Transport Phenomena of Non-Newtonian Fluids
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Lecture - 10
Rotational Viscometers - III

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids, the title of this lecture is Rotational Viscometers part III. In this week we have started discussing about rotational viscometers, what are the working principles and then what are the equations that we can make use in order to know the rheology of a an unknown fluid. Before going into todays lecture what we will be doing? We will be having a kind of recapitulation of what we have studied in last two lectures.

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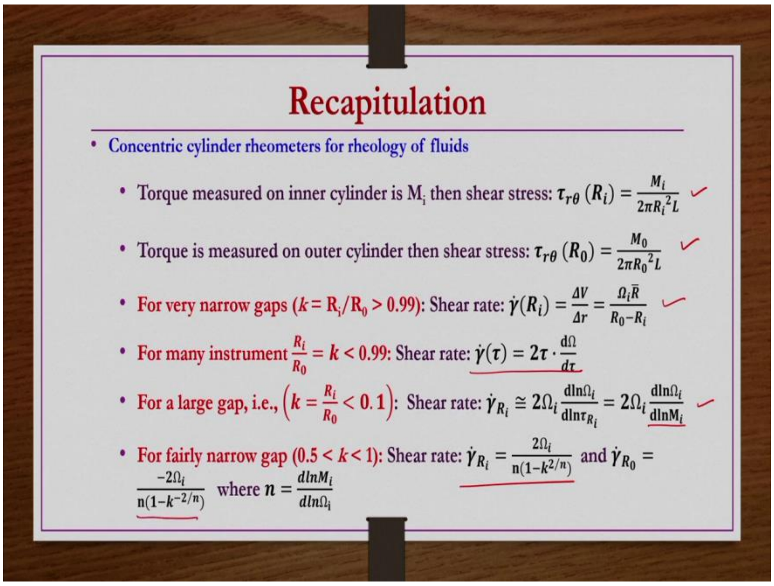

Under the category of rotational viscometers we started with concentric cylinder rheometers for measuring the rheology of fluids. And then let us say if we measure the, if the torque is measured on the inner cylinder is M_i then shear stress we can calculate using this expression that is $\frac{M_i}{2\pi R_i^2 L}$ whereas, the torque is measured on the outer cylinder then shear stress can be calculated using this equation $\frac{M_0}{2\pi R_0^2 L}$ that is what we have seen.

Remember the geometry that we started with to develop these equations, we started we have considered the inner cylinder rotating; but however, for the either of the cases or both the cases rotating how the equations to be interchanged how the equations are to be corrected and then appropriately use that also we have discussed. Coming to the shear rate it largely depends on the gap between the two cylinders.

So, then we started taking initial case where very narrow gap is there that is $\frac{R_i}{R_0}$ is greater than 0.99 that is they are both the cylinders are almost touching to each other. So, then under such conditions we assume that the curvature effect is negligible and then we got this shear rate expression.

But however, in reality always we may not have such very narrow gap rheometers, in general we may because of the nature of concentrated multi-phase emulsion suspensions nature of many of the non-Newtonian fluids you may not, you may not be able to take such narrow gap concentric rheometers.

So, then when the gap is less than 0.99 or when the k value that is $\frac{R_i}{R_0}$ is less than 0.99 then shear rate we obtain as this expression $\dot{\gamma}(\tau)$ is nothing but $2\tau\frac{d\Omega}{d\tau}$. Ω is nothing but the rotational velocity at which the inner cylinder is rotating.

For very large gap cases where that is when $\frac{R_i}{R_0}$ less than 0.1 then shear rate we obtained as this expression $\dot{\gamma}_{R_i}$ is nothing but 2 $\Omega_i\frac{dln\Omega_i}{dln\tau_{R_i}}$, which we can also write as a 2 $\Omega_i\frac{dln\Omega_i}{dlnM_i}$ because τ is directly proportional to M and then R h L etcetera are constant in the expression of the shear stress right.

So, but if you have a fairly narrow gap then we found the shear rate is given by this one for a inner cylinder and then outer cylinder shear rate is given by this one where, n is nothing but $\frac{dlnM_i}{dln\Omega_i}$ this we obtained by applying the Maclaurin series. After discussing about the concentric cylinder rheometers and its working principles then what we have done? We have taken a case where the geometry is, where the geometry is cone and plate geometry.

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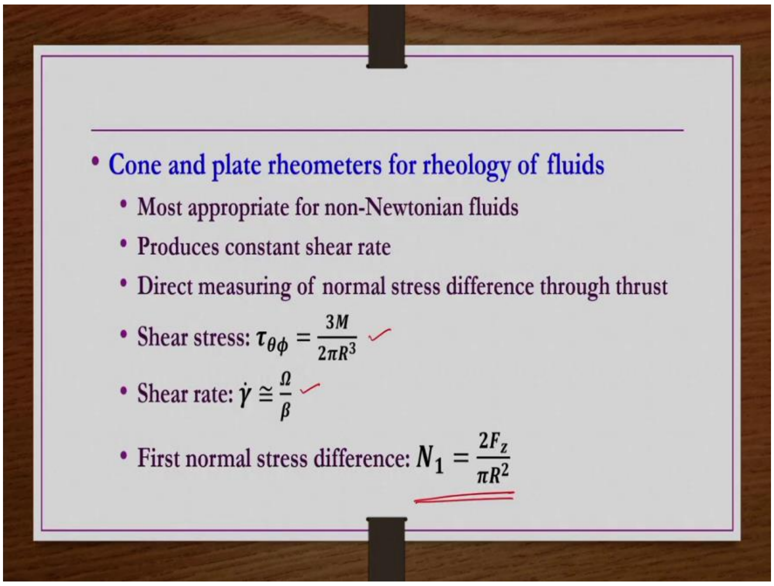

So, cone and plate rheometers for measuring the rheology of fluids is most appropriate especially when the fluids are expected to have non-Newtonian fluids and they produces constant shear rate because of the low cone angle in general and then it is also have a provision that we can directly measure the normal stress differences through the thrust.

Thrust directly we can measure and then from there we can measure the normal stress differences directly using this cone and plate rheometer. That is one of the important reason that for viscoelastic material or the material which are expected to display elastic behavior one prefers to go for cone and plate rheometers.

So, then under this category shear stress expression we got like this that is $\tau_{\theta\varphi} = \frac{3M}{2\pi R^3}$ whereas, the shear rate, $\dot{\gamma}$ we got it as $\frac{\Omega}{\beta}$; β is the cone angle it is true only when the cone angle is very very small and in most of the cone and plate rheometers operate with a low cone angle device.

