Transport Phenomena of Non-Newtonian Fluids

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Lecture - 14
Time Independent Fluids Flow through Pipes

Welcome to the MOOC's course Transport Phenomenon of Non-Newtonian Fluids. The title of this lecture is Time Independent Fluids Flow through Pipes. Till now what we have seen? We have seen different aspects of non-Newtonian behavior and then how to measure the rheology of unknown fluid those things we have seen. Then also we have seen a few basics of transport phenomena and then we also derived conservation equations of mass, momentum and energy in previous lecture till now right.

So, now what we are going to do from this lecture onwards? We are going to apply those principles and then trying to obtain the transport phenomena of different types of non-Newtonian fluids.

So, what we are going to start with? We are going to start with transport phenomena of time independent non Newtonian fluids when they are flowing through pipes. So, within the category of time independent non Newtonian fluids or generalized Newtonian fluids we take a few cases of power law fluids and then Bingham plastic fluids, Herschel Bulkley fluids or even Ellis model fluids also we take. And then we are going to take a different geometries like flow these non-Newtonian fluids flowing through pipes and then they are sliding down through inclined plates etcetera or between two infinite parallel plates etcetera different types of geometries we are going to take.

And then subsequent course we are going to handle the heat and mass transfer phenomena associated with this non-Newtonian fluids as well. So, we start with the case of flow of a power law fluid through circular tubes due to pressure difference.

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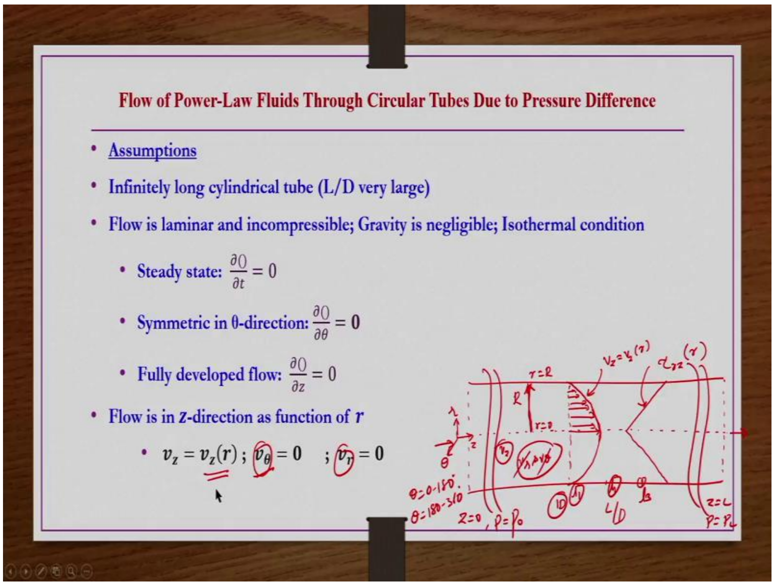

That is we have a circular tube we are having which is having a very large you know L/D ratio that is you know a length is very large compared to the diameter; compared to the diameter of the circular tube right.

So, L/D is very large we are taking so that we can have the fully developed flow. So, at one end you know we have you know fluid coming in and then at the other end fluid going out. So, what is the velocity distribution for this case that we have to see and then what is the volumetric flow rate.

And then what is the friction factor because now fluid has changed. So, then friction factor will also be changing. So, then what is the friction factor? So, under what range of Reynolds number the flow is going to be laminar, under what range it is going to be turbulent all those things we are going to see one by one in you know coming lectures as well right.

So, let us say this is the center of the circular tube alright. Now, the coordinate system if you consider. So, this is your z direction and then this is your r direction. The radius of this circular tube is R our diameter is D. So, now, the fluid is flowing through this one because of the pressure difference.

So, at z = 0 what we do? We take pressure is equals to P_0 and then at z = L we are going to take pressure = P_L and then the L is nothing, but the length of the circular tube that we have taken right. So, any fluid that is flowing. So, now, we are taking a case where there is no slip existing.

So, at the wall the velocity is going to be 0 and then at the center the velocity is going to be maximum, we are taking one dimensional motion only ok under laminar conditions. So, those assumptions we are listing out anyway. So, then what happens you know at the center you are expecting to have a kind of maximum velocity right.

So, let us say you have a Newtonian fluid. So, then what will be the velocity? Maximum velocity at the center would be 2 times to the average velocity that is flowing through. So, then as we move up what happens? The velocity gradually decreases the velocity gradually decreases as we move towards the wall like this and then we have a kind of parabolic profile something like this ok right.

So, now the velocity profile something like this right gradually as you move from center to the wall the velocity decreases ok then shear stress also we have seen till now, the shear stress is 0 at the center right. And then as we move towards the wall, it increases and then it increases linearly for one dimensional flow in the case of a flow through circular tube like this.

So, then this is you know distribution of the shear stress and then this is the distribution of the velocity. Now, the velocity here only v_z velocity is existing one dimensional motion we are taking, the flow is pre-dominating in the z direction. So, v_z is dominating v_r and then v_θ are you know they are very small compared to v_z. So, then we can take them 0.

So, then this v_z velocity is function of now what we understand it? It changes with r it is maximum at r = 0 and then it is 0 at r = R that is at the wall right. So, this what is the shear stress that we are expecting here in this case because one dimensional motion. So, then τ_{rz} we are going to have and then that is also function of r and then that function is a linear function that we understand through our you know in a capillary viscometer case also indeed this is completely same like capillary viscometer, but the context that we are seeing is different there the context was to obtain the shear stress shear rate expression.

So, that to get the rheological behavior and then make required adjustments because of the any sources of errors etcetera those things were the contents there. Now, here we are purely trying to find out what is the volumetric flow rate how it is changing with respect to the pressure drop, what is the velocity distribution, how it is changing with a radial position all those things we are going to see now in this lecture right.

So, this is the physically the problem. So, then what we are going to see? We have to have a kind of enlist of a restrictions of the flow because now here we are saying the laminar flow what are the other assumptions that we are taking. So, let us say we have taken horizontal pipes. So, then gravity we cannot take and then we are taking isothermal conditions. So, all these kind of several restrictions may also be there. So, then we are going to enlist all those things right. So, first we do that one.

So, assumptions involved in this derivation that is derivation for the volumetric flow rate for the power law fluid flowing through a pipe and then how that volumetric flow rate is changing with respect to the pressure difference that is what we are going to see. So, for this derivation the assumptions are like this. So, first most important that infinitely long cylindrical tube that we are taking.

So, that L/D is very large, it is more than 150 or something like that so that end effects are not there; so that end effects etcetera would not be there right. So, then flow is laminar and incompressible, gravity is negligible, isothermal conditions we are taking because we are not considering heat or mass transfer simultaneously along with this momentum transfer problem we are taking only momentum transfer part.

So, this isothermal conditions we are taking, then steady state we are taking steady state because of the steady state $\frac{\partial}{\partial t}$ of anything should be 0 and then what we are taking? The velocity profile is going to be symmetric. So, now this is the two dimensional view of you know cylindrical tube. If you have a kind of a complete picture so then θ d whatever the θ is there in the θ direction the flow is symmetric that is what we are going to say the other one.

The symmetric in θ direction so that is whatever the flow, nature, behavior or distribution flow distribution or velocity distribution is there between θ = 0 to 180, the same that one is taking place between 180 to 360 degrees ok.

So, whatever the flow is there in the θ direction or the distribution that is there velocity distribution from θ direction point of view, $\frac{\partial}{\partial \theta}$ of anything is 0; that means, whatever the flow or velocity distribution is there between 0 to 180 degrees the same thing is there between 180 to 360 degrees right. Then fully developed flow L/D we have taken very large. So, that fully developed flow is taking place. What does mean by fully developed flow?

So, let us say at this location we have a, let us say this location we are calling l_1. So, this is the velocity profile, let us say you take this location is l_2. Again, you draw the velocity profile it would be exactly same thing whatever it was at l_1 and then let us say if you draw the velocity profile other location l_3. So, then at that position also velocity profile would be exactly same like at locations l_1, l_2 that means, along the flow direction the velocity profile does not change; does not change. So, that is the reason along the direction flow direction; flow direction is z. So, here.

So, that is the reason $\frac{\partial}{\partial z}$ of anything any flow variable is 0 not kind of thing scalars or temperature pressure kind of thing or concentration kind of thing this is valid for only flow properties like velocity, shear stress etcetera for those things only it is valid right fully developed flow that where $\frac{\partial}{\partial z}$ of anything is 0 that is what we are having.

That is in the flow direction velocity distribution is not going to change ok right, that is another assumption. And then this fully developed flow assumption is valid in the region away from the entry and then away from the exit because entry and exit effects are there. So, then we are not taking the domain to get the velocity profile within this range of entry and exit, we are taking far away from entry far away from the exit. So, somewhere in between so, that fully developed flow region is developed in which at any location if you measure the velocity profile you are going to get the same profile that is what mean by $\frac{\partial}{\partial z}$ of any flow variable is 0 ok.

That is the other constraint and then flow is predominating in z direction and then it is changing as function of r; obviously, that is clear from the physics of the problem. So, v_z is function of r whereas, v_θ and then v_r very small compared to the v_z. So, then we can strike off them, we do not need to consider right. So, now, these are the assumptions.

So, now what are we going to do by list enlisting these assumptions? We are trying to actually what we have to do? The you have to find out the so called the stress distribution and then velocity distribution etcetera for a power law fluids, whatever they have drawn in the previous slide that is only for the you know the velocity profile is only for the Newtonian fluid right.

