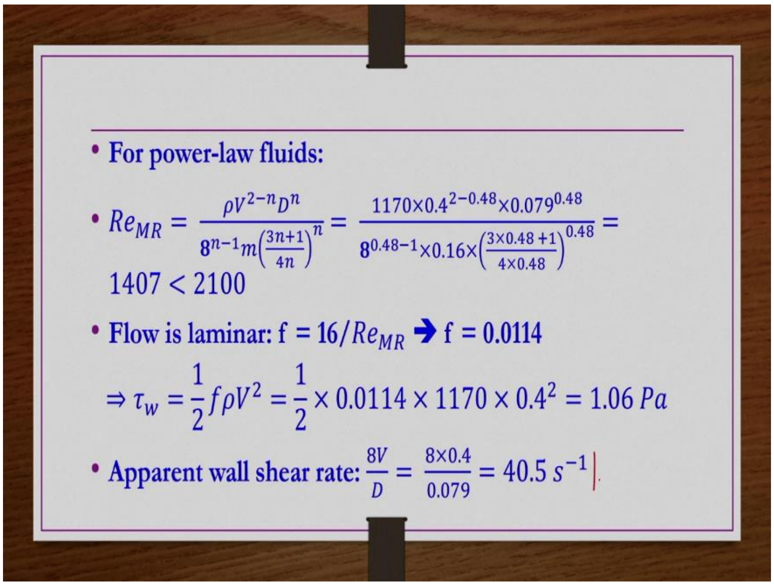

For Reynolds number for the case of power-law fluid, we have this expression Re_{MR}, and then here in this expression ρ, V, D, and m everything is given. So, then when you substitute all of them you get Re_{MR} 1407 which is less than 2100. So, then flow is under the laminar flow conditions, alright.

Then, what is f? f is nothing but $\frac{16}{Re_{MR}}$. So, then you get f = 0.0114 for the case of power-law fluid. So, once f is known, τ_w you can find out. When you do it you get τ_w is 1.06 pascals. Whereas, the apparent wall shear rate remaining same as in the previous case, because here V D are also same. So, now, what we do? We go into the details of velocity profiles measurement in turbulent flow of power-law fluids in pipes.

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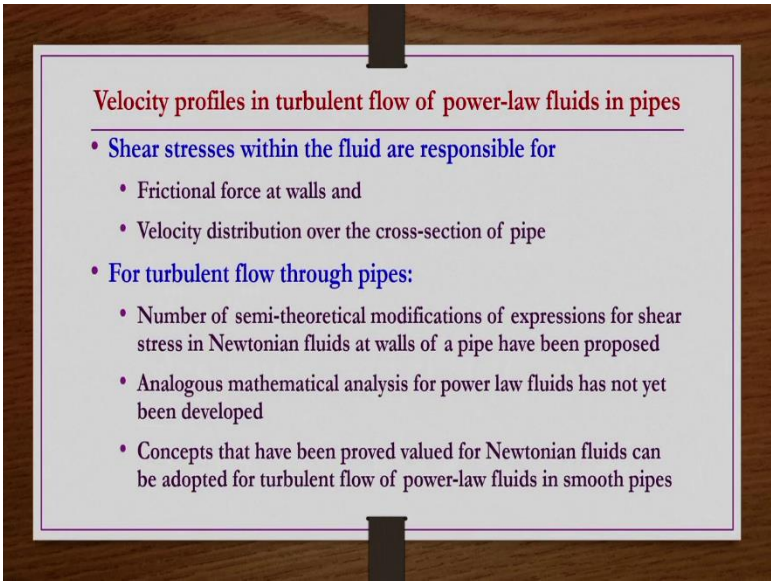

So far, we have taken only pipe flow geometries. Under the pipe flow geometric conditions if the flow is laminar. So, different types of generalized Newtonian fluids, power-law fluid, Bingham plastic fluid, Herschel-Bulkley fluid, Ellis model fluid, etcetera when they are flowing through pipes under laminar flow conditions, we have seen how to obtain the velocity profile volumetric flow rate expression and then friction factor expressions, etcetera, those things we have seen.

But what if the flow is not laminar, if it is under turbulent flow conditions, how to find out the velocity profiles? Is it really possible? That is what we have to see analytically, is it really possible. If it is not possible, how we can make some kind of an approximations and then try to get a velocity profile for the case of turbulent flows as well, right.

So, actually turbulent flows are very chaotic kind of flows and then there are a different size of eddies are there. So, they lead the flow to be very randomly changing in all 3 directions of the geometry. So, then it is very difficult to find out the velocity profile for such conditions even numerically. So, analytically it becomes even much more difficult.

But however, what happens in general, the gradients are much more important. At the pipe wall what is the velocity profile that is much more important in general, compared to the what is the overall velocity distribution across the cross section and then across the flow direction, etcetera. So, rather than that one, what is the velocity gradient at the wall is much more important from engineering point of view?

So, now, that point we take under consideration, and then making thus making that point under consideration, we try to make some simplifications, some assumptions, or divide the flow into different categories and then we try to obtain the velocity profile for the case of turbulent flow. But, the velocity profile close to the wall only we are going to find out.

Why? Because close to the wall because as I already mentioned the velocity gradients are in general much more important compared to the individual velocity at different locations so right so, from engineering applications point of view, right.

Similarly, if it is heat transfer so, then temperature gradient at the wall is much more important compared to the individual temperature in the remaining area because at the wall whatever the temperature gradient is there, that is going to show influence on the overall heat transfer rate, etcetera.

Same thing here in the case of moment on transfer, what is the velocity gradient at the wall that is much more important, ok. So, that concept we keep in mind and then try to develop some simplification to get some velocity profiles even for the turbulent flow, but the velocity profile close to the wall that is what we are going to do.

So, in general, shear stresses within the fluid are responsible for frictional forces or friction factor whatever that you wanted to measure, you know you need to know the shear stress. Once you know the shear stress then only you can find out. And then also if you know the shear stress you can also find out the velocity distribution across the cross sectional area, right.

So, but how to get the shear stress? If it is simple laminar flow, then you know shear stress expression that we have already seen for different types of fluids. So, then it they can also they can easily be used and then get the flow rate. You can get we can easily get the velocity distribution, and then friction factor etcetera, those things we can get. But can we use the same shear stress expressions here also or is there any additional term is coming into the picture, those things we have to discuss if the flow is turbulent.

So, for turbulent flow through pipes, in the case of Newtonian fluids number of semi-theoretical modifications of expression for the shear stress at the walls of pipe have been proposed, right. So, what are those that we have to see, right. We can we cannot go all those semi-theoretical modification.

We take one expression for the shear stress for the Newtonian fluids at the wall, in the case of turbulent flow through pipes. So, that expression we try to take and then we try to simplify. But such kind of information for the case of a power-law fluids has not yet been developed in law.

However, whatever the concepts that have been applied for the case of a turbulent flow of a Newtonian fluid through pipes are there. So, then those concepts may also be analogously used for the case of power-law fluids and then we can develop certain kind of velocity profiles as we have done, as we are going to do for the Newtonian fluids as well.

So, what we are going to do? We are going to how; we are going to use the well-established concepts of a turbulent flow of a Newtonian fluids, what is the corresponding shear stress at the walls when the fluid is flowing through pipes. So, those expressions we try to write, and then simplify for the case of Newtonian fluids and then get the expression for the velocity profile. So, those concepts we are analogously going to use for the case a power-law fluid also.

So, first what we are going to do? We are going to see basics of this you know flow regions etcetera for the case of turbulent flow through pipes, especially with the aim to get the frictional forces or the shear stress at the wall, right or close to the wall, ok. From those point of view, we are going to have a discussion. We are not going to have a discussion in the rigorous manner of whatever the rigorous chaotic turbulent flow is taking place.

So, then in general what happens in the case of Newtonian fluid flow through pipes, and then flow is under turbulent conditions. What we can have? We can divide the flow into, we can divide the flow into 3 categories or 3 regimes we can categorize.

