Transport Phenomena of Non-Newtonian Fluids

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Lecture - 13
Equations of Change for Non-Isothermal Systems

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of this lecture is Equations of Change for Non-Isothermal Systems. Before going into the details of equations of change for non isothermal system and then derivation of respective conservation equations etcetera, what we do?

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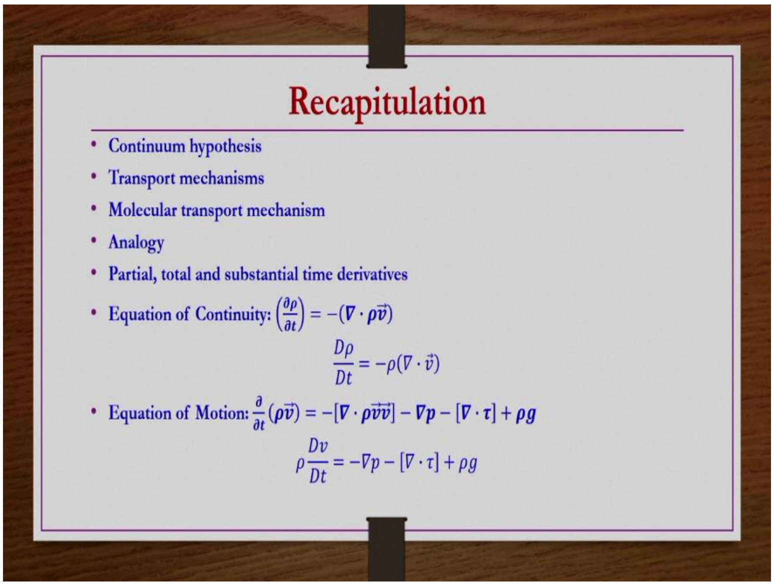

We are going to have a kind of recapitulation of what we have seen in last two lectures. In last two lectures we have seen a few basics of continuum hypothesis. Then we also recapitulated transport mechanisms and out of which we have seen the details of molecular transport mechanism concerned with the momentum transfer, heat transfer and mass transfer.

Then we have also seen analogy amongst the momentum transfer, heat transfer and then mass transfer especially when the transport mechanism is governed by the molecular transport. Then we also seen a few basic details of partial time derivative, total time derivative and substantial time derivative. And what are the difference amongst these thing we have seen with an example of a fish concentration in a river stream ok.

Then we have also derived equation of continuity that is nothing but conservation of mass and then that we got $\left(\frac{\partial \rho}{\partial t}\right) = -(\nabla . \rho\vec{v})$. This equation we have written rectangular coordinates, cylindrical coordinates and spherical coordinates.

If the same equation if we write in substantial time derivative form then $\frac{D\rho}{Dt} = -\rho(\nabla . \vec{v})$ that is what we get. Then we also derive the equation of motion that is nothing but conservation of momentum for a very generalized system irrespective of the nature of the fluid, irrespective of the nature of the flow region we have derived the equation of motion.

That is nothing but $\frac{\partial}{\partial t}(\rho\vec{v}) = -[\nabla . \rho\vec{v}\vec{v}] - \nabla \text{p} - [\nabla . \tau] + \rho g$ this is what we have seen. If additional forces or you know additional terms are also causing momentum transfer, so, then those term should also be added in the right hand side that is what we have seen.

Then the same equation if you write in a substantial time derivative form then we got this $\rho\frac{Dv}{Dt} = -\nabla \text{p} - [\nabla . \tau] + \rho g$, this is what we have seen in last two lectures.

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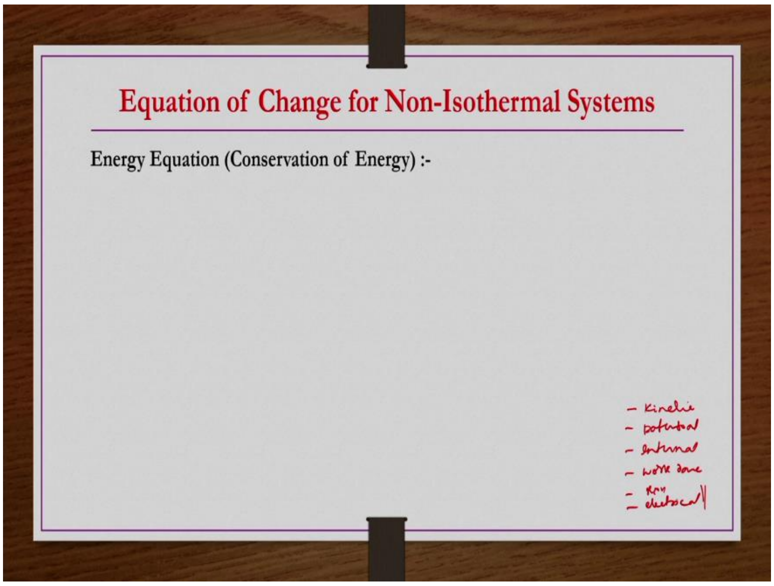

Now, in this lecture we are going to derive equation of change for non isothermal systems. So, that is nothing but conservation of energy that is what we are going to derive. So, now when we talk about the conservation of energy, what we try to include? We try to include all the forms of energy and then we also include the transport of energy because of all possible mechanism + in addition we also include energy because of any additional work done because of the molecular stresses or because of the external forces like gravity etcetera all those terms we include.

And then very generalized energy equation we are going to develop that is what we are going to do in this lecture. Then subsequently what we are going to do? We are going to simplify this equation so that to get the final energy equation in a very simplified form where we can have this energy equation in the form of temperature. Why in the form of the temperature? Because you know temperature is a measurable quantity.

So, then application of energy equation would be more appropriate if you write that energy equation in the form of temperature. So, what are the types of different possible energies that may be associated with the system? So, we may have the kinetic energy, we may have the potential energy, we may have the internal energy, we may have the work done on the system because of you know molecular stresses or because of the external force like gravity etcetera.

Or because of the reaction, all those things may be causing some kind of work done on the system and then that may be including some energy. So, that all those terms may be there ok. Then like that you know these different possible energies let us say reaction energy or you know if you have a electrical energy associated with the electrical you know electric gradient electrical potential gradient etcetera.

If you have the energy associated with the electrical potential gradient etcetera those things, different forms of energies are all possible right. So, we are going to see how generalized way we can include most of these terms, if not all that is what we are going to see.

And then this energy may be transporting because of the different mechanism both by the convective transport mechanism as well as the molecular transport mechanism. So, that is what we are going to have. So, then we are going to make a balance for you know energy of the system which includes most of the which includes all forms of the energy majority of the energy.

Because you know let us say electrical energy associated with the reaction and our energy associated with the electromagnetic potential etcetera those terms may be added up in the right hand side of the equation. So, then we are not going to include them in a kind of generalized equation of conservation for energy. Because, we are developing a kind of a generalized equation not specific to any problem any one particular problem, ok. However, what we are going to have?

We can we are going to have a kind of a simplification where we can use this equation for any specific problem as well, so, which we are going to see in subsequent lectures.

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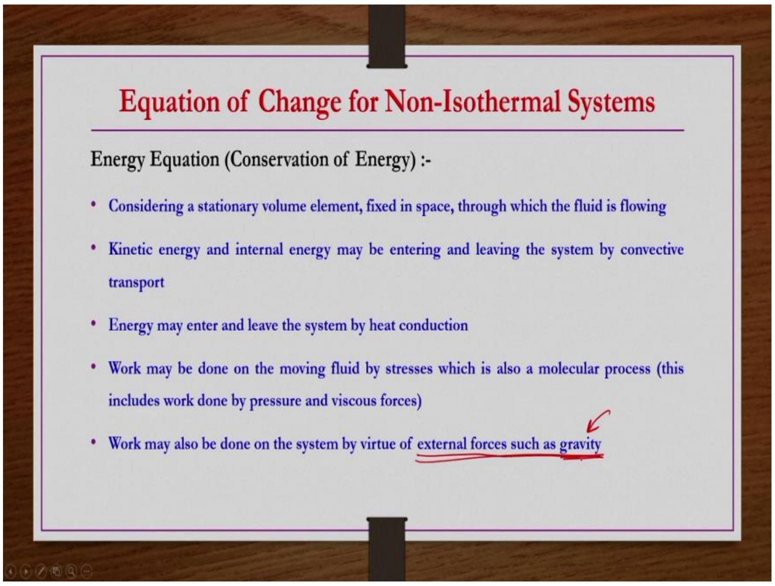

So, when it comes to the conservation of energy the, what we do? We consider a stationary volume element which is fixed in space and through which a fluid is flowing, so, that because we should also able to include the convective transport. So, that is the reason we are taking the fluid element volume element in a moving fluid and then that volume element is fixed in space right.

So, that is the reason we are considering a stationary volume element fixed in space through which the fluid is moving so, that the generalized energy equation can be developed. Then kinetic energy and internal energy may be entering and then leaving the system by convective transport and there may be energy entering and leaving the system by heat conduction.

And there may be work done on a moving fluid by stresses something like because of the molecular stresses that is like you know work done because of the pressure or work done due to the pressure or the work done because of the viscous forces etcetera those terms may also be included as already mentioned. And then there may be work done on system by virtue of external forces.

So, external forces we are taking only gravity as of now. Let us suppose if you have additional terms like you know energy associated with a reaction or energy because of the electromagnetic potential etcetera. So, those terms may be added up in the right hand side of the balance equation that we are going to develop ok.

So, but here now as an external force we are taking only gravity only because there is one reason for this. So, when we include the gravity or the work done on the system by virtue of the gravity, so, then that will also take care whatever the potential energy part is there. So, that is the reason we are taking only the gravity as the part.

