Mass Transfer Coefficients
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Lecture – 10
Dimensionless groups and correlations for convective mass transfer coefficients

Welcome to the second lecture of module 2 on Mass Transfer operation.

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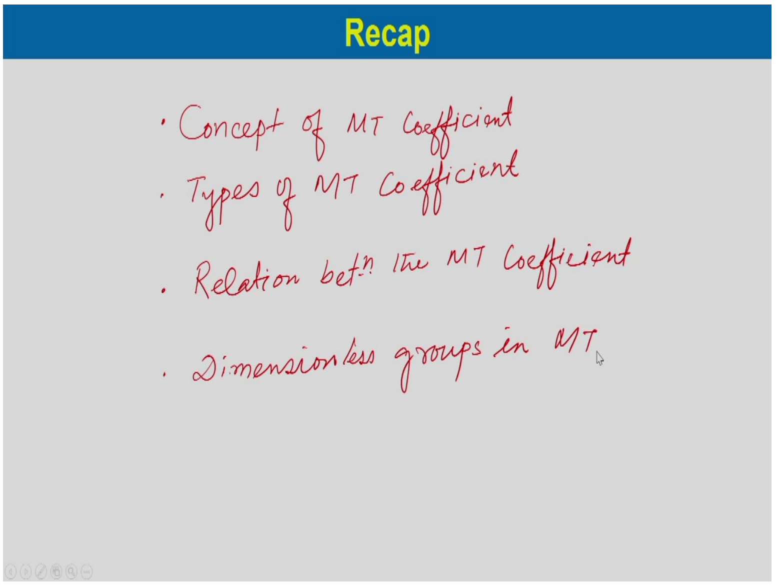

In the last lecture, we have discussed on the concept of mass transfer coefficient under which we have discussed the different types of mass transfer coefficient. And, then we have discussed the relation between the mass transfer coefficient. In this lecture, we will discuss the dimensionless group involve in mass transfer.

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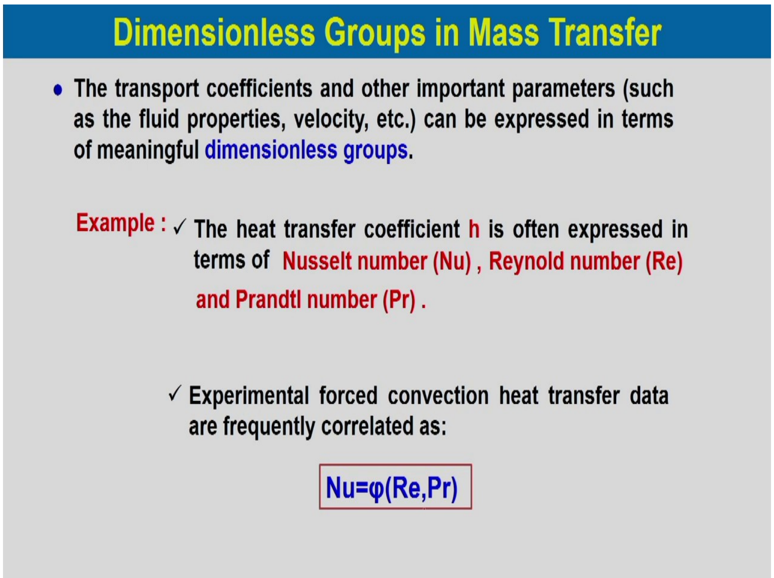

The transport coefficient and other important parameters such as fluid properties, velocity etc. can be expressed in terms of meaningful dimensionless groups. So, for example, the heat transfer coefficient h is often expressed in terms of the Nusselt number and also Reynolds number and Prandtl number. Experimental forced convection heat transfer data are frequently correlated as Nusselt number is a function of Reynolds number and Prandtl number. This is very well known equations we know.

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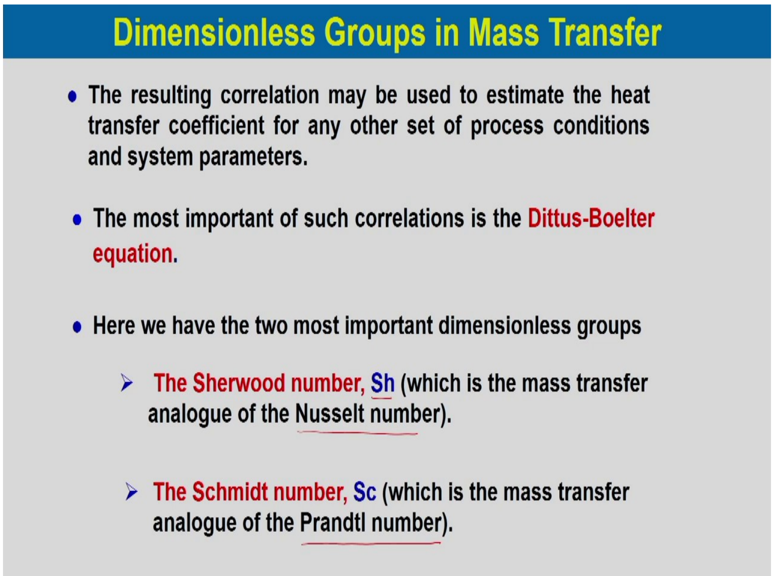

The resulting correlation may be used to estimate the heat transfer coefficient for any other set of process conditions and system parameters. The most important equations which relates the Nusselt number with the Reynolds number and Prandtl number is the Dittus-Boelter equation. Here we have two most important dimensionless group in case of mass transfer similar to the heat transfer. One of them is the Sherwood number which is the mass transfer analogue of the Nusselt number.

So, so in case of mass transfer it is Sherwood number which is analogue to the Nusselt number in case of the heat transfer. Another important dimensionless group in mass transfer is the Schmidt number. This is also the mass transfer analogue of the Prandtl number in case of the heat transfer. So, these two important dimensionless groups also relates with the heat transfer. We have some more dimensionless group, we will discuss at the later part of this lecture.

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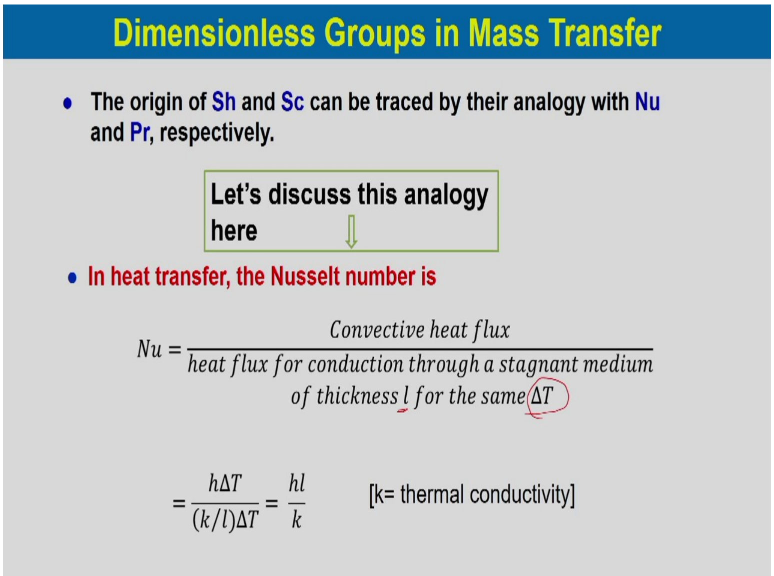

The origin of Sherwood and Schmidt can be traced by analogy with Nusselt and Prandtl respectively. So, like let us considered or discuss this analogy here. In case of heat transfer the Nusselt number we can define is the convective heat flux divided by the heat flux for conduction through a stagnant medium of thickness l with a delta T is the temperature gradient the driving force. So, it is a ratio between the convective heat flux and then heat flux due to conduction. So, if you just write the convective heat flux is h into delta T and then the conduction heat flux we can write k by l delta T where, l is the thickness of the stagnant medium. So, this will lead to h l by k where, k is the thermal conductivity.

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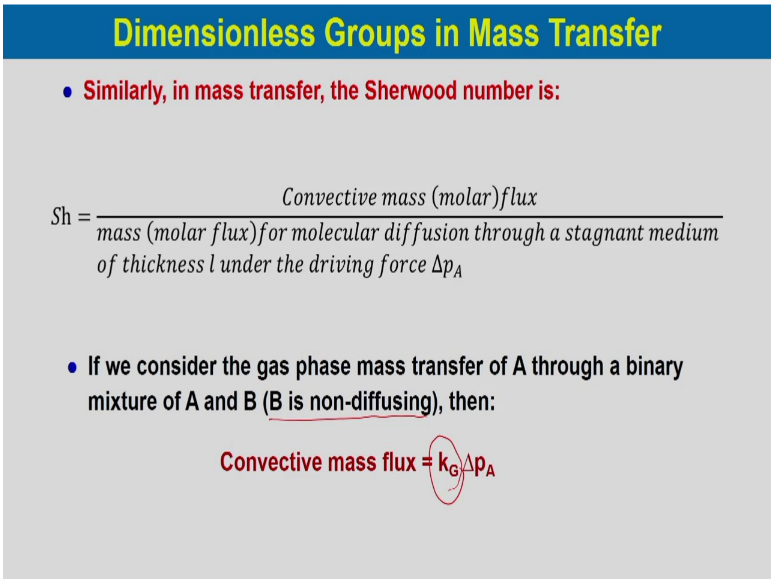

Now, if we look into the mass transfer the Sherwood number can be defined by convective mass flux or molar flux divided by the mass or molar flux for molecular diffusion through a stagnant medium of thickness l under the driving force of delta p A. Delta p A is the partial pressure driving force.