And then first normal stress difference N_1 can be obtained from the total thrust F_z by using this equation $N_1 = \frac{2F_z}{\pi R^2}$. So, in the last two lectures this is what we have seen about the concentric cylinder rheometers and cone and plate rheometers. Now, in this lecture we will be discussing another type of rotational viscometer where the geometry is nothing but parallel disk right. So, the today's lecture would be on parallel disk rheometers.

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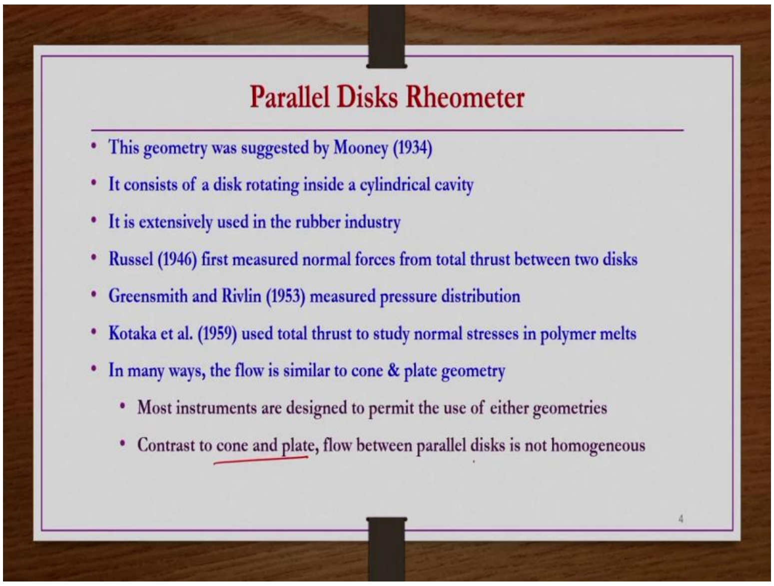

So, this geometry was also suggested by Mooney in 1934, it consist of a disk rotating inside a cylindrical cavity we will be seeing the schematic in next slide. Then it is extensively used in rubber industry. Russell have used the parallel disk rheometer in order to obtain the normal stress differences from total thrust between two disks.

Greensmith and Rivlin measured the pressure distribution using this parallel disk rheometer. Kotaka et al used total thrust to study the normal stresses in polymer melts. And then some, these are the some of the applications the some of the studies where you know this cone and this where this parallel disk rheometer has been extensively used right.

Further this geometry is many way similar to cone and plate geometry except that you know in the case of a cone and plate geometry we have a cone placed in a cylindrical in a plate. Now, we will be having you know in a cylindrical cavity two parallel disks could be there.

In many ways the flow is similar to cone and plate geometry especially most instruments are designed to permit the use of either of the geometries that is cone and plate and parallel disk rheometers both can be you know applied, you know are the instruments are designed such a way that the both the geometries can be utilized in one single device.

And then contrast to cone and plate rheometer, flow between parallel disk is not homogeneous ok. Now we will be discussing schematically, what is this parallel disk geometry that is used for measuring the rheology of the fluid. And then what are the constraints that we are going to be using in order to simplify the equation. So, that to get the equations, simplified equations for the shear stress shear rate and, or the normal stress differences, that is what we are going to see now.

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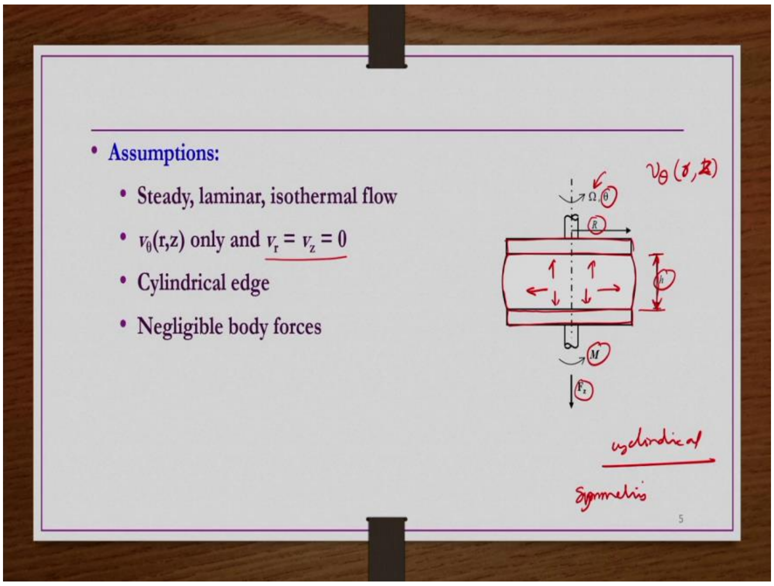

So, now we have a cylindrical cavity like this ok. Now here, two disk, parallel disk are placed such a way that the distance between these two disk is h as given here, the distance between these two disk parallel disk is h whereas, the radius of the disk is R right, the material whatever the material or the fluid for which you wanted to measure the rheology that is confined between these two disks.

So, then this material is being squeezed in the radial direction ok as we will as because of this height you know the variations velocity variations would also be there in the other direction also, that is possible right. The top disk is rotating in the θ direction right it is rotating at *Ω* velocity ok.

Now, since the rotation is in theta direction. So, a velocity component in θ direction would be predominant and then since it is being squeezed out in the radial direction so this it would be a function of r. And then because of the height in the distribution, velocity distribution is varying in the vertical direction also. So, then this v_θ would also be a function of z, that is v_θ is function of both r and z ok. So, the torque is M and then thrust is F_z is provided ok as shown here ok.

So now this is the schematic. So, then what we have? We need to have a proper assumption simplification so that we can simplify the momentum equations. Now, here the momentum equations we should take in cylindrical coordinates because of the cylindrical cavity that we have taken ok.

So, what are the assumptions? In general, we take steady laminar isothermal flow and then v_θ as explained here it is function of r and z. Since the rotation is there in θ direction so velocity component would be predominant in the θ direction, but compared to that velocity component, velocity components in other two directions would be very small.

So, v_r and v_z would be negligibly small compared to v_θ. So, what we can take? v_r is 0 and then v_z is also 0. Further we are taking cylindrical edge and then negligible body forces and then also we are taking symmetry in θ direction ok.

