Mass Transfer coefficient
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Lecture –15
Interphase mass transfer and material balance for operating line

Welcome to the 7th lecture of module 2, on Mass Transfer Operation. Before going to this lecture, let us have a brief recap on our last lecture. We have started with interface mass transfer and we have discussed the fundamental concept of interfacial mass transfer. We considered major component over here is equilibrium between the phases and then we considered Roult's law and the third thing we considered is Henry's law.

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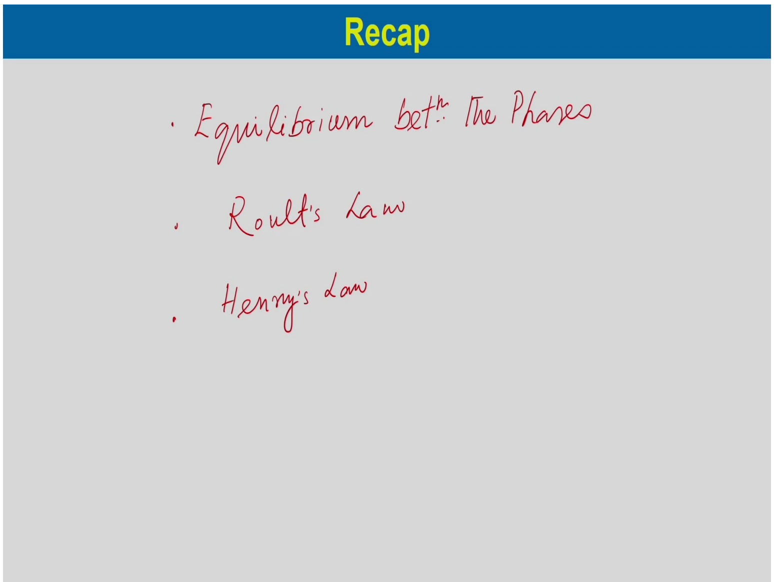

Recap

- Equilibrium bet^{n} The Phases
- Roult's Law
- Henry's Law

We will continue our discussion on Interface Mass Transfer. In this lecture, we will consider Lewis and Whitman Two-film Theory for the mass transfer between the phases.

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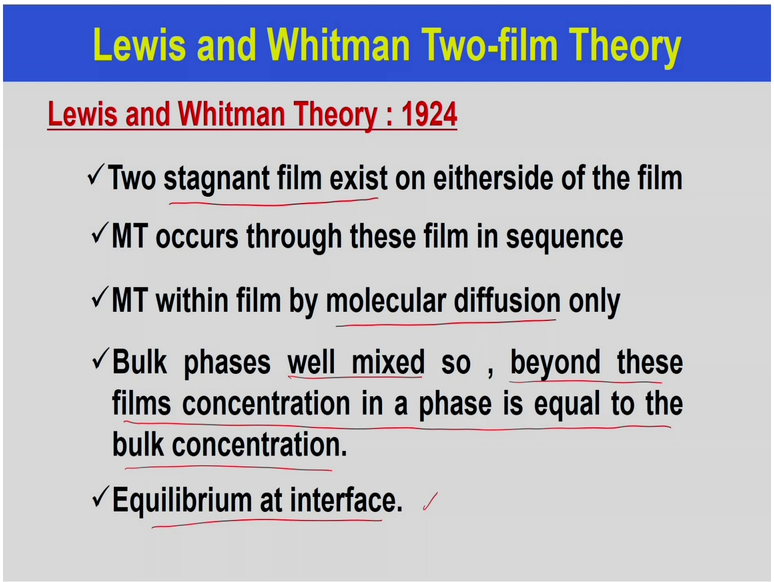

Lewis and Whitman Two-film Theory

Lewis and Whitman Theory : 1924

- ✓Two stagnant film exist on eitherside of the film
- ✓MT occurs through these film in sequence
- ✓MT within film by molecular diffusion only
- ✓Bulk phases well mixed so , beyond these films concentration in a phase is equal to the bulk concentration.
- ✓Equilibrium at interface.

Lewis and Whitman proposed this theory in 1924 for the interface mass transfer or it is known as two film theory. Here the following you know things has been considered two stagnant film exist on either side of the film.

So, first thing is considered two stagnant film. So, note that the films are considered as stagnant, on either side of the film or either side of the interface. Mass transfer occurs through the film in sequence. Mass transfer within the film by molecular diffusion only, so molecular diffusion occurs within the film. Bulk phases are considered well mixed and beyond this the films concentration in a phase is equal to the bulk concentration. So, beyond the film that is know both gas side film as well as the liquid side film beyond this films the gas phase or in the liquid phase both are considered now well mixed and the concentration which will reach there is the bulk concentration of that component in that phase, equilibrium at interface.

So, these are the things or assumptions considered under Lewis and Whitman theory which is proposed in 1924.

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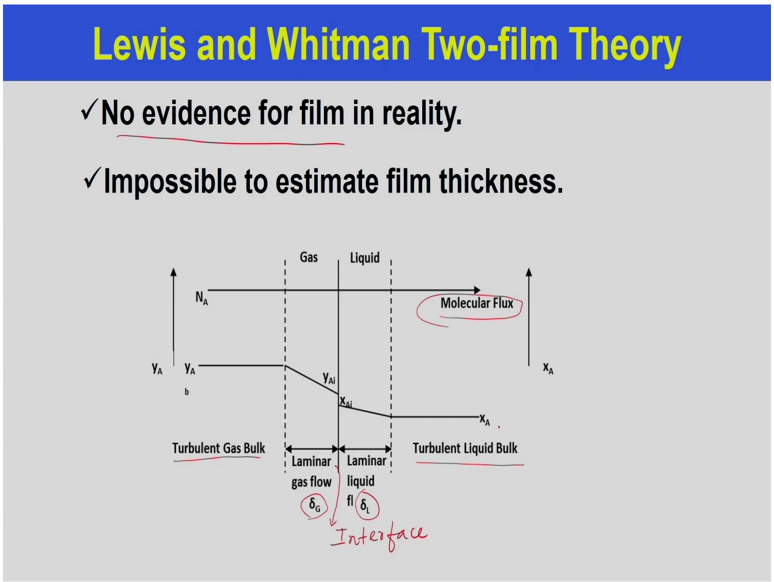

Lewis and Whitman Two-film Theory

- ✓No evidence for film in reality.
- ✓Impossible to estimate film thickness.

But now in reality there is no evidence of the film. So, which we could not see the film exist between the phases and it is impossible to estimate the film thickness, very difficult to measure the film thickness. So, the transfer mechanism which are proposed is you can see this is a know bulk, gas bulk and this is the liquid bulk, and liquid bulk is considered as turbulent liquid bulk, gas bulk is considerable as turbulent gas bulk and this is the interface. So, this is interface.

And on the liquid side there is a laminar liquid film which is the thickness, it is considered as delta L. In the gas side there is also laminar gas film which is considered as delta G. There is a flux gas is absorbing and we considered the solute a is the diffusing or is transferring from the gas phase to the liquid phase. And in the film the diffusion or mass transfer happens through molecular diffusion or molecular flux. You can see the concentration at the bulk is y Ab and concentration at the interface is y Ai in the gas side in the liquid side is x A i and the liquid bulk is x A.

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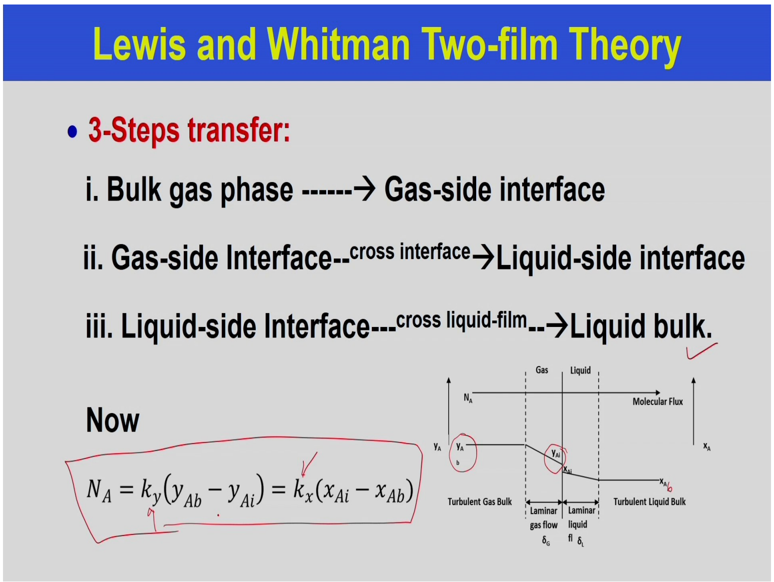

Lewis and Whitman Two-film Theory

- **3-Steps transfer:**

  i. Bulk gas phase ------→ Gas-side interface

  ii. Gas-side Interface--$^{\text{cross interface}}$→Liquid-side interface

  iii. Liquid-side Interface---$^{\text{cross liquid-film}}$--→Liquid bulk.

Now

$$\boxed{N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab})}$$

So, the 3-step transfer which we already discussed over the bulk gas phase to the gas side interface. Gas side interface then it transfer or know transfer through the interface, so it goes to the liquid side interface. So, that is the interfacial transfer. Then liquid side interface to the liquid film through the liquid film it goes to the liquid bulk. So, these are the 3 transfer steps generally involved for transfer of the component from the gas phase to the liquid phase.

Now, N A is the flux and if k y is the mass transfer coefficient in the gas phase based on the mole fraction unit k y which into the driving force in the gas phase that is y Ab minus y Ai that is the bulk concentration over here and y Ai which is over here. So, this is the driving force into the mass transfer coefficient in the gas phase.

Similarly, in the liquid phase it will be the flux would be equal to k x is the liquid side mass transfer coefficient into the concentration gradient that is concentration of component A at the gas liquid interface that is x Ai would be higher than the bulk concentration of that solute in the liquid. So, the driving force is x Ai minus x Ab. So, the bulk concentration and this since there is no resistance at the interface or interface at equilibrium, it is considered as instantaneous transfer and hence, we can write the flux of the both phases would be equal, so that is related with this relation.

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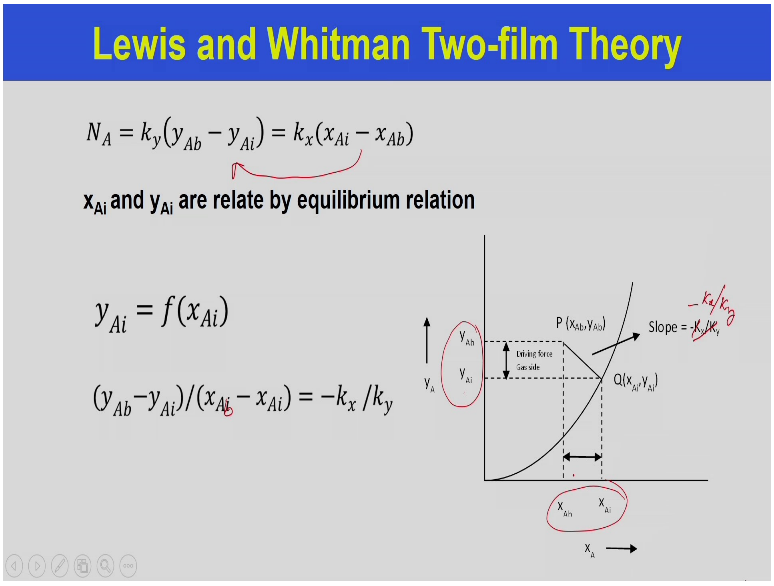

Lewis and Whitman Two-film Theory

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab})$$

x_{Ai} and y_{Ai} are relate by equilibrium relation

$$y_{Ai} = f(x_{Ai})$$

$$(y_{Ab} - y_{Ai})/(x_{A{i}_{b}} - x_{Ai}) = -k_x/k_y$$

Now, here x Ai and y Ai are relate by the equilibrium relation. So, equilibrium relation is y Ai is equal to the function of x Ai. So, the interfacial concentration as we considered in the Lewis and Whitman theory there is equilibrium at the interface. So, y Ai would be function of x Ai. If we rearrange this relation then we can get y Ab minus y Ai divided by x Ab minus x Ai would be equal to minus k x by k y.

