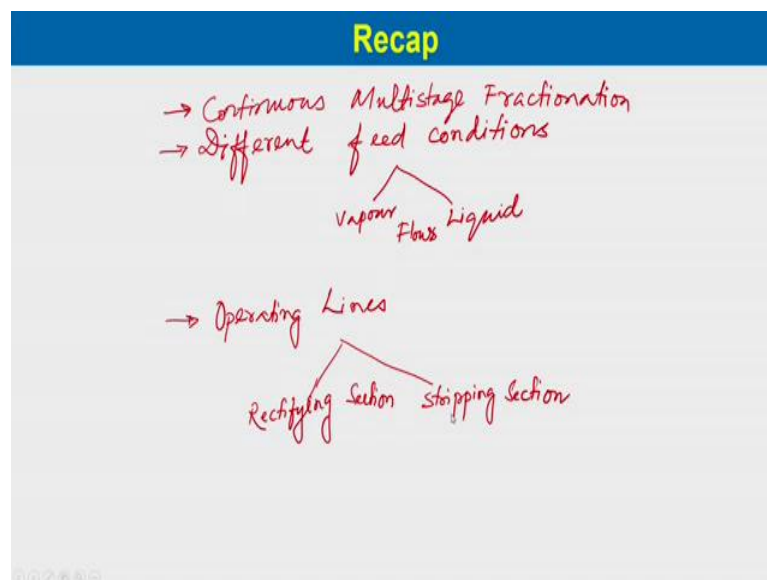


Mass Transfer Operations -I
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Lecture - 31
Number of trays by McCabe & Thiele for distillation

Welcome to the 6th lecture of module 5 of Mass Transfer Operation. In this model, we are discussing distillation operations. So, before going to this lecture let us have a brief recap on our previous lecture.

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In our previous lecture, we have considered continuous multistage fractionation. The second thing we have considered is different feed conditions under which we have considered both cases of vapor and liquid flows. And third thing we have considered operating lines, this is also for both rectifying section and the stripping section.

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Module 5: Lecture 6

- Equimolal Overflow
- Feed Tray & Feed Line
- McCabe-Thiele Method to determine number of ideal trays
- Limiting Cases ← Total Reflux
Minimum Reflux
- Fenske Eqⁿ

So, in this lecture, we will mainly consider equimolal overflow, then we will consider feed tray and feed line. The third thing we will consider McCabe-Thiele method to determine number of ideal trays. We will also consider different limiting cases such as total reflux and minimum reflux. We will consider one analytical method to find out the number of trays using Fenske equation.

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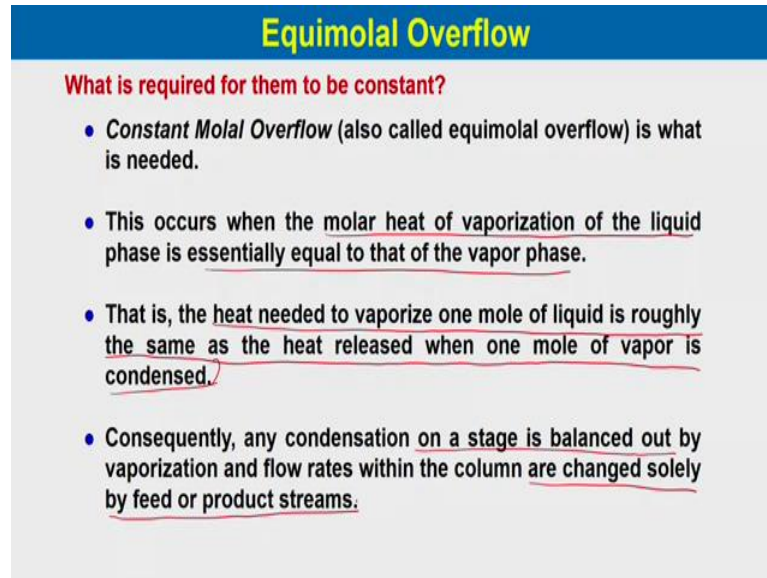
Equimolal Overflow

- Calculations using the equations discussed in previous lectures are much more convenient if the two operating equations are straight lines.
- This is true only if the liquid and vapor flows do not change in a given section of the column.

So, let us start with equimolal overflow. Calculation using the equations discussed in our previous lectures are much more convenient if the two operating equations are straight

lines. Both rectifying section operating line and the stripping section operating line if there are straight line it would be much easier for the calculation. This is true only if the liquid and the vapor flows do not change in the given section of the column.

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Equimolal Overflow

What is required for them to be constant?

- *Constant Molal Overflow* (also called equimolal overflow) is what is needed.
- This occurs when the molar heat of vaporization of the liquid phase is essentially equal to that of the vapor phase.
- That is, the heat needed to vaporize one mole of liquid is roughly the same as the heat released when one mole of vapor is condensed.
- Consequently, any condensation on a stage is balanced out by vaporization and flow rates within the column are changed solely by feed or product streams.

Then what is required for them to be constant that is the vapor and the liquid flow, they remains constant. Constant molal overflow also called equimolal overflow is what is needed to be constant for the liquid and the vapor flows. This occurs when the molar heat of vaporization of the liquid is essentially equal to that of the vapor phase. That is, the heat needed to vaporize one mole of the liquid is roughly the same as that of the heat released when one mole of vapor is condensed. And consequently, any condensation on a stage is balanced out by vaporization and flow rates within the column are changed solely due to the feed and product streams.

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Equimolar Overflow

What is required for them to be constant?

- Quickest way to check the validity of this assumption is to compare the heats of vaporization of the components. If their ratio is roughly 1:1, the assumption is probably acceptable.
- When $x=x_D$, then $y=x_D$ as well. This means that the point (x_D, x_D) lies on the rectifying line.
- If we assume this, then rectifying OL can be drawn using only this point and the slope.
- Stripping OL line can be drawn with point (x_W, x_W) and the slope.

The quickest way we can check the validity of this assumption is to compare the heats of vaporization of the components. If the ratio is roughly 1 is to 1, then the assumption is probably acceptable. The ratio of the heat of vaporizations of the two components if that is 1 is to 1 then these assumption of equimolar overflow will be probably acceptable or will be constant. When x is equal to x_D , then y would be equal to y_D as well. This means that the point x_D and x_D lies on the rectifying line. If you assume these, then rectifying operating line can be drawn using only this point and the slope. Similarly, for stripping operating line can be drawn with the point x_W , x_W and the slope.

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Feed Tray

- It may be useful to examine the steady state balances on a feed tray.
- **The total material balance on feed tray is**

$$F + L + \bar{G} = \bar{L} + G$$

Energy Balance:

- Assume the change in enthalpy of a phase as it passes through the feed plate is small

$$FH_F + LH_L + \bar{G}H_G = \bar{L}H_L + GH_G$$

$$\Rightarrow (\bar{L} - L)H_L = (\bar{G} - G)H_G + FH_F$$

Now, let us consider feed tray. So, this is a feed tray, where feed enters with the composition z_F and flow rate molar flow rate is F . Having the enthalpy H_F and the liquid which is entered into this tray is L , and gas which entered in this tray is G_{bar} and liquid which comes out from the tray is L_{bar} and gas which goes out from the tray is G . So, these having the enthalpy H_F ; this is having $H_{L_{bar}}$ this is $H_{G_{bar}}$; this is H_G ; and this is having H_L . Now, it may be useful to examine the steady state balances on a feed tray.

