

Transport Phenomena of Non-Newtonian Fluids
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Lecture - 11
Continuum Hypothesis and Transport Mechanisms

Welcome to the MOOCS course Transport Phenomena of Non-Newtonian fluids, the title of this lecture is Continuum Hypothesis and Transport Mechanisms.

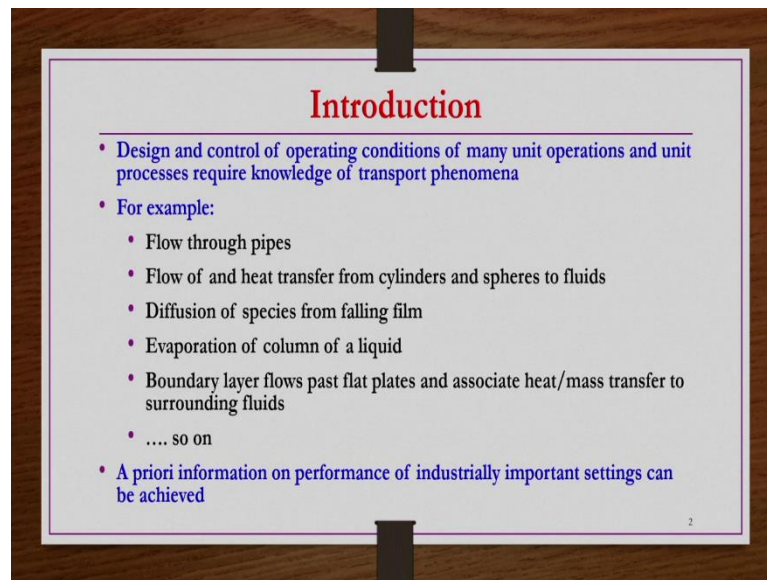
Till now in last three weeks what we have seen? We have seen different aspects of non-Newtonian fluids, their classifications, their examples and then mathematical models for non-Newtonian fluids then how to measure the rheology of an unknown fluid. If the fluid is non-Newtonian and then viscoelastic how to measure the normal stress. And then in addition to the normal stress conventional shear stress versus shear rate information how to measure by different types of rheology and all that. Those things we have seen with the appropriate example problems wherever required, right.

So, because till now what we have seen? We have seen the basics required knowledge of non-Newtonian fluids because this course is on transport phenomena of non-Newtonian fluids. We have seen adequate the information about non-Newtonian fluids which would be more than sufficient with respect to these contents of this course, right.

Now before going into the details of applying the transport phenomena concept for these non-Newtonian fluids then what we will have now? We will be having a kind of a recapitulation of a few basic of transport phenomena. Like transport phenomena you know continuum hypothesis, transport mechanisms, equations of change for isothermal system, equations of change for non-isothermal systems etcetera, the derivation of those equations of change. All those things we are going to see in this particular week.

So, now we have a kind of basics of transport phenomena in this particular week starting with the concepts of continuum hypothesis and then transport mechanisms. Why do we need to study the transport phenomena?

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Basically, if you see because if you have the knowledge of transport phenomena then designing or controlling the operating of any of the unit operations or unit processes or the entire plant could become very convenient to the engineers. That is the reason transport phenomena knowledge is very much essential for chemical engineering grads.

So, because of that reason we are having a UG level core transport phenomena course. If the same transport phenomena course is advanced level, like advanced transport phenomena or like transport phenomena of non-Newtonian fluids that is again a kind of a core course for the PG level chemical engineering students right.

So, how it is essential? Why that much essential having the knowledge of transport phenomena for a chemical engineering grads? That we see with a few basic examples. Let us say you have a flow through pipes, in the pipe a fluid is flowing. So, for a required flow rate how much pressure drop should you maintain; if you wanted to calculate you need to have a knowledge of transport phenomena.

If you have a several pipes connecting you know different types of unit operations and unit processes, then what is the friction factor or what are the frictional losses because of these connections etcetera. Those things if you wanted to understand you need to have a knowledge of a transport phenomena.

Then let us say you have a vertical column and then that column is packed with some kind of material, like you know like a beads, glass beads or crust glasses or sand etcetera, such kind of material is used for that packing for column. And then a fluid is flowing through that column because of some requirement of either reactions or you know heat transfer purpose or whatever it may be.

So, you there also you need to have a kind of knowledge what is the frictional pressure drop or in order to maintain a certain flow rate how much pressure drop you have to maintain, what is the friction factor all those things you know if you have the knowledge, so then you can comfortably design the packed columns or you comfortably make modifications to the operating conditions if at all required.

So, not only for the design, but also controlling the operation of any of the unit operations unit processes or the entire plant the chemical engineering grads need the knowledge of transport phenomena ok.

So, let us say if the flow through pipes or packed column if you also have a kind of temperature gradients. So, then how that temperature gradient is going to affect this pressure drop versus volumetric flow rate or a friction factor etcetera, how that is going to be affected because of the variations in you know temperature from one location to another location. Then again you need to have a knowledge of transport phenomena, ok.

So, likewise if you take some other examples like you know flow of a fluid along a spherical particles, along cylindrical particle and then associate heat transfer is also very common phenomena in majority of chemical engineering plans. So, then there also you need to have the knowledge of transport phenomena. So, that you can properly operate it or tune the operating conditions or you know you can even design them if required.

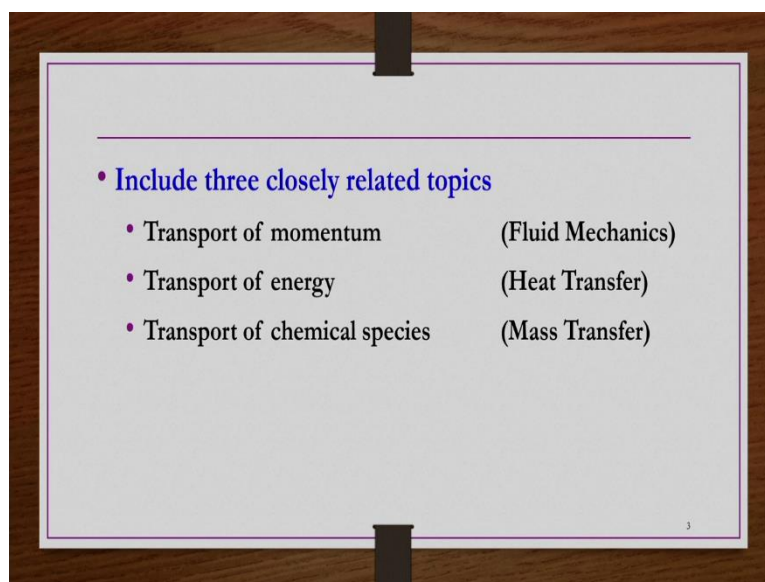
Then diffusion of species from falling film one another example. In polymeric industries usually it is often required to have a polymeric films of very small thickness 1 mm or even less. So, then usually these dilute polymer solutions are allowed to flow aligned inclined or vertical surface. And then while they are while they are flowing along these inclined surfaces etcetera they are also dragged, so that we can have a kind of thin film kind of things are you know very common examples.

So, at what rate this polymeric solution should be allowed to flow, so that required thickness film should form. That if you wanted to know then you should know the evaporation rate, diffusion rate etcetera all those things are required. So, if you wanted to understand those concepts then again you required to have the knowledge of transport phenomena.

Similarly like evaporation of column of liquid, then boundary layer flows pass, flat plates, and associate heat and mass transfer to surrounding fluids, wherever you need to have the rate of heat transfer, rate of mass transfer etcetera those kinds of calculations. So, then these transport phenomena knowledge is going to be very essential.