So, it is going to be different for power law fluids. So, how it is going to be different that depends on the value of n power law index. So, that we have to develop that relation that v_z as a function of r that we have to develop which is again dependent on the value of n right. So, how it is dependent that if you wanted to find out you have to do the momentum balance right?

So, now what we do? Since this course is at advanced level course rather doing fundamental shell balance for every problem what we are going to do? We are going to use the conservation equation if you solving for the momentum transfer. So, then equation of motion we use and then we apply the constraints of the problem to some to simplify those conservation equations of motion so that you know you can get some kind of a simplified equation.

Those equations you can solve to get the velocity profile and then shear stress distribution etcetera that is what we are going to do. Similar kind of approach we are going to follow for the heat transfer and then mass transfer problems also in the later course, in the due course of the semester ok.

So, that is what we are going to do. We are not going to do the shell balance every time as we have done in the UG course right because whatever the momentum equations are there, they are nothing but the momentum balance equation for control volumes specified control volume right.

So, such momentum equations are already generalized momentum equations are already available. So, those things we are going to use and then we are going to apply the constraints of each and every problem that we are going to consider. So, the constraints of this problem we have enlisted in the previous slide. So, now, what we are going to do? We are going to simplify the continuity equation and then momentum equations so that to get the required velocity profile. Once you have the velocity profile you can get any way volumetric flow rate.

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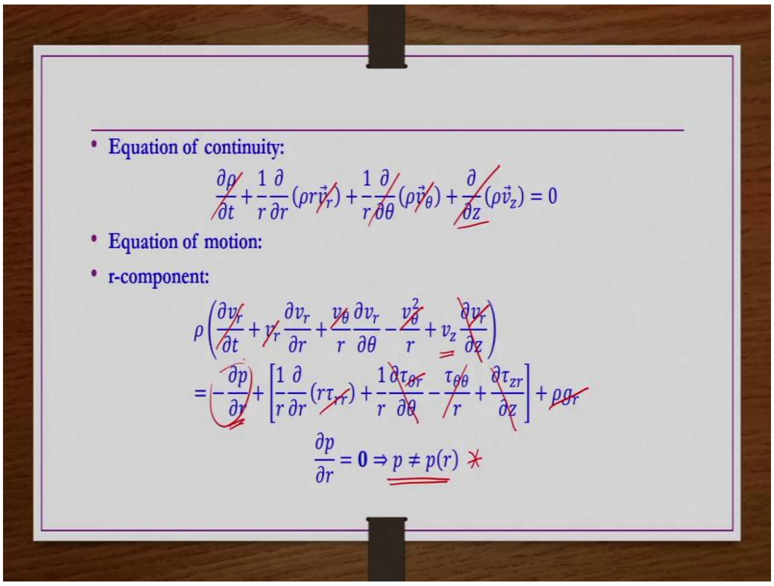

So, equation of continuity in cylindrical coordinates because we have taken circular tubes so which is a cylindrical geometry. So, then what happens? The continuity equation that we have already seen in one of the previous lecture is $\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r\vec{v}_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho\vec{v}_\theta)+\frac{\partial}{\partial z}(\rho\vec{v}_z)=0$. If you know the solution what I said that importance of the continuity equation is that whatever the distribution velocity distribution that you get that must satisfy the continuity equation right otherwise the solution is not reliable ok.

So, now we apply the constraints, what are the constraints? We have the steady state. So, then the first term we can cancel out and then compared to v_z, v_r and v_θ are 0. So, then second, third terms are also 0 because of the symmetry also this third term is 0 anyway you can cancel out. And then because of the fully developed flow $\frac{\partial}{\partial z}$ of any flow variable is 0 so; that means, the constraints are also consistent and then we are getting that continuity is being satisfied here.

So, this is one point of simplifying continuity equations. In some problems what happens you may not able have a conclusions like whether the flow is symmetric or whether the flow is fully developed something like that. So, then in such conditions you can simplify the continuity equation to get one of the conditions right ok.

So, let us say you know the flow is only symmetric, but you do not know whether the flow is fully developed or not. So, then if you simplify this equation, then whatever the condition that you get from that you can understand ok. So, that is the other point of simplifying the equation of continuity, but anyway that we will do later course anyway for this problem it is not required.

Then equation of motion cylindrical coordinates r, θ, z components all of them we are going to simplify. So, r component of equation of motion this is what we have. By the way you do not need to remember all these equations these equations are available in standard textbooks if at all you need to solve this problem for your assignments or for your you know exams these equations would be provided ok.

So, now for r component of momentum equation steady state you cancel out the first term $v_r = 0$, $v_\theta = 0$, $v_\theta = 0$. So, these all these stems are 0 first 4 times now cancelled out v_z is not 0 it is function of r, but $v_r = 0$. So, then that way also it is 0 can last term in the LHS cancelled out and then because of the fully developed flow also we can cancel out the last term right.

And then pressure in general we do not know any information about the you know pressure boundary conditions, limitations on the pressure in general for most of the fluid flow or momentum transfer problems. So, we cannot make any judgment whether that particular term should be included or not right. So, then we will not say anything about it, we just retain it as it is then only τ_{rz} is existing here for this case or here τ_{rr} would function of v_r only.

So, that way also it is cancelled out then because of symmetry this one is cancelled out or τ_θ is r also not there. So, that way also it is cancelled out $\tau_{\theta\theta}$ would be having the terms of only v_θ. So, which is 0. So, that is cancelled out similarly here you know because of the fully developed flow $\frac{\partial p}{\partial r}$ of anything is 0. So, that is also cancelled out. And then we have taken a horizontal tube. So, this term is also 0 gravity we are not considering.

So, what are we finding out by simplifying this r component of momentum equation? We are finding that $\frac{\partial}{\partial z}$ is 0; that means, pressure is not function of r pressure is not function of r this is what we understand by simplifying the r component equation. It depends on problem to problem you know some other geometry, some other problem when you simplify the r component of equation.

So, then you may not you may be having additional terms also right. It is not like that every time whenever you simplify the r component of momentum equation you are going to get $\frac{\partial p}{\partial r} = 0$ no it is not like that, this is only for this problem because all this canceling out of these terms we have done based on the constraints of this particular problem that we have taken which is nothing but the flow of a power law liquid through circular tubes because of the pressure difference.

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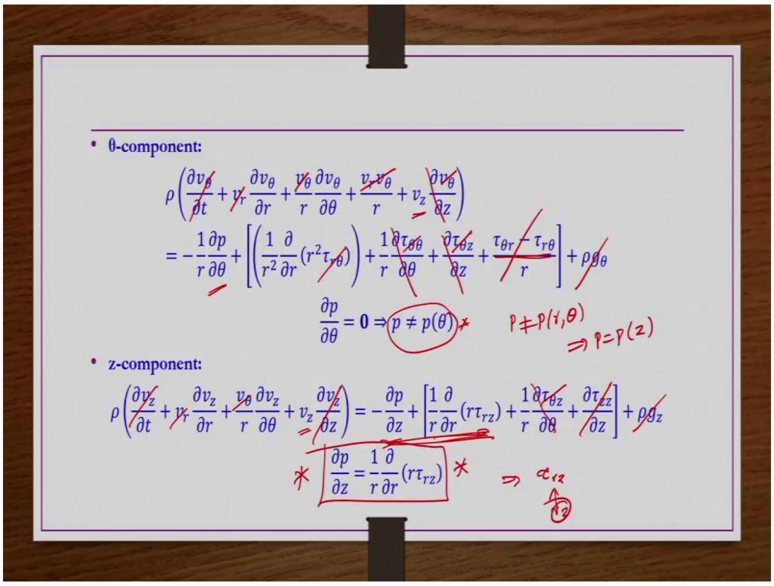

Then θ component of momentum equation this is what we have. So, here steady state. So, this time is 0, v_r is 0, v_θ is 0, then v_r v_θ both are 0, v_z is not 0, but v_θ is 0 as well as the fully developed flow $\frac{\partial}{\partial z}$ of anything is 0 and then this thing we cannot say anything whether it is because pressure we do not know conditions whether it is independent of θ or not, how it is dependent on the θ that we do not know right and also as I mentioned symmetric flow conditions etcetera they are for the vectors not vectors or tensors only that is not for the scalars. So, we cannot say this one ok.

So, just retain it as it is and then only τ_{rz} is existing, $\tau_{r\theta}$ is not existing for this flow problem it is one dimensional flow that we have taken. So, now, this is cancelled out or this $\tau_{r\theta}$ would be having the terms related to the v_θ and then v_r, v_θ function of r, v_r function of v_θ maybe we are having right v_r and v_θ terms maybe there v_r function of θ and then v_θ function of r maybe there. So, both v_r v_θ 0. So, then this should also be 0.

And then the same way this is also 0 because you know otherwise you know because of the symmetry also this is 0. This is not there, $\tau_{\theta z}$ is not existing or because of fully developed flow also $\frac{\partial}{\partial z}$ of anything is 0 and then for symmetric laminar flow these two quantities are going to be same. So, then their difference is going to be 0 anyway.

We are not considering gravity in this problem. So, then what we understand from here? $\frac{\partial p}{\partial \theta} = 0$; that means, pressure is not function of θ also. So, now, by solving this r and θ components of momentum equation what we understand? Pressure is not function of r and θ. In fact, we do not need to worry about this one also because it is already given in the problem statement, the flow is because of the pressure difference and then pressure difference is in the z direction at z = 0 pressure is equals to p_0, at z = l the pressure = p_l ok. So, that is what we have anyway.