Consider all these parameters feature in small x and small y term. So, there is a type of. So, slope is minus small k x by small k y and the driving force in the know gas side you can see the gas side driving force is y Ab minus y Ai. So, this is the gas side driving force and this is the liquid side driving force.

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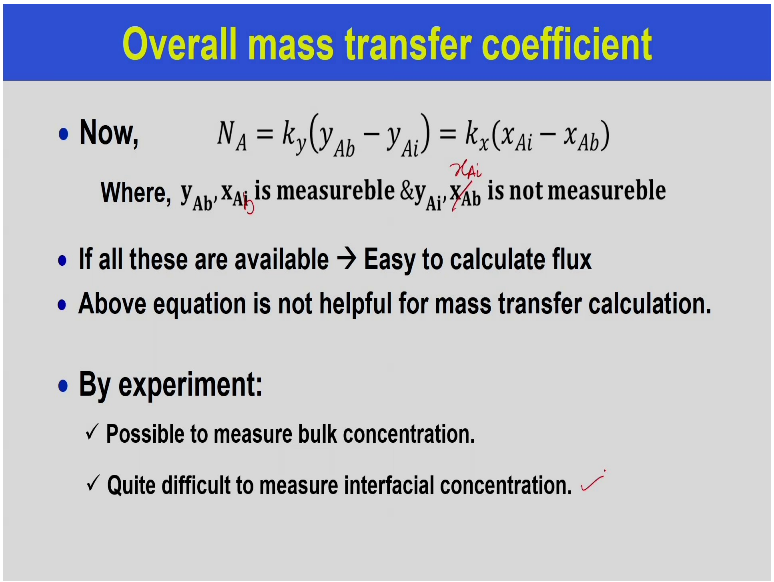

Overall mass transfer coefficient

- **Now,** $N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab})$

  Where, y_{Ab}, $\underset{b}{x_{Ai}}$ is measureble & y_{Ai}, $\overset{x_{Ai}}{\cancel{x_{Ab}}}$ is not measureble
- If all these are available → Easy to calculate flux
- Above equation is not helpful for mass transfer calculation.
- **By experiment:**
  - ✓ Possible to measure bulk concentration.
  - ✓ Quite difficult to measure interfacial concentration. ✓

So, if you consider this relation where y Ab and x Ab these are measurable quantity whereas y Ai and x Ai is not measurable quantities because these are interfacial concentration. So, it is very difficult to measure these interfacial concentration. If all these are available, then easy to calculate the flux. So, if I can calculate the interfacial concentration x Ai and know y Ai then it would be easy to use this flux to calculate the flux, but these equations is not helpful for mass transfer calculation because the interfacial concentration is not known to us. So, we have to depend on the measurements to calculate the flux and it is possible to measure experimentally the bulk concentration.

So, if we can relate the flux equation with respect to only bulk concentration or the measurable quantity it would be easy for us to calculate the flux. So, as we said the it is quite difficult to measure the concentration.

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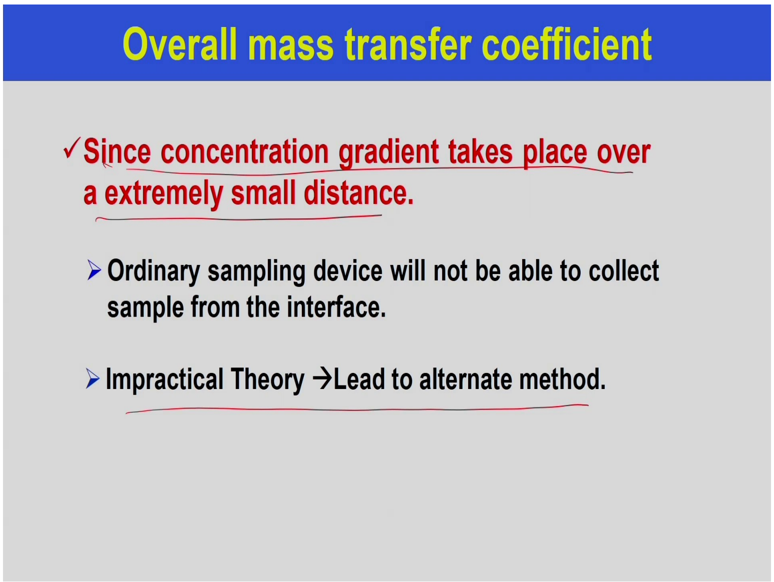

Overall mass transfer coefficient

- ✓ Since concentration gradient takes place over a extremely small distance.
  - Ordinary sampling device will not be able to collect sample from the interface.
  - Impractical Theory →Lead to alternate method.

So, what to do in that case? Since, the concentration gradient takes place over a extremely small, very small distance it is the ordinary device cannot measure the interfacial concentration because of this region this theory is impractical and which lead to follow some alternate method.

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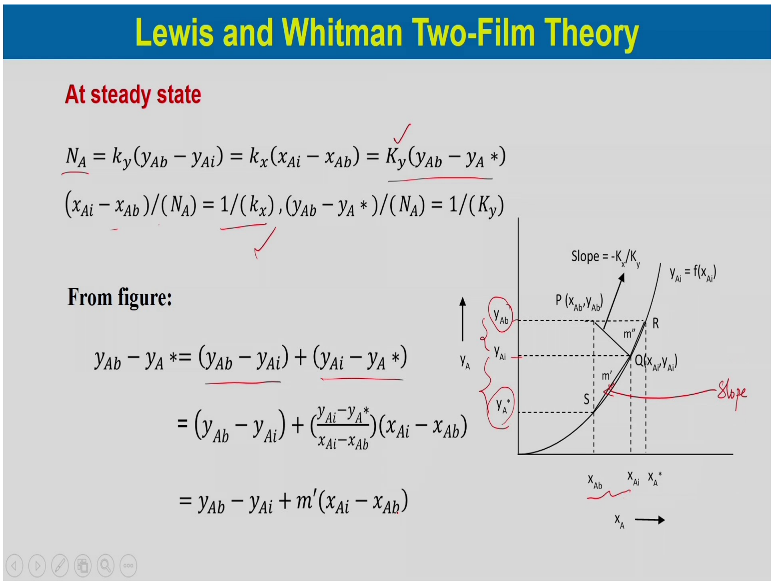

Lewis and Whitman Two-Film Theory

At steady state

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab}) = K_y(y_{Ab} - y_A*)$$

$$(x_{Ai} - x_{Ab})/(N_A) = 1/(k_x), (y_{Ab} - y_A*)/(N_A) = 1/(K_y)$$

From figure:

$$y_{Ab} - y_A* = (y_{Ab} - y_{Ai}) + (y_{Ai} - y_A*)$$

$$= (y_{Ab} - y_{Ai}) + \left(\frac{y_{Ai} - y_A*}{x_{Ai} - x_{Ab}}\right)(x_{Ai} - x_{Ab})$$

$$= y_{Ab} - y_{Ai} + m'(x_{Ai} - x_{Ab})$$

Now, if we consider at steady state what happens? The N A we can write k y into y Ab minus y Ai would be equal to k x x Ai minus x Ab would be equal to capital K y into y

Ab minus y A star because our transfer is happening from the gas phase to the liquid phase.

So, if we can get the equilibrium values, the overall driving force from the gas phase to the liquid phase is from y Ab to y A star. So, that is why we can write, so this capital K y include the the mass transfer coefficient both in the gas phase and in the liquid phase or the resistance involve in both the phases. So, from this relation we can just rearrange this equation we can write from here is x Ai minus x Ab by N A which we can write 1 by k x and y Ab minus y A star divided by N A from this we can write and this we can write y Ab minus y A star by N A would be equal to 1 by capital K y.

If you look into this figure y Ab minus y A star that is this one and this one would be equal to y A y Ab minus y Ai, so the distance from here to here plus y Ai minus y A star. So, this distance and this distance. So, if you just rearrange y Ab minus y Ai, plus if we multiply both numerator and denominator by x Ai minus x Ab, and this will be y Ai minus y A star divided by x Ai minus x Ab into x Ai minus x Ab. From the figure we can see y Ai minus y A star divided by x Ai minus x Ab that is from this. So, this represents the slope of this line, so which is m dash. So, we can write y Ab minus y Ai plus m dash x Ai minus x Ab.

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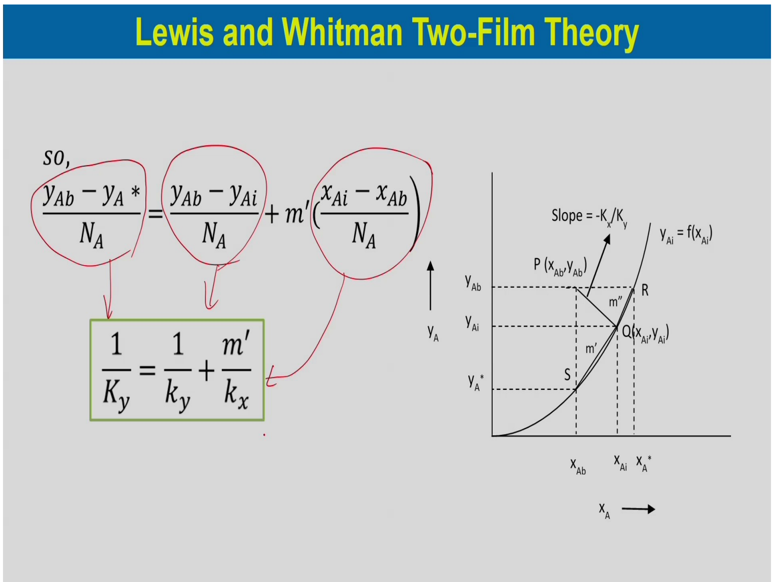

Lewis and Whitman Two-Film Theory

so,

$$\frac{y_{Ab}-y_A^*}{N_A}=\frac{y_{Ab}-y_{Ai}}{N_A}+m'\left(\frac{x_{Ai}-x_{Ab}}{N_A}\right)$$

$$\boxed{\frac{1}{K_y}=\frac{1}{k_y}+\frac{m'}{k_x}}$$

Now, y Ab minus y A star by N A would be equal to y Ab minus y Ai divided by N A plus m dash x Ai minus x Ab divided by N A. From this we can write this is equal to 1 by capital K and this is 1 by small k y and this is 1 by small k x into m dash. So, 1 by K y capital K y would be equal to 1 by small k y plus m dash by k x small k x.

So, this is if we get the overall mass transfer coefficient. So, this is the relation between the overall mass transfer coefficient and the individual mass transfer coefficient that is mass transfer coefficient in each phase. And this is basically in terms of the resistance in series model, so 1 by capital K y is the overall resistance for the mass transfer from the gas phase to the liquid phase; 1 by k y is the gas phase mass transfer resistance and m dash by k x is the resistance in the liquid phase.

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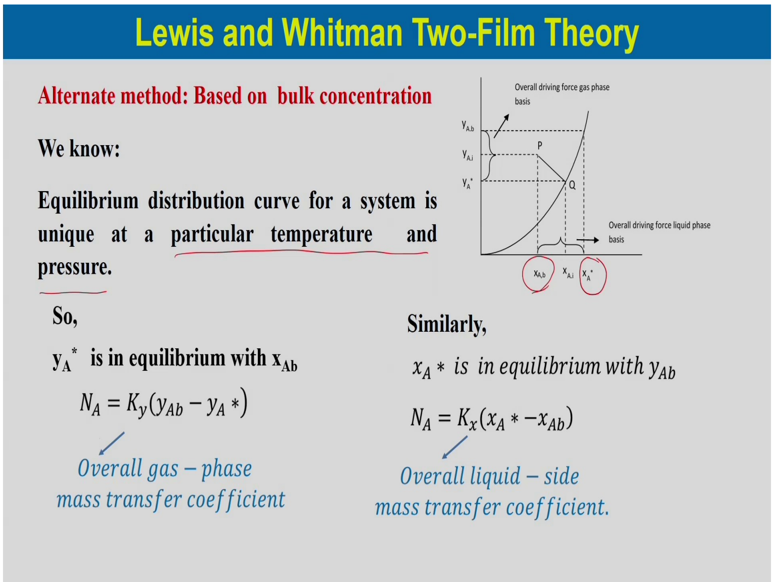

Lewis and Whitman Two-Film Theory

Alternate method: Based on bulk concentration

We know:

Equilibrium distribution curve for a system is unique at a particular temperature and pressure.

