Mass Transfer Coefficients
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Lecture – 11
Mass transfer coefficient in laminar flow

Welcome to the 3rd lecture of module 2 on mass transfer operation. We are discussing Mass Transfer Coefficient, let us have a brief recap on the previous lecture which we have covered.

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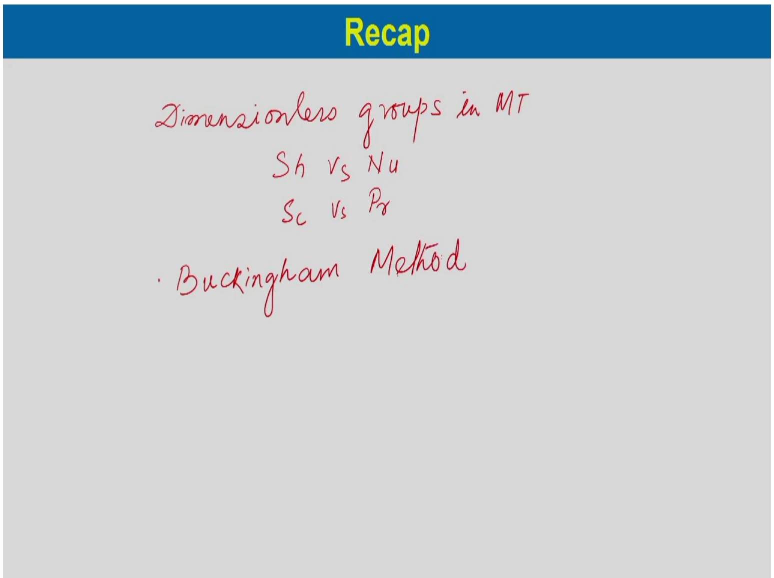

In our previous lecture we have discussed the dimensionless groups in mass transfer. Here actually we have discussed the analogous dimensionless group which is exist in heat transfer between heat and mass transfer. We have discussed among them like Sherwood number versus Nusselt number, Schmidt number versus Prandtl number in heat transfer we have also discuss the determination of the dimensionless group using Buckingham method.

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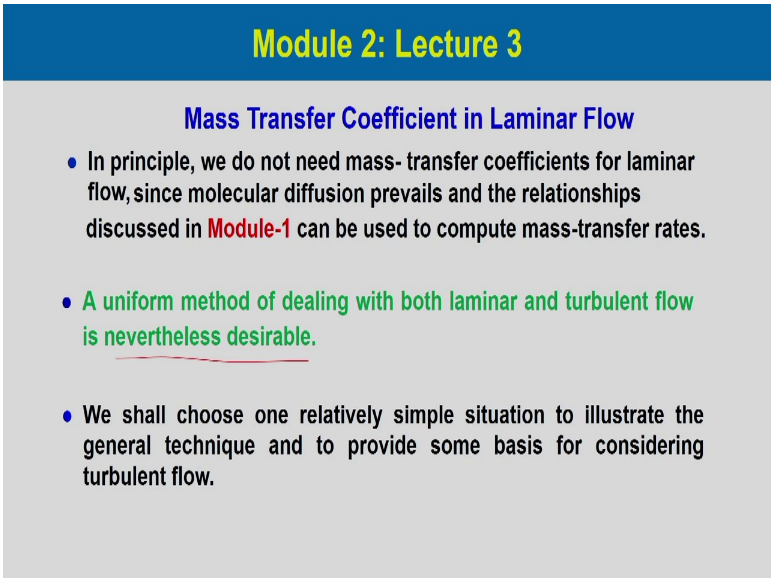

In this lecture we will discuss the mass transfer coefficient in laminar flow condition. In principle if we consider laminar flow where basically the diffusion takes place under laminar flow condition and in that respect we do not need to study the mass transfer coefficient in laminar flow conditions. Because, we have already discussed the molecular diffusions and which we have discussed in module 1, which can be used to compute the mass transfer coefficient. But a uniform method of dealing both laminar and turbulent flow is never the less desirable so, there should be a uniform policy to study both laminar and turbulent flow. We shall choose one relatively simple situation to illustrate the general technique and to provide some basis for considering turbulent flow.

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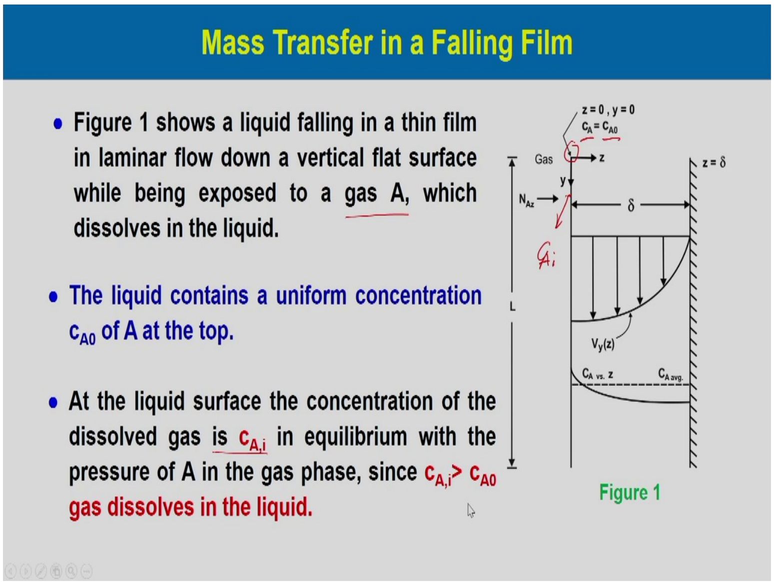

Let us consider falling film as shown in this figure 1, there is a falling film which is falling through a vertical surface and under laminar flow conditions and it is exposed to a gas A, which dissolves in the liquid. So, now, the liquid contains a uniform concentration C A naught of A at the top. So, at this location the C A is C A naught that is the liquid is dissolved with component A at a concentration C A naught. At the liquid surface the concentration of the dissolved gas is C A i. So, on this surface the concentration of A is C A i; that means, which is in equilibrium with the pressure of a in the gas phase. So, if gas is at atmospheric pressure so, it will be in equilibrium with that pressure at the gas liquid interface so, which is at the surface.