So like that if you keep on listing you can have n number of applications in chemical industries or almost all chemical industries you will find the requirement or knowledge of transport phenomena almost like you know ubiquitous for any part of plant you consider ok. So that means, you know a priori information on performance of industrially important settings can be achieved if you have an adequate knowledge of transport phenomena, ok.

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These transport phenomena include three closely related topics. The first one is transport of momentum related to the fluid mechanics part, transport of energy related to the heat transfer part and transport of chemical species related to the mass transfer part, ok. So, these are the processes like transport of momentum, transport of energy, transport of chemical species you know essential.

So, if you see only a fluid mechanics point or heat transfer point or mass transfer point all these courses we have already done before during the transport phenomena courses in general any university UG curriculum. So, then why are we doing this transport phenomena course again, as a core course that also because they occur simultaneously.

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Handwritten notes on the slide:

- $x \rightarrow z$
- $y \rightarrow y$
- $w \rightarrow z$

Why these transport phenomena should be studied together?

- Simultaneous occurrence
- Governing equations are closely related
- Mathematical solution approaches are similar
- Molecular mechanisms are very closely related

Handwritten equations:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + F_x$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + R$$

Because they occur simultaneously or rather having only one transport process or transport of only one momentum or only transport of energy or only transport of chemical species is a kind of exception rather than a rule.

But in this fluid mechanics heat transfer or mass transfer wherever you take, you take individually like fluid mechanics you know you take only you know macroscopic transport phenomena; that also without considering the effect of you know concentration variations, temperature variations, how it is going to change the flow phenomena etcetera, those kind of things we in general do not take into the picture.

Similarly, when it comes to the heat transfer only heat transfer part, only we have studied in the heat transfer course in general. So, that is indirectly the variations in the velocity, velocity distribution how it is going to affect the overall heat transfer phenomena or local heat transfer phenomena; that those things we have not studied.

So, that means, you know the heat transfer combined with the momentum transfer is not studied in heat transfer course in general, even if it is studied maybe a small bit, not the in detail. So, similarly the same is true for the mass transfer part also.

We see specific to the transport of you know chemical species without having the knowledge of, how the momentum transfer is going to affect the transfer of this chemical species from one location to the other locations. But in reality majority of industrial settings these transport phenomena problems they occur simultaneously rather than individually.

Then governing equations are also very closely related right. So, if you write the transport of I mean equations of change for momentum for energy, for concentration of species way then you can compare. So, these equations are closely related; very much closely related we are going to derive those equations in the next class anyways. So, there we can realize.

In fact, if you remember from your UG level course so, then what we have? Momentum transfer equations you know let us say u component we have. So, let us say x component of momentum equation. And then in the x direction velocity component if you designate with u then x component of momentum equation in Cartesian coordinates if you write this is what you have.

So, u is, in the x direction v is in the y direction and w is in the z direction; the velocity components in three different directions are u v w then you this is what you have. Let us say you have the heat transfer part equation if you see. So, then ρC_p ; for simplicity I write these equations like this by taking this u outside and then hear by taking this v outside and then hear by taking this w outside of this differentiation.

So, now if you see these equations here, so all these equations they are quite similar. So, the temporal variations and then space variations in these properties whether it is temperature or momentum they are very same except this you know pressure term is additional term here in the momentum equation, but without that if you compare the momentum equation and energy equation they are very similar to each other very close to in the form, form wise they are very similar; they are very similar.

And then if you write the species concentration equation then that will be even much more you know similar; the energy equation in species concentration equation would be very much similar to each other.

So, that is the reason the governing equations; these are the you know equations generalized equation where the transport is there because of the convection and then molecular transport etcetera all those things are there. So, then these equations are closely related to each other and they are similar to each other, ok.

Since they are the closely related and similar to each other their solution approach would also be similar it is quite possible. Then molecular mechanisms are also very closely related. So, whether the heat transfer or momentum transfer these terms, whatever these three terms are there they are indicate the transport because of the convection, right.

So, similarly here these three terms they indicate transport of the energy because of the convection ok. Whereas, these right hand side these three terms you know they indicate the transport of momentum because of you know molecular mechanism right. Whereas, here again these three atoms in the energy equations you know they indicate transport of the energy because of you know molecular mechanism that is by conduction ok.

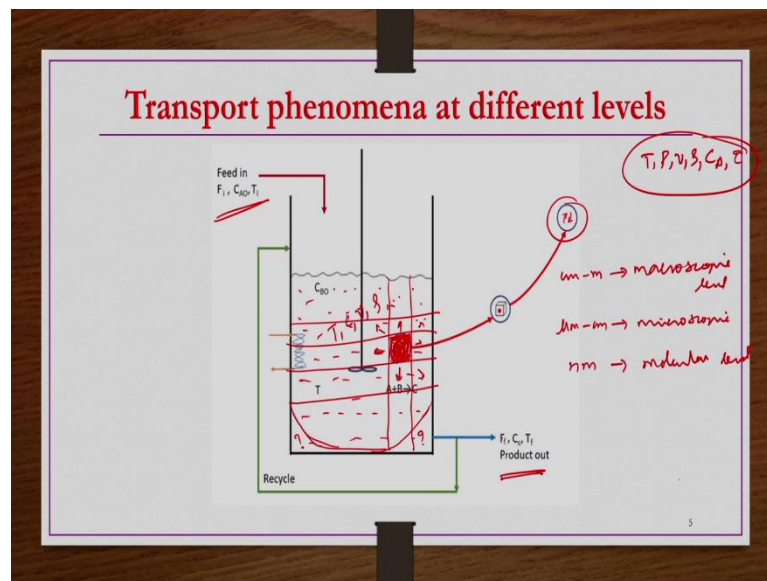
So, the mechanisms which are leading for the transport of whether it is momentum, transport of whether it is energy, transport of whether it is concentration, the molecular mechanisms are also quite similar. Only thing that you know in the molecular part the material properties also coming into the picture, but convective part you know the nature of the material is not coming into the picture at all.

Only thing that the bulk motion, what is the velocity? What is the velocity in x direction? What is the velocity in y direction? What is the velocity component with the z direction? That is only coming into the picture if you take the convection part only.

Assume that all these equations are you know already known to you in your UG level course. So, then I have directly written. But we are going to derive these equations again we are going to discuss them in detail wherever we required. So this is what we see. So, now, what we understand from these equations are similar.

They are occurring simultaneously equations are similar, so then solution approaches are also similar. And then also molecular mechanism which are leading the transport of these properties are also similar right. Because of this reason so we need to study transport phenomena as a separate course you know despite we have studied the fluid mechanics heat transfer and then mass transfer individually.

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So, transport phenomena at different levels; what that what does it mean? So, from this slide we will also realize that you know transport phenomena is how it is different from the fluid mechanics heat transfer or mass transfer part that we have studied previously. So, for that purpose what we do? We take you know continuous stirring reactor we take here.

So, in which we are already having B component at concentration C_{B0} . So, to this reactor C_{A0} or a component A is added at a concentration C_{A0} . The temperature of that component feed material coming in is T_i , and then material is coming at F_i flow rate ok.

So, then when it comes and joins the B component in the reactor. So, then A + B giving rise to C reaction is taking place and then product is being collected out because C is product now here. So, then that product is collected out at T_f temperature and then at flow rate F_f .

So, whatever the unreacted A and B are there. So, those things again we are separating and then putting them back into the reactor as a recycle. So this is the process here. So, this

transport phenomena is possible at three different level; macroscopic level, microscopic level and then molecular level.

In the macroscopic level what we worried about we worried about or we consider only the quantities that are entering and then that are leaving; at what rate, at what they are entering to the system, at what rate they are leaving. Or the, we are worried about these properties whichever property maybe. Properties when I call when it can be temperature pressure, velocity, density, concentration, stress etcetera anything it may be, it may be any quantity any property that you wanted to measure.