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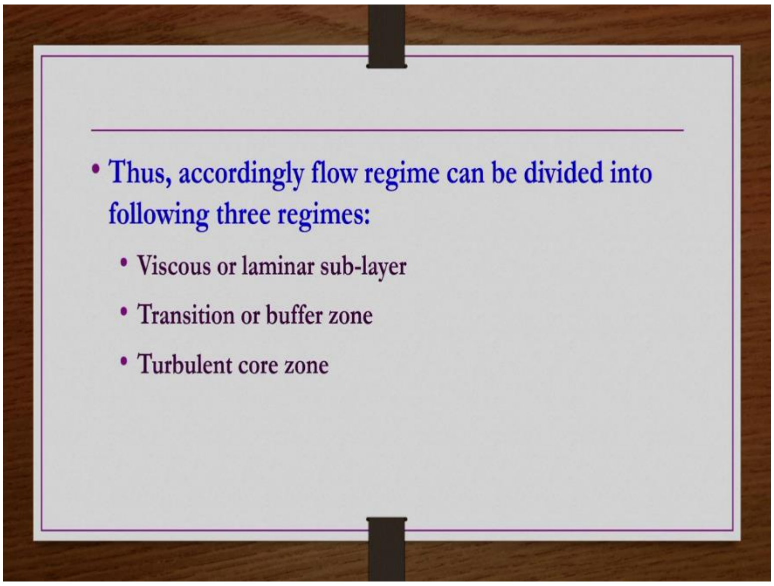

One is the laminar or viscous sub-layer which is at the wall which is close to the wall of the pipe. Another one is the turbulent region which is at the center central core of the pipe or the central most of the central core of the pipe cross section is under turbulent conditions.

And then between this turbulent region and then laminar sub-layer regions there is a transition region. So, transitional or buffer zone, and then finally, turbulent core zone. These three are there in general. So, pictorially if you see we can have like this, right.

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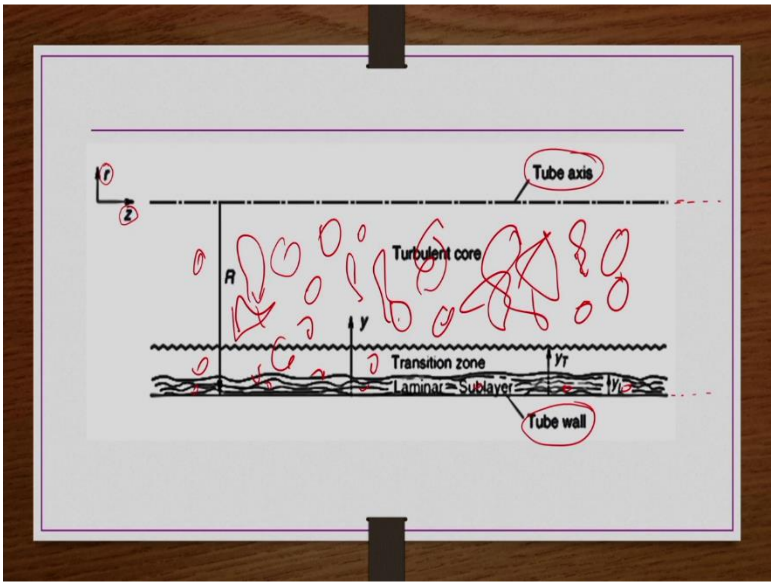

So, now, what we have? We have a pipe the tube. So, center axis of the tube is this top line. We have taken only half of the side of the tube, half cross section only we are we are taking two-dimensional and then we are out of this two-dimensional only, we are out of this 2D pictorial representation also, we are taking only lower half of the pipe.

So, now coordinate system if you see the flow direction is z-direction and then vertical direction is r-direction. And then bottom line whatever is there that is nothing but the tube wall. Top line is nothing but the tube axis. In between the flow is taking place, right. So, now the flow is prevailing flow under these conditions are you know turbulent conditions. So, then you have a bigger bigger eddies, you know smaller eddies kind of things are in general possible everywhere within the pipe, right.

So, they are very different in the size, shape, etcetera you cannot generalize them, right. So, they may be forming immediately breaking down those kind of things are in general possible when it comes to the turbulent eddies like this, right. So, then not necessarily be circles are shown, they very different way, it is, we cannot generalize them.

So, these eddies are everywhere in the pipe cross section. It is not that they are only at the turbulent core, in the middle of the turbulent core. But their frequency, their influence is more at the central cross section of the pipe that is towards the turbulent core. Whereas, their intensity is there and then frequency is very less at the laminar sub-layer, ok. So, that we may or we are going to make use, ok.

So, now, this turbulent eddies whatever are there, they are more frequent and then very chaotic in the main turbulent core. But they are infrequent and their influence is very small in the laminar sub-layer.

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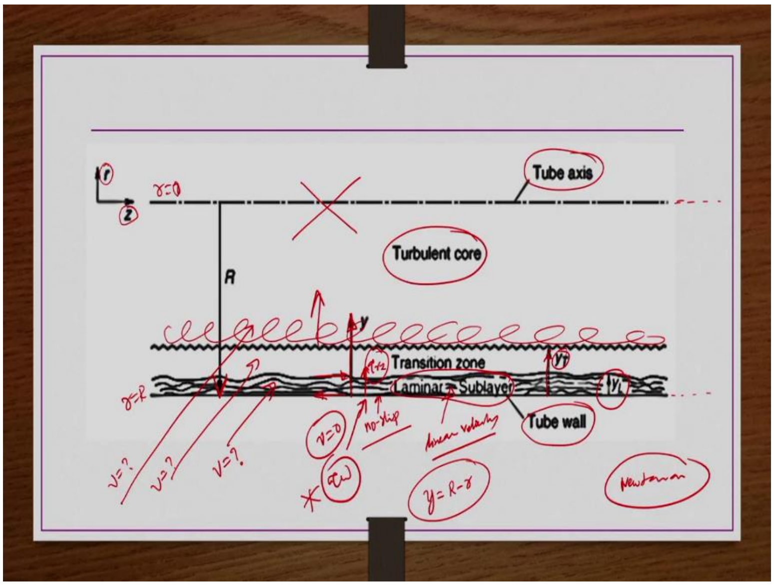

Why it is? Because, so at the tube wall what we have in general? We have a no slip condition; that means, the very first layer attached to the tube wall is having velocity 0, right. So, even the velocity at the center turbulent core is you know very high velocity, but at the wall the very first layer of the fluid attached to the wall is having 0 velocity, right. So, irrespective of what is the other velocity at the center of the pipe, ok.

So, even at the center of the pipe we have the chaotic turbulent flow, at the wall we are having 0 velocity because of the no slip conditions, right. And then the flow is in the z-direction here, ok. So, now, what happens? As you move along in the radial direction from wall towards the center, you know velocity gradually increases.

But there is some region, so then where the velocity is very very small, right. Rather, velocity is very very small what we can say; so, in there is a region where the shear stress is you know change because now the flow is in z-direction. So, shear stress whatever this τ_{rz} is there, so that is also gradually changes as we move, as we change the value of R, from r = R to r = 0 if you move, in terms of r if you write this is r = 0, this is r = R.

So, from r = R shear stress whatever is that τ_w is maximum value that we know. This maximum value gradually decreases as you move towards the center of the pipe. But we can separate out the layer where the change in the shear stress some maximum tau values to decreasing, that decrement is very very small to certain layer only, right. So, that layer we are separating out and then we are calling it laminar sub-layer which is having certain thickness y_L, right.

So, this y coordinate we are drawing you know in a reverse direction of R, ok. So, R is you know increasing to from center to the wall, whereas the y is increasing from wall to the center, ok. So, just for our convenience for easiness we are drawing like this way, ok.

So, now, at the close to the wall of the pipe, what we are having? We are having this laminar sub-layer under which the change in shear stress, whatever the change in shear stress is very very small, because the flow is in z-direction and then the velocity at the wall is 0. And then that velocity is gradually increasing, but the close to the wall the layers fluid layers are having a very small velocity.

So, then what we have this, that; why they why the fluid layers close to the wall, they are having very small velocities? Because the change in shear stress is very very small very very small. Compared to the τ_w value compared to τ_w it gradually decreases, but that decrement is very very small that we can neglect. So, that we can assign one single constant value of tau w are shear stress tau value for that laminar sub-layer.

If the shear stress is constant for a given region, so then obviously, shear rate is also going to be constant. If both of them are constant, so then velocity profile is expected to be linear, linear velocity profile, right. So, that means, whatever the retarding force at the wall is there, so the retarding shearing force at the wall is these.

So, that is counter balanced by the accelerating shearing force at the top layer of the laminar sub-layer, right. So, that is counter balance and the net shear stress is going to be negligible within this laminar sub-layer because the flow is dominating in the z-direction and then within this close to the wall, this region the change in shear stress is very very small. And then net shear rate or net shear stress is 0 here. So, if net shear stress is 0; that means, we can assign one single constant value of the shear stress. Let us call it that τ_w, ok.

