Transport Phenomena of Non-Newtonian Fluids
Prof. Nanda Kishore
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 18
Transition and Turbulent Flow of GNF in Pipes

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of this lecture is Transition and Turbulent Flow of Generalized Newtonian Fluids in Pipes. Before going into the details of today's lecture what we will do? We will have a kind of recapitulation of what we have studied in last 3 to 4 classes.

What we have studied in last week is that you know when a generalized Newtonian fluid is flowing through pipe because of the pressure difference how to derive the velocity profile and then how to get the equations for volumetric flow rate etcetera, friction factor etcetera those kind of derivations we have seen the all of them are for a laminar flow.

We have taken power law fluids, we have taken Ellis model fluids and then we have also taken Bingham plastics and then Herschel-Bulkley fluids which are under viscoplastic category, right. So, all of them are generalized Newtonian fluids because we know that under certain limiting conditions they reduced to Newtonian behavior, that is the region those fluids are known as the generalized Newtonian fluids.

So, what we do? Now we have a kind of recapitulation of some of those equations that we have derived in previous lectures, right. So, that you know we will have a kind of continuity of a subject.

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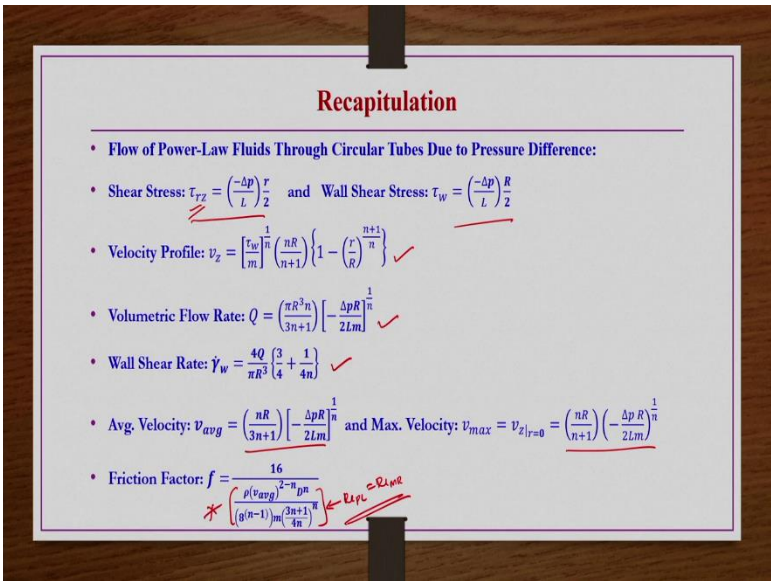

So, when a power law fluid is flowing through circular tubes due to pressure difference and then flow is laminar fully developed. The shear stress that we have derived is this one and then of course, we also realized that to get the shear stress we have not applied any kind of rheological information of the fluid.

So, that indicates that it is also valid for all other any type of non-Newtonian fluids provided that the flow has to be laminar fully developed flow, ok. So, in this equation if we substitute r = R then we get the wall shear stress then using this equation we got a velocity profile for different fluids because τ_{rz} we have substituted as per the different types of non-Newtonian fluid rheological behavior.

Power law fluid we have substituted $\tau_{rz} = m\left(\frac{-\,dv_z}{dr}\right)^n$ and then got the information. After substituting $\tau_{rz} = m\left(\frac{-\,dv_z}{dr}\right)^n$ in this equation here and then after doing the integration and obtaining the integration constant followed by some simplification steps we get this velocity profile. Then we got volumetric flow rate equation like this.

And then similarly wall shear rate we got by doing $\frac{-\,dv_z}{dr}$ and then r = R. So, that expression is nothing but this one. We have represented in terms of volumetric flow rate rather representing in terms of the average velocity, ok. If you represent this one in average velocity it will be $\frac{8V}{D}\left(\frac{3n+1}{4n}\right)$.

Then average velocity we got this expression, maximum velocity we got this expression. Friction factor we got this expression. We have written this expression in the form 16 by some expression that is nothing but the $R_{e\ PL}$ which is also known as the $R_{e\ MR}$. So, that we have in a similar form like $\frac{16}{Re}$ which is for Newtonian fluid, ok so, but now here R_e for power law fluid is defined in a different way using the effective viscosity.

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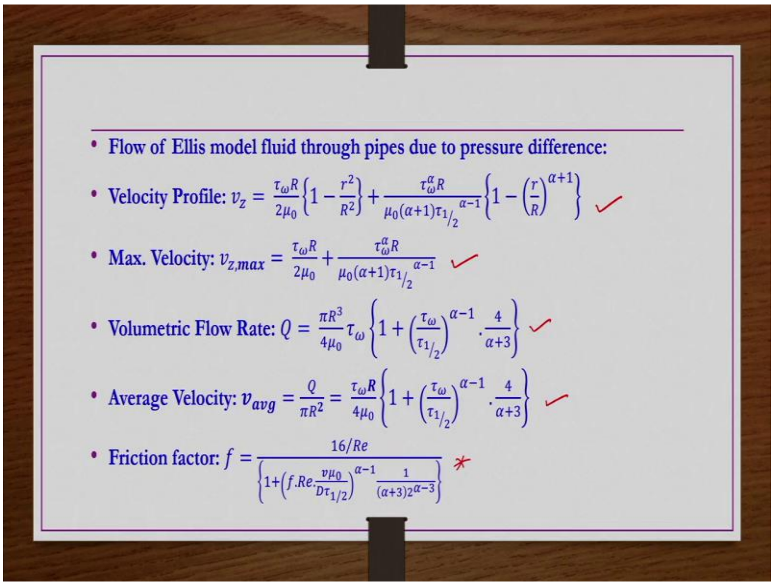

Then if the fluid is Ellis model fluid and then we do the similar analysis. What velocity profile we got? We got this velocity profile and then by substituting r = 0 in this above equation we got maximum velocity this one. And then volumetric flow rate equation we got this one, then we got average velocity this one by dividing the volumetric flow rate with πR^2 sand then friction factor we got this expression.

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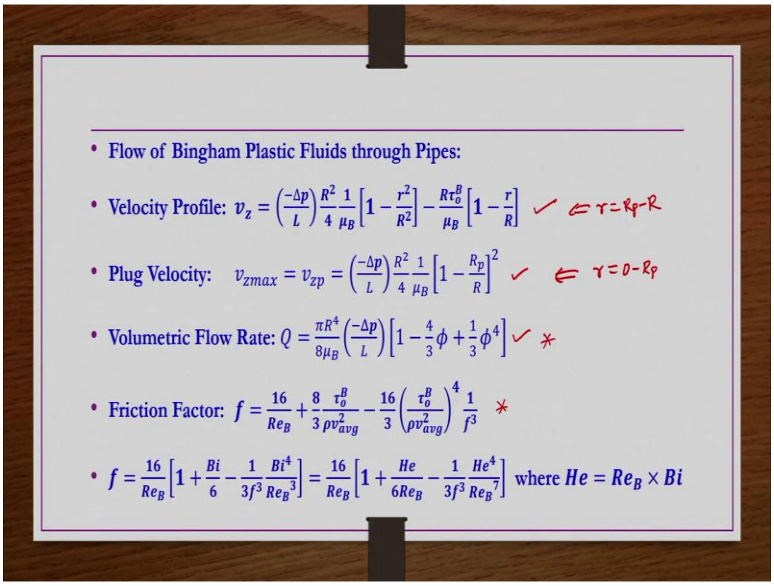

Then we moved on to the case of a viscoplastic fluids under which we have taken Bingham plastic as well as the Herschel-Bulkley fluids. When a Bingham plastic fluids flowing through pipe and then we got the velocity profile which is valid only for the range of R_p to R which is deforming region, ok.

Then plug velocity that is valid for r = 0 to R_p, remember for the viscoplastic fluids the flow area is divided into two parts; one is the plug solid like portion another one is the deforming fluid like portion.

Why that happens? Because applied stress when it is less than the yield stress the material does not deform and then that region the material flows like a plug solid plug. So, that velocity is this one. And then volumetric flow rate equation we got this one and then this is for the entire flow rate, it is not for an individual plug or individual you know deforming region the volumetric flow rate for the entire region, this is what we got.