So,

y_A^* is in equilibrium with x_{Ab}

$$N_A = K_y(y_{Ab} - y_A *)$$

Overall gas – phase mass transfer coefficient

Similarly,

$x_A *$ is in equilibrium with y_{Ab}

$$N_A = K_x(x_A * - x_{Ab})$$

Overall liquid – side mass transfer coefficient.

Now, alternate method based on the bulk concentration. Equilibrium distribution curve for a system is unique at a particular temperature and pressure. So, because once the temperature and pressure is specified and we can get the equilibrium distribution curve at the temperature for a particular system. So, y A star is in equilibrium with x Ab, this x Ab would be in equilibrium with the equilibrium concentration that is y A star. So, we can write flux N A would be equal to capital K y into y Ab minus y A star. So, this capital K y is the overall gas phase mass transfer coefficient.

Now, similarly in case of the liquid phase we can write x A star is in equilibrium with y A b. So, we can write N A would be equal to K x x A star minus x Ab and this capital K x is the overall liquid side mass transfer coefficient. So, we can define the flux with respect to the measurable quantities, that is the equilibrium concentration and the bulk concentration.

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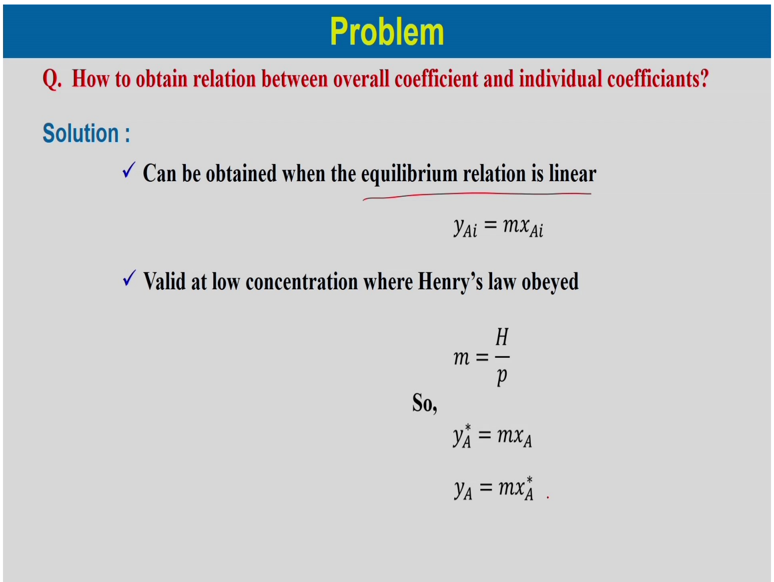

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution :

- ✓ Can be obtained when the equilibrium relation is linear

$$y_{Ai} = mx_{Ai}$$

- ✓ Valid at low concentration where Henry's law obeyed

$$m = \frac{H}{p}$$

So,

$$y_A^* = mx_A$$

$$y_A = mx_A^*$$

Now, how to obtain relation between the overall coefficient and individual coefficient? We have already discussed before the relations are derived the relations between the individual mass transfer coefficient and the overall mass transfer coefficient and this can be possible only when the equilibrium relation is linear. So, if the equilibrium relations is linear then only we can relate with the overall mass transfer coefficient. Like y Ai would be equal to m into x Ai, and it is valid at low concentration where the Henry's law is obeyed that means, m is equal to H by partial pressure, or that component. So, y A star would be equal to mx A and we can write y A is equal to mx A star.

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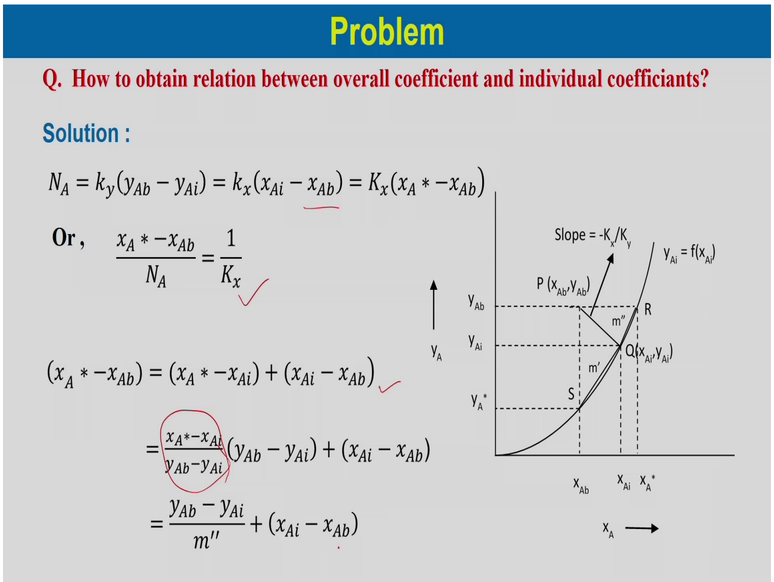

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution :

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab}) = K_x(x_A * - x_{Ab})$$

Or , $$\frac{x_A * - x_{Ab}}{N_A} = \frac{1}{K_x}$$

$$(x_A * - x_{Ab}) = (x_A * - x_{Ai}) + (x_{Ai} - x_{Ab})$$

$$= \frac{x_A * - x_{Ai}}{y_{Ab} - y_{Ai}}(y_{Ab} - y_{Ai}) + (x_{Ai} - x_{Ab})$$

$$= \frac{y_{Ab} - y_{Ai}}{m''} + (x_{Ai} - x_{Ab})$$

Now, from the same relation we can just rearrange from this figure N A is equal to k y y Ab minus y Ai is equal to k x x Ai minus x Ab is equal to capital K x x A star minus x A b. So, we can rearrange these x A star minus x Ab by N A would be equal to 1 by capital K x. So, similarly like earlier case we have derived we can write this relation x A star minus x Ab would be equal to x A star minus x Ai from this figure in the liquid side plus x Ai minus x Ab. Now, if we multiply both numerator and denominator with y Ab minus y Ai then we can write x A star minus x Ai divided by y Ab minus y Ai into y Ab minus y Ai plus x Ai minus x Ab.

So, this ratio is the nothing but over here you can see is m double dash. So, m double dash is y Ab minus y Ai into divided by x A star minus x Ai. So, we can substitute over here and we will obtain y Ab minus y Ai divided by m double dash plus x Ai minus x Ab.

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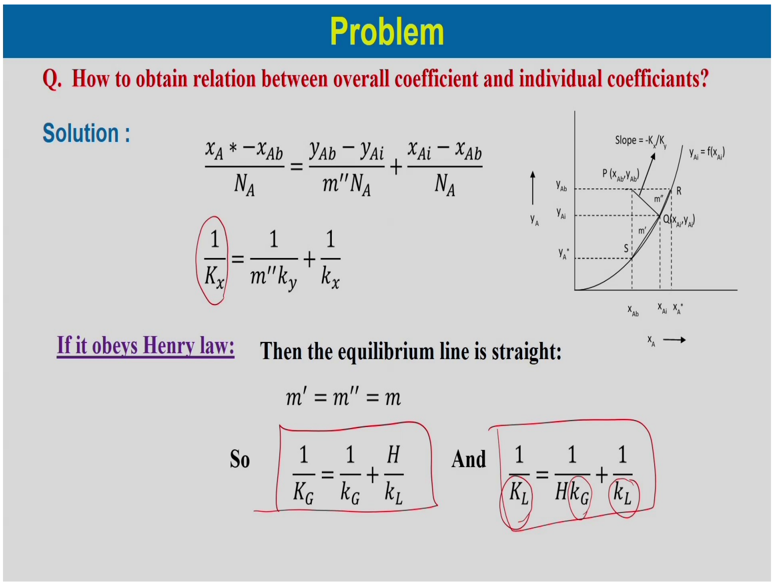

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution :

$$\frac{x_A* - x_{Ab}}{N_A} = \frac{y_{Ab} - y_{Ai}}{m''N_A} + \frac{x_{Ai} - x_{Ab}}{N_A}$$

$$\frac{1}{K_x} = \frac{1}{m''k_y} + \frac{1}{k_x}$$

If it obeys Henry law: **Then the equilibrium line is straight:**

$$m' = m'' = m$$

So $$\boxed{\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L}}$$ **And** $$\boxed{\frac{1}{K_L} = \frac{1}{Hk_G} + \frac{1}{k_L}}$$

So, from this relation we can just write the equation of individual mass transfer coefficient and overall mass transfer coefficient based on the liquid phase mass transfer coefficient. So, over all mass transfer coefficient based on liquid phase that is 1 by capital K x would be equal to 1 by m double dash k y plus 1 by K x. If it obeys Henry's law then the equilibrium line is straight and this m dash would be equal to m double dash equal to m. So, in that case we can write 1 by capital K G would be equal to 1 by small k G plus H by k L.

So, when the know the equilibrium line is straight in that case this relations will be valid m dash would be equal to m double dash would be equal to m and we can get the relation between the overall gas phase mass transfer coefficient k G would be equal to 1 by K G capital K G would be equal to 1 by small k G plus H by small k L. Or similarly, we can write in terms of the overall mass transfer coefficient in the liquid phase a capital K L that is 1 by capital K L would be equal to 1 by H small k G plus 1 by small k L. That is individual mass transfer coefficient liquid phase and this is individual gas phase mass transfer coefficient, this is overall liquid phase mass transfer coefficient.

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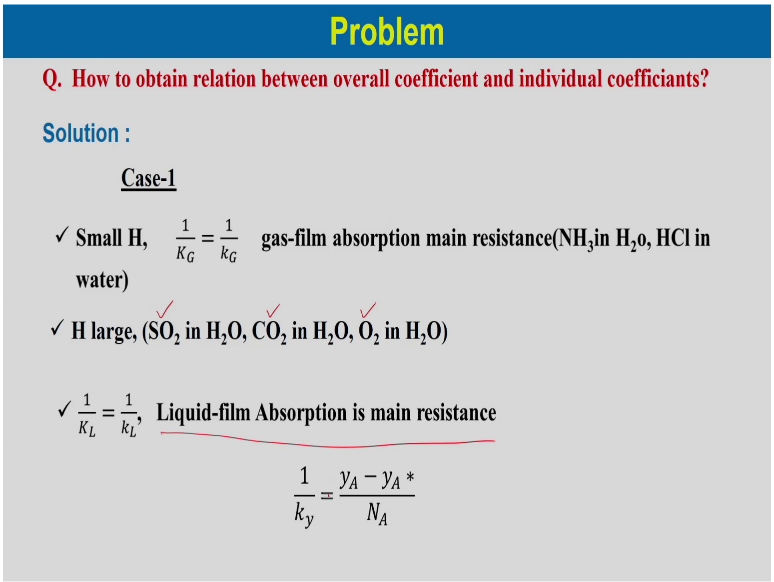

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution :

Case-1

- ✓ Small H, $\frac{1}{K_G} = \frac{1}{k_G}$ gas-film absorption main resistance(NH_3in H_2o, HCl in water)
- ✓ H large, (SO_2 in H_2O, CO_2 in H_2O, O_2 in H_2O)
- ✓ $\frac{1}{K_L} = \frac{1}{k_L}$, Liquid-film Absorption is main resistance

$$\frac{1}{k_y} = \frac{y_A - y_A *}{N_A}$$

Now, consider case 1, where H is small that is Henry's constant is small from the earlier equations we can write 1 by capital K G would be on by small k G. That means, gas film absorption main is the main resistance that is ammonia in water or HCl in water. In that case what will happen? The resistance will lie in the gas phase only but not in the liquid phase.