So, if the shear stress is constant, shear rate would also be constant and then accordingly the velocity profile has to be linear. If both shear stress and then shear rate has to be constant, so then velocity profile has to be linear, then only it is possible as long as the laminar flow conditions prevail.

So, even at the center of the core, would we have the turbulent flow, at the tube wall, at the pipe wall, the velocity is very small that you are having almost laminar flow conditions. Why? Because of the no slip condition at the wall, and then a few layer close to the wall they are still having very small velocity. So, then we can call that, that layer is a laminar or viscous sub-layer.

Then, immediately after crossing this laminar zone it is not possible the turbulent zone would be there. So, there will be a kind of buffer region or transition zone is there. So, let us call that thickness of that region is y_T. And then after this y_T whatever is there, so that is nothing but the turbulent core. So, that is nothing but the turbulent core, ok.

So, now, what are we going to see now here? So, we are going to develop velocity profile for this region, what is V? What is V in this region transition zone? And then what is V in this turbulent region? But, all these close to the wall. In the turbulent region also, we are going to take a region which is close to the transition zone, so that we can say that that is close to the wall that is close to the wall.

So, whatever the velocity profile that we are going to develop for the turbulent core, so that is valid in the turbulent core region, but that towards the wall region, towards the wall region only it is valid. Towards the center region it is not valid, ok.

So, with this understanding I think we can start moving to the mathematical representation of these things and then get the equations. So, first we do for the case of Newtonian fluids, ok. So, now, this pipe geometry we have taken. So, though it is r-z coordinates. So, then now we are interested to know the velocity profile towards the wall. So, we have defining another coordinate, y coordinate here, so which is nothing but $y = R - r$, right.

If this r is varying value of R value from 0 to R. If $r = R$ that is tube wall, so then $y = 0$. So, this r and then y are in two opposite directions opposite to each other. So, whatever we discussed in the previous slide have been provided here.

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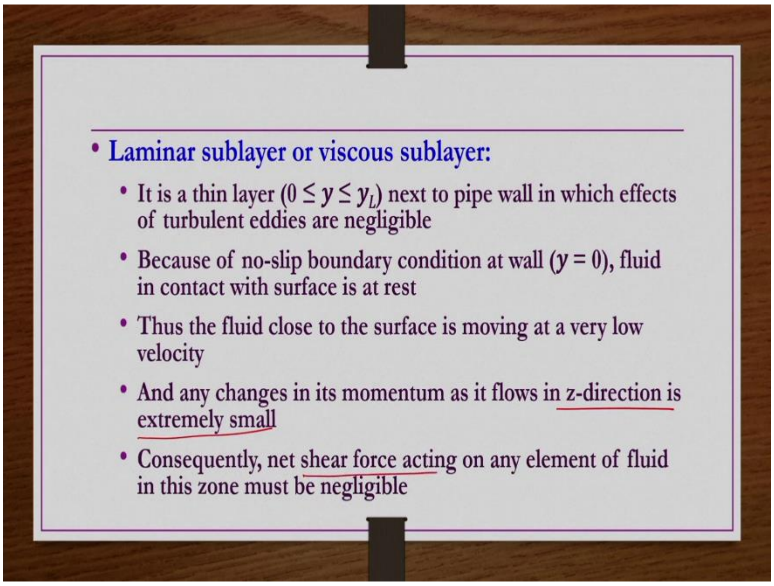

So, that is laminar sub-layer or viscous sub-layer is a thin layer which extended between 0 to y_L values, which is next to the pipe wall in which effects of turbulent eddies are negligible. Negligible only we are saying. They are existing, but they are not you know there is no reason in the pipe without eddies. Only thing that in the laminar layer are close to the wall, the frequency of eddies or the size of eddies or their effect is very very small because of the no slip conditions at the wall, ok.

So, because of no slip boundary condition at wall, fluid in contact with surface is at rest. Thus, the fluid close to the surface is moving at a very low velocity and any changes in momentum as it flows in z-direction is extremely small, is extremely small because the fluid close to the surface is moving at very low velocity, and then that momentum that momentum that fluid velocity is also in the z-direction.

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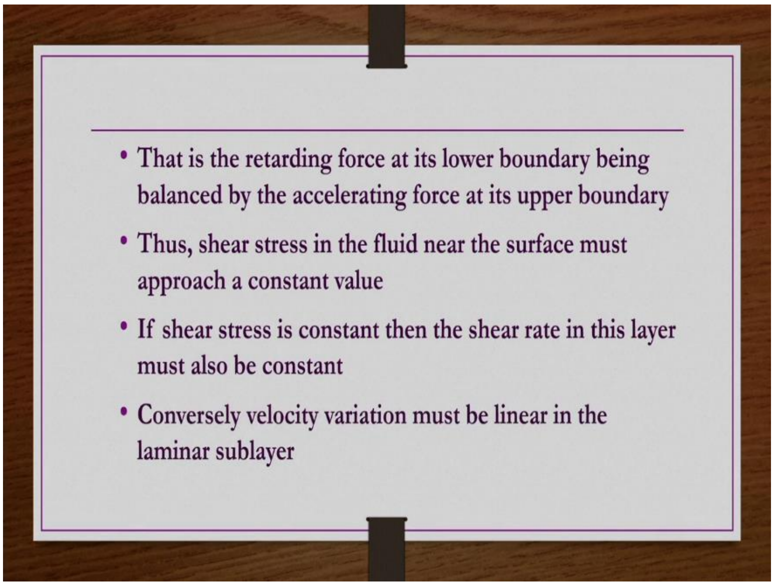

So, consequently whatever the net shear force acting on any element of fluid in this zone must be negligible, that is the retarding force at its lower boundary being balanced by the accelerating force at its upper boundary. So, thus, the shear stress in the fluid near the surface must approach a constant value. It is not one single constant value. It is very small. The change is very small.

So, then we can assign one’s constant value and then we can assign wall shear stress the as that constant value. So, if the shear stress is constant then the shear rate in this layer must also be constant. So, then, obviously, velocity variations must be linear in the laminar sub-layer all that we have discussed in the picture.

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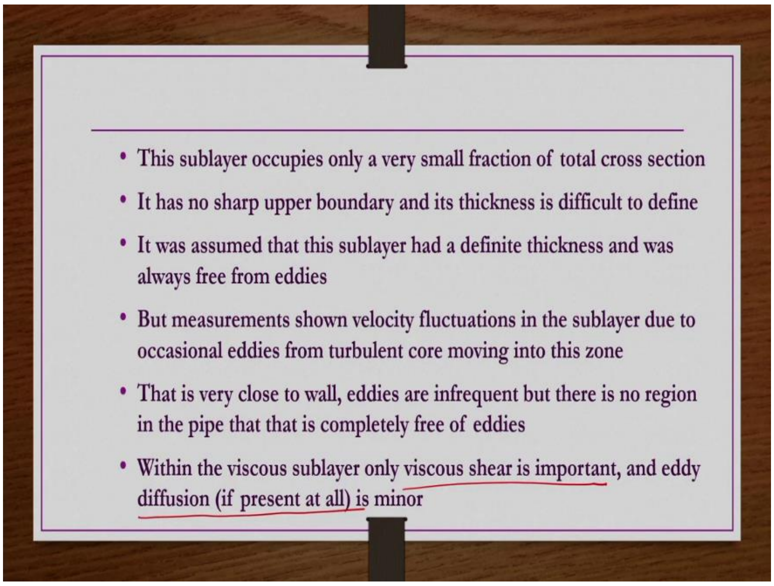

So, this sub-layer occupies obviously very small fraction of total cross section area. The y_L is very very y_L thickness, y_L whatever is there that is very small. It has no sharp upper boundary and its thickness is difficult to define in general. And then it was assumed that this sub-layer had a definite thickness and was always away from eddies at layer before having you know sophisticated advanced measurement techniques.

But after advancement of a sophistic velocity measurement techniques, it was found that velocity fluctuations in the sub-layer also there because of the occasional eddies that are coming from the turbulent core and then moving into the laminar sub-layer. So, that is very close to the wall, eddies are infrequent but there is no region in the pipe that is completely free from eddies. So, within the viscous laminar sub-layer viscous shear is important, and eddy diffusion if any present that is very small or minor, ok.