Because we have to if you are developing an equation conservation of energy equation then we should also include the not only kinetic energy and then we should include not only the kinetic energy and internal energy, but also we should include the potential energy right. So, let us say we have enlisted all this possible energy associated with a given system in a very generalized form. So, then we are also enlisted what are you know modes of transport possible.

So, now for a moving system and then in that moving system in a fixed location, if you take a volume element in which the fluid is moving, for that if you make a balance equation for conservation of energy, what you have?

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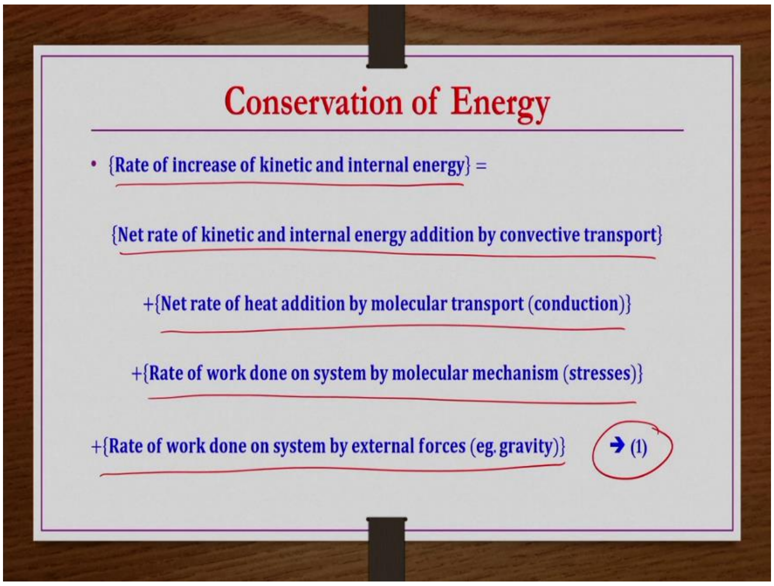

You will be having rate of increase of kinetic and internal energy should be balanced by the net rate of kinetic and internal energy addition by convective transport + net rate of heat addition by molecular transport that is by conduction + rate of work done on system by molecular mechanism something like stresses + rate of work done on system by external force like something like gravity right. So, now, this includes almost every possible way of energy associated with a given system except the reaction.

So, then that can also be added if we want as a kind of additional term in the right hand side right. So, now, this is what we are having. For any specific problem you have taken any if you are solving any specific problem or you are developing an energy balance equation for a specific problem, so, then some of them involved and some of these terms may not be involved.

So, but we are developing a very generalized conservation of energy equation, so that that is the reason we are including all the terms here ok. Now, what we do? Next part of the lecture is that we write a mathematical form of each and every term that has been written in this equation number 1. What is how it how do you represent mathematically that left hand side term and then how do you represent mathematically the right hand side terms those that is what we are going to do individually.

And then after doing them we are substituting them here in this equation so that we can have a generalized conservation of energy equation ok.

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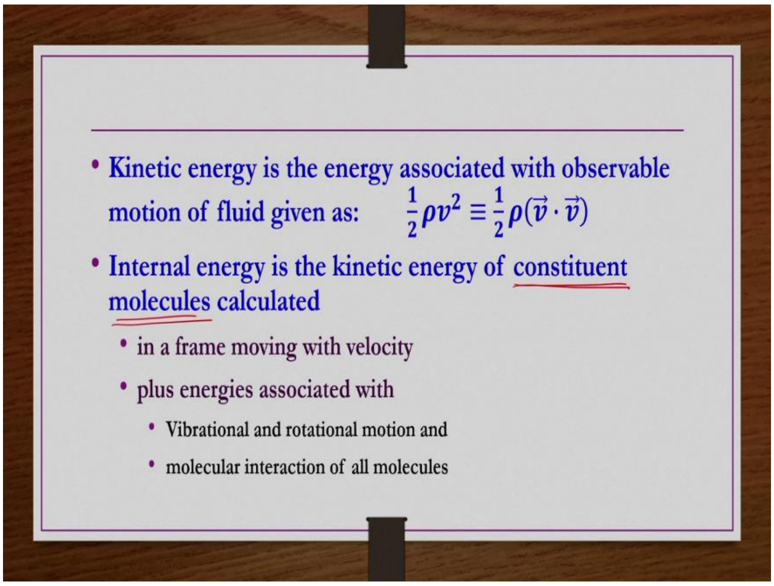

So now, the definition of individual terms that we are going to see. The kinetic energy is the energy associated with the observable motion of the fluid which is given by $\frac{1}{2}\rho v^2 \equiv \frac{1}{2}\rho(\vec{v}.\vec{v})$ ok. Whereas, the internal energy is the kinetic energy of the constituent molecules in; whatever the molecules constituting that fluid whatever the moving fluid is there that molecules whatever the molecules are there, they may be constituting some kinetic energy.

So, that internal energy of that system is nothing but the kinetic energy of constituent molecules calculated in a frame moving with velocity plus energies associated with the vibrational and rotational motion along with the intermolecular interactions of all molecules. So, all these things are included in the internal energy part ok.

However, we are not going to you know write individual part, what is the contribution of vibrational energy in the internal energy, what is the contribution of the rotational motion in the internal energy, what is the you know contribution of inter molecular interactions in the internal energy part, all those individual things we are not going to see anyway right. So, let us not worry about that part.

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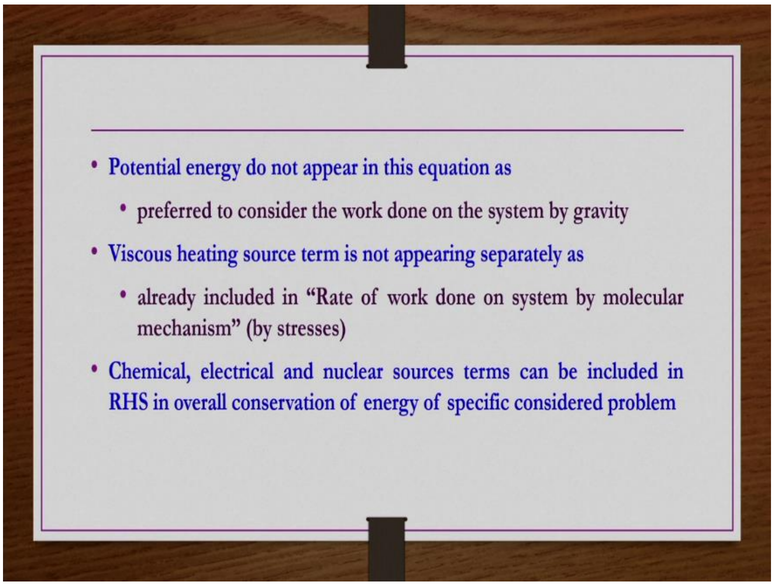

So, now, we are not writing that potential energy explicitly in the conservation equation number 1 that we have written in a previous slide. Because you know we are preferring to consider the work done on the system by gravity which will also take care of the potential energy part, subsequently we are going to see in coming slides, right. In addition, in most of the chemical engineering problems viscous dissipation or you know viscous heating a kind of important physical process right.

So, then that should also be included, but that also we have not written separately in equation number 1. Why because viscous heating part has already been taken care in the term rate of work done on system by molecular mechanism. How it is taken care that we are going to do when we write a mathematical form of this individual terms of equation number 1 ok.

Further, any additional energy source terms like a (Refer Time: 13:06) because of the chemical, electrical and then nuclear sources they all can be added in the RHS of the equation number 1, ok.

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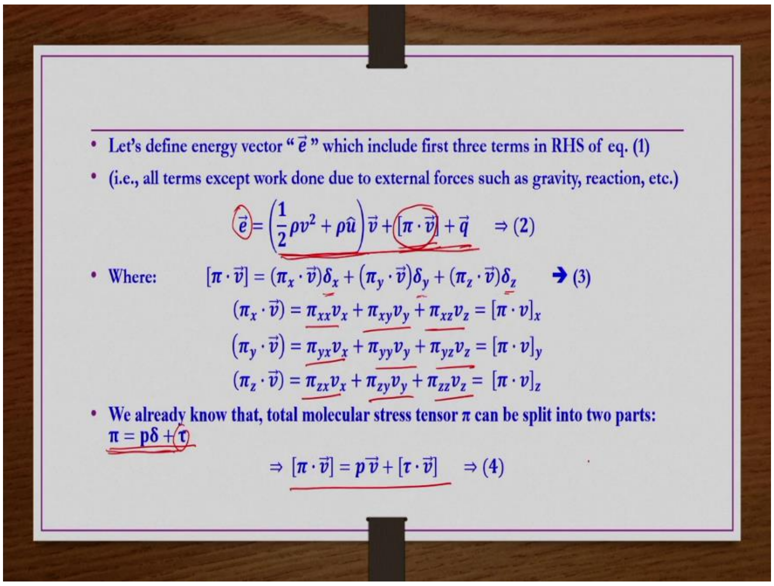

So, now what we are going to do? We are defining a energy vector $\vec{e}$ which includes first three terms in RHS of equation number 1. What are the first three terms in the RHS of equation number 1 are that is except the work done due to the external forces that is except the terms which are having gravity, reaction etcetera. All other terms in the RHS of equation number 1 should be added or you know together and then we have written them as a kind of a vector $\vec{e}$.

So, that is whatever the internal energy plus kinetic energy that part right. Or whatever the net rate of heat addition, net rate of an addition of internal energy and then kinetic energy because of the convection that part plus net rate of heat addition because of conduction then rate of work done because of the molecular transport mechanisms something like molecular stresses. These are the three terms all these three terms are now included here ok. So, why are we writing?

Because we are going to do your balance, when we are doing going to do a balance, if you have you know three dimensional like you know all three directions we are considering the balance, so, then what happens? Let us say if you take the conduction whatever the rate of heat in at x because of the conduction and then whatever the rate of heat out at x + Δ x because of the conduction likewise rate of heat in at x because of the convection.