Then friction factor we got this one which can also be represented in terms of the Bingham number and Reynolds number or Hedstrom number and then Reynolds number as given here, ok.

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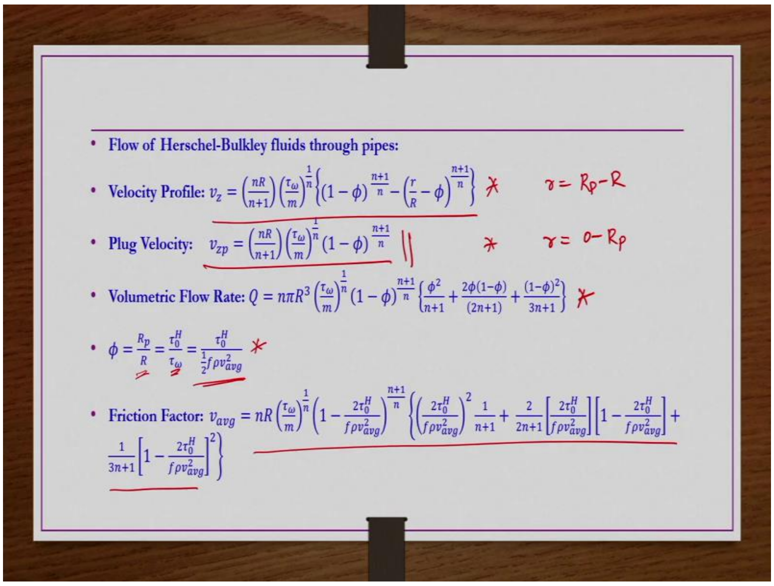

Then in the last class we have taken the case of Herschel-Bulkley fluids flowing through pipes. So, for that also the flow area flow region is divided into two portions solid plug like region where the applied stress is less than the yield stress. And then deforming fluid like region where the applied stress is more than the yield stress of the material.

And then for that region velocity profile we got this one which is for between R_p to R region R_p is nothing but the plug radius and then R is nothing but the radius of the tube. Then plug velocity we got which is valid from 0 to R_p region only that is from the centre to the plug region whatever that solid plug like region.

So, that velocity is this one that plug is moving with this constant velocity, this is one value whereas, this one is varying with r, this is one single value v_{zp} is one single constant maximum value. Then volumetric flow rate we got this one which is again for the entire region not like individual plug like region or the deforming fluid like region it is for the entire fluid.

Whatever you know whatever the fluid is flowing in the entire cross section that Herschel-Bulkley fluid so, that is for this one. It is not for individual deforming region or individual plug region, but it is entire region. Where in this equations ϕ is nothing but $\frac{R_p}{R}$ that is for the Bingham plastic fluids also same. So, which is nothing but the yield stress divided by the wall stress.

And then in this equation in place of wall stress, wall shear stress if you write $\frac{1}{2}\rho v_{avg}^2$ and then substitute this one here in the average velocity expression. Then we get this particular expression which is nothing but the friction factor we understand that both for the case of a Bingham plastic fluids and then Herschel-Bulkley fluids expression for the friction factor is implicit we have to do we have to go for trial and error and error approach, ok.

So, for these cases we have also taken a few example problems right. So, now, we move on to the today's lecture. Today's lecture what we are going to discuss? First we discuss the criteria; criteria for a transition from laminar to turbulent if the fluid is non Newtonian and then that fluid is flowing through pipe. We are taking only generalized Newtonian fluid regions that is whatever the time independent non Newtonian fluids are there for those fluids only we are seen, right.

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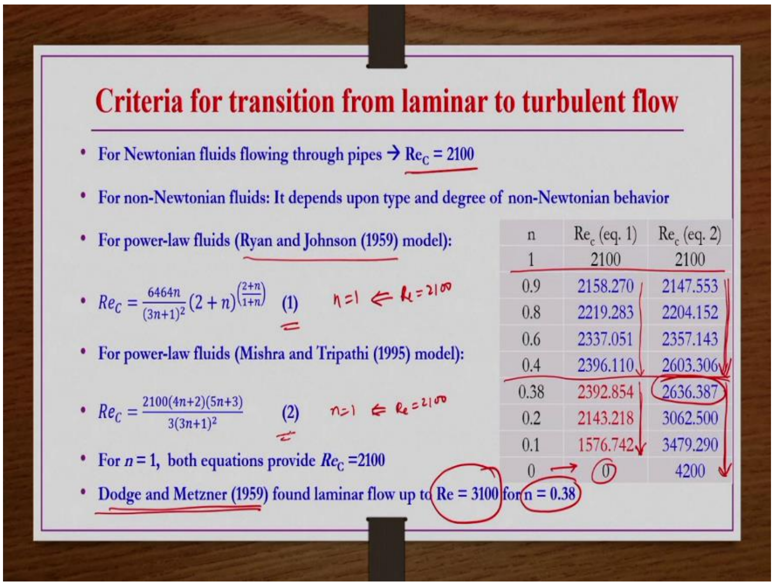

So, criteria for transition from laminar to turbulent flow, for Newtonian fluids flowing through pipes we know that the critical Reynolds number is 2100 right below which the flow would be under laminar conditions above which flow would be under turbulent conditions that we know, right.

So, below which the flow would be under laminar conditions if you cross this Reynolds number or the flow volumetric flow rate or the average velocity or such a way that if that over all Reynolds number crosses this value then the flow would not be laminar anymore and then we cannot apply the equations that we have derived in last four classes, ok.

So, for non-Newtonian fluids it depends upon type and degree of non-Newtonian behavior so; obviously, depending on the type of the non-Newtonian fluid we have a different expressions for the critical Reynolds number or different approaches to get the critical Reynolds number ok.

However, for the case of non-Newtonian fluids the available information about the critical Reynolds number are primarily based on the experimental information and that whatever the experimental results are there. So, they have been converted in the form of an empirical equations, ok.

Whatever the experimental results are there they have been converted into a different forms of an empirical equation. So, we see a few of them for a power law fluids, Bingham plastic fluids and then Herschel-Bulkley fluids, which are mostly accepted amongst the researchers. So, for power law fluids Ryan and Johnson they have proposed this correlation $Re_c = \frac{6464n}{(3n+1)^2}(2+n)^{\left(\frac{2+n}{1+n}\right)}$.

So, this equation if you substitute n = 1 you will get $Re_c = 2100$ that is it is reducing to Newtonian results, right. Similarly Mishra and Tripathi also they have done several experiments and their results they have presented in a form of an empirical correlation for the critical Reynolds number that is given like this $Re_c = \frac{2100(4n+2)(5n+3)}{(3n+1)^2}$ right.

Here also if you substitute n = 1 you will be getting critical Reynolds number value 2100 which is nothing but the case for the case of Newtonian fluid ok. If n = 1; that means, it is Newtonian behavior ok. And then when n is equals to 0.38 Dodge and Metzner they have found that the flow is remaining laminar even up to Reynolds number of 3100 for highly shear thinning fluid n 0.38; that means, it is highly shear thinning fluid ok.

So, now. So, what we understand? Now, this for at least from this equation 1 and 2 what we understand, critical Reynolds number is strong function of the rheological parameters of the fluid whichever we have taken. So, here in the case of power law fluid it is just a strong function of n power law behavior index. So, now, what we do? We take different values of n and then we obtain corresponding Re_C value. So, that to check whether are we getting same values or you know similar values or close to each other values these correlations are there is a deviation those kind of things we can see.

So, when we do this what we can see? Though at when n = 1 both of them are reducing giving the critical Reynolds number value of 2100, but you know as n decreases they have you know given different results, the difference is more increases more and then even trends also changes, right. Let us say equation number 1 Ryan and Johnson, if you see up to n is equals to 0.4 as n decreases what happens? This critical Reynolds number is increasing.

But after that trend is changed it is decreasing up to 0.4 critical Reynolds number value, expected critical Reynolds number value increasing compared to do compared to the 2100 value for n = 1, but after that it is decreasing and there is no region why it is happening as per them.

And then surprisingly when n = 0, that means, it's a very highly shear thinning and very small values of n that is n tends to 0 this critical Reynolds number is tending to 0 which is highly inconsistent not acceptable kind of results, ok.