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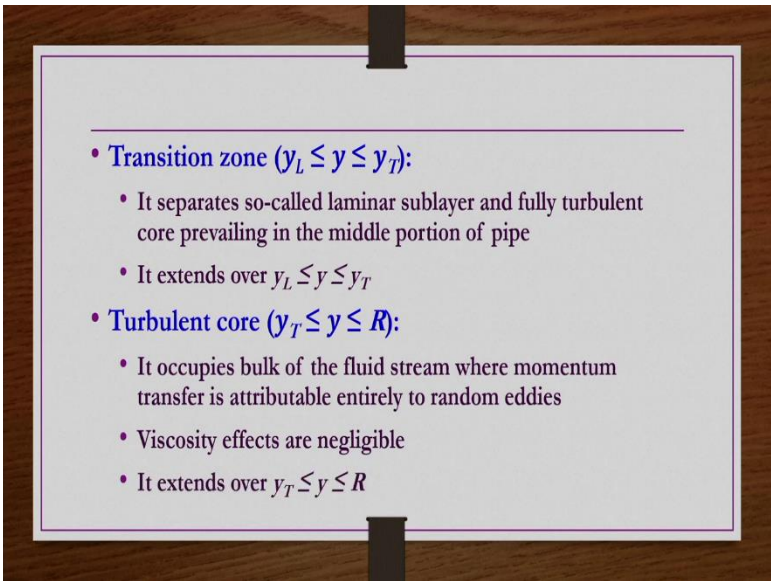

So, transition zone it extends between y_L to y_T. It separates the so called laminar sub-layer and then fully turbulent core. Whereas, the turbulent core is extend from y_T to R value that is towards the center of the axis. It occupies bulk of the fluid stream where momentum transfer is attributable entirely to random eddies. And then viscosity effects are negligible, right. So, it extends between y_T to R values.

So, now, what we understand here? We understand in the laminar sub-layer only molecular viscosity is dominating, right. Whereas, in the turbulent core, but close to the wall we are taking consideration of the eddy viscosity is predominating or you know its contribution is much higher compared to the molecular viscosity.

So, then we take only eddy viscosity under consideration to get the velocity profile for the case of turbulent flow, but close to the wall, ok. So, these two things we are going to take under consideration.

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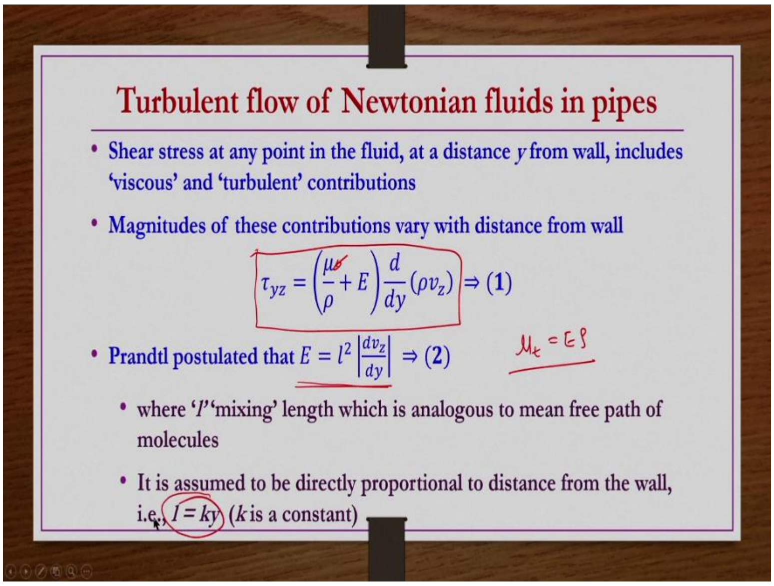

So, turbulent flow of Newtonian fluids in pipes that we take as a basis, so that it will be easy for us to move on to the power-law fluids once we have the basics of the Newtonian fluids. So; obviously, shear stress at any point in the fluid, at any distance y from the wall includes viscous and turbulent contributions because the flow is turbulent flow here. So, magnitude of these contributions vary with distance from wall. So, we need to have an expression for the shear stress.

So, τ_{yz} is this expression that we can have. So, this μ whatever is there. So, now $\frac{\mu}{\rho}\frac{d}{dy}(\rho v_z)$ if you take ρ constant. So, then $\frac{dv_z}{dy}$. This μ indicates the viscous contribution to the shear stress whereas, this E ρ, E x ρ whatever is this. So, that gives the turbulent contribution for this shear stress expression, ok.

So, there are several semi-analytical theoretical approaches are there. So, this is one of the best approach that has been found to be you know valid compared to the experimental results. As we are going to see further anyway in the coming slides, ok.

So, Prandtl postulated this E is nothing but $l^2|\frac{dv_z}{dy}|$. So, E x ρ is nothing but the turbulent viscosity, alright. That is turbulent viscosity is nothing but $\rho l^2 \left(\frac{dv_z}{dy}\right)^2$, if $\frac{dv_z}{dy}$ is positive, ok, where l here is nothing but mixing length which is analogous to the mean free path of molecules and then it is assumed to be directly proportional to the distance from the wall, so l = ky, k is a constant, right.

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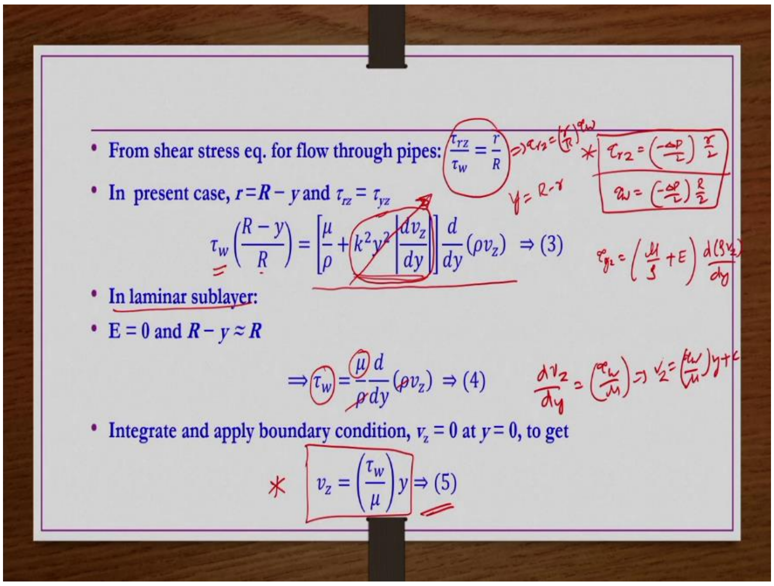

So, now shear stress equation for flow through pipes, what we have developed? When we are deriving the velocity profiles for the laminar flow through pipes, what we have developed; τ_{rz} is nothing but $\left(\frac{-\Delta p}{L}\right)\frac{r}{2}$ this is what we have seen. And then from here we get $\tau_w = \left(\frac{-\Delta p}{L}\right)\frac{R}{2}$ this is what we have seen.

And then what we have seen further? This expression when you developed, especially this one $\tau_{rz} = \left(\frac{-\Delta p}{L}\right)\frac{r}{2}$, we have not taken any concentration about you know nature of the fluid etcetera. So, then it is valid for all the fluids. And then from this expression what we get? $\frac{\tau_{rz}}{\tau_w} = \frac{r}{R}$, right.

So, that means, r if you write in terms of y coordinates because we are writing everything all the equation in terms of y and z coordinates because y coordinates is close to the from; we are defining from the wall and then at the wall what is happening, how the velocity profile are being varied that we can easily calculate as per the analysis that we have discussed previously. So, that is the reason r we are representing in terms of y.

So, how we have defined y we have defined as R - r. So, then in place of r we can write R - y. And then τ_{rz} should be replaced by τ_{yz}. Then, whatever that previous equation that we have τ_{rz} = $\left(\frac{\mu}{\rho} + E\right)\frac{dv_z}{dy}$ this expression here.

So, now here what are we writing? This from here, we are writing this expression as τ_{rz}. From here, what we are writing? $\tau_{rz} = \frac{r}{R}\tau_w$. So, that is τ_w and then r is nothing but $\frac{R-y}{R}$, right is equals to τ_{rz} that is nothing but τ_{yz}. So, τ_{yz} is nothing but what we get in the previous slides. We have written τ_{yz} = $\left(\frac{\mu}{R} + E\right)\frac{d\rho v_z}{dy}$. This is what we have written.