So, now, since C A i should be obviously, more than which was z is equal to 0 that is C A is equal to C A 0. So, if C A i greater than C A naught; that means, gas dissolve in the liquids.

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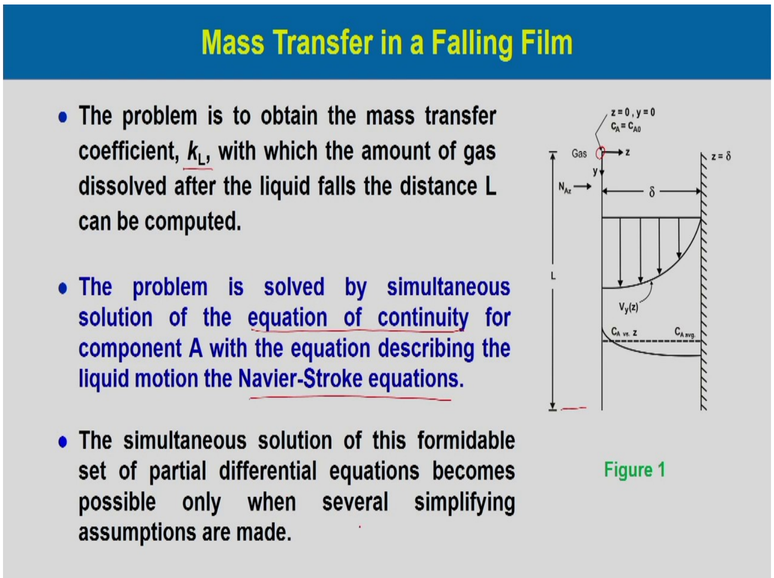

The problem is to obtain the mass transfer coefficient that is k L in the liquid phase with which the amount of gas dissolve after the liquid falls a certain distance L. Suppose the liquid falls from this location at z is equal to 0 and it goes to z is equal to 0 and y is equal to 0 at this location and it goes down to the length of l so, that need to be calculated.

The problem is solved by simultaneous solution of equation of continuity for the component A, with the equation which describes the liquid motion that is the Navier Strokes equations. We need to solve 2 equation simultaneously that is equation of motion and that is Navier Strokes equation and the equation of continuity for component A, which is diffusing to the liquid phase. The simultaneous solutions of the formidable set of partial differential equations become possible only when several simplifying assumptions are made.

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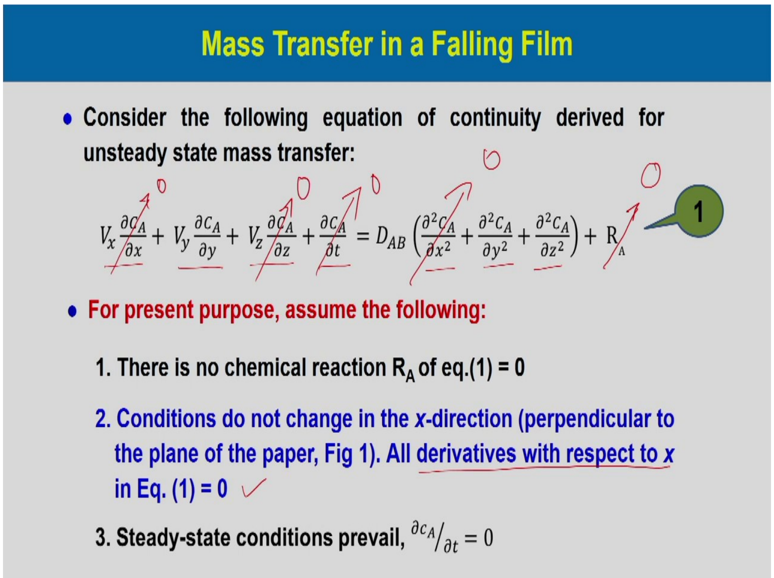

Now, let us consider the following equations of continuity which is derived for unsteady state mass transfer incase of module 1, we have already derived the unsteady state mass transfer that is using Fick's second law with chemical reactions. So, basically this is the unsteady state mass transfer equations which is V x del C A del x plus V y del C A del y plus V z del C A del z plus del C A del t is equal to D AB del 2 C A del x 2 del 2 C y del del y 2 plus del 2 C A del z 2 plus R A. So, this is the equation of continuity derived for unsteady state mass transfer. Now, for the present purpose let us assume that there is no chemical reactions in the systems so, R A of this equation 1 should be 0, this should be 0.

Now, the 2nd assumptions is that conditions do not change in the x direction that is perpendicular to the plane of the paper. So, as we have seen before in figure 1, if it is perpendicular to the plane it is in x directions. So, all derivatives with respect to x in equation 1 should be 0. The 3rd assumption is steady states, with respect to the assumption 2 this part would be 0, if we considered steady state condition; that means, del C A del t would be equal to 0 so, this part would be equal to 0.

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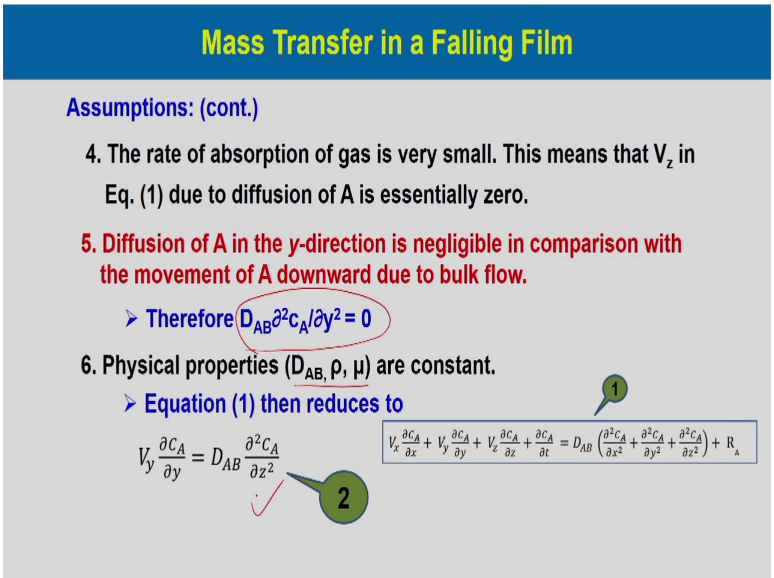

Another assumption is that the rate of absorption of gas is very small this means that V z in equation 1 due to diffusion of A is essentially zero. So, if V z is 0, then this should be equal to 0.