So, the property how much it is entering, how much it is leaving that much only we are concerned in the case of macroscopic level. And then the difference whatever instead that we relate to the accumulation if at all some any accumulation, right.

So, we do not worry what is happening inside the reactor, whether the concentration is uniform changing, how it is changing; one location to another location that we do not at all worried about those kinds of thing in macroscopic level right.

And then scale is in general you know some centimeters to meters in the case of macroscopic level. But what we understand whether it is temperature, whether it is concentration, whether it is velocity, whether it is density whatever you take any property you take is not going to be uniform at each and every location. At each and every location it is not going to uniform that we know; that we know actually right.

So, we realize that is the reason you know we realize; that is the reason you know sometimes you know you may get the different values of different when you repeat the experiments, right. We, but you know what happens the properties they are not uniform they are not uniform in you know entire reactor, they are changing from one location to the other location.

So, if you wanted to consider those changes then what you do? You subdivide the system into the small-small control volumes like this right. And then see that properties whatever the property is there that you assume that you know that is uniform within the small control value, because that is the reason. You know the variations are there, but variations are varying from one location to the other location.

So, now you take a region of a certain location that we call a small control volume. So, within each control volume these properties may be same, but the property may change when we move to the neighboring control volume, right.

The properties may be changing. It may be increasing, decreasing depending on the system and the change may be large, change may be small we cannot say, but there may be change. So, that change you consider by you know writing the equations for this microscopic level right.

So, that is what we say in the microscopic level, what we do? We subdivide the system into the small-small control volumes and then within each control volume we say that the properties are uniform; the properties are uniform, but when we move from one control volume to the other control volume properties may change and then that change may be small or large depending on the gradients, right. Such a way that we write the constitutive equations and solve them to get the interior details; what is happening ok. All those things we see.

And then this is very much essential because in sometimes many a times at corners you know the mixing may not be proper, at the corners mixing would be you know inefficient. So, at this locations primary the already whatever the B present that is only there the C and they may be very less at this corner. So, those kind of inequalities would be there.

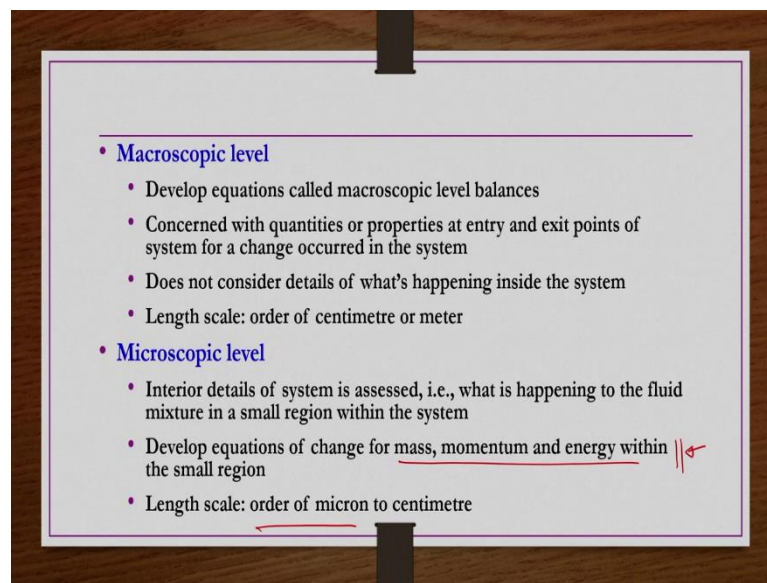
So, if you wanted to understand those kind of things then you have to do this microscopic level transport phenomena where the length scale is few microns to centimeters right. So, that is the microscopic level transport phenomena.

But within the each control volume we have the large number of molecules, huge number of molecules would be there. And then so all those large number of molecules are interacting amongst themselves. So, when they are interacting there may be change or there may be transport of property from one molecule to the other molecule maybe because of the vibration, because of the rotation, because of the interaction or you know oscillation whatever the nature of molecule, whatever the thing that is happening between the molecules.

Whether interaction, because of the vibration, because of the rotation, because of oscillations, because of just translation there may be interaction and then because of that interaction transport or change of property may be taking place right.

So, if you are considering those level you know changes in the properties then we call that transport phenomena at molecular level, where the scale usually few nanometers or microns.

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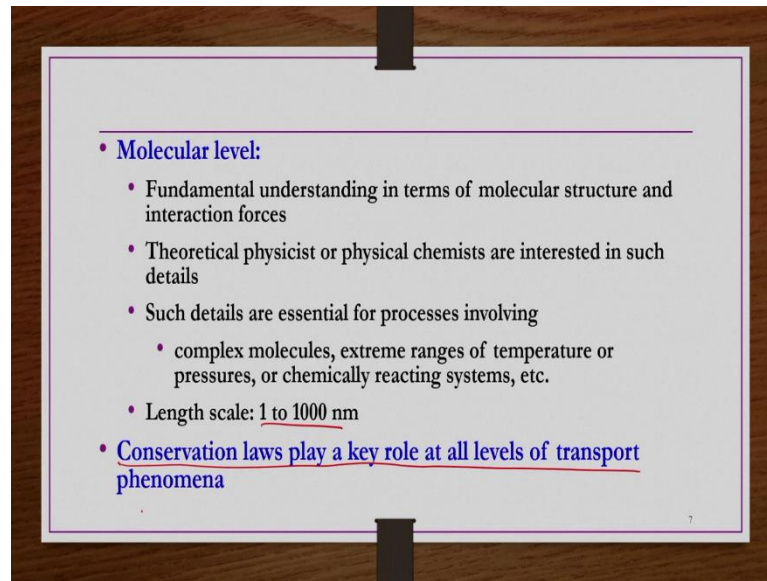


So, in the macroscopic level transport phenomena we develop equations called macroscopic level balances, where we are concerned with quantities or properties at entry and exit points of system for a change occurred in the system. And it does not consider details of what is happening inside the system. Length scale is in general order of centimeter or meter.

Then in the microscopic level, we consider interior details of system where we try to find out what is happening to the mixture in the small region with the within the system. Then we develop equations of change for mass, momentum and energy within this small region and then that we integrate to the overall system right if required. So that is what we do.

So, the primarily the course that we are going to study is handling you know developing of these equations and then using these equations to solve engineering transport phenomena problems. So, here the length scale is usually order of micron to centimeters.

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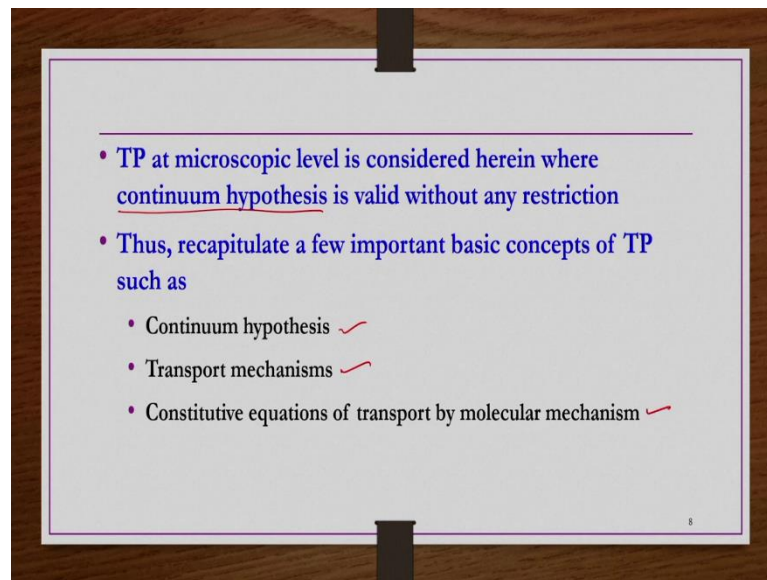


In the molecular level what we have fundamental understanding in terms of molecular structure and interaction forces. And then these details are usually important from the theoretical physicist or physical chemist point of view. But however, such details are also essential in processes where engineering grads are involved.