Similarly, rate of heat out at x + Δ x because of the convection like that if you write individual terms there may be so many terms and then that balance equation will become very clumsy. So, that is the reason what we are writing. We are defining an energy vector which includes all these three contributions; the conduction, the molecular stresses or the energy associated with the molecular stresses and then internal energy and then kinetic energy.

All these parts you know we have added together and then defined a one energy vector so that we write energy whatever the energy entering at x whatever the energy leaving at x +Δx like that we write a balance and then do the simplification right. After doing that simplified equation in terms of $\vec{e}$ then, what we do? We substitute this equation further to expand so that all these terms will appear in that equation.

It will be very easy process or you know simpler process rather writing individual terms entering in leaving out in all three directions that becomes very complicated and then so many terms would be there. If you miss out any term and then anywhere in place of plus if you write minus or in place of minus, plus if you write, so, things may become very complicated and confusing. So, then that is the reason we can have an energy vector defined like this ok.

Now, here this pi dot v that because of the viscous stresses or molecular stresses that we already know that it is having 9 components. So, those 9 components you know individually you can write 3-3 components combination that is $(\pi_x.\vec{v})\delta_x + \left(\pi_y.\vec{v}\right)\delta_y + (\pi_z.\vec{v})\delta_z$ right. This is how we can have this $\pi.\vec{v}$. Then again an individual $\pi_x.\vec{v}$, $\pi_y.\vec{v}$, $\pi_z.\vec{v}$ we can write as like this.

So, that you know we have all 9 components; 1, 2, 3, 4, 5, 6, 7, 8, 9. So, this particular term is having all 9 components. Now, imagine these 9 components you are writing you know in all three direction leaving entering and leaving. So, many terms will become very complicated ok. Further, we also know that this molecular stresses total molecular stress tensor pi whatever is there that is having two contributions, two parts that is τ and then p δ right.

So, that is if i = j then $\delta_{ij} = 1$. If i ≠ j then $\delta_{ij} = 0$ and then this part is nothing but the viscous stress ok. So, now what we do? We substitute this $\pi = p\delta + \tau$ in this equation number 2, so, that we can now in place of $\pi.\vec{v}$ we can write $p\vec{v} + \tau.\vec{v}$ we can write $p\vec{v} + \tau.\vec{v}$ we can write ok.

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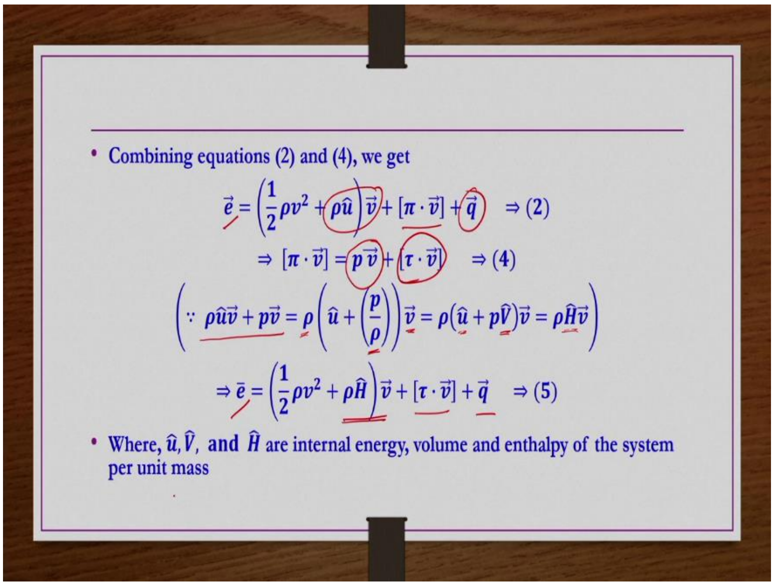

So, that when you do this one so in place of this is equation number 2 in place of $\pi.\vec{v}$ if you write a $p\vec{v} + \tau.\vec{v}$, so, then what we have? So, these two terms are there right. So, what; we are going to do some kind of rearrangement. So, then what we do? This $\rho\hat{u}\vec{v}$ and then this $p\vec{v}$ ok, we combined together we write as a one term right and then remaining terms like you know $\tau.\vec{v}$ and then q these things are separately we are going to write as a kind of one term ok.

So, now here from $\rho\hat{u}\vec{v} + p\vec{v}$ what I am trying to do? I separated out $\vec{v}$ which is common and then from $\rho\hat{u}$ and then p terms I am taking ρ common. So, then $\hat{u} + \frac{p}{\rho}$ I am going to have and then $\frac{1}{\rho}$ I can write $\hat{V}$. So, this $\hat{u}$ and then $\hat{V}$ are nothing but the internal energy per unit mass and then volume of the system per unit mass.

So, u + p v we can write it as $\hat{H}$. So, that is enthalpy per unit mass that is what we can write. So, that this equation number 2 after combining with equation number 4, we get this equation. In place of $\rho\hat{u}\vec{v} + p\vec{v}$ we can write $\rho\hat{H}\vec{v}$. So, that is this part that is $\rho\hat{H}\vec{v}$ and then remaining terms $\tau.\vec{v} +$ q and then $\frac{1}{2}\rho v^2$ as it is ok.

So, in a balance equation after doing the simplification you know we are we have not written the balance shell balance equation we have not written. So, after writing that shell balance equation for in place of $\vec{e}$ we can use either this equation or we may use this equation subsequent subsequently. Where, here $\hat{u}$, $\hat{V}$ and $\hat{H}$ are nothing but internal energy, volume and enthalpy of the system per unit mass.

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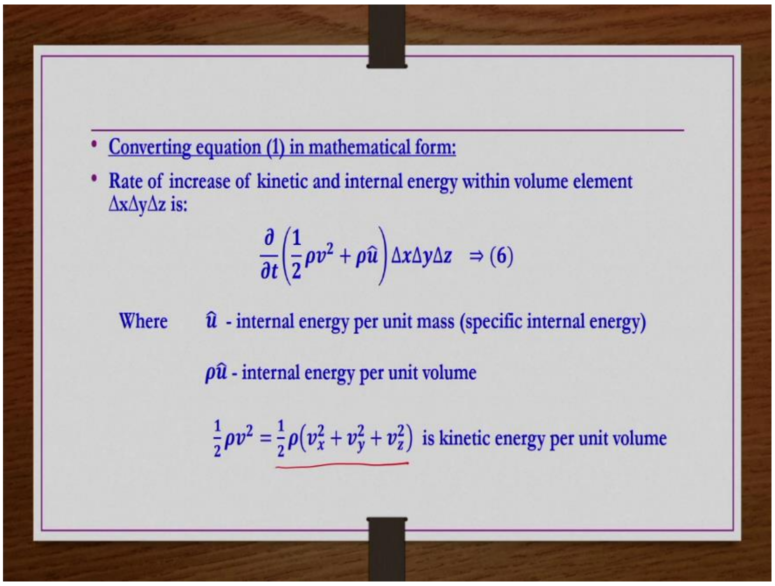

Now, what we are going to do? Converting equation 1 in mathematical form. In the first part in the left hand side of equation number 1, what we have? Rate of increase of kinetic + internal energy within volume element Δx Δy Δz. So, that if you write that is nothing but $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{u}\right)$ Δx Δy Δz.

Here $\hat{u}$ is internal energy per unit mass, sometimes it is also referred as the specific internal energy and then $\rho\hat{u}$ is nothing but internal energy per unit volume. Whereas, the $\frac{1}{2}\rho v^2$ with this which is nothing but when you expand $\frac{1}{2}\rho\left(v_x^2 + v_y^2 + v_z^2\right)$, which is nothing but the kinetic energy per unit volume.

So, all these quantities we are writing per unit volume. Remember v this is vector $\vec{v}$. Many times here it is clearly written as specifically like vector kind of thing right. So, if it is not written, so, then v it is explicitly; that means v is standing for the v vector here.

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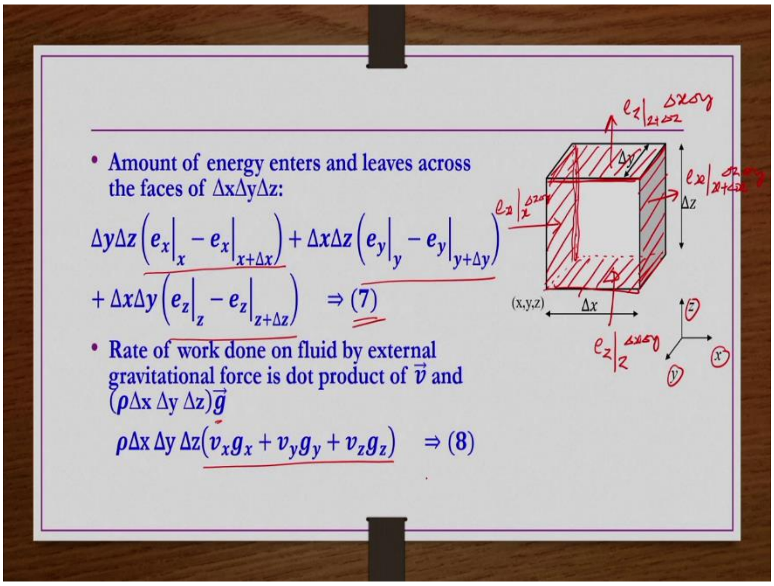

Now, what we do? We take this control volume like in derivation of equation of continuity and then equation of motion that we have done. So, we have taken a control volume of size Δx Δy Δz. So, the same way we have taken here. So, let us say horizontal direction is x direction, vertical direction is z direction and the third direction is the y direction.

So, now in the horizontal direction at x location the control volume is having Δx Δy Δz ok that is the size of the control volume. So, in the x direction, the size of the control this face is the size of the face is nothing but Δx right.