Then z component of momentum equation if you simplify. So, this is the equation. So, steady state this term is 0, v_r is 0, v_θ is 0, v_z is not 0, but $\frac{\partial}{\partial z}$ of any flow variable is 0 because of the fully developed flow. So, left hand side all the terms are cancelled out, $\frac{\partial p}{\partial z}$ we cannot say the pressure right. So, whatever the fully developed flow condition etcetera is there that is also for flow variables not for the scalars like temperature and pressure.

So, then we cannot cancel out this one right and then τ_{rz} is existing because it is having v_z and then v_z is function of r. So, we cannot cancel out. So, then this term would be there and then $\tau_{\theta z}$ is not existing because it will be having v_θ v_z, v_z is function of r only it is not function of θ so that way also this term is cancelled out or it will be 0 or because of the symmetry also this term is 0. Then because of the fully developed flow $\frac{\partial}{\partial z}$ of any flow variable τ_{zz} is flow variable it is not scalar. So, then that is cancelled out and then we are not taking gravity. So, that is cancelled out.

So, only these two terms are remaining; that means, $\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$. So, this equation we can solve. So, now, what we understand now here? Pressure is function of z; pressure is function of z it is already clear from the previous two simplifications also, but now the same thing is realized here also pressure is function of z right. So, now, further you know these momentum equations we are simplifying for a Cauchy's momentum equations we are simplifying.

We are not simplifying Navier Stokes equations; Navier Stokes equations are explicitly for Newtonian fluids right. Cauchy's momentum equations are generalized for any fluid Newtonian or non-Newtonian fluid. Only that in the place of τ if you substitute respect to equation for you know nature of the fluid or rheology of fluid. So, then that equation would be for that fluid ok right.

So, now, this equation if you solve then you can get expression for τ_{rz} and then after that you further simplify this one depending on the nature of the fluid Newtonian or power law or Bingham plastic anything, then from here we can get the velocity profile v_z ok. How? That we are going to see now.

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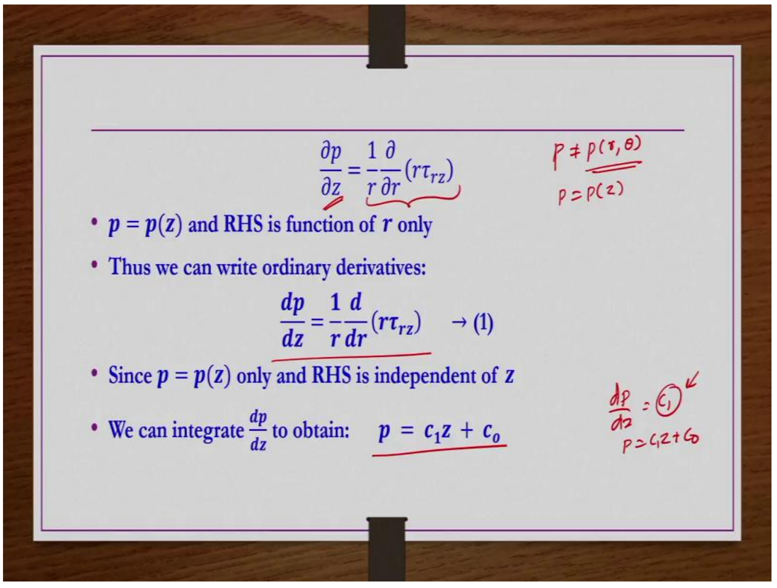

So, this is what we get. What we understand by simplifying r and θ momentum equation p is not function of r and θ right.

So, left hand side it is $\frac{\partial p}{\partial z}$ it is function of z only, but right hand side we are having all you know out of r, θ, z only r coordinates are there here right. So, only r coordinates are there. So, then for left hand side term the right hand side term is nothing, but is a constant term kind of thing because left hand side term the pressure term is there that we already realized that it is independent of r.

So, that is the point of you know simplifying all equations of motion. So, you get certain information definitely which is going to be useful right. So, now, we understand only it is only function of z from this z component of momentum equation. So, right hand side term it is all function of r. So, then for left hand side term the right hand side term is a constant. Similarly right hand side term it is all function of r and then τ_{rz} it is having v_z and then it is function of r only it is not function of z.

So, for the right hand side term the left hand side term whatever $\frac{\partial p}{\partial z}$ term or $\frac{\partial p}{\partial z}$ is there that is constant; that means, we can integrate this equation independent of each other and then get the solution ok. So, that is what we are going to do right. So, because of this nature we can also write this equation in ordinary derivatives like this $\frac{dp}{dz} = \frac{1}{r}\frac{d}{dr}(r\tau_{rz})$. Now, we can integrate this $\frac{dp}{dz}$ because it is function of z only and then RHS is independent of z.

So, then when you solve this $\frac{dp}{dz}$ is equals to some constant c_0, then you when you saw take this $\frac{dp}{dz}$ is equals to some constant c_1. So, then you get $p = c_1 z + c_0$. So, that is what you have. What is this constant c_1 let us not worry about it ok right. So, that is what we have.

So, now for this you apply the boundary condition and get the pressure distribution whether it is linear or non-linear in the z direction that is what you get.

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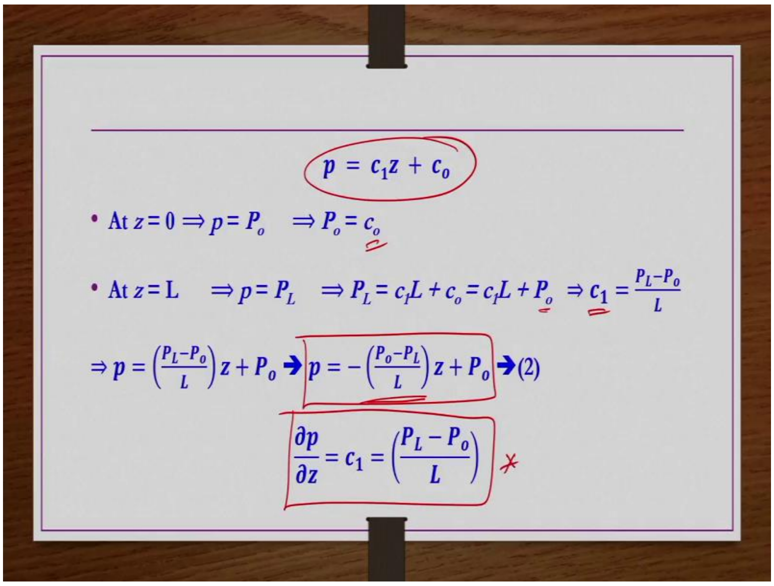

So, at z = 0, we got $p = P_0$. So, that if you substitute you get $c_0 = P_0$ one constant you got at z = L pressure is nothing, but P_L that is given schematically that in the first slide we had so; that means, $P_L = c_1 L + c_0$. So, c_0 you already got P_0 then c_1 is nothing, but from this equation $\frac{P_L - P_0}{L}$.

So, now you got both c_0 and then c_1 expression. So, that you substitute here in this equation. So, that you get this expression $p = -\left(\frac{P_0 - P_L}{L}\right) z + P_0$ this is what you get. This is nothing, but you can take it at $\frac{-\Delta P}{L}$ ok. So, what you understand from here? You understand that pressure is a linear function of z.

It is function of z, but this function is a linear function that is what you understand ok or $\frac{\partial p}{\partial z} = \frac{-\Delta P}{L}$ that is what you get from this equation. So, this is going to be useful anyway right. So, now, in the equation number 1 what we have?

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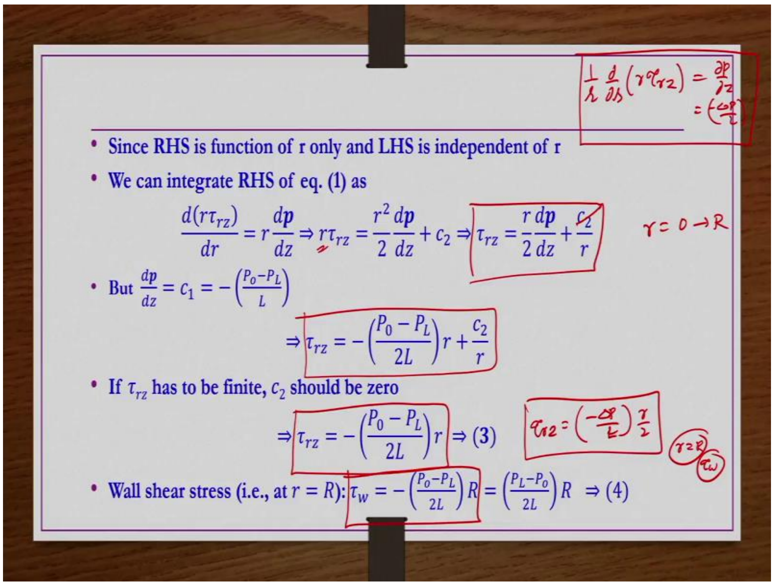

$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = \frac{\partial p}{\partial z}$. Now, $\frac{\partial p}{\partial z}$ is nothing, but c_1 that we already got which is nothing, but $\frac{-\Delta P}{L}$ right. So, this is $\frac{-\Delta P}{L}$. So, now, this equation you can integrate you can integrate.