The total material balance on the on the feed tray we can write the entering components entering flows that is F is the feed plus the liquid from the top tray is L and the vapor which is entering from the bottom tray of the feed tray is G_{bar} which is equal to the liquid leaving from the tray is L_{bar} plus the gas or vapor leaving the tray is G . So, this is the total material balance on feed tray.

Now, if you do the energy balance assume that the change in enthalpy of a phase as it passes through the feed point is small. So, in that case, we can write $F H_F$ plus $L H_L$ plus $G_{bar} H_{G_{bar}}$ would be equal to $L_{bar} H_{L_{bar}}$ plus $G H_G$. So, in this case, the enthalpy of the vapor which is coming to this tray is considered as H_G , and the enthalpy of the liquid which is coming down from the tray is also considered as H_L . So, this is due to these assumptions that the change in enthalpy of the phases as it passes through the feed tray is small. So, we can write $L_{bar} H_{L_{bar}} - L H_L$ is equal to $G_{bar} H_{G_{bar}} - G H_G$ plus $F H_F$.

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Feed Tray

- Now using the total material balance equation we have:

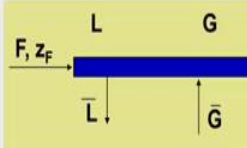
$$F + L + \bar{G} = \bar{L} + G \Rightarrow \bar{G} - G = \bar{L} - L - F$$

$$(\bar{L} - L)H_L = (\bar{G} - G)H_G + FH_F$$

$$(\bar{L} - L)H_L = (\bar{L} - L - F)H_G + FH_F = (\bar{L} - L)H_G + F(H_F - H_G)$$

$$\Rightarrow (\bar{L} - L)(H_L - H_G) = F(H_F - H_G) \checkmark$$

$$\Rightarrow \frac{(\bar{L} - L)}{F} = \frac{H_F - H_G}{H_L - H_G} = \frac{H_G - H_F}{H_G - H_L} = q$$
- But $\bar{L} - L$ = increase in the liquid flow rate across the feed tray as a result of introduction of feed
 = rate of input of liquid with the feed
 So, q is the fraction of liquid in the feed.



Now, using the total material balance equation, we can write this is the total material balance equation F plus L plus \bar{G} is equal to \bar{L} plus G . From here, we can write \bar{G} minus G is equal to \bar{L} minus L minus F . So, now if we come consider the earlier equation enthalpy balance equation or the energy balance equation, which is \bar{L} minus L into H_L is equal to \bar{G} minus G into H_G plus $F H_F$. So, if you substitute this \bar{G} minus G is \bar{L} minus L minus F , so in place of these, if we substitute this one, then we will have \bar{L} minus L into H_L is equal to \bar{L} minus L minus F into H_G plus $F H_F$ which is equal to \bar{L} minus L into H_G plus F into H_F minus H_G . From here we can rearrange and we can write \bar{L} minus L into H_L minus H_G is equal to F into H_F minus H_G .

And again we can write \bar{L} minus L by F is equal to H_F minus H_G divided by H_L minus H_G . Now, if we just rearrange this one, we can write H_G minus H_F divided by H_G minus H_L which we can define as q . We can see that \bar{L} minus L is equal to increase in the liquid flow rate across the feed tray as a result of introduction of feed. So, this is why we can write it as the rate of input of the liquid with the feed. So, q is defined as the fraction of the liquid in the feed.

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Feed Line

- Another significance is as follows:
$$\frac{(\bar{L}-L)}{F} = \frac{H_F - H_G}{H_L - H_G} = \frac{H_G - H_F}{H_G - H_L} = q$$

$$q = \frac{\text{heat required to convert 1 mol feed to saturated vapor}}{\text{molar heat of vaporization of the saturated liquid}}$$
- It would be nice to know where the rectifying line and the stripping line intersect.
- The point of intersection (x,y) must be satisfied by rectifying and stripping section lines.

Another significance is as follows. This is the equation $\bar{L} - L$ divided by F is equal to $H_G - H_F$ divided by $H_G - H_L$ is equal to q . So, in previous slides, we have said that the q is defined as the fraction of the liquid in the feed that is in the left hand side $\bar{L} - L$ by F . So, $\bar{L} - L$ is the liquid which is available in the feed. So, it is the fraction of the liquid in the feed that is q . And from the right hand side, this $H_G - H_F$ by $H_G - H_L$ we can write the heat required to convert one mole of feed to saturated vapor divided by the molar heat of vaporization of the saturated liquid.

We can see that this is the enthalpy of the saturated liquid and H_G is the enthalpy of the vapor. So, $H_G - H_L$ is the molar heat of vaporization of the saturated liquid. And $H_G - H_F$ is the heat required to convert 1 mole feed to saturated vapor. So, this is the ratio of heat required to convert 1 mole of feed to saturated vapor divided by molar heat of vaporization of the saturated liquid. It would be nice to know where the rectifying line and the stripping section line intersect.

The point of intersection which is x, y must be satisfied by rectifying and stripping section line because that is also in the column. The feed tray is inside the column. So, the feed point must be satisfied that is x, y feed point must be satisfied with the rectifying, both the rectifying section and the stripping section operating lines.

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Feed Line

Where the rectifying line and the stripping line intersect?

- Point of intersection (x, y) must be satisfied by the material balance equations of both rectifying and stripping section.
- If the point of intersection is (x,y) then:
 - Rectifying balance:** $Gy = Lx + Dx_D$ ✓
 - Stripping balance:** $\bar{G}y = \bar{L}x - Wx_W$ ✓

So, the point of intersection x, y must be satisfied by the material balance equations of both rectifying and stripping section operating lines. If the point of intersection is x, y, then rectifying section we can write G y would be equal to L into x plus D into x D; x D is the composition of the distillate. Stripping balance we can write G bar into y would be equal to L bar into x minus W x W. So, x W is the composition of the bottom product.

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Feed Line

- Subtracting the stripping balance from the rectifying balance yields

$$(G - \bar{G})y = (L - \bar{L})x + (Dx_D + Wx_W) \quad \checkmark$$
- Now, using overall material balance equation the above equation reduces to:

$$(G - \bar{G})y = (L - \bar{L})x + Fz_F \quad \checkmark \quad \underline{Fz_F = Dx_D + Wx_W}$$
- Now if we divide by F to the total material balance equation and use the definition of q then:

$$\frac{\bar{G} - G}{F} + 1 = \frac{\bar{L} - L}{F} = q$$

$$\frac{-(\bar{G} - G)}{F}y = \frac{-(\bar{L} - L)}{F}x + z_F \Rightarrow -(q-1)y = -qx + z_F \Rightarrow y = \frac{q}{q-1}x - \frac{z_F}{q-1}$$

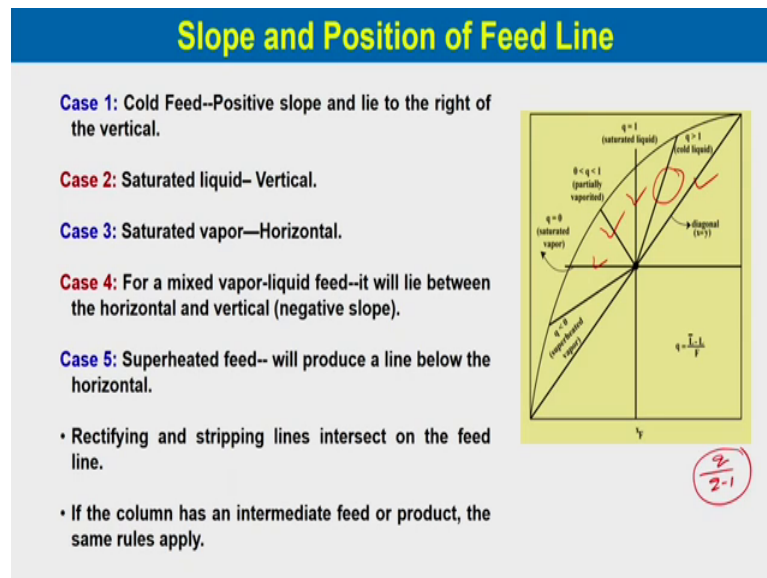
Now, if you subtract the stripping balance from the rectifying balance which will yield G minus G bar into y is equal to L minus L bar into x plus D into x D plus W into x W.