So, in this case convective mass transfer, ratio of convective mass transfer and the mass transfer due to the molecular diffusion will give you the Sherwood number. So, if we considered a gas phase, mass transfer of A through a binary mixture of A and B in which we considered B is not diffusing. In that case we can write the convective flux as k G into delta p A where, k G is the mass transfer coefficient in the gas phase.

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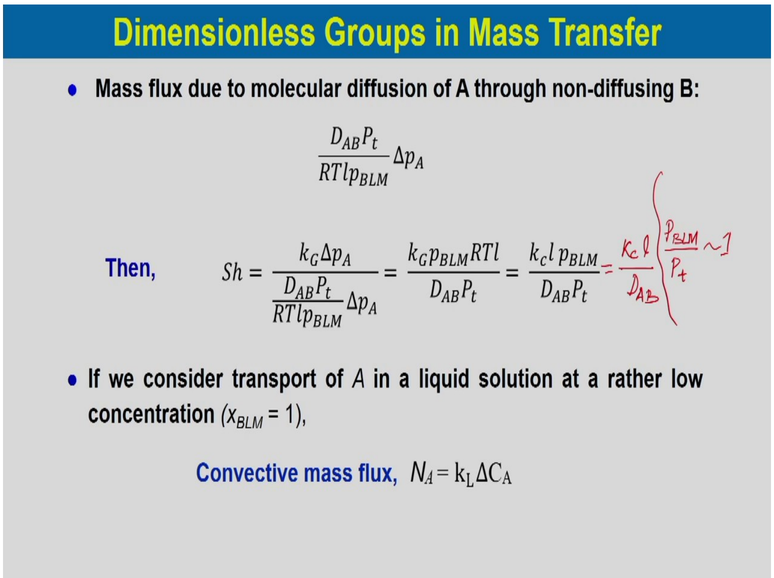

The mass flux due to molecular diffusion of A through non-diffusing B we have derived earlier in the module 1 which is D AB P t by RT l p BLM into delta p A. D AB is the mutual diffusion coefficient of A into B and P t is the total pressure, R is the universal gas constant. T is any temperature, l is the distance through which the diffusion takes place and p BLM is the log mean pressure difference. So, we can write then Sherwood number would be the k G delta p A that is the convective flux divided by the diffusive flux which is written over here. So, it would if we simplify it would be k G p BLM RTl divided by D AB P t which we can write k c l p BLM by D AB P t.

So, in case of know very dilute solution p BLM by P t is approximately 1. So, in that case we can write k c l divided by D AB. Now, if we considered transport of A in a liquid solution at a rather low concentration where X BLM is 1, in that case we can write the convective flux N A would be equal to k L, k L is the liquid phase mass transfer coefficient and delta C A is the concentration driving force in the liquid phase.

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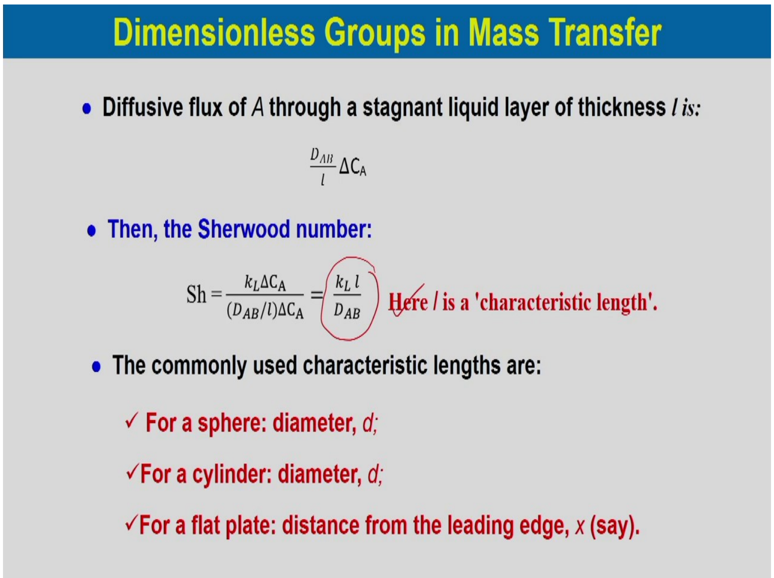

Now, the diffusive flux of A through a stagnant liquid layer of thickness l we can write D AB by l into delta C A. So, the Sherwood number we can calculate k L delta C A divided by D AB by l into delta C A. So, which will lead to k L into l D AB it is the Sherwood number which is analogous number of the heat transfer that is Nusselt number.

So, here l is the characteristic length. The commonly used characteristics lengths in different cases as you can see, if we considered a sphere of diameter d then it is the characteristic length. In case of cylinder the diameter d is the characteristics length and for a flat plate the distance from the leading edge x is the characteristic length. These are the commonly used characteristic length.

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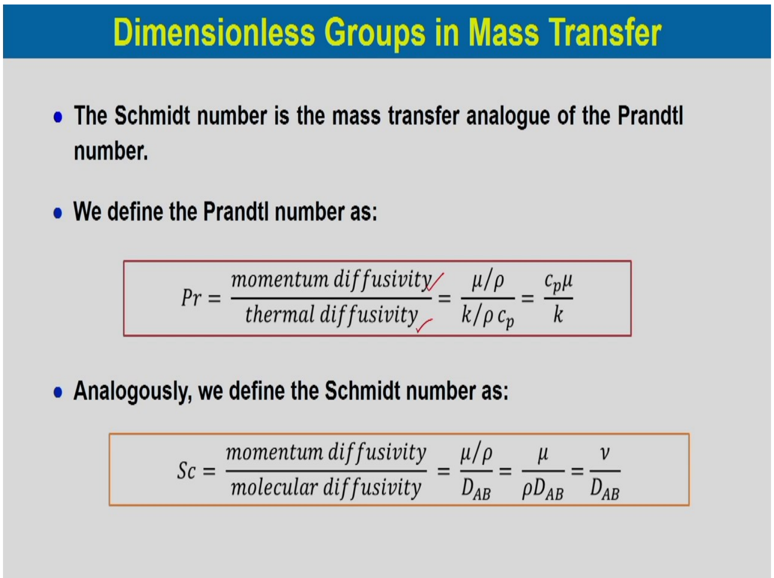

Now, Schmidt number is the mass transfer analogue of Prandtl number as we have said. This Schmidt number we can define like in case of heat transfer, we define the Prandtl number is the momentum diffusivity divided by the thermal diffusivity. Momentum by thermal; so, momentum diffusivity is mu by rho mu is the viscosity and rho is the density divided by k by rho c p that is the thermal diffusivity.

So, it is c p mu by k; so, Prandtl number we can write c p mu by k. Similarly, analogously we can write the Schmidt number in case of mass transfer which is momentum diffusivity divided by the molecular diffusivity. So, momentum diffusivity mu by rho divided by the molecular diffusivity is D AB. So, we can write mu by rho D AB or mu by D AB. So, similarly this is the Schmidt number and an analogous number of heat transfer is Prandtl number.

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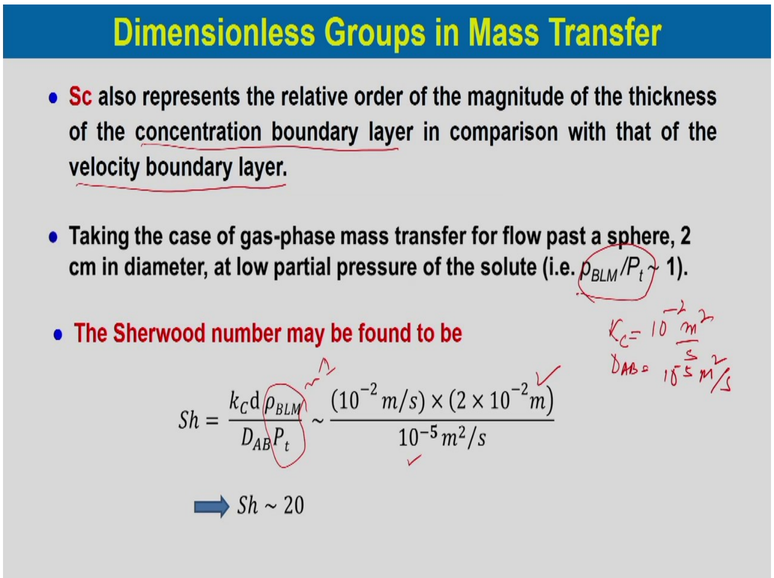

Now, Schmidt number also represents the relative order of magnitude of the thickness of concentration boundary layer in comparison with that of the velocity boundary layer. So, this taking the case of gas phase mass transfer for flow past a sphere, if we take 2 centimetre in diameter of the sphere and at low partial pressure of the solute; as we said for low partial pressure of the solute this p BLM by P t would be approximately equal to 1. So, in that case the Sherwood number we can write if we have k C is 10 to the power minus 2 metre per second and D AB is 10 to the power minus 5 metre square per second.

