

Chemical Engineering Thermodynamics
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Lecture – 08
Entropy and Lost Work

Hello and welcome back. We are going to continue our discussion on the second law entropy. And, today we are going to learn about a new concept known as Lost Work, but before we do that let us quickly recap what we have done so far.

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Recap

- The ONLY effect of an apparatus cannot be to completely convert heat into work
- No process can solely transfer heat from T_c to T_H
- Maximum efficiency for transfer of heat to work occurs in a reversible process (Carnot Engine)
- $\eta_{carnot} = 1 - \frac{|Q_c|}{|Q_H|} = 1 - \frac{T_c}{T_H}$ $\eta_{max} = \eta_{Carnot} = 1 - \frac{T_c}{T_H}$

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Cutting to the second law, the ONLY effect of an apparatus cannot be to completely convert heat into work. And, then no process can transfer heat from a cold body to a hot body alone that needs to be something else that needs to be done. So, there is no process that is possible ritually results solely in transfer of heat from T C to T H. The maximum efficiency for converting heat into work occurs when we have a Carnot engine; it is given by 1 minus Q C by Q H or T C by T H. Let us quickly solve one other problem for entropy change calculations, before we start introducing the concept known as lost work.

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Problem

- Consider a heat exchanger with counter current flow of the hot and cold fluids. The fluids on both sides have same flow rates and specific heat capacity ($C_p = 1500 \text{ J kg}^{-1} \text{ K}^{-1}$). Assume that there is no heat loss to the surroundings. The cold fluid enters at 300 K and leaves at 400 K.

- a) Calculate the entropy changes of two streams and hence for the total system, if the hot fluid enters at 600 K.
- b) Calculate the entropy changes of two streams and hence for the total system, if the hot fluid enters at 450 K.
- c) Discuss the difference between the two results.

(a)

$$\Delta S_{\text{cold}} = C_p \ln \frac{T_2}{T_1} = 1500 \times \ln \frac{400}{300} = 431.5 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\Delta S_{\text{hot}} = C_p \ln \frac{T_2}{T_1} = 1500 \times \ln \frac{500}{600} = -273.5 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\Delta S_{\text{system}} = 431.5 - 273.5 = 158 \text{ J kg}^{-1} \text{ K}^{-1}$$

We have a heat exchanger as you know in a heat exchanger there is going to be two sides to the heat exchanger, one side is going to be the hot fluid and on the other side we have the cold fluid. So, let us say that red one is the hot fluid and the blue one is the cold fluid right; it is operating in a counter current fashion. So, the flow rates and the heat capacities are same for both the sides, for the time being we are going to assume that the flow rates and the heat capacities of the two streams are same and those numbers are given to us. So, this is the blue one is the cold side and the hot one is the red. So, let us represent the hot fluid comes in goes out and the blue one comes in a counter current flow comes in and goes out.

The cold fluid is going to enter at 300 Kelvin and leaves at 400 Kelvin. So, $T_{C \text{ in}}$ is 300 $T_{C \text{ out}}$ is 400, the hot fluid enters at 600 Kelvin. In first case, in case a $T_{H \text{ in}}$ is 600 Kelvin alright. And we do not know what $T_{H \text{ out}}$ is, but what we do know is the flow rates and the specific heat capacities are same for both the streams. We want to calculate the entropy changes of the two streams and hands for the total system. This is the first case here what we are looking at right case a, this is how it looks entering at 600 the cold stream enters at 300 leaves at 400. Because, the mass flow rates and the specific heat capacities are same; if the temperature change for the cold fluid is 100 Kelvin then for the hot fluid also it needs to be 100 Kelvin which means $T_{H \text{ out}}$ is also equal to ΔT for $T_{H \text{ in}}$ is 100 Kelvin or $T_{H \text{ out}}$ is equal to 500 Kelvin.

That is the only way it can work because, the specific heat is same and the flow rates are same both the fluids alright. If that be the case, what we are interested in finding out is the entropy change for the two streams enhance for the total system. Delta S for the cold stream is going to be $C_p \ln \frac{T_2}{T_1}$ and what we have for T 2 is 400 T 1 is 300. So, it will be 1500 times ln of 400 over 300 or that would be 431.5 joule per kg per Kelvin. On the other hand, for the hot fluid it is going to be $C_p \ln \frac{T_2}{T_1}$ that is going to be 1500 times ln of 500 over 600. So, that will be negative 273.5 joule per kg per Kelvin. And, for the total system these are the specific quantities per kilogram of fluid.