However, Mishra and Tripathi's correlation what we can see as n decreases the critical Reynolds number value is increasing right. And then it is further increasing for all values of n not like that you know for certain range it is increasing and then decreasing like an equation 1. Equation 2 it is all increasing it shows increasing trend with decreasing n so, which is at least consistent, ok.

And further for n 0.8 what we can see? Here it is 2600 it is showing which is acceptable also by the trend wise also from Dodge and Metzner results also what we can realize? We can realize; that means, if n decreasing that critical Reynolds number should be more than the critical Reynolds number of a Newtonian fluid case. So, that trend at least maintained by this equation number 2 provided by Mishra and Tripathi.

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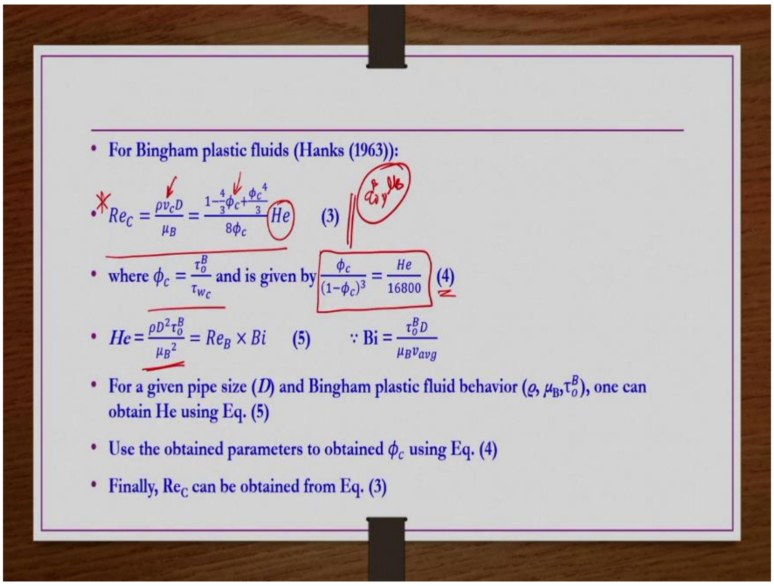

Now, for a Bingham plastic fluids Hanks provided this expression for the critical Reynolds number, it is expression for the critical Reynolds number it does not give like it give the what depending on the; depending on the values of you know what is τ_0^B etcetera and then what is μ_B based on these values what we understand you know this expression provides the critical Reynolds number value which not necessarily be 2100 or less than 1 or greater than 1.

Because according to them this critical Reynolds number is strong function of the rheological parameters in addition to the geometry and then velocity conditions etcetera all those thing. So, then; obviously, we cannot assign one single value for the critical Reynolds number according to them.

So, depending on these values material properties and then geometry geometrical information etcetera. So, what we can understand this Reynolds critical Reynolds number is going to be different, right. We see an example and we realize.

So, what they have provided they have provided it in terms of Hedstrom number and then ϕ_c is nothing but whatever the $\frac{R_p}{r}$ or $\frac{\tau_0^B}{\tau_w}$, but at critical velocity value, right. So, $\phi_c = \frac{\tau_0^B}{\tau_{w_c}}$ and then τ_{w_c} is nothing but the wall shear stress when the velocities at the critical velocity. Further we have to know this ϕ_c value in addition to the Hedstorm number then only we can get this Re_C.

So, for that region they have provided one expression between Hedstrom number and then ϕ_c and then that is given by equation number 4. Hedstrom number we already know it is nothing but $\frac{\tau_0^B \rho D^2}{{\mu_B}^2}$. So, for a given fluid density is available and then rheological parameters τ_0^B μ_B are available from experimental results.

D diameter of the tube or pipe whatever we are taken that also known. So, Hedstrom number is known, we do not need the average velocity pressure drop etcetera to get this one ok, right. So, once now if you wanted to get this Re_C value first what you have to do? For a given fluid rheological parameters that is τ_0^B and then μ_B you find out what is Hedstrom number and then use the equation number 4 to get the ϕ_c, right.

Once Hedstrom number is available then you can solve this equation number 4 to get the ϕ_c value then once ϕ_c is also known then you can use the equation number 3 get in order to get the critical Reynolds number value. And then once the critical Reynolds number is there is known so, then you can find out what is v_c that will give the average velocity or critical average velocity below, which flow is going to be laminar that information is available, that information can be obtained.

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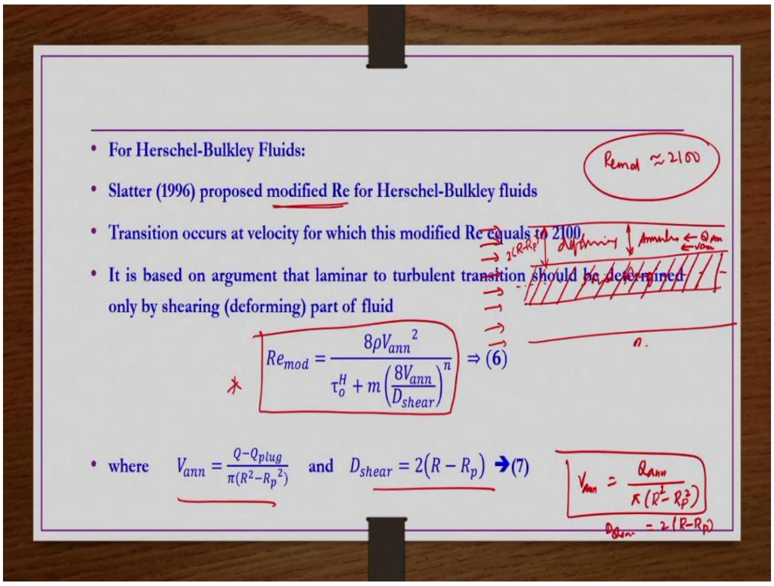

Now, for Herschel-Bulkley fluids, Slatter proposed at modified Reynolds number, ok. So, he proposed it is based on the region which is deforming actually you know Bingham plastic what we know now we realize that certain portion of the pipe is flowing with a constant plug velocity right. So, that plug is like this like a solid plug right and then this is deforming region.

So, now what they what is the argument of Slatter is that, if I have to find out the Reynolds number critical Reynolds number to define whether the flow is laminar or not. So, then I should take the region which is deforming only not the entire region not the entire flow region. So, that deforming region he called it annulus region and this region whatever the Q annulus is there are whatever the V annulus is there. So, those values he found out. And then based on those he defined the Reynolds number.

How to find out this Q annulus V annulus etcetera, that we see right. And then also the dimension the length dimension that is required for defining the Reynolds number that is R or D that we use. So, that also he has taken only this portion because within this portion only the deformation is taken. So, then 2 (R - R_p) he is taking as a kind of shearing region diameter for defining the Reynolds number ok fine.

So, now we see details of their thing. So, and then he said that corresponding to that one you have to adjust or you have to do the calculations or find out the velocity such a way that this Re modified has to be 2100 for the flow to be critical if the fluid is Herschel-Bulkley fluid according to this model by Slatter, ok.

Now, that modified Reynolds number now this is what proposed right. So, this is V annulus and then this D shear is nothing but as I mentioned 2 only 2 times the deforming region R - R_p whatever is there that he is taking D shear.

So, these things are to be calculated in order to find out this modified Reynolds number. So, the velocity for which this modified Reynolds number is close to 2100 up to which up to that velocity the flow is going to be laminar and then after that the flow is not going to be laminar according to this case.

And then once again reiterating whatever the things that we have derived in our previous 3 to 4 lecture all of them are only for the laminar flow. So, we cannot go for we cannot use those equations beyond the laminar flow conditions. So, Q_{ann} we can get as Q - Q_p once we have these values V_{ann} we can find it out as $\frac{Q_{ann}}{\pi(R^2-R_p^2)}$. So, this was this is also known and then D shear we already seen that D shear are the effective shearing region is 2 (R - R_p).

