# Mass Transfer Coefficients

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## Lecture – 12
## Boundary Layer Theory and mass transfer coefficients in turbulent flow

Welcome to the 4th lecture of module 2 on mass transfer operation. In module 2 we are discussing the Mass Transfer Coefficient. So, before going to this lecture let us have small recap on our earlier lecture. In our last lecture we discussed, no mass transfer coefficient in laminar flow. In the laminar flow conditions we have considered two cases; one case is mass transfer coefficient in laminar falling film through a vertical solid surface.

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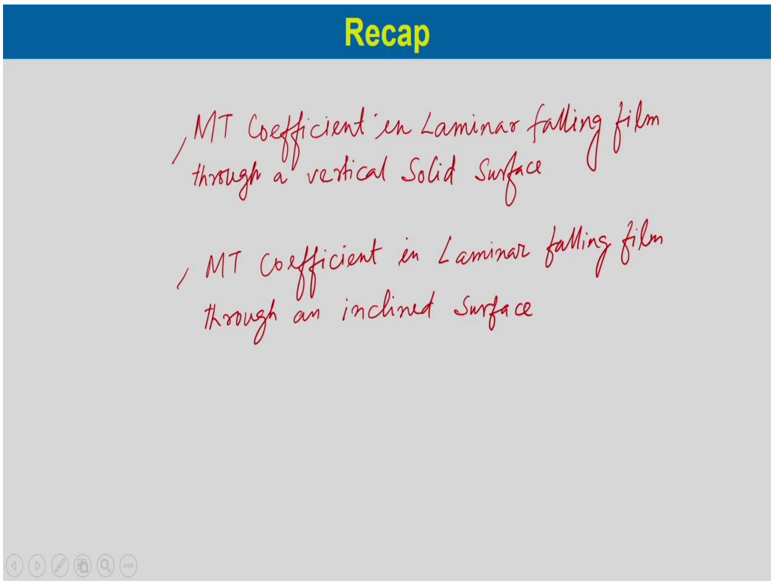


The second case we have considered mass transfer coefficient in laminar flow condition falling film through an inclined surface.

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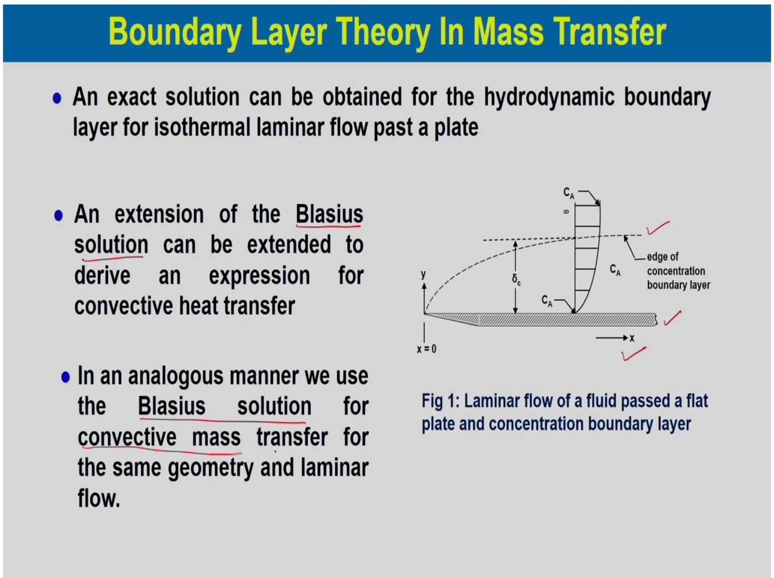


In this lecture, we will consider boundary layer theory in mass transfer and then we will consider mass transfer coefficient in turbulent flow. If we considered a case the flow through a solid surface in a horizontal surface and then we see that the when the fluid pass the solid surface there is a gradient of the velocity and also the gradient of concentration over the surface, near to the surface and the we call a layer which is formed where the concentration variation exist is called the boundary layer.

So, let us see the boundary layer, how it is formed. We have a solid surface over here and when the fluid flow horizontally through the x direction where x is equal to 0 at the start of the velocity profile or the boundary layer and then we see that; no there is a concentration gradient over the surface and this is the edge of concentration boundary layer and then the thickness of this boundary layer is delta C.

So, an exact solution can be obtained for the hydrodynamic boundary layer for isothermal laminar flow past a plate, if it is a plate. An extension of the Blasius solutions can be extended to derive an expression for convective heat transfer. In the analogous manner we can use the Blasius solutions for convective mass transfer as well for the same geometry and laminar flow.

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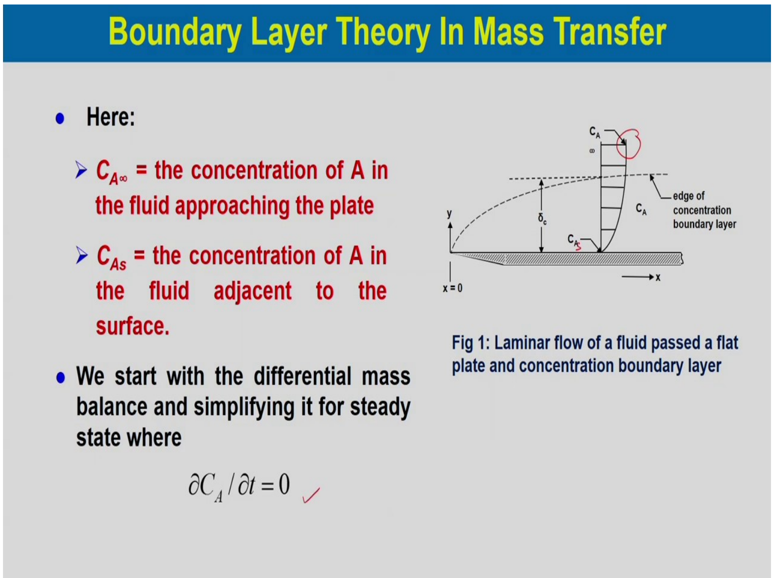


Now in this case to define the system C A infinity is the concentration of A in the fluid approaching the plate. So, concentration is C A infinity at this location and this is the concentration variation and C As which is at the surface it is C As concentration of A in the fluid adjacent to the surface so, which is over here.