So, when that you do it you get $\frac{d}{dr}(r\tau_{rz}) = r\frac{dp}{dz}$; that means, $r\tau_{rz} = \frac{r^2}{2}\frac{dp}{dz}$; $\frac{dp}{dz}$ we are not substituting we can substitute later on there is no issue at all plus integration constant c_2. So, then what you get? $\tau_{rz} = \frac{r}{2}\frac{dp}{dz} + \frac{c_2}{r}$ right.

Now, this r is varying between 0 to R right. So, if r = 0. So, then what happens? This constant by 0 it is going it is not going to be defined it is going to be infinite right. So, that is not possible shear stress that cannot be infinite because of the continuum hypothesis no flow variable or physical properties can be infinite or undefined at any location that is what we have seen one of the previous lectures so; that means, this c_2 has to be 0.

Then what you get? $\tau_{rz} = \frac{r}{2}\frac{dp}{dz}$ and then $\frac{dp}{dz}$ is nothing, but $\frac{-\Delta P}{L}$ that is $\tau_{rz} =- \frac{-\Delta P}{L}\frac{r}{2}$ this is what you get right. So, the same thing you can write $\tau_{rz} = \frac{-\Delta P}{L}\frac{r}{2}$ right this is the shear stress distribution. And then what we understand? $\frac{\Delta P}{L}$ is constant.

So, what we understand? The shear stress is linearly changing with r location it is 0 at the center and then it is maximum at r = R that is what we can understand from this equation number 3. So, what is that maximum value of shear stress? If you substitute r = R, then you get that maximum value of shear stress that is nothing, but the wall shear stress right. So, that is $\tau_w = \frac{-\Delta P}{L}\frac{R}{2}$ right.

So, you got the shear stress distribution, but your interest is not just a shear stress distribution, it is also to get the velocity distribution and then subsequently finding out the volumetric flow rate.

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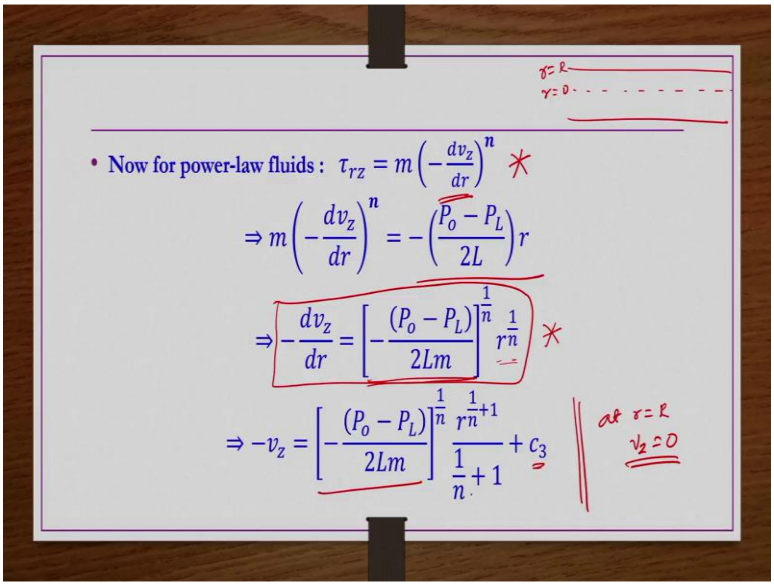

So, for power law fluids what is τ_{rz}? Is nothing, but $m\left(-\frac{dv_z}{dr}\right)^n$ we have $-\frac{dv_z}{dr}$ we have taken because what happens the pipe circular tube whatever we have taken the velocity is maximum at the center right.

And then it is decreasing as we move towards the wall at from r = 0 to r = R so; that means, as r increasing the velocity is decreasing. So, then this gradient velocity gradient is going to be negative that is the reason we have to take minus here, this whether should you take minus or plus that depends on the flow geometry like this ok. So, $\tau_{rz} = m\left(-\frac{dv_z}{dr}\right)^n$ is the expression for power law fluids ok. The relation between shear stress and shear rate for power law fluids is given by this equation this we have seen in the introduction of non-Newtonian fluids right.

So, then this equation τ_{rz} just now you got it as $\frac{-\Delta P}{L}\frac{r}{2}$. So, then that we have written here. So, next step what we are going to do? We are taking this m to the right hand side and then both sides we are taking power $\frac{1}{n}$ so, that we get $-\frac{dv_z}{dr} = \left[\frac{-\Delta P}{2Lm}\right]^{\frac{1}{n}} r^{\frac{1}{n}}$ this is what you are going to have.

So, this expression also we are this is nothing, but the variation of the shear rate. How the shear rate is varying with r because this now this parenthesis whatever the term is there this entire thing is constant for a given fluid, for a given power law fluid of known m and n value this is a constant.

So, it is a shear rate is changing as $r^{\frac{1}{n}}$; $r^{\frac{1}{n}}$it is changing ok. So, now, this is we are going to use later also for other purpose of simplifications right; however, first what we have to do now? We have to integrate this equation to get the v_z; to get the v_z that we can do it when you do $-v_z$ is equals to this is all constant and then integration of $r^{\frac{1}{n}}$ is nothing but $\frac{r^{\frac{1}{n}+1}}{\frac{1}{n}+1}$ + plus integration constant c_3. Now, the velocity at r = R is 0 alright. So, that boundary condition if you use then you get c_3. So, at r = R v_z is 0.

So, c_3 should be you know whatever this term is there minus of this term $\frac{R^{\frac{1}{n}+1}}{\frac{1}{n}+1}$ that is what you get.

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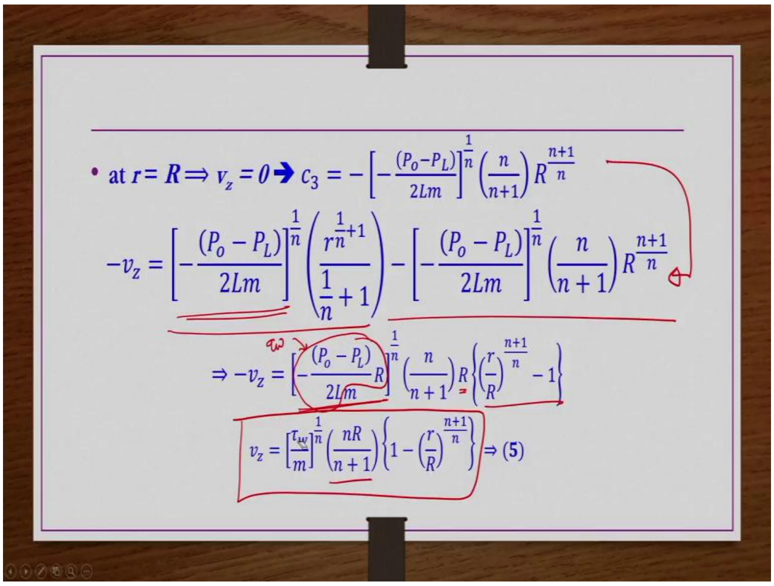

So, this is the c_3. Now, this c_3 you are going to substitute in the minus v_z is equals to this particular term and then plus c_3 was there in place of plus c_3 this is the term that we got right.

So, now here what we do? Except this equation we rearrange such a way that we have $-v_z$ one side and then minus whatever these square parenthesis terms are there that we are taking common and then also we are taking $R^{n+\frac{1}{n}}$ also common from the both the terms, from these two terms.

So, out of that $R^{\frac{1}{n}}$ we are combining with this $\left[\frac{-\Delta P}{2Lm}\right]^{\frac{1}{n}}$ term. So, that we can have $\left[\frac{-\Delta P}{2Lm}R\right]^{\frac{1}{n}}$ and then remaining R we are right here. So, that this is what we are having. So, this is $-v_z$. So, then $+v_z$ would be this particular term here. So, this would be $1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}$ it will be there what we have seen $\frac{-\Delta P}{L}\frac{R}{2}$ is nothing but τ_w it is nothing, but τ_w in the previous slide only we have seen right.

So, that is what we have seen. So, in place of $\frac{-\Delta P}{L}\frac{R}{2}$ I have written $\left[\frac{\tau_w}{m}\right]^{\frac{1}{n}}$ and then $\frac{nR}{n+1}\left\{1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right\}$.

So, for shear thinning fluids if you substitute n < 1 whatever the n value is there. So, then you get the velocity profile for a shear thinning fluids flowing through pipes right circular tubes or pipes right. If you have shear thickening fluids if you substitute n > 1. So, then you get the velocity profile for that fluid flowing through a pipe.

So, that you can understand you can calculate from this one. So, you can use this equation either in terms of $\frac{-\Delta P}{L}$ or you can use the τ_w terms as well anyway you can write ok.