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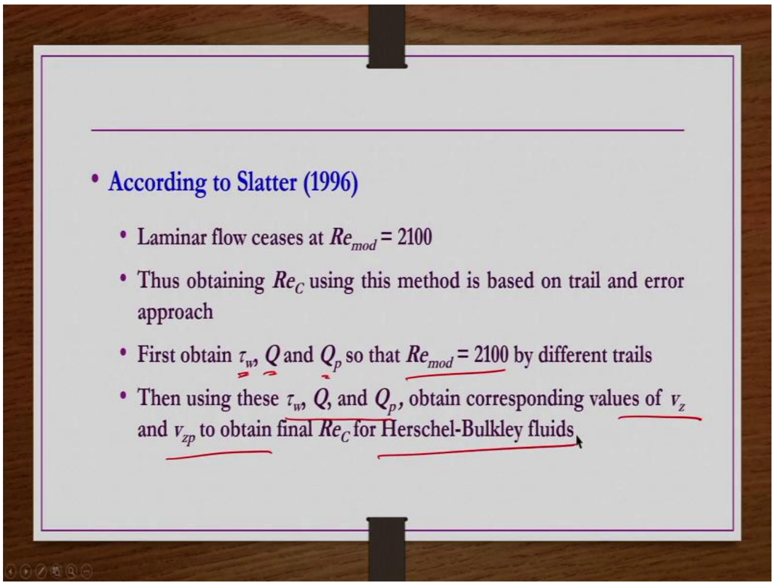

So, according to Slatter, laminar flow ceases at Re_{mod} = 2100. Thus obtaining Re_C is based on trial and error approach because we do not know V annulus and then Q_p etcetera. So, obtain the τ_w, Q and then Q_p. So, that Re_{mod} 2100 by different trials then using these values obtain corresponding values of v_z and v_{zp} to find out final Re_C for Herschel-Bulkley fluids, ok.

Now, what we do? In order to understand are how to use the equations for the Bingham plastic fluids and then Herschel-Bulkley fluids both the models Hanks and Slatter model we take an example problem here, ok.

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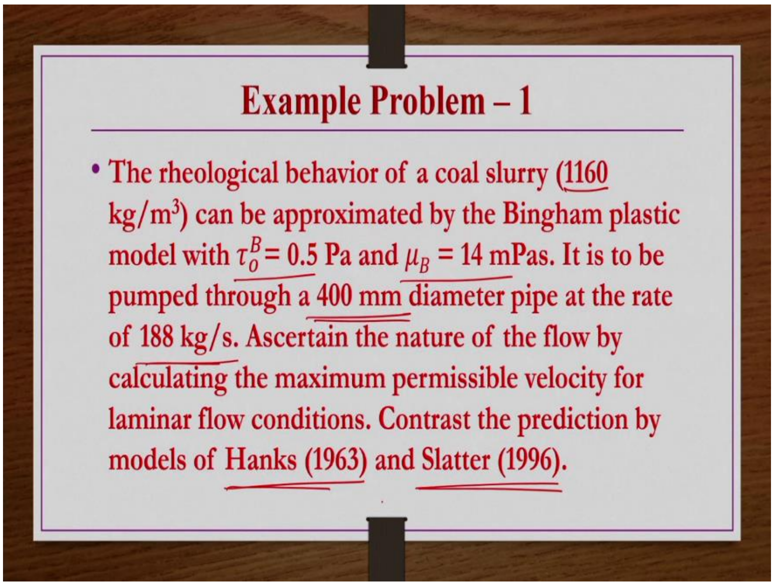

So, rheological behavior of a coal slurry of density 1160 kg per meter cube can be approximated by Bingham plastic model with this $\tau_0$value this $\mu_B$value. The material is pumped through 400 mm diameter pipe at the rate of 188 kg per second. So, we have to find out the maximum permissible velocity for laminar flow conditions. Contrast the prediction by models of Hanks and Slatter; that means, we have to use both the models and then find out these values.

So, we know the actual velocity we can know because the mass rate is given density is given right. If you do the mass rate divided by the density you will be getting the volumetric flow rate. And then volumetric flow rate if you divide by the cross section area you will get the average velocity that is actual velocity that is present in the pipe right.

So, now, as per these two models we have to find out what is the critical velocity and then compare that critical velocity with the actual velocity which is calculated based on the mass rate right.

So, then if the actual velocity is less than the critical velocities that we get by these two models then flow would be under laminar conditions otherwise flow would not be under laminar conditions.

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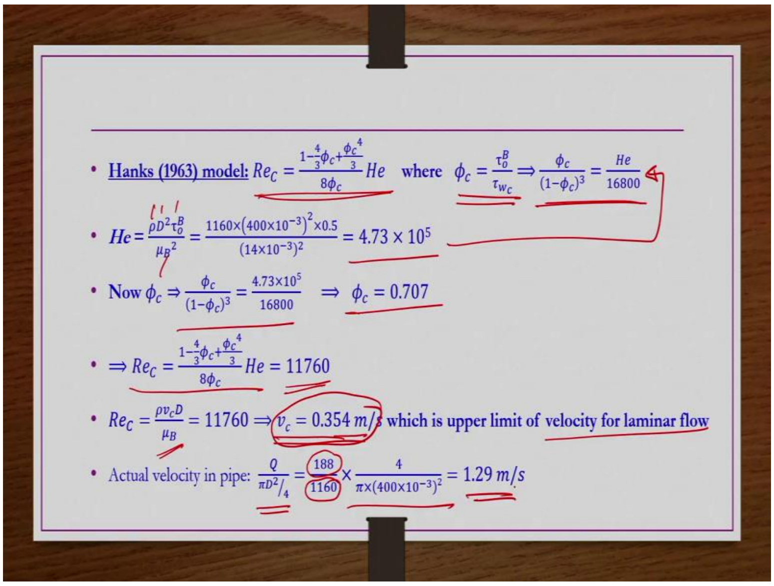

So, Hanks model Re_C this is the expression is given and then ϕ_c is this one whereas, the ϕ_c is related to Hedstrom number using this equation. So, first we have to find out the Hedstrom number which is nothing but $\frac{\rho D^2 \tau_0^B}{{\mu_B}^2}$. ρ is given, D is given, τ_0^B is given, μ_B is given everything is given. So, you substitute you get Hedstrom number 4.73 x10^5. So, now, this value if you substitute here then we get this expression from where we get ϕ_c as 0.707.

So, then Re_C because now Hedstrom number and ϕ_c's both of them are known so, then we can use this equation $Re_C = \frac{1-\frac{4}{3}\phi_C+\frac{{\phi_C}^4}{3}}{8\phi_C}$ Hedstrom number then we get 11760. Then according when we use $Re_C = \frac{\rho v_c D}{\mu_B}$ and then get the critical velocity critical velocity is 0.354 meter per second which is the upper limit for the velocity to be laminar, right.

So, now here if you change τ value changes or μ_B value changes, accordingly Hedstrom will number will change accordingly ϕ_c value will change accordingly Re_C will also change. That is what the important thing about this Hanks model that they have incorporated these parameters into the model and then that Reynolds number is not fixed like other models it changes.

So, Re_C if you maintain up to this one. So, then your flow rate is going to be. So, then your flow is going to be laminar. So, now, what is the actual velocity? Actual velocity as I mentioned $\frac{Q}{\pi D^2/4}$. So, Q is $\dot{m}$ is given $\dot{m}$ is 188 kg per second and then density is given $\dot{m}$ by density if you do 1160 is the density kg per meter cube you will get Q in meter cube per second.

And then multiplied by $\frac{4}{\pi D^2}$ so then this is what? D is 400 mm. So, you get 1.29 meter per second. So, which is much higher than the critical velocity according to Hanks model for this fluid geometry fluid the critical velocity is going to be this one, but the actual velocity is much higher than this one. So, then flow is not going to be laminar according to Hanks model ok.