We start with the differential mass balance and simplifying it for steady state process. If we consider the differential mass balance and if we considered the steady state, for steady state we considered del C A del t is equal to 0. So, del C A del t this is equal to 0. Now for the flow which is we considered only in x and y directions.

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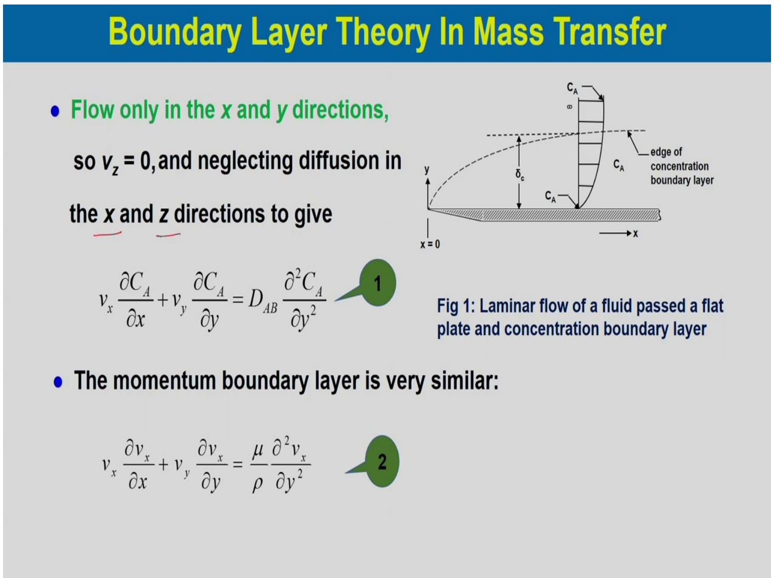


Then the velocity component that is v z would be 0 in the z direction and if we neglect the diffusion which is happening towards the x and z direction. So, since the fluid is flowing through the flat surface in the x direction, the convective mass transfer is much prominent than the diffusion. Hence, we can neglect the diffusion part in the x and z direction.

So, we will be know arising out of this know equations which is v x del C A del x plus b v y del C A del y is equal to D AB del 2 C A del z. So, this we can derive from the fixed second law equations without no chemical reactions and understudy state condition. The momentum layer is also very similar. It will be v x del v x del x plus v y del v x del y is equal to mu by rho del 2 v x del y 2. Here mu is the viscosity and rho is the density.

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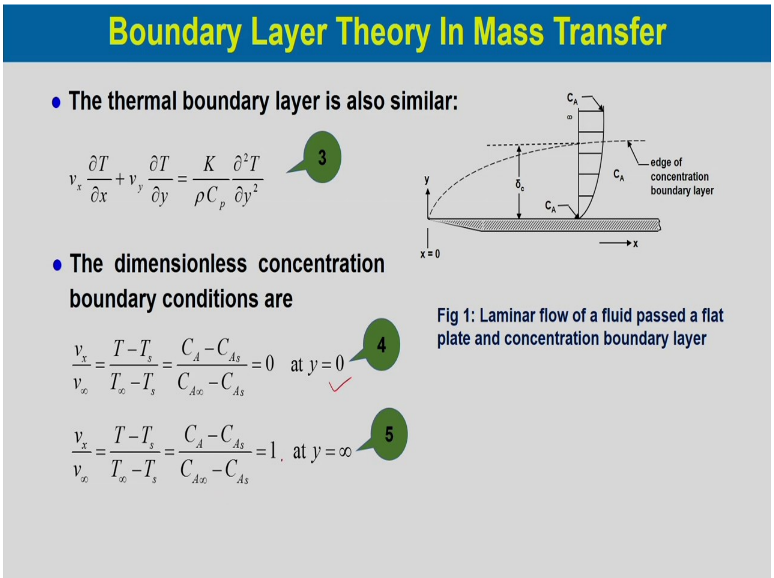


Now, thermal boundary layer also would be similar which is v x del T del x plus v y del T del y is equal to k by rho C p del 2 T del y 2. Now, if we write in terms of the dimensionless concentration in that case we can write v x by v infinity. This v infinity is the know free stream velocity over here is equal to T minus T s divided by T infinity minus T s which would be equal to C A minus C As divided by C A infinity minus CAs would be equal to 0 at y equal to 0. So, under this conditions, we can write this boundary condition in a dimensionless form. Now similarly we can write at y equal to infinity at a very long distance from the surface v x by v infinity would be equal to T minus Ts divided by T infinity minus Ts would be equal to CA minus CAs divided by CA infinity minus CAs is equal to 1.

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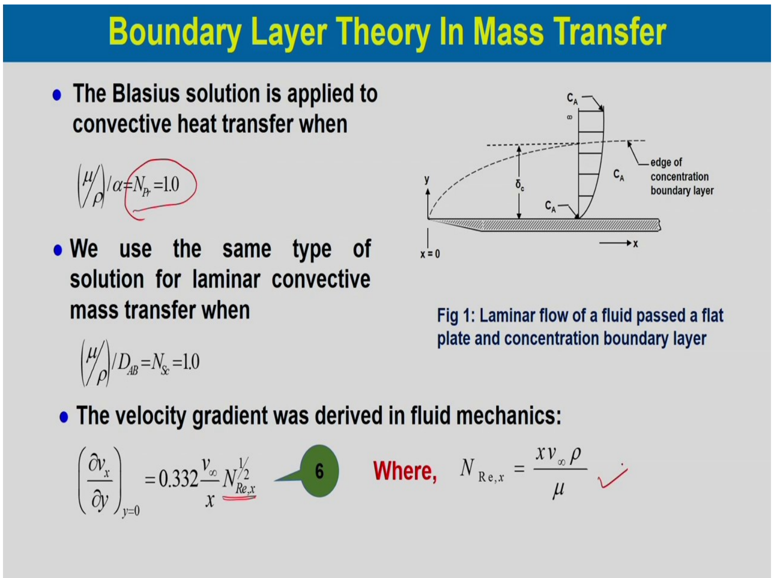


Now, the Blasius solution is applied to convective heat transfer when mu by rho by alpha that is the prandtl number which is equal to 1. So, when the prandtl number equal to 1, we can apply the Blasius solutions for convective heat transfer. Similarly in case of mass transfer we use for the laminar convective mass transfer when the Schmidt number equal to 1. If the Schmidt number equal to 1, that is mu by rho D AB is equal to 1, if that is 1 then we can apply the Blasius solutions.