Now, if we do the overall material balance over the mole fractionators, the above equations will reduce to $G - \bar{G}$ into y would be equal to $L - \bar{L}$ into x plus F in to z F . The overall balance equations is $F z F$ would be equal to $D x D$ plus $W x W$. So, if we substitute here $F z F$, this equation will reduce to $G - \bar{G}$ into y is equal $L - \bar{L}$ into x plus $F z F$.

Now, if we divide by F to the total material balance equation and use the definition of q , then we can write $G - \bar{G}$ by F plus 1 is equal to $L - \bar{L}$ by F which is equal to q . So, this is the definition of the q line. Now, if we substitute this definition of q in the material balance equation over here in this equation, we can write minus $G - \bar{G}$ divided by F into y is equal to minus $L - \bar{L}$ by F into x plus $z F$. So, this is nothing but q , so minus $q - 1$ into y is equal to minus q into x plus $z F$.

And if you just simply this will give y is equal to q by $q - 1$ into x minus $z F$ by $q - 1$. So, this is the equation of the q line or the feed line having the slope q by $q - 1$, and intercept is minus $z F$ by $q - 1$.

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So, now this is the feed conditions or the positions of the feed line and their slopes depending on the values of q or the feed conditions. If case 1, if it is cold feed, then the positive slope and lie to the right of the verticals. Cold feed means the feed is completely liquid. So, q will be the amount of feed plus the vapor which will be condensing the feed tray because of the introduction of the cold feed and some vapor will condense and

increase the temperature of the feed to the boiling point. So, the q would be greater than 1; and slope will be positive. And it will lie on the right of the vertical line. So, this is for cold feed line with a positive slope. And it is the diagonal line, x is equal to y .

Now, case 2, if we have saturated liquid, so if saturated liquid, the feed is the completely liquid at its bubble point, and we need not to heat it up or the vapor which will come from the bottom of the tray will not be condensed on the feed tray. So, the q would be 1 and the slope that is q by q minus 1 the slope will be infinite. So, it will be a vertical line shown over here. Case 3 is the saturated vapor. And the for saturated vapor the feed is the q is 0, that means, the complete feed is the vapor feed. So, in that case when q is 0, then the slope is 0 and it is horizontal line.

For case 4, that is for a mixture of vapor liquid feed it will lie between the horizontal and vertical line because the slope is negative for q less than 1 and greater than 0. So, this is the partially vaporized feed. So, the slope is negative. And this will lie in between vertical and horizontal line. Case 5 is the superheated feed which will produce a line below the horizontal, because the q is less than 0. So, for superheated vapor, it will lie between the horizontal line and the diagonal line. Rectifying and stripping line intersects on that feed line. If the column has an intermediate feed or the product, the same rules also apply. So, these are the slope and position of they feed lines.

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Calculations

- Before beginning most distillation calculations, a decision must be reached:

1) Does [equimolar overflow](#) apply?

If so, the operating equations are straight lines and you have one set of options -- notably the [McCabe-Thiele](#) method.

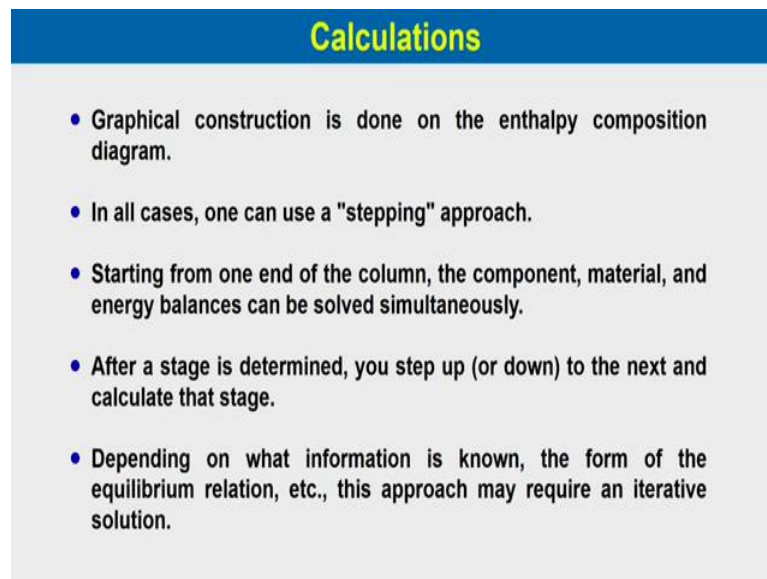
2) If not, [energy balances](#) must be explicitly considered.

- There are several ways of incorporating the energy effects.

➤ The [Ponchon-Savarit](#) method is a graphical approach that does not require an assumption of equimolar overflow.

Before beginning most distillation calculations, a decision must be reached. What are they that does the equimolar overflow apply for this process or for this application? If so the operating equations are straight line and you have one set of option that is they McCabe-Thiele method of determining the number of stages that is graphical method; if not then energy balances must be explicitly considered there are several ways of incorporating the energy effects one search method is the Ponchon and Savarit method which is a graphical method that does not require an assumption of equimolar overflow.

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Calculations

- Graphical construction is done on the enthalpy composition diagram.
- In all cases, one can use a "stepping" approach.
- Starting from one end of the column, the component, material, and energy balances can be solved simultaneously.
- After a stage is determined, you step up (or down) to the next and calculate that stage.
- Depending on what information is known, the form of the equilibrium relation, etc., this approach may require an iterative solution.

Now, graphical construction is done on the enthalpy composition diagram. In all cases, one can use a stepping approach. Starting from one end of the column, the component, materials, and energy balances can be solved simultaneously. After a stage is determined, you can step up or step down to the next and calculate that stage. Depending on what information is known, the form of equilibrium relations and other things, this approach may require an iterative solutions. So, regarding the determination of the number of trays using the Ponchon and Savarit method, we will considered in the later part of our discussion.

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McCabe-Thiele Method

Topic of discussion

- McCabe-Thiele graphical construction
- Determination of N and X_w
- Total reflux
- Example

In this lecture, we will consider graphical method of determination of number of ideal tray by McCabe-Thiele method. So, the topic which we will considered here is the McCabe-Thiele graphical construction, determination of number of ideal tray and the mole fractions of the bottoms that is x_w , the total reflux and example.