So, now at x location at x = x whatever the energy that is entering by all means, by all means like you know by convection and then you know molecular transport is nothing but e x entering at x multiply by the area through area of the face through which it is entering.

Area of the face through which it is entering is nothing but this is nothing but Δz and then this is nothing but Δy. So, that is Δz Δy. So, in the x direction at location x +Δx how much energy is leaving out? And the rate of energy leaving out at x +Δx is nothing but $e_x|_{x+\Delta x}$ multiplied by the area of the face through which it is living that is Δz Δy.

Likewise in the z direction, if you take the energy that is entering at z is nothing but $e_z|_z$ multiplied by the area of the face through which it is entering that is Δx Δy. And then energy that is leaving at location z+Δz is nothing but $e_z|_{z+\Delta z}$ multiplied by the area of the face through which it is living that is nothing but this is Δz this is Δy that is this is Δx this is Δy. So, that is Δx Δy.

Likewise in the y direction also we can write. So, that is amount of energy enters and leaves across the faces of Δx Δy Δz control volume in all three direction if you write $(e_x|_x - e_x|_{x+\Delta x})$ Δy Δz + $(e_y|_y - e_y|_{y+\Delta y})$ Δx Δz +$(e_z|_z - e_z|_{z+\Delta z})$ Δx Δy right.

So, now in the conservation equation that we have written in equation number 1 left hand side term we have written that is equation number 6 in the previous equation that is $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{u}\right)$ Δx Δy Δz that is the left hand side term. In the right hand side term there were four terms we have written in equation number 1.

So, that is the net rate of heat addition, net rate of addition of internal energy + kinetic energy because of the convection + net rate of heat addition because of the conduction + rate of work done on system because of the molecular stresses. All those three terms are included in the energy vector e and then in the form of e that how much what is the net rate that we are writing here in the form of equation number 7.

So, in the RHS of equation number 1 first, second, third term whatever are there, so, those terms are represented by this equation number 7. And then last term in the equation number 1 is nothing but the rate of work done on fluid by external force. In the external force here we are taking gravity that you can get by taking the dot product of $\vec{v}$ and $(\rho\ \Delta x\ \Delta y\ \Delta z)\vec{g}$. So, that if you do you get ρ Δx Δy Δz $\left(v_x g_x + v_y g_y + v_z g_z\right)$.

So, now, this equation numbers whatever 6, 7, 8 are there, so, they are representing the mathematical form of different terms that are appearing in equation number 1 that is the basic conservation energy equation.

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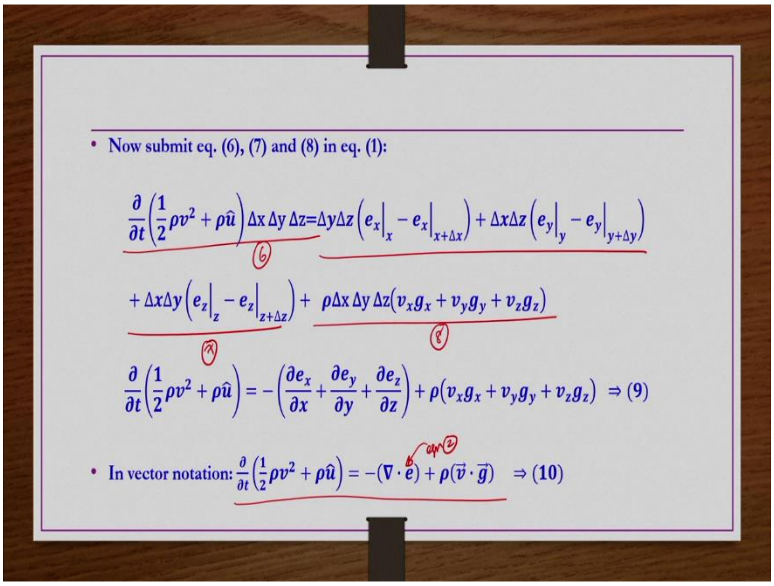

So, if we substitute this equation number 6, 7, 8 in equation number 1 then we have this equation. This part is nothing but your equation number 6 and then this part this, three terms these are nothing but the equation number 7 and this is nothing but equation number 8, ok. Now, what you do? Both sides you divide by Δx Δy Δz and then you take limit Δx → 0, limit Δy → 0, limit Δz → 0. Then what you have?

You have $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho\hat{u}\right)=-\left(\frac{\partial e_x}{\partial x}+\frac{\partial e_y}{\partial y}+\frac{\partial e_z}{\partial z}\right)+\rho\left(v_x g_x+v_y g_y+v_z g_z\right)$ right. So, the same equation if you write in vectorial form how you can write? You can write $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho\hat{u}\right)=-(\nabla.e)+\rho(\vec{v}.\vec{g})$right.

So, now, in this equation this e we are going to substitute from equation number 2, from equation number 2 whatever e that we have written because this e includes three terms right. Rate of addition of internal energy and kinetic energy because of the convection and then net rate of heat addition by your conduction and then rate of work done on the system by molecular stresses all three terms are there in this e.

So, those three terms we have written in the form of equation number 2. So, from equation number 2 we are going to substitute here in this equation.

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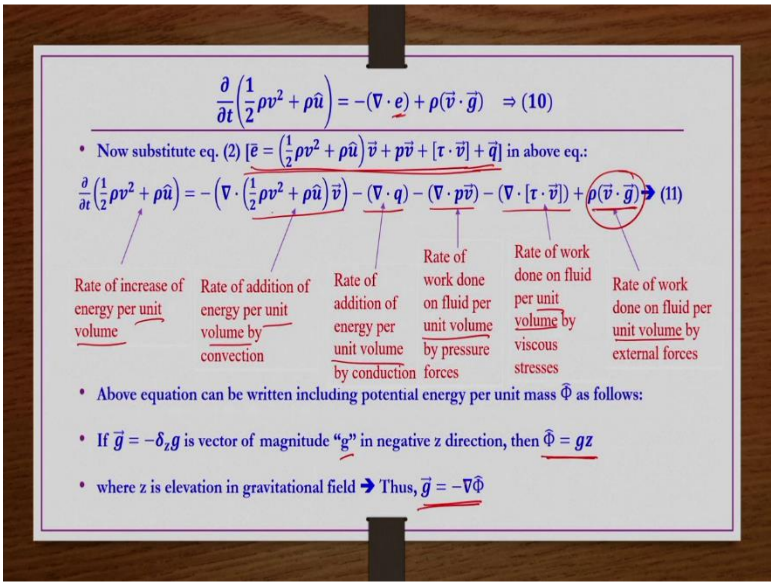

So, in this equation in place of e if you substitute e is equals to this form from equation number 2, what we will have? Left hand side as it is, right hand side $-\left(\nabla.\left(\frac{1}{2}\rho v^2 + \rho\hat{u}\right)\vec{v}\right)$ one term and then $-\nabla.p\vec{v}$ another term second term $-\nabla.[\tau.\vec{v}]$ is a third term and then $-\nabla.q$ is the fourth term.

So, all four terms 1, 2, 3, 4 terms have come into the picture in the form of $\nabla.$ as a kind of dot product and then whatever last term $\rho(\vec{v}.\vec{g})$ is as it is right. So, now, what this term indicates? This term indicates rate of increase of energy per unit volume. What is this term indicates? This term indicates rate of addition of energy per unit volume by convection.

What is this term indicates? This term indicates rate of addition of energy per unit volume by conduction and then this term indicates rate of work done on fluid per unit volume by pressure forces. And then this term indicates rate of work done on fluid per unit volume by viscous stresses. Last term is nothing but a rate of work done on fluid per unit volume by external forces right.

So, now you can see this equation whatever is there the energy we are writing per unit volume everything per unit volume ok that should be carefully observed ok fine. So, now, this above equation we can also include the potential energy because if you include the potential energy part then only it will become like you know complete conservation of energy equation right. Because now only the kinetic energy and internal energy have been included here, but the potential energy has not been included.

So, but that is already included actually, it is not explicitly shown that is already included here you know in the form of this external force by gravity. So, now, what we do? We are going to simplify this term further so that the potential energy will also come into the picture. So, what we do? Let us say if $\vec{g} = -\delta_z g$ is a vector of magnitude g in the negative z direction then this potential energy per unit mass $\hat{\phi}$ we can write it as g z.

That means, we can write $\vec{g} = -\nabla.\hat{\phi}$. This tilde indicates per unit mass, ϕ indicates the potential energy. So, now, this in play in the equation number 1 in place of $\vec{g}$ we are writing $-\nabla\hat{\phi}$ and then doing some simplification. What simplification are we doing?

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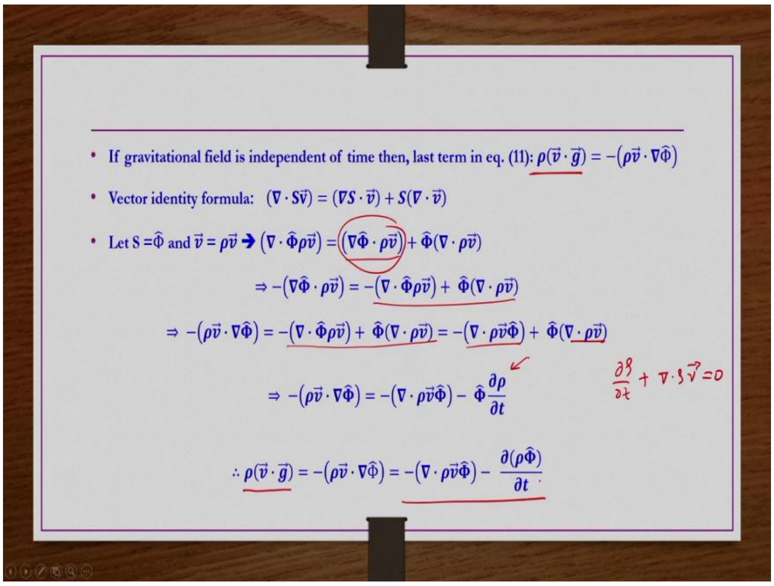

So, in equation number $\rho(\vec{v}.\vec{g})$term is this. So, here this in place of g we are writing $-\nabla\hat{\phi}$, so that the potential energy part can also come into the final conservation energy equation ok. So, that now we have. So, the $\rho(\vec{v}.\vec{g})$we can write $-(\rho\vec{v}.\nabla\hat{\phi})$. This is how we can write.