If H is large like SO 2 in water or carbon dioxide in water or oxygen in water that means the affinity of this component sulphur dioxide, carbon dioxide or oxygen in water is very less. So, you have a very high H value. In that case what will happen 1 by capital K L we can write is equal to 1 by k L small k L that is liquid film absorption is the main resistance. So, 1 by k y would be equal to y A minus y A star by N A. If we can just get the gas side mass transfer coefficient we can just calculate the flux.

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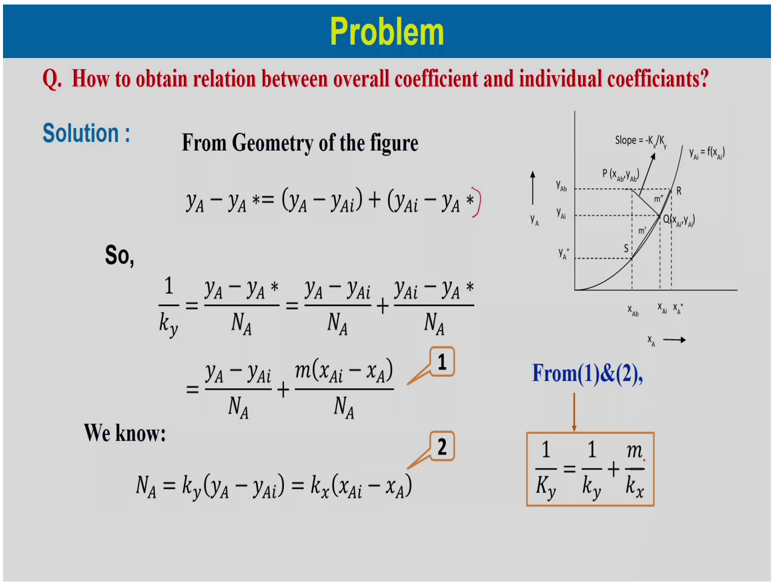

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution : From Geometry of the figure

$$y_A - y_A^* = (y_A - y_{Ai}) + (y_{Ai} - y_A^*)$$

So,

$$\frac{1}{k_y} = \frac{y_A - y_A^*}{N_A} = \frac{y_A - y_{Ai}}{N_A} + \frac{y_{Ai} - y_A^*}{N_A}$$

$$= \frac{y_A - y_{Ai}}{N_A} + \frac{m(x_{Ai} - x_A)}{N_A} \quad ①$$

We know:

$$N_A = k_y(y_A - y_{Ai}) = k_x(x_{Ai} - x_A) \quad ②$$

From(1)&(2),

$$\boxed{\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}}$$

Now, from the geometry of the figure we can write once again it is y A minus y A star is equal to y A minus y Ai plus y Ai minus y A star. So, we can rearrange it is 1 by small k y would be equal to y A minus y A star divided by N A which is equal to y A minus y Ai divided by N A plus y Ai minus y A star divided by N A. And we can write y A minus y Ai divided by N A plus m into x Ai minus x A divided by N A.

Now, we know that N A is equal to k y y A minus y Ai would be equal to k x x Ai minus x A. So, from equation 1 and 2 if we just correlate among them then we can get 1 by capital K y would be equal to 1 by small k y plus m by k x. Similarly, in case of the liquid phase we can obtain in the similar manner.

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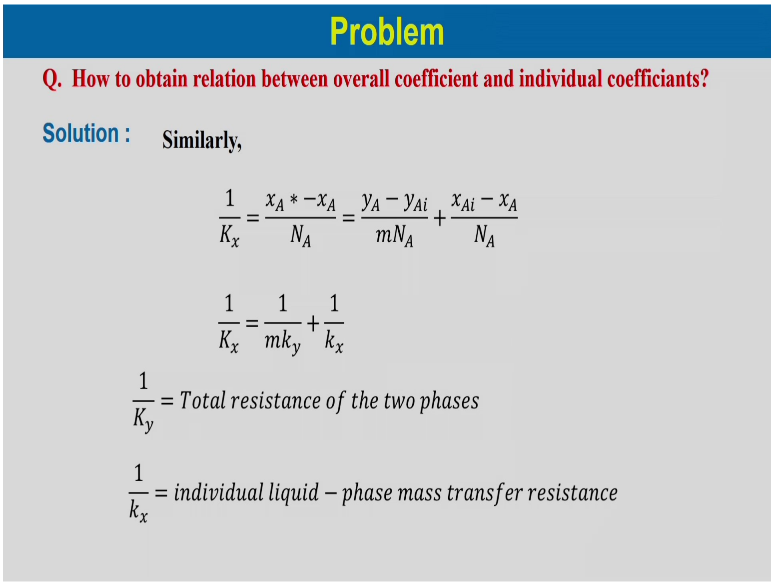

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution : Similarly,

$$\frac{1}{K_x} = \frac{x_A * -x_A}{N_A} = \frac{y_A - y_{Ai}}{mN_A} + \frac{x_{Ai} - x_A}{N_A}$$

$$\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

$$\frac{1}{K_y} = Total\ resistance\ of\ the\ two\ phases$$

$$\frac{1}{k_x} = individual\ liquid - phase\ mass\ transfer\ resistance$$

So, 1 by k y is the individual gas mass transfer coefficient, m by k x is the individual liquid mass transfer coefficient.

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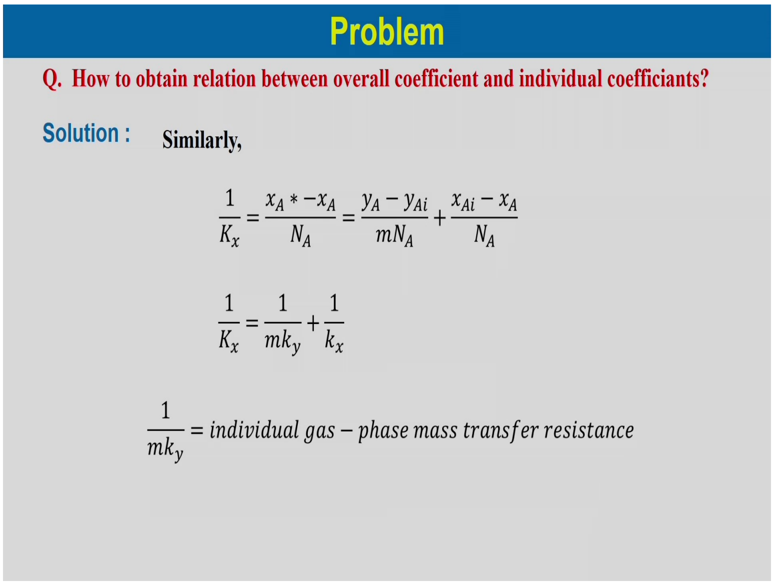

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution : Similarly,

$$\frac{1}{K_x} = \frac{x_A * -x_A}{N_A} = \frac{y_A - y_{Ai}}{mN_A} + \frac{x_{Ai} - x_A}{N_A}$$

$$\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

$$\frac{1}{mk_y} = individual\ gas - phase\ mass\ transfer\ resistance$$

1 by capital K y is the total resistance of the two phases, and 1 by small k x is the individual liquid mass transfer coefficient, 1 by m k y is the individual gas phase mass transfer resistance.

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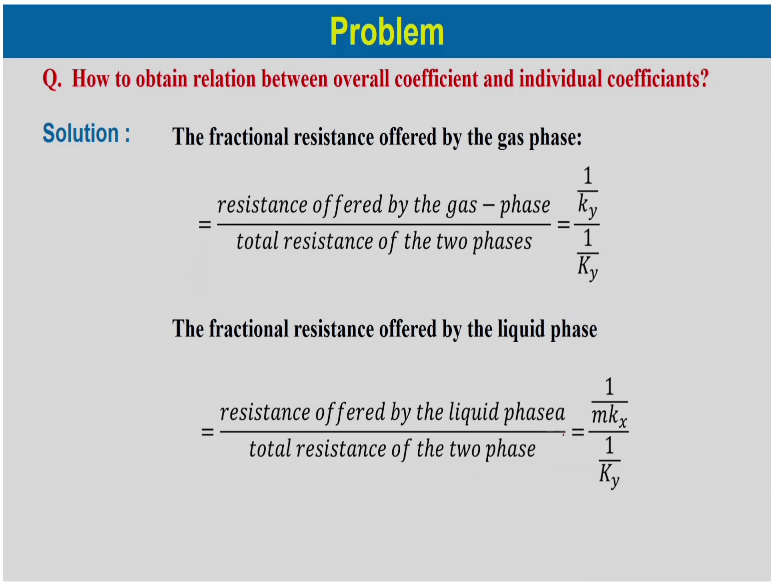

Problem

Q. How to obtain relation between overall coefficient and individual coefficiants?

Solution : The fractional resistance offered by the gas phase:

$$= \frac{resistance\ offered\ by\ the\ gas-phase}{total\ resistance\ of\ the\ two\ phases} = \frac{\frac{1}{k_y}}{\frac{1}{K_y}}$$

The fractional resistance offered by the liquid phase

$$= \frac{resistance\ offered\ by\ the\ liquid\ phasea}{total\ resistance\ of\ the\ two\ phase} = \frac{\frac{1}{mk_x}}{\frac{1}{K_y}}$$

Now, the fractional resistance offered by the gas phase we can write or we can define is the resistance offered by the gas phase divided by the total resistance of the two phases. So, the fractional resistance would be 1 by small k y divided by 1 by capital K y.

Similarly, in case of the liquid phase the fractional resistance offered by the liquid phase we can write resistance offered by the liquid phase divided by the total resistance of the two phase. So, this is equal to 1 by m small k x divided by 1 by capital K x.

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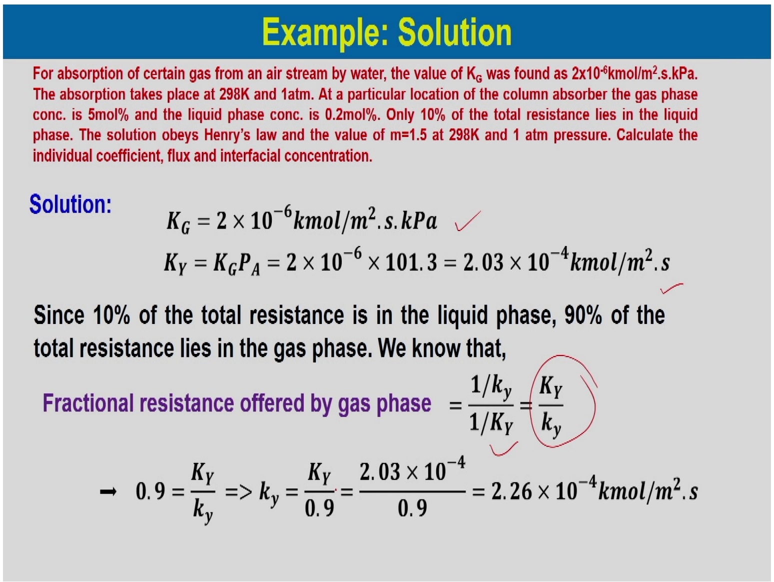

Example: Solution

For absorption of certain gas from an air stream by water, the value of K_G was found as $2x10^{-6}kmol/m^2.s.kPa$. The absorption takes place at 298K and 1atm. At a particular location of the column absorber the gas phase conc. is 5mol% and the liquid phase conc. is 0.2mol%. Only 10% of the total resistance lies in the liquid phase. The solution obeys Henry's law and the value of m=1.5 at 298K and 1 atm pressure. Calculate the individual coefficient, flux and interfacial concentration.