And diffusion of A in the y direction is negligible in comparison with the movement of A outward due to bulk flow. So, if the diffusion in the y direction is negligible then the above equation we the del D A B del 2 C A del y 2 would be equal to 0. So, once we substitute this in the earlier equation this part would be equal to 0 and then all the physical properties in this case D A B rho mu all are constant. So, if these are constant then this equations will reduce to this equation 2 considering these six assumptions all this terms will be cancelled out and we will have V y del C A del y is equal to D A B del 2 C A del z 2.

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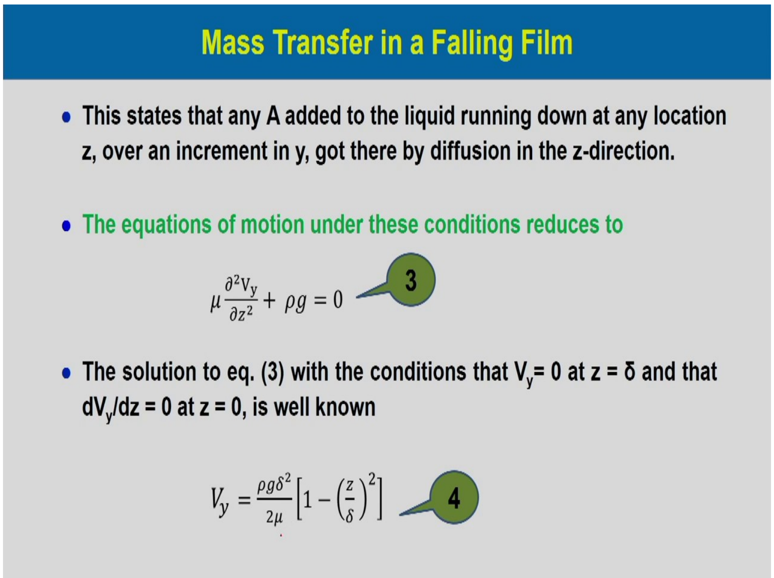

This equation states that A added to the liquid running down at any location z over an increment in y, got there by diffusion in the z direction. So, this is the statement we can made from this equation 2. The equation of motion under this condition will be again reduced to equation 3 which is mu del square V y divided by del z 2 plus rho g equal to 0, rho is the density and g is the gravitational acceleration and V y is the velocity z is the distance from the gas liquid interface to the surface mu is the viscosity.

The solution to this equation with the conditions V y is equal to 0 at z is equal to delta and that d V y d z is equal to 0 at z is equal to 0. So, this is very well known solutions known to all of us, we have studied in the fluid mechanics which is V y equal to rho g delta square divided by 2 mu 1 minus z by delta whole square. So, this is we can derived from this equation of motion with this condition.

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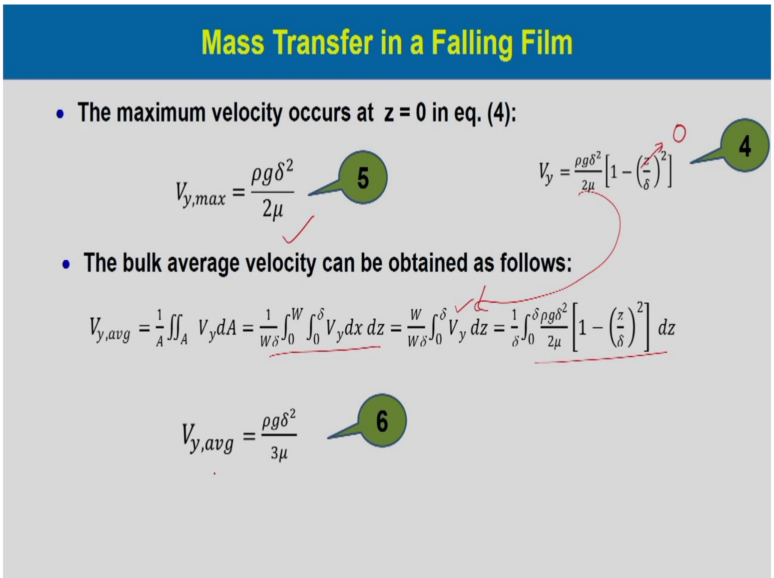

Now, the maximum velocity which occurs at z is equal to 0 in equation 4 would be we can write maximum velocity will happen when z is equal to 0 that is at the gas liquid interface. So, at when z is equal to 0 in this equations if we put z is equal to 0 so, we will obtain V y max is equal to rho g delta square by twice mu. The bulk average velocity can be obtained as follows, like we know V y average would be equal to 1 by A integral over the area V y into d A which is equal to 1 by W delta integral 0 to W integral 0 to delta V y d x d z.

And if you just integrate sequentially it will give W by W delta integral 0 to delta V y d z and then again if we integrate it will be 1 by delta integral 0 to delta rho g delta square by twice mu into 1 minus z by delta whole square; that means, if we substitute V y from this equations over here so, you will obtain this relations. Now, if we just integrate this you will obtain V y average would be equal to rho g delta square by 3 mu.

