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Lecture – 09 Mass transfer coefficient concept and classifications

Welcome to the second module of the course mass transfer operation. In this module we will discuss Mass Transfer Coefficients, before going to the today's first lecture let us have small recap on our previous lecture. In our previous know module particularly we have discussed diffusion mass transfer.

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Recap Diffusion Mans Transfer
Concept of Molecular Diffusion
Convective Diffusion
Diffusion Measurements
- Gas
- Liquid
- Solid

Under which we have a discussed concept of molecular diffusion and convective diffusion. We have also considered diffusion measurements, gas then liquid and solid, we have also discussed its prediction.

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So, in this lecture, we will consider concept of mass transfer coefficient. In the first lecture we will mostly concentrate on the what is mass transfer coefficient, type of mass transfer coefficient, diffusion of A through non diffusing B and in that case what should be the mass transfer coefficient. Then we will discuss equimolar counter diffusion of A and B and we find out the different mass transfer coefficient related to the system and finally, we will discuss the relation between the gas phase mass transfer coefficients K G and K Y.

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The mass transfer coefficient is defined as the rate of mass transfer as we know is proportional to the concentration driving force that is the difference in concentration and also the rate of mass transfer is proportional to the area of contact between the phases.

So, if we consider W A is the rate of mass transfer in terms of kilomole per second of particular solute A and if the concentration driving force between the two points is delta C A, that is C A 1 and C A 2, C A 1 minus C A 2 delta C A is the concentration driving force and the area of mass transfer is a in that case we can write W A is proportional to a into delta C A concentration driving force. So, from this we can write W A would be equal to k c a into delta C A. This k c over here is basically the proportionality constant and it is called the mass transfer coefficient.

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So, this is the mass transfer coefficient. Now, if we considered N A is the molar flux which is expressed in kilomole per metre square second we may write W A would be equal to a into N A because W A is the molar flow rate that is kilomole per second. So, if we multiplied by the area a then the unit would be molar flow rate that is k mole per second. So, then if you substitute N A, it would be k c a delta C A and from this we can write mass transfer coefficient. So, mass transfer coefficient k c would be equal to from this relation we can write k c would be equal to N A by delta C A; that means, the molar flux divided by the concentration driving force.

For the purpose of comparison, we may recall the definition of heat transfer coefficient. We know for heat transfer coefficient h, which is the heat flux divided by the temperature driving force. So, it is in the similar fashion we can write the mass transfer coefficient is the molar flux divided by the concentration driving force.

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The inverse of mass transfer coefficient is a measure of the mass transfer resistance. So, 1 by mass transfer coefficient would be the resistance for mass transfer. If the driving force is expressed as the difference in concentration that is kilomole per metre cube say, then we can write the unit of mass transfer coefficient would be meter per second. So, this is a unit of velocity. So, we can also write in terms of centimetre per second or in feet per second with no different length in it which is the same of the unit of the velocity.

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Now, if the mass transfer coefficient is expressed as the ratio of the local flux and the local driving force, then it is called the local mass transfer coefficient. So, the local mass transfer coefficient can be written as local flux divided by the local driving force. Now, when it is expressed as the ratio of the average flux that is over a surface if the flux changes and if we take the average then the average flux divided by the average driving force then it should be named as average mass transfer coefficient. So, average mass transfer coefficient would be equal to average flux divided by the average driving force.

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Convective mass transfer can occur in a gas or in a liquid medium, but it does not occur in the solid medium. So, in these two cases a few choices of the driving force can be written, one is the difference in concentration. So, in case of concentration driving force it diffuses from the high concentration to the low concentration. So, the solute moves from higher concentration to the lower concentration, we can also write the driving force in terms of the partial pressure difference of a particular component. So, partial pressure is another driving force. We can also write in terms of the molar fraction.

So, mole fraction can be written as the driving force. But in case of the heat transfer the temperature difference is the only driving force.