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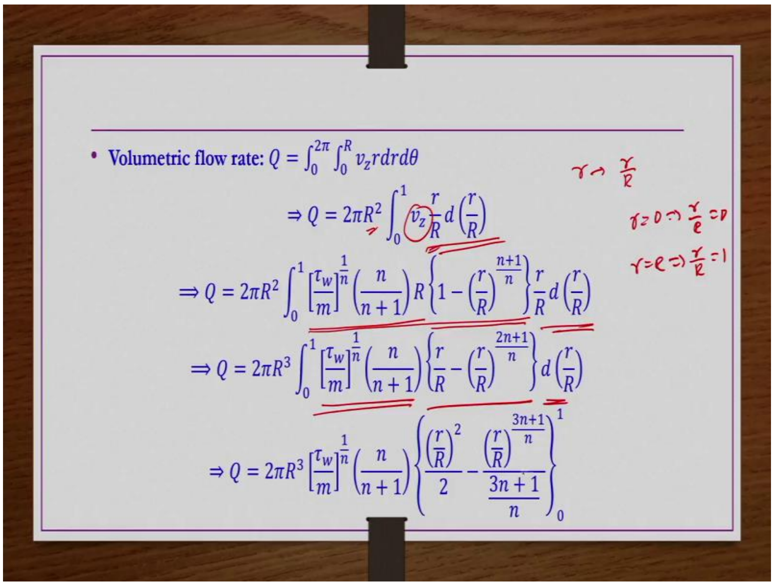

So, now volumetric flow rate if you wanted to find out you can do it $\int_0^{2\pi} \int_0^R v_z r dr d\theta$ simply because v_z is not is now known v_z as function of r is known yeah. So, it is not function of θ. So, then that we can write $2\pi \int_0^R v_z r dr$ we can write right. So, that we have written here and then further what we are doing? We this we are r we are writing $\frac{r}{R}$. So, that d also we can write $d\left(\frac{r}{R}\right)$.

So, we are dividing by R^2 here. So, then we have to multiply by R^2 here why are we doing because v_z is having something like you know terms $\frac{r}{R}$. So, then if you do the integration with respect to $\frac{r}{R}$ it will be easier otherwise there is no particular reason that writing in this form. So, v_z just now we have seen $\left[\frac{\tau_w}{m}\right]^{\frac{1}{n}} \frac{nR}{n+1} \left\{1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right\} \frac{r}{R} d\left(\frac{r}{R}\right)$ is as it is here ok. So, this term is nothing, but your v_z terms integration you are taking changing from r to $\frac{r}{R}$.

So, then; obviously, this limits also will change from 0 to 1 because at r = 0 $\frac{r}{R}$ is also going to be 0 at r = R $\frac{r}{R}$ is going to be 1 right. So, now, what we do? We multiply this $\frac{r}{R}$ or bring this $\frac{r}{R}$ inside this parenthesis and then we are going to do the integration. So, when you do it? So, because all these things are constant. So, then we can take out as a common outside of the integration.

So, we have the integration $\frac{\left(\frac{r}{R}\right)^2}{2} - \frac{\left(\frac{r}{R}\right)^{\frac{3n+1}{n}}}{\frac{3n+1}{n}}$ and then 0 to 1 are the limits; when you substitute 0 to 1 when you substitute when $\frac{r}{R}$ is 0. So, then lower limit both the both these terms are 0 upper limits when you substitute $\frac{1}{2} - \frac{1}{\frac{3n+1}{n}}$ that is $\frac{n}{3n+1}$ that is what you get.

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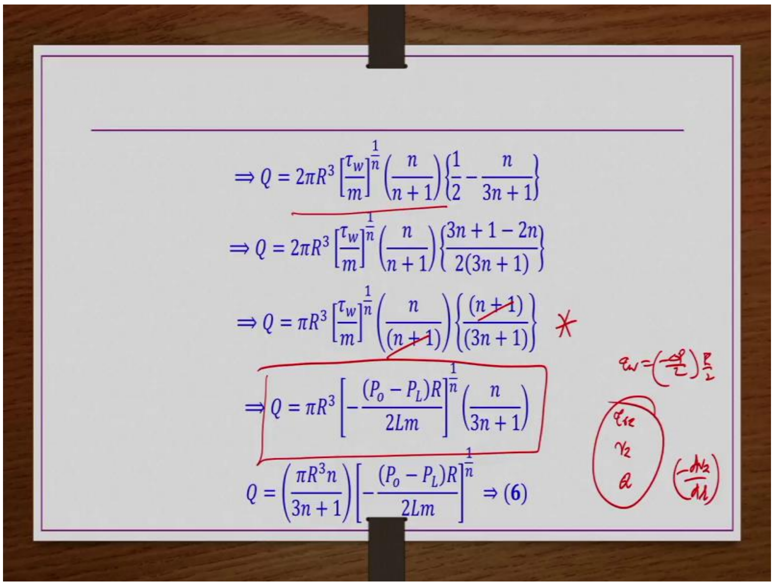

So, $\frac{1}{2} - \frac{n}{3n+1}$ you get remaining all other terms are constant. Now, this you can do LCM further simplify to get this Q value as this one.

So, this n + 1, n + 1 also you can cancel out. So, this is the final volumetric flow rate this you can write in terms of τ_w as well as you can write in place of τ_w you can write $\frac{-\Delta P}{L}\frac{R}{2}$ so that you can write this equation as like this fine. So, now, here we are combining all the terms.

So, then we have $\left(\frac{n\pi R^3}{3n+1}\right)\left[\frac{-\Delta PR}{2Lm}\right]^{\frac{1}{n}}$ is volumetric flow rate right. So, now, you have a v_z expression you have a Q expression you have a τ_{rz} expression also you got it right. So, further what we are going to do? We are getting $\left(-\frac{dv_z}{dr}\right)$ how much it is. So, that to get you know shear rate also and then shear rate also we will be representing in terms of Q and then after that we will be doing the simplification what is the maximum velocity, average velocity etcetera.

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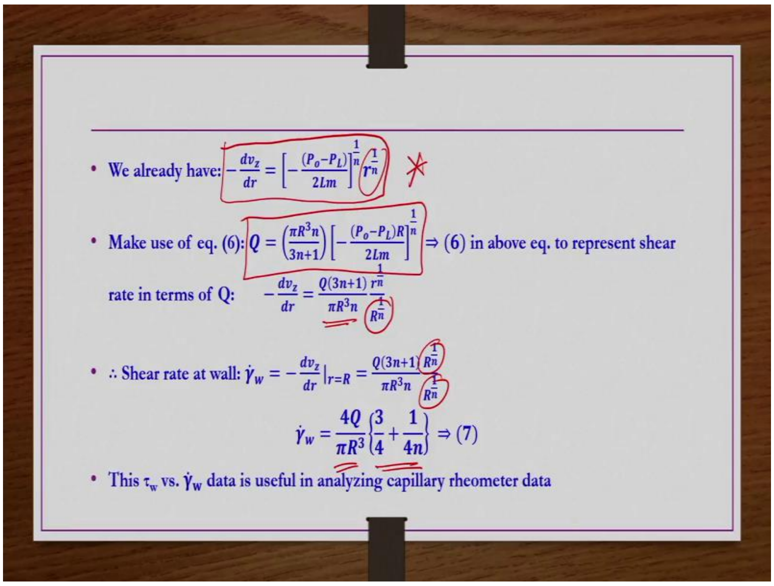

So, as I already mentioned at the beginning this equation that we have already derived that is we are going to reuse somewhere later.

So, this is the point we wanted to find out the shear rate as function of Q. So, now, this is what we have already seen one of the previous slides. Now, in this just now previous side Q we got this expression equation number 6 right. So, now, here what you do? Wherever this $\left[\frac{-\Delta PR}{2Lm}\right]^{\frac{1}{n}}$ is there in that place in this equation you can write $\frac{Q\,(3n+1)}{n\pi R^3}$ that is what you can write and then divided by this whatever this R is there you know this $R^{\frac{1}{n}}$ that is here and then $r^{\frac{1}{n}}$ is as it is here.

So, that is what we are doing. So, simply this equation number 6 we are using in this above equation here. So, that $\left(-\frac{dv_z}{dr}\right)$we can represent in terms of volumetric flow rate that is the; that is the thing that we are doing because in general you know the volumetric flow rate. So, then that directly you can substitute and get the shear rate information.

So, at the wall if you wanted to know the shear rate what you have to substitute? You have to substitute r = R. So, that you substitute. So, then $\frac{R^{\frac{1}{n}}}{R^{\frac{1}{n}}}$ you get. So, then both are same. So, you have wall shear rate is nothing but $\frac{(3n + 1)Q}{n\pi R^3}$.

So, then this equation what you do? You multiply by 4 and divide by 4 so that you can write $\frac{4Q}{\pi R^3}\frac{3n+1}{4n}$ this is how you can write the wall shear rate ok. So, now, you got this shear rate expression also in addition to the shear stress.

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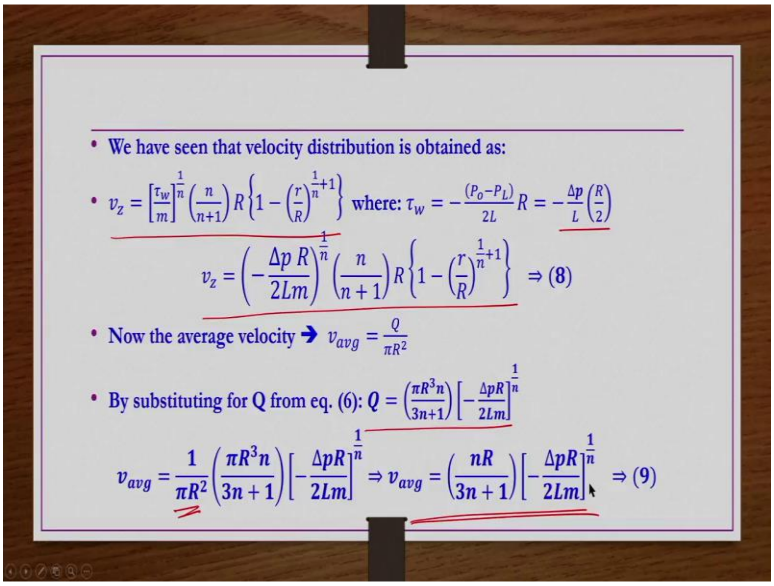

Now, what we got till now? We got the velocity distribution, volumetric flow rate expiration, shear stress distribution, shear rate distribution etcetera all those things we have seen. So, now, what we are going to do here? We are going to plot the velocity distribution because if a Newtonian fluid is flowing through infinitely long circular cylinder we know that the velocity profile in the fully developed flow region is a parabolic profile; is it the same for a fluids like you know a shear thinning fluids or the different flow rate or the different distribution is going to be there that is what we are going to see now here right.