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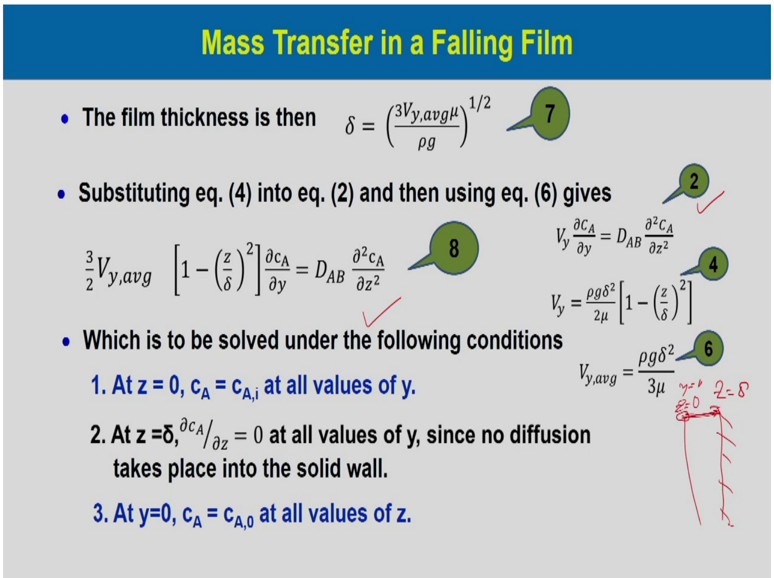

The film thickness is then we can calculate delta would be equal to 3 V y average into mu by rho g to the power half so, this just rearrangement of equation 6. So, if we rearrange this equations we can get the film thickness. Now substituting equation 4 into equation 2 and then if we use equation 6. So, equation 4 is substituted V y in equation 2, if we substitute and then if we use the V y average equation 6 we can obtain 3 by 2 V y average into 1 minus z by delta whole square into del C A del y would be equal to D A B del 2 C A del z square.

So, this is the equations we can obtain equation 8 and then which is to be solved under the following conditions. The condition 1 is at z is equal to 0 C A is C A i at all values of y, at z is equal to delta del C A del z would be 0 at all values of y since no diffusion takes place in the solid wall. And in the 3rd one is at y is equal to 0 C A is C A naught at all values of z; that means, when we take the falling film. So, it is at z is equal to 0 and y is equal to 0 it is at z is equal to delta. So, when z is equal to delta del C A del z would be 0 for all values of y since no diffusion takes place in the solid wall.

So, in this case and at y is equal to 0; that means, at this location C A is C A naught. So, for all values of the z that is at this surface at the beginning.

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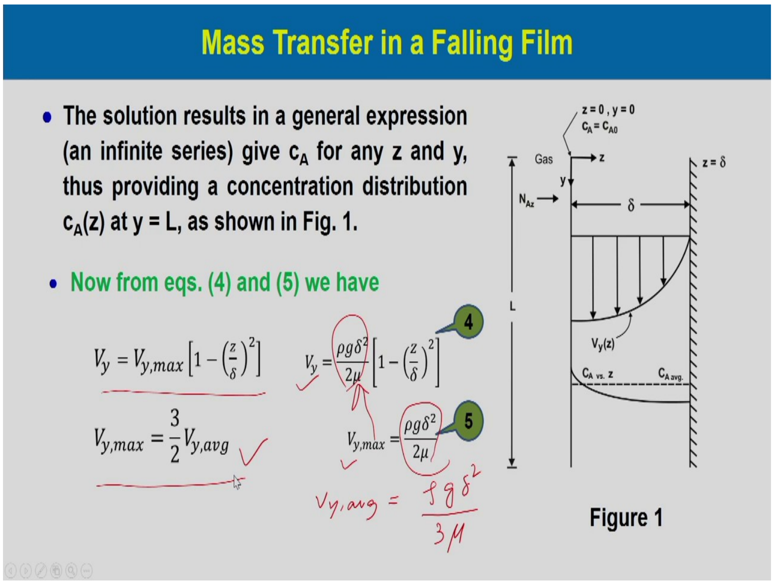

Now, the solution results in a general expression an infinite series gives C A for any z and y thus providing a concentration distribution that is at C A z at y is equal to L and which is shown over here. So, this is the concentration distribution, we will obtain out this equation. Now from equation 4 and equation 5 we have so, this is equation 4 and this is equation 5. So, that is V y equation 4 is V y rho g delta square by twice mu into 1 minus z by delta square.

This is we have already derived an equation 4 is y max which is rho g delta square by twice mu. Now we just compare, we can write V y is equal to V y max into 1 minus z by delta whole square; that means, we can substitute this one over here. So, we will obtain this relation. Now we know that V y average is equal to rho g delta square by thrice mu. Now, if we compare among these 2 then we can write V y max would be is equal to 3 by V y average. So, this two relations we can obtain from the earlier relations.

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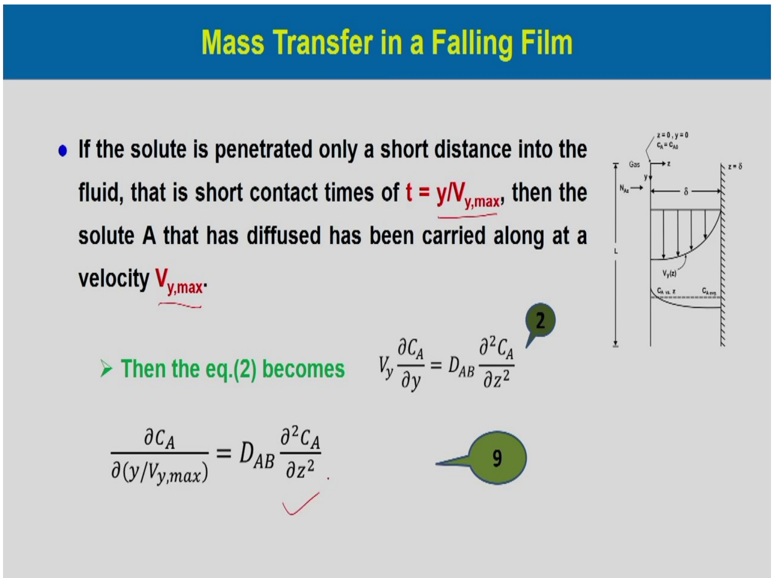

If the solute is penetrated only very small distance into the fluid from the surface this is possible when the contact time is very small that is t is equal to y by V y max, then the solute A that has diffused has been carried along at a velocity V y max. So, then the equation 2 this equations V y del C A del y is equal to D A B del 2 C A del z 2 would become del 2 C A divided by del y V y max would be equal to D A B del 2 C A del z square so, this is equation 9.