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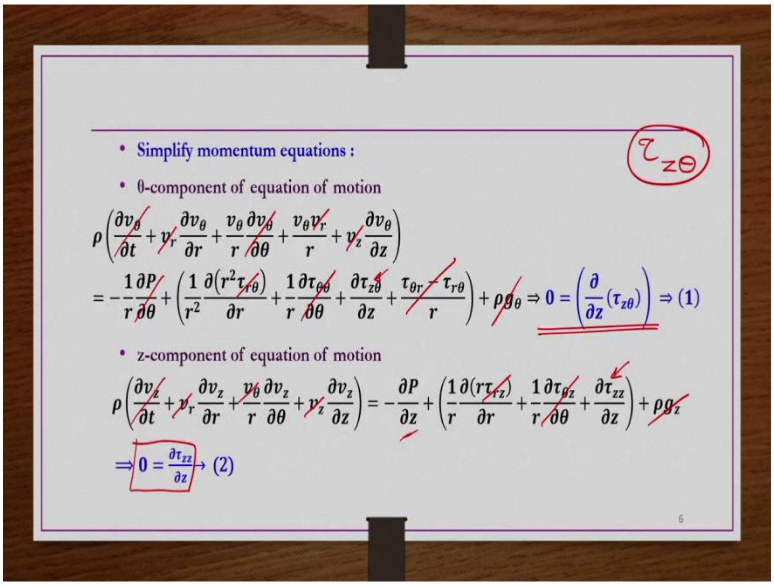

So now, what we do? We simplify momentum equations; we start with θ component of equation of motion right. So now, what we have to understand from the schematic that you know the rotation is in the θ direction and then variations are in z and r directions also, but h is having a significant role on the shear rate. So, then what we have? If we have the narrow gap parallel disk so then only shear stress that would be acting is $\tau_{z\theta}$, other component of shear stress would be negligible or very small ok.

And then as in previous two cases here also we cannot say normal stress components or extra stress component or 0 because there is no logic for cancelling out extra stress component for non-Newtonian fluids. We do not know nature of the fluids so then we cannot say that you know since it is a Newtonian fluid.

So, then $\tau_{rr}, \tau_{\theta\theta}$ etcetera are 0 or their difference is 0 like that we cannot say ok. So, then you know applying the assumptions or the constraint that we have enlisted in the previous slides we simplify momentum equations now ok. So, since we have taken steady state first term is 0 v_r is 0, v_θ is not 0, but because of symmetry this term is 0 v_r is 0, v_z is 0.

So, left hand side all the terms are 0 and then because of symmetry we can take this one also 0 and then only shear stress existing in this current geometry or the situation flow situations we have taken such a way that only $\tau_{z\theta}$ is existing. So, this is 0 and then because of symmetry this is 0 right.

So, this term we cannot say 0 it exists and then, but laminar flow at least for the laminar symmetric flow conditions you know these two quantities should be equal to each other. So, it is 0 and then we are not taking body forces in this geometry not required. So, then we have simplified θ component of equation of motion is nothing but $\frac{\partial}{\partial z}(\tau_{\theta z})$ or $\frac{\partial}{\partial z}(\tau_{z\theta}) = 0$ right.

So, this equation gives that the shear stress at least in the θ direction, you know at least in the z direction it is constant right, shear stress is constant in the z direction that is what we can understand from this equation ok. Then only $\frac{\partial}{\partial z}(\tau_{z\theta})$ is would become 0 ok. Now, z component of equation of motion; similarly, if you simplify then what we get, the steady state. So, this term is 0, v_r is 0, v_θ is 0, v_z is 0, pressure we do not know let us keep it as it is and then only $\tau_{\theta z}$ is existing. So, this is 0 because of symmetry this is 0 and then τ_{zz} extra stress component, extra stress component for non-Newtonian fluids it would be there so, but we cannot say whether it is function of z only it is function of r or it is function of θ as of now you know we cannot say, so then we cannot cancel out this term right.

So, further next you know body forces we are not considering. So, then last term is also cancelled out. So, in absence of pressure you know pressure the squeezing out of the fluid is taking place in the radial direction. So, then pressure variation in the z direction we can take it would be very small or negligible. So, then that can also be taken out. So then finally what we have? $\frac{\partial \tau_{zz}}{\partial z} = 0$; That means, extra stress component τ_{zz} is independent of z direction that is what we can say whereas, the shear stress is independent of z direction.

So, both in the z directions what we can understand? That shear stress and then extra stress component both of them are independent, they are not varying, and they are constant ok that is what we can say that is what we can say. So, it is very important for understanding because you know based on these assumptions only or based on these considerations only we can develop velocity profile ok.

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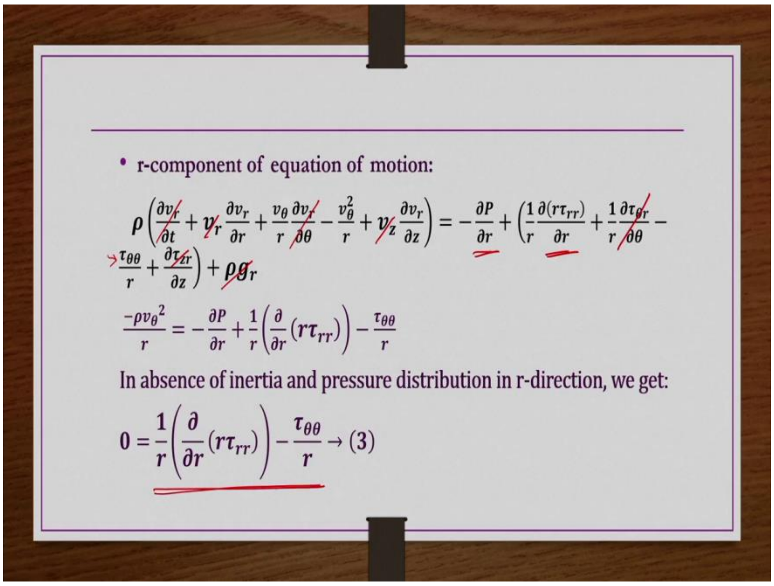

So, now r component of equation of motion similarly if you simplify what we have? Steady state. So, this term is 0 v_r is 0, v_θ is existing, but you know symmetry this term is 0, v_θ is existing v_z is not there pressure we do not know anything. So, this is the extra stress component so we cannot cancel out.

So, $\tau_{\theta r}$ is not existing as we well as because of the symmetry this entire term is 0, extra stress component we cannot say how much it would be without knowing the nature of the fluid. So, then it has to be there and then these τ_{zr} also not existing because only $\tau_{z\theta}$ is existing and there is no body force in this case.

So, then what we have? We have right hand side three terms and the left hand side one term. So, that is $-\rho \frac{v_\theta^2}{r} = -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau_{rr})}{\partial r} - \frac{\tau_{\theta\theta}}{r}$. So, now, in the absence of inertia and then we if you assume that the pressure distribution in the radial direction is constant then what we can have? We can have this equation because this equation will provide us information about the normal stress differences ok.

So now r θ z component of equations of motion we have simplified and then we got some information from those equations. What is the important thing that we realize from the z and θ components of equation of motions after simplifying? We realize that the shear stress and then extra stress component τ_{zz} are independent of z direction.