So, some examples like you know; if we have a complex molecules, extreme ranges of temperature or pressures or chemically reacting systems etcetera. Then we also need to have the knowledge of this molecular level and then length scale usually up to 1000 nanometers.

So, whether it is macroscopic level or microscopic level or molecular level, the conservation loss play a key role at all levels of transport phenomena. So, the maintaining of conservation of momentum or energy or species whatever is there that is very essential at all levels of transport phenomena.

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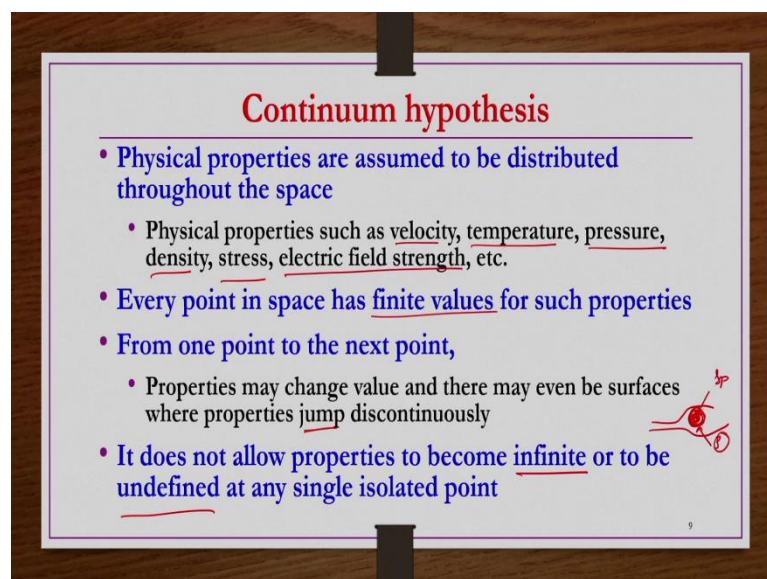
Slide 8 contains a list of bullet points in blue text. The first point states that transport phenomena at the microscopic level is considered where the continuum hypothesis is valid without any restriction. The second point asks to recapitulate a few important basic concepts of transport phenomena, such as:

- Continuum hypothesis ✓
- Transport mechanisms ✓
- Constitutive equations of transport by molecular mechanism ✓

The slide is numbered 8 in the bottom right corner.

So, that means, transport phenomena at microscopic level is considered herein where we are considering the continuum hypothesis is valid without any restrictions ok. So, that part only we are considering in this course. So, obviously, a recapitulation of few basic concepts of transport phenomena such as; continuum hypothesis, transport mechanisms, and then constitutive equations of transport by molecular mechanisms are essential. So, that is what we are going to see now.

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Slide 9 is titled "Continuum hypothesis" in red text. It contains a list of bullet points in blue text:

- Physical properties are assumed to be distributed throughout the space
 - Physical properties such as velocity, temperature, pressure, density, stress, electric field strength, etc.
- Every point in space has finite values for such properties
- From one point to the next point,
 - Properties may change value and there may even be surfaces where properties jump discontinuously
- It does not allow properties to become infinite or to be undefined at any single isolated point

The slide is numbered 9 in the bottom right corner. There is a small red handwritten mark on the right side of the slide.

So, what is continuum hypothesis? Continuum hypothesis it says that physical properties are assumed to be distributed throughout the space, physical properties means what? What kind of properties? It may be velocity, it may be temperature, it may be pressure, and it may be densities, stress, and electric field strength etcetera. It may be any property that is associated with you know system whichever we are considering ok.

And then we are assuming as per the continuum hypothesis these properties are assumed to be distributed throughout the space without any you know; without any locations where these properties are being infinite or not being defined. So, we had the continuum hypothesis does not allow it ok. They have to be distributed throughout the space and then every point in space should have finite values for such properties whichever we are considering.

And from one point to the next point the properties may change; the property may change, the velocity may change from one point to the other point within the system, the temperature may change from one point to the other point within the system. There may also be discontinuity of the properties the whether temperature, pressure or velocity stress density whichever it may be.

There may be discontinuities may also be possible right. Or discontinuities in because of the jump what does it mean by how the jump of the fluid will take place. Let us say you have a spherical particle right and then there is a fluid flowing around it, right.

So, the density of the particle ρ_p is usually very large you know compared to the density of fluid of you know any fluid that you take. So, right at the surface the density is jumping from very high density ρ_p to some moderately low some other density low density ρ .

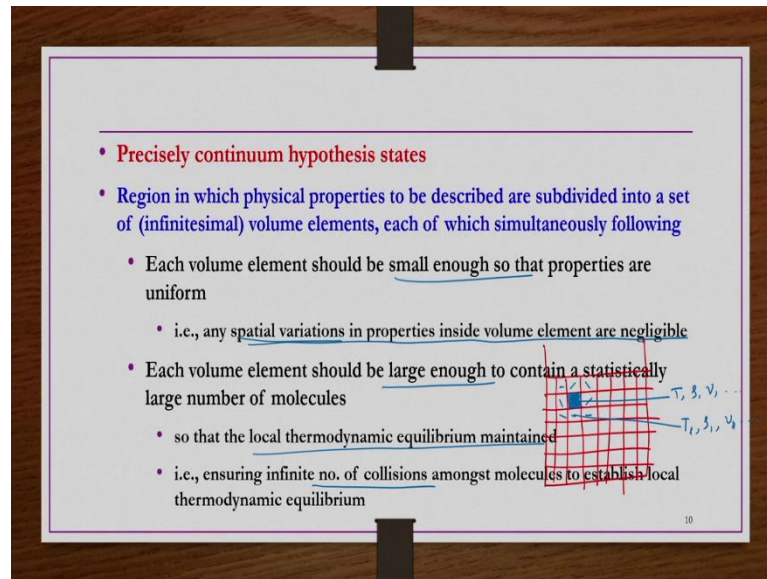
So, the jump is you know is very high. So, that such kind of discontinuities may be there. But we cannot have the case where the properties becoming infinite or to be undefined at any single stage. So, that is not allowed. This is what about you know basic requirements of continuum hypothesis right.

We assume the physical properties are distributed throughout the space and then every point the property has the finite values. And then from one point to the other point when you move the property may change and there may even be discontinuous jump as defined

here. But we cannot afford to have any location the property being in finite or not being defined ok.

So, precisely in simpler language if you wanted to explain this continuum hypothesis let us say the same reactor that we are taking here, ok.

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So, this is the system only this part where the fluids are being interacting or the reaction is taking place let us say. So, this space we have subdividing into very you know small-small control volumes like this.

So, the entire space whatever is there that we have subdivided into the small infinitesimally small spaces. So, that the property here let us say at this property. So, that at this level property may be something right. At this level it may be something, at this level it may be something, at this level may be something, something and different-different values it may be there compared to the property at this level, ok.

But within the control volume; within each control volume the property is going to be uniform is going to uniform, what does it mean by? Let us say this completely shaded control volume completely filled control volume here, whether you take the take at the center of the control volume, whether at the corner of the control volume, at the wall of the control volume. Any location within the control volume the property whichever the

property you are taking the temperature or density, velocity anything stress anything that you are trying to measure they are going to be uniform right.