Solution:

$$K_G = 2 \times 10^{-6} kmol/m^2.s.kPa$$

$$K_Y = K_G P_A = 2 \times 10^{-6} \times 101.3 = 2.03 \times 10^{-4} kmol/m^2.s$$

Since 10% of the total resistance is in the liquid phase, 90% of the total resistance lies in the gas phase. We know that,

Fractional resistance offered by gas phase $= \frac{1/k_y}{1/K_Y} = \frac{K_Y}{k_y}$

$$\rightarrow\ 0.9 = \frac{K_Y}{k_y} \Rightarrow k_y = \frac{K_Y}{0.9} = \frac{2.03 \times 10^{-4}}{0.9} = 2.26 \times 10^{-4} kmol/m^2.s$$

Now, let us take an example for gas absorption of certain gas from an air steam by water the value of K G was found as 2 into 10 to the power minus 6 kilo mole per metre square second kilo Pascal. The absorption takes place at 298 Kelvin and at one atmosphere pressure at a particular location of the column the gas phase concentration is 5 mole and liquid phase concentration is 0.2 mole percent.

Only 10 percent of the total resistance lies in the liquid phase. So, it is given only 10 percent of the total resistance lies in the liquid phase. The solution obeys Henry's law and the value of Henry's law constant m is given as 1.5 at the temperature and pressure conditions mentioned.

Now, we need to calculate the individual mass transfer coefficient, flux and interfacial concentration. So, K G is given over here which is 2 into 10 to the power minus 6 kilo mole per metre square second kilo Pascal. So, k y is K G into Pa which is equal to K G is given and Pa is 1 atmosphere which is 101.3 kilo Pascal. So, this is coming out to be 2 into 10 to the power minus 6 into 101.3 which is equal to 2.03 into 10 to the power minus 4 kilo mole per metre square second.

Now, since the 10 percents of the total resistance is lies in the liquid phase and hence so 90 percent of the total resistance should be in the gas phase. Now, we know that the fractional resistance offered by the gas phase is equal to 1 by small k y divided by 1 by capitals K y. So, it would be capital K y by small k y. So, this ratio of the mass transfer coefficient that is over all mass transfer coefficient and divided by the individual mass transfer coefficient based on the gas phase.

And now, if we just put the values which is 90 percent. So, it will be 0.9 would be equal to capital K y by small k y and so small k y would be equal to capital K y by 0.9. So, capital K y we have calculated here which is 2.03 into 10 to the power minus 4 divided by 0.9. So, we will have 2.26 into 10 to the power minus 4 kilo mole per metre square second. So, k y small k y is now calculated.

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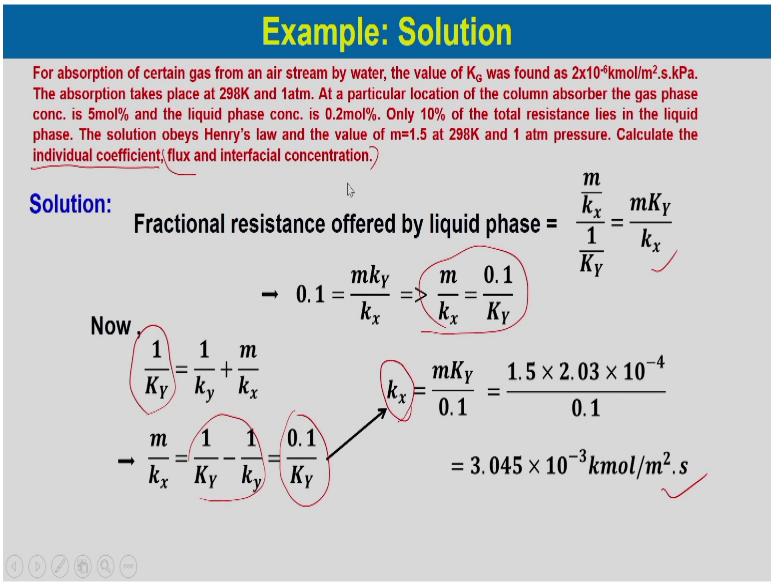

Example: Solution

For absorption of certain gas from an air stream by water, the value of K_G was found as 2×10^{-6}kmol/m^2.s.kPa. The absorption takes place at 298K and 1atm. At a particular location of the column absorber the gas phase conc. is 5mol% and the liquid phase conc. is 0.2mol%. Only 10% of the total resistance lies in the liquid phase. The solution obeys Henry's law and the value of m=1.5 at 298K and 1 atm pressure. Calculate the individual coefficient, flux and interfacial concentration.

Solution:

Fractional resistance offered by liquid phase = $\dfrac{\frac{m}{k_x}}{\frac{1}{K_Y}} = \dfrac{mK_Y}{k_x}$

$$\rightarrow 0.1 = \frac{mk_Y}{k_x} \Rightarrow \frac{m}{k_x} = \frac{0.1}{K_Y}$$

Now,

$$\frac{1}{K_Y} = \frac{1}{k_y} + \frac{m}{k_x}$$

$$\rightarrow \frac{m}{k_x} = \frac{1}{K_Y} - \frac{1}{k_y} = \frac{0.1}{K_Y}$$

$$k_x = \frac{mK_Y}{0.1} = \frac{1.5\times2.03\times10^{-4}}{0.1}$$

$$= 3.045\times10^{-3}\,kmol/m^2.s$$

Now, fractional resistance offered by the liquid phase we know it is m by small k x divided by 1 by capital K y which is equal to m capital K y divided by k x. So, as we know the values of capital K y and m which are given so and also the fractional resistance offered by the liquid phase we know. So, you would be able to calculate small k x.

So, 10 percent resistance which lies in the liquid phase would be equal to m k y divided by k x, from there we can write m by k x would be equal to 0.1 by k y. So, now, we know that 1 by capital K y would be equal to 1 by small k y plus m by k x. So, this is the resistance in series models. We can co-relate with the as you have learned the overall mass transfer resistance is related with the individual mass transfer resistance of the films or the phases.

If we just rearrange this equation m k x would be equal to 1 by capital K y minus 1 by small k y. So, this would be equal to so, as we know from here m by k x is equal to 0.1 divided by capital K y. So, we can substitute over here this part would be equal to 0.1 divided by capital K y. So, from here we can write k x would be equal to m capital K y divided by 0.1.

So, if you substitute the values which is no, m is 1.5 into 2.03 into 10 to the power minus 4 divided by 0.1. So, this will give 3.045 into 10 to the power minus 3 kilo mole per metre square second. So, this is the individual mass transfer coefficient in the liquid phase. And we have also calculated the individual mass transfer coefficient in the gas phase. So, this part we have done. Now, you have to calculate the flux and interfacial concentration.

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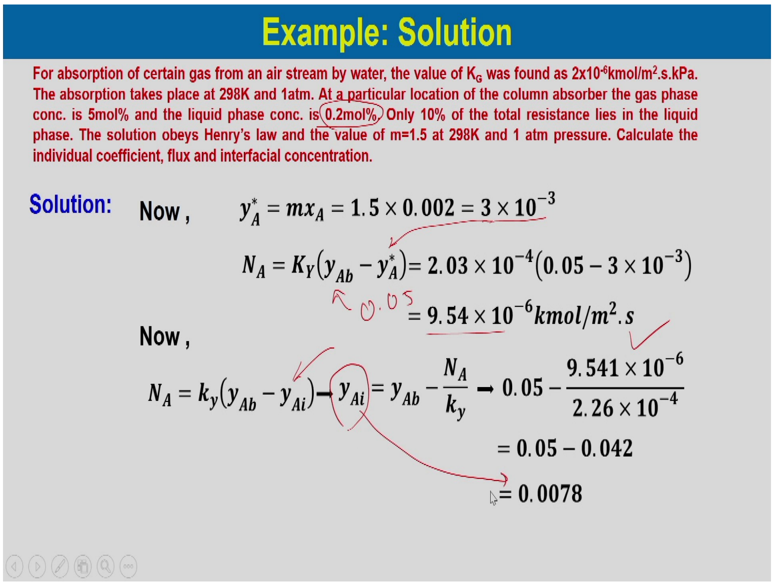

Example: Solution

For absorption of certain gas from an air stream by water, the value of K_G was found as 2x10^{-6}kmol/m^2.s.kPa. The absorption takes place at 298K and 1atm. At a particular location of the column absorber the gas phase conc. is 5mol% and the liquid phase conc. is 0.2mol%. Only 10% of the total resistance lies in the liquid phase. The solution obeys Henry's law and the value of m=1.5 at 298K and 1 atm pressure. Calculate the individual coefficient, flux and interfacial concentration.

Solution: Now,

$$y_A^* = mx_A = 1.5 \times 0.002 = 3 \times 10^{-3}$$

$$N_A = K_Y(y_{Ab} - y_A^*) = 2.03 \times 10^{-4}(0.05 - 3 \times 10^{-3})$$

0.05

$$= 9.54 \times 10^{-6} kmol/m^2.s$$

Now,

$$N_A = k_y(y_{Ab} - y_{Ai}) \rightarrow y_{Ai} = y_{Ab} - \frac{N_A}{k_y} \rightarrow 0.05 - \frac{9.541 \times 10^{-6}}{2.26 \times 10^{-4}}$$

$$= 0.05 - 0.042$$

$$= 0.0078$$

Now, the equilibrium relations y A star would be equal to m into x A. So, m is 1.5 into 0.002 because it is 0.2 mole percent which are given over here the liquid phase concentration is 0.2 mole percent. So, it is 3 into 10 to the power minus 3. So, we can calculate the flux is equal to capital K y into y Ab minus y A star.

So, y Ab is given gas phase concentration is 5 molar and so it is 0.05, capital K y is known 2.03 into 10 to the power minus 4 into 0.05 minus 3 into 10 to the power minus 3. So, if you substitute the values over here and this is 0.05, so we will be able to obtain 9.54 into 10 to the power minus 6 kilo mole per metre square second. So, this is the flux which we can calculate.

Now, N A the flux is equal to small k y into y Ab minus y Ai. So, you have to calculate the interfacial concentration, y Ai is the concentration of gaseous solute at the gas liquid interface. So, from this equation we can write y Ai would be y Ab minus N A by small k y. This is no 0.05 minus 9.541 into 10 to the power minus 6 divided by 2.26 into 10 to the power minus 4 and from there we can calculate it is 0.05 minus 0.042 and which is equal to 0.0078 mole fraction. So, the interfacial concentration is 0.0078.

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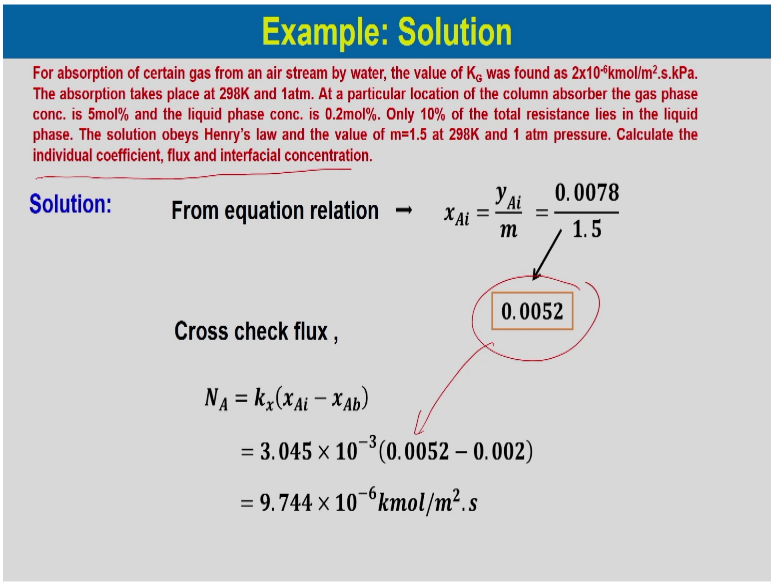

Example: Solution

For absorption of certain gas from an air stream by water, the value of K_G was found as $2x10^{-6} kmol/m^2.s.kPa$. The absorption takes place at 298K and 1atm. At a particular location of the column absorber the gas phase conc. is 5mol% and the liquid phase conc. is 0.2mol%. Only 10% of the total resistance lies in the liquid phase. The solution obeys Henry's law and the value of m=1.5 at 298K and 1 atm pressure. Calculate the individual coefficient, flux and interfacial concentration.