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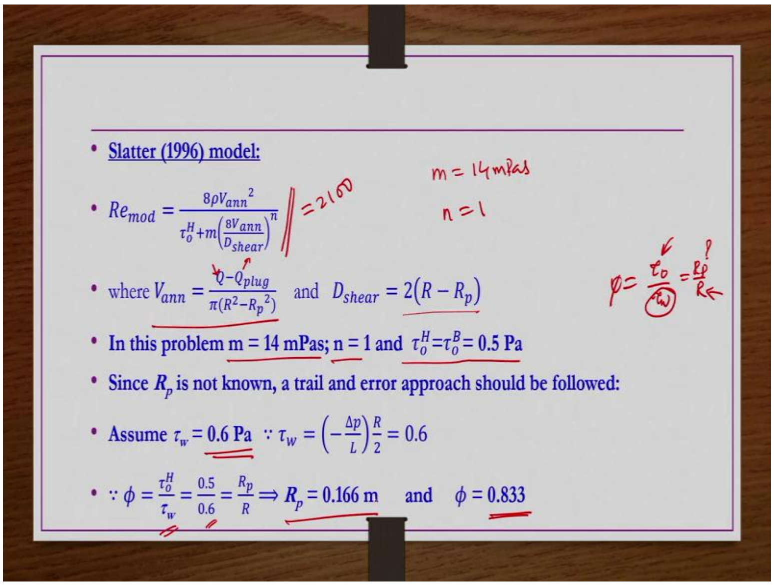

So, now Slatter model according to Slatter we have to find out the conditions where Re_{mod} = 2100, Re modified expression is given by this one, alright. So, now in this equation we analyze we do not know D_{shear} we do not know because R_p value is not known right. R_p value plug radius is not given diameter of the tube is given, but plug radius is not given right.

So, what we have to do? We have to find out V_{ann} and then D_{shear} first and then since this is for the Herschel-Bulkley fluids m is nothing but 14 milli pascal seconds here and then n is nothing but 1 because its Bingham plastic fluid that we are using, ok. So, $V_{ann} = \frac{Q - Q_p}{\pi(R^2 - R_p{}^2)}$ and then D_{shear} is nothing but 2 (R - R_p), ok.

So, in this problem m is 14 milli pascal second n = 1 and then τ_0^H is nothing but τ_0^B which is nothing but 0.5. So, in order to get the V annulus you have to know this Q and then Q plug also Q is given for this case right, but we do not go by the that Q whatever is given that is the true volumetric flow rate.

So, but this we have to find out according to in order to get the Reynolds number close to 2100 if the modified Reynolds number to be 2100 how much Q should be there. So, that we have to find out. So, then we have to use the equation that we have derived in one of the previous lecture, ok.

So, which is function of ϕ, ϕ is nothing but $\frac{R_p}{R}$ and then R_p also we do not know. So, if you wanted to know R_p then what we have to do? This ϕ is nothing but $\frac{\tau_0^H}{\tau_w}$.

So, now τ_0^H is known R is known if you wanted to know the R_p you have to know what is τ_w which is not known. So, that is the region first we have to assume one τ_w value and then find out the R_p and then using and then find out the ϕ for this $\tau_w$value and then for this ϕ value you have to obtain the volumetric flow rate all that we have to do.

So, first we assume τ_w 0.6 pascal right, then ϕ is nothing but $\frac{\tau_0^H}{\tau_w}$ that is 0.5 is τ_0^H for the first trial τ_w we assumed as 0.6. So, then it is coming out to be you know 8, 0.833 and then corresponding R_p is nothing but 0.833 x 200 mm so, which is nothing but 166 mm or 0.166 m meters, ok. So, now ϕ is known τ_w is known, known in the sense for assumed values ϕ is known.

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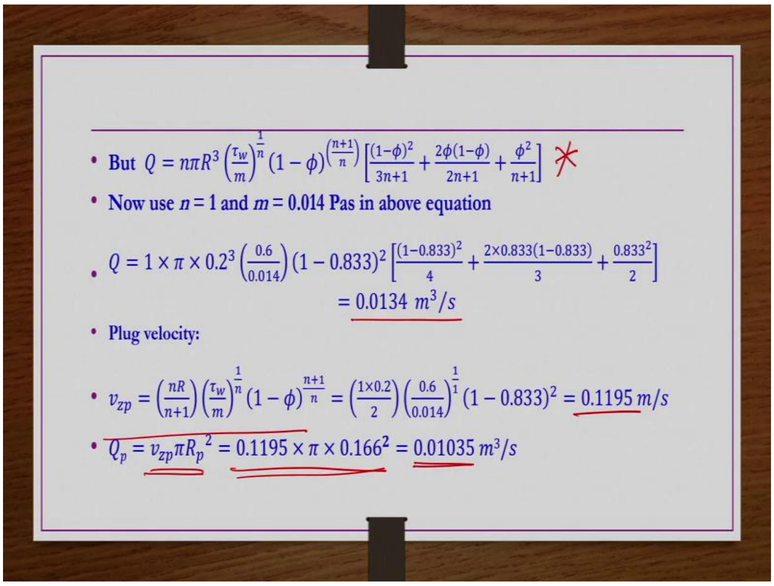

So, once it is known what you can do? You can use this equation for volumetric flow rate that we have derived in yesterday's previous lecture. So, in this equation τ_w ϕ m n R values you substitute. So, then you get Q = 0.0134 meter cube per second. Now, having known Q is not sufficient you should also know Q_p then only you can know V_{ann} or V annulus.

So, Q_p you can know once you know the plug velocity, plug velocity v_{zp} we have seen this expression we derived yesterday. So, here also everything is known now. So, you substitute so, then you get 0.1195 meter per second. So, Q_p would be nothing but this multiplied by $\pi\ R_p{}^2$, right. So, then you have substitute all these values. So, then you get Q_p = 0.01035 meter cube per second. So, now, Q is known, Q_p is known, R_p is also known for the first trial.

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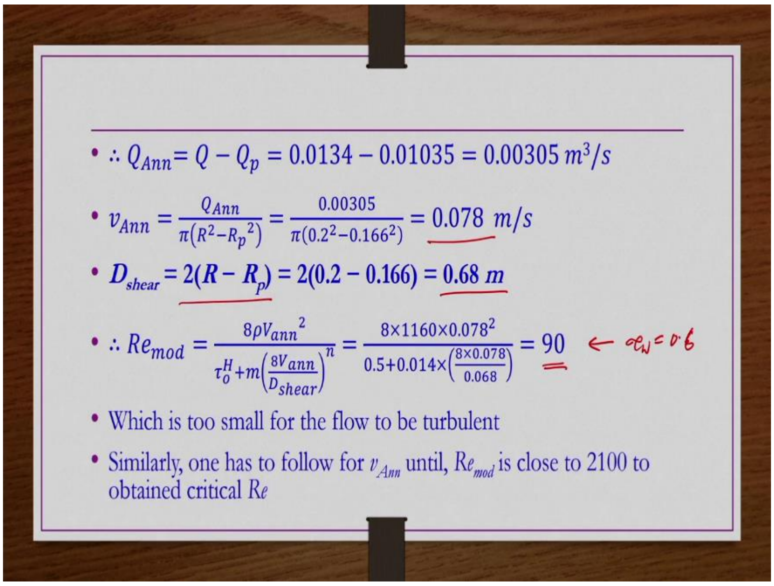

So, V annulus you can easily find out. Q annulus would be $\frac{Q-Q_p}{\pi(R^2-R_p{}^2)}$ you do you get V annulus 0.078 meter per second. Then D_{shear} is nothing but 2 (R - R_p) that is coming out to be 0.68 meters right.

So, Re modified you can know for the first trial you substitute all the values when you assume tau w is equals to 0.6 and then obtain all these values and then after obtaining all these V_{ann}, D_{shear} etcetera you when you substitute them here Re_{mod} is 90.

But the flow to be laminar according to Slatter model this Re_{mod} = 2100. So, this Re modified is too less for an assumed value of τ_w of 0.6. So, what you have to do? You have to assume another value of τ_w and then do the repeated repetitive calculations until this Re_{mod} = 2100.