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So, different type of mass transfer coefficient have been defined depending upon the driving force and also it depends on whether the mass transfer occur in the gas phase or in the liquid phase. And second is choice of driving force, as we said before it is also depends on the driving force and the third one is whether it is a case of diffusing of component A through non diffusing B or whether it is a equimolar counter current diffusion or the counter diffusion. If the transport of mass occur through a stagnant film of thickness delta, then we can write down the flux as mass transfer coefficient into driving force.

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So, the mass transfer in the gas phase in that case we can write N A would be equal to k G p a 1 minus p a 2 which would be equal to k y, y A 1 minus y A 2 is equal to k c, C A 1 minus C A 2. So, this is termed as equation 1. In this case the k G, k y and k c these are the gas phase mass transfer coefficient; k G that is when we define the driving force in terms of the partial pressure difference, when we define the driving force as a mole fraction unit then it is k y and when we define the concentration unit we define the mass transfer coefficient as k c.

So, three different mass transfer coefficient is defined over here in case of gas phase depending on the driving force we use. The unit of mass transfer coefficient k y we can easily calculate from these flux equation and which is kilomole per metre square second divided by the driving force. Here delta y stands for the driving force in mole fraction unit. In case of the liquid we can write N A is the flux is equal to $k \times n$ into $x \times A$ 1 minus $x \times A$ 2 is equal to k L C A 1 minus C A 2. So, this is equation 2 for the mass transfer in the liquid phase.

So, the mass transfer coefficient k x and k L these two are the mass transfer coefficient in the liquid phase and the subscript for both the equations, subscript 1 and 2 refer to the two position in the medium it is considered.

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Now, if we consider the gas phase is ideal in that case, the concentration term in equation one which is this the concentration term C A we can write as p A by R T, the partial pressure of component a divided by R T. So, in that case p A is the partial pressure of component A. Now, suppose that the distance between the two locations A and 1 and 2 is delta that is the film thickness.

The expression of mass transfer coefficient can be obtained by comparing equation 1 with this equation, this we have already defined in case of diffusion N A would be equal to D AB p t, p t is the total pressure by R T x 2 minus x 1 P BLM into p A 1 minus p A 2 would be equal to D AB P t divided by R T l p BLM, here l is the thickness over here x 2 minus x 1. Later we will define as delta into p BLM into partial pressure difference where, the p BLM is the logarithmic mean partial pressure of species B. We can define it p B 2 minus p B 1 divided by ln p B 2 by p B 1.

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So, the expression of mass transfer coefficient can be obtained by comparing equation 2 with this equation; equation 2 in the liquid phase with the other equation in case of the diffusion that is N A would be equal to D AB rho by M average divided by l x BLM into x A 1 minus X A 2. In this case x BLM it would be logarithmic mean of mole fraction of component B not partial pressure mole fraction of species B which is x B 2 minus x B 1 divided by ln x B 2 by x B 1.

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In case of gas phase we can define k G is equal to D AB P t by R T delta p BLM. Similarly, for k y in terms of the mole fraction unit of the component gas phase mole fraction unit that is equal to D AB P t square divided by R T delta p BLM and k c would be equal to D AB P t by delta p BLM. So, this is equation 3. The relation among the three types of gas phase mass transfer coefficient that is k G k y and k c among these 3 can easily be obtained from these 3 relations. In case of liquid phase, we can write k x would be equal to D AB rho by M average divided by delta x BLM and k L would be D AB by delta x BLM.

So, this is equation 4 and here we have considered the l the thickness between the two points where the diffusion is occurring as delta the diffusion length. The relation between the two types of liquid phase mass transfer coefficient similarly to the gas phase we can obtained from k x and k L relations in equation 4 among these two. So, k c would be equal to R T k G, k y would be equal to P t k G and k x would be rho by M average k L. So, this is the final relations we can obtained among these mass transfer coefficient.