So, now what we do? We take a vector identity formula that is $(\nabla.S\vec{v})$ that is the dot product of ∇ and then multiplication of a vector scalar. So, then we what this we can represent as $(\nabla S.\vec{v}) + S(\nabla.\vec{v})$ this is how we can write. So, let S = $\hat{\phi}$ and then $\vec{v} = \rho\vec{v}$. So, that we can write $(\nabla.\hat{\phi}\rho\vec{v}) = (\nabla\hat{\phi}.\rho\vec{v}) + \hat{\phi}(\nabla.\rho\vec{v})$.

So, now what I am trying to do? Next step I am trying to take this term to the left hand sides and then this term to the right hand side term to the right hand side. So, that I can write, so that I can write $-\left(\nabla\hat{\phi}.\rho\vec{v}\right) = -\left(\nabla.\hat{\phi}\rho\vec{v}\right) + \hat{\phi}(\nabla.\rho\vec{v})$, this is what I can write ok. So, now because, why I am writing?

Because I need this term, minus of this term is required and now this is a dot product. So, then I can write whether it is a.b or b.a it is same because it is a dot product. So, then left hand side I am what I am writing? $-\frac{1}{2}\rho\vec{v}.\nabla\hat{\phi}$ and then right hand side it is going to be as it is like here. In this next step what I am doing? Rather writing $\nabla.\hat{\phi}\rho\vec{v}$, I am just writing $\nabla.\rho\vec{v}\hat{\phi}$ that is what I am writing ok. In the next step what we have?

This $\nabla.\rho\vec{v}$ from continuity equation we can have it is nothing but it is nothing but $-\frac{\partial\rho}{\partial t}$ that is $\frac{\partial\rho}{\partial t} + \nabla.\rho\vec{v}\hat{\phi} = 0$ from the equation of continuity that we know. So, in place of $\nabla.\rho\vec{v}$, I am writing $-\frac{\partial\rho}{\partial t}$ right, $\hat{\phi}$ is as it is.

So, then in place of $\rho(\vec{v}.\vec{g})$ we can write we can write it as $-\nabla.\rho\vec{v}\hat{\phi} - \frac{\partial\rho\hat{\phi}}{\partial t}$ that is what we can write. So, in equation number 11, this is what we are going to substitute.

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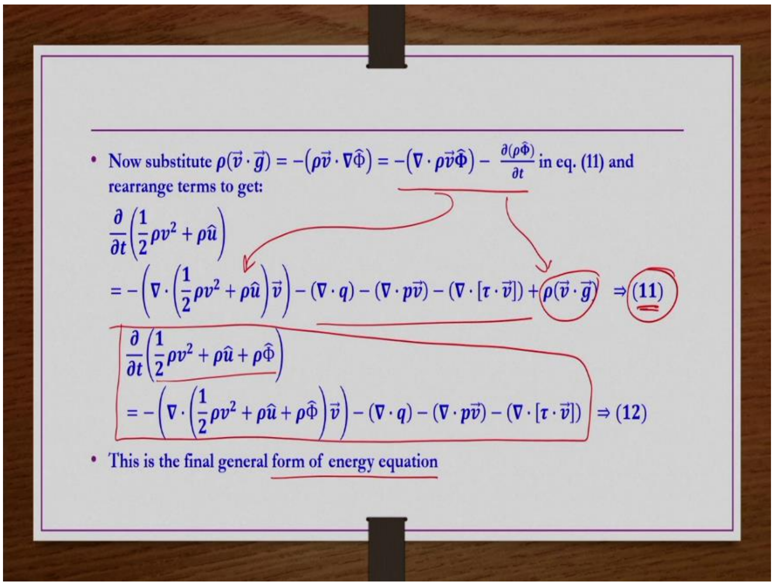

So, what is your equation number 11? So, this is your equation number 11 that we have already derived in the previous slide. So, now, in place of $\rho(\vec{v}.\vec{g})$ we can add these two terms $-\nabla.\rho\vec{v}\hat{\phi} - \frac{\partial\rho\hat{\phi}}{\partial t}$ right. So, this what I am doing?

This $\frac{\partial}{\partial t}$ of this term these two terms comes here in place of $\rho(\vec{v}.\vec{g})$. Out of these two terms $-\frac{\partial\rho\hat{\phi}}{\partial t}$ I am taking to the left hand side, so that can be added with the along with the kinetic energy and then internal energy ok.

And whereas, this part is there whatever $\nabla.\rho\vec{v}\hat{\phi}$ that I am adding to the first term in the RHS of this equation number 11. So, then what I can have? $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{u} + \rho\hat{\phi}\right) = -\left(\nabla.\left(\frac{1}{2}\rho v^2 + \rho\hat{u} + \rho\hat{\phi}\right)\vec{v}\right) -$ remaining terms three terms as it is that is $-\nabla.\mathrm{q} - (\nabla.\rho\vec{v}) - (\nabla.[\tau.\vec{v}])$ right.

So, now all three forms of energy are coming into the picture; internal energy, kinetic energy and potential energy. So, this is the final general form of energy equation. So, if you are asked what is the energy equation, so, this is what you have to write because it is generalized equation right. So, now, what we see? It is in the form of you know potential energy kinetic energy and internal energy and then rate of heat addition or any work done on the system etcetera those additional terms are there right.

So, the problem is that it is not in final usable form. The equation there any equation mathematical form of any physical system should be such a way that it should be in a measurable quantity or you know it should be in measurable property form that is how it should be ok. So, but you know you can measure temperature, pressure, concentration, density etcetera directly of a system without any difficulty.

But if you wanted to measure internal energy, potential energy, kinetic energy then you have to use some of these measurable quantities either temperature, pressure, density, viscosity, velocity etcetera and then you calculate these things. So, it becomes very you know lengthy process. So, then what we try to do now? This equation we further simplify. How do we simplify you know, we now included all the terms in the energy like internal energy, potential energy and kinetic energy.

Now, we are going to simplify this equation, so that we can have a equation of change for a enthalpy equation of change for internal energy, equation of change for enthalpy and then equation of change for temperature. The, equation of change for temperature that form is going to be final useable form, easy form for the engineering applications, so, that is what we are going to do. How are we going to do? First we are going to write you know equation for you know internal energy. So, for that what you have to do?

From this equation number 1, so, you subtract the equation of change for kinetic energy that is possible if we use the equation number 11. If we use the equation number 12 from this equation, if you subtract the equation of change for a kinetic energy + potential energy then the only term would be remaining is equation of change for an internal energy.

So, either of two ways we can do. So, then what we are going to do? We are going to make use of this equation number of this equation number 11 and then from this equation number 11 we are going to subtract equation of change for a kinetic energy.

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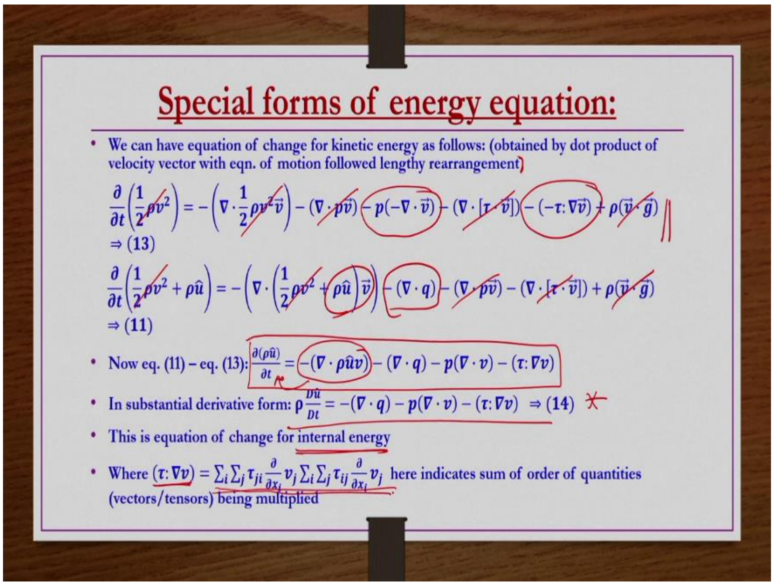

So, that equation of change for kinetic energy that takes into consideration all mechanical aspects of the you know in the motion whatever the because of the motion because of the mechanical aspects whatever the energy is there that part is nothing but this one right. So, this we can obtain by dot product of velocity vector with equation of motion and then followed by some rearrangement. So, then what you get? This one you get, so this is we are not going to do, we are just adapting it. This how you get?

Equation of motion whatever we have derived in the previous lecture that equation of motion you take and then velocity vector u and then you take velocity vector $\vec{v}$. You do the dot product of these two and then do some simply simpler, but lengthier the rearrangement, you get this equation ok. This equation indicates nothing but the equation of change for kinetic energy. So, now we are writing this equation as equation number 13. Equation number 11 already we are having this one.

So, then what we are going to do? We are going to subtract the equation number 13 from equation number 11 so that similar terms may be cancelled out and then finally, you have an equation of change for an internal energy. So, here that if you do then this $\frac{1}{2}\rho v^2$ this $\frac{1}{2}\rho v^2$ may be cancelled out. From here also $\frac{1}{2}\rho v^2$ $\frac{1}{2}\rho v^2$of $\nabla.$; $\nabla.\frac{1}{2}\rho v^2$ $\nabla.\frac{1}{2}\rho v^2$ are same.