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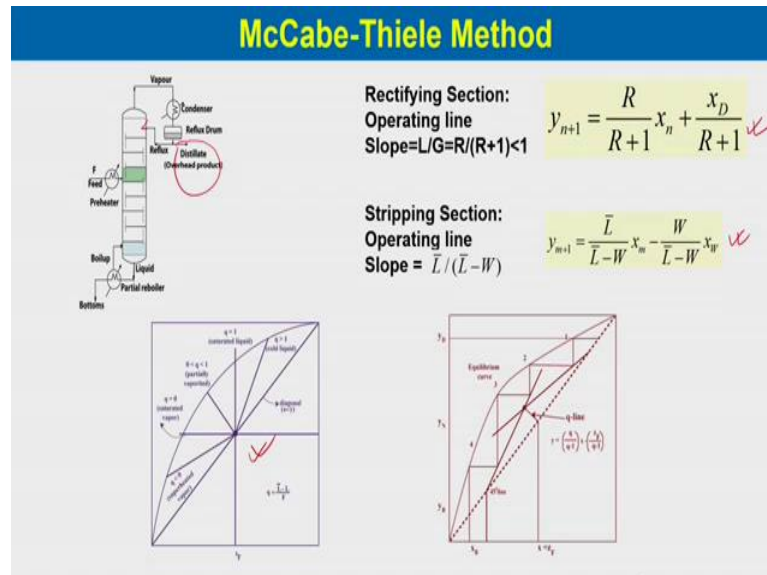
McCabe-Thiele Method

- The graphical *McCabe-Thiele Method* can be used to determine the number of ideal stages and feed tray location.
- To do this, you make a plot showing:
 - ✓ The equilibrium curve
 - ✓ Feed line
 - ✓ Operating lines for the rectifying and stripping sections (all on the same axes)
- Then find answers by graphical construction.

The graphical McCabe-Thiele method can be used to determine the number of ideal stages and feed tray locations. To do this, you make a plot showing the equilibrium

curve, then the feed line, then the operating line for the rectifying and stripping sections or on the same axis, then point answers by graphical construction.

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So, this is a typical fractionator or distillation column where feed is entered at a certain location or certain tray, and then the vapor goes up and it condenses, and then it is taken to the reflux drum, part of the product is taken as distillate. And part of that is return back acid reflux to the column, the liquid comes down. And there is a there is a partial reboiler, where it is boiled up the boil off vapor is goes to the tray return to the column and the product is taken as bottoms. So, this is already we have discussed. Depending on the feed conditions, we have feed line. The rectifying section operating line slope is L by G which is equal to R by R plus 1, which is less than 1. This is the equation of the rectifying section operating line y_{n+1} would be equal to R by R plus 1 into x_n plus x_D by R plus 1.

Stripping section operating line slope is equal to \bar{L} by \bar{L} minus W, and the stripping section operating line equation is y_{m+1} equal to \bar{L} by \bar{L} minus W into x_m minus W by \bar{L} minus W into x_W . So, this is tripping section operating line equation; this is rectifying section operating line equation. And this is the different feed line depending on the feed condition. So, we can locate different points, we can draw the equilibrium curve, we can draw the operating line for both the sections, and we can draw the feed line. And then by this graphical method we can calculate the number of trays.

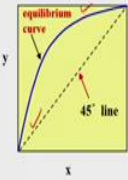
So, we will come step by step how to determine the number of ideal trays required following McCabe-Thiele method of graphical construction.

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McCabe-Thiele Method

- ✓ Step 1: Plot **equilibrium curve** and 45° line.
- ✓ Step 2: Locate and plot given compositions (x_p , x_w , and x_D)
- ✓ Step 3: Calculate **slope** $[q/(q-1)]$ of q-line and draw **q-line** using $x=z_F$ and the slope
- ✓ Step 4: Calculate y-intercept $[x_D/(R+1)]$ of the rectifying line and draw operating line (OL) for **Rectifying section**
- ✓ Step 5: Draw OL for **Stripping section** using point (x_w, x_w) and intersection between rectifying OL and q-line
- ✓ Step 6: From x_D locate x_1 and y_1 drawing a **horizontal line** to the **equilibrium line** for stage 1.
- ✓ Step 7: Find y_2 drawing a **vertical line** to the **rectifying OL** locate the mass balance condition between x_1 and y_2 .
- ✓ Step 8: From y_2 draw a **horizontal line** to the **equilibrium line** for stage 2 to locate x_2 .
- ✓ Step 9: Return to step 7 and cycle through steps 7 and 8 until $x_i < x_p$. Draw subsequent **vertical lines** to the **stripping OL**.
- ✓ Step 10: End after predetermined number of stages, or when x_i is less than x_w .


Step 1



Step 2



Step 3



So, these 10 steps we need to follow to determine the number of ideal stage by this method. In the first, plot the equilibrium curve and the 45 degree line. So, this is the equilibrium curve, and this is the 45 degree diagonal. So, this is the first step we have to do. In the second step, locate and plot the given composition that is z_F , x_W and x_D . So, you have to locate these points on the curve. So, this is z_F which is on the 45 degree diagonal, then x_D is over here and then x_W is over here. So, the location of these points on the 45 degree diagonal should be known or should be located. Step 3 calculate slope that is q by q minus 1 of the q line, and draw q line using x is equal to z_F and the slope. So, this is the point at z is equal to z_F , and slope is q by $q - 1$ q by q minus 1. So, if you know this is the slope and this point, we can draw this q line. So, this is the q line.

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McCabe-Thiele Method

- ✓ Step 1: Plot **equilibrium curve** and 45° line.
- ✓ Step 2: Locate and plot given compositions (x_F , x_W , and x_D)
- ✓ Step 3: Calculate **slope** $[q/(q-1)]$ of q-line and draw **q-line** using $x=x_F$ and the slope
- ✓ Step 4: Calculate y-intercept $[x_D/(R+1)]$ of the rectifying line and draw operating line (OL) for **Rectifying section**
- ✓ Step 5: Draw OL for **Stripping section** using point (x_W, x_W) and intersection between rectifying OL and q-line
- ✓ Step 6: From x_D locate x_1 and y_1 drawing a **horizontal line** to the **equilibrium line** for stage 1.
- ✓ Step 7: Find y_2 drawing a **vertical line** to the **rectifying OL** locate the mass balance condition between x_1 and y_2 .
- ✓ Step 8: From y_2 draw a **horizontal line** to the **equilibrium line** for stage 2 to locate x_2 .
- ✓ Step 9: Return to step 7 and cycle through steps 7 and 8 until $x_i < x_F$. Draw subsequent **vertical lines** to the **stripping OL**.
- ✓ Step 10: End after predetermined number of stages, or when x_i is less than x_W .

Step 4

Step 5

The next step is to draw the rectifying section operating line that is to draw the rectifying section operating line calculate y intercept that is x_D by R plus 1 of the rectifying line and draw the operating line OL for rectifying section. Since x_D is known to ask, so we can locate this point, we can calculate we know the x_D and we know the reflux ratio. So, we can calculate easily x_D by R plus 1.

So, we can locate the intercept on the y-axis and then we can join this two points to obtain the rectifying section operating line. So, this is much easier to draw instead of using this point and the slope of the operating line. Similarly, for the step 5, we can draw operating line for stripping section using point x_W and x_W which is located over here, and the intersection between the rectifying section operating line and the q line. Since the rectifying section operating line and q line intersects over at this point the stripping section would also pass through this intersection point because this line also valid in the feed line equations. So, the all the three lines should intersect at the same point. From x_D locate x_1 and y_1 drawing a horizontal line. So, from x_D , we can draw a horizontal line over here and locate the values at x_1 and y_2 . So, this is for stage 1.