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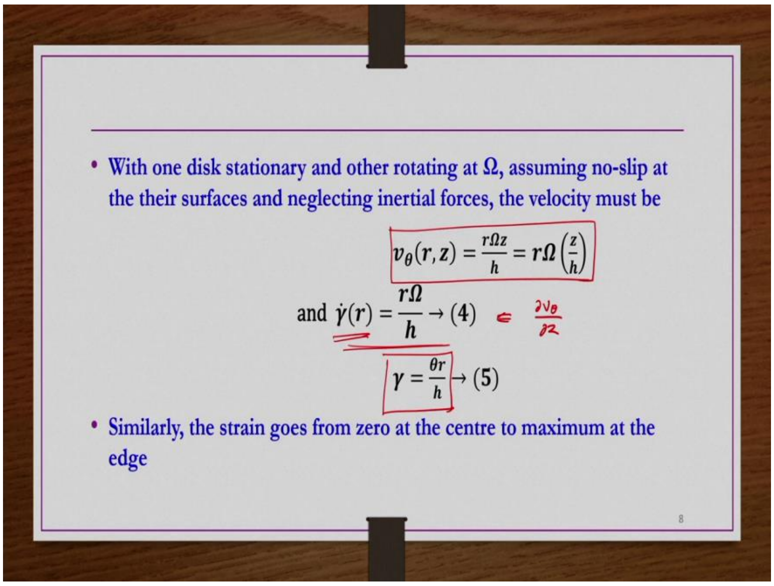

So, by making that assumptions you know we can develop a velocity profile. So, what do we say? With one disk stationary and other one rotating at *Ω*. So, bottom one let us fix stationary and then top one let it is assume it is rotating at *Ω* and assuming no slip at their surfaces and neglecting inertial forces the velocity must be this one; v_θ (r, z) should be nothing but $\frac{r\Omega z}{h}$. It should be a linear profile as long as this h is very small right ok.

So, this is the velocity profile we can have the best possible velocity profile this one of course, we can also derive it from the $\frac{\partial \tau_{\theta z}}{\partial z} = 0$ and expression that expression we can use and then simplify that expression for a given fluid and then that we can do. But that we can do only when we know the nature of the fluid ok so, but let us not go into those details.

Now, having the information from equation number 1 that is you know θ component of equation of motion simplification that is $\frac{\partial \tau_{\theta z}}{\partial z}$ or $\frac{\partial \tau_{z\theta}}{\partial z} = 0$, what we can understand that shear stress is independent of the z direction right.

From the second equation that is z component of equation of motion when we simplified we got $\frac{\partial \tau_{zz}}{\partial z} = 0$, that is extra stress component τ_{zz} is also independent of the z direction. So, then what we can say? We can by taking these two as a kind of basis and then what we can understand that the velocity profile is going to be the linear one as long as this h value is going to be small, is going to be small.

So, then what we take? Within now we take one disk stationary and other one rotating at omega, and then we assume the no slip boundary condition at the surface then we can after neglecting the inertial forces we can get v_θ as function of r and z which is nothing but $\frac{r\Omega \mathrm{z}}{h}$, this is what we can have ok.

So, once you have the velocity profile. So, then similarly what you can write? You can write the shear rate $\dot{\gamma}\ (r)$ which is nothing but $\frac{r\Omega}{h}$ because this shear rate we are measuring in the z direction between two cylinders. That is $\frac{\partial v_\theta}{\partial z}$ that is nothing but this $\dot{\gamma}$ and then it is function of r only, because we have already seen the shear stress is independent of z direction so; obviously, the shear rate would also be independent of the z direction ok.

So, then this is nothing but $\frac{\partial v_\theta}{\partial z}$ and then which is nothing but function of r ok. So, this equation from this equation once you have this one, similarly strain also we can get $\gamma = \frac{\theta r}{h}$ similar to shear rates we can have the strain also like $\frac{\theta r}{h}$. So, what we understand? Strain also goes from 0 at the center to maximum at the edge at r = 0 both strain as we well as the shear rate both of them are 0 and they are maximum at the edge.

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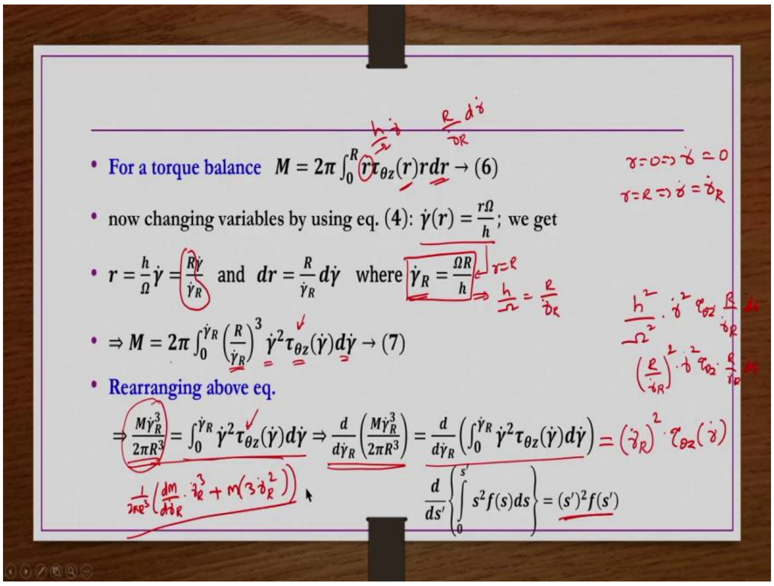

Now, we need to have information about the shear stress as well. So, that we can get by doing a torque balance. So, shear stress multiplied by the surface area on which it is acting that should be balanced by the force due to the torque right. So, then that if you write for differential quantities and then integrate then you get $M = 2\pi \int_0^R r\tau_{\theta z}(r) r dr$.

So, now, this equation though from the balance between torque and then shearing forces we have a relation for the shear stress, but we cannot use this equation right because though the torque is known for us from the experimental conditions, the right hand side integration what function of r is the shear stress that we do not know.

So, we cannot simplify this equation further. So, then what we do? We change the variables by using equation number 4; equation number 4 what we have $\dot{\gamma}(r) = \frac{r\Omega}{h}$. So, from here what we can have? $r = \frac{h}{\Omega\dot{\gamma}}$ right. So now, the from this equation if you substitute r = R then whatever the $\dot{\gamma}$ is there, let us say if we call it $\dot{\gamma}_R = \frac{\Omega R}{h}$ right.

So now, in place of $\frac{h}{\Omega}$ you can write, from this equation in place of $\frac{h}{\Omega}$ you can write $\frac{R}{\dot{\gamma}_R}$. So, that I am writing here in this equation here. So, $\dot{\gamma}\frac{R}{\dot{\gamma}_R}$ and then dr is nothing but $\frac{R}{\dot{\gamma}_R}$ $d\dot{\gamma}$.