So, the velocity gradient was derived in fluid mechanics, you have already derived that equations the velocity gradient that is del vx del y at y equal to 0 is equal to 0.332 v infinity by x Reynolds number to the power half N Re x it is Reynolds number. So, Reynolds number to the power half. So, this N Re x is x v infinity rho by mu, which is Reynolds number.

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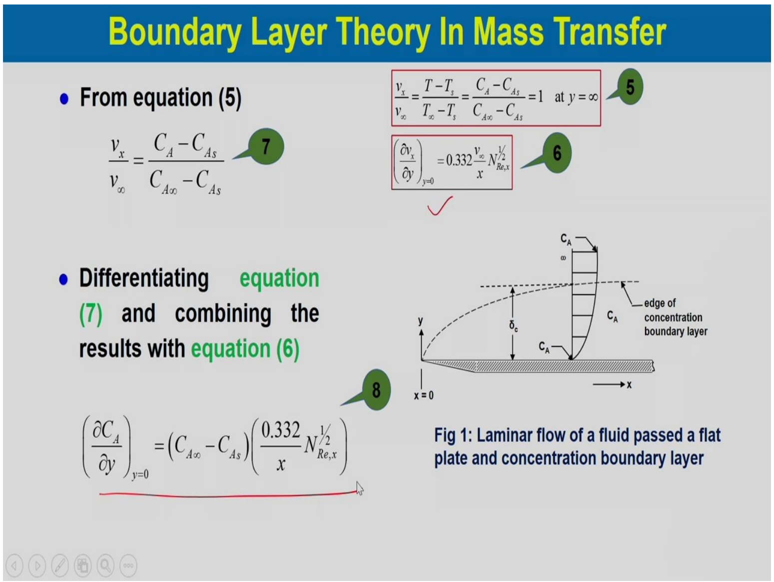


Now, from this equation know 5 which the boundary conditions which we have written in terms of the dimensionless concentration or dimensionless velocity or dimensionless temperature at y equal to infinity, we can write v x by v infinity would be equal to C A minus C As divided by C A infinity minus C As. Now if we differentiate equation 7, this equations if we differentiate and combined with the results of equation 6. So, we can write del C A del y at y equal to 0 would be equal to C A infinity minus C As into 0.332 by x Reynolds number to the power half. So, I think this is very clear for you to understand by differentiating this equation and then substituting this equation 6 in that you will be able to obtain this relation.

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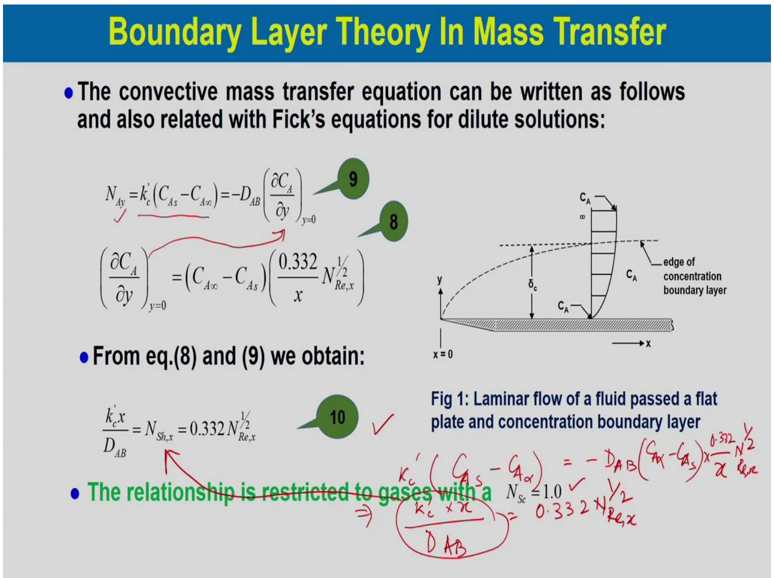


Now, the convective mass transfer equation can be written as follows and also related with the fix equation for dilute solutions. So, for convective mass transfer, we write the flux N A at any position y would be equal to k c dash into the concentration gradient that is C As minus C A infinity. So, this is the convective mass flux. Now, if we for a very dilute solution we can relate with the diffusive flux which is minus D AB del C A del y at y equal to 0.

Now, we know equation 8 which we have derived which is del C A del y at y equal to 0 is equal to C A infinity minus C As into 0.332 by x Reynolds number to the power half. So, if we substitute this know del C A del y at y equal to 0 over here so, we will obtain k c dash x by D AB would be equal to Sherwood number, which is equal to 0.332 Reynolds number to the power half.

So, substituting this equations over here like we can write k c dash C As minus C A infinity would be equal to minus D AB into C A infinity minus C As into 0.332 by x N Re x to the power half. Now, this if we rearrange this will canceled out and we will get k c dash into x divided by D AB would be equal to 0.332 N Re x to the power half. So, this is nothing but say sherwood number with respect to x. So, we can obtain this equation 10. So, this relation is valid when this schmidt number is 1; so, which we have said before.