Then we can calculate from this equation Sherwood number equation k C 10 to the power minus 2 metre per second and d which is given 2 centimetre is in diameter. So, 2 into 10 to the power minus 2 metre is d and then the diffusion coefficient D AB is 10 to the power minus 5 metre square per second. So, if we substitute them then and as we said p BLM by P t this term is 1 approximately 1. So, the Sherwood number would be equal to 20.

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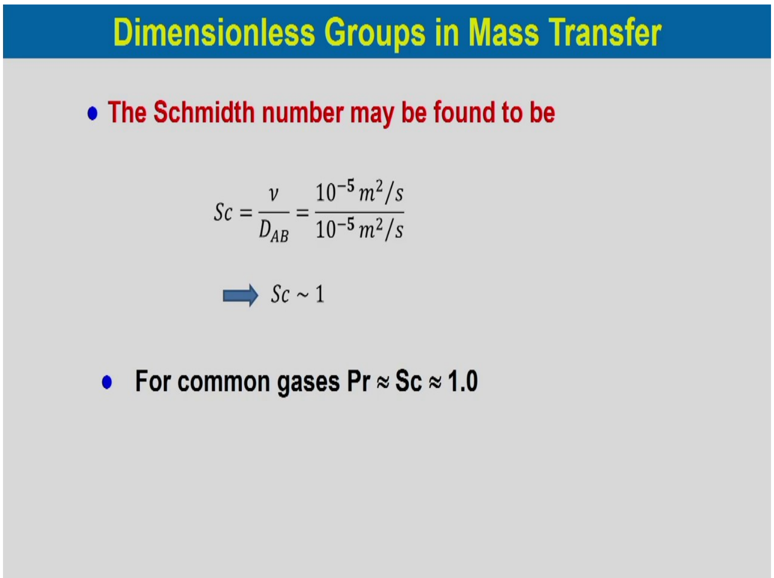

The Schmidt number may be found to be as follows nu is the viscous kinematic viscosity which is 10 to the power minus 5 metre square per second. And which is given and D AB is also given. So, you can calculate a Schmidt number is about 1. So, for common gases the Prandtl number approximately equal to Schmidt number and would be approximately equal to 1.

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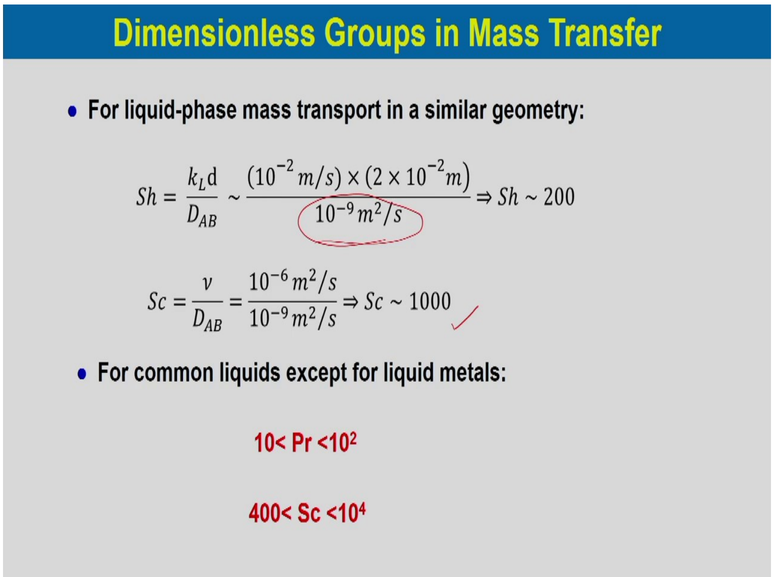

Now, for liquid phase mass transport in a similar geometry we can calculate Sherwood number k L d by D AB. So, k L is 10 to the power minus 2 metre per second and d is given 2 into 10 to the power minus 2 metre. And, then diffusivity in the liquid phase is in the range of 10 to the power minus 9 metre square per second. So, which is about 4 order less compared to the gas phase diffusion coefficient.

So, if we substitute in case of liquid phase the Sherwood number is approximately 200. Now, Schmidt number also we can calculate it is about 1000. So, basically the Schmidt number for most of the common liquids its and the Prandtl number you can see the range between 10 to 100. And, in case of Schmidt number it varies between know 400 to 10000.

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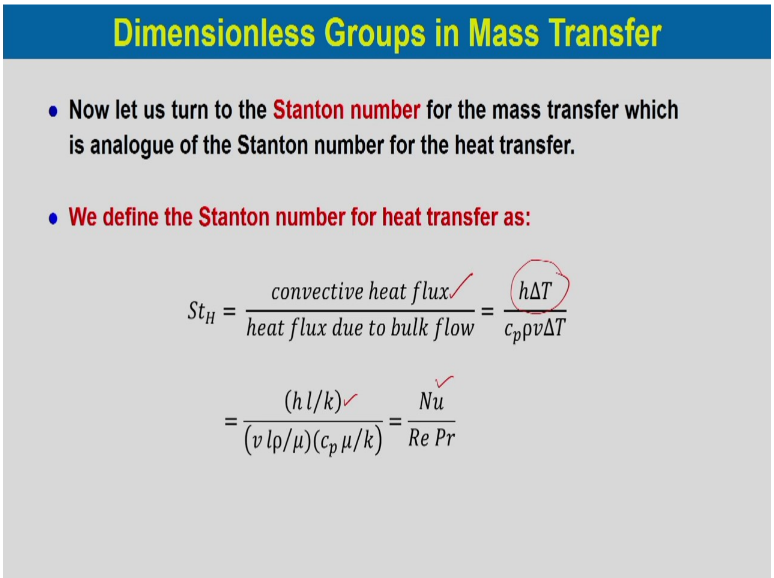

Now, let us turn to the Stanton number for the mass transfer which is also analogous number you know Stanton number for heat transfer. So, we define Stanton number in case of the heat transfer, it is the convective heat flux by heat flux due to bulk flow. So, convective heat flux is h delta T and then the heat flux due to bulk flow is c p rho v delta T. So, we can just rearrange this if we just multiply l know both side in the numerator and the denominator.

And, then if we just manipulate with rho by mu over here we can write this know ratio as h l by k divided by v l rho by mu into c p mu by k. So, this is represented in terms of the different dimensionless number we have studied. So, that is Nusselt number divided by Reynolds number into Prandtl number. So, the Stanton number can be defined with this dimensionless number.

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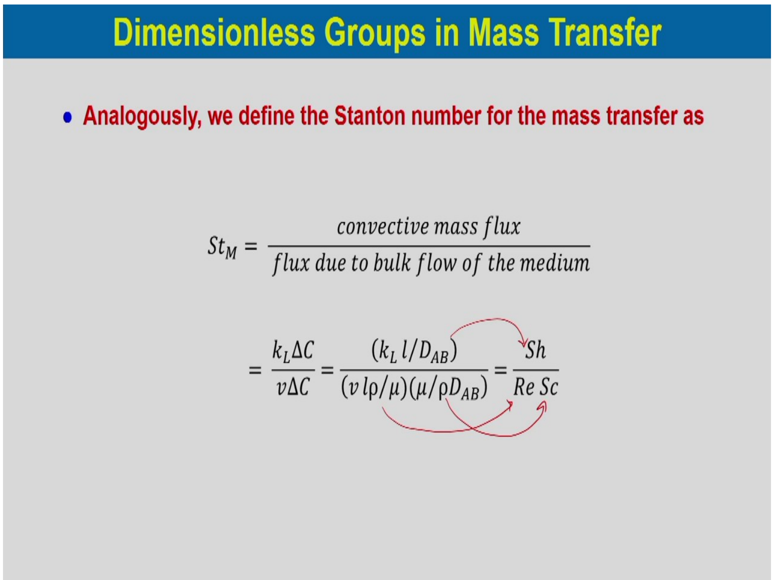

Similarly, in case of mass transfer we can also write Stanton number for mass transfer which is convective mass flux divided by flux due to bulk flow of the medium. So, convective mass flux as we know k L into delta C divided by v into delta C that is the flux due to bulk flow. And, if we rearrange it is k L C k L into small l divided by D AB whole divided by v l rho by mu into mu by rho D AB.

So, this is Sherwood number and this is basically Reynolds number and this one is Schmidt number. So, we can write the Stanton number for mass transfer in terms of the Sherwood number and Reynolds number and Schmidt number.