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McCabe-Thiele Method

- ✓ Step 1: Plot **equilibrium curve** and 45° line.
- ✓ Step 2: Locate and plot given compositions (x_F , x_W , and x_D)
- ✓ Step 3: Calculate **slope** $[q/(q-1)]$ of q-line and draw **q-line** using $x=x_F$ and the slope
- ✓ Step 4: Calculate y-intercept $[x_D/(R+1)]$ of the rectifying line and draw operating line (OL) for **Rectifying section**
- ✓ Step 5: Draw OL for **Stripping section** using point (x_W, x_W) and intersection between rectifying OL and q-line
- ✓ Step 6: From x_D locate x_1 and y_1 , drawing a **horizontal line** to the **equilibrium line** for stage 1.
- ✓ Step 7: Find y_2 drawing a **vertical line** to the **rectifying OL**. locate the mass balance condition between x_1 and y_2 .
- ✓ Step 8: From y_2 draw a **horizontal line** to the **equilibrium line** for stage 2 to locate x_2 .
- ✓ Step 9: Return to step 7 and cycle through steps 7 and 8 until $x_i < x_F$. Draw subsequent **vertical lines** to the **stripping OL**.
- ✓ Step 10: End after predetermined number of stages, or when x_i is less than x_W .

Now, find y_2 drawing a vertical line to the operating line so come vertically to the operating line, and then from this point locate the mass balance conditions between x_1 and y_2 . So, if we locate the mass balance equation x_1 and y_2 , we can calculate the values for y_2 . From y_2 draw a horizontal line to the equilibrium line for stage 2 to locate the x_2 . So, from here again go to the equilibrium line this is for stage 2, and then come down to the vertically to the operating line. And then again go to the equilibrium line come down to the operating line in the stripping section, and then it will continue until the terminal point is reached.

So, this way the end after predetermined number of stages or when x_i is less than x_W . So, at this location this is crossing x_W . So, the it should close from over here. So, the number of stages required for from this, we can see it is less than 4. Please note that while drawing the staircase arrangement on the stages graphically the shifting from rectifying section operating line to the stripping section operating line will happen in the feed line. You can see this is the intersection; this is the feed line. So, this plate two over here is the feed plate.

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Example 1

A mixture of 45 mole % n-hexane and 55 mole % n-heptane is subjected to continuous fraction in a tray column at 1 atm total pressure. The distillate contains 95% n-hexane and the residue contains 5% n-hexane. The feed is saturated liquid. A reflux ratio of 1.5 is used. The relative volatility of n-hexane in mixture is 2.36. Determine the number of ideal trays required.

Solution

- The distillate contains 95% n-hexane
- Distillate (x_D, x_D) is (0.95, 0.95)
- Feed mixture contains 45 mole % n-hexane
- Feed (x_F, x_F) is (0.45, 0.45)

Now, let us take an example. A mixture of 45 mole percentage n-hexane and 55 mole percent n-heptane is subjected to continuous fractionation in a tray column at 1 atmosphere total pressure the distillate contains 95 percent n-hexane, and the residue contains 5 percent n-hexane. The feed is saturated liquid. A reflux ratio of 1.5 is used. The relative volatility of n-hexane in the mixture is 2.36. Now, we have to determine the number of ideal trace required.

So, let us start with the conditions which are given. So, distillate is having ninety five percent n-hexane it is given so 95 percent n-hexane. So, the distillate composition or the location is x_D, x_D would be 0.95, 0.95 feed mixtures contains 45 mole percent n-hexane. So, the feed condition is x_F, x_F on the 45 degree diagonal which is 0.45, 0.45. So, both distillate and feed are known.

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Example 1

A mixture of 45 mole % n-hexane and 55 mole % n-heptane is subjected to continuous fraction in a tray column at 1 atm total pressure. The distillate contains 95% n-hexane and the residue contains 5% n-hexane. The feed is saturated liquid. A reflux ratio of 1.5 is used. The relative volatility of n-hexane in mixture is 2.36. Determine the number of ideal trays required.

Solution

- Residue contains 5% mole of n- hexane (x_W, x_W) is (0.05, 0.05)
- Relative volatility of n- hexane is 2.36
- Reflux ratio is 1.5

Then residue contains 5 mole percent of n-hexane. So, x_D , x_W and x_W is also known on the 45 degree diagonal which is point naught five, point naught five. Relative volatility of n-hexane is given which is 2.36 reflux ratio is 1.5.

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Example 1

A mixture of 45 mole % n-hexane and 55 mole % n-heptane is subjected to continuous fraction in a tray column at 1 atm total pressure. The distillate contains 95% n-hexane and the residue contains 5% n-hexane. The feed is saturated liquid. A reflux ratio of 1.5 is used. The relative volatility of n-hexane in mixture is 2.36. Determine the number of ideal trays required.

Solution

- Intersect $\frac{x_D}{R+1} = \frac{0.95}{1.5+1} = 0.38$

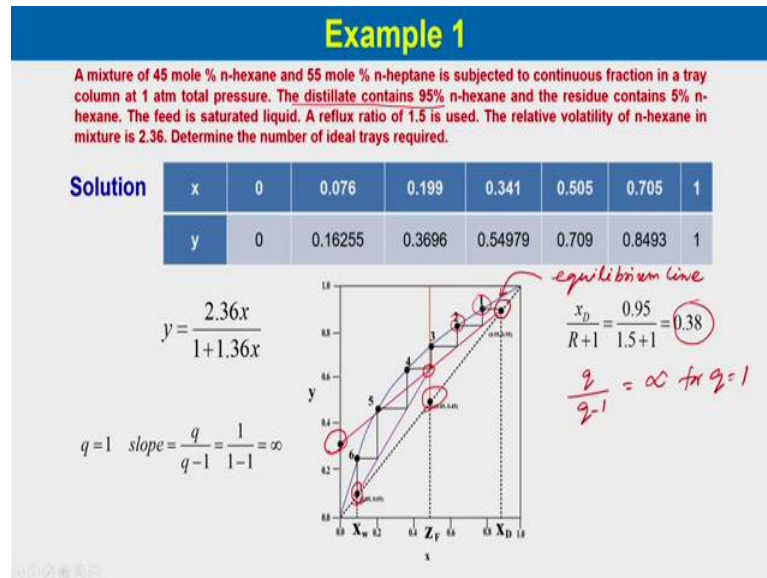
$q=1$ slope = $\frac{q}{q-1} = \frac{1}{1-1} = \infty$

$$y = \frac{\alpha x}{1 + (\alpha - 1)x} \quad \alpha = 2.36$$

Now, from the data given, we can calculate the intercept of the rectifying section operating line which is x_D by R plus 1. So, x_D by R plus 1 x_D is given as 0.95 and r is 1.5. So, 0.95 by 1.5 plus 1 and which is equal to 0.38. Now, we have to calculate the slope of the q line. Since, in this case, the feed is saturated liquid since feed is saturated

liquid q is equal to 1. So, we can calculate slope is equal to q by q minus 1 which is 1 by 1 minus 1, which is infinite. So, the slope is infinite. So, it will be a vertical line from the feed location of the feed on the 45 degree diagonal. The relative volatility equations we know y is equal to αx divided by 1 plus α minus 1 into x . The relative volatility over here α is given as 2.36.