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Now, the set of notations for mass transfer coefficient used in case of equimolar counter diffusion of a and b. So, if we considered equimolar counter diffusion here we have defined the mass transfer coefficient with a sign of prime. So, like in gas phase, we can write N A would be equal to k G dash into the partial pressure driving force p A 1 minus p A 2 which is equal to k y dash into the mole fractions driving force that is y A 1 minus

y A 2 is equal to k c dash into the concentration driving force C A 1 minus C A 2 this is equation 6. In case of liquid phase, similarly we can write $N A$ would be equal to k x dash x A 1 minus x A 2 is equal to k L dash C A 1 minus C A 2.

Now if we consider equation 6, for the gas phase transport with this relations N A equal to D AB P t by R T l y A 1 minus y A 2 is equal to D AB by R T l p A 1 minus p A 2, then we can also compare equation 7 for the liquid phase transport with this relation N A would be equal to D AB rho by M average divided by l x A 1 minus x A 2.

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Then we can have the following expression for the mass transfer coefficient in this case. Gas phase we will get k G dash would be equal to D AB by R T into delta, k y dash would be equal to D AB P t by R T and k c dash would be D AB by delta.

Similarly, this is equation 8. Similarly, we can get for liquid phase k x dash would be equal to D AB rho by M average by delta and k L dash would be D AB by delta. So, this is equation 9. Now, if we convert among them then we will get k c dash would be equal to R T by P t k y dash which would be equal to R T k G dash and k x dash would be equal to rho by M average k L dash would be equal to C average k L dash.

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Now, if the concentration of A is expressed in mole ratio unit, the mass transfer coefficient k capital Y and k capital X are expressed as N A would be equal to k capital Y Y A 1 minus Y A 2 for the gas phase.

That means here k Y A 1 and capital Y A 2 they are not mole fraction unit they are mole ratio unit and similarly for the liquid phase N A would be k capital X X A 1 minus X A 2 here, capital X A 1 minus X A 2 are the mole ratio unit in the liquid phase. So, Y A and Y X A capital Y A and capital X A are the concentration of A in the gas or in the liquid phase in mole ratio unit, note that similar expression can be written using the mass ratio unit that is mass of A by mass of B unit as well. So, here in this case, as we said it is mole ratio unit; that means, it is a capital Y A would be equal to small y A divided by 1 minus y A.

That is the solute free basis, this we need to consider when we consider different know systems or applications of mass transfer coefficient for different mass transfer operations. Similarly for liquid phase capital X A would be equal to small x A divided by 1 minus small x A.

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Now, different types of mass transfer coefficient as we have discussed for both diffusion of A through non diffusing B and equimolar counter diffusion of A and B, we will summarize them in a tabular format for gas phase mass transfer the flux we write k G into the partial pressure unit say p A 1 minus p A 2 and its mass transfer coefficient k G is D AB P or P t divided by R T delta p B M.

And in case of equimolar counter diffusion of A and B, the flux is kG dash p A 1 minus p A 2 and the mass transfer coefficient is k G dash is equal to D AB by R T into delta. The unit of mass transfer coefficient over here is mole per time in per area per partial pressure gradient. Similarly, if we write in terms of the mole fraction unit k y the flux in case of diffusion of A through non diffusing B, we can write the flux k y is N A would be equal to k y y A 1 minus y A 2 and its know mass transfer coefficient in this case is D AB P t square divided by R T delta p B M.

And in case of equimolar counter diffusion, flux N A would be k y dash into y A 1 minus y A 2 and then, in this case the mass transfer coefficient k y dash D AB P t by R T delta. In this case, the unit of mass transfer coefficient would be mole per unit time per unit area per the mole fraction difference. Similarly, in terms of the concentration we can write k c C A 1 minus C A 2, k c would be equal to D AB P t by delta p B M and in case of equimolar counter diffusion, it is k c dash C A 1 minus C A 2 and k c dash is equal to D AB by delta.