And then, we have also seen that this E is nothing but $k^2 y^2 \frac{dv_z}{dy}$, r we have found this E is nothing but $l^2 \frac{dv_z}{dy}$, and then l is nothing but k x y and that is what we have seen. So, all that we substitute. So, then this is what we get.

Now, in the laminar sub-layer this contribution is negligible. This contribution whatever, this multiplied by ρ whatever is there, this multiplied by ρ whatever is there this is nothing but eddy viscosity or turbulent viscosity contribution. So, that is very negligible if the if we are considering the laminar sub-layer which is the layers few fluid layers which are attached to the wall.

So, we can strike out for we can take up this one for the case of laminar sub-layer. So, then we have E = 0 and then R - y approximately close to the R. Then τ_w = $\left(\frac{\mu}{\rho}\right)\frac{d\rho v_z}{dy}$ only, we are going to have for the laminar sub-layer.

So, now here τ_w is a constant value. It is not changing. τ_{rz} or τ_{yz}, it is changing with the y or r directions, but τ_w is not changing, it is one constant value. μ is constant, ρ is constant. So, now, from here what you can do? The ρ is not changing with you know it is incompressible fluid. So, then ρ, if you cancel out so then what you get from here? τ_w by $\mu = \frac{dv_z}{dy}$. So, that means, $v_z = \frac{dv_z}{dy} = \frac{\tau_w}{\mu}$ you are having.

So, when you integrate it, you will get $v_z = \left(\frac{\tau_w}{\mu}\right) y + c$. But at y = 0, v_z is 0. So, if you substitute y = 0 in this equation, right-hand side 0 + c and then at y = 0 that is at the wall

v_z is 0. So, left-hand side is also 0. That means, constant c is 0; that means, we get $v_z = \left(\frac{\tau_w}{\mu}\right) y$.

So, what we understand here? As we discussed previously, if the change in shear stress within the laminar sub-layer is very very small or negligible, so then we can assign a one constant value of the shear stress. And then if we assign the wall shear stress is as that constant value of the shear stress, then velocity profile has to be linear and then that linear velocity profile you can get using this expression.

Remember this is only for laminar sub-layer close to the wall. Very few fluid layers at the wall which are moving with very very small flow rate, even if we have a turbulent flow at the center of the pipe, ok.

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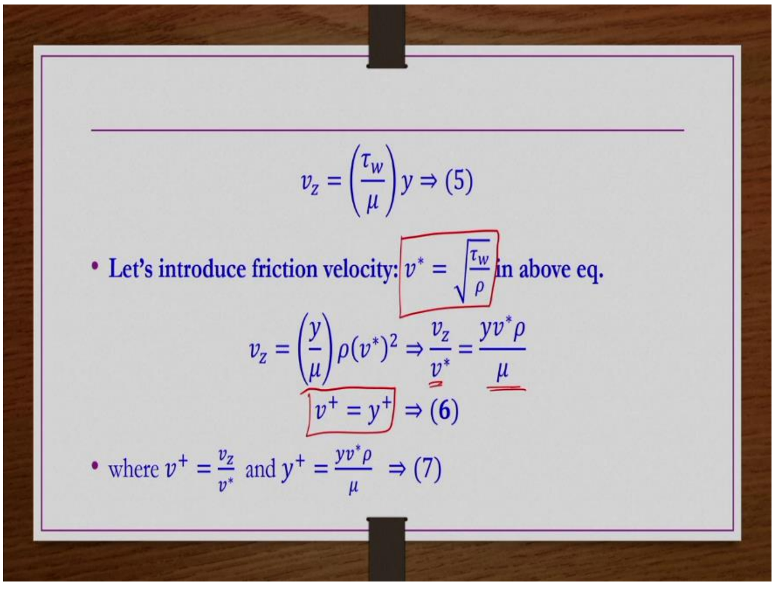

Now, this equation what we do? We try to introduce frictional velocity or we are just trying to non-dimensionalize it. Usually, velocities are being non-dimensionalized using the free stream velocities or upcoming you know inlet velocity condition whatever in general, but here the velocity profile we are developing towards the wall, towards the wall only it is not for the entire cross section area of the pipe. It is towards the wall, close to the wall only we are developing.

At the wall, whatever the wall shear stress is there or the frictional forces are there they are important. So, that is the reason, the velocity we are trying to non-dimensionalize using the frictional velocity or frictional velocity v star which we are defining as square root of tau w by rho, because it has to be known thing, right.

For a given for a given $\frac{\Delta P}{L}$value, tau w is known constant value, right. So, tau w is known, rho is known. So, frictional velocity we are defining or non-dimensionalizing of the velocity we are doing using friction velocity v* and that is defined as $\sqrt{\frac{\tau_w}{\rho}}$.

We are not non-dimensionalizing the velocity using the free stream velocity or inlet velocity in general. Why are we not doing that one? Because all these velocity profile that we are going to develop whether it is laminar flow or turbulent zone, but that is all close to the wall only. And then at the wall, frictional forces at the wall shear stress is more important engineering parameter that is the reason.

Now, in this equation what we can have? $v_z = \left(\frac{y}{\mu}\right)$ we can write as it is. In place of τ_w, I can write rho $\rho(v^*)^2$. And then, next term $\frac{v_z}{v^*}$, I am taking from the, right-hand to the left-hand to the left-hand side. So, then $\frac{v_z}{v^*} = \frac{yv^*\rho}{\mu}$. This is what we have, right.

So, now, this is dimensionless, and then right-hand side this is also dimensionless distance. So, let us designate them as v^+ and then y^+. This y^+ may be seen as a kind of Reynolds number in terms of y values, y, in terms of y coordinates and then frictional velocity, ok.

So, this is the velocity profile in terms of a dimensionless coordinates, dimension dimensionless velocity and then dimensionless distance if you have for a given condition. So, then velocity profile for the laminar or viscous sub-layer is nothing but $v^+ = y^+$. And then what are this v^+? $v^+$is nothing but $\frac{v_z}{v^*}$ and then y^+ is nothing but $\frac{yv^*\rho}{\mu}$.

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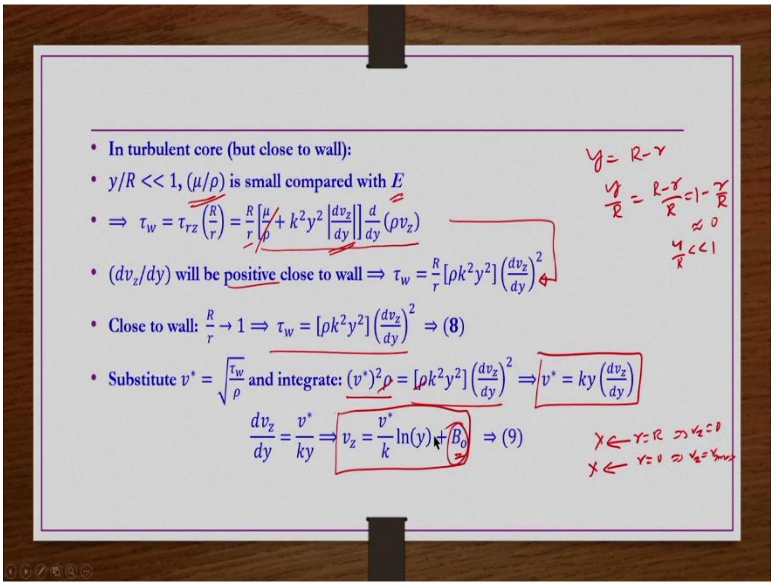

Now in turbulent core, but close to the wall, still close to the wall. As I mention in the picture one of the previous slides, turbulent core also velocity profile we are developing, but that we are taking the region close to the wall that is just after the transition layer, ok.

So, for that also we can take $\frac{y}{R}$ is very very less than 1 because y is nothing but R - r. So, then $\frac{y}{R} = \frac{R-r}{R}$, so that is $1 - \frac{r}{R}$. If it is closed to the wall, so then r should be close to the R value, so then it is coming approximately 0. So, that means, $\frac{y}{R}$ is very very less than 1, we can write.

And then $\frac{\mu}{\rho}$ that is the viscous contribution in the shear stress is going to be very small compared to the E or you know E ρ which is nothing but the turbulent contribution in the shear stress expression, right.