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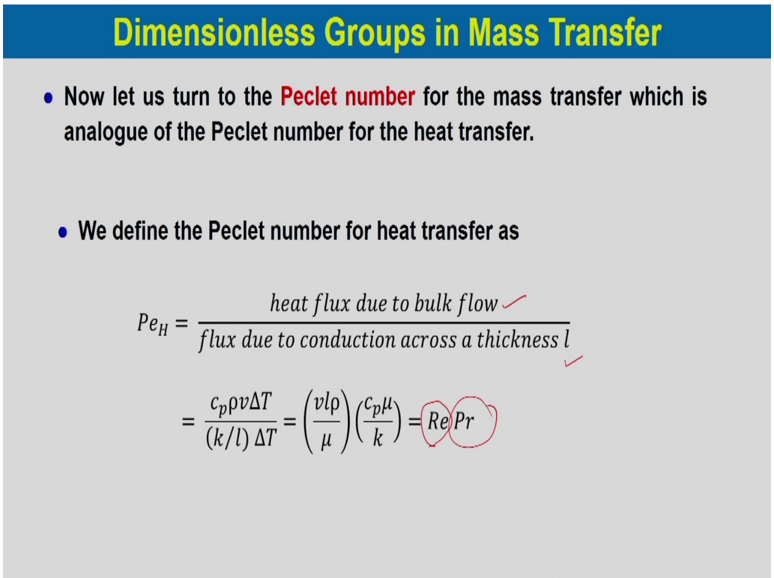

Now, let us turn to the Peclet number for the mass transfer which is analogue of the Peclet number in case of the heat transfer. We define Peclet number in case of heat transfer as Peclet number is equal to heat flux due to bulk flow and flux due to conduction across a thickness l. So, if we just write the heat flux due to bulk flow c p rho v into delta T divided by k by l into delta T, we can just group them into two different dimensionless number; it is v l rho by mu into c p mu by k.

So, it is we can write Reynolds number and Prandtl number. So, Peclet number in case of heat transfer we can define in terms of Reynolds number and a Prandtl number.

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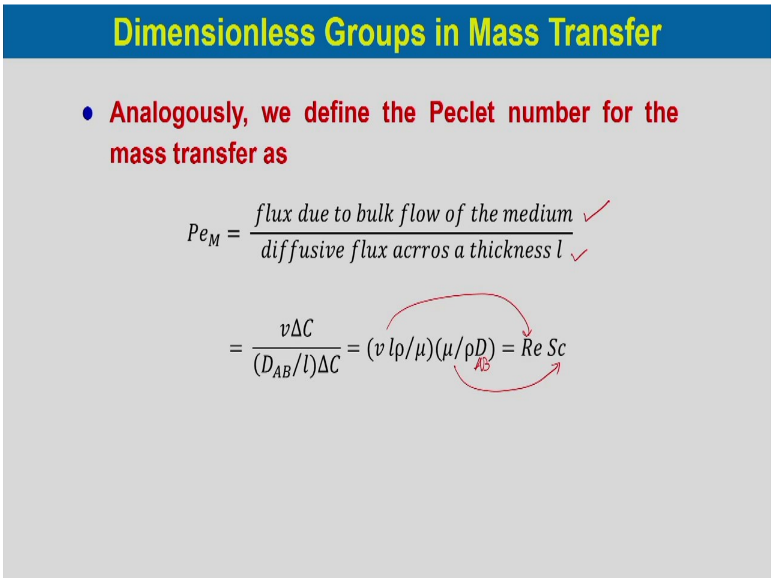

So, similar way we can write the analogous number for mass transfer. So, Peclet number for the mass transfer we can define flux due to the bulk flow of the medium divided by the diffusive flux across a thickness l. So, if we just write the flux equation v into delta C is the flux due to the bulk flow of the medium divided by the diffusive flux across a thickness is D AB by l into delta C.

So, if we just group them it will be v l rho by mu into mu by rho D or D AB. So, this is Reynolds number and this is Schmidt number. So, Peclet number we can define in terms of the two dimensionless number is Reynolds number and Schmidt number.

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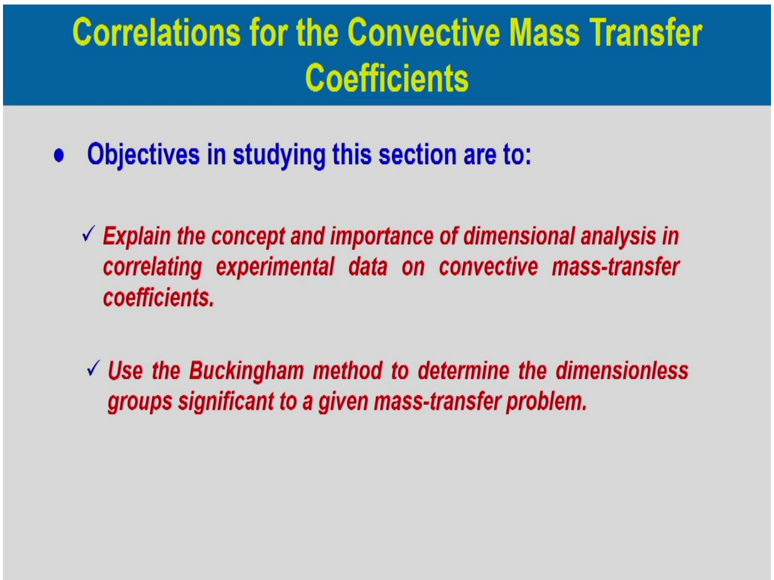

Now, correlation for the convective mass transfer coefficients. So, the objective in studying the section are to explain the concept and importance of the dimensional analysis in correlating experimental data on convective mass transfer coefficient. And, the another one is used Buckingham method to determine the dimensionless group significant to a given mass transfer problems. So, with these two objectives let us continue our discussion.

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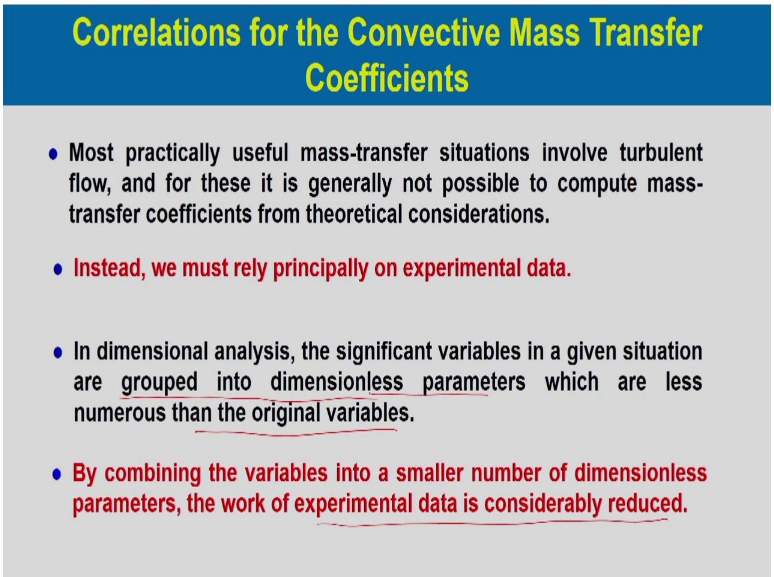

Most practically useful mass transfer situations involve turbulent flow and for this it is generally not possible to compute mass transfer coefficient from theoretical consideration. So, instead we must rely principally on experimental data. So, if we rely on the experimental data then we need to have the dimensional analysis that relates the significant variable in a given situation that are grouped into the dimensionless parameters which are less numerous than the original variables.

So, if we can make the dimensionless group then it would be much more helpful to correlating the experimental data in case of turbulent flow. By combining the variable into a smaller number of dimensionless parameter the work of experimental data is considerably reduced. So, this will help us to reduce the experimental work.

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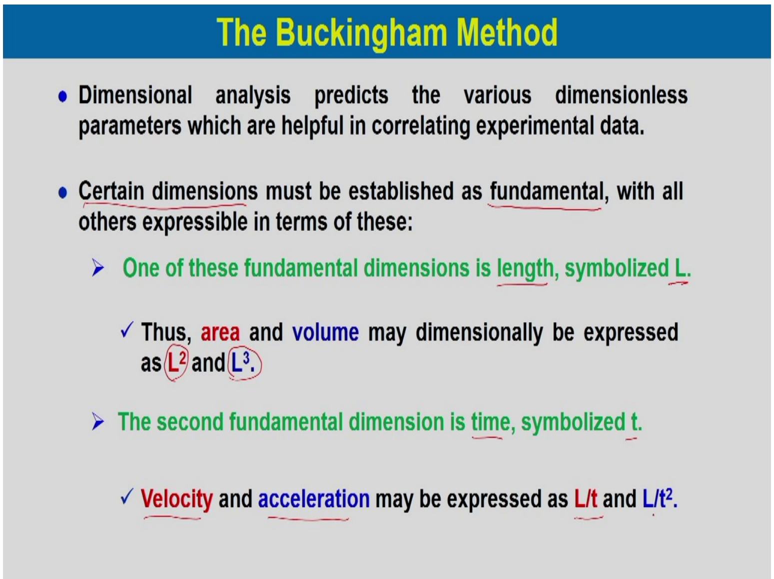

Dimensional analysis predicts the various dimensionless parameters which are helpful in correlating experimental data. Certain dimensions must be established as fundamental. So, we need to have a fundamental dimensions with which we can express the other terms. So, that is those terms are considered as fundamental, like one of these fundamentals dimension is the length. So, length we can use symbol L. Now, if we want to define area and the volume we may dimensionally be expressed which know area and volume as length square and length cube. So, the volume can be expressed in length cube and area can be expressed in plane square. Similarly, the second fundamental dimension is time like which is which we can symbolize with t. So, like if we want to define the velocity or acceleration we can write in case of velocity it is length per time and acceleration length per time square. So, this velocity acceleration or area and volume these are based on certain fundamental dimensions which is over here is length time.