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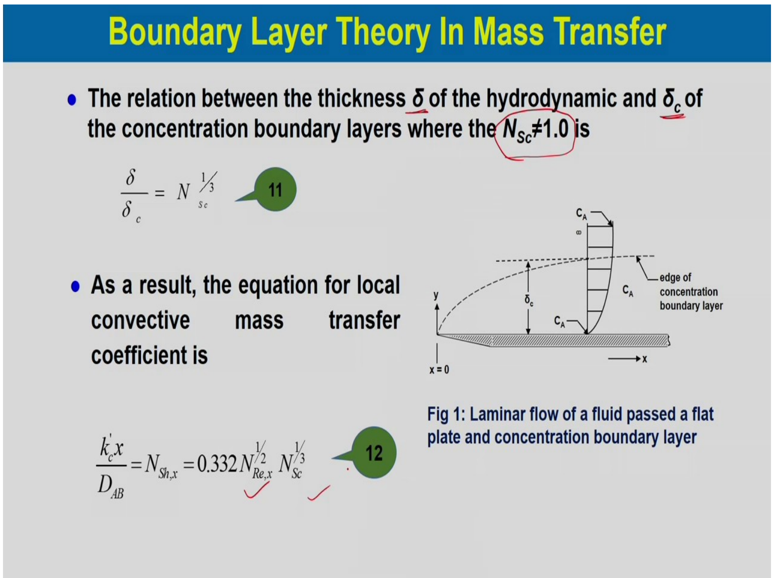


Now, the relations between the thickness delta of the hydrodynamic and the delta c of the concentration boundary layer. So, the hydrodynamic boundary layer thickness is delta and the concentration boundary layer thickness is delta c. When Schmidt numbr is not equal to 1, if Schmidt number is not equal to 1, then there is a relations between the hydrodynamic boundary layer thickness and the concentration boundary layer thickness and which is delta by delta c would be equal to Schmidt number to the power one-third. The equation for local convective mass transfer coefficient that is k c dash x by D AB which is sherwood number x with respect to x would be equal 0.332 reynolds number to the power half into Schmidt number to the power one-third. So, this is question 12.

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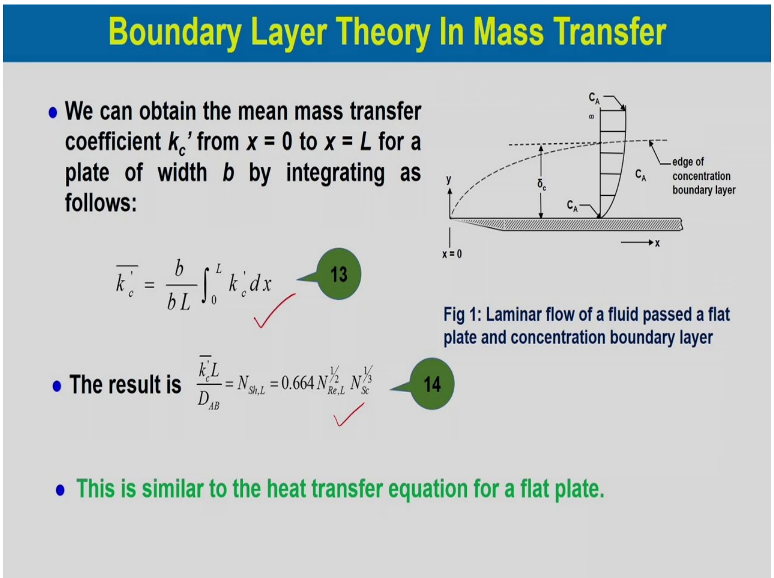


We can obtain the mean mass transfer coefficient k c dash from x is equal to 0 to x is equal to L for a plate of width b by integrating as follows. So, we can integrate k c dash would be equal to b by b L integral 0 to L k c dash d x. Now if we substitute k c dash which we have derived before k c dash from here then, we would be able to obtain k c dash L divided by D AB is equal to sherwood number N with respect to sherwood number would be equal to 0.664 Reynolds number to the power half into Schmidt number to the power one-third. So, the derivation after substituting k c dash from the equation 13 to 14 is left to you as a part of your homework, you can do it very easily. So, this is similar to the heat transfer equations for a flat plate.

(Refer Slide Time: 16:45)

## Example 1

A large volume of pure water at 25 °C is flowing parallel to a flat plate of solid benzoic acid, where $L$ = 0.244 m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kmol/m$^3$. The diffusivity of benzoic acid is $1.245\times10^{-9}$ m$^2$/s. Calculate the mass transfer coefficient $k_{L,avg}$ and the flux $N_A$. Given that μ = $8.71\times10^{-4}$ kg/m s and ρ=996 kg/m$^3$

Now, let us take an example, a large volume of pure water at 25 degree centigrade is flowing parallel to a flat surface of solid benzoic acid, where L is 0.244 meter in the direction of flow. The water velocity is 0.061 meter per second. The solubility of benzoic acid in water is 0.02948 kilo mole per meter cube. The diffusivity of benzoic acid is 1.245 into 10 to the power minus 9 meter square per second. We need to calculate the average mass transfer coefficient k L average and the flux N A and the viscosity and the density are given.

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## Example 1: Solution

A large volume of pure water at 25 °C is flowing parallel to a flat plate of solid benzoic acid, where $L$ = 0.244 m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kmol/m$^3$. The diffusivity of benzoic acid is $1.245\times10^{-9}$ m$^2$/s. Calculate the mass transfer coefficient $k_{L,avg}$ and the flux $N_A$. Given that μ = $8.71\times10^{-4}$ kg/m s and ρ=996 kg/m$^3$

**Solution:**

- The Schmidt number is

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{8.71\times10^{-4}}{996(1.245\times10^{-9})} = 702$$

- The Reynold's number is

$$N_{Re,L} = \frac{L u \rho}{\mu} = \frac{0.244(0.061)(996)}{8.71\times10^{-4}} = 1.7\times10^{4}$$

So, now we need to calculate the Schmidt number. So, Schmidt number is mu by rho D AB, mu is given 8.71 into 10 to the power minus 4 kg per meter second, rho is 996 and diffusivity is given. So, you can calculate the Schmidt number. Similarly, we can calculate the Reynolds number which is L u rho by mu L is given 0.244 meter and then we know the velocity 0.061 meter per second. So, velocity is given L is known and rho is given 996 kg per meter cube and also the viscosity. So, we can calculate know the Reynolds number 1.7 into 10 to the power 4, which is in the turbulent region.