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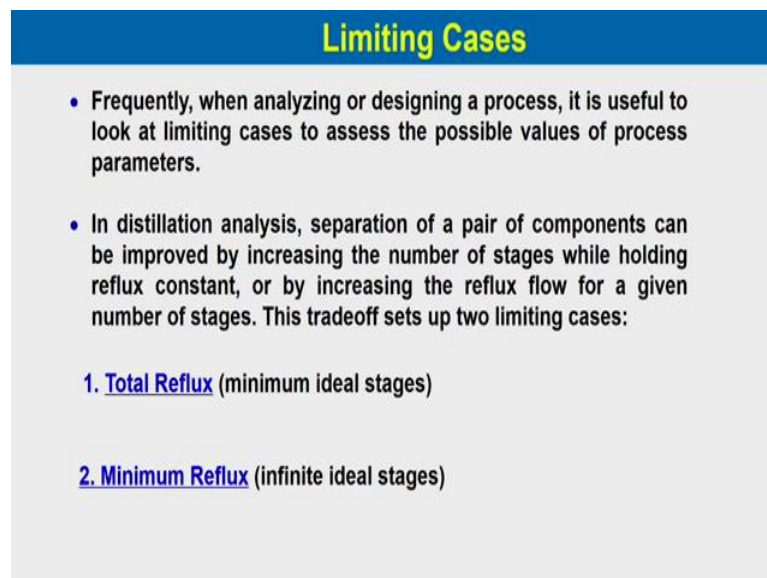
From the equations which is given y is equal to if we substitute α which is 2.36, so this equation would be y is equal to 2.36 x divided by 1 plus 1.36. So, from here we add different values of x , we can calculate y , and this is the data which is required to plot the equilibrium curve. So, with data this is the, the blue line represents here is the equilibrium line. And we know the intercept of the rectifying section operating line, so we locate on the $y = 1$ curve which is 0.38, we locate the intersection point. And composition of the distillate is given is 95 percent n-hexane. So, we know the point on the 45 degree diagonal which is 0.95, 0.95. So, with this we can plot the operating line of the rectifying section.

We also know the feed point that is Z_F , Z_F , 0.45, 0.45 on the 45 degree diagonal, and the slope of the q line is q by q minus 1 is equal to infinite for q is equal to 1. So, we can plot a vertical line over here which will intersect at this location. Since the bottom composition that is the residue composition is 5 mole percent n-hexane is known. So, point naught five, point naught five, this location is known and the intersection point

between the q line and the rectifying section operating line is known. So, we will join these points and get the rectifier stripping section operating line.

From this point, we can do the stairs case do the you know we can plot the horizontal line from the 45 degree diagonal, it will go to the if we draw the horizontal line from this 45 from this distillate points that is x_D , x_D it will go to the equilibrium curve. So, this will give the first tray and then go vertically to the operating line and then horizontally this will give step stage 2. And similar way if we just proceed the staircase arrangement, we will obtain about 6 number of ideal trays required for the separation.

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Limiting Cases

- Frequently, when analyzing or designing a process, it is useful to look at limiting cases to assess the possible values of process parameters.
- In distillation analysis, separation of a pair of components can be improved by increasing the number of stages while holding reflux constant, or by increasing the reflux flow for a given number of stages. This tradeoff sets up two limiting cases:
 1. **Total Reflux** (minimum ideal stages)
 2. **Minimum Reflux** (infinite ideal stages)

Now, we will considered limiting cases. Frequently, when analysing or designing a process, it is useful to look at limiting cases to assess the possible values of process parameters. In distillation analysis, separation of a pair of components can be improved by increasing the number of stages while holding the reflux constant or by increasing the reflux slope for a given number of stages. This tradeoff sets up two limiting cases. One is total reflux that is minimum number of stages, and another one is minimum reflux infinite number of stages.

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Limiting Cases

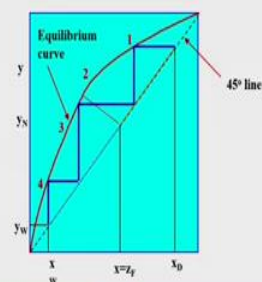
- The design tradeoff between reflux and stages is the standard economic optimization problem chemical engineers always face.
- Balancing capital costs (the number of trays to be built) vs. the operating cost (the amount of reflux to be recirculated).
- A good design will operate near a cost optimum reflux ratio.

The design tradeoff between reflux and the stages is the standard economic optimization problem for the chemical engineers always face. Balancing capital cost that is the number of trays to be built versus the operating cost that is the amount of reflux to be recirculated, so this is required. A good design will operate near a cost optimum reflux ratio. So, you have to find out the optimum reflux ratio for the particular operation.

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Total reflux

- If the liquid from the overhead condenser is totally recycled to the column as reflux
- i.e. no distillate is removed from the reflux drum ($D=0$)
- The column is said to run at 'total reflux'.

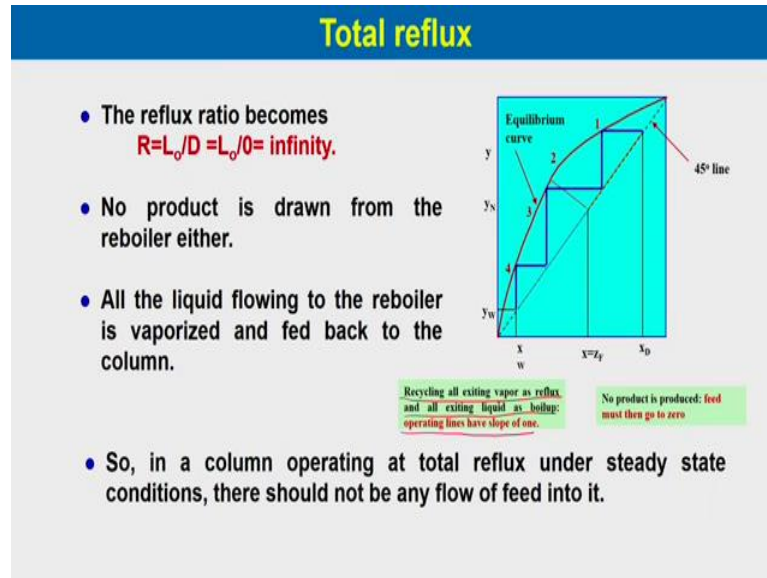


Recycling all exiting vapor as reflux and all exiting liquid as boilup: operating lines have slope of one.

No product is produced: feed must then go to zero

If the liquid from the overhead condenser is totally recycled to the column as reflux that is no distillate is removed from the reflux drum that is D is equal to 0. So, in that case, the column is said to run a total reflux.

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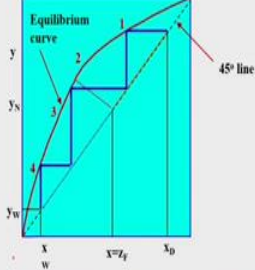


So, at total reflux, what happens the reflux ratio becomes infinite because R is equal to L naught by D . And D we are not taking any product out all the vapor condensed and return back as reflux. So, no distillate is taken out. So, the reflux ratio is infinite. So, no product is drawn from the reboiler either. All the liquid flowing to the reboiler is vaporized and feed back to the column. So, in a column operating at total reflux under steady state conditions, there should not be any flow of the feed into it. So, you can see over here, recycling all existing vapors as a reflux and all exiting liquid as boil off. So, operating line have slope of one, no product is produced that is feed must then go to 0. So, we should not give any feed during this operation.

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Total reflux

- At total reflux, the slope of the rectifying section operating line $[R/(R+1)]$ for $R \rightarrow \infty$.
- i.e. the slope of the line is unity and it passes through the point (x_D, x_D) on the diagonal. $\frac{R}{R+1} = 1$ at $R \rightarrow \infty$
- Therefore, the operating line coincides with the diagonal.



Recycling all exiting vapor as reflux and all exiting liquid as boilup: operating lines have slope of one.