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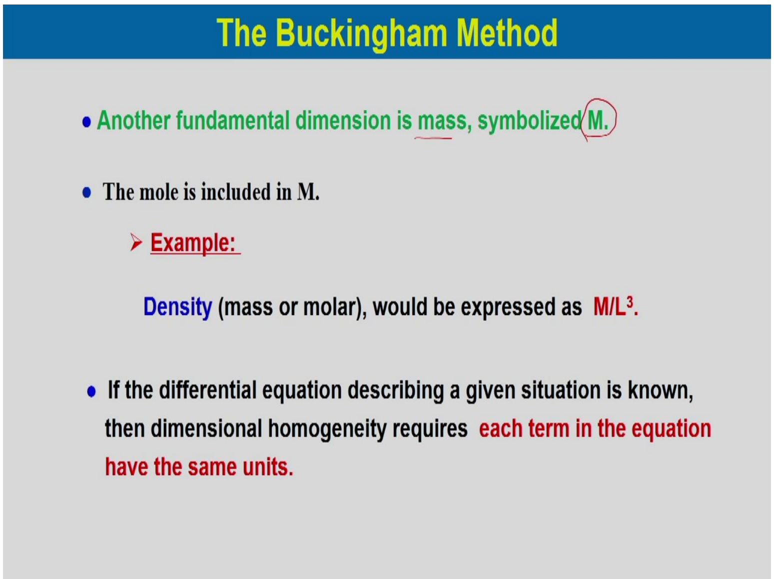

Now, another fundamental dimension is mass which is symbolised as M. So, M is symbolised as mass. So, also mole is also included in this, the mole also included in case of the dimension M. Like take an example density which is mass density or molar density and it is expressed in terms of M per length cube, that is mass or mole M per L cube. If the differential equation describing a given situation is known then dimensional homogeneity requires each term in the equations have the same units.

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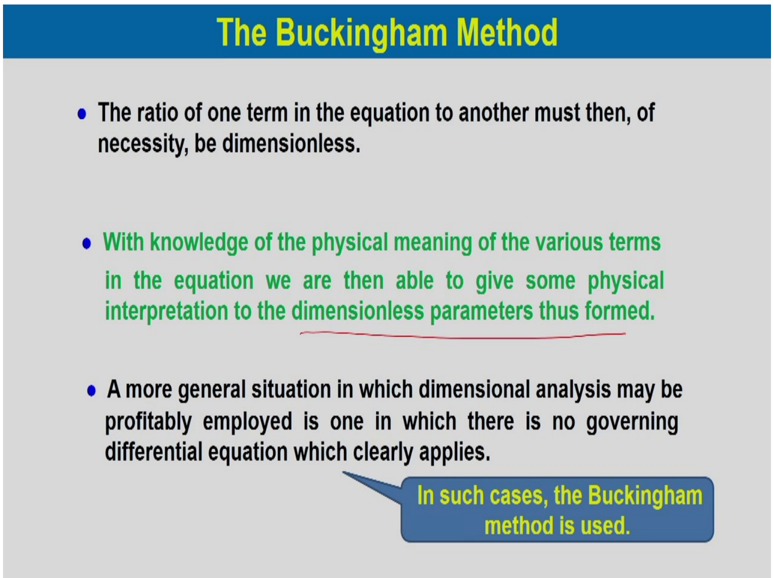

So, the ratio of one term in the equation to another mass then be of necessity be dimensionless. So, in this case with knowledge of physical meaning of the various terms in the equation, we are then able to give some physical interpretation to the dimensionless parameter thus formed. So, we can give a physical interpretation for the various terms in the equation to some physical systems.

A more general situation in which dimensional analysis may be profitable employed is one in which there is no governing differential equation which clearly applies. So, where when there is no governing equations available, differential equation available or that does not applicable to that particulate system; then this dimensional analysis is a much helpful in that situation. So, in such cases the Buckingham method is used for the dimensional analysis.

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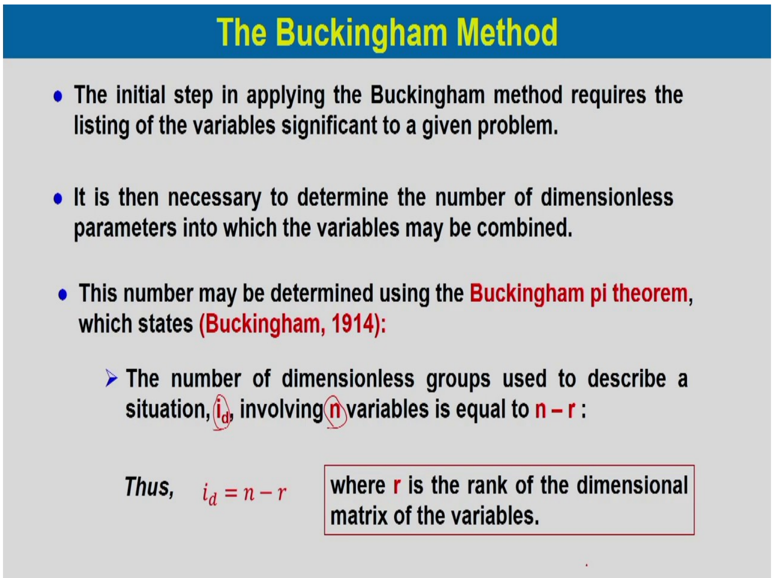

The initial step in applying that Buckingham method request, the listing of the variables significant to a given problem. So, once the problem is given and then we have to identify the significant variable for that particular problem. It is then necessary to determine the number of dimensionless parameters into which the variables maybe combined. This number may be determined using Buckingham pi theorem and this theorem states that the number of dimensionless groups used to describe a situation that is i d is the number of dimensionless group involving n variables should be equal to n minus r; that means, i d would be equal to n minus r where, r is the rank of the dimensional matrix of the variables.

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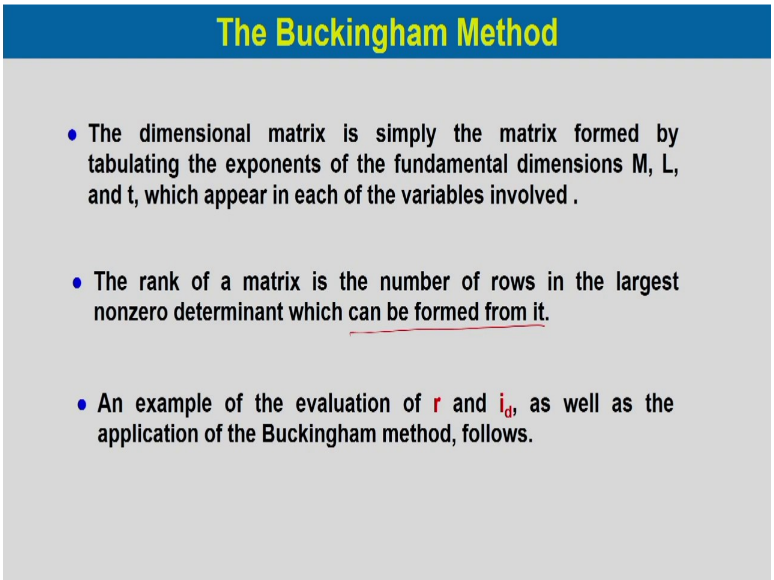

The dimensionless matrix is simply the matrix formed by tabulating the exponent of the fundamental dimension M, L and t which appear in each of the variable involved. So, if we make the dimensionless group and the fundamental dimensions raised to the power some values those form the dimensionless matrix. And, the rank of the matrix is the number of rows in the largest non-zero determinant which can be formed from it. So, let us know take an example of the evaluation of r and i d as well as the application of Buckingham method.