Solution: From equation relation ➞ $x_{Ai} = \frac{y_{Ai}}{m} = \frac{0.0078}{1.5}$

$$\boxed{0.0052}$$

Cross check flux ,

$$N_A = k_x(x_{Ai} - x_{Ab})$$

$$= 3.045 \times 10^{-3}(0.0052 - 0.002)$$

$$= 9.744 \times 10^{-6} kmol/m^2.s$$

Now, similarly for the liquid phase concentration we can co-relate with its x Ai would be equal to y Ai by m. So, m is the Henry's law constant. H you know the gas liquid interface we can create with the Henry's law constant to calculate the interfacial concentration. So, if we apply that this would be 0.0075 divided by 1.5, so it is about 0.0052.

So, to cross check the flux with this if we use the interfacial concentration in the liquid phase which is k x into x Ai minus x Ab we can write the concentration bulk concentration is know 0.002 and the k x is 3.045 into 10 to the power minus 3, and then the interfacial concentration which we have obtained over here we can substitute we will get the values of the flux and which is 9.744 into 10 to the power minus 6 kilo mole per metre square second.

Hence, the values for the flux which we have obtained using the interfacial concentration is same as based on the overall mass transfer coefficients. So, the calculation is correct. So, finally, we are able to calculate the individual coefficient flux and the interfacial concentration.

So, now, we will move to a know contacting device where the mass transfer takes place between the gas and the liquid phase. And we try to do the material balance for the component which is transferred and try to generate the operating line.

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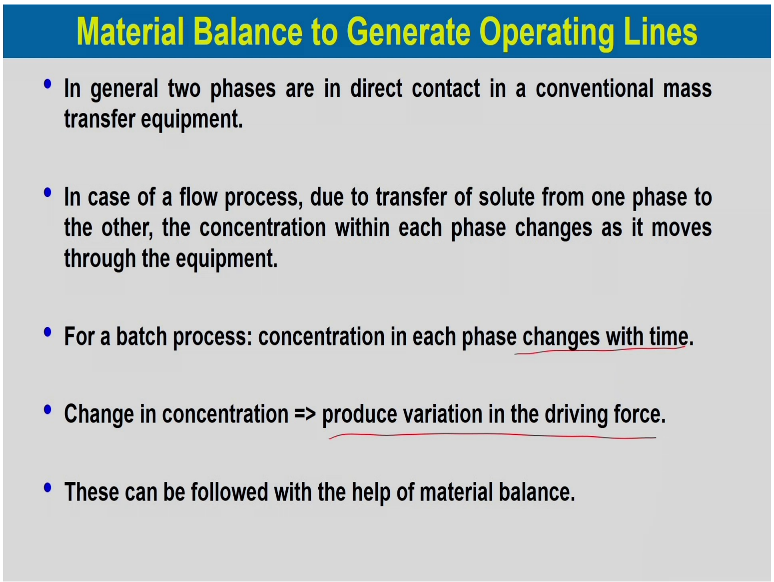

Material Balance to Generate Operating Lines

- In general two phases are in direct contact in a conventional mass transfer equipment.
- In case of a flow process, due to transfer of solute from one phase to the other, the concentration within each phase changes as it moves through the equipment.
- For a batch process: concentration in each phase changes with time.
- Change in concentration => produce variation in the driving force.
- These can be followed with the help of material balance.

In general two phases are in direct contact in conventional mass transfer equipment. In case of a flow process, due to transfer of solute from one face to the other, the concentration within each phase changes as it moves through the equipment. So, for a batch process concentration in each phase changes with time because once the absorbtion batch wise, then the concentration in the gas phase as well as liquid phase changes. If the concentration changes during the process then what happens? It produce the variation in the driving force. Since the driving force is also changes, this can be followed with the help of material balance how the concentration driving force changes for the mass transfer and which will give us the operating line.

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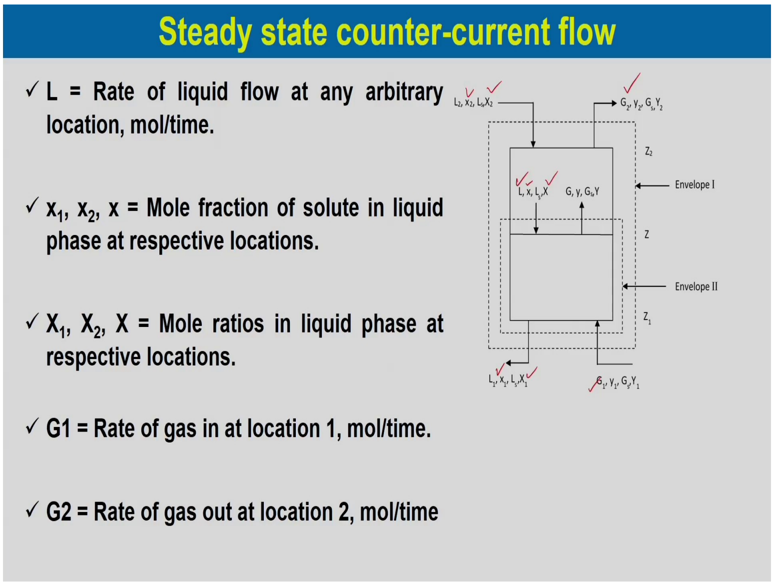

Steady state counter-current flow

- ✓ L = Rate of liquid flow at any arbitrary location, mol/time.
- ✓ x_1, x_2, x = Mole fraction of solute in liquid phase at respective locations.
- ✓ X_1, X_2, X = Mole ratios in liquid phase at respective locations.
- ✓ G1 = Rate of gas in at location 1, mol/time.
- ✓ G2 = Rate of gas out at location 2, mol/time

Three modes of operations happens generally in the industrial equipments, one is counter current operation and then co-current operation and cross-current operations. Let us consider counter current operation this is a column in which there is a inlet gas and at the bottom of the column and then inlet liquid at the top of the column, and then outlet gas at the top of the column, and outlet liquid at the bottom of the column.

So, here L is the rate of liquid flow at any arbitrary location in mole per time. So, L is the rate of liquid flow at arbitrary location in mole per time. This small x 1, x 2 and x. So, x 1, x 2 and x at any location these are the mole fraction of solute in liquid phase at respective locations. Capital X 1, X 2 and X, capital X 1, capital X 2 and capital X which is over here these are mole ratios in liquid phase at respective locations.

G1 the rate of gas in at location 1. So, G 1 is the gas flow rate at location 1 in mole per time and G 2 is the gas rate gas out at location 2, rate of gas out at location 2. So, the bottom of the column is considered or the location at the bottom considered as 1 and top is considered as 2.

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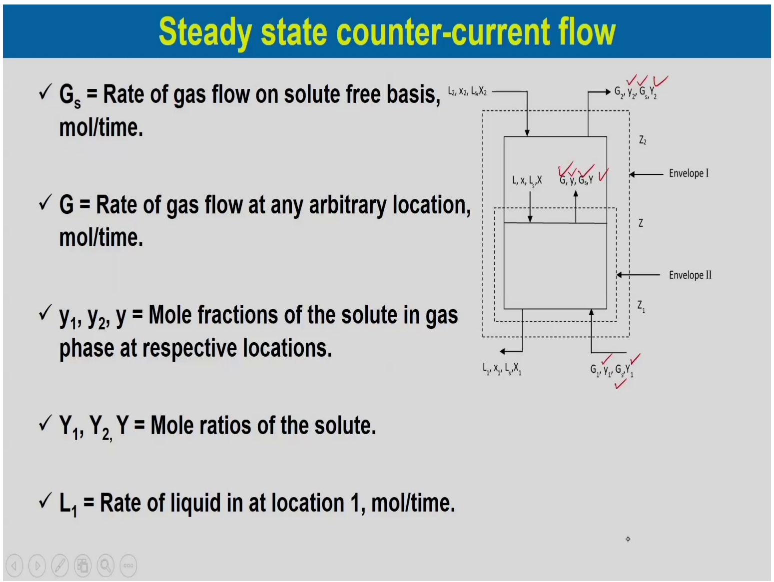

Steady state counter-current flow

- ✓ G_s = Rate of gas flow on solute free basis, mol/time.
- ✓ G = Rate of gas flow at any arbitrary location, mol/time.
- ✓ y_1, y_2, y = Mole fractions of the solute in gas phase at respective locations.
- ✓ Y_1, Y_2, Y = Mole ratios of the solute.
- ✓ L_1 = Rate of liquid in at location 1, mol/time.

Now, G s the rate of gas flow on solute free basis which is the defined over here G s on solute free basis. So, it does not change when passes through the column that is the inert for the absorption process or for the mass transfer. So, G s is the rate of gas flow on solute free basis in mole per time and G is the rate of gas flow at any arbitrary location. So, which is given over here small y 1, y 2 and y are the mole fraction of the solute in the gas phase at respective locations and capital Y 1, Y 2 and Y are the mole ratios of the solute. L 1 is the rate of liquid at location 1.

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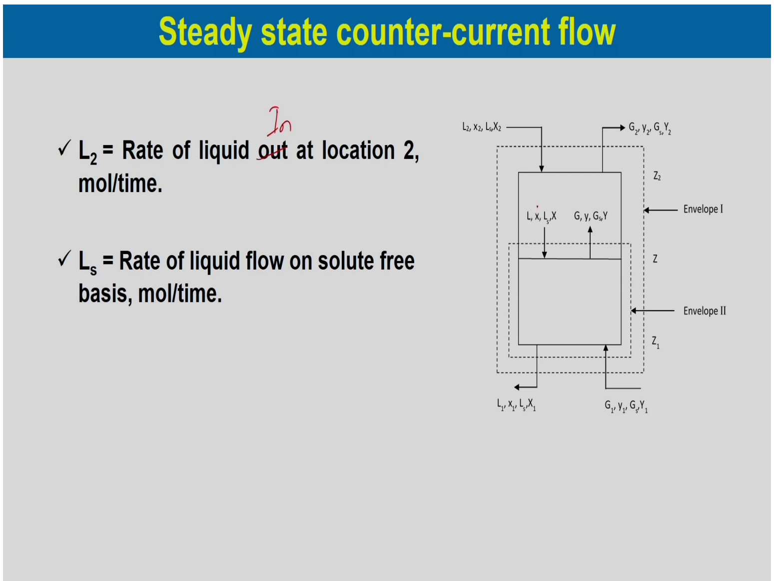

Steady state counter-current flow

- ✓ L_2 = Rate of liquid ~~out~~ In at location 2, mol/time.
- ✓ L_s = Rate of liquid flow on solute free basis, mol/time.

And L 2 is the rate of liquid in at location 2. And L s is the rate of liquid flow on solute free basis in mole per time.

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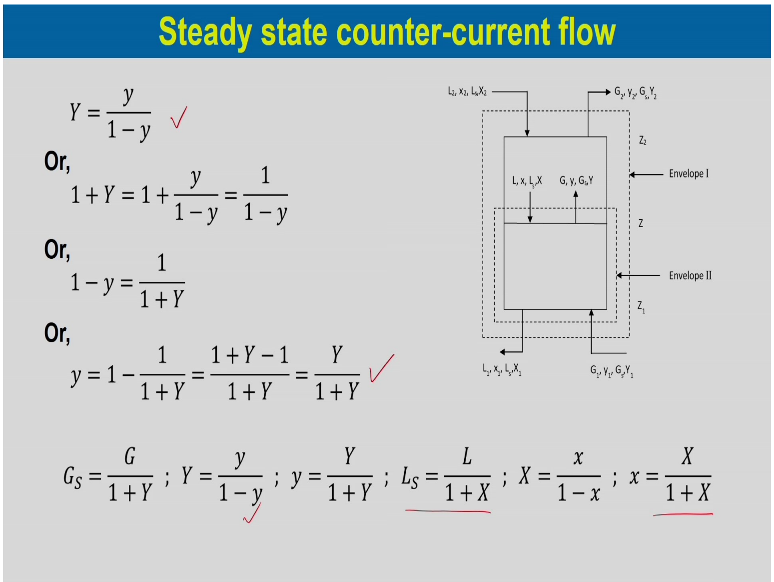

Steady state counter-current flow

$$Y = \frac{y}{1-y} \quad ✓$$

Or,

$$1 + Y = 1 + \frac{y}{1-y} = \frac{1}{1-y}$$

Or,

$$1 - y = \frac{1}{1+Y}$$

Or,

$$y = 1 - \frac{1}{1+Y} = \frac{1+Y-1}{1+Y} = \frac{Y}{1+Y} \quad ✓$$

$$G_S = \frac{G}{1+Y}\ ;\ Y = \frac{y}{1-y}\ ✓\ ;\ y = \frac{Y}{1+Y}\ ;\ \underline{L_S = \frac{L}{1+X}}\ ;\ X = \frac{x}{1-x}\ ;\ \underline{x = \frac{X}{1+X}}$$

We should know the relations between know small y and capital Y, small x and capital X basically capital is mole ratio unit that is on solute free basis and small y is the mole fractions unit. So, the relations between capital Y and small y is that capital Y is equal to y by 1 minus y that means, the solute which excluding the solute it is on solute free basis. From this relation we can rearrange 1 plus Y would be equal to 1 plus y by 1 minus y which would be equal to 1 by 1 minus y. And from there we can write 1 minus y would be equal to 1 by 1 plus y. So, from here we can write y would be equal to 1 minus 1 by 1 plus Y and we can get the relations between small y and capital Y, which is capital Y divided by 1 plus Y.