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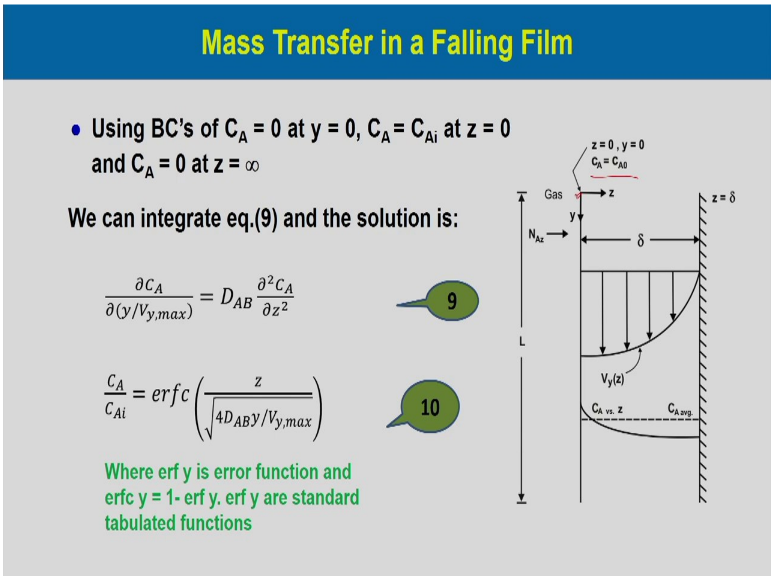

Now, if we use the boundary conditions that is at C A would be 0 at y is equal to 0 here and C A would be C A i at z is equal to 0 at the surface and C A would be 0 at z is equal to infinity. So, if we use this boundary conditions we can integrate equation 9, this is the equation 9 and if we integrate we will get C A by C A i would be equal to complementary error function of z by root over 4 D A B y V y max. If we define y is a function and then erfc y is the complementary error function of y which we can write as 1 minus error function of y. And error function of y are the standard tabulated function we can get from the table the value of error function and we can calculate the complementary error function.

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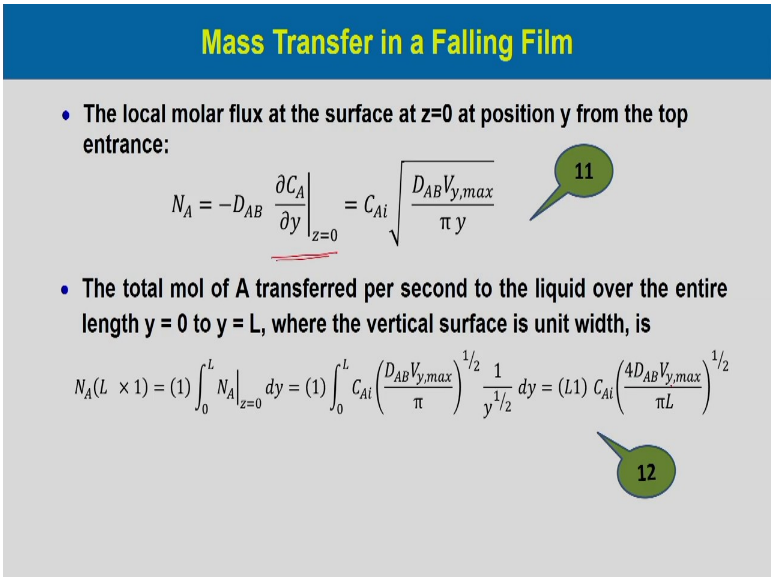

Now, the local molar flux at the surface at z is equal to 0 at any position y from the top of the entrance we can write N A would be equal to minus D A B del C A del y at z is equal to 0. So, local molar flux at the surface of the falling film at any position y from the top of the entrance; that means, we have to calculate the flux at the gas liquid interface. So, it is minus D A B del C A del y at z is equal to 0. So, if we just substitute the concentration profile we have obtained before over here C A by C A i. So, if we just substitute and differentiate this we will get C A i root over D A B V y max by pi y so, this is equation 11. The total mol of A transferred per second to the liquid over the entire length y is equal to 0 to y is equal to L, where the vertical surface is unit width which can be calculated as follows.

N A L into 1 would be equal to 1 integral 0 to L N A at z is equal to 0 d y which is equal to 1 integral 0 to L C A i D A B V y max by pi to the power half 1 by y to the power half d y, which is equal to L into 1 C A i 4 D A B V y max by pi L to the power half.

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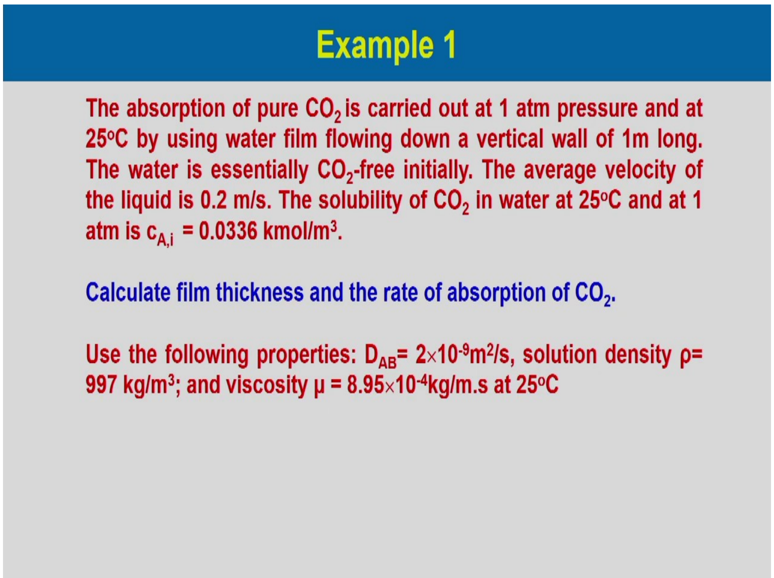

So, this is equation 12 and from this if you just simplify it will be NA would be equal to C A i 4 D A B V y max by pi L to the power half. Now let us take an example the absorption of pure carbon dioxide is carried out at 1 atmospheric pressure and at 25 degree centigrade by using water film flowing down a vertical wall of 1 meter long.