So, here instead of all other remain sames it will be the concentration driving force in case of the unit of mass transfer coefficient remaining all other same.

> Different types of mass transfer coefficients Diffusion of A through Equimolar counterdiffusion Unit of the mass non diffusing B of A and B transfer coefficient Flux, N_A Mass transfer Flux, N_A Mass transfer coefficient coefficient Liquid -phase mass transfer $k_L(C_{A1} - C_{A2})$ $k_{L}(C_{A1} - C_{A2})$ D_{AB} mol D_{AB} $\overline{\delta x_{RM}}$ $\overline{\mathbf{r}}$ $\overline{(time)(area)(\Delta C_4)}$ $k_X(x_{A1} - x_{A2})$ $D_{AB}C$ $k_X(x_{A1} - x_{A2})$ CD_{AB} mol $\overline{\delta x_{DM}}$ $\overline{(time)(area)(\Delta X_A)}$ Conversion $k_GRT = \frac{RT}{P_c} k_y = k_c; k_L = \frac{k_x}{C_{cm}}$ $k_c = RTk_c = \frac{RT}{P} k_y$; $k_L = \frac{k_x}{C_{av}}$

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Now, if we consider the liquid phase mass transfer, similarly for liquid phase also we have considered k L and k x unit which is given over here k L into the concentration driving force then k X into the mole fraction driving force and then their mass transfer coefficient is written in the respective terms. And here also it is the unit of mass transfer coefficient will depend on the driving force we are considering for the mass transfer.

Now, if we convert among these all these mass transfer coefficient we can write k G R T would be equal to R T by P t k y would be equal to k c and k L would be equal to k x by C average. In case of the liquid phase, we can write k c dash would be equal to R T k capital G dash would be equal to R T by P t into k y dash and k L dash would be equal to k x dash by C average. So, this is the conversion among the mass transfer coefficient both in the gas phase as well as in the liquid phase.

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The former mass transfer coefficient; that means, in case of the know diffusion of component A through non diffusing B that is k G, k y, k c, k x and k L these are inherently associated with the log mean concentration of the other species B which is non-diffusing. So, if we consider the species B is non-diffusing then, these mass transfer coefficients would be associated with the log mean concentration gradient. Accordingly these type of mass transfer coefficient has a dependence on concentration because of the term p BLM or x BLM.

This dependence can however, be ignored at low concentration. So, if we considered very low concentration this dependency of the logarithmic term or the p BLM or x BLM term in case of gas and the liquid phase we can ignore when the concentration of diffusing species that is component A is very low. On the other hand, in case of the equimolar counter diffusion of component A and B in that case the coefficient k G dash, k y dash, k c dash, k x dash and k L dash do not have dependence on concentration. The second type of mass transfer coefficient or like k c dash is called Colburn Drew mass transfer coefficient.

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Now there is another type of mass transfer coefficient which is called F-type coefficient and proposed by Treybal and Benitez in 1980 and then 2002, this is F-type mass transfer coefficient. This coefficient similar to the Colburn Drew mass transfer coefficient and is not concentration dependent even in case of the diffusion of A through non diffusing B ok. So, if it is on for both the cases that is equimolar counter diffusion of component A and B and also the case where diffusion of A through non diffusing B both cases it is independent.

This F-type mass transfer coefficient is independent on the concentration terms. Now if we integrate over a film of thickness delta from this F-type mass transfer coefficient we can obtain N A would be equal to N A by N A plus N B D AB P t by R T ln N A divided by N A plus N B minus y A 2 divided by N A by N A plus N B minus y A 1. So, we can write N A by N A plus N B into F which is this part is written as F into ln NA by N A plus N B minus y A 2 divided by N A by N A plus N B minus y A 1. So, this is equation 10; y A is the mole fractions it is p A by P t. So, partial pressure of component a divided by the total pressure and D AB P t by R T is F.