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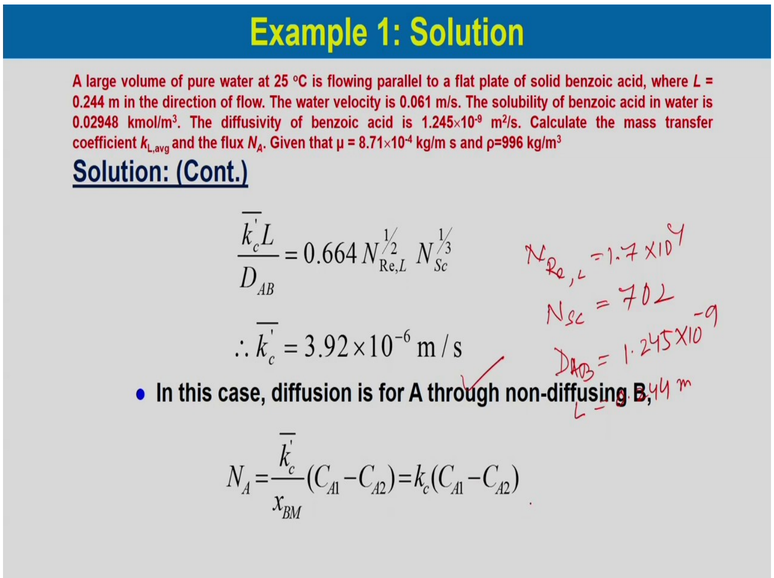


Now, if we just use the know value of N Reynolds number L which is know 1.7 into 10 to the power 4 and then Schmidt number which is 702, substituting this we would be and then the diffusion coefficient is known to us D AB is given 1.245 into 10 to the power minus 9 meter square per second and then the L which is given 0.244 meter. So, if we substitute we will get k c dash average would be 3.92 into 10 to the power minus 6 meter square per second.

In this case, diffusion of A through non diffusing B we can write the flux, N A is k c dash average by x BM C A1 minus C A2 would be equal to k c C A1 minus C A2.

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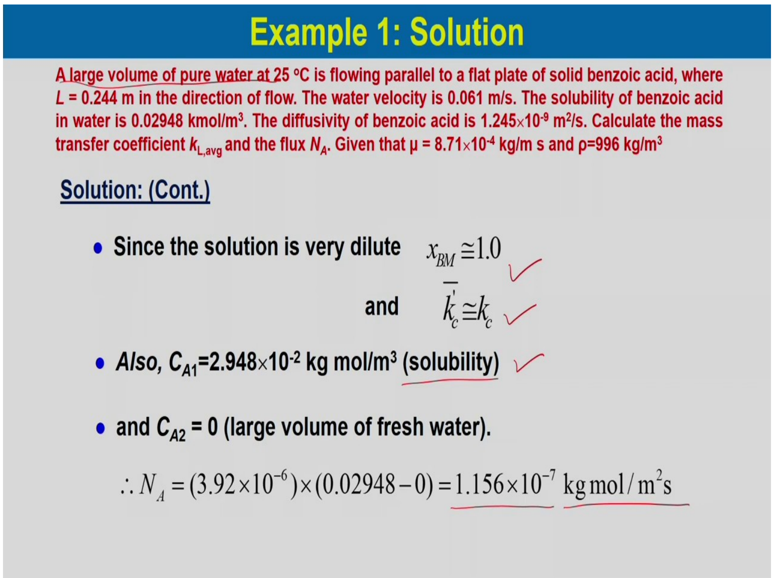


Now, if we considered very dilute solutions x BM would be approximately equal to 1 and k c dash approximately equal to k c. So, the concentration C A1 which is 2.948 into 10 to the power minus 2 kg mole or kmol per meter cube which is given a solubility over here. So, C A1 is known and since the large volume of fresh air it is said a large volume of pure water once it said pure water. So, CA 2 would be equal to 0. So, then we can write the flux would be equal to k c into the concentration gradient 0.02948 minus 0. So, which would be equal to 1.156 into 10 to the power minus 7 kg mole per meter square second.

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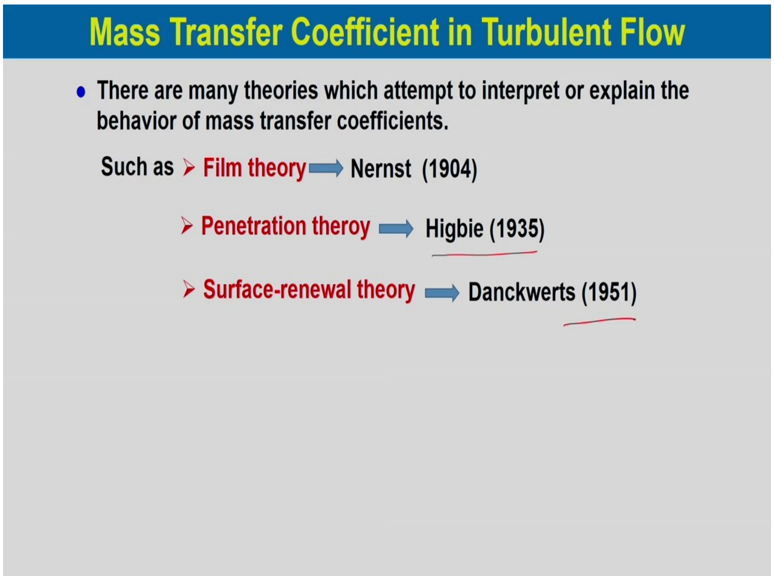

Now, we will consider mass transfer coefficient in turbulent flow. There are many theories which attempt to interpret or explain the behavior of mass transfer coefficient. So, we will try to cover few theories which are applicable in case of the turbulent flow. One of them is film theory which is proposed by Nernst in 1904. The second theory turbulent flow turbulent mass transfer coefficient is penetration theory, which is proposed by Higbie in 1935. So, it is popularly known as Higbie's theory and then third one is surface renewal theory which is proposed by Danckwerts and it is in 1951. So, this is you know evolution of this theory film theory, penetration theory and surface renewal theory we will try to explain them sequentially.