So, the length of the no vertical wall is 1 meter, the water is essentially CO 2 free initially; that means, C A naught is equal to 0 at y is equal to 0. The water is essentially CO 2 free initially, the average velocity of the liquid is 0.2 meter per second, the solubility of CO 2 in water at 25 degree centigrade and at 1 atmosphere pressure that is at C A i the interfacial concentration is given is 0.0336 kilo mole per meter cube. Now we need to calculate the film thickness and the rate of absorption of carbon dioxide.

Use the following properties the diffusivity of component A in B is given 2 into 10 to the power minus 9 meter square per second, solution density rho is equal to 997 kg per meter cube and viscosity mu is given 8.95 into 10 to the power minus 4 kg per meter second at 25 degree centigrade.

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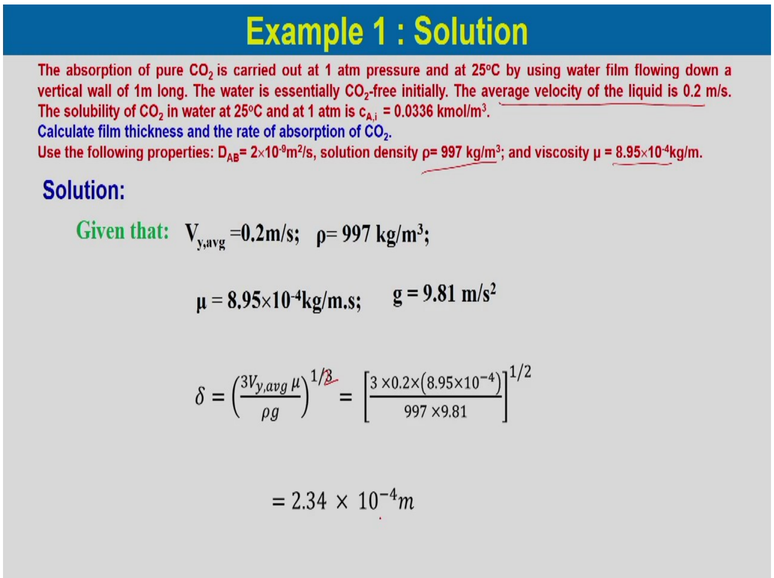

Now, let us consider V y average is 0.2 meter per second which is given the average velocity of the liquid 0.2 meter per second, density is 997 kg per meter cube. Then viscosity is given 8.95 into 10 to the power minus 4 kg per meter second, g is known to us 9.81 meter per second square. Now the film thickness if we use this relation which we have derived delta is equal to 3 V y average mu by rho g to the power half. So, if you substitute the values it is 3 into 0.2 into 8.95 into 10 to the power minus 4 divided by 997 into 9.81 to the power half. So, this will lead to 2.34 into 10 to the power minus 4 meter per second.

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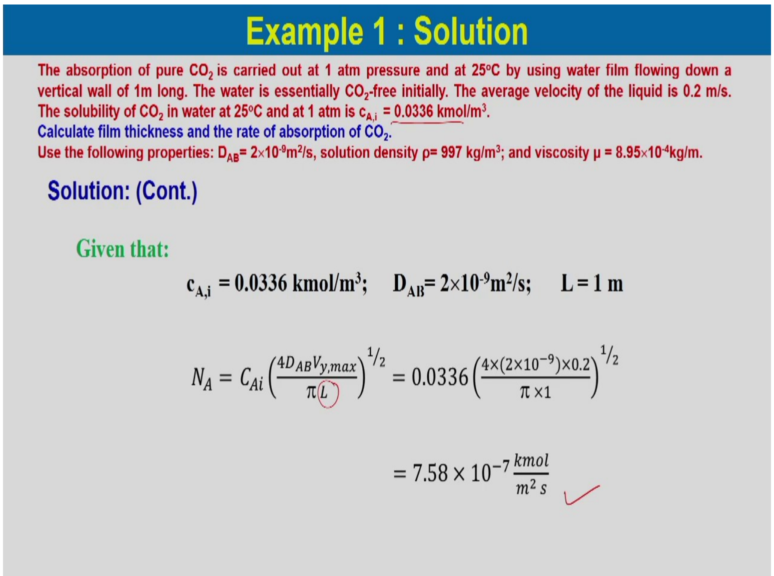

Now, for the rate of absorption of carbon dioxide we need to know the concentration driving force. So, C A i which is given C A i is given and 0.336 kilo mole per meter cube, D A B is 2 into 10 to the power minus 9 meter square per second, L is 1 meter. So, we can write the equation of flux NA would be equal to C A i 4 D A B V y average by pi L to the power half.

So, if we substitute it is 0.0336 into 4 into 2 into 10 to the power minus 9 into 0.2 divided by pi into 1 so, since length is 1 so, whole to the power half. So, this will give 7.58 into 10 to the power minus 7 kilo mole per meter square second. So, this is a very simple examples which relates to the problem which you have formulated based on the theory we discussed so far for a falling film which is through a vertical surface and it is under laminar flow conditions.