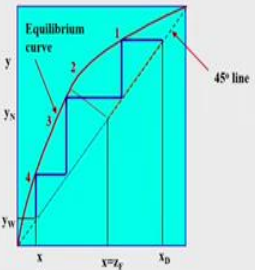
No product is produced: feed must then go to zero

At total reflux the slope of the rectifying section operating line is R by R plus 1 for R tends to infinite would be that is the slope of the line is unity. And it passes through the point x_D, x_D on the diagonal. So, slope of the operating line that is R by R plus 1, when is equal to 1 at R tends to infinite. Therefore, the operating line coincides with the diagonal. So, both the operating line both rectifying section as well as the stripping section operating line will fall on the 45 degree diagonal.

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Total reflux

- So does the stripping section operating line.
- With the operating lines on the diagonal, they are as far as they can get from the equilibrium curve.
- So if the numbers of plates are stepped off using the diagonal and the equilibrium curve, the number of theoretical stages will be a minimum.
- This gives the *theoretical minimum number* of stages to achieve a given separation.

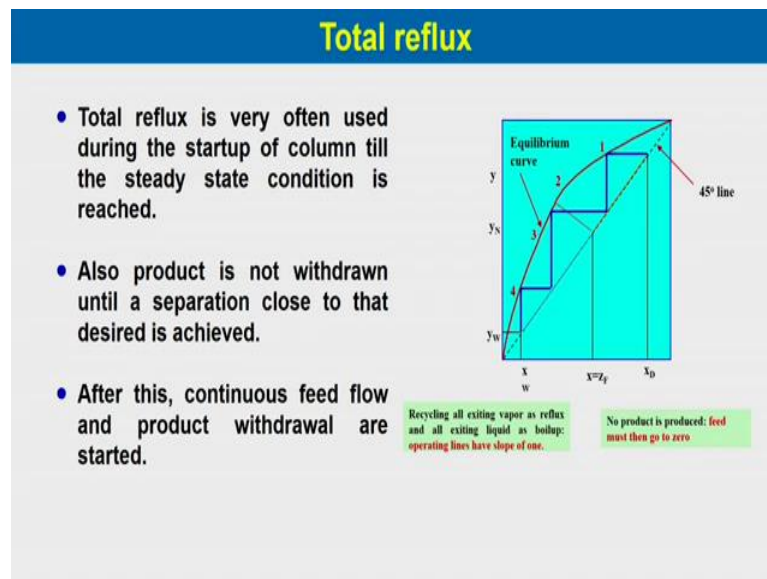


Recycling all exiting vapor as reflux and all exiting liquid as boilup: operating lines have slope of one.

No product is produced: feed must then go to zero

So does the stripping section operating line. With the operating lines on the diagonal they are far as they can get from the equilibrium curve. If the number of plates are stepped off using the diagonal and the equilibrium curve, the number of theoretical stages will be the a minimum, because of the maximum driving force that is 45 degree diagonal and the equilibrium line that is the maximum driving force, so the number of trays required in this case will be total reflux minimum under conditions. This gives the theoretical minimum number of stages to achieve a given separation.

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Total reflux is very often used during the startup of column till the steady state condition in reached. Also product is not withdrawn until a separation close to that desired is achieved. After this, continuous feed flow and product withdrawal are started.

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Fenske equation

- The *Fenske Equation* is another method for determining the minimum number of trays required for a given separation.
- It is an example of a "shortcut" distillation method.
- There are a number of these approximate methods available to get initial estimates of distillation requirements.
- This equation can be used to theoretically calculate minimum number of trays if the relative volatility remains reasonably constant.

Now, we will consider Fenske Equation is another method for determining the minimum number of trays required for a given separation. This is the analytical method. It is an example of shortcut distillation method. There are a number of these approximate methods available to get initial estimate of distillation requirements. This equation can be used to theoretically calculate minimum number of trays if the relative volatility remains reasonably constant throughout the column.

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Fenske equation

- Let N_m be the minimum number of trays in the column. Besides, there is a *total reboiler*.
- If α_w is the relative volatility of A at the reboiler temperature and pressure
- x_w and y_w are the equilibrium liquid and vapour concentration in the reboiler, then by definition:

$$\frac{y_w}{1-y_w} = \alpha_w \frac{x_w}{1-x_w}$$

Now, let N_m be the minimum number of trays in the column. Besides, there is a total reboiler. If α_W is the relative volatility of A at the reboiler temperature and pressure; x_W and y_W are the equilibrium liquid and vapor concentration in the reboiler, then by definition we can write y_W by $1 - y_W$ would be equal to α_W into x_W by $1 - x_W$.

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Fenske equation

- The vapour leaving the reboiler and entering the lowest tray (tray number N_m in this case) has a mole fraction y_w of the component A.
- The liquid leaving this tray has a composition x_{N_m}
- So the point (x_{N_m}, y_w) lies on the operating line.
- Because the operating line coincides with the diagonal at total reflux: $x_{N_m} = y_w$

The vapor leaving the reboiler and entering the lowest tray that is the tray number n_m in this case has a mole fractions of y_W of the component A. The liquid leaving this tray has also composition x_{N_m} . So, the point x_{N_m} and y_W lies on the operating line. Because the operating line coincides with the diagonal at total reflux, so we can write x_{N_m} would be equal to y_W .

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Fenske equation

Now, $\frac{y_W}{1-y_W} = \alpha_W \cdot \frac{x_W}{1-x_W}$ Becomes: $\frac{x_{N_m}}{1-x_{N_m}} = \alpha_W \cdot \frac{x_W}{1-x_W}$ ✓

- Applying the same procedure to the case of tray number N_m

$$\frac{y_{N_m}}{1-y_{N_m}} = \alpha_{N_m} \cdot \frac{x_{N_m}}{1-x_{N_m}} = \alpha_{N_m} \cdot \alpha_W \cdot \frac{x_W}{1-x_W}$$

- Similarly for tray number $N_m - 1$

$$\frac{y_{N_m-1}}{1-y_{N_m-1}} = \alpha_{N_m-1} \cdot \frac{x_{N_m-1}}{1-x_{N_m-1}} = \alpha_{N_m-1} \cdot \alpha_{N_m} \cdot \alpha_W \cdot \frac{x_W}{1-x_W}$$

Now, if we substitute this in this relation x is equal to x_{N_m} is equal to y_W , then this equation becomes $x_{N_m} / (1 - x_{N_m})$ would be equal to $\alpha_W \cdot x_W / (1 - x_W)$. Now, applying the same procedure to the case of tray number N_m , we can write y_{N_m} from this equation, $y_{N_m} / (1 - y_{N_m})$ would be equal to $\alpha_{N_m} \cdot x_{N_m} / (1 - x_{N_m})$. Now, if we substitute this x_{N_m} by $1 - x_{N_m}$ from this we can write $\alpha_{N_m} \cdot (1 - x_{N_m}) / (1 - x_{N_m}) = \alpha_W \cdot x_W / (1 - x_W)$. Similarly, for tray number $n - 1$, we can write $y_{N_m - 1} / (1 - y_{N_m - 1})$ is equal to $\alpha_{N_m - 1} \cdot x_{N_m - 1} / (1 - x_{N_m - 1})$, which is equal to $\alpha_{N_m - 1} \cdot \alpha_{N_m} \cdot \alpha_W \cdot x_W / (1 - x_W)$.

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Fenske equation

Note that

- ✓ The point (x_{Nm-1}, y_{Nm})
 - Lies on the operating line which coincides with the diagonal
- ✓ Therefore, $x_{Nm-1} = y_{Nm}$
 - Continuing the procedure up to the top tray (where $y_1 = x_D$)

The point x_{Nm-1}, y_{Nm} lies on the operating line which coincides with the 45 degree diagonal. Therefore, x_{Nm-1} would be equal to y_{Nm} continuing the procedure up to the top tray where y_1 is equal to x_D .