So now, in place of r I will be writing $\frac{h}{\Omega\dot{\gamma}}$ and then in place of dr I will be writing $\frac{R}{\dot{\gamma}_R}$ d $\dot{\gamma}$. Remember $\dot{\gamma}_R$ is nothing but the shear rate at the edge; at the edge in the sense at r = R ok, that this at the edge of the disk ok.

So now when you do this one, then you have $M = 2\pi$ integral in place of R you know, so we have r multiplied by r. So, then we have $\frac{h^2}{\Omega^2}$. $\dot{\gamma}^2$ and then $\tau_{\theta z}$ as it is, then dr is nothing but $\frac{R}{\dot{\gamma}_R}$ d$\dot{\gamma}$ that is what we have ok.

So now, in place of $\frac{h}{\Omega}$ what I can write? I can write $\left(\frac{R}{\dot{\gamma}_R}\right)^2$ I can write then this $\dot{\gamma}^2$is as it is, $\tau_{\theta z}$ is as it is, $\frac{R}{\dot{\gamma}_R}$ we are having d$\dot{\gamma}$. So, then $\left(\frac{R}{\dot{\gamma}_R}\right)^3$ $\dot{\gamma}^2$ $\tau_{\theta z}$ right. Now which is nothing but function of $\dot{\gamma}$ and then d$\dot{\gamma}$ that is what we are having.

So, the limits also we have seen from equation number 4, when r = 0 $\dot{\gamma}$ function of r is nothing but 0, and then at r = R we have taken $\dot{\gamma}$ is nothing but some maximum value of $\dot{\gamma}$ that is $\dot{\gamma}_R$ we are writing. So, then this equation we are having right.

So, now this equation we rearrange such a way that this $2\pi R^3$ we get it to the left hand side and then $\dot{\gamma}_R^3$ also we take to the left hand side. So then left hand side what we have? $\frac{M\dot{\gamma}_R^3}{2\pi R^3}$ that is what we have in the left hand side. In the right hand side what we will be having? $\int \dot{\gamma}^2\tau_{\theta z}d\dot{\gamma}$.

So, now this $\tau_{\theta z}$ we have written in as a function of a $\dot{\gamma}$ rather as a function of r because we do not know what it is as a function of r. In fact, what it is as function of $\dot{\gamma}$ also we do not know unless we know the fluid, but let us not worry about that one. So, this is equation number 1 when you rearrange so we have this equation. Now this equation what we will be doing?

We will be doing differentiation both sides using you know $\dot{\gamma}_R$ ok. So, then left hand side you know this is as it is so we are not doing anything. Right hand side whatever this term is there so that is having you know in the Leibniz form, in the Leibniz form it is there. So, then this should be having this particular solution.

So, right hand side term you will be getting answer as $\dot{\gamma}_R^2\ \tau_{\theta z}\ (\dot{\gamma})$, that is what you have ok for the right hand side. You will be having left hand side $\frac{1}{2\pi R^3}$ common. And then what you will be having? $\frac{dM}{d\dot{\gamma}_R}.\dot{\gamma}_R^3 + M(3\dot{\gamma}_R^2)$ this is what you have left hand side.

So, further what you do? Now, whatever this left hand side term is there so, that we are equating to this right hand side term. So, and then simplifying.

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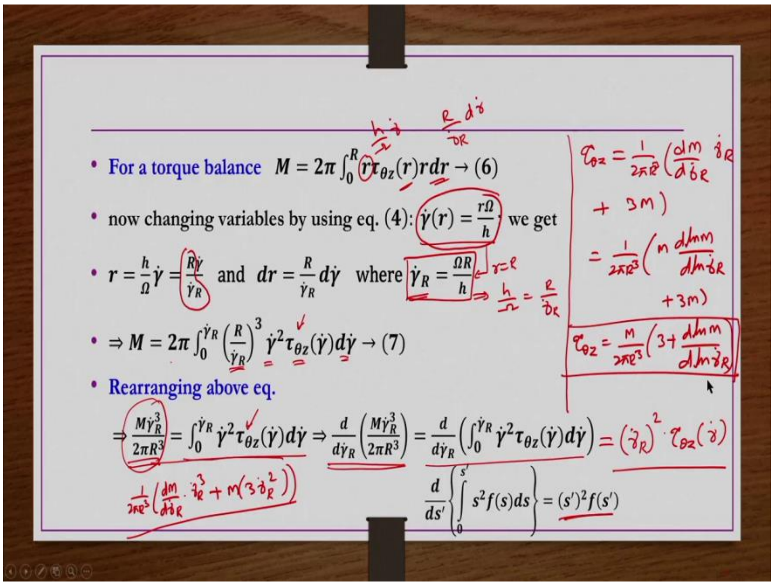

So, in the right hand side I am keeping the $\tau_{\theta z}$ as it is one side and then whatever the $\dot{\gamma}_R^2$ is there that I am taking to the other side. So, then I can have $\frac{1}{2\pi R^3}$ then $\frac{dM}{d\dot{\gamma}_R}\ \dot{\gamma}_R$ only will be there + 3M would be there. So, next step what we can do? $\frac{1}{2\pi R^3}$. Here what I am doing? I am multiplying and dividing by M here so then I can write $\frac{dln\ M}{dln\dot{\gamma}_R}$, that is for only first term and then second term as it is.

So then, $\tau_{\theta z} = \frac{M}{2\pi R^3}\left(3 + \frac{dln\ M}{dln\dot{\gamma}_R}\right)$ this is what we get. This is what we get expression for the shear stress previous slide equation number 4 we got shear rate expression.

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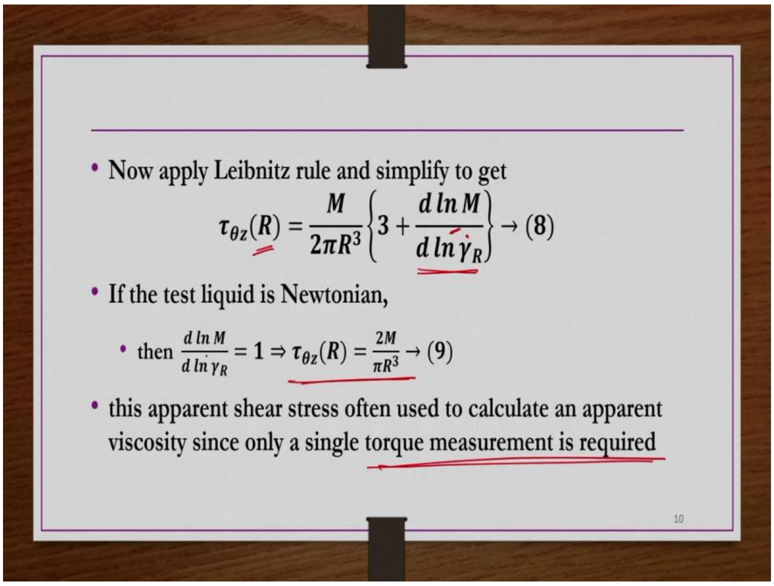

Now, shear stress expression we got this one the same equation. So, after simplifying the same equation is written here as equation number 8. So, $\tau_{\theta z}$ function of $\dot{\gamma}$ it should be function of $\dot{\gamma}_R$ and then that is obtained as $\frac{M}{2\pi R^3}\left\{3 + \frac{dln\ M}{dln\ \dot{\gamma}_R}\right\}$ ok.