But, the very next control volume if you consider, then the property may be different T_1 , ρ_1 for example, V_1 I am saying it is not a different fluid same fluid. The property from this control volume to this control volume has changed there is a possibility that change may be small or large that depends on the gradient, but that is changing.

But any control volume you take you know within the control volume the property is going to be uniform, the property is going to uniform that is one thing. So, that is how you have to divide the system into the small-small. That means, you cannot have you know very bigger control volume, if you take very bigger control volume obviously, from one corner to the other corner of control volume the properties may change; you may not have the uniformity, right.

So, but again you cannot take very very small control volume that you know within the control volume you may be able to accumulate only 5 or 10 molecules like that only. Why we cannot have such small control volumes? Because if you wanted to have the uniform properties within the control volume the molecules should go you know in finite number of interactions amongst themselves. So, that local thermodynamic equilibrium should be maintained.

That local thermodynamic equilibrium when it is maintained then only the properties are going to be uniform within the control volume. So, that is the important thing. You have to have a control volume very small size, but it cannot be very very small that you cannot have a sufficient large number of molecules.

Again, you cannot have the control volume very big size that you know the property is changing from one location to the other location within the same control volume. From one control volume to the other control volume it will anyway change ok. So, those are the important thing about this continuum hypothesis.

That is precisely the reason in which physical properties to be described or subdivided into a set of infinitesimal volume elements each of which simultaneously following that each volume element should be small enough, so that properties are uniform that is any spatial

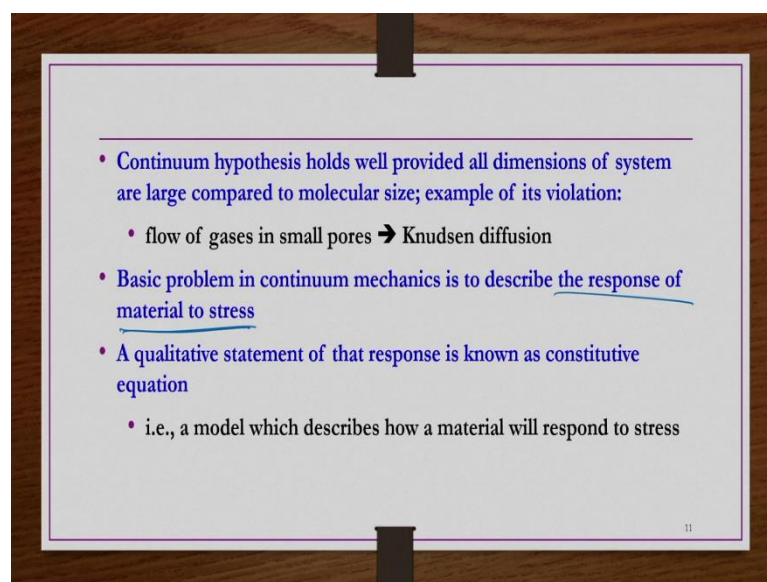
variations in properties inside the control volume or volume element are negligible. That is within each control volume or volume element the properties are going to be uniform.

Spatial variations are not allowed within the control volume. From one control volume or from one volume element to the other volume element it is possible. That is how we are doing. Then each volume element should be large enough to contain statistically large number of molecules.

Why we need large number of molecules? Because the local thermodynamic equilibrium should be maintained. So, what is the connection between local thermodynamic equilibrium and large number of molecules, because when you have the large number of molecules infinite number of collisions amongst those molecules possible which will establish the local thermodynamics, right?

So, such a way you divide the system and then for each control volume you write the balance equations then you develop the differential equations. Those equations are nothing but the so called momentum equations, equations of change for momentum energy and then species concentration that we are going to discuss in transport phenomena course in general, ok.

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So, continuum hypothesis holds well provided all dimensions of system are large compared to the molecular size ok. That is how the control volumes are being made in

small-small sizes right, but its violation is possible. Let us say if you have you know diffusion of a gas in a porous media in a pore. So, the size of the pore is very very small and then the molecule size is size of the molecules are anyway small. So, then under such conditions the dimensions of the system and then molecular size of these components are you know of the same size comparable to each other, right.

So, but under such conditions we cannot have the local thermodynamics maintained indeed local non-thermodynamic equilibrium would be there, so because of that one under such small systems we cannot impose the continuum hypothesis. That is flow of gases in small pores where Knudsen diffusion is occurring that is one of the example where we cannot say the continuum hypothesis holds with.

Then basic problem in continuum mechanics is to describe the response of material to stress. A qualitative statement of that response is known as the constitutive equation ok. So, when a material is apply certain kind of force. So, then you know or how to a applied stress how the material is responding right. So, that is what we try to understand.

We try to understand in continuum mechanics the response of material to applied stress. So, that response if you write as a qualitative statement we call them as a constitutive equation that is a model which describes how a material will respond to stress ok.

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Transport mechanisms

- If T or C in any material are perturbed so that they become non-uniform, then
 - Gradients tend to disappear over time
 - Same is true for velocity variations within a fluid
 - Spontaneous dissipation of such gradients is a fundamental and far-reaching observation
- i.e., energy, concentration of material and momentum of a fluid stream tend to
 - Move from regions of higher to lower intensity levels

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So, now we see the transport mechanisms. So, whether temperature or chemical concentration of the material, if you are perturbing it so that they become non-uniform, then what happens? Whatever the gradient develop because of the perturbation that will tend to disappear over time ok.

Then the same is true for velocity variations within the fluid. In terms of temperature concentration it is very easy to realize. So, then same is true for the momentum as we well with a fluid right. And then spontaneous dissipation of such gradients of such gradients is a fundamental and far reaching observation, ok.

So, that is the energy, concentration of material and then movement of fluid. What is happening according to these statements? When there is a perturbation has been given then you know there is a non-uniformity, that non uniformity whatever is there that causing the gradients and those gradients are tending to disappear over time.

So, those gradients are you know going to disappear over time from which region to which region? From high intensity region to the low intensity region they are going to transport. That is they move from regions of higher to lower intensity levels.

The transport of those properties whether it is energy, concentration or momentum, how it is going to occur right? How it is going to move from one region to the other region? So, that depends on the transport mechanism. So, there are the two contributions are there.

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- Fluxes of those quantities, i.e., energy, concentration and momentum, each have two contributions
 - Convective transport which accompanies any bulk motion
 - Molecular or diffusive transport which originates from intermolecular forces and relative motion of molecules
- Convection part can be described in a general manner
- Molecular part of transport processes are specific to given material or class of materials, thus their constitutive eqs. relate
 - Molecular fluxes to material properties and
 - Gradients of temperature, concentration and velocity

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So, convective transport which accompanies any bulk motion. And then molecular or diffusive transport which originates from intermolecular forces and relative motion of the molecules, right.

So, the momentum equation and then energy equation I have written a few slides back. So, there we can see the convective transport it has taken, consider the bulk motion that is velocity components only because of the velocity, how the property is changing how the temperature is changing that part is only included in the energy equation convective part ok.

So, whereas, the molecules or diffusive transport which originate from intermolecular forces and relative motion of the molecules that is there in the right hand side of this equations that we see anyway.

So, convection part can be described in a general manner. So, if you get back those equations momentum equation and energy equation and then species concentration equation that we have written few slides back. You know whatever the convection part they are very similar right; $\rho C_p \rho v$ those kind of terms were there in the left-hand side ok.

And then their spatial variations were there right. So, that is only velocity, spatial variations in the velocity or the spatial variations in the temperature because of the bulk motion velocity. Those things are only there. So that means, you know convection part can be described in a general manner.

So, what does mean by in general manner? Whether the fluid is benzene and or water or cyclohexane or toluene that part is not coming into the picture in the left-hand side of those equations momentum equations or energy equation whichever you take right. That part is not coming into the picture.