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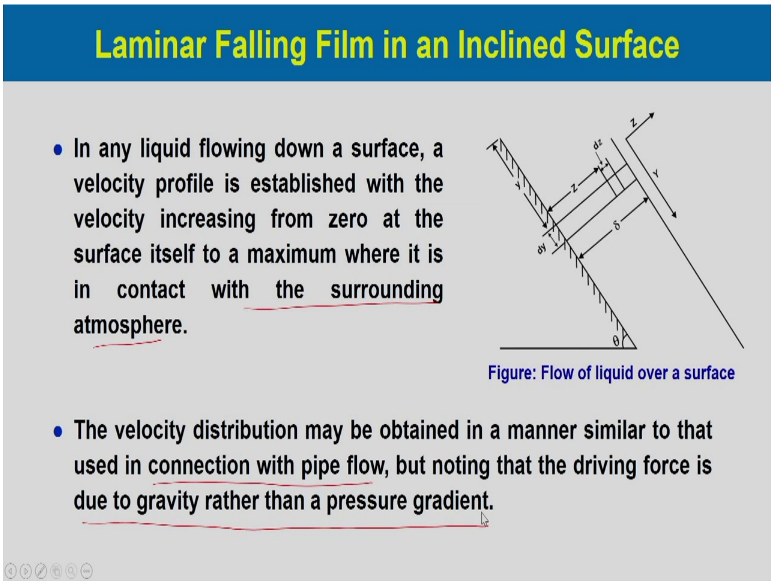

If we have a laminar falling film, but the surface is an inclined surface in that case how we can calculate the mass transfer coefficient. In any liquid flowing down a surface a velocity profile is established with the velocity increasing from 0 at the surface itself to a maximum where it is contact with the surrounding atmosphere.

This is already we have discussed. The velocity distribution may be obtained in a manner similar to used in connection with the pipe flow. So, the velocity distribution it will be similar to the pipe flow, but noting that the driving force is due to gravity rather than the pressure gradient in this case.

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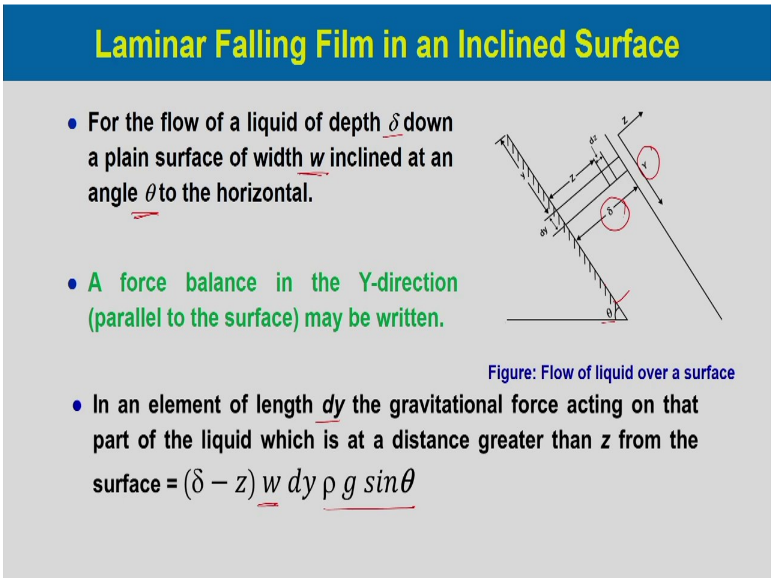

Now, for a fluid of a liquid depth delta, this one is the solid surface and the depth of the fluid is delta and then the width of the solid surface is w; w is the width of this solid surface and delta is the liquid depth on the solid surface and surface is inclined at a angle of theta to the horizontal.

So, if with the horizontal it makes a theta angle then a force balance in the Y direction that is the parallel to the surface. So, this Y directions we considered parallel to the surface which we can write the force balance on it. So, in an element of length if we considered no delta y, length is very small length delta y the gravitational force acting on that part of the liquid which is at a distance greater than z from the surface. So, we can write delta minus z into w d y rho g sin theta. So, this is the no gravitational force acting on that part of the liquid at a distance greater than z from the surface.

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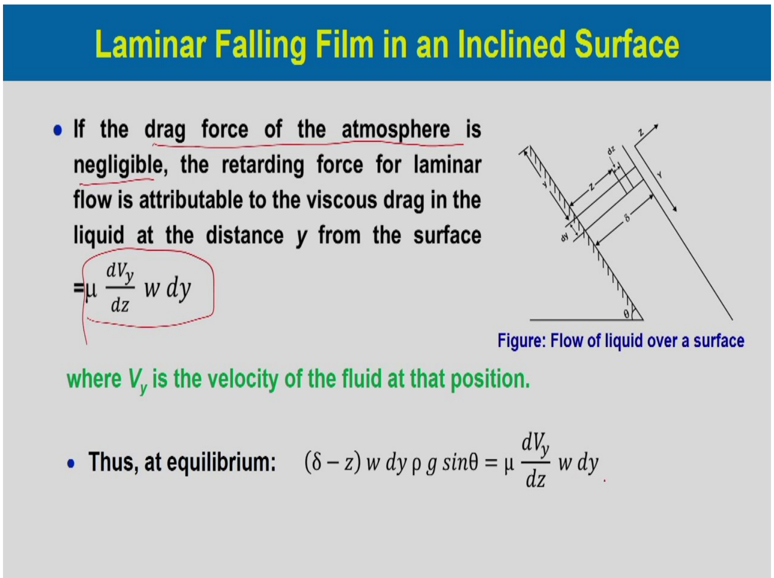

Now, if the drag force of the atmosphere is negligible. So, we assume that the drag force of the atmosphere is negligible in that case then the retarding force for laminar flow considered to the viscous drag in the liquid at a distance y from the surface. So, that is that we can write mu d V y d z w d y. So, this is the viscous drag which is of the liquid at a distance y from the surface, V y is the velocity of the fluid at that position.

Now, at equilibrium this two forces as we have calculated one is the gravitational force, another is this drag force or the viscous force which is applicable over here viscous drag they will be under equilibrium. So, we can write delta minus z w dy rho g sin theta would be equal to mu d V y d z w d y.