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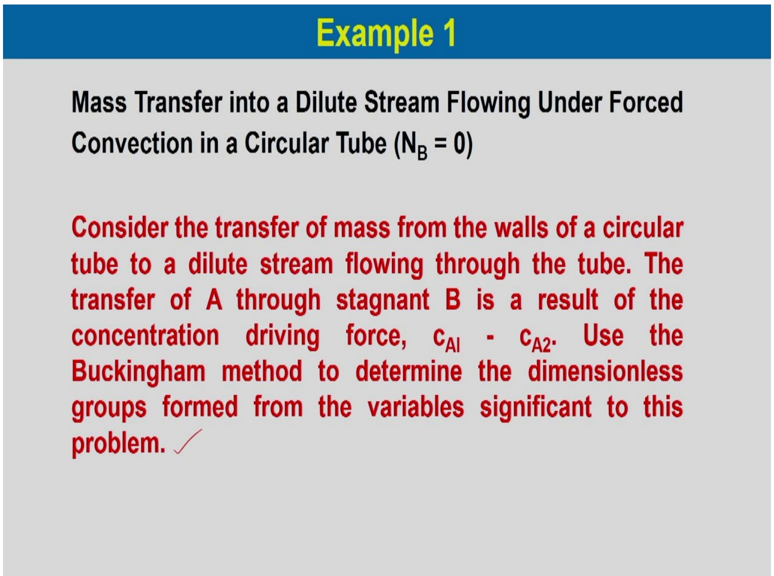

So, the example is mass transfer into a dilute stream flowing under forced convection in a circular tube. So, here N B is equal to 0 that is stagnant B. Now, consider the mass transfer from the walls of a circular tube to a dilute stream flowing through the tube. The transfer of A through stagnant B is a result of the concentration driving force, that is C A1 minus C A2. Use the Buckingham method to determine the dimensionless group formed from the variable significant to these problems. So, we have taken an example problem and we will see how the Buckingham method can be applicable to find out the dimensionless groups.

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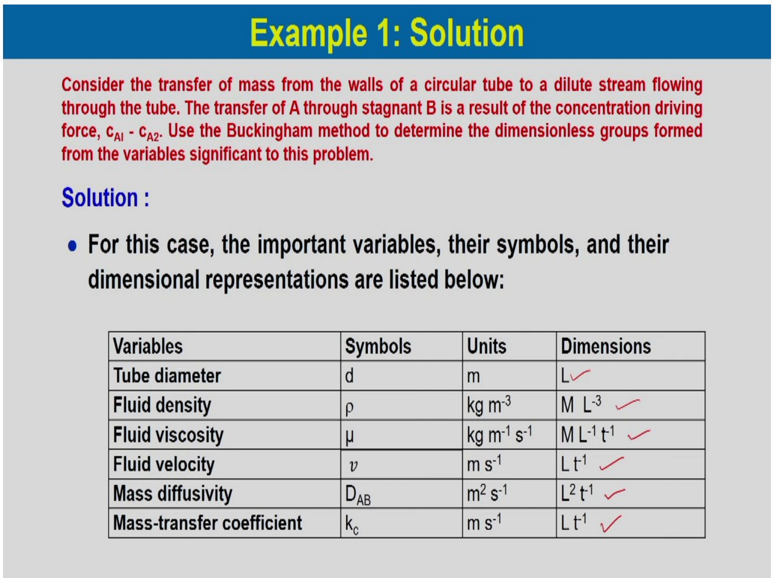

Variables	Symbols	Units	Dimensions
Tube diameter	d	m	L
Fluid density	ρ	kg m^{-3}	M L^{-3}
Fluid viscosity	μ	kg m^{-1} s^{-1}	M L^{-1} t^{-1}
Fluid velocity	v	m s^{-1}	L t^{-1}
Mass diffusivity	D_{AB}	m^2 s^{-1}	L^2 t^{-1}
Mass-transfer coefficient	k_c	m s^{-1}	L t^{-1}

For this case the important variables and their symbols, their dimensional representation are listed. So, that has to be listed first as you can see. The variables here the tube diameter d and its units is metre and then fundamental dimension is L. Similarly, fluid density rho which is kg per metre cube which is the fundamental dimension is M L to the power minus 3, fluid viscosity mu kg per metre second.

And then fundamental dimension is M L inverse and t inverse. Fluid viscosity fluid velocity v which is metre per second is length into time inverse, mass diffusivity D AB is metre square per second. So, it is length square per time, mass transfer coefficient k C is metre per second which is length per time. So, this know all these variables for a particular situations are described in terms of the dimensionless number.

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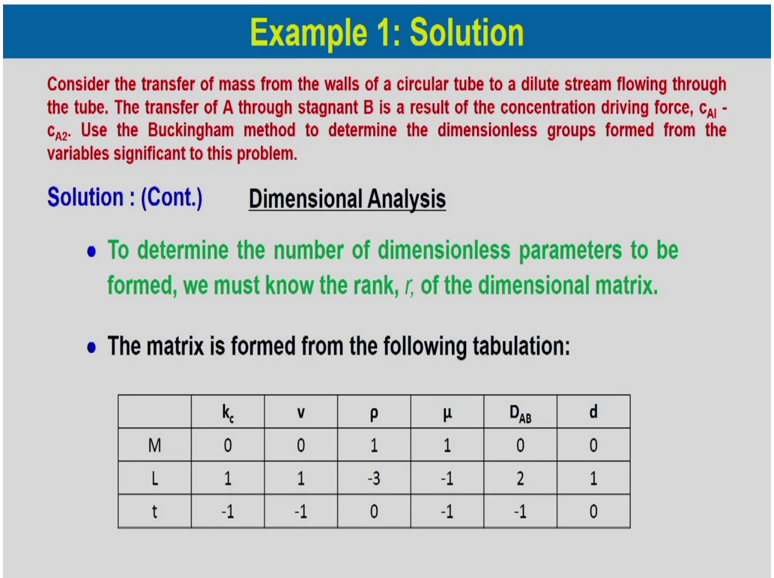

Now, to determine the number of dimensionless parameters to be formed we must know the rank r of the dimensional matrix. The matrix is formed in the following tabulation; you can see that if we write k c which is know length per time. So, it is L is 1, per time is minus 1. Similarly, the velocity is length per time is metre, length is a 1 and time is minus 1 and like density is kg per metre cube or the mass is 1 per metre cube is length to the power minus 3. So, similar way viscosity, diffusivity and diameter these are defined in terms of the fundamental dimensions M L and t.

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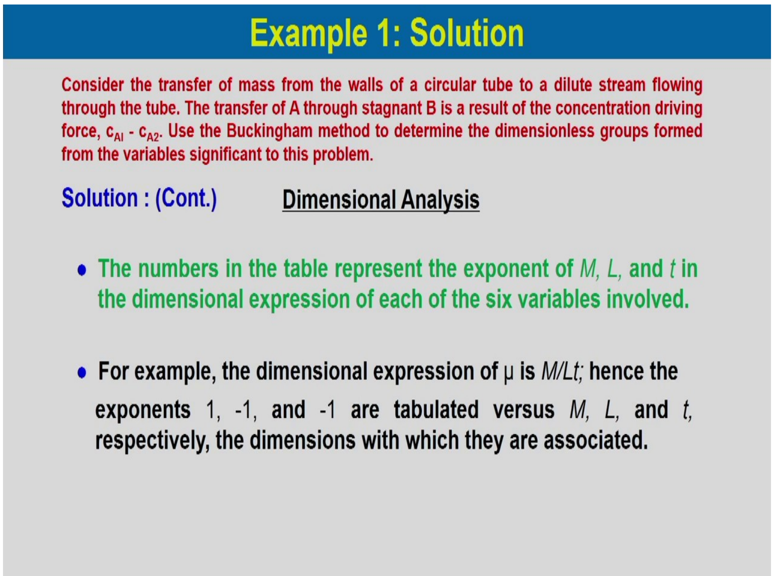

Now, the numbers in the table represents the exponent of M, L and t in the dimensional expression of each of the six variables involved. Now for example, that I am as we said the dimensional expression of mu is M per litre into time. Hence, the exponent 1, minus 1 and minus 1 are tabulated versus M, L and t respectively. The dimensions with which they are associated so, as we have already discussed.

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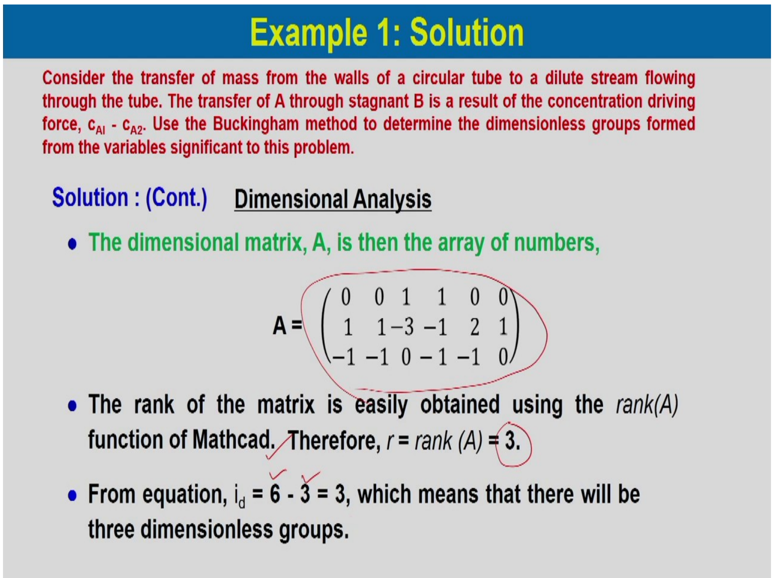

Now, if we want to make a dimensional matrix A is then the array of numbers which we can write A matrix would be equal to as shown in the tabulation. This is the A matrix, the rank of this matrix can easily be obtain using the rank a function of Mathcad or any other software you can use to calculate the rank of the matrix or mathematical; you can use or MATLAB you can use to calculate the rank of the matrix. So, in this case the rank a of A matrix is 3; that means, from the equation which we have said i d is equal to A n minus r. So, in this case a rank is r is 3 and so, i d would be equal to number of variables is 6 and rank is 3. So, i d would be 3 which means that there will be three dimensionless groups.