So, only part is coming into the picture at what velocity is it moving given fluid is moving that part is only coming into the picture. That is the reason convection part can be described in a very general manner. However, molecular part of transport processes are specific to given material or class of materials. Like in the right hand side you know whatever the viscosity μ is there.

So, it depends on the material what fluid it is newton, whether it is water, benzene, and cyclohexane. So, accordingly its viscosity is coming into the picture. Similarly, energy equation the thermal conductivity that K is coming into the picture so, it is going to be dependent on the material from one material to the other material like that ok.

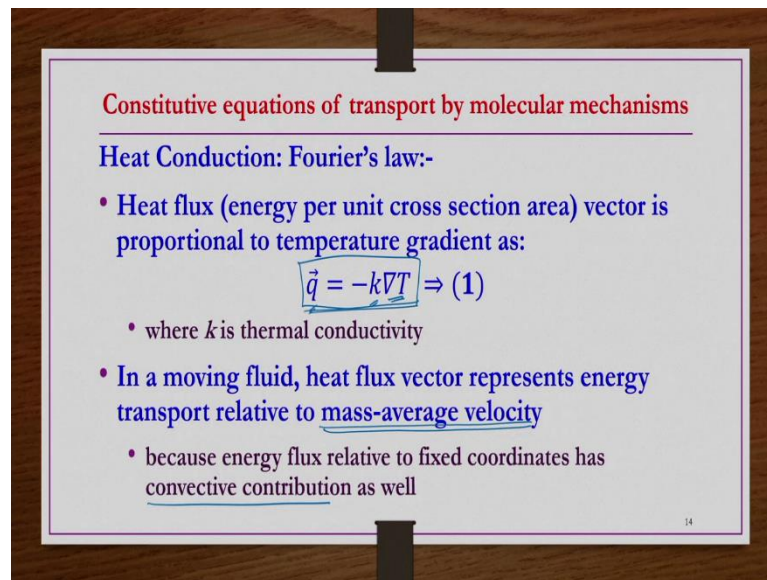
So, whatever the right hand side part of those equations are there. So, that is because of the molecular part you know those are dependent on the nature of the material or group of the materials ok.

So, these equations the molecular fluxes related to the material properties whatever the transport because of the molecular mechanism is occurring. So, that molecular flux is related to the material properties and gradients of temperature, concentration and elasticity if at all they are involved.

So, molecular fluxes are related to the material properties and gradients of temperature concentration and velocity all three; if all three transport phenomena are occurring ok. So now, that is the reason we are going to see now some of the equations.

You know molecular mechanism; by molecular mechanism how they change in or the transport of energy is taking place, because of the molecular mechanism how the diffusion or the mass transfer is taking place, because of the molecular mechanism how the momentum is transfer momentum transfer is taking place that is what we are going to see now.

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Constitutive equations of transport by molecular mechanisms

Heat Conduction: Fourier's law:-

- Heat flux (energy per unit cross section area) vector is proportional to temperature gradient as:
$$\vec{q} = -k\nabla T \Rightarrow (1)$$
- where k is thermal conductivity
- In a moving fluid, heat flux vector represents energy transport relative to mass-average velocity
- because energy flux relative to fixed coordinates has convective contribution as well

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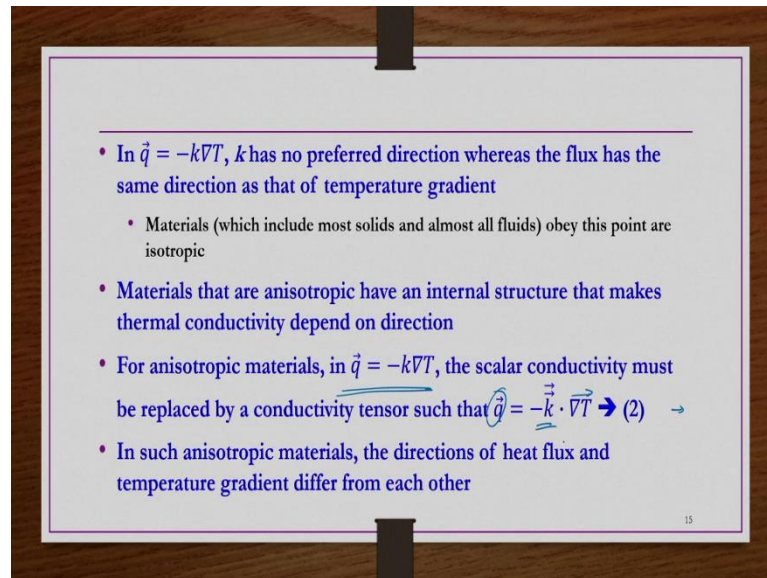
Constitutive equations of transport by molecular mechanisms. Heat conduction Fourier's law, if you remember it is heat flux that is energy per unit cross section area vector, it is a vector is proportional to temperature gradient as $\vec{q} = -k\nabla T$. ∇T is the temperature gradient, k is the thermal conductivity, q is the heat flux; it is vector and then temperature gradient that is also a vector.

That is also a vector individual temperature is not vector, but gradient of it is a vector ok. So, both of them are having moving in the same direction; in the same direction q and ∇T both will be having the same direction ok.

So, in a moving fluid heat flux vector represents energy transport relative to the mass average velocity. Otherwise if you have a kind of definition of this or you know heat flux related to the fixed coordinates then convective contribution will also come into the part. So, if you wanted to write an equation only for you know transport, because of the molecular mechanism then the flux has to be represented related to the mass average velocity only.

If you take the fixed coordinate relative to the fixed coordinates then convective contribution will also come into the picture.

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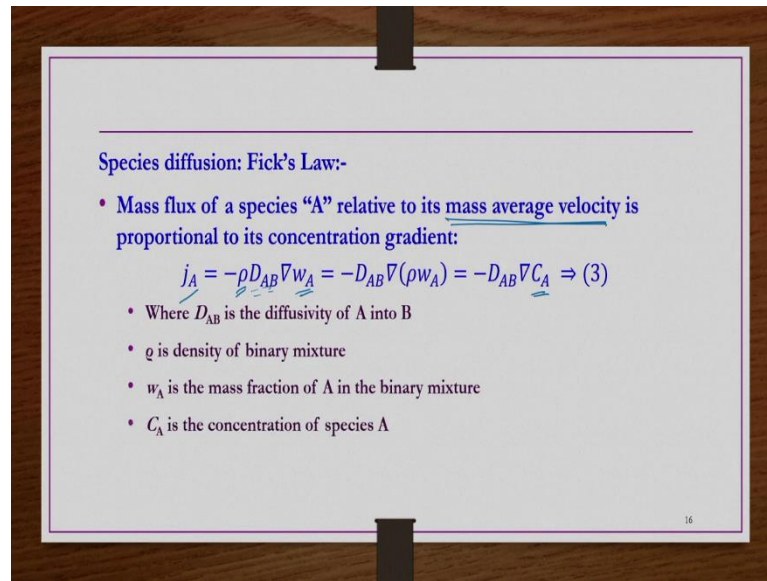


In this equation k has no preferred direction, whereas, the flux has the same direction as that of temperature gradient. So such kind of materials are known as the isotropic materials which include most of the solids and are almost all fluids. And materials that are anisotropic have an internal structure that makes thermal conductivity depend on the direction also.

So, that in this equation wherever this k is there that k is scalar k is replaced by the tensor $\underline{\underline{k}}$. So, dot product between tensor and then vector because this ∇T is nothing but a vector T is a scalar, but ∇T is a vector. So, dot product between tensor and then a vector will give a vector. So, then that is the reason q is still having the direction this one.