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The F type mass transfer coefficient which is independent of the concentration of the diffusing species and if we compare equation 10 with equation 3 for the case of diffusion of A through non diffusing B, it is very easy to find out F is equal to k G p BLM and this is the equation 3 and 10. So, we can from there if we compare we can write F would be equal to k G p BLM.

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Now, for equimolar counter diffusion, F and k G are k G dash are related as F is equal to k G dash P t, which is independent of partial pressure of diffusing species.

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Now, what is the relation between the k G and k y. So, now, we will find out the driving force in case of mole ratio unit Y that is k capital Y between the two points 1 and 2 we can write as capital Y A 1 minus capital Y A 2 would be equal to p A 1 by P t minus p A 1 minus p A 2 divided by P t minus p A 2. Now if we just re solve it then we will get P t p A 1 minus p A 2 p A 2 minus P t p A 1 plus p A 1 p A 2 divided by P t minus p A 1 into P t minus p A 2 which would be equal to P t p A 1 minus p A 2 by p B 1 into p B 2. So, this we will obtain.

Now, if we just rearrange this relations just to calculate the partial pressure difference we can write p A 1 minus p A 2 would be equal to capital Y A 1 minus Y A 2 this part and then multiplied by over here p B 1 p B 2 divided by P t. So, after rearranging we will obtain this partial pressure difference.

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Now, we have N A is equal to k G p A 1 minus p A 2. Now, if we substitute that p A 1 minus p A 2 which we have obtained we will get D AB P t divided by R T delta p BLM p A 1 minus p A 2. So, we will have finally, k Y into Y A 1 minus Y A 2.

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Since N A is equal to k Y capital Y A 1 minus Y A 2, we may write capital k Y would be equal to D AB p B 1 into p B 2 divided by R T p BLM would be equal to k G p B 1 into p B 2 divided by P t. A similar relations between k G dash and k Y dash in case of the equimolar counter diffusion applies.

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Relation between \mathbf{k}'_0 and \mathbf{k}'_Y **We know that** $p_{A1} - p_{A2} = (Y_{A1} - Y_{A2}) \frac{p_{B1}p_{B2}}{P_{A2}}$ **Solution: Now we have** $N_A = k'_{G}(p_{A1} - p_{A2}) = \frac{D_{AB}}{RT\delta}(p_{A1} - p_{A2})$ $= \frac{D_{AB}}{RT\delta} \left(Y_{A1} - Y_{A2} \right) \frac{p_{B1}p_{B2}}{p_t} = \left(Y_y (Y_{A1} - Y_{A2}) \right)$ since $N_A = k'_{Y}(Y_{A1} - Y_{A2})$, we may write $k'_{Y} = \frac{D_{AB} p_{B1} p_{B2}}{R T \delta} = k'_{G} \frac{p_{B1} p_{B2}}{p_t}$

Now, we know that $p A 1$ minus $p A 2$ would be equal to capital Y A 1 minus Y A 2 $p B 1$ into p B 2 by P t. So, we have N A is equal to k G dash p A 1 minus p A 2 is equal to D AB R T delta p A 1 minus p A 2.

So, if we substitute D AB by RT delta into capital Y A 1 minus Y A 2 into p B 1 into p B 2 divided by P t. So, this part we can write k Y dash into Y A 1 minus Y A 2, from this we can write y A dash N A is it since N A is equal to k dash Y Y A 1 minus Y A 2 we can write y k Y dash would be equal to this D AB p B1 into p B2 divided by RT into delta. So, we can get this would be equal to k G dash p B1 into p B2 by P t.

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Conversion among the gas phase mass transfer coefficient, we can write F would be equal to k G p BLM would be equal to k y p BLM divided by P t which is equal to k c p BLM by R T.