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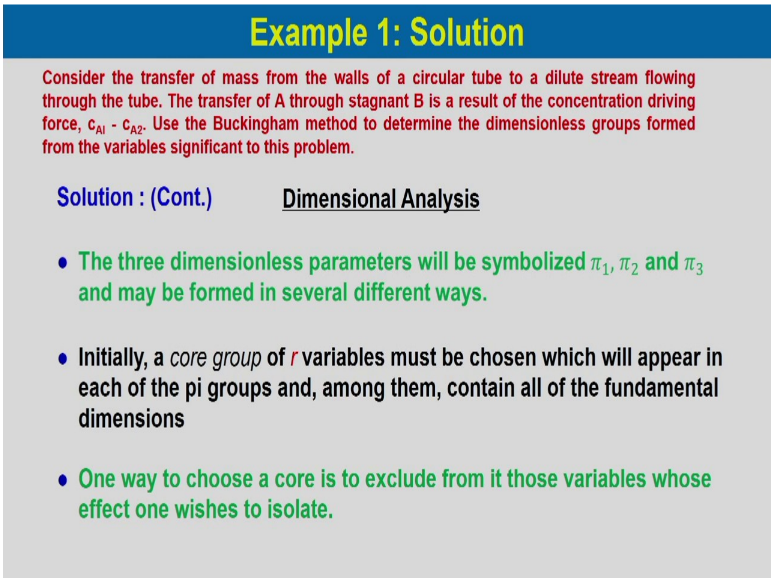

The three dimensionless parameters which will be symbolised as pi 1 pi 2 pi 3 and may be formed in several different ways. So, initially a core group of r variables must be chosen which will appear in each of the pi groups and among them contain all of the fundamental dimensions. One way to choose a core is to exclude from it those variables whose effect one wishes to isolate. So, if we want to isolate effect of some variables we will exclude them from the core groups.

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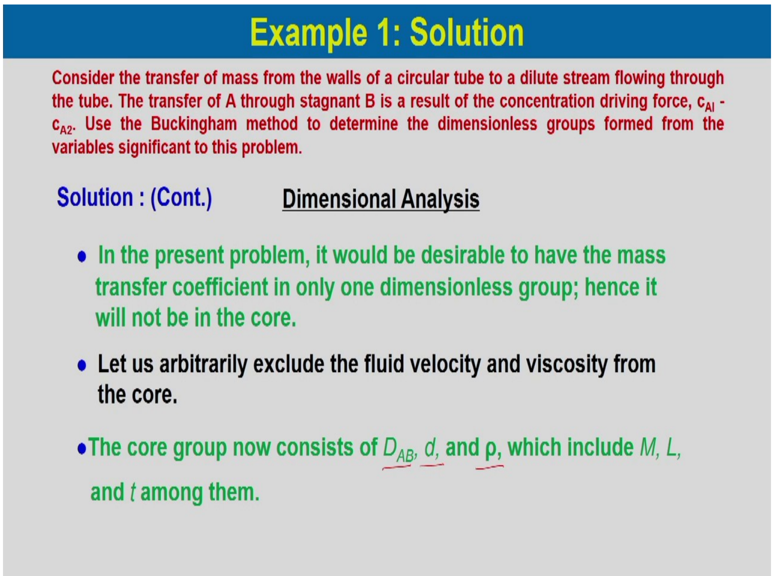

Like in this case it would be desirable to have mass transfer coefficient in only one dimensionless group. Hence, it will not be in the core one. Similarly, let us arbitrarily exclude fluid velocity and viscosity from the core group. The core group is now consists of D AB, d, rho and which include M, L and t among them; if we just look into the dimension of these 3 parameters.

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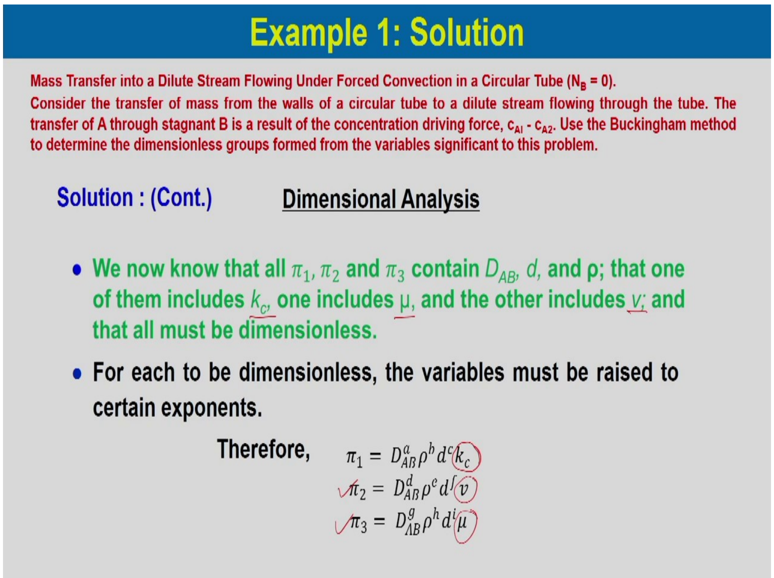

Now, we know that all pi 1 pi 2 and pi 3 contain D AB, d and rho that one of them include k c, one include mu and then other include v which we have excluded from the core group; k c, mu and v and then all must be dimensionless. So, let us write the for each dimensionless group the variables must be raised to certain exponents.

So, let us write the different terms pi 1 would be equal to D AB to the power a rho to the power b and d to the power c into k c. So, we have included k c in this know fundamental core group along with k c which we have excluded. Similarly, in pi 2 we have excluded v, in pi 3 we have included know mu. So pi 1, pi 2, pi 3 we can write in this form.

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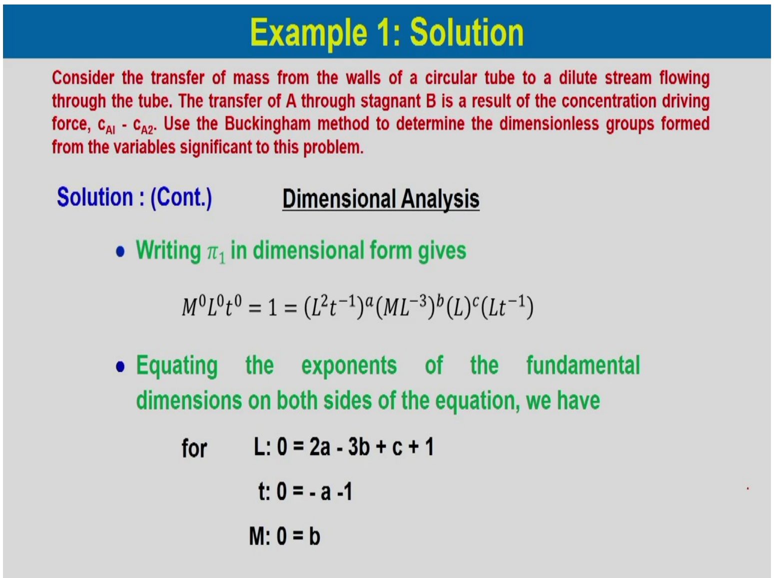

Now, writing pi 1 in dimensionless form gives M to the power 0 L to the power 0 and t to the power 0 would be equal to 1 which is equal to length square t to the power minus 1 whole to the power a into M into L to the power minus 3 whole to the power b L to the power c and L into t to the power minus 1. So, basically the pi 1 which is D AB, rho, d and k c are written with the fundamental dimension. Now, if we equate the exponent of the fundamental dimension on both sides of the equations we can just get the value of L and then t and M.