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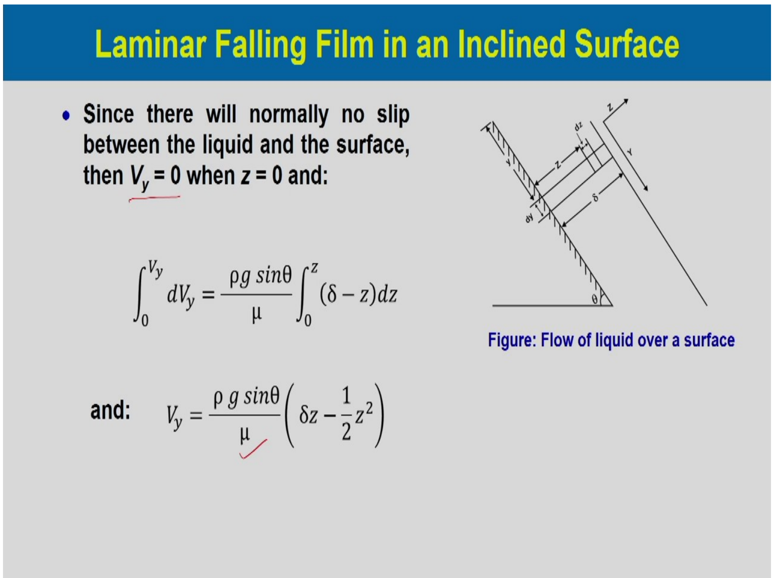

Now, since there will be normally no slip between the liquid and the surface. So, at the surface there will be no slip between the liquid and the surface. So, the velocity V y at z is equal to 0 which is over here and we can write know integral 0 to V y d V y is equal to rho g sin theta by mu integral 0 to z delta minus z d z. So, with this if we integrate this relation we will get V y would be equal to rho g sin theta divided by mu into d z minus 1 by 2 z square.

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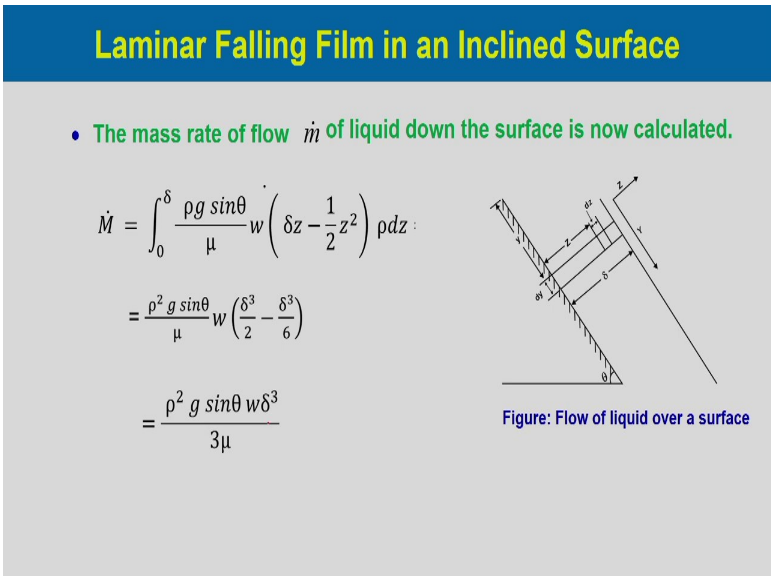

Now, we can calculate the mass rate of flow that is m dash of liquid down the surface can be calculated which is m dash would be equal to integral 0 to delta rho g sin theta by mu into w into d z minus 1 by 2 z square into rho d z. So, this is the mass rate. So, if we just integrate it will be rho square g sin theta divided by mu w delta cube by 2 minus delta cube by 6. So, this would be rho square g sin theta w delta cube by 3 mu.

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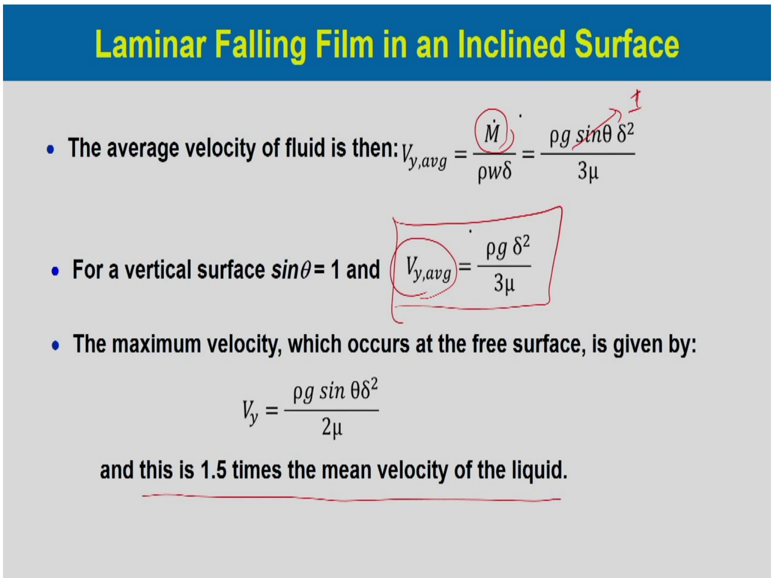

Now, the average velocity of fluid we can calculate V y average would be equal to m dash by rho w into delta. So, this is the average velocity. So, if you just substitute the values of know mass rate which we have determined before this one we can obtain rho g sin theta delta square by 3 mu. Now for a vertical surface where sin theta would be 1, in that case this equation will reduce to rho g delta square by 3 mu, which we have derived in case of the vertical surface. The maximum velocity which occurs at the free surface can be obtained by V y would be equal to rho g sin theta delta square by 2 mu and this is at 1.5 times the mean average velocity of the liquid.

So, this is the mean average velocity and this is the maximum velocity. So, the maximum velocity which occurs at the surface can be calculated V y would be rho g sin theta delta square divided by 2 mu. And then we can just see the mean velocity and the maximum velocity over here and they we can see that it is 1.5 times the mean velocity of the fluid.

Thank you.