So, in this case the q and ∇T may not be having the same direction they may be differing from each other ok. So, such kind of equation is true for anisotropic material.

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Species diffusion: Fick's Law:-

- Mass flux of a species "A" relative to its mass average velocity is proportional to its concentration gradient:

$$j_A = -\rho D_{AB} \nabla w_A = -D_{AB} \nabla (\rho w_A) = -D_{AB} \nabla C_A \Rightarrow (3)$$

- Where D_{AB} is the diffusivity of A into B
- ρ is density of binary mixture
- w_A is the mass fraction of A in the binary mixture
- C_A is the concentration of species A

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Similarly, species diffusion Fick's law if you write; mass flux of species relative to its mass average velocity again, why mass average velocity? Because you are writing the equation only for the molecular transport mechanism only. If you take fixed coordinate related to the fixed coordinate then contribution because of the convection will also come into the picture. That is the reason mass flux of species here related to its mass average velocity is proportional to its concentration gradient.

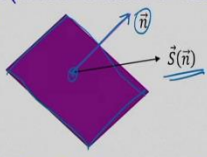
And then that is written here j is the; j_A is the mass flux of A ρ is the density of the mixture, it is not density of individual A or B, but it is a mixture density. D_{AB} is nothing but diffusivity of A and B and then w_A is nothing but the mass fraction of this component. The same a equation if you take the ρw_A then that we can be written as a C_A that is nothing but concentration of that component A ok. So, D_{AB} is the diffusivity of A into B ρ is density of binary mixture w_A is the mass fraction of A in the binary mixture C_A is the concentration of species A.

So, that is about the molecular transport of energy and then species concentration by molecular mechanism. Now, what about the molecular mechanism or transport of momentum by molecular mechanism that is what we are going to see. So, stress and then momentum flux.

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Stress and momentum flux

- Local rates of momentum transfer in a fluid are determined by stresses
- Consider a point within a fluid through which an imaginary surface having an orientation is described by unit normal $\vec{n}$
- Stress at that point (w.r.t. orientation of test surface) is given by stress vector $\vec{S}(\vec{n})$



The diagram shows a purple diamond-shaped surface element. A unit normal vector $\vec{n}$ points from the center of the diamond towards the upper right. A stress vector $\vec{S}(\vec{n})$ points from the same center towards the right, parallel to the surface. Below the diamond, there are blue arrows pointing left and right, representing the fluid on either side of the surface.

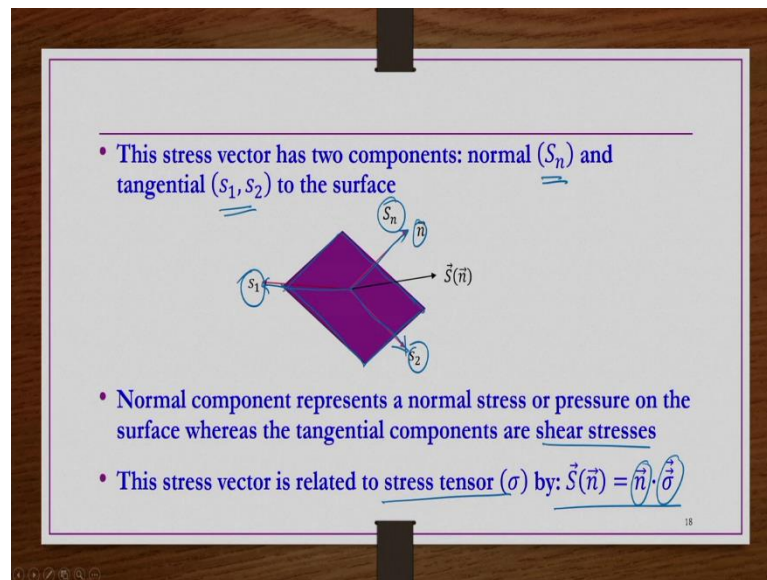
- This stress is defined as the force per unit area on the test surface exerted by the fluid toward which $\vec{n}$ points

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Local rates of momentum transfer in a fluid are determined by the stresses in general. So, consider a point within a fluid through which an imaginary surface having an orientation is described by a unit normal $\vec{n}$. Stress at that point with respect to the orientation of stress surface is given by stress vector $\vec{S}(\vec{n})$. So, let us say this is the surface you are taking and this surface you are representing by some normal vector that normal vector is $\vec{n}$.

So, at this point on the surface if you take. So, at this point whatever this stress vector is this, so that is nothing but $\vec{S}$ ok. So, this stress is defined as the force per unit area on the stress surface exerted by the fluid toward which $\vec{n}$ points ok.

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So, it has two components; normal component and then tangential component. There is only one single normal component, but there are two tangential component. So, pictorially if you see the same surface whatever we have taken in the previous slide which is you know described by this normal vector n .

So, then in the normal direction whatever the stress is there that we call S_n that is normal stress and then two tangential stresses whatever are there. So, in this direction S_2 and then in this direction S_1 , right. So, $\vec{S}(\vec{n})$ is nothing but normal stress S_1 S_2 are nothing but tangential stresses ok.

So, normal component represent a normal stress or pressure on the surface whereas, the tangential components are nothing but the shear stresses ok. This stress vector is related to stress tensor σ by this equation. $\vec{S}(\vec{n}) = \vec{n} \cdot \vec{\sigma}$ between a normal vector and then stress tensor, so that will be giving a vector stress vector S ok.

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• In Cartesian coordinates

$$\vec{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} -p + \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -p + \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -p + \tau_{zz} \end{bmatrix}$$

• $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$ where $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$

• For example, σ_{xy} is the force per unit area on a plane perpendicular to y axis; acting in x-direction and exerted by fluid at greater y

• σ_{ij} has three parts

- $j \rightarrow$ specify a reference plane
- $i \rightarrow$ direction in which it is acting
- and a sign convention

Handwritten notes in blue ink:
 $i=x, j=y$
 $\sigma_{xx} = -p + \tau_{xx}$
 $\sigma_{ij} = 0$
 $\sigma_{xy} = \tau_{xy}$
 $\sigma_{yx} = \tau_{yx}$

So this sigma in Cartesian coordinate if you write you have this these 9 components: σ_{xx} σ_{xy} σ_{xz} , σ_{yx} σ_{yy} σ_{yz} , σ_{zx} σ_{zy} σ_{zz} like this. So, this have two components you know one is the isotropic pressure component another one is the viscous component.

So, the same thing we can write $-p + \tau_{xx}$ in place of sigma xx and then in place of σ_{xy} simply we can write τ_{xy} . And then in place of σ_{xz} we can write simply τ_{xz} . Likewise $\tau_{yx} - p + \tau_{yy}$ τ_{yz} τ_{zx} $\tau_{zy} - p + \tau_{zz}$.

That is this is we can represent sigma we can represent as sigma the component whatever the 9 components are there general way we can represent as $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$. And then if $i = j$ then you know the δ_{ij} would be equals to 1. That means, when x is equals when $i = x$ and then $j = x$, then we have the $\sigma_{xx} = -p + \tau_{xx}$ as we have written here.

Let us say if $i = x$ and then $j = y$ then δ_{ij} is equals to 0 then σ_{xy} is nothing but τ_{xy} as we have written here. That is a generalized representation of each component σ_{ij} , right.

For example, σ_{xy} is the force per unit area on a plane perpendicular to y axis acting in x direction and exerted by fluid at greater y. That means, it has three parts j specify a reference plane and then i specify the direction in which it is acting and there would be a sign convention whether it positive or negative. Positive or negative that is depends direction, moving backward or forward that way we have these directions.