We can write k G dash P t would be equal to k y dash which is equal to k c dash P t by R T. Now conversion among the liquid phase mass transfer coefficient we can write F is equal to k x x BLM would be equal to k L x BLM into C is equal to k L dash C would be equal to k L dash rho by M is equal to k x dash. So, this is in case of the liquid phase and this is in case of the gas phase, for both the systems one is equimolar counter diffusion and another one is diffusion of A through non diffusing B.

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Now, typical values for the mass transfer coefficient and the film thickness, for gas phase mass transfer coefficient it is k c is around 10 to the power minus 2 metre per second and for a film thickness of delta of 1 millimetre. In case of liquid phase mass transfer coefficient k c is equal to or k L is equal to 10 to the power minus 5 metre per second and the film thickness is close to delta is would be about 0.1 millimetre.

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Now, let us take an example large volume of pure nitrogen gas at atmospheric pressure is flowing over a pool of liquid of methanol, which is evaporating. Nitrogen is assumed to

be insoluble in liquid. The gas phase mass transfer coefficient of methanol which is given k G is 2 into 10 to the power minus 5 kilomole per metre square second kilopascal.

Assume vapour pressure of methanol at 298 Kelvin is 10 kilopascal. Now, we need to calculate k y k c capital k Y and F. If the diffusivity of methanol at 298 Kelvin is 2 into 10 to the power minus 5 metre square per second calculate the thickness of the gas film. So, here the diffusivity values are also given which is equal to the mass transfer coefficient in the gas phase.

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Now, in this case the diffusion of methanol occurs through non diffusing nitrogen. So, the first thing it is given that the pure nitrogen is diffusing at atmospheric pressure. So, the total pressure P t is 1 atmosphere which is 101.3 kilopascal. Now, R is known to us is 0.082 metre cube atmosphere per kilomole Kelvin which is we can write in terms of the metre cube kilopascal kilomole Kelvin in this unit.

And which is 8.3066 metre cube kilopascal per kilomole Kelvin. So, and temperature is given is 298 Kelvin. So, k y is equal to k G P t. So, we can just substitute k G which is given and p t is known to us. So, k y would be equal to 2 into 03 into 10 to the power minus 3 kilomole per metre square second delta y.

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k c would be equal to k G into R T which is equal to 2 into 10 to the power minus 5 kilomole per metre square second kilopascal into R into T. If we substitute R and T then we can get is 0.0495 metre per second. So, the partial pressure of component A that is nitrogen is given as 10 kilopascal. So, we can calculate p B 1 which P t minus p A 1 which is 91.3 kilopascal and p A 2 is 0 that is p B 2 would be equal to P t which is equal to 101.3 kilopascal. So, both p A 1 then p A 2 p B 1 and p B 2 are known to us.

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So, we can calculate capital k Y would be equal to k G p B 1 into p B 2 by P t. So, if we substitute this would be equal to 1.83 into 10 to the power minus 3 kilomole per metre square second into the driving force delta Y. F is equal to k G P BLM. So, if we substitute we can calculate P BLM and then if we substitute over here it would be equal to 2 into 10 to the power minus 5 and it is 1.92 into 10 to the power minus 3 kilomole per metre square second.

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So, F is calculated and we need to calculate the thickness of the gas film for k G as we know it is D AB P t by R T delta, delta is missing over here delta and from here we can write, if we rearrange delta would be equal to D AB P t by R T k G p BLM.

So, delta from here we can write delta would be equal to D AB P t divided by R T k G p BLM. So, D AB is given to us and is 2 into 10 to the power minus 5 metre square per second P t is 101.3 kilopascal and R is known to us, temperature is also known to us and k G is given. So, if we substitute D AB P t R T k G and p BLM then.

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P BLM as we have soon earlier p BLM we can calculate for in this case it is 96.213. So, we can calculate delta using the earlier equations which is equal to 0.425 millimetre.

Thank you very much for attending this lecture. In the next lecture we will continue for dimensionless analysis of the mass transfer coefficient.