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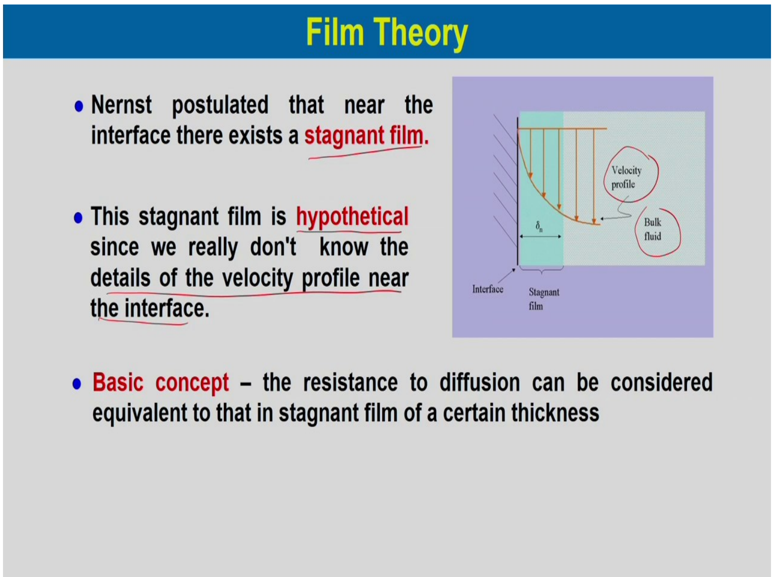


Now, you can see there is an interface over here and there is a stagnant film over here which is given as delta n and you could see the fluid bulk and the velocity profile look like this. The Nernst postulated that near the interface there exist or stagnant film. So, this film is considered as stagnant film at a gas liquid interface towards the liquid site. The stagnant film which is hypothetical this is considered as hypothetical since we really do not know the details of the velocity profile near the interface. So, this is considered as hypothetical.

The basic concept the resistance to diffusion can be considered equivalent to that in the stagnant film of a certain thickness. So, most of the resistance which create for mass transfer is due to the stagnant film. So, most of the resistance is considered in this stagnant film and has a certain thickness of delta n considered over here.

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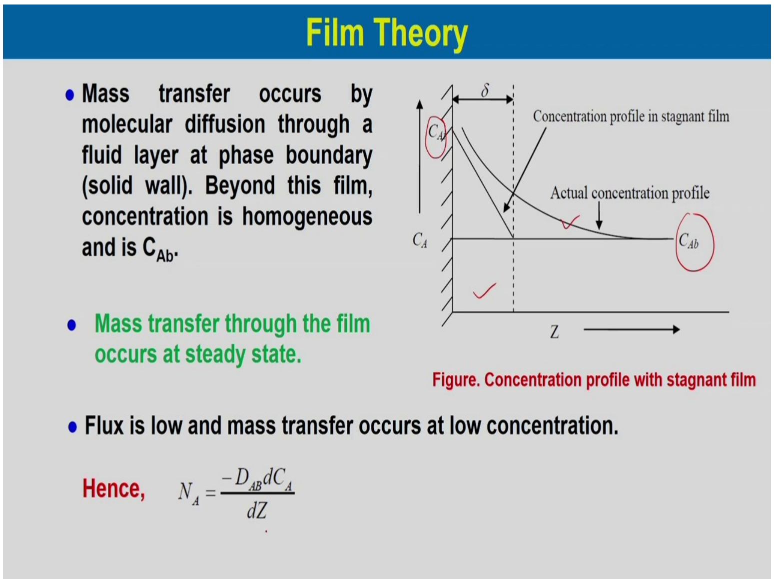


Now, you can see the concentration profile with the stagnant film how it is varying, this is delta the thickness and the concentration is varying CA at the interface the concentration is CA i and at the bulk the concentration is C Ab. The stagnant film concentration you can see this is a linear drop in concentration, but you can see the actual profile look like this, this is the actual profile, we will discuss know this things over here.

Mass transfer occur by molecular diffusion through the fluid layer at phase boundary that is at solid wall. Beyond this film the concentration is homogeneous and is C Ab. So, at the face boundary know the molecular diffusion takes place and beyond at a certain distance delta concentration is homogeneous that is considered as C Ab. Mass transfer through stagnant film which occurs in this film is basically steady state. So, flux in this film is very low and mass transfer occurs at very low concentrations. Hence, we can write N A would be equal to minus D AB d C A dZ.

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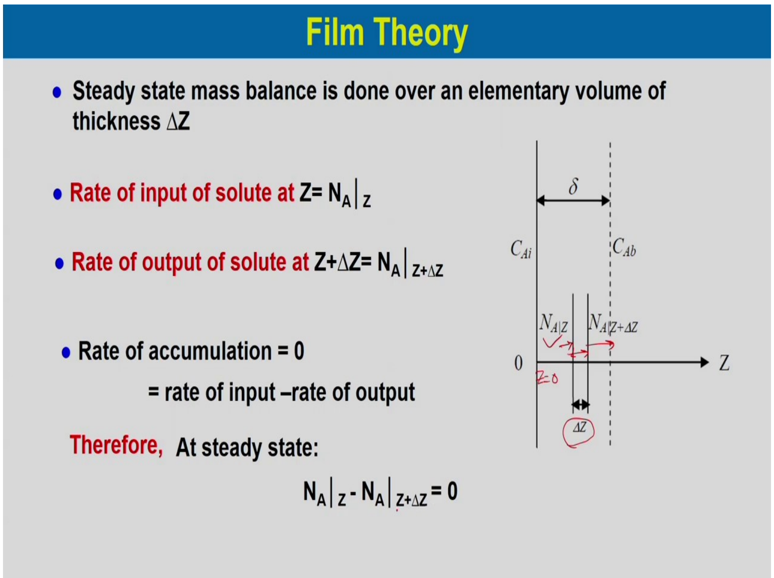