Further this if you know different way people represent some books you know they are written they are used in a different way maybe I specifying a reference plane and specifying the direction in which this stress is acting, ok.

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Stress may be interpreted as momentum flux in the case of fluids

- Part of stress caused solely by fluid motion is termed as viscous stress
- This viscous stress contribution to stress tensor is denoted by $\underline{\tau}$
- For Newtonian fluids with constant density, viscous stress is related to local velocity gradient: $\underline{\tau} = -\mu[\nabla\vec{v} + (\nabla\vec{v})^T] \Rightarrow (4)$
 - Where μ is viscosity of fluid
 - $\nabla\vec{v}$ is velocity gradient
 - $(\nabla\vec{v})^T$ is transpose of velocity gradient
- For a unidirectional flow with $v = v_x(y)$: $\tau_{yx} = -\mu \frac{dv_x}{dy} \Rightarrow (5)$

So, stress may be interpreted as momentum flux in the case of fluids. Parts of stress caused solely by fluid motion is termed as viscous stress and this viscous stress contribution to stress tensor is denoted by τ .

And for Newtonian fluid with constant viscosity; and Newtonian fluid with constant density and then viscosity this viscous stress is related to the local velocity gradient by this equation $\underline{\tau} = -\mu[\nabla\vec{v} + (\nabla\vec{v})^T]$. Where μ is the viscosity of the fluid, $\nabla\vec{v}$ is the velocity gradient, $(\nabla\vec{v})^T$ is the $(\nabla\vec{v})^T$ is nothing but transpose of the velocity gradient.

And then if you take unidirectional flow where only one component of velocity v_x is existing and then it is function of y then we will be having only one stress component $\tau_{yx} = -\mu \frac{dv_x}{dy}$. This is what in general we write for definition of Newtonian fluid basically, but complete definition is this one.

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By analogy

- $q = -k \frac{dT}{dy} = -\alpha \frac{d(\rho C_p T)}{dy} \Rightarrow (7)$
 - Where $\alpha = \frac{k}{\rho C_p}$
- $j_A = -\rho D_{AB} \nabla w_A = -D_{AB} \nabla (\rho w_A) = -D_{AB} \nabla C_A \Rightarrow (8)$
 - $= -D_{AB} \frac{dC_A}{dy}$
- $\tau_{yx} = -\mu \frac{dv_x}{dy} = -\vartheta \frac{d(\rho v_x)}{dy} \Rightarrow (9)$
 - Where $\vartheta = \frac{\mu}{\rho}$
- α, D_{AB} and ϑ are by analogy indicate thermal diffusivity, binary molecular diffusivity and kinematic viscosity respectively
- All having units of m^2/s

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So, now we see analogy of these molecular transport mechanisms for the case of momentum transfer, for the case of energy transfer, for the case of species concentration change ok.

So, let us say heat transfer we have this equation that same thing we are writing $-\alpha \frac{d(\rho C_p T)}{dy}$ ok; where $\alpha = \frac{k}{\rho C_p}$. Then mass transfer we have this equation this we can write it as $-D_{AB} \frac{dC_A}{dy}$. And τ_{yx} we are writing $-\vartheta \frac{d(\rho v_x)}{dy}$ where ϑ is nothing but $\frac{\mu}{\rho}$.

Now, in these three equations what we have the gradients $\frac{d}{dy}$ is there ok all three cases, and then in the case of heat transfer we have the property change in property or gradient of $\rho C_p T$. And then in the case of mass transfer the gradient or change in concentration with respect to variations in y direction that is what we have. Here in the momentum flux ρv_x we are having.

So, they are very similar right, $\frac{d}{dy}$ of whatever the properties changing in the energy in the mass transfer and in momentum transfer right. So, this whatever this α , this D_{AB} , and then this ϑ we are having they are similar. They are dependent on the material property because $\alpha = \frac{k}{\rho C_p}$ ρ k or depending on the material to material. And diffusivity of A may not be

same; diffusivity of A in B is not going to be same for the case of diffusivity of A in C.

So, that depends on material to material. Same is here viscosity and then density of a fluid are changing from one fluid to the other fluid. So, then this is going to be different. However, all of them are having you know same units of meters per second. So, that is the analogy that thermal diffusivity α , binary molecular diffusivity D_{AB} and then kinematic viscosity ν all of them are having units of meter square per second ok.

So, what is the important of writing this analogy? Now we understand, right.

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• Their ratios provide comparison of intrinsic rates of different transport processes if more than one transport is occurring simultaneously
 • Rate of viscous momentum transfer relative to heat conduction is Prandtl number:

$$Pr = \frac{\nu}{\alpha} = \frac{(\mu/\rho)}{(k/(\rho C_p))} = \frac{\mu C_p}{k}$$
 • Rate of viscous momentum transfer relative to species diffusion is Schmidt number:

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$
 • Ratio of diffusivities that arises in the analysis of simultaneous heat and mass transfer is Lewis number:

$$Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}} = \frac{Sc}{Pr}$$

So, that is the ratios will provide a comparison of intrinsic rates of different transport processes if more than one transport is occurring simultaneously. At the beginning of the lecture we said that transport phenomena more than one transport process may be occurring simultaneously. That is the reason you know if you wanted to solve the problem, which transport do you need to prioritize which you do not you cannot; you do not need to worry about or you really worried about all the transport processes to be occurring simultaneously really worthy to consider simultaneously.

All those analysis if you wanted to understand you can have the ratios of these you know thermal diffusivity, molecular diffusivity and then kinematic viscosity etcetera. So, then you will have a kind of a rational which is important, which is more, which is less, like

that which type of transport or transport of which property is more essential to be considered those kinds of things you can realize.

Let us say ratio of viscous momentum transfer related to the heat conduction is Prandtl number, where ν by α is nothing but Prandtl number that you can rate $\frac{\mu C_p}{k}$. Let us say in a process the momentum transfer is taking place and then heat transfer is also taking place.

So, in the process whether consider the interaction between these two processes important or not that you can realize from this number. If it is very large then you know then you can say that momentum transport is going to be leading the process. If it is very small then you can say the conduction heat transfer due to conduction is going to lead leading the process. That kind of understanding you can have.

Similarly, rate of viscous a momentum transfer relative to species diffusion is Schmidt number. So, where $\frac{\nu}{D_{AB}}$ is nothing but Schmidt number that we can write $\frac{\mu}{\rho D_{AB}}$, how much important is the role of viscosity and density compared to the role of diffusivity in a given transport process? That you can realize or you understand from this Schmidt number ok.

Or should you worry strongly about the diffusion part in the momentum transport part or not should, or should you worry about the role of diffusion mass diffusion or molecular diffusion in a process where both momentum and then must answer are occurring. That you can consider by taking this $\frac{\mu}{\rho D_{AB}}$ number ok.

If it is; if it is very large then you can say the role of diffusivity is very very small and then you can consider the transport phenomena or momentum transfer without worrying the effect of diffusion in the transport of momentum, like that you can analyze. Similarly, ratio of diffusivities that arises in the analysis of simultaneous heat and mass transfer is Lewis number that is $\frac{\alpha}{D_{AB}}$ ok.

So, that is the reason that we have written the analogy for transport of momentum energy and then species concentration by molecular mechanism. So, that we can have a kind of rational understanding which transport process is going to lead and then how much essential given a transport process if more than one transport phenomena is occurring simultaneously.

In the next class, we are going to discuss the derivation of equations of change for isothermal systems.

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The references for this lecture this one. The primarily the entire lecture is prepared from this reference book Analysis of Transport Phenomenon by Deen. But however, other books would also be equally useful especially this Panton and Bird's, Stewart, Lightfoot; these books will also provide adequate knowledge about the, whatever the things basics that we have discussed in this particular lecture.

Thank you.