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Fenske equation

$$\frac{x_D}{1-x_D} = \frac{y_1}{1-y_1} = \alpha_1 \alpha_2 \dots \alpha_{Nm} \cdot \alpha_W \cdot \frac{x_W}{1-x_W}$$

$$\Rightarrow \frac{x_D}{1-x_D} = (\alpha_{av}^{Nm+1}) \cdot \frac{x_W}{1-x_W} \Rightarrow \alpha_{av}^{Nm+1} = \frac{x_D(1-x_W)}{x_W(1-x_D)}$$

$$i.e. \quad Nm + 1 = \frac{\log \frac{x_D(1-x_W)}{x_W(1-x_D)}}{\log \alpha_{av}} \Rightarrow Nm = \frac{\log \frac{x_D(1-x_W)}{x_W(1-x_D)}}{\log \alpha_{av}} - 1$$

Here α_{av} is the average relative volatility of the more volatile component.

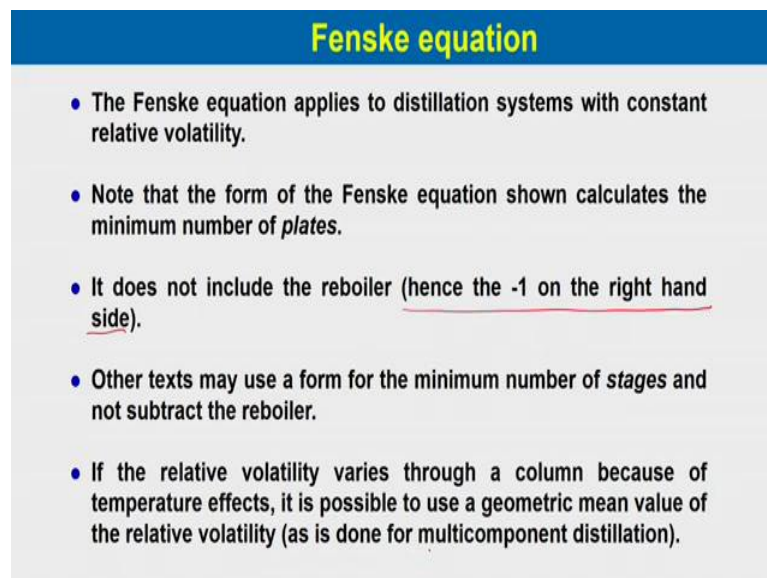
- The above equation is called *Fenske's equation*, which is useful for the calculation on the minimum number of trays.

We can write x_D by $1 - x_D$ who is equal to y_1 divided by $1 - y_1$ and we can write this would be equal to $\alpha_1 \alpha_2 \dots \alpha_{Nm} \alpha_W$ into x_W by $1 - x_W$. We can simplify it as x_D by $1 - x_D$ is equal to α_{av}^{Nm+1} into x_W divided by $1 - x_W$ that if you just

rearrange this equation, this would be alpha average to the power $N + 1$ is equal to x_D into $1 - x_W$ divided by x_W into $1 - x_D$.

Now, taking log for the both sides, we can write $N + 1$ would be equal to \log of x_D into $1 - x_W$ divided by x_W into $1 - x_D$ whole divided by \log alpha average, and also this we can also write as N is equal to $\log x_D$ into $1 - x_W$ divided by x_W into $1 - x_D$ by \log alpha average minus 1. Here alpha average is the average relative volatility of the more volatile component. The above equation is called Fenske's equation, which is useful for the calculation on the minimum number of trays.

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Fenske equation

- The Fenske equation applies to distillation systems with constant relative volatility.
- Note that the form of the Fenske equation shown calculates the minimum number of *plates*.
- It does not include the reboiler (hence the -1 on the right hand side).
- Other texts may use a form for the minimum number of *stages* and not subtract the reboiler.
- If the relative volatility varies through a column because of temperature effects, it is possible to use a geometric mean value of the relative volatility (as is done for multicomponent distillation).

The Fenske equation applies to distillation systems with constant relative volatility. Please note that the form of the Fenske equation shown calculates the minimum number of plates. It does not include the reboiler hence minus 1 on the right hand side of the equations derived earlier. Other texts may use a form for the minimum number of stages and not subtract the reboiler. If the relative volatility varies through a column because of temperature effects, it is possible to use a geometric mean value of the relative volatility as is done for multicomponent distillation. So, this is possible to use the geometric mean values of the relative volatility.

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Fenske equation

- The Fenske equation is used to get an approximate value for the number of stages.

The Fenske equation is used to get an approximate value for the number of ideal stages.

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Example 2

A mixture of 45 mole % n-hexane and 55 mole % n-heptane is subjected to continuous fraction in a tray column at 1 atm total pressure. The distillate contains 95% n-hexane and the residue contains 5% n-hexane. The feed is saturated liquid. The average relative volatility is 2.36. Determine the number of ideal trays using Fenske equation.

Solution

- The distillation contains 95% n-hexane
- Distillate (x_D, x_D) is (0.95, 0.95) ✓
- Feed mixture contains 45 mole % n-hexane
- Feed (x_F, x_F) is (0.45, 0.45) ✓
- ✓ Residue contains 5% mole of n-hexane (x_W, x_W) is (0.05, 0.05)
- ✓ Average relative volatility of n-hexane is 2.36.

Let us considered an example. A mixture of 45 mole percent n-hexane and 55 more percent n-heptane is subjected to continuous fraction in a tray column at 1 atmosphere total pressure. The distillate contains 95 percent of n-hexane and the residue contains 5 percent n-hexane. The feed is saturated liquid. The average relative volatility is 2.36. Now, determine the number of ideal trays using Fenske equation.

The distillation which distillates which contains 95 percent of n-hexane distillate compositions will be x_D , x_D which is 0.95, 0.95 feed mixture contains 45 mole percent n-hexanes. So, feed point would be at x_F , x_F which is 0.45, 0.45, these two are non residue contents 5 mole percent n-hexane. So, x_W , x_W is known that is point naught five, point naught five. And average relative volatility of n-hexane is given which is 2.36.

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Example 2

A mixture of 45 mole % n-hexane and 55 mole % n-heptane is subjected to continuous fraction in a tray column at 1 atm total pressure. The distillate contains 95% n-hexane and the residue contains 5% n-hexane. The feed is saturated liquid. The average relative volatility is 2.36. Determine the number of ideal trays using Fenske equation.

Solution • Fenske Equation

$$N_m + 1 = \frac{\log \frac{x_D(1-x_W)}{x_W(1-x_D)}}{\log \alpha_{av}}$$

$$N_m + 1 = \frac{\log \frac{x_D(1-x_W)}{x_W(1-x_D)}}{\log \alpha_{av}} = \frac{\log \frac{0.95(1-0.05)}{0.05(1-0.95)}}{\log 2.36} = 6.8$$

Fenske equation, we can write $N_m + 1$ is equal to $\log \frac{x_D(1-x_W)}{x_W(1-x_D)}$ divided by $\log \alpha_{av}$. If we substitute the values for x_D is 0.95, for x_W is point naught five, and for α_{av} is 2.36. So, if we calculate it will give 6.8. So, the number of trays including reboiler would be 6.8, excluding reboiler it will be 5.8. This is the way by which we can use the Fenske equation and calculate the number of ideal trays required for a particular distillation operation.

Thank you for your patience hearing. And we will continue our discussion on distillation in the next lecture.