So, this v_z we have already derived where τ_w is nothing, but $\frac{-\Delta P}{L}\frac{R}{2}$. So, in terms of $\frac{-\Delta P}{L}$ if you wanted to write the same expression you can write like this ok. Now, average velocity if you wanted to find out you have to divide the volumetric flow rate by cross section area of the pipe through which the fluid is flowing. So, then $\frac{Q}{\pi R^2}$ if you do you get the v_{avg}.

So, that is Q just now we got this expression equation number 6, if you divide this $\frac{1}{\pi R^2}$ and then do the simplification v_{avg} you get $\left(\frac{nR}{3n+1}\right)\left[\frac{-\Delta PR}{2Lm}\right]^{\frac{1}{n}}$ this is what we have right.

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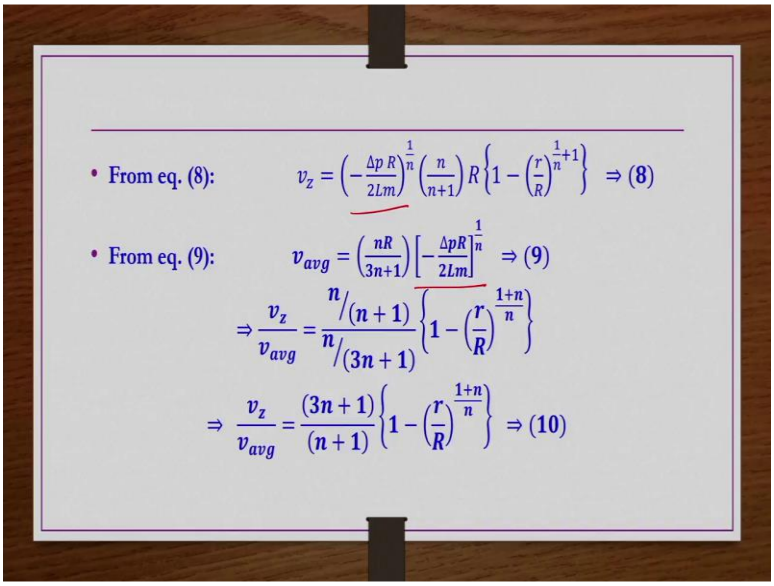

So, rewriting that equation number 8 that is for velocity distribution in terms of $\frac{-\Delta P}{L}$ and R then v_{avg} just now we got this expression. So, then when we do $\frac{v_z}{v_{avg}}$. So, then what will happen? So, this term this term are same alright. So, then remaining terms if you rearrange you get $\frac{v_z}{v_{avg}}$ is nothing, but $\frac{3n+1}{n+1}\left\{1-\left(\frac{r}{R}\right)^{\frac{1+n}{n}}\right\}$ that is what you get right. So, this expression we are going to use it.

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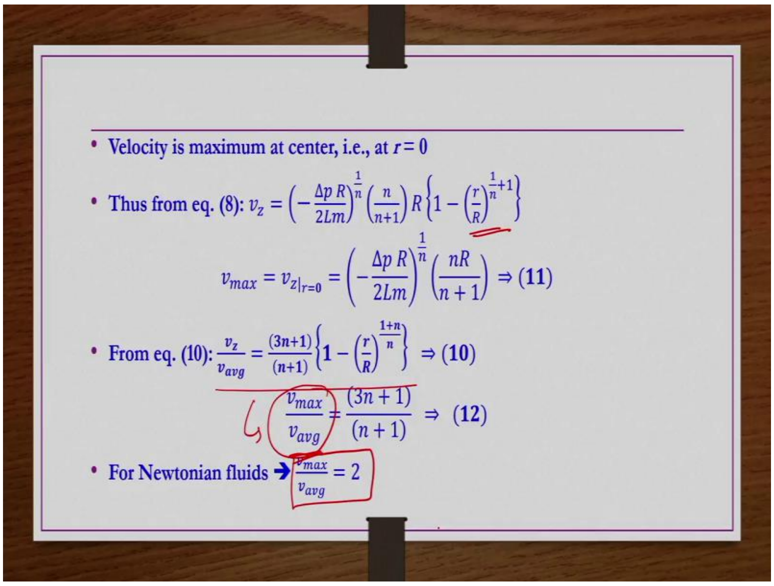

Now, anyway and that is average velocity. If you wanted to find out the maximum velocity; velocity is going to be maximum at the center of the circular pipe that we have taken. So, then we have to substitute in the velocity profile distribution r = 0 in the velocity profile distribution equation whatever we derived that is equation number 8 if you substitute r = 0 then you get maximum velocity.

If r = 0. So, then this term is 0 right so; that means, v_{max} you are going to have $\left[\frac{-\Delta PR}{2Lm}\right]^{\frac{1}{n}}\left(\frac{nR}{n+1}\right)$ that is what you are going to have. So, now, $\frac{v_z}{v_{avg}}$ we already got it in the previous slide this is what we got. Now, if you wanted to find out $\frac{v_{max}}{v_{avg}}$ what you have to do? $\frac{v_{max}}{v_{avg}}$ if you wanted to find out in this equation number 10.

If you substitute it r = 0, then you get $\frac{v_{max}}{v_{avg}}$ right then you get $\frac{v_{max}}{v_{avg}}$ is nothing, but $\frac{3n+1}{n+1}$. So, let us say if it is Newtonian fluids n should be equals to 1; that means, $\frac{v_{max}}{v_{avg}} = 2$ that we already know right. So, that is what we get $\frac{v_{max}}{v_{avg}} = 2$ for the Newtonian fluid, but it is $\neq 2$ for shear thinning or shear thickening fluid it is different depends on the n value.

So; that means, velocity profile is going to be different for shear thinning or shear thickening fluid. We are not going to get parabolic profile as well for the shear thinning and shear thickening fluid flowing through pipes that we get parabolic profile we get only for Newtonian fluids.

So, how do the profile look like for shear thinning and share thickening fluid that is what we have to see now.

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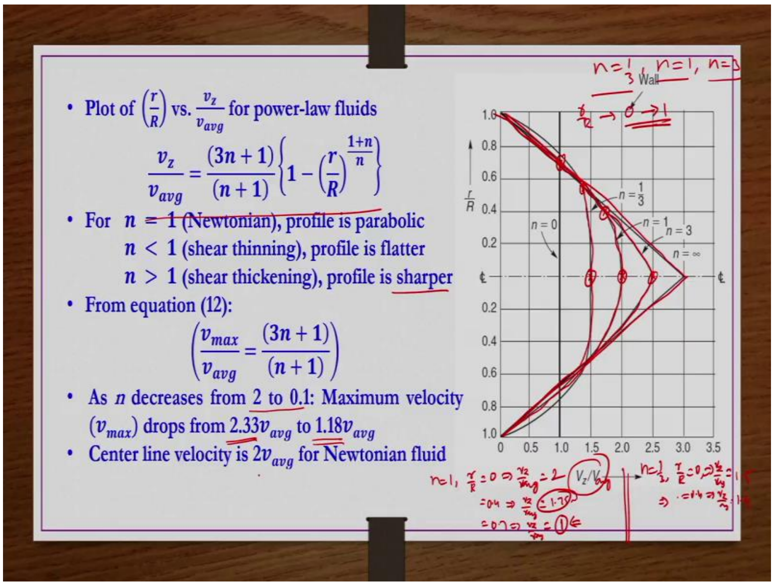

So, this is what we are taking. So, for that we are plotting $\frac{r}{R}$ versus $\frac{v_z}{v_{avg}}$. So, $\frac{v_z}{v_{avg}}$ that equation number 10 the just now we had, that is $\frac{v_z}{v_{avg}}$ we had $\frac{3n+1}{n+1}\left\{1 - \left(\frac{r}{R}\right)^{\frac{1+n}{n}}\right\}$.

Now, what you do? You take different values of n different values of n you take; you take n = 1, you take n = 3, then you take n = $\frac{1}{3}$ also. So, this is shear thinning, this is Newtonian, this is shear thickening and then you take different $\frac{r}{R}$ values between 0 and 1 because $\frac{r}{R}$ it is the radial coordinate is divided by the radius of the tubes. So, then maximum value $\frac{r}{R}$ is going to be 1 only.

So, between 0 and 1 you take different values of $\frac{r}{R}$ for each n value and then you find out the values and then tabulate them then you plot them. So, what you get you get here. Let us say if r = 0, or if $\frac{r}{R}$ = 0 and then n = 1, $\frac{r}{R}$ = 0; that means, this $\frac{v_z}{v_{avg}}$ here you know if it is 0. So, then we are going to get $\frac{3n+1}{n+1}$ only n = 1. So, then $\frac{4}{2}$ is 2. So, n = 1 when $\frac{r}{R}$ = 0, $\frac{v_z}{v_{avg}}$ is 2. So, that data points is here fine.