So, this relation we should remember while defining mole ratio and mole fraction unit or while changing this know parameters. So, the G s which is the gas flow rate on solute free basis we can write the total gas flow rate divided by 1 plus Y and capital Y as we said is y by 1 minus y, small y would be equal to capital Y by 1 plus Y. Similarly, L s would be L divided by 1 plus capital X and capital X would be equal to x by 1 minus x and small x would be equal to capital X divided by 1 plus X.

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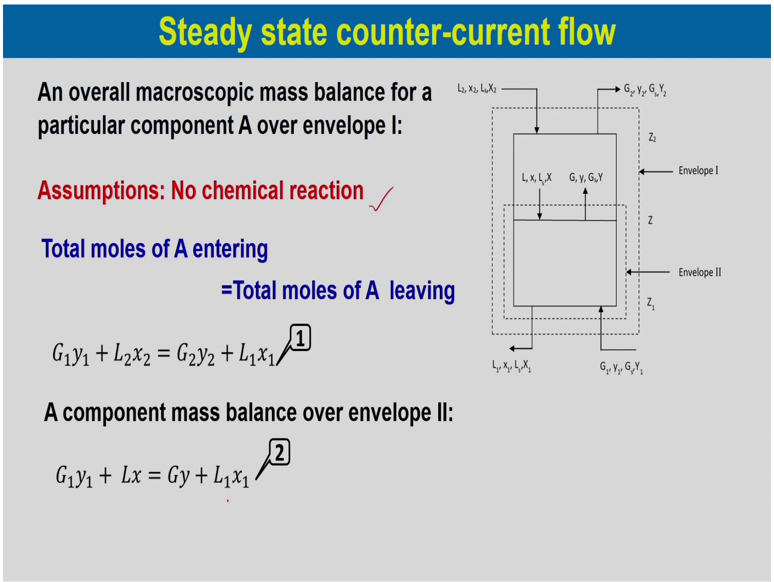

Steady state counter-current flow

An overall macroscopic mass balance for a particular component A over envelope I:

Assumptions: No chemical reaction ✓

Total moles of A entering

=Total moles of A leaving

$$G_1y_1 + L_2x_2 = G_2y_2 + L_1x_1 \quad \boxed{1}$$

A component mass balance over envelope II:

$$G_1y_1 + Lx = Gy + L_1x_1 \quad \boxed{2}$$

So, based on this conversion term we can develop an envelope know for we can do the overall macroscopic mass balance for particular component A over the envelope I.

So, envelope I consider the whole know column come over here and if we write if we take the assumptions that is there is no chemical reaction. So, in that case the total moles of A entering would be equal to the total moles of A leaving. So, we can write G 1 y 1 which is entering plus L 2 x 2 would be equal to G 2 y 2 plus L 1 x 1. So, entering moles would be equal to exit moles.

If we do the component balance over envelope II over here and taking at any location, so we can get G 1 y 1 plus L x would be equal to G y plus L 1 x 1, this is equation 2.

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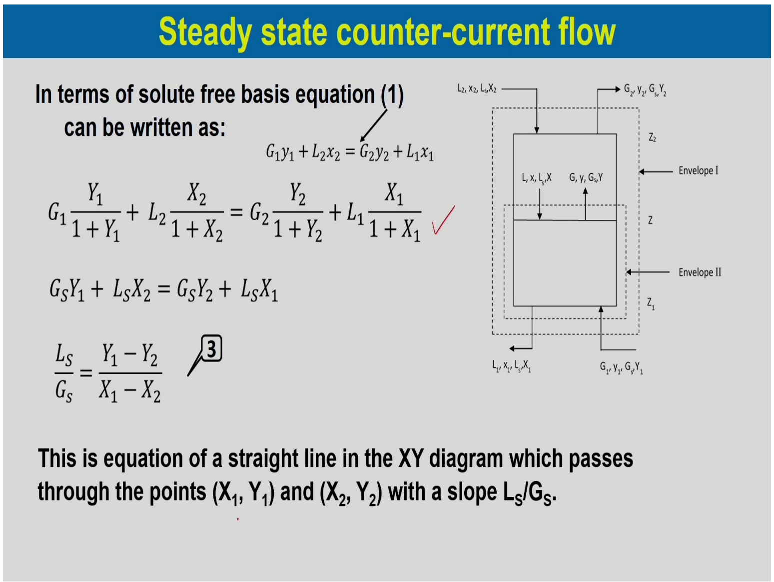

Steady state counter-current flow

In terms of solute free basis equation (1) can be written as:

$$G_1 y_1 + L_2 x_2 = G_2 y_2 + L_1 x_1$$

$$G_1 \frac{Y_1}{1+Y_1} + L_2 \frac{X_2}{1+X_2} = G_2 \frac{Y_2}{1+Y_2} + L_1 \frac{X_1}{1+X_1} \checkmark$$

$$G_S Y_1 + L_S X_2 = G_S Y_2 + L_S X_1$$

$$\frac{L_S}{G_S} = \frac{Y_1 - Y_2}{X_1 - X_2} \quad \boxed{3}$$

This is equation of a straight line in the XY diagram which passes through the points (X_1, Y_1) and (X_2, Y_2) with a slope L_S/G_S.

Now, in terms of the solute free basis equation 1 can be written as G 1 into y small y 1 we can substitute capital Y 1 divided by 1 plus y 1 plus L 2 into x 2 capital X 2 divided by 1 plus capital X 2, would be equal to G 2 capital Y 2 divided by 1 plus capital Y 2 plus L 1 capital X 1 divided by 1 plus X 1. So, we substitute the know conversion terms small x and capital X, small y and capital Y.

Now, if we write, so this is basically G s G 1 by 1 plus Y 1 we can write G s Y 1 plus L s X 2 would be equal to G s Y 2 plus L s X 1. Now, from here if you just rearrange, we can get L s by G s would be equal to Y 1 minus Y 2 divided by X 1 minus X 2. So, this is equation 3. We can see it is a equation of straight line, in the x y diagram which passes through the points X 1 capital X 1 capital Y 1 and capital X 2 capital Y 2 with a slope L s by G s.

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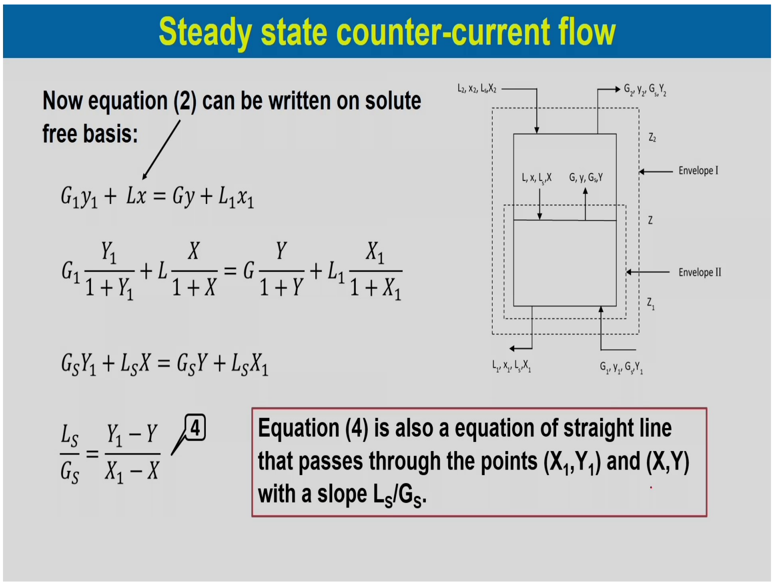

Steady state counter-current flow

Now equation (2) can be written on solute free basis:

$$G_1 y_1 + Lx = Gy + L_1 x_1$$

$$G_1 \frac{Y_1}{1+Y_1} + L\frac{X}{1+X} = G\frac{Y}{1+Y} + L_1\frac{X_1}{1+X_1}$$

$$G_S Y_1 + L_S X = G_S Y + L_S X_1$$

$$\frac{L_S}{G_S} = \frac{Y_1 - Y}{X_1 - X} \quad \boxed{4}$$

Equation (4) is also a equation of straight line that passes through the points (X_1,Y_1) and (X,Y) with a slope L_S/G_S.

Now, equation 2 can be written equation on solute free basis. This if we write on solute free basis we substitute the terms it will be G 1 Y by 1 plus Y 1 plus L into capital X by 1 plus X equal would be equal to G into Y by 1 plus Y plus L 1 into capital X 1 divided by 1 plus X 1. So, from here we can write G s into capital Y 1 plus L s into capital X would be equal to G s into capital Y plus L s into capital X 1. So, if we rearrange it will be L s by G s would be equal to capital Y 1 minus capital Y divided by capital X 1 minus capital X. So, this is a equation of straight line that passes through the points X 1 Y 1 and X Y with a slope L s by G s.

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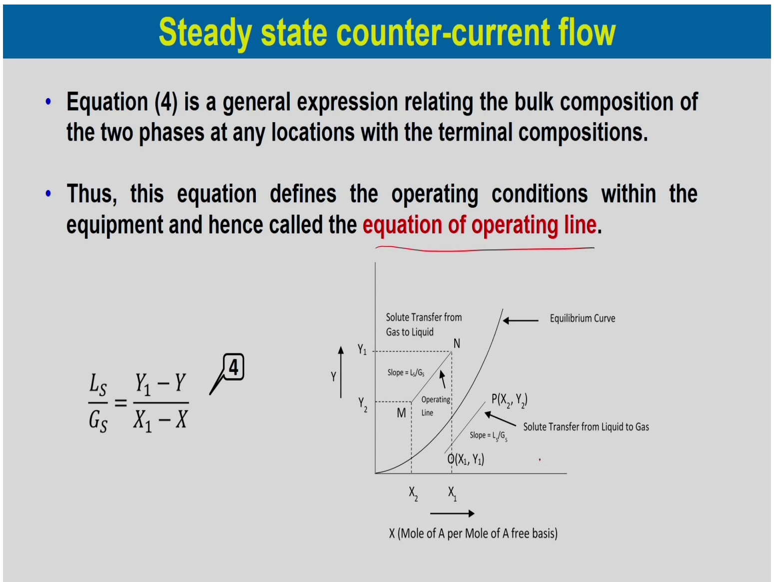

Steady state counter-current flow

- Equation (4) is a general expression relating the bulk composition of the two phases at any locations with the terminal compositions.
- Thus, this equation defines the operating conditions within the equipment and hence called the equation of operating line.

$$\frac{L_S}{G_S} = \frac{Y_1 - Y}{X_1 - X} \quad \boxed{4}$$

This equation 4 as you can see it passing through any point inside the column X Y, any points. So, the equation 4 is a general expression relating to the bulk composition of the two phases at any location with the terminal composition. Thus, this equation defines the operating conditions within the equipment and hence is called the equation of operating line, ok. So, the equation 4 is the equation of operating line.