So obviously, if the test liquid is Newtonian liquid. So, then what we will be getting? This slope $\frac{dln\ M}{dln\dot{\gamma}_R}$ you will be getting value 1. So, then we get 3 + 1 = 4, $\frac{4M}{2\pi R^3}$ that is nothing but $\frac{2M}{\pi R^3}$ if it is a Newtonian fluid. If it is Newtonian fluid then only, otherwise we have to use this equation number 8.

This apparent shear stress often used to calculate an apparent viscosity, since only a single torque measurement is required, that is the reason this is mostly used in a many of the rubber industries in situ kind of thing.

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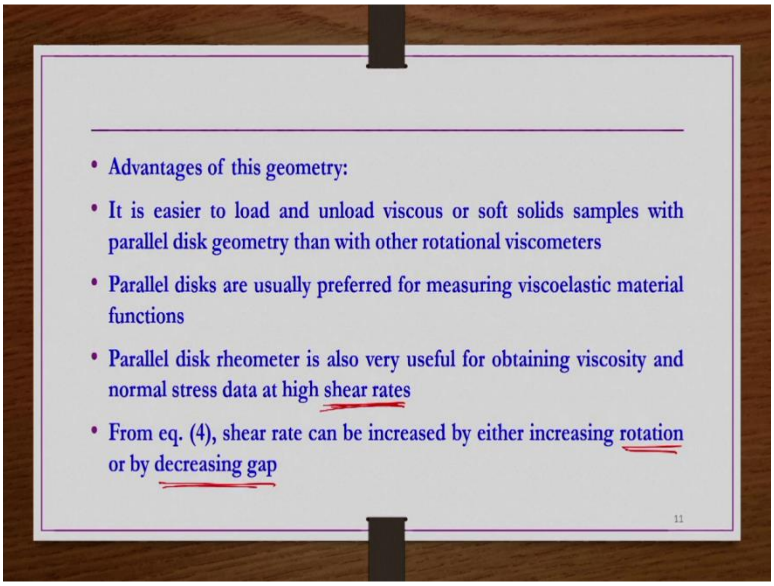

Some of the advantage of this parallel disk rheometer are provided here. It is easier to load and unload viscous or soft solid samples with parallel disk geometry than other two rotational viscometers that we have studied. And then parallel disks are usually preferred for measuring viscoelastic material functions, because directly from the thrust we can get the normal stress differences.

This is also very useful for obtaining viscosity and a normal stress data at high shear rates also and then not only low shear rates, but high shear rates also we can get because high shear rate that equation whatever the equation number 4 according to that equation number 4 you can get by increasing the rotational speed or decreasing the gap or by decreasing the gap we can get the high shear rate. So, then even at high shear rate also we can get the viscosity and then and normal stress differences ok.

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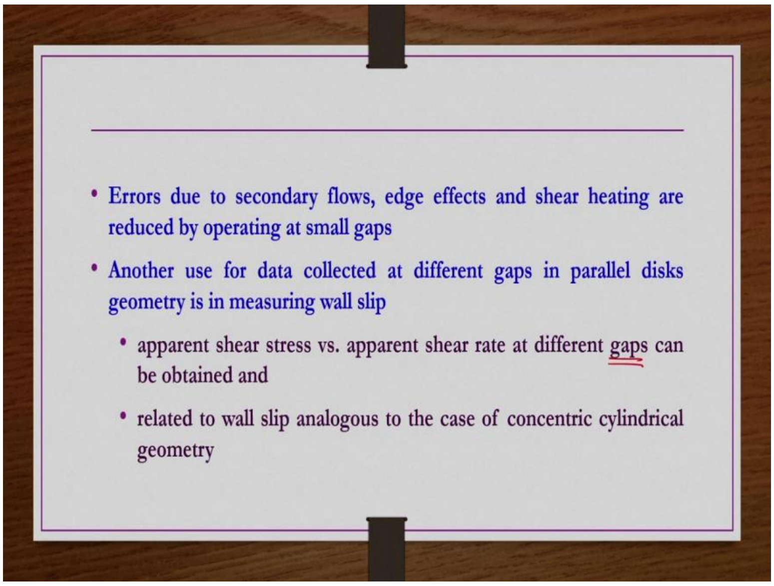

So, in general errors due to the secondary flows edge effects and then shear heating etcetera are reduced by operating at small gaps that is small h values. And then another use for data collected at different gaps in parallel disk geometry is in, is useful in measuring the wall slip.

How it is? Apparent shear stress versus apparent shear rate at different gaps one can obtain, and then related to the wall slip analogous to the case of concentric cylindrical geometry right. So, now we see equation number 3 and then we try to get normal stress differences from that say equation number 3.

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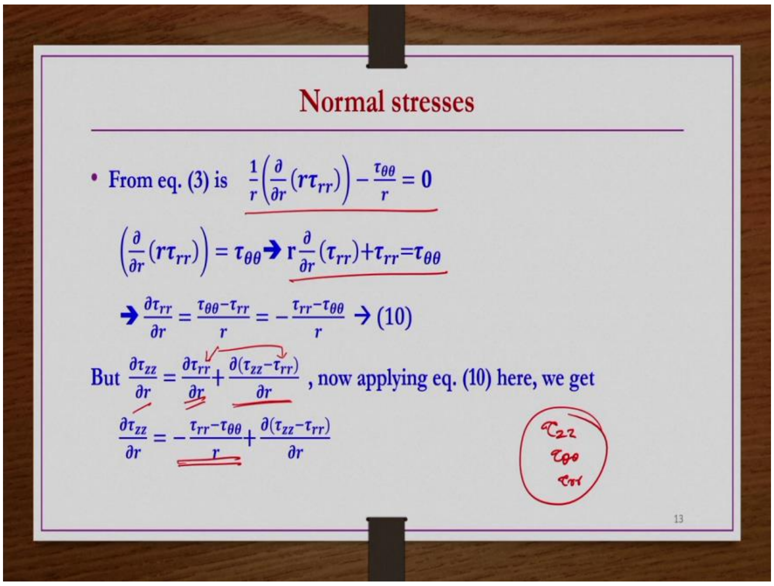

So, equation number 3 after neglecting the inertial forces and then assuming the pressure is independent in the r direction or pressure drop is constant in the r direction then that equation number 3 is this one ok. Now, that r we can cancel out and then we can rearrange this equation like this. So, then $\frac{\partial \tau_{rr}}{\partial r}$ we can write it as $\frac{-\tau_{rr} - \tau_{\theta\theta}}{r}$.