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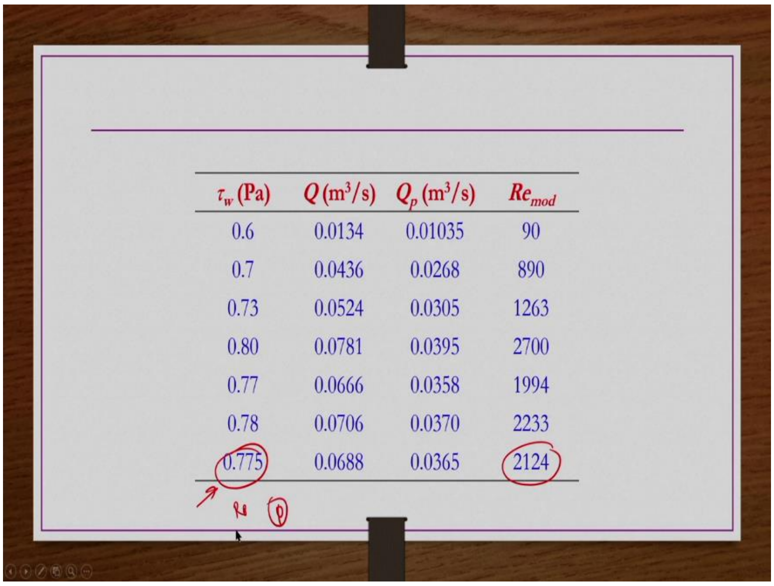

τ_w (Pa)	Q (m^3/s)	Q_p (m^3/s)	Re_{mod}
0.6	0.0134	0.01035	90
0.7	0.0436	0.0268	890
0.73	0.0524	0.0305	1263
0.80	0.0781	0.0395	2700
0.77	0.0666	0.0358	1994
0.78	0.0706	0.0370	2233
0.775	0.0688	0.0365	2124

So, when you do and then make it evolution you can understand that when τ_w = 0.775 then Re_{mod} is close to 2124. So, now, for this τ_w now we got the true τ_w value for which this Re_{mod} is 2100 or close to that right. Now, τ_w is known. So, then from here R_p you can find out from here you can also find out the p. So, then you can get the true values of the average velocity.

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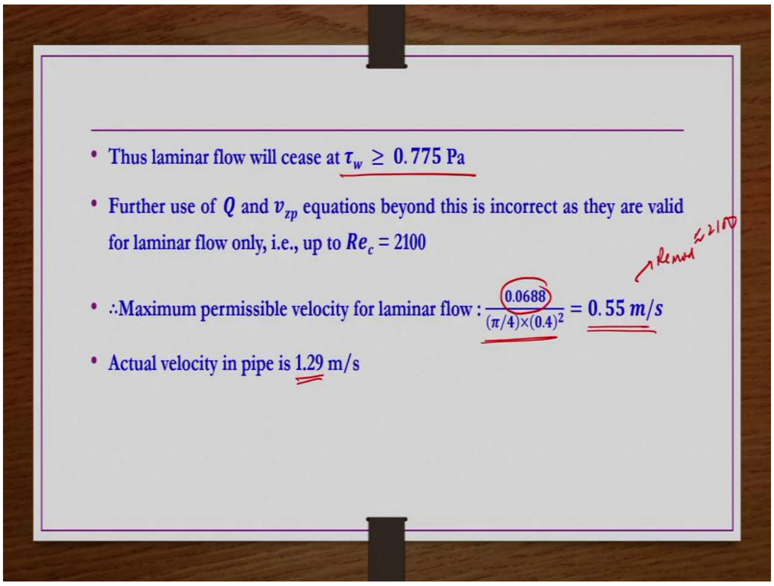

So, the laminar flow so, what we understand? Now, the laminar flow ceases when $\tau_w \geq$ 0.775 pascals. So, this corresponds to volumetric flow rate. Corresponds to this Q value what is the volumetric flow rate? Q is nothing but 0.0688 meter cube per second and then that you divide by the cross section area of the pipe. So, then you get the average velocity in the pipe which is the maximum permissible velocity for the flow to be laminar.

Because when you use this velocity Re_{mod} is going to be approximately 2100, but actual velocity we calculated it as 1.29 meter per second which is given which is obtain based on the mass rate of the fluid flowing through the pipe etcetera right density etcetera we are used.

So, now here also what we understand the actual velocity is much higher than the critical velocity right. So, then what is the conclusion? Whether you are use the Hanks model or Slatter model for this problem you are going to have if you are going to apply or if you are going to have the conditions as given in the problem statement. So, you are going to have a only turbulent flow you are not going to have a laminar flow.

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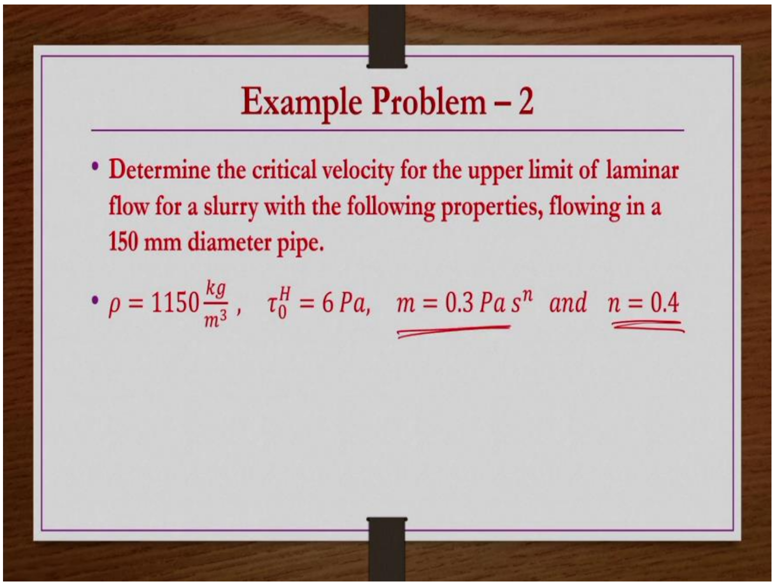

So, now we take another example problem which is quite similar. So, density τ_0^H, m, n values are given right. So, in the previous case n = 1 and then m = 0.014 pascal second something like that now it is n = 0.4 and then m is 0.3 pascal second power n ok so; that means, it is quite similar to previous problem ok.

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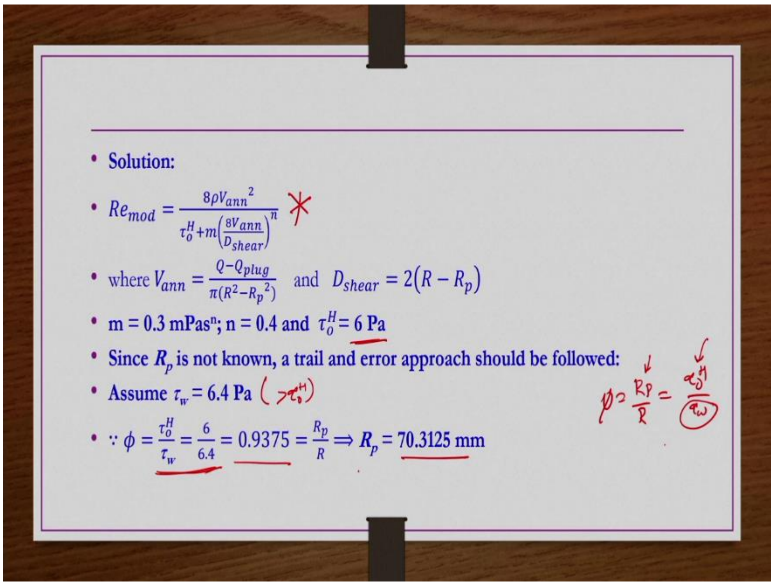

Now, we have to find out a combination of τ_w etcetera all those values where this Re_{mod} = 2100 and then corresponding velocity we have to take it as a kind of critical velocity. So, again V_{ann} D_{shear} we already know these expressions. So, we do not know what is R_p here. So, we cannot use any of these equations.

So, if you wanted to know the R_p you have to know the τ_w right because $\phi = \frac{R_p}{R}$and then ϕ is nothing but $\frac{\tau_0^H}{\tau_w}$ right. So, that is the region $\phi = \frac{R_p}{R}$. R is known R_p if you wanted to know ϕ has to be known. So, then ϕ is given by $\frac{\tau_0^H}{\tau_w}$. So, τ_0^H is given, but τ_w is not known.

So, you assume some τ_w value. What should be the starting assumption of τ_w value? This assumed τ_w value assumed τ_w value should be greater than τ_0^H then only deformation will take place and then all these values expressions whatever given by the Slatter they are only for the deforming region. So, that is the region whatever the assumed τ_w values in this trial and error approach solution methodology that assumed τ_w value should be higher than the τ_0^H value.