So, then they will be cancelled out. $\nabla.\mathrm{q}$ is already there, so $-\nabla.\rho\vec{v}$ here and then $-\nabla.\rho\vec{v}$ here also there. And then $-(\nabla.[\tau.\vec{v}])$ is here and then $-(\nabla.[\tau.\vec{v}])$ is here also and then $\rho(\vec{v}.\vec{g})$ & $\rho(\vec{v}.\vec{g})$ here also. So, then it will be cancelled out.

So, then remaining terms in the left hand side you have $\frac{\partial}{\partial t}\rho\hat{u}$ only, right hand side you have minus of this term then + of this term and then + of this term right. So, that you write; $\frac{\partial}{\partial t}\rho\hat{u} = -\nabla.\rho\hat{u}v$ this term is also there $-\nabla.\mathrm{q}$.

And then here minus of minus + is there and then minus of this term is $-p(\nabla.v)$ and then here also minus of minus +. So, then +, but minus of this entire term is $-\tau:\nabla\mathrm{v}$. This term is nothing but it indicates the viscous dissipation. So, the finally, equation of change for internal energy you get this equation. So, now, this particular part if you take to the left hand side, so, and then added up with the $\frac{\partial}{\partial t}\rho\hat{u}$.

Then you can make use of a substantial time derivative and then you can write this equation as $\rho\frac{D\hat{u}}{Dt} = -\nabla.\mathrm{q} - p(\nabla.v) - \tau:\nabla\mathrm{v}$ ok. So, this is nothing but an equation of change for internal energy ok. So, initially we have written for an equation of change including all three energies; internal energy, potential energy and kinetic energy. Now, we are writing equation of change only for internal energy.

This is what we have derived. Now, we further simplify this equation, so, that we can write the same equation for an equation of change in the form of an enthalpy. So, that is we are going to derive equation of change for enthalpy of the system, so that we can get. So, this tau dot $\tau\!:\!\nabla v$ whatever is there, this indicates the viscous dissipation part and then it is having 9 terms. It is not 9, 1 term it is having 9 terms and in all those 9 terms in a vectorial form in vector summation form it is represented it here like here.

It indicates the sum of order of quantities, quantities like vectors and tensors being multiplied. So, all those terms are here. So, then when you expand these terms you will get 9 terms that are available in any standard textbooks. We do not need to worry about them as of now.

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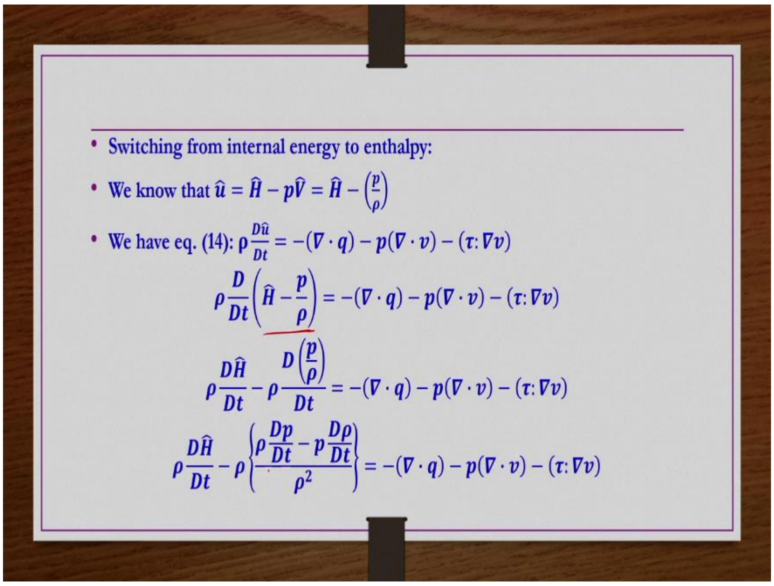

Now, switching from internal energy to enthalpy, so, we know that $\hat{u} = \hat{H} - p\hat{V}$ or $\hat{H} = \hat{u} + p\hat{V}$. So, that $\hat{u} = \hat{H} - p\hat{V}$. $\hat{V}$ we can write $\frac{1}{\rho}$. So, equation 14 what you have? $\rho\frac{D\hat{u}}{Dt} =$ right hand side this term as it is. So, now in place of $\hat{u}$ you are going to write $\hat{H} - \frac{p}{\rho}$. So, that is what you are having.

So that means, you are going to do the substantial time derivative of $\hat{H} - \frac{p}{\rho}$. Substantial time derivative has to be done similar way as the total derivative being done in general. So, that is what we are going to do in several times in the subsequent slides also. So, when you do this one what you get? $\rho\frac{D\hat{H}}{Dt} - \rho\frac{D\left(\frac{p}{\rho}\right)}{Dt}$.

So, this if you further do the substantial time derivative then what you have? $\frac{\rho\left(\frac{Dp}{Dt}\right)-p\left(\frac{D\rho}{Dt}\right)}{\rho^2}$ and then right hand side terms are as it is. So that means, this we multiply this ρ also if you bring inside. So, ρ^2 and then this ρ^2 will be cancelled out and then here you have + pρ $\frac{D\rho}{Dt}$. That is what we are having.

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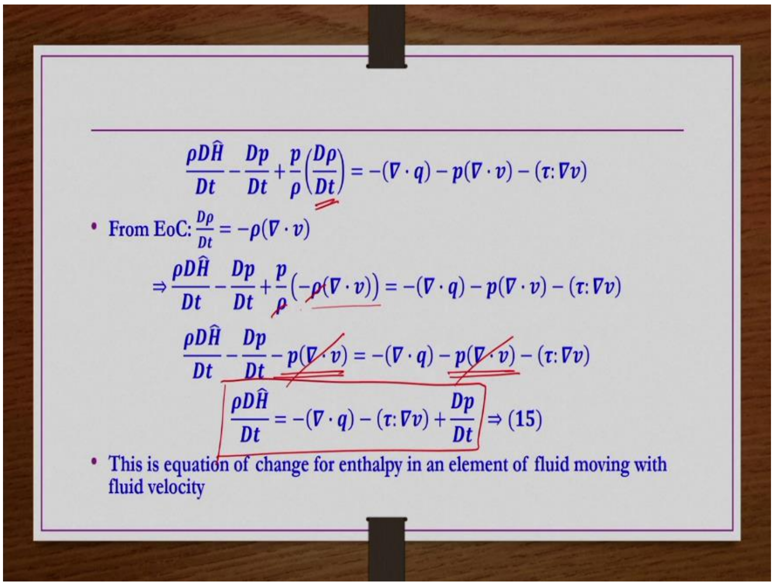

So, $\frac{\rho D\hat{H}}{Dt}-\frac{Dp}{Dt}+\frac{p}{\rho}\left(\frac{D\rho}{Dt}\right)$ = right hand side terms as it is we are not doing any changes in the right hand side term. Now, from equation of continuity we can write $\frac{D\rho}{Dt}$ is nothing but $\left(-\rho(\nabla.v)\right)$, this is what we can write. This we have derived in the last class. So, that if you substitute here, so, $\frac{p}{\rho}-\rho(\nabla.v)$. So, this ρ this ρ is cancelled out. So, that you can write $-p\nabla.v$ here rest all other terms as it is.

So, that this $-p\nabla.v$ in the left hand side and this $-p\nabla.v$ in the right hand side can be cancelled out and then this $-\frac{Dp}{Dt}$ if you bring to the right hand side then you have $\frac{\rho D\hat{H}}{Dt}=-\nabla.q-\tau:\nabla\mathrm{v}+\frac{Dp}{Dt}$. And this is nothing but equation of change for enthalpy in an element of fluid moving with fluid velocity ok.

So, now we got equation of change for enthalpy. Now, what we do? This enthalpy we write in the form of temperature, pressure and then we try to write the equation of change for temperature that is what we are going to do now. How are we going to do?

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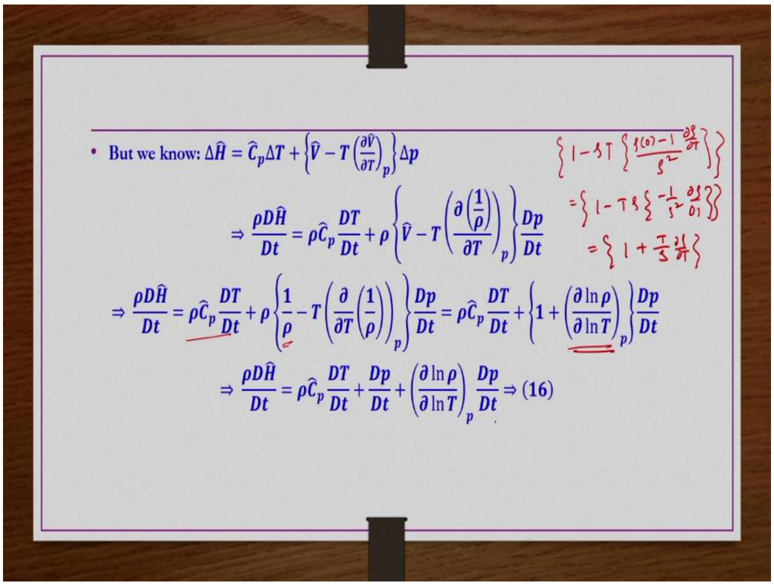

We have to make use this relation that is $\Delta\hat{H} = \hat{C}_p \Delta T + \left\{\hat{V} - T\left(\frac{\partial \hat{V}}{\partial T}\right)_p\right\}\Delta p$, this equation we have to make use ok. Now, here this equation both sides, first what you do? You do the substantial time derivative and then multiply both sides by ρ.