But $\frac{\partial \tau_{zz}}{\partial r}$ we can write it as $\frac{\partial \tau_{rr}}{\partial r} + \frac{\partial(\tau_{zz} - \tau_{rr})}{\partial r}$, because this we can write it as $\frac{\partial \tau_{zz}}{\partial r} - \frac{\partial \tau_{rr}}{\partial r}$. So, then that $+\frac{\partial \tau_{rr}}{\partial r}$ and then this $-\frac{\partial \tau_{rr}}{\partial r}$ would be cancel out so then only $\frac{\partial \tau_{zz}}{\partial r}$ will be remaining. This is just a mathematical playing around with equation to get the required things right.

So, now, in place of $\frac{\partial \tau_{rr}}{\partial r}$ what we write here? From equation number 10 we write $\frac{-\tau_{rr} - \tau_{\theta\theta}}{r}$ here so that this term is like this here ok. Now, this equation we cannot further integrate or simplify. So, that to get this $\tau_{zz} - \tau_{rr}$ or $\tau_{rr} - \tau_{\theta\theta}$ information they are nothing but the normal stress differences.

So, we can get only what function are, what function are these τ_{zz} or $\tau_{\theta\theta}$ or τ_{rr} right. So, how are these; how are these τ_{zz} $\tau_{\theta\theta}$ τ_{rr} depending on radial coordinate r we do not know, when we know then only we can simplify this equation ok. And they are independent of the z direction because that we have already found from equation number 1 and 2 ok. So, then what we do? We do you know some kind of mathematical rearrangement.

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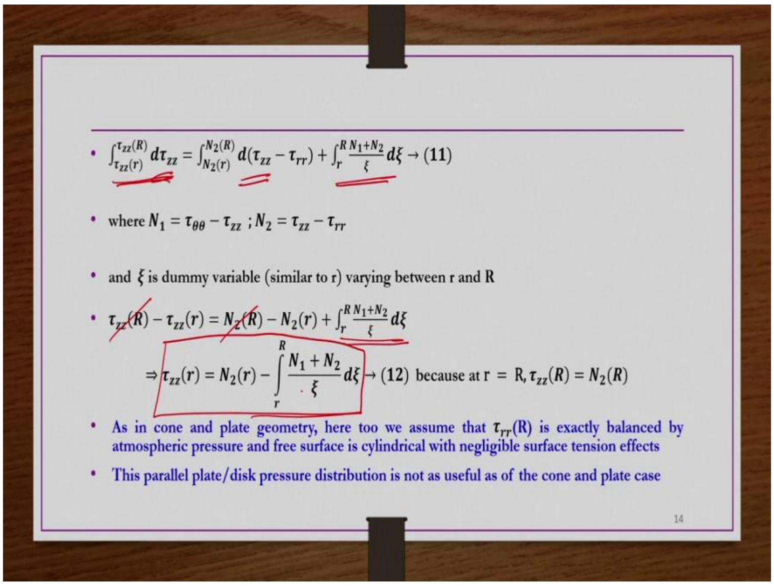

So, here now the same equations we are writing in the form of integral form like this. So, left hand side whatever $\frac{\partial \tau_{zz}}{\partial r}$ was there. So, the dr we have taken to the right hand side and then we are integrating it. So, then we have this equation ok. So, this equation is simple this equation is also simple, but this part third part whatever is there that we have written in terms of dummy variables ξ ok which is similar to r.

So, here N_1 is nothing but $\tau_{\theta\theta} - \tau_{zz}$ and then N_2 is nothing but $\tau_{zz} - \tau_{rr}$. So, when you integrate the first part then what you get ξ is dummy variable similar to r varying between r to R. So, now, when you integrate this term, you get $\tau_{zz}(R) - \tau_{zz}(r) = N_2(R) - N_2(r) +$ this one we are not integrating because we do not know what is that N_1 and N_2 function of zeta those things we do not know.

So but $N_2(R)$ and zz at R they are same. So, then this equation we can write $\tau_{zz}(r)$ $= N_2(r) -$ this integration ok. So now, this equation you know will provide us some information about normal stress differences right. So, whatever this N_2.2πr dr and then this integral whole integral $\int$ 2πr dr and then integration of this entire thing with a negative sign would be giving you the thrust.

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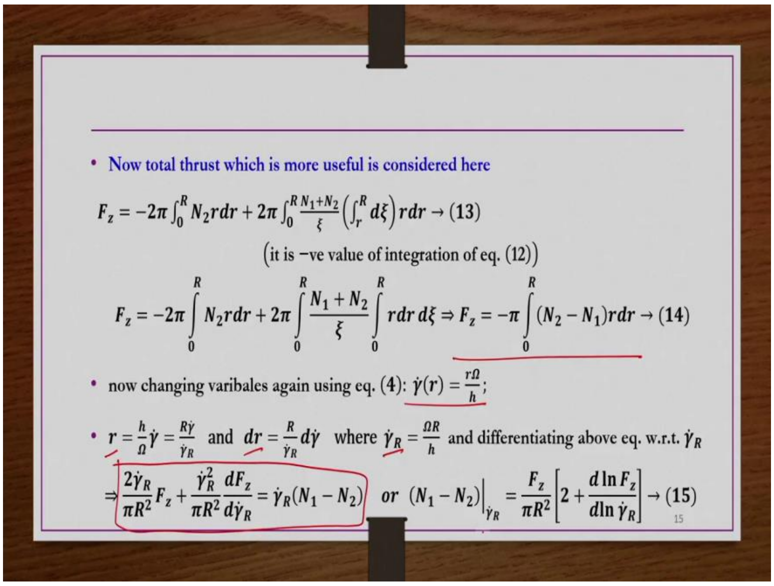

So, that equation is written here. So, $F_z = -2\pi \int_0^R N_2 r dr + 2\pi \int_0^R \frac{N_1+N_2}{\xi}\left(\int_r^R d\xi\right) r dr$ this is what we have. So as mentioned it is negative value of integration of equation of, equation number 12.