Since τ_0^H is 6 pascal we are taking first assumption 6.4 pascal as τ_w right. Then $\frac{\tau_0^H}{\tau_w}$ is nothing but $\frac{6}{6.4}$ that is 0.9375 which is equals to $\frac{R_p}{R}$ because $\frac{\tau_0^H}{\tau_w} = \frac{R_p}{R}$ that we have derived in the previous lecture anyway. So, from here R_p is 70.3125 mm right. So, now you have the R_p. So, then you can find out all these values.

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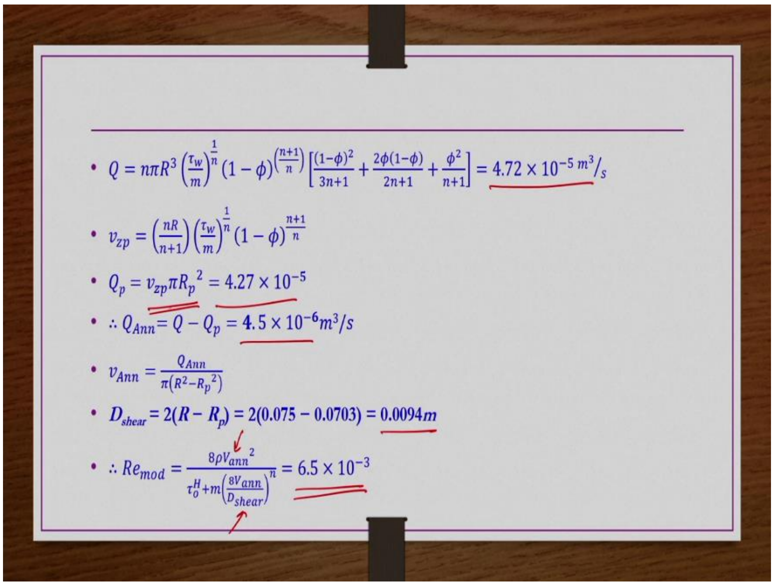

So, first what you have to find out? We have to find out Q this expression we derived. So, when you substitute the first assumed value tau w corresponding phi etcetera then you get Q is equals to this value v_{zp} expression we have this one. So, and then you substitute all these values and then multiplied by $\pi R_p{}^2$if you do you will get Q_p which is nothing but this value.

So, then Q_{ann} is nothing but Q - Q_p that comes out to be 4.5 x 10^{-6} meter cube per second once you have this one you can get the V annulus that is nothing but $\frac{Q_{ann}}{\pi(R^2 - R_p{}^2)}$ ok. So, D shear is nothing but 2 (R - R_p). So, for the first case it is 0.0094 meters.

So, now everything is known. So, Re modified you get here V annulus for the first trial whatever you obtain that you substitute here, D shear also you substitute m n τ_0^H are given ρ are given. So, then you get Re_{mod} 6.5 x 10^{-3} which is very very less than the 2100. So, then you have to, what you have to do? You have to assume another higher value of τ_w and then repeat the process until you get Re_{mod} = 2100.

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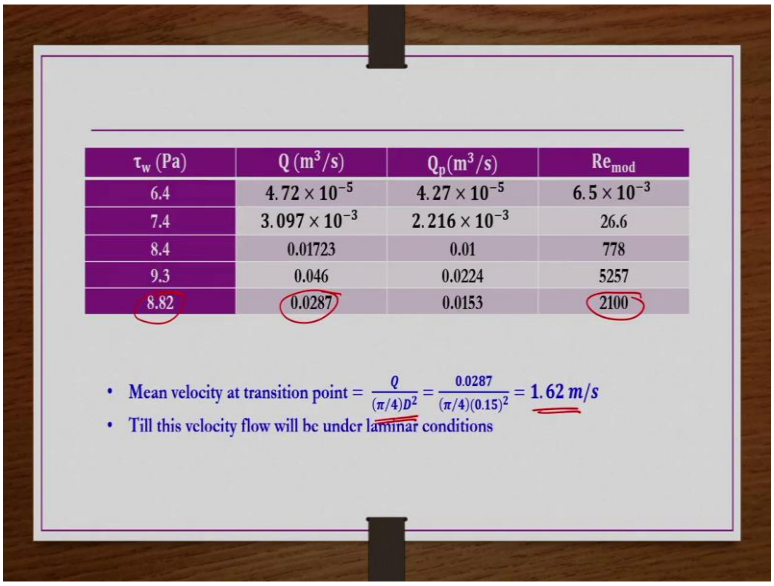

So, that we get here when tau w is equals to 8.82 Pascals you get Re_{mod} is 2100 and then corresponding flow rate is 0.0287. So, that if you divide by $\frac{4Q}{\pi D^2}$ then you get maximum velocity or mean velocity at transition point or at critical point.

So, that is 1.62 meter per second. So, up to this maximum value of velocity you can change the operating conditions or up to this higher velocity you can go if you wanted to operate in laminar conditions ok.

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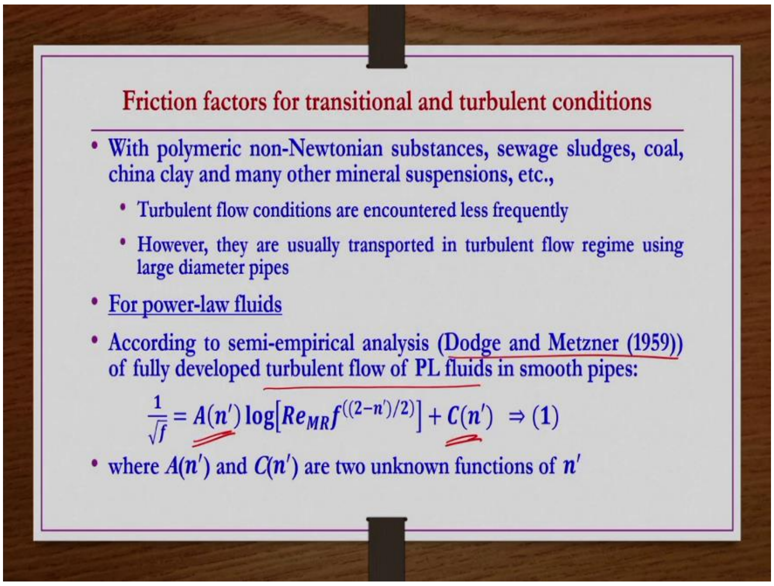

So, what we have seen till now? We have seen how to obtain the critical Reynolds number or how to obtain the critical velocity when a fluid is Bingham plastic fluid or Herschel-Bulkley fluid that we have seen using two different models Hanks model and then Slatter model.

Now, but what is another important factor in engineering applications is friction factor. So, what is the friction factor for generalized Newtonian fluids are flowing through pipe and then flow is not under the laminar conditions, if the flow is under transition or turbulent conditions what should be the friction factors should we use or what are the expressions available those things we are going to see.

So, usually many polymeric non Newtonian fluids you know sewage, sludge, coal slurry, china clay etcetera you know you very rarely get the turbulent flow conditions very rarely, but; however, if you suppose to transport them under turbulent conditions what you can do? You can transport them in higher diameter pipes that is quite possible right. So, then for under such conditions if the flow if the fluid is power law fluid. So, then what is the friction factor?

So, again here these many of the friction factors that we are going to see they are either semi-empirical or empirical kind of things only ok. So, this case Dodge and Metzner they proposed for a fully developed turbulent flow of power law fluids following friction factor they proposed, where A and C are some unknown functions; unknown functions of n’ is nothing but power law behavior index.