So, then you have left hand side $\frac{\rho D\hat{H}}{Dt} = \rho\hat{C}_p \frac{DT}{Dt} + \rho\left\{\hat{V} - T\left(\frac{\partial \hat{V}}{\partial T}\right)_p\right\}$ I am writing $\frac{1}{\rho}$ and then $\frac{Dp}{Dt}$ that is what we are having right. So, now, this left hand side as it is, right hand side first term is as it is. This term here again what I am writing? $\hat{V}$ again I am writing as $\frac{1}{\rho}$.

So, now this whatever you multiply inside this ρ if you bring in, so, what you are going to have? You are going to have $1 - T\rho$ and then $\left(\frac{\partial\rho}{\partial T}\right)_p$. So, then ρ^2 then we have $\rho^2, \rho(0) -$ 1 and then $\frac{\partial\rho}{\partial T}$ that is what you are going to have. So, then that we can write $1 - T\rho$ and this is $-\frac{1}{\rho^2}\frac{\partial\rho}{\partial T}$.

So, that I can write $1 + \frac{T}{\rho}\frac{\partial\rho}{\partial T}$. So, $\frac{T}{\rho}\frac{\partial\rho}{\partial T}$, I can write $\frac{\frac{\partial\rho}{\rho}}{\frac{\partial T}{T}}$. So, that I can write $\frac{\partial ln\rho}{\partial lnT}$. So, that is 1+ $\left(\frac{\partial ln\rho}{\partial lnT}\right)_p$, remaining terms are as it is right. That means, $\frac{\rho D\hat{H}}{Dt} = \rho\hat{C}_p \frac{DT}{Dt} + \frac{Dp}{Dt} +$

$\left(\frac{\partial ln\rho}{\partial lnT}\right)_p \frac{Dp}{Dt}$. This $\frac{D}{Dt}$ indicates substantial time derivative as we have seen in the previous lecture right.

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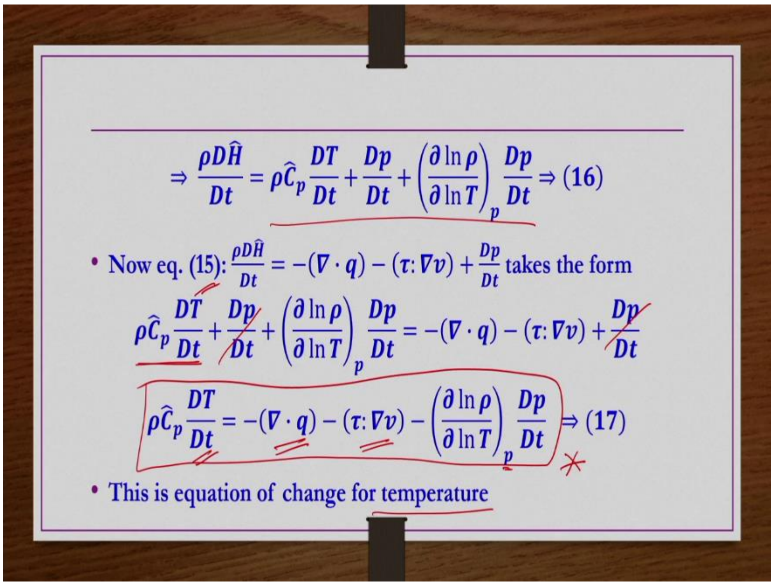

So, now what we do? This part we are going to substitute in equation number 14. This equation number 16 whatever we just derived that we are going to substitute in equation number 15 ok. So, in place of $\frac{\rho D\hat{H}}{Dt}$ we are going to write this part from equation number 16. So, that you have this equation right.

So, now here left hand side $\frac{Dp}{Dt}$ and then right hand side $\frac{Dp}{Dt}$ can be cancelled out and then left hand side we keep only this term this term and then whatever the $\frac{\partial ln\rho}{\partial lnT}$ term is there that we take to the right hand side. So, that $\rho\hat{C}_p \frac{DT}{Dt} = -\nabla. q - \tau : \nabla v - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \frac{Dp}{Dt}$.

So, this is nothing but equation of change for temperature. So, now, here this term indicates the rate of increase of temperature the temporal term and then it also include includes the rate of addition of temperature. Because of the convection this term indicates the rate of addition of temperature because of the conduction.

This term indicates the rate of addition of temperature because of the viscous dissipation and then this part indicates the rate of addition of temperature because of the work done because of the pressure forces all those terms are included right. But in general we do not use this entire equation in the complete form right.

So, now, this is the final energy equation in the form of temperature right. So, all these equations of change whatever we have seen they are energy equations only. First equation is that including all three forms of energy that is kinetic energy, potential energy and internal energy. Then we have written for the separately internal energy then separately for enthalpy then now separately for in the form of temperatures.

So, it is different forms simplified one after the other we are you know reducing the complexities and then finally, come to one form of equation where the terms are having something which are you know measurable right.

If at all if you wanted to do the validation with experimental parts, so, you should be in a measurable terms. So, that is temperature, density, velocity all these things are measurable. So, that is the advantage of this form of equation of change for energy ok for non isothermal systems.

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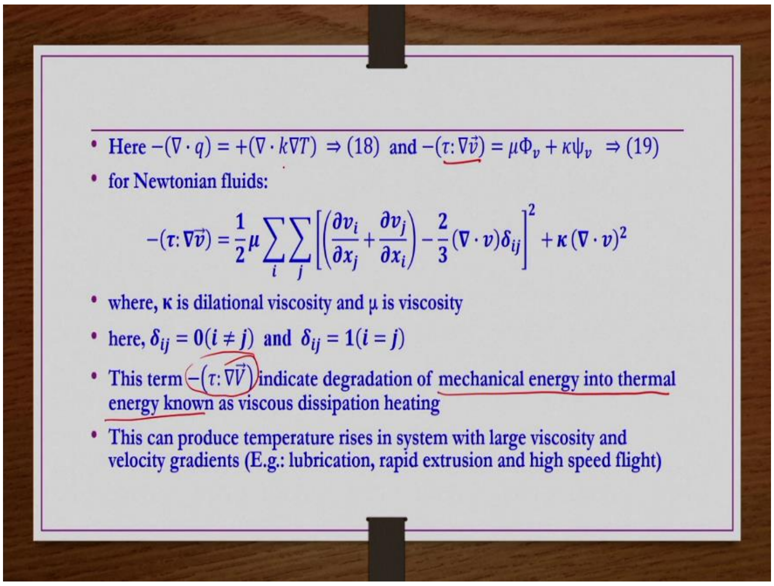

So, now here $-\nabla . q$ is nothing but $\nabla . k\nabla T$ right and then $-\tau{:}\nabla\vec{v}$ is nothing but $\mu\phi_v + \kappa\psi_v$. Here for Newtonian fluids this part whatever is there that can be written as this equation. Here κ is the dilational viscosity and then μ is the viscosity. If the δ if i = j then $\delta_{ij} = 1$. If $i \neq j$ then $\delta_{ij} = 0$ in this equation right.

So, this term indicates degradation of mechanical energy into thermal energy which is known as the viscous dissipation heating or energy due to the viscous dissipation ok. And this usually how the importance when we have a high viscous fluids which are moving at high velocity. Usually high viscous fluids, does not move at high velocities in general.

So, then in most of the application the contribution of this viscous dissipation is very small; however, there are many cases it is also included ok. So, now, what we do? We take a case where the conductivity is constant and then where we take the case the viscous dissipation heating is negligible.

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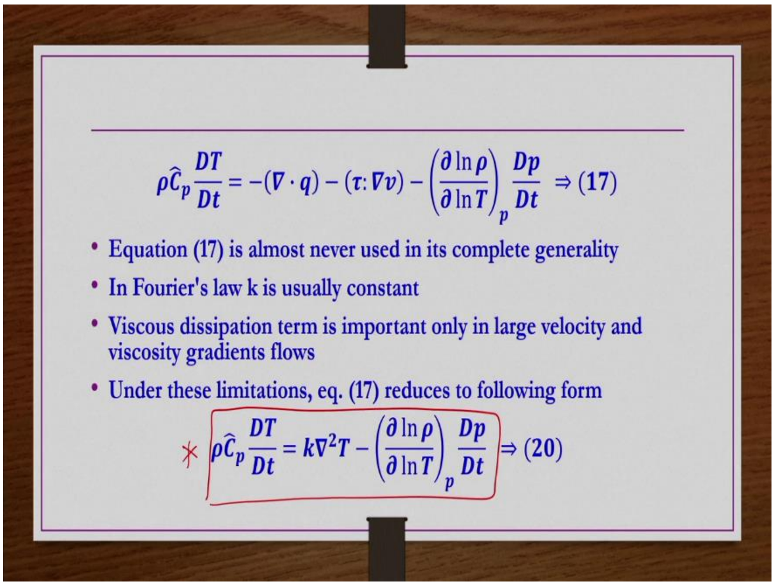

So, when we take those assumptions then this equation number 17 we can write in this form that is $\rho\hat{C}_p \frac{DT}{Dt} = k\nabla^2 T - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \frac{Dp}{Dt}$ ok. This is even further simplified compared to the previous equation number 17. Because now we have taken in Fourier's law k is constant and then viscous dissipation terms we have neglected. Now, we take a few further simplifications of these equations. So, what we do?