Now, if we do the steady state mass balance over an elementary volume of thickness delta Z. So, considered this is a elemental volume of delta Z this one thickness and flux at Z, Z is varying from the surface interface towards this side. So, at Z is equal to 0 over here at Z is equal to Z the flux is considered N AZ that is the rate of input of solute at Z. So, over here at this location and the rate which is out over here is N A at Z plus delta Z. So, rate of output of solute at Z plust delta Z would be equal to N A Z plus delta Z. The accumulation is considered here is 0, the rate of accumulation is 0. So, which is equal to rate of input minus rate of output. So, from this we can write at steady state N A Z minus N A Z plus delta Z at Z plus delta Z would be equal to 0.

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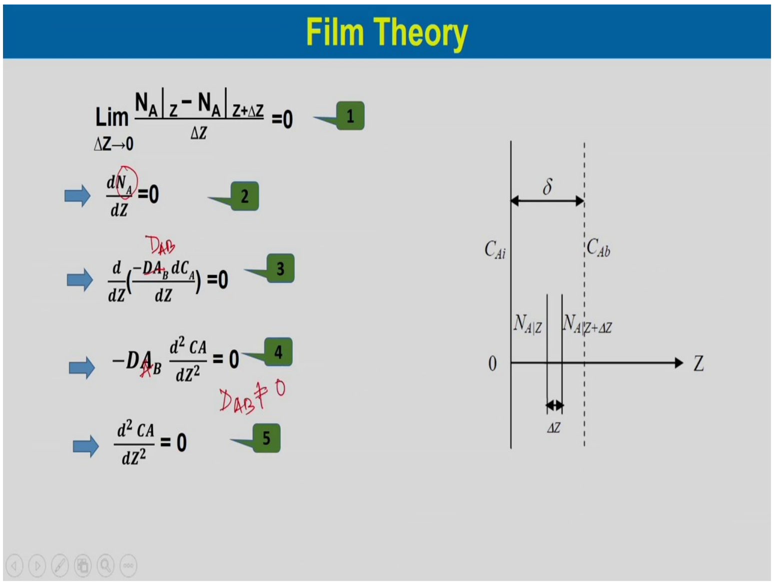


So, if we considered very small thickness delta Z and taking limit del Z tends to 0 we can write this equation N A at Z minus N A at Z plus delta Z by delta Z would be equal to 0. So, we can write dN A dZ would be equal to 0. Now N A if we substitute this N A from the Fick's law which is minus D A B d C A dZ which is equal to 0. So, we can write minus DA B del 2 CA dZ square would be equal to zero since, DA B is not equal to 0.

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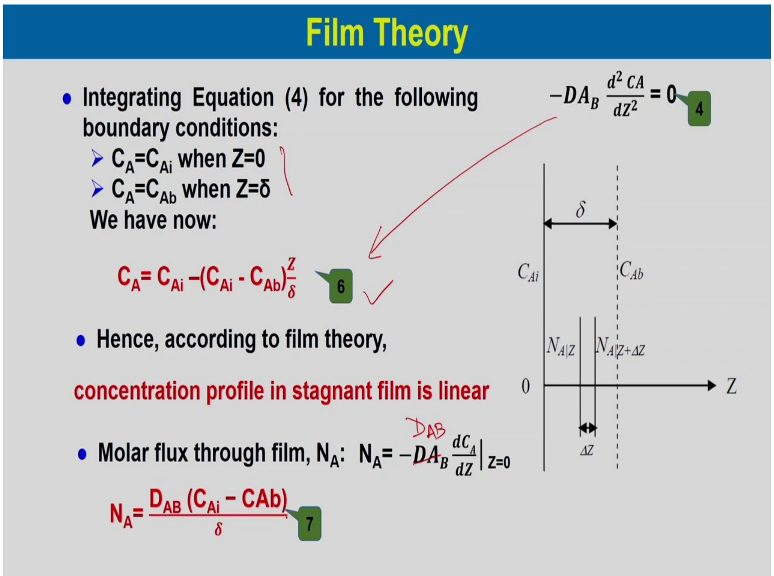


Now, if we integrate this equation 4 that is minus DA B d 2 CA dZ 2 equal to 0 with the boundary conditions that is at C A would be C Ai at Z is equal to 0 at the interface it will be C Ai that is at Z is equal to 0 and C A would be C Ab when Z is equal to delta. So, that is after the thickness it will remain C Ab the bulk concentration. So, we have now if we just integrate these equations we can obtained C A would be equal to C Ai minus C Ai minus C Ab into Z by delta. So, the integration with the boundary conditions given over here is left to you as a homework. So, you can just know integrate this equation and we will get this concentration profile.

Now, according to film theory we can write the concentration profile is in stagnant film is linear. As per film theory the concentration profile over here would be linear and molar flux through the film N A we can write N A would be equal to minus DA B dC A dZ at Z is equal to 0. So, if we differentiate this equation 6 and then substitute in this relation we will get N A would be equal to D AB C Ai minus CAb by delta.