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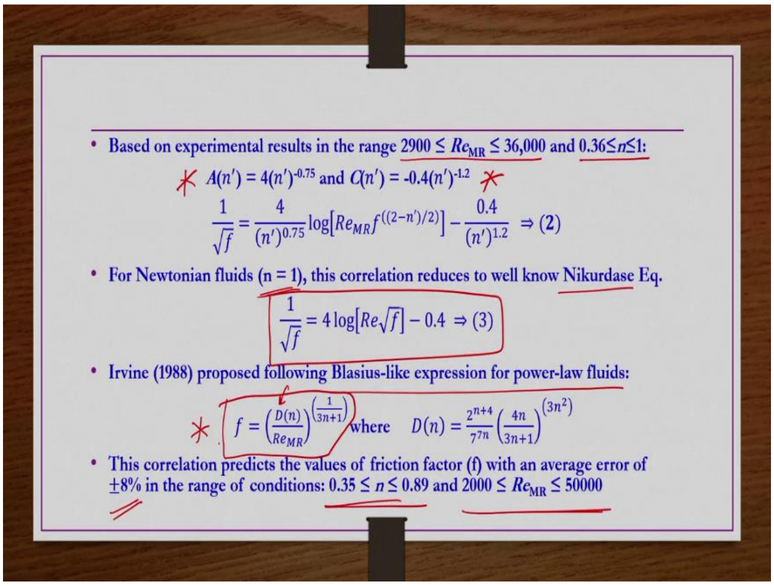

And they have conducted large number of experiments comprising the range of Reynolds number given this one and then n in this range. Then what they found when they are doing the curve fitting etcetera for this friction factor versus Reynolds number. And then try to find out what is A(n') are C(n'). Then they find that A(n') is nothing but $4(n')^{-0.75}$ or $\frac{4}{(n')^{0.75}}$ that is what.

And then C(n') is nothing but $-0.4(n')^{-1/2}$ that is what they got the these are based on the experimental values right. So, by using semi-empirical analysis they could get the form of the equation that we have seen in the previous slide for $\frac{1}{\sqrt{f}}$, but that is having 2 unknown functions A and C which are functions of n'. So, those functions they got based on the experimental results.

So, whatever this correlation after substituting A function of n' C function of n'. So, this is having the validity with experimental results also. So, it is more reliable right. Now in this equation if you substitute n = 1 you will get the famous Nikurdase equation that is nothing but this equation $\frac{1}{\sqrt{f}} = 4log\left[Re\sqrt{f}\right] - 0.4$.

Then Irvine proposed following Blasius-like expression for power law fluids friction factor when the fluid is flowing under turbulent condition. So, that f is given by this expression which is having another function D function of n. So, that is given by this one ok. So; however, this correlation predicts the values of friction factor with an average error of plus or minus 8 percent with the experimental results which is a very good one which is very good; that means this equation is also very much reliable.

However the condition shearer n is varying between 0.35 to 0.89 and then Re_{MR} is varying between 2000 & 50000 under this range of conditions only you can use this equation.

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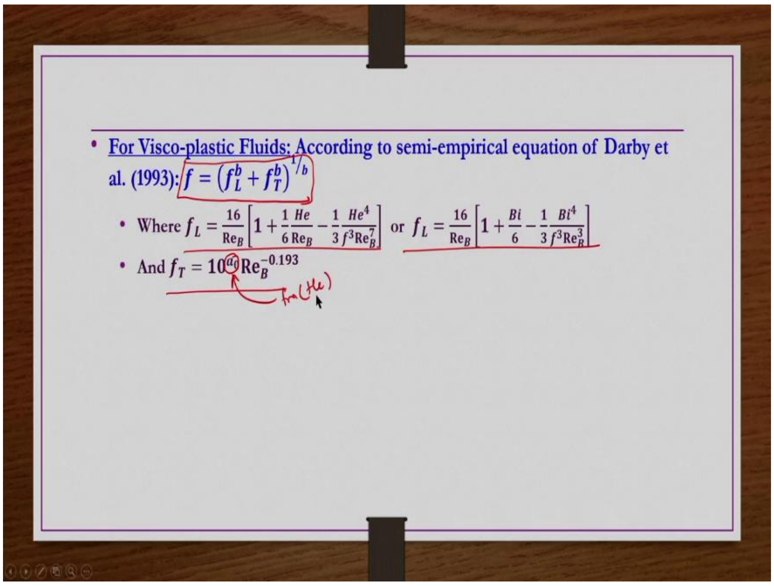

For viscoplastic fluids according to semi-empirical equation of Darby they have given $f = \left(f_L^b + f_T^b\right)^{1/b}$ this expression they have given. This f_L is nothing but the friction factor under the laminar conditions that we have already derived in previous class yesterday's class or day before yesterday's class right f_T^b they are they have given an expression here.

So, f_L this we have derived in one of the previous class ok which can also be written like this either in terms of Hedstrom number and Reynolds number or Bingham number and Reynolds number as shown in these two equations. And then this f_T is given by this expression $10^{a_0} Re_B^{-0.193}$ whatever this a naught is the that is function of Hedstrom number that is function of Hedstrom number.

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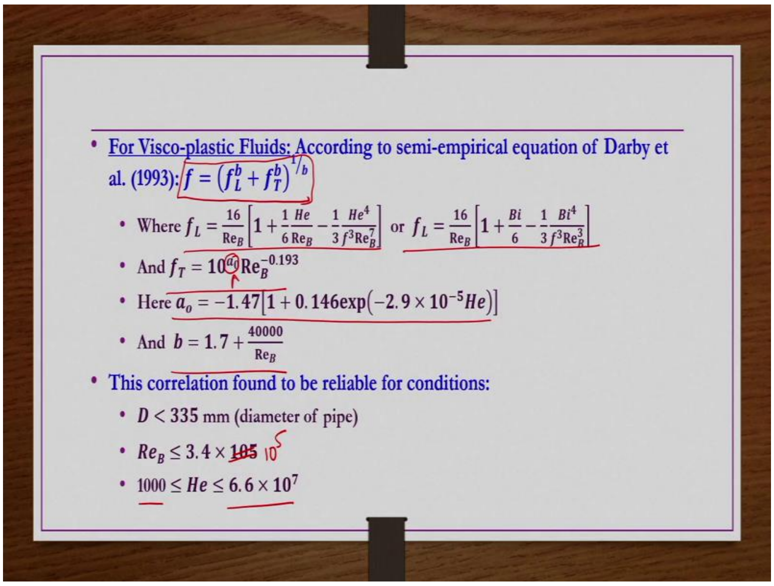

And then that is given by in this equation here. So, Hedstrom number and Reynolds number known. So, then you can calculate f $_L$ and f $_T$ as well then after that also you can you cannot get the friction factor for you know transition and turbulent region because still you need to know b. What is that b? That b is nothing but given by this one which is again function of Reynolds number ok.

This correlation found to be reliable for following conditions because the consistency check has been done with experimental results people found when the conditions are in this range. So, this is 10^5 $Re_B \leq 3.4$ x 10^5 and then Hedstrom number between 10^3 and then 6.6 x 10^7, ok.

So, in the next class what will be doing we will be taking one or two example problems on these things on this particular you know case. So, that to get the friction factor for a viscoplastic fluids flowing through pipe and then flow is under transition or turbulent condition, ok.

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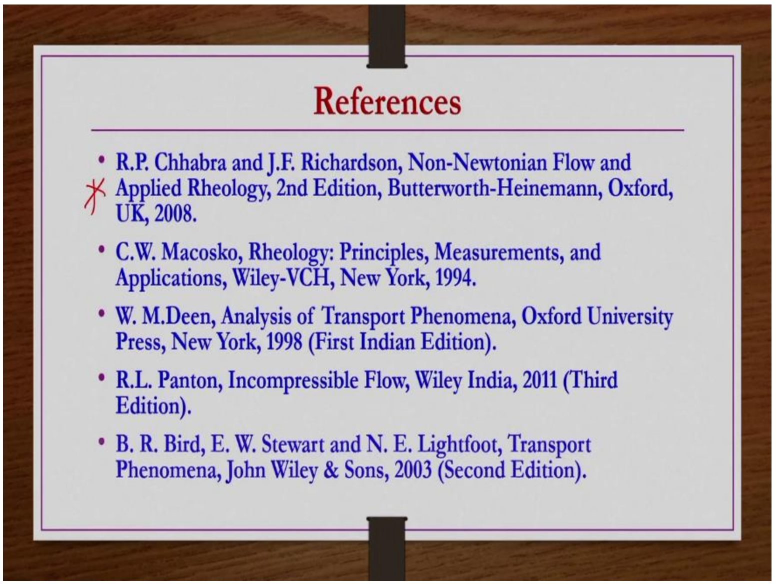

The references for this lecture this lecture is prepared from this excellent book by Chhabra and Richardson other useful references are given here.

Thank you.