So, then here again we can do the simplifications then we get F_z is this one. Again we cannot simplify these equations you know without knowing this N_2 and N_1 information, N_2 how it dependent on r, N_1 how it dependent on the r value that we do not know. So, then straightforward we cannot simplify this equation. So, we are in the same position as in the case of shear stress measurements. So, then what we have done there? We have used the shear rate expression to change the variables.

So here also we will be doing the changing the variables using equation number 4, then we have this r is equals to this one, dr is this one and then $\dot{\gamma}_R$ is this one and then differentiating above equation with respect to $\dot{\gamma}_R$. And then doing the all simplification that we have done for the case of shear stress by applying the Leibniz formula and all that.

Then finally, what we get? You get this expression. This you can try yourself we can get without any difficulty or $N_1 - N_2$ at constant $\dot{\gamma}_R$ is nothing but $\frac{F_z}{\pi R^2}\left[2 + \frac{dlnF_z}{dln\dot{\gamma}_R}\right]$ ok. So that is how we can get the shear stress shear rate and then normal stress differences using parallel disk rheometers. Now, we take an example problem to understand how to use these equations.

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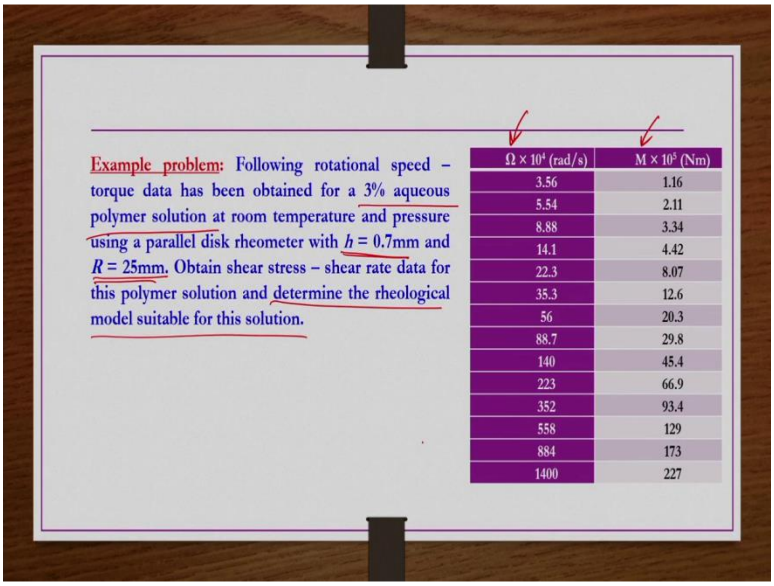

So, the rotational speed and torque data for an aqueous polymer solution is given here at a certain temperature and pressure. Height is the gap between this parallel disks is 0.7 mm that is very small and then radius of the disk is nothing but 25 mm. The question is obtain shear stress versus shear rate data for this polymer solution and determine the rheological models suitable for this solution.

So, some 3 percent aqueous polymer solution for that you know that solution has been confined between two parallel disk and then one of the disk was rotating. And then so called the rotational velocity and then torque have been measured right. So, that data has been provided here.

So, the gap between two disks is 0.7 mm and then the radius of the disk is 25 mm right. So, all the information is given actually in order to get the shear stress shear rate information. We were not asked to get the normal stress differences, if we are about to if we are supposed to get the normal stress differences also then we should also been given some information about the total thrust. So, that is not required now.

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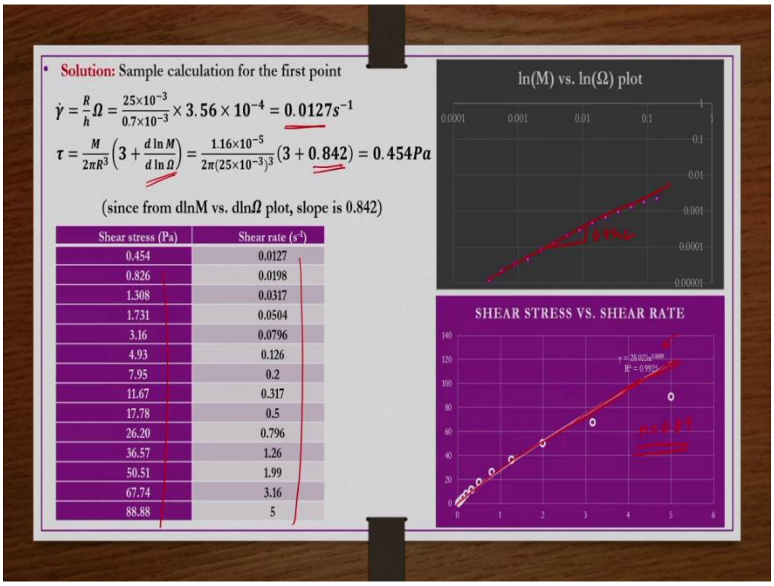

So, now sample calculation for the first data point $\dot{\gamma} = \frac{R}{h}\Omega$. Because now all these τ and then shear rate etcetera we are calculating at the edge, that is it r = R ok. So, R is 25 mm h is 0.7 mm and then *Ω* first data point 3.56 x 10^{-4} then when you simplify these numbers we will get $\dot{\gamma}$ 0.0127 s^{-1}.

Similarly, tau we got this expression $\tau = \frac{M}{2\pi R^3}\left(3 + \frac{d\ln M}{d\ln\Omega}\right)$. So, first data point M is given as 1.16 x 10^{-5}Nm and then 2 π R is nothing but the 25 mm ok, multiplied by 3 this M versus *Ω* data is given.

So, that data you have to plot on a log-log graph sheet and then get the slope right. So, that slope is nothing but 0.842 right. So, then you get first data point shear stresses 0.454 Pascal. So, whatever the torque versus rotational velocity was there so that we have converted in terms of you know shear stress and then shear rate ok.

So that we tabulated here right. So, actually ln M versus ln *Ω* plot we have here. So, then slope of this one is 0.842. So, likewise for all the data points of M versus *Ω* corresponding shear stress versus shear rate values are calculated and tabulated here ok. So, now, if you plot them what you can understand? This data is passing through the origin and then it is slightly non-linear it is not completely linear and then it is not very highly non-linear. So, what we can see? It obeys a power law behavior of n = 0.89, that is very mild shear thinning behavior is there ok.

So, this is about how to generate, how to develop the equations for the shear stress shear rate and then normal stress differences using parallel disk rheometers and then how to solve problem that is what we have seen now.

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The references for this lecture; the entire lecture is prepared from this reference book Rheology: Principles, Measurements, and Applications by Macosko; however, some data and then example problem have also been taken from this book by Chhabra and Richardson. Other useful reference books are provided here.

Thank you.