So, this is what $\tau_w = \tau_{rz}\frac{R}{r}$ we are having $\frac{R}{r}$, we keep as it is. τ_{rz} in terms of τ_{yz} expressions this is what we have already seen in the previous slides. We are substituting here. Now, this $\frac{\mu}{r}$ is very very small, so then we can strike off in the first case. And in second case is that you know as y increasing, y increasing what happens velocity is increasing; that means, $\frac{dv_z}{dy}$ is going to be positive close to the wall.

So, then we can take of the modulus from this modulus of $\frac{dv_z}{dy}$. So, then we can have $\left(\frac{dv_z}{dy}\right)^2$ by taking that ρ as a constant, by taking that rho as a constant. So, then what we get? This expression $\tau_w = \frac{R}{r}[\rho k^2 y^2]\left(\frac{dv_z}{dy}\right)^2$ right.

So, now, close to the wall $\frac{R}{r}$ is approximately 1, if it is close to the wall $\frac{R}{r}$ approximately we can take close to the 1. Then, we have, τ_w this value. Now, we introduce frictional velocity $v^* = \sqrt{\frac{\tau_w}{\rho}}$ or in place of τ_w we can write $(v^*)^2\rho$. Right-hand side is as it is. So, now, this ρ and then ρ if you cancel out from both the sides, then all the terms are having square.

So, then when you remove the square. So, then you have $v^* = ky\frac{dv_z}{dy}$, that if you take like $\frac{dv_z}{dy} = \frac{v^*}{ky}$, now you can integrate. So, then you get $v_z = \frac{v^*}{k}\ln(y) + B_0$. But this B_0you know you cannot evaluate, you cannot evaluate, because you do not have the boundary condition.

You have the boundary condition right at the wall, right; you have the boundary condition at the center of the wall. That is at r = R, you are having boundary condition $v_z = 0$, at r = 0, you know you have the $v_z = v_{z\ max}$ boundary condition.

But this relation is not at this location is not at this location. It is close to the wall, but not at the, right at the wall. So, then you do not have any boundary condition to get this constant. So, then what we have to do?

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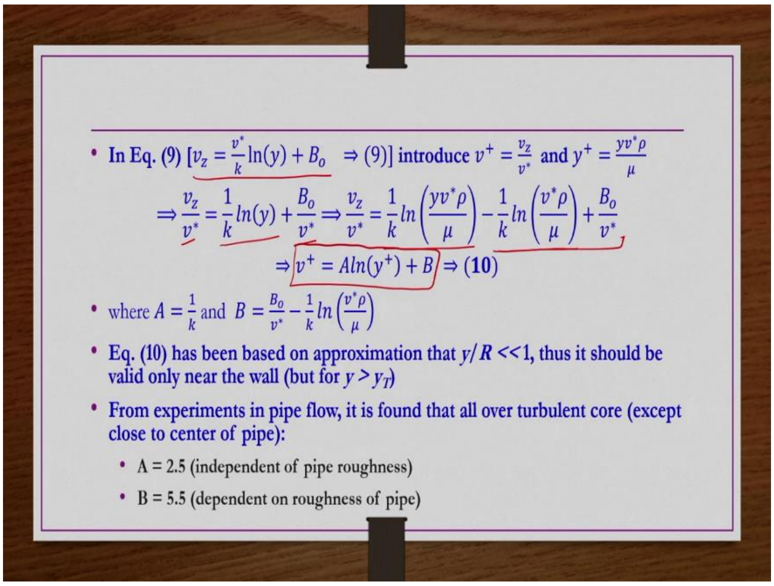

We introduce here again whatever the non-dimensionalization is there.

So, that is $\frac{v_z}{v^*}$ we write as v^+ and then wherever $\frac{yv^+\rho}{\mu}$ is there, we are going to write v^+ in this equation, which is the last equation of the previous slide. So, first what we do? We divide both sides by v^+. So, then we have $\frac{v_z}{v^*} = \frac{1}{k}\ln(y) + \frac{B_0}{v^*}$, right.

So, left-hand side is ok. In the right-hand side, what we do? We subtract $\frac{1}{k}\ln\left(\frac{v^*\rho}{\mu}\right)$ and then we add $\frac{1}{k}\ln\left(\frac{v^*\rho}{\mu}\right)$ and then whatever the added part is there, that is added with $\frac{1}{k}\ln(y)$. So, then we can write $\frac{1}{k}\ln\left(\frac{yv^*\rho}{\mu}\right)$ right. And then last term $\frac{v_0}{v^*}$ is as it is.

So, now, why we are writing? Because we wanted to write in terms of y^+. We do not want to write in y. So, now, this term we can write as y^+. So, $v^+ = \frac{v_z}{v^*}$ is nothing but $v^+ = A$ ln$\left(\frac{yv^*\rho}{\mu}\right)$ is nothing but y^+ + B. So, now, A is nothing but $\frac{1}{k}$ and then B is nothing but these two terms, $\frac{B_0}{v^*} - \frac{1}{k}\ln\left(\frac{v^*\rho}{\mu}\right)$ is nothing but B.

Now, this is the velocity profile for the turbulent core, but close to the wall, but close to the wall. But still this A and B constants are not known. So, now, what we do? We compare with experimental results and then try to get this A and B constants, ok. So, from the experimental observation it has been found that A is nothing but close to the 2.5 and then

B is nothing but close to the 5.5 value. A is independent of pipe roughness whereas, B is dependent on the pipe roughness, ok, right.

So, that is for the turbulent core. But transition zone, there are no such analysis, because in the transition zone neither we can take off the viscous contribution and shear stress nor we can take off the eddy or the turbulent viscosity contribution in the shear stress expression. Both we have to include, when we include both we cannot do theoretical analysis, analytical analysis to get the velocity profile, ok. That is not possible.

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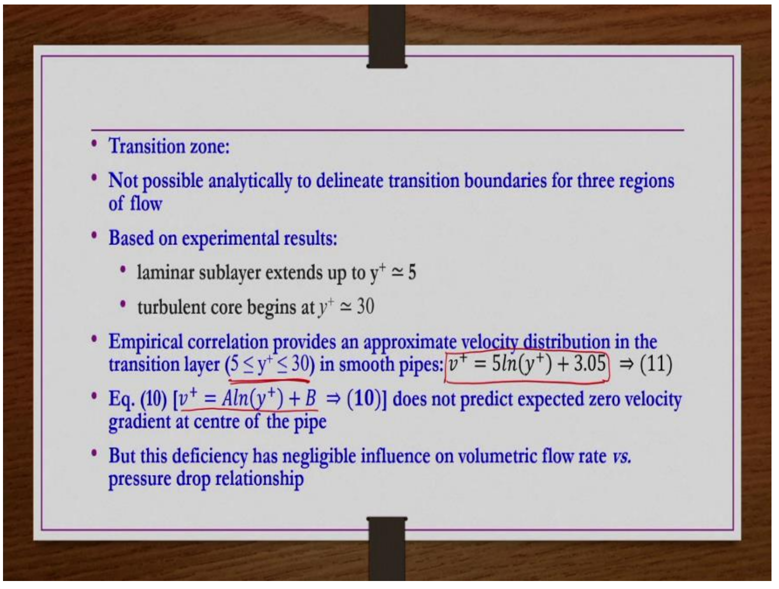

However, based on the experimental results what people have found that laminar sub-layer extends up to y^+ is approximately 5 and then turbulent core begins at y^+approximately 30. So, between y^+values 5 to 30 whatever is there, so that region is taken as a transition zone and then in that zone $v^+ = 5\ln(y^+) + 3.05$ is found to be suitable velocity profile.

This is from the experimental results when they have represented in terms of empirical correlations. So, for the transition zone also velocity profile is having the same form as a turbulent core velocity profile, but only that constants A and B are different here, ok.

So, the previous equation for turbulent core whatever is there that is $v^+ = A(y) + B$, does not predict expected 0 velocity gradient or maximum velocity at the center of the pipe. At the center of the pipe velocity has to the maximum, has to be maximum or its gradient has to be 0. But this all this analysis we are doing close to the wall. We are not doing at the center of the pipe. So, they are not predicting that 0 velocity gradient at the center of the pipe. However, this is not going to have any influence on volumetric flow rate versus pressure drop relationships anyway.

So, now, that is for the turbulent flow of a Newtonian fluid flowing through pipe. If the fluid is power-law fluids, so then what we have to do?