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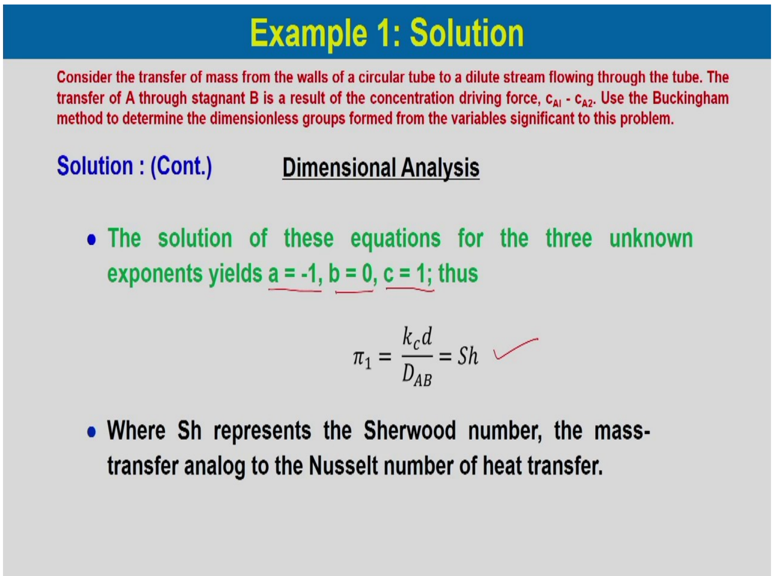

So, from this 3 relation we can obtain a is equal to minus 1. So, the solution of this equation for the 3 unknown exponent yield a is equal to minus 1, b is equal to 0 and c is equal to 1. Thus, we can write pi 1 would be equal to k c d by D AB which is Sherwood number. The Sherwood number represents the mass transfer analogue of the Nusselt number of heat transfer.

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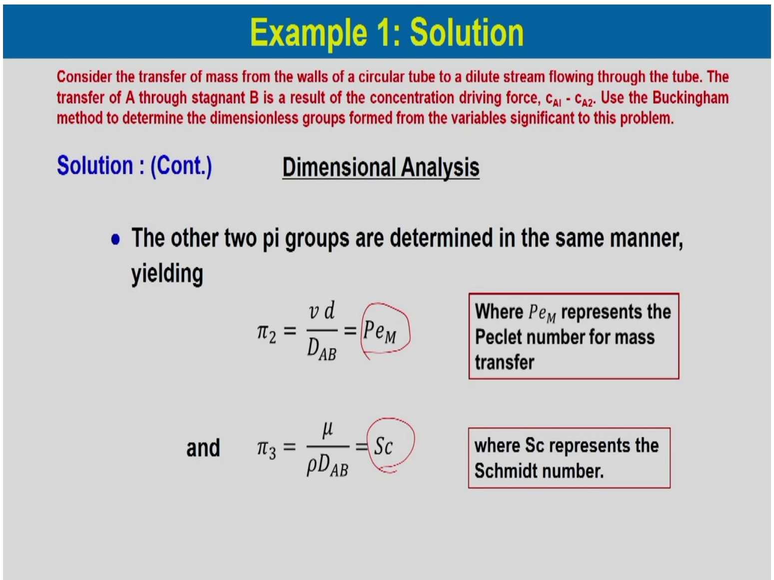

So, the other two pi groups are determined in the same manner and that will yield pi 2 would be is equal to v d by D AB which is Peclet number in case of mass transfer and pi 3 is mu by rho D AB which is Schmidt number. So, here S c represent the Schmidt number in case of the mass transfer. So, we can get similar with three dimensionless numbers.

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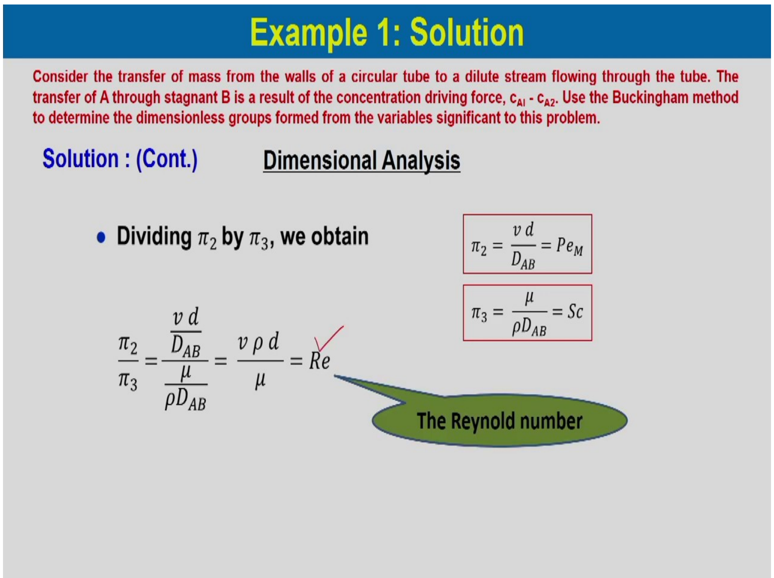

If we divide know pi 2 by pi 3, pi 2 is the Peclet number and pi 3 is the Schmidt number; if we divide them to we will get the Reynolds number ok.

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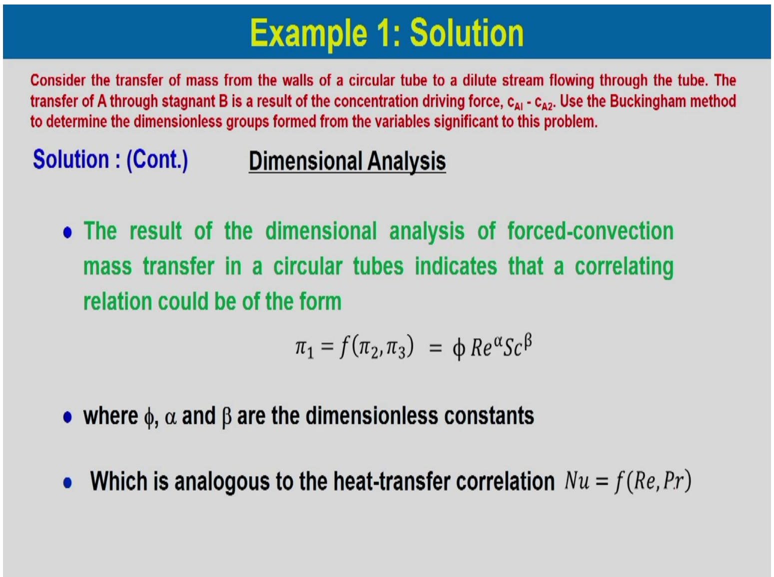

The result of the dimensional analysis of forced convection mass transfer in a circular tube indicates that a correlating relations could be of the form pi 1 is a function of pi 2 and pi 3. So, that is phi Reynolds number to the power alpha and Schmidt number to the power beta. So, in this case phi alpha beta are the dimensionless constants and which is analogous to the heat transfer correlation, Nusselt number is a function of Reynolds number and Prandtl number.

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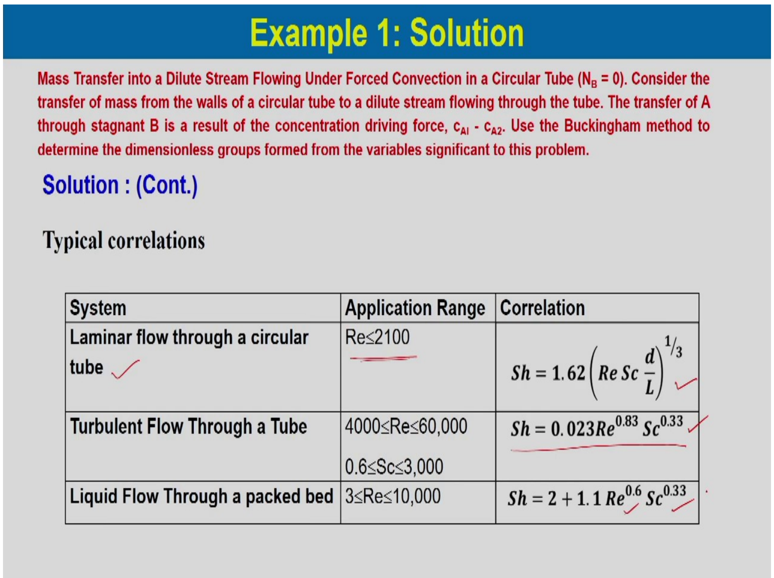

So, the typical correlations pi 1 is Sherwood number as we have derived earlier and then typical values for different systems are tabulated over here. Like if we considered laminar flow through a circular tube, the Reynolds number is less than equal to 2 and 2100. The Sherwood number is 1.62 into Reynolds number into Schmidt number d by L to the power one-third. Similarly, for turbulent flow through a tube where Reynolds number is in the range of know 4000 to 60,000.

And, the Schmidt in between 0.6 to 3,000; the Sherwood number can be correlated with Sherwood number is equal to 0.023 into Reynolds number to the power 0.83 and Schmidt number to the power 0.33. Like liquid flow through a packed bed where Reynolds number in between 3 to 10,000 we can correlate Sherwood number as 2 plus 1.1 Reynolds number to the power 0.6 and Schmidt number to the power 0.33.

Thank you, for hearing this lecture. And, in the next lecture we will continue to the discussion of the mass transfer coefficient in laminar flow condition.