So, now what you do? You take $\frac{r}{R} = 0.4$, then $\frac{v_z}{v_{avg}}$ you find out how much it is. It will be roughly 1.75. So, that data point is here right. So, next what you do? Like that you keep changing the data points let us say you take you know the $\frac{r}{R} = 0.7$ or something like that then. So, when $\frac{r}{R} = 0.4$ $\frac{v_z}{v_{avg}}$ for Newtonian case we are getting something roughly like you know 1.75 or 1.8 something like that roughly.

So, that point is this one similarly if you take our $\frac{r}{R}$ = something around 0.7, then $\frac{v_z}{v_{avg}}$ you will get approximately value of 1. So, these are approximate values that I am saying. So, then that data point would be here. So, like that you take different data points and you plot. So, then what you find? You find for the Newtonian case you are going to get a parabolic profile like this right.

So, let us say if you have n = $\frac{1}{3}$, then $\frac{r}{R} = 0$. Then what is your $\frac{v_z}{v_{avg}}$? So, $\frac{r}{R}$ is 0. So, then this term is 0 if $\frac{n}{3}$ if n = $\frac{1}{3}$. So, this is you know how much? $\frac{1}{3}$ * 3 that is 1 + 1 that is 2; 2 divided by you know this is $\frac{4}{3}$ that is 4 8; 8 divided by. So, that is you get roughly you are going to get that value close to 1.5. So, that is the data point here right.

Likewise, if you change the $\frac{r}{R}$ value to 0.4 here in next level, then this $\frac{v_z}{v_{avg}}$ you are going to get roughly something like 1.4 or something like that. So, that data point is here. So, like that if you plot. So, for n = $\frac{1}{3}$ you are going to get a flatter profile like this the second this curve; that means, for Newtonian fluids it is a parabolic profile, but for shear thinning fluid it is a flatter kind of profile you can get here this profile you can see there is a flatter kind of profile you can get.

And then similarly if you do for shear thickening fluid let us say if you take n = 3, then at $\frac{r}{R}$ = 0, $\frac{v_z}{v_{avg}}$ you are going to get 2.5 that data point is here. So, likewise other cases also $\frac{r}{R}$ if you take and then for n = 3, if you obtain the data points and then plot them together here. So, you are going to get a steeper profile like this.

So, shear thickening fluid profile is sharper and then as n increases it becomes very sharper. So, the velocity profile of a shear thinning fluid flowing through a pipe and the if the flow is fully developed flow, the velocity profile is going to be flatter one and then as n increases it flatness gradually decreases and then for n = 1 the profile becomes parabolic and then further if you increase n value where shear thickening behavior started n > 1. Then the profile becomes gradually sharper and then n = infinity it becomes a straight lines like this, straight line profile like this you can get here as shown here ok.

So, now we realize that if the fluid rheology changes the velocity profile is going to be very different. So, then; obviously, the volumetric flow rates are also going to be different or for a fixed of volumetric flow rate of Newtonian fluid whatever the pressure drop is there, the same pressure drop may not be giving the same volumetric flow rate if the fluid is a non-Newtonian shear thinning or shear thickening fluid. We are going to see with an example problem also.

So, from equation number 12, this is what we get $\frac{v_{max}}{v_{avg}} = \frac{3n+1}{n+1}$; that means, maximum velocity n as n decreases from 2 to 0.1 the maximum velocity drops from 2.33 average to 1.18 v_{avg} velocity.

So, when n decreases from 2 to 0.1 v max velocity decreases from 2.33 times v_{avg} to 118 times v_{avg} velocity ok. So, that much important is the rheological nature of the fluid to be consider while designing any unit operation where the non-Newtonian fluids are being handled or even for you know tuning the operational parameter as well ok.

So, central line velocity for Newtonian fluids is nothing, but 2 times the average velocity that we have seen anyway.

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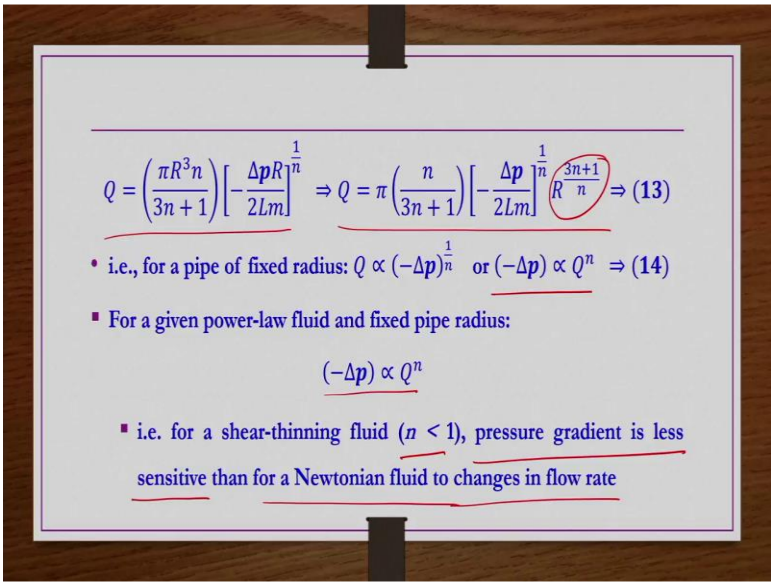

So, now we see a few more details because whatever the volumetric flow rate versus Δ p that is important from a engineering applications point of view, that information now we are going to analyze for a power law fluids here in this case.

Volumetric flow rate we got this expression previously in one of these slides we have derived it, this equation rearranging this way that is it doing nothing we are just rearranging so, that all the r terms are grouped together. So, that for a fixed value of r how the Q is changing for a change in Δp that is what we are going to see how it is changing Q is proportional to $[-\Delta p]^{\frac{1}{n}}$ or $-\Delta p$ is proportional to Q^n that is what we are getting here right.

So, for a given power law fluid and fixed pipe radius minus delta p is proportional to Q^n; that means, for a shear thinning fluid the pressure gradient is less sensitive than for Newtonian fluid to changes in flow rate that is what we can understand because n < 1. So, then this sensors the pressure drop would be less sensitive compared to the case when n = 1 that is what we can understand.

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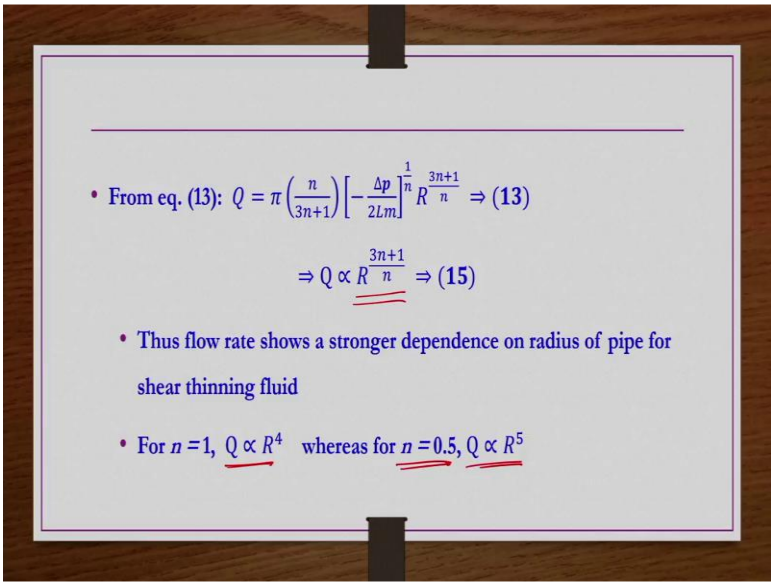

The same equation 13 again. So, now, let us say for a fixed pressure drop for if the pressure drop if you are keeping fixed then Q is proportional to $R^{\frac{3n+1}{n}}$; that means, flow rate shows a strong dependence on radius of pipe for shear thinning fluid. For shear thinning fluid the flow rate shows a less dependence on the pressure drop, but it shows a strong dependence on the radius of the pipe because it is for a fixed pressure drop Q is proportional to $R^{\frac{3n+1}{n}}$ whereas, for a fixed R value Q is just Δp is just proportional to Q^n only.

So, the flow rate is strongly dependent on the radius of pipe for shear thinning fluid rather than the pressure drop that is what we can understand here. So, for n = 1 that is Q is proportional to R^n whereas, for n = 0.5 Q is proportional to R^5 that is what we have here. So, the radius of pipe is going to have more influence on the flow rate for the case of shear thinning fluids.

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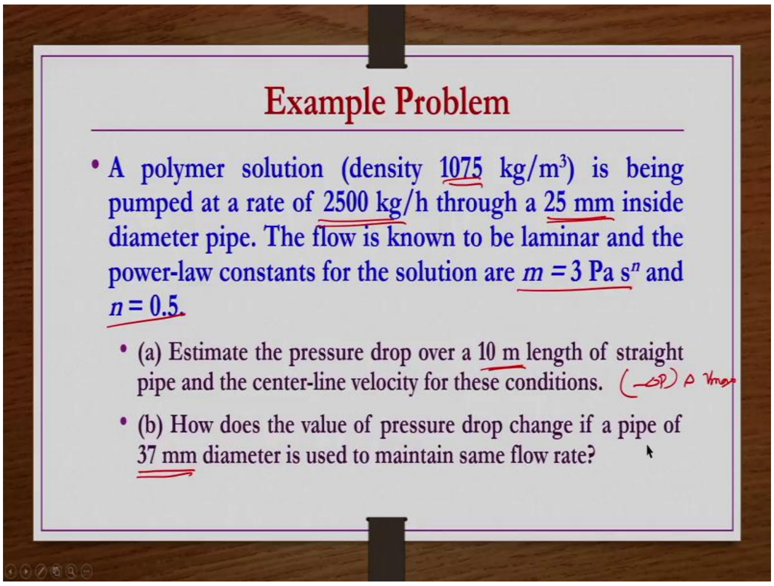

So, before winding up this class we will take a simple example problem. So, there is a polymer solution of certain density is being pumped at a certain mass rate through a pipe of certain diameter and then flow is known to be laminar and power law and the power law constant for this fluid whatever the power law fluid is there that rheology is you know represented by the power law of nature.