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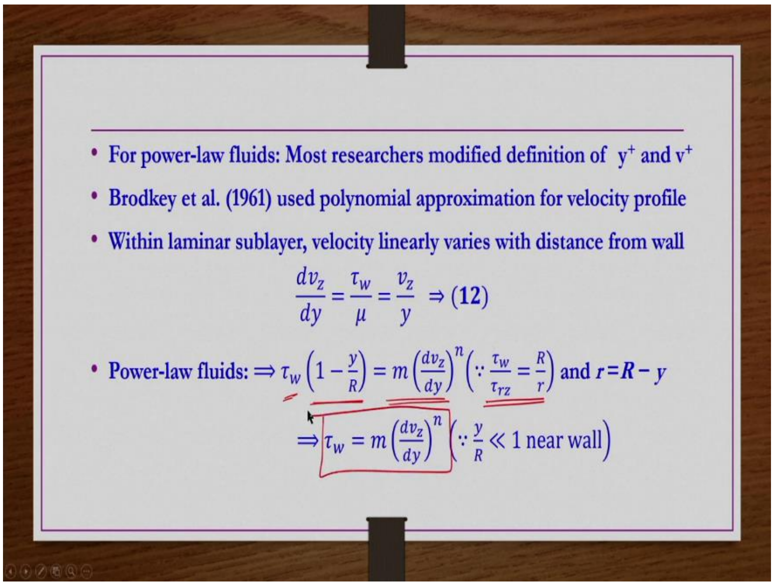

So, many people, many researchers, what they have done? They have tried to modify the definitions of y^+ and v^+ Brodkey tried to provide polynomial approximation for velocity profile. But, however, we take a similar analysis whatever we have done for the case of Newtonian fluid still now, and then try to get the expressions for the power-law fluids also.

So, within the laminar sub-layer velocity linearly varies with the distance from wall. So, then what we have seen $\frac{dv_z}{dy} = \frac{\tau_w}{\mu}$ or that same we can write it as $\frac{v_z}{y}$ or v_z we got $\frac{\tau_w}{\mu} y$, right. So, for the power-law fluids what we have? This expression, $\frac{\tau_w}{\tau_{rz}} = \frac{R}{r}$.

So, from here what we can write? $\tau_{rz} = \tau_w \frac{R}{r}$. So, $\frac{R}{r}$ we can write $\frac{1-y}{R}$ and then τ_w is as it is because we are writing in terms of y coordinates. And then right-hand side, τ_{rz} we have to write in terms of τ_{yz} for the power-law fluids. For the power-law fluids τ_{yz} is nothing but the m $\left(\frac{dv_z}{dy}\right)^n$, ok.

So, now, here what we can write? Since, close to the wall $\frac{y}{R}$ is very very smaller or close to the 0 value. So, then $\frac{1-y}{R}$ is going to be approximately close to 1. So, then we can have $\tau_w = \text{m}\left(\frac{dv_z}{dy}\right)^n$.

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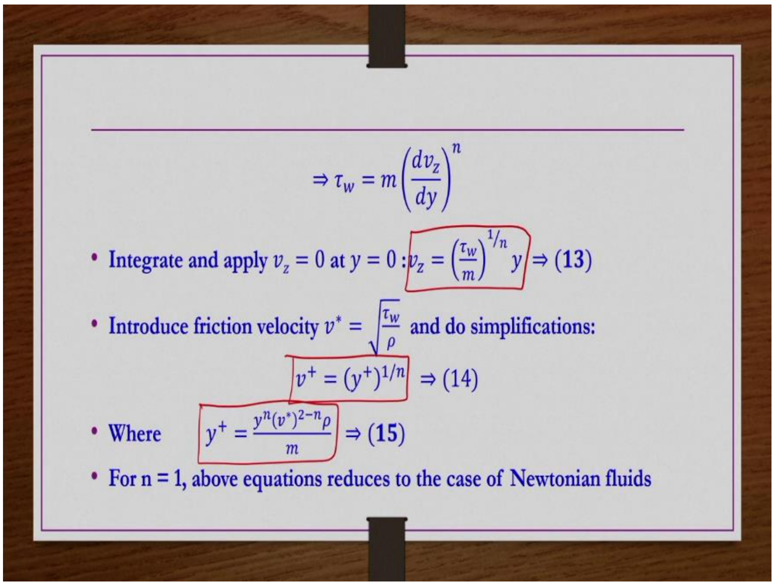

Now, again, we can do the integration and then apply the boundary condition y = 0, at y = 0, $v_z = 0$, then we get $v_z = \left(\frac{\tau_w}{m}\right)^{1/n}$ y. Now, here again if you apply frictional velocity $v^* = \sqrt{\frac{\tau_w}{\rho}}$ in order to non-dimensionalize this above equation and then do some simplification, what you get? You get $v^+ = (y^+)^{1/n}$.

Whereas, this v^+ is nothing but $\frac{v_z}{v^*}$, but y^+ is nothing but $\frac{y^n (v^*)^{2-n}\rho}{m}$ which is again may be taken as a Reynolds number for the power-law fluids, but defined in terms of a friction velocity and then y coordinate, ok. So, now, here if you substitute n = 1. So, then we are going to get $v^+ = y^+$ which is nothing but same as a kind of you know Newtonian case. Previously we have done.

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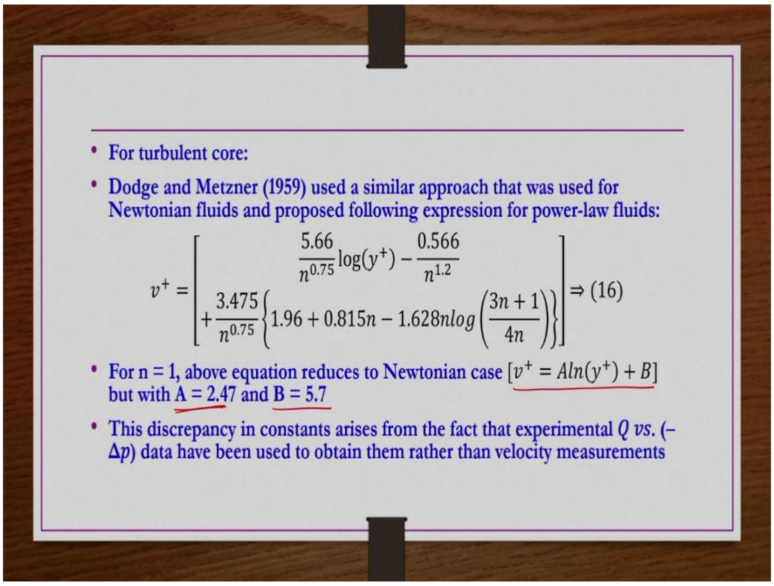

So, for turbulent core Dodge and Metzner used a similar approach that was used for the Newtonian fluids and then provided this correlation. In this correlation, when they substituted n = 1 they got this Newtonian case expression that is $v^+ = A\ln(y) + B$, but A they got 2.47, rather getting 2.5 and then B they got 5.7 rather getting 5.5.

But, this small discrepancies are coming because they have used experimental Q versus $-\Delta p$ information to get this correlation rather than using the velocity measurements, ok because of that small discrepancies are coming.

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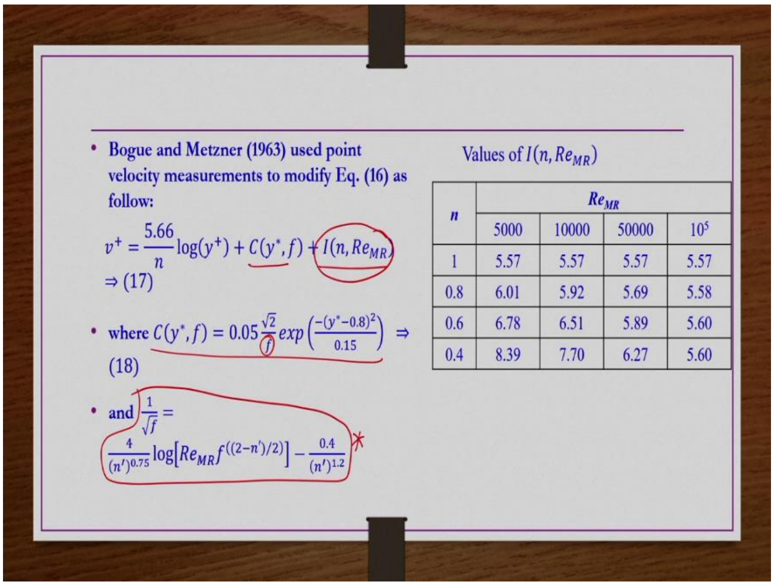

Then, Bogue and Metzner, they used velocity measurements to modify equation number 16 because velocity measurement if you are going to use directly to get the velocity profile expressions empirically, so that is going to be more reliable rather than using Q versus $-\Delta p$ expression.