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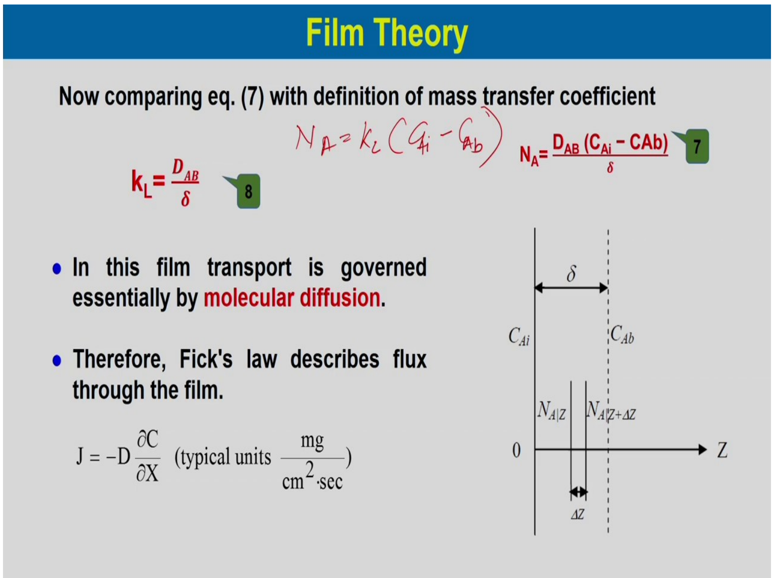


Now, if we compare equation 7 with definition of mass transfer coefficient. So, this is equation 7 and if we compare with the mass transfer coefficient equations that is know mass transfer coefficient is k L and the flux equation we can write N A is equal to k L into C Ai minus C Ab. So, if we compare these two relation we can calculate k L would be D AB by delta. So, in this film transport is governed essentially by molecular diffusion. Therefore, Fick's law describes flux through the film and we can write J is equal to minus D dC dX and typically you need for this is milligram per meter square second, it is mass flux.

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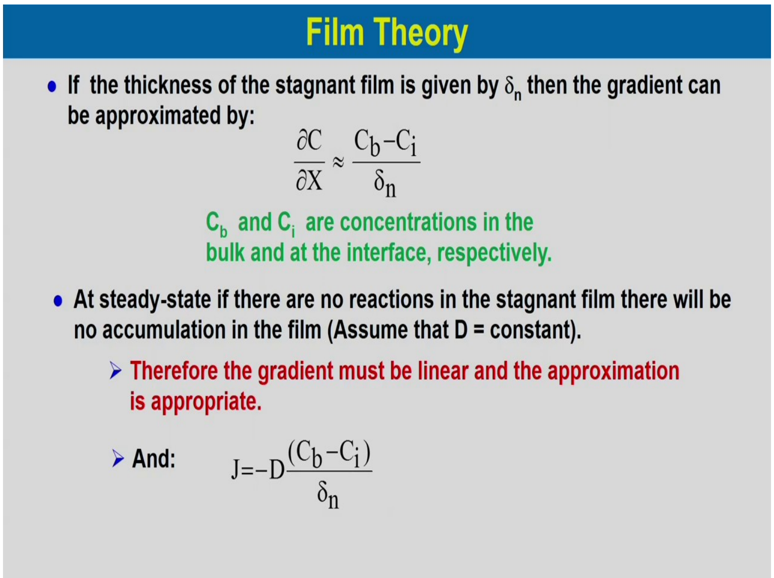


If the thickness of the stagnant film is given by delta n then the gradient can be approximated by dC dX which is equal to C b minus C i by delta n. And C b and C i are concentration in the bulk and at the interface. So, we can add any components. So, we can write this equation at steady state if there is no reactions in the stagnant film there will be no accumulation in the film that is D is constant and therefore, the gradient must be linear and then the approximation is appropriate which we have considered. So, we can write J would be equal to minus D C b minus C i by delta n.

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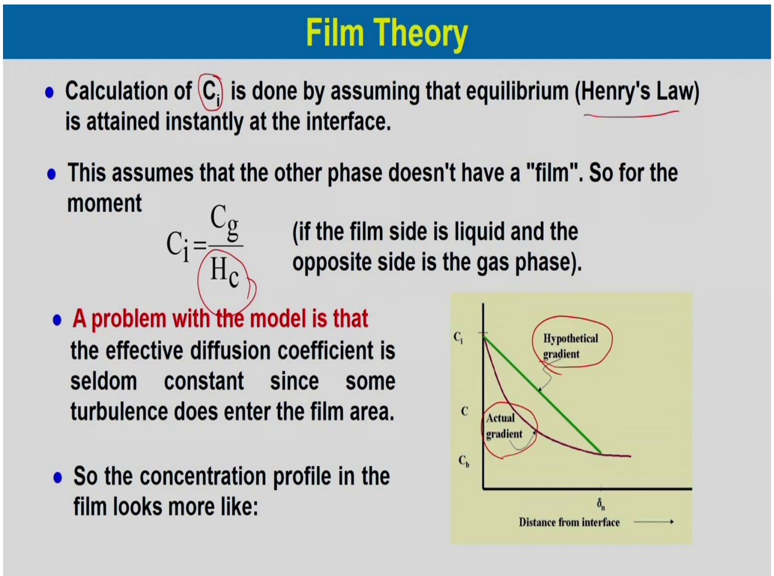

So, calculation of C i is done by assuming that the equilibrium that is the Henry's law is attained instantly at the interface. So, when the know if we assume the equilibrium which will attained at the interface very instantaneously then we can apply the Henrys law at the interface to calculate the interfacial concentration. So, C i we would be able to calculate by Henrys law if the equilibrium achieved at the interface instantaneously. This assumes that the other phase does not have a film. So, for the moment C i would be equal to C g by H c Henry's law H c is Henry's law constant and C g is the gas phase concentration of that component. If the film side is liquid and the opposite side is the gas phase. So, C i the interfacial concentration we can calculate C g by H c. So, this essentially considered that the gas there is no resistance in the gas phase. A problem with the model is that, the effective diffusion coefficient is seldom constant since some turbulence does occurred the film area.

So, the film theory which states that in the film the concentration profile is linear, but actually it is not linear because the defective diffusion coefficient is not constant because it varies due to the turbulence in the film, which actually exist show. So, the concentration profile in the film will look like actual gradient will look like this because of the turbulence which exist, but the hypothetical gradient as per the film theory is linear.

Thank you for hearing this lecture and we will continue with the mass transfer coefficient in turbulent flow in the next lecture.