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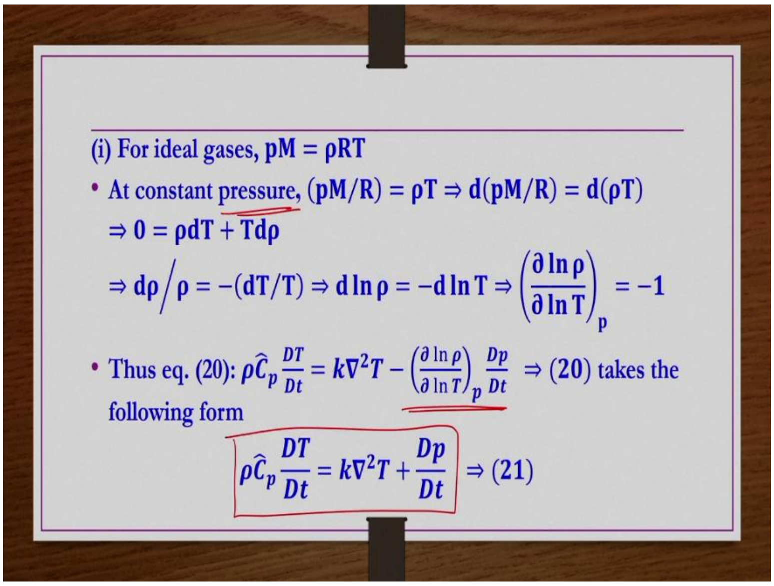

We take an ideal gas. For ideal gas p M = ρ R T at constant pressure what if we rearrange this equation p M/ R = T ρ. And then if you take the differentiation both sides in the left hand side M is constant for a given system, R is constant, p is also constant because we are doing at constant pressure. So, left hand side is equals to 0 and the right hand side term we have ρ Dt + T Dρ.

So, that if you simplify you get $\left(\frac{\partial ln\rho}{\partial lnT}\right)_p$ is nothing but -1, ok. So, this if you substitute in equation number 20 here, so, then what we have? $\rho\hat{C}_p\frac{DT}{Dt} = k\nabla^2 T + \frac{Dp}{Dt}$ this is what we are having.

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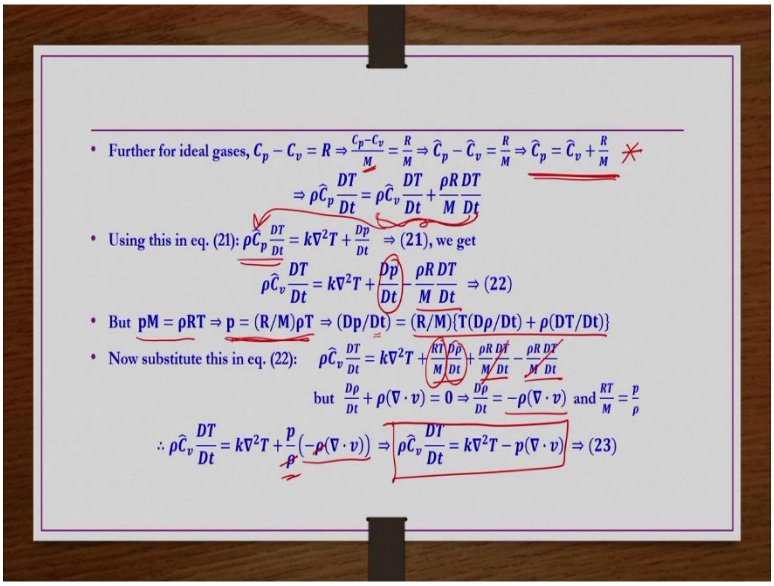

Now, for the same ideal gas what we have? $C_p - C_v = R$. Both sides if you divide by molecular weight of the gas whatever you have taken then you have $\hat{C}_p - \hat{C}_v = \frac{R}{M}$ because this tilde indicate per unit mass. So, that $\hat{C}_p = \hat{C}_v + \frac{R}{M}$ we can write. So, what are we going to do now here?

The previous equation we have written in terms of $\hat{C}_p$, now we are going to write in terms of $\hat{C}_v$. Further then you know $\rho\hat{C}_p$this equation from here as per this making use of this relation what we can write? $\rho\hat{C}_p \frac{DT}{Dt}$= $\rho\hat{C}_v \frac{DT}{Dt} + \frac{\rho R}{M}\frac{DT}{Dt}$ that is nothing but this equation we are multiplying by ρ and then multiplying by $\frac{DT}{Dt}$ both sides.

Now, in equation number 21, wherever $\rho\hat{C}_p \frac{DT}{Dt}$ is there you make use of this particular thing here. So, that you get the equation in terms of $\hat{C}_v$ ok. So, that is $\rho\hat{C}_v \frac{DT}{Dt} = k\nabla^2 T + \frac{Dp}{Dt}$ as it is. Whatever $\frac{\rho R}{M}\frac{DT}{Dt}$ is there that we are taking to the right hand side. So, then we are getting $-\frac{\rho R}{M}\frac{DT}{Dt}$right. So, further we have p M = ρ R T for the same ideal gas that we can write p = $\frac{R}{M}$T ρ.

Why are we doing here? Because we wanted to just get rid of these additional terms here in the right hand side. So, now, this equation if you do substantial time derivative both sides $\frac{Dp}{Dt} = \left(\frac{R}{M}\right)\left\{T\left(\frac{D\rho}{Dt}\right) + \rho\left(\frac{DT}{Dt}\right)\right\}$ that is what you get. So, in place of $\frac{Dp}{Dt}$ here in this equation number 21, you substitute this expression here.

So, that you do $\rho\hat{C}_v \frac{DT}{Dt} = k\nabla^2 T$ as it is and then $\frac{RT}{M}\frac{D\rho}{Dt} + \frac{\rho R}{M}\frac{DT}{Dt}$ and then $-\frac{\rho R}{M}\frac{DT}{Dt}$ of equation number 22 as it is. So, this term, this term you can cancel out because one is + another one is minus. Then further from the continuity equation in place of $\frac{D\rho}{Dt}$ you can write it as $-\rho\nabla.\mathrm{v}$ and then for a same ideal gas $\frac{RT}{M}$ we can write $\frac{p}{\rho}$. So, in place of $\frac{RT}{M}$ I have written $\frac{p}{\rho}$ here.

And then in place of $\frac{D\rho}{Dt}$ I have written $-\rho\nabla.\mathrm{v}$ from for this part. So, that you know this ρ and then this ρ is further cancelled out and then we have this equation $\rho\hat{C}_v \frac{DT}{Dt} = k\nabla^2 T -$

$p\nabla.\mathrm{v}$ this is what we have right. So, the same equation in different forms in different variables we are writing that is it.

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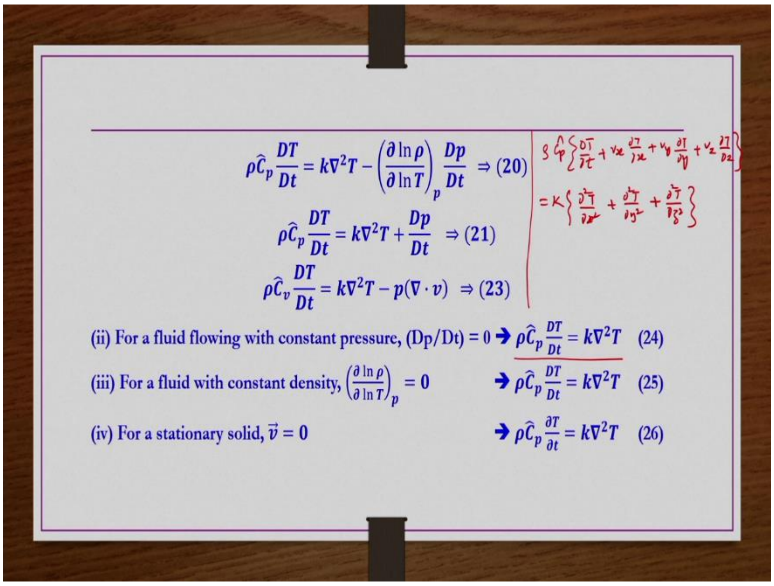

So, as a summary what we have? In terms of temperature equation of change for temperature we have this equation, we also have this equation for ideal gas system and then we also have in terms of $\hat{C}_v$ this equation. These are the things just now we have derived ok. These are some special forms. We are writing the same equation for different cases ok. So, this is what all we have written for you know different cases.

Now, let us say if we have a fluid flowing with constant pressure, if the pressure is constant then $\frac{Dp}{Dt}$ is going to be 0. So, this equation here if you write $\frac{Dp}{Dt} = 0$ then you have only $\rho\hat{C}_p \frac{DT}{Dt} = k\nabla^2 T$. Then if you have a fluid with constant density then again $\left(\frac{\partial ln\rho}{\partial lnT}\right)_p = 0$.

So, this part is 0 anyway then again we have $\rho\hat{C}_p \frac{DT}{Dt} = k\nabla^2 T$. Remember this equation is the general equation this is not for a this is not for a ideal gas. These are for ideal gas, but this is not for ideal gas this is general any fluid right. So, now, finally, for a stationary solid if the solid is there that is the fluid is not moving or if you have a stationary solid then vector v is 0.

Then in the left hand side $\rho\hat{C}_p$ in place of a partial in place of substantial time derivative you have to write a partial time derivative. Because substantial derivative in the substantial derivative if the v is 0 that is nothing but you know partial derivative, you get only partial derivative term right. So, these are the equation.

So, this equation if we expand then we have $\rho\hat{C}_p\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}=k\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}$. This is what we have you know in Cartesian coordinates likewise we can have the equation in cylindrical and spherical coordinates ok.

So, now we have seen basics of a transport phenomena part also, the derivation of continuity equation, momentum equation and energy equation and then previously we have seen so many details of non-Newtonian fluids. So, from next class onwards what we are going to see? We are going to see a few problems associated with the transport phenomena of non-Newtonian fluids ok.

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The reference for this lecture; the entire lecture is prepared from this standard textbook Bird, Stewart and Lightfoot that is Transport Phenomena Second Edition. But similar details may also be found in these two books that is Deen and then Panton, additional reference are may be available in these two books as well.

Thank you.