So, the same thing this Bogue and Metzner, they have done they have used the velocity measurements and then they modified the Dodge and Metzner equation, and then they got this expression. Whereas, here you know we have some C and then I. So, they are given as C is given by this expression. C function of y star f is given by this expression. Whereas, in this expression we have this f, so that f is given by this correlation that we have seen, this you know previous lecture, right.

This is nothing but friction factor for power-law fluids flowing through pipes, if the flow is under transitional or turbulent flow conditions. Whereas, this I which is function of n and Re_{MR}, they have given tabulated values like this. For different n values as Re_{MR} changing how this I value is changing that is given here, right.

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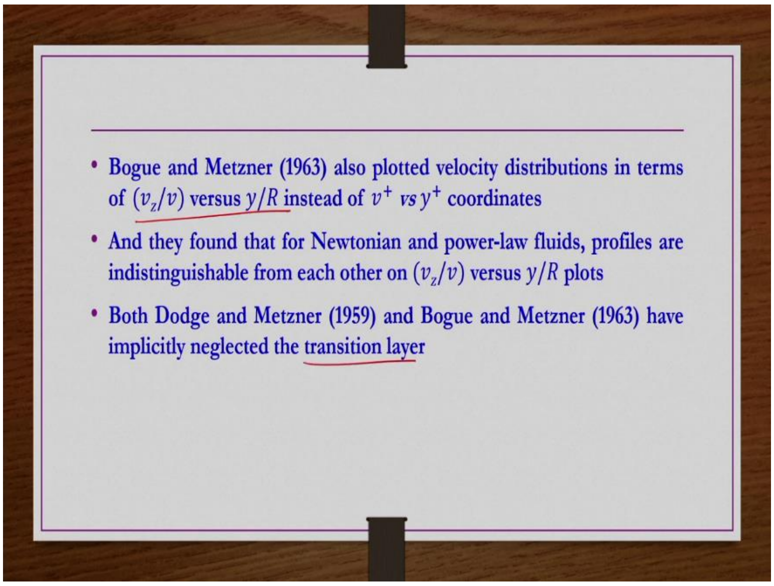

Finally, Bogue and Metzner also plotted this velocity distribution rather plotting them in terms of y^+ versus, rather plotting the velocity distribution in terms of v^+versus y^+only. They have also plotted v_z/v versus y/R as well. And then, they found when they plotted v_z/v versus y/R there is no distinguishable difference between Newtonian and power-law fluid velocity profiles, ok.

But, however, both the analysis whatever Dodge and Metzner, and Bogue and Metzner have done, they have did not consider any transition layer at all.

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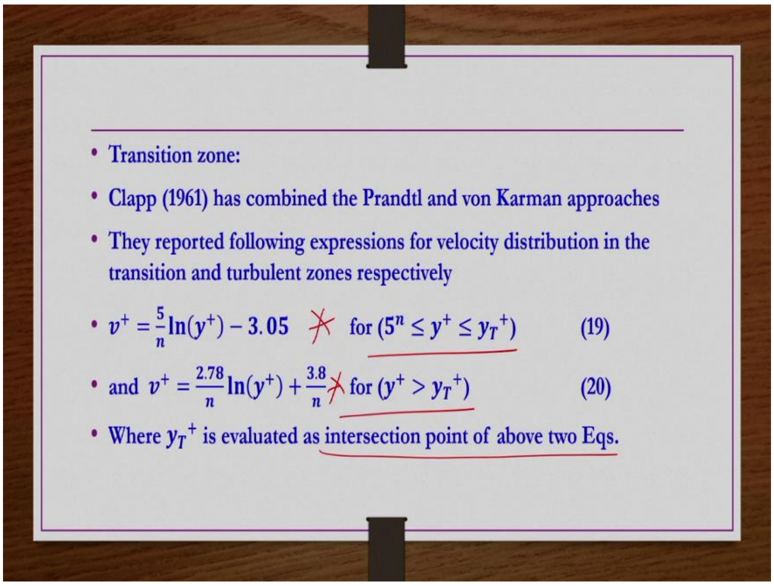

So, for the transition zone finally, the Clapp combined Prandtl and von Karman approaches, and they reported following expression for the velocity distribution for the transition and turbulent regions. This is for the transition region and this is for the turbulent region, right, where $y_T{}^+$ in this equation you know you have to evaluate as intersection point of about two equations, ok. Most of them are empirical, ok.

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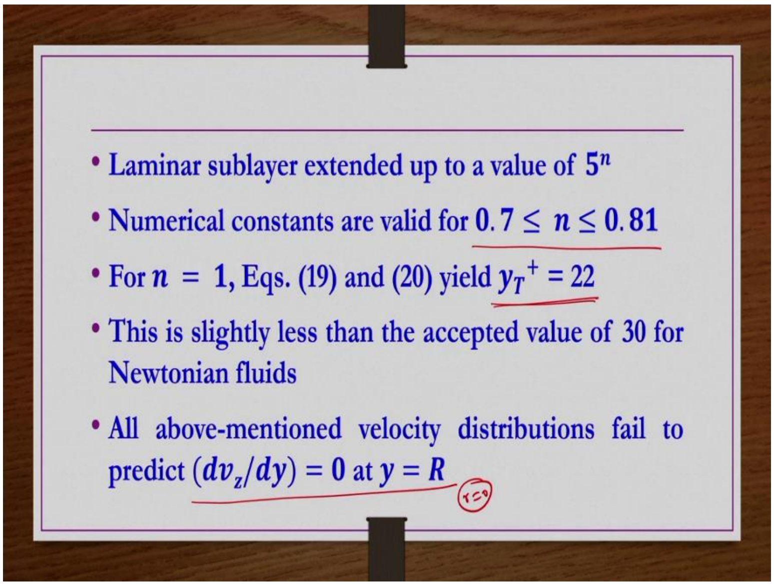

So, finally, what we understand laminar sub-layer extended up to a value of 5^n according to Clapp, and then these values numerical constants whatever are provided in their analysis they are valid between narrow range of n between 0.7 to between 0.7 and 0.81, for n = 1 ${y_T}^+$ they are getting approximately 22. But it has to be 30, if you are comparing with the Newtonian counterparts. So, then you know slightly lesser value, ok.

However, any of this analysis whether Newtonian or the power-law cases that we have seen in today's lecture, they are failed to report $\frac{dv_z}{dy} = 0$ at r = R or at r = 0, r = 0 because these all this analysis are at R close to the wall region only. They are not valid towards the center of the pipe.

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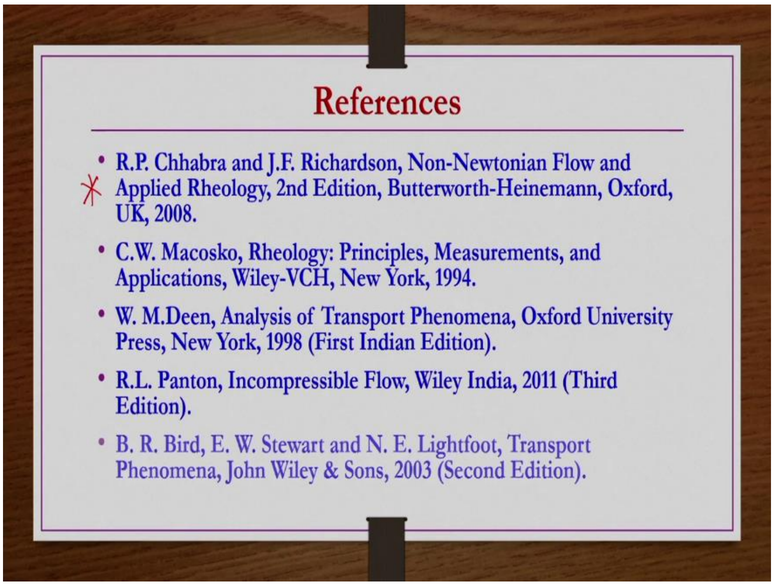

References; the entire lecture is prepared from this excellent book by Chhabra and Richardson. Other useful references are given here.

Thank you.