So, then m is 3 Pa s^n and n is 0.5. So, the question is estimate the pressure drop over a 10 meter length of straight pipe and the center line velocity for these condition that is $-\Delta p$ and v_{max} we have to find out? The second part of the question how does the value of pressure drop change if pipe of a 37 mm diameter is used to maintain the same flow rate?

If you maintain Q same and then you increase the diameter of the pipe how much pressure drop is going to change that is the second part. Very simple straightforward calculations; actually Q versus minus delta p that relation we have already developed. So, Q is given actually $\dot{m}$ is given density is also given. So, $\dot{m}$ by density if you do you get the Q value that Q is nothing, but this value.

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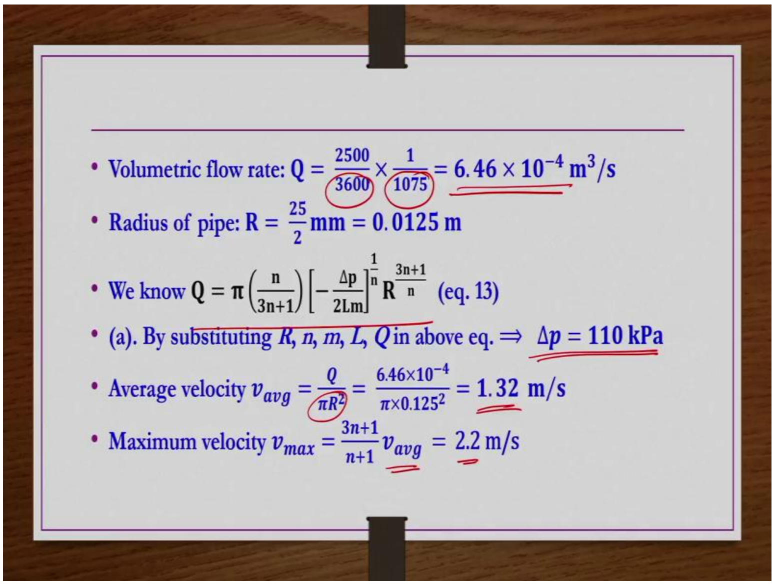

2500 kg per hour. So, per hour we have written 3600 seconds and then dividing by density 1075 kg per meter cube. So, then volumetric flow rate you get 6.46 multiplied by 10^{-4} cube per second.

And then R is given as 25 by 2 mm that is 0.0125 meters and this equation 13 we derive this equation just now in one of the previous slides. So, in this equation except the minus delta p everything is known. So, you substitute all the values R, n, m, L, Q in above equation because n and m also given in the problem then you simplify. So, you get minus delta p is nothing but 110 kilopascals right.

And then average velocity Q is already obtained. So, this Q is if you divide by πR^2 then you get the average velocity 1.32 meter per second, it is not part of the question, but we can do anyway. Then maximum velocity is nothing, but $\frac{3n+1}{n+1}$ times the average velocity that we have already seen. So, that you substitute n here, v_{avg} you got 1.32 meter per second here. So, then 2.2 meter per second. So, the pressure drop as well as the central line velocity or the maximum velocity we got the first part is done right.

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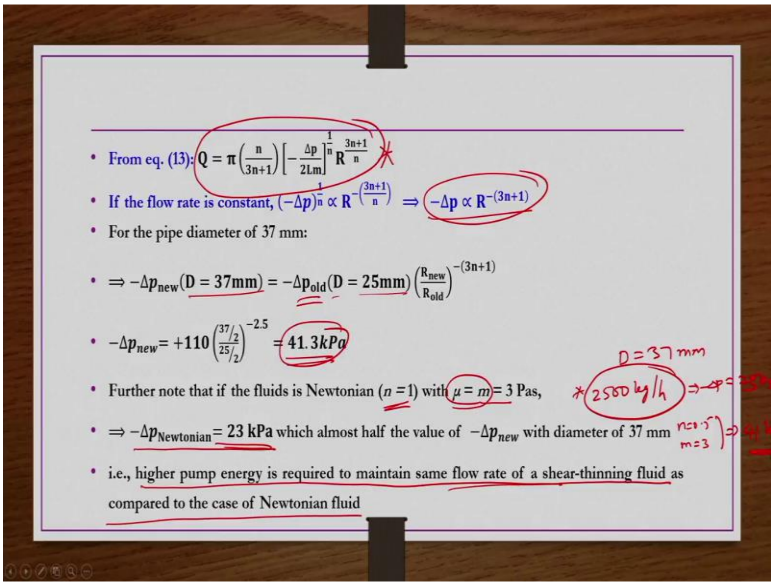

Now, second part is that, you know if you change the diameter of the pipe it is rather increased. So, then how the pressure drop is going to change. So, then same equation number 13 we are using it here. So, between Q versus R what is the relation? Q is proportional to $R^{\frac{3n+1}{n}}$ that is what we have right we are having constant flow rate ok.

So, otherwise what we get from here $[-\Delta p]^{\frac{1}{n}}$ is proportional to R^{-3n+1} this is what you have r - Δp is proportional to $R^{\frac{3n+1}{n}}$ you are having.

Why are we getting this relation? It is because if your radius is changing you know how much pressure drop is changing that is what you wanted to find out. Though the same equation you can use as it is by substituting R = $\frac{37}{2} * 10^{-3}$ meters directly, but we do this way.

So, for pipe diameter of 37 mm - Δp_{new} that is for D = 37 mm divided by that should be equals to whatever the - Δp_{old} which is nothing for the D = 25 mm and then that should be multiplied by $\left(\frac{R_{new}}{R_{old}}\right)^{-3n+1}$. New stands for the case of 37 mm diameter old stands for the case of 25 mm diameter.

So, 25 mm diameter Δp_{old} we already calculated previous slide 110 kilopascals that you substitute R_{new} R_{old} are known right then Δp_{new}. So, that is for the case of when you that is for the case of diameter of 37 mm pipe - Δp is going to be 41.3 kilopascals.

So, it has decreased substantially by increasing the radius of the pipe. This is the second part of the problem one last addition let us say if the fluid is Newtonian then n = 1 and then whatever the m value is there that should be taken as mu right. Then you get minus delta p new you are going to get if you use the same equation this equation just n = 1 you substitute and then m is equals to you substitute 3 and then do this simplification you get 23 kilopascals which is almost half of this value right.

That means, higher pump energy is required to maintain. So, to maintain same flow rate of shear thinning fluid as compared to the case of Newtonian fluid. So, whatever the volumetric flow rate that 6.46* 10^{-4} meter cube per second that volumetric flow rate that is given here; that means, you are using 37 mm diameter pipe.

And then in order to maintain 2500 kg per hour of mass rate, what is the pressure drop for a Newtonian fluid is just 41, is just 23 kilopascals, but if you maintain the same flow rate for the shear thinning fluid of n = 0.5 and then m = 3 pascal second, then it increases to 41 roughly 41 kilopascals right.

So, that is what to maintain the same flow rate you need to provide the higher pump energy in the case of shear thinning fluids compared to the case of a Newtonian fluids.

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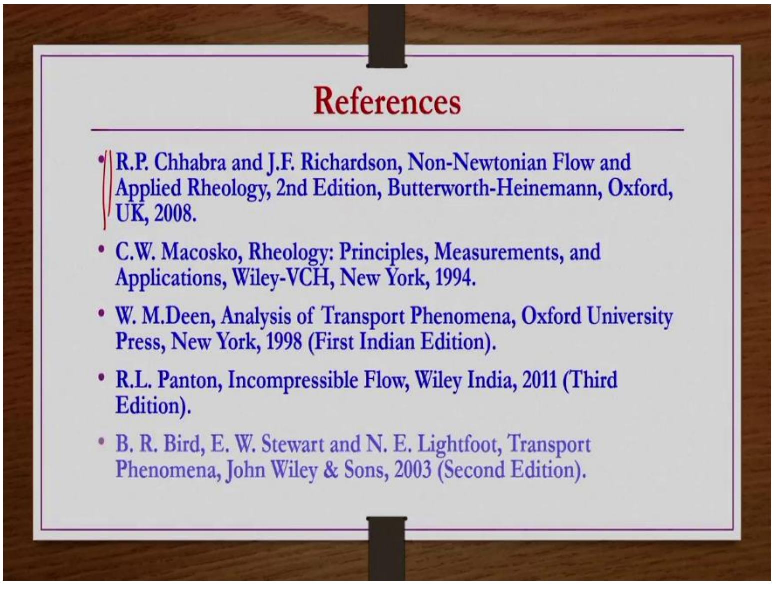

So, the references for this lecture the entire lecture is prepared from this reference book by Chhabra Richardson, other reference books are provided here.

Thank you.