So, this is the equation and you can see when there is transfer from the solute from the gas to the liquid the slope of the curve is this that is between m and n if you plot. So, you will get the slope of the curve is L s by G s, and similarly if solute transfer from the liquid to the gas slope of the curve you will get between this two which is below the equilibrium curve.

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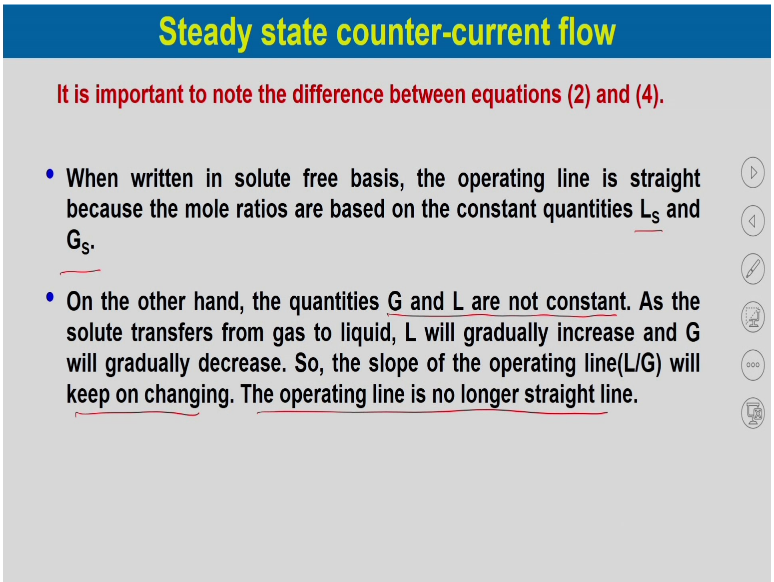

Steady state counter-current flow

It is important to note the difference between equations (2) and (4).

- When written in solute free basis, the operating line is straight because the mole ratios are based on the constant quantities L_S and G_S.
- On the other hand, the quantities G and L are not constant. As the solute transfers from gas to liquid, L will gradually increase and G will gradually decrease. So, the slope of the operating line(L/G) will keep on changing. The operating line is no longer straight line.

Now, it is important to note the difference between the question 2, which is based on the mole fraction unit and equation 4 which is the mole ratio unit. When written in solute free basis the operating line is straight because the mole ratios are based on the constant quantities L s and G s. So, when we write equation 4 it is on solute free basis the operating line are basically straight line because it is based on the constant quantities L s and G s. But, on the other hand the quantities G and L which is know variable inside the column it is changing because of the solute transfer between the phases as the solute transfer from the gas to liquid L will gradually increase and gas will gradually decrease.

So, the slope of the operating line L by G will keep changing. The operating line will no longer be straight line.

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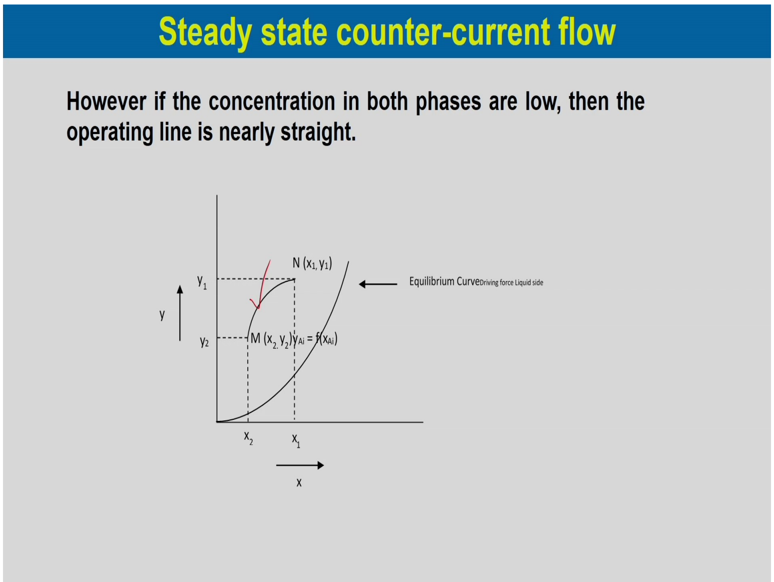

Steady state counter-current flow

However if the concentration in both phases are low, then the operating line is nearly straight.

But if we consider very dilute solution in case of the concentration of the both phases are very low then the operating line is nearly straight and we can consider straight line, but it is not straight it looks like this the operating line will be curve like this, ok. So, this is the operating line when we write in terms of the this is small y mole ratio unit and when the concentration changes know between the phases are high in that case this is not the straight line the operating line is not the straight line it is a curved line.

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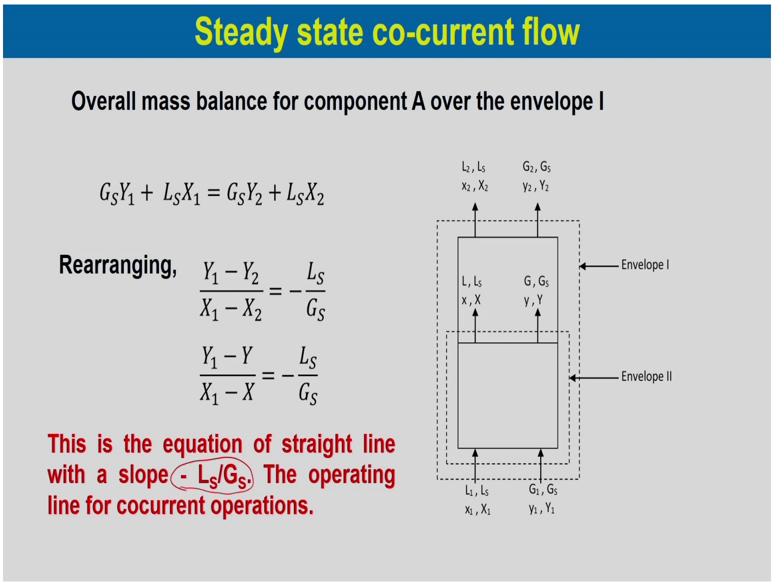

Steady state co-current flow

Overall mass balance for component A over the envelope I

$$G_S Y_1 + L_S X_1 = G_S Y_2 + L_S X_2$$

Rearranging, $$\frac{Y_1 - Y_2}{X_1 - X_2} = -\frac{L_S}{G_S}$$

$$\frac{Y_1 - Y}{X_1 - X} = -\frac{L_S}{G_S}$$

This is the equation of straight line with a slope - L_S/G_S. The operating line for cocurrent operations.

So, if you look into the overall mass balance of component A over the envelope, this is the envelope for the co-current operation we have done for the counter current operation. If we do with a co-current operation the nomenclatures are same but the flow is different. In this case the gas and liquid both are entering at the bottom and it is exiting at the top.

So, if we do the know more balance we will get the similar equations and if we rearrange them we will get Y 1 minus Y 2 divided by X 1 minus X 2 is equal to minus L s by G s. The slope over here is negative. Similarly, any concentration inside the column we can write capital Y 1 minus capital Y divided by X 1 minus capital X would be equal to minus L s by G s. So, this equations of straight line with a slope of minus L s by G s, the operating line for counter current operation.

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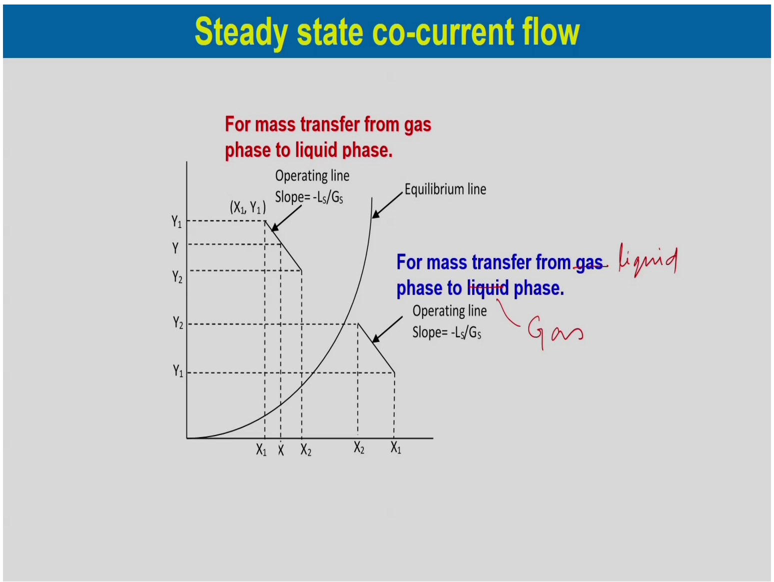

Steady state co-current flow

For mass transfer from gas phase to liquid phase.

For mass transfer from ~~gas~~ liquid phase to ~~liquid~~ Gas phase.

So, for operating line for counter current operation the slope will be negative and we can just plot them and you can see that if the mass transfer from the gas phase two the liquid phase happens the slope of the operating line is negative. So, it is the slope the curve will look like straight line it is a straight line because it is based on the mole ratio unit capital Y and capital Y X. So, this is the curve look like and similarly, for the mass transfer from the gas phase to the liquid phase to the gas phase. So, then the operating line will lie below the equilibrium line as it is in case of know counter current operation, but in this case the slope is also negative, so the curve will operating line curve will look like this.

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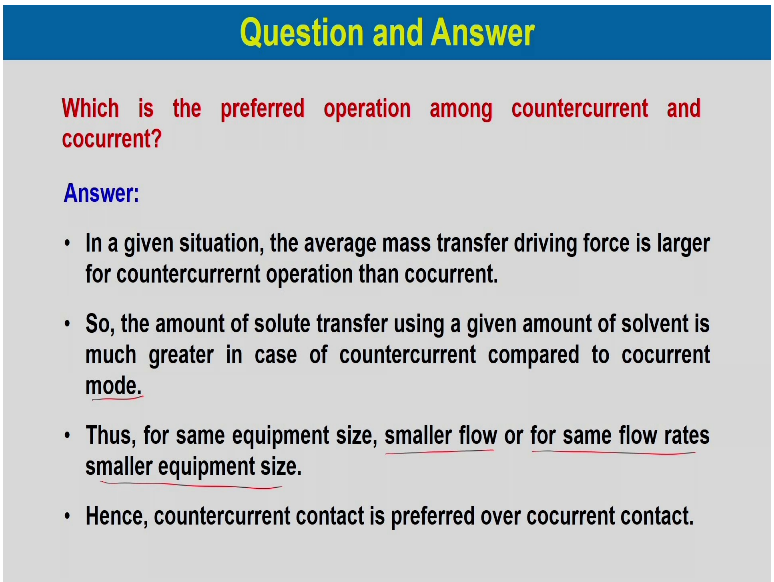

Question and Answer

Which is the preferred operation among countercurrent and cocurrent?

Answer:

- In a given situation, the average mass transfer driving force is larger for countercurrernt operation than cocurrent.
- So, the amount of solute transfer using a given amount of solvent is much greater in case of countercurrent compared to cocurrent mode.
- Thus, for same equipment size, smaller flow or for same flow rates smaller equipment size.
- Hence, countercurrent contact is preferred over cocurrent contact.

Now, we have to consider say, two system we have considered one is counter, counter current operation another is co-current operation. Among this two which one is best for the know for the operation or which would be economical.

So, in a given situation you can see the average mass transfer driving force is larger for counter current operations than co-current, because in case of co-current you can see the operating line equation as it is reaching closer to the equilibrium line, the driving force is gradually decreasing. The amount of solute transfer using a given amount of solvent is much greater in case of the counter current compared to the co-current mode.

Thus, for the same equipment size smaller flow or for same flow rates smaller equipments size, hence counter current contact is preferred over co-current operation. So, for the same equipment size if we take, we need to have smaller flow rate to complete the specific job and for the same flow rates we need smaller equipment size to complete the same job in case of the counter current operation compared to the co-current. Hence counter current contact is much preferred over the co-current operation.

Thank you for hearing this lecture. And we will continue with our interface mass transfer topic in our next lecture.