And, for the total system either kilogram of cold or hot fluids that is coming in because, both quantities are same it does not matter which one we consider. Delta S for the system is going to be 431.5 minus 273.5, we simply add these 2 quantities and this turns out to be 158 joules per kilogram of fluid coming in per Kelvin; this is delta S for the system. So, this will be the total entropy change if we have a counter current fluid operating in this fashion. Now, we have the second case; the only difference is that the hot fluid enters now at 450 Kelvin, the cold fluid is still entering at 300 and leaving at 400. The hot fluid instead of entering at 600 now, enters at 450 Kelvin.

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(b) $\Delta S_{\text{cold}} = C_p \ln \frac{T_2}{T_1} = 1500 \times \ln \left(\frac{400}{300} \right) = 431.5 \text{ J kg}^{-1} \text{ K}^{-1}$

$\Delta S_{\text{hot}} = C_p \ln \frac{T_2}{T_1} = 1500 \times \ln \left(\frac{350}{450} \right) = -376.97 \frac{\text{J}}{\text{kg K}}$

$\Delta S_{\text{system}} = 431.5 - 376.97 = 54.53 \frac{\text{J}}{\text{kg K}}$

(c)

The diagram shows a rectangular heat exchanger. The cold fluid (blue) enters from the bottom at 300 K and exits from the top at 400 K. The hot fluid (red) enters from the top at 450 K and exits from the bottom at 350 K. The heat exchanger is labeled with a 90-degree angle.

So, what would be the entropy change in such a scenario? We repeat our exercise for the cold fluid nothing has changed. So, $C_p \ln \frac{T_2}{T_1}$. So, that is 1500 times ln of T 2 is 400 over 300 and what we get is 431.5 joule per kg per Kelvin right. Maybe we should

just draw the scenario here, that is for the hot fluid, this one is for the cold fluid; let us use the same directions as we have used earlier. So, the cold fluid enters at 300 Kelvin and leaves at 400 Kelvin. Whereas, the hot fluid now enters at 450 Kelvin and because the flow rates are same it should leave at 350 Kelvin.

So, ΔS then for the hot fluid is going to be $C_p \ln \frac{T_2}{T_1}$ or $1500 \ln$ of $\frac{350}{450}$. So, that number there is negative 376.97 joule per kg Kelvin and ΔS for the system then, per kilogram of fluid flowing is going to be 431.5 minus of 376.97 which will be 54.53 joule per kilogram Kelvin. So, the difference between these two scenarios, we have discussed is, in one case the cold fluid or in both cases the cold fluid enters and leaves at the same conditions. One case we are using a hot fluid at 600 Kelvin and in the other case we are using a hot fluid at 450 Kelvin. Turns out that the entropy change is higher when the temperature gradient is higher, if you look at this the temperature gradient here is 200, on this side is 200 this side also is 200 Kelvin right; the difference between the two sides. On the other hand the temperature gradient here is only 50 Kelvin in this scenario.

So, when the temperature gradient is lower the total amount of entropy change for this whole system the both the hot and the cold fluid, its put together is much lower that is because its driving forces smaller, the irreversibility is smaller and hence the total entropy change for the system is smaller in the second scenario. So, we have seen how to handle entropy or calculate entropy for an ideal gas, when we have a liquid and how to calculate entropy for the system or a combination of system and surroundings for the total system or universe right. What we are going to do next is talk about a quantity called as lost work. And, to be able to do that we first derive an equation for entropy balance in case of an open system.

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Entropy balance for open system

Energy balance $\frac{d(mU)_{cv}}{dt} + \Delta \left[\left(h + \frac{1}{2}u^2 + gz \right) \dot{m} \right] = \dot{Q} + \dot{W}_s$

Entropy balance

Entropy out - Entropy in + Entropy of surrounding } = Entropy generated
 + Δ Entropy change for control volume

$$\Delta(\dot{m}s)_{fs} + \frac{dS_{surr}}{dt} + \frac{d(m s)_{cv}}{dt} = \dot{S}_G \quad \dot{S}_G \geq 0$$

$$\Delta(\dot{m}s)_{fs} + \frac{d(m s)_{cv}}{dt} - \frac{\dot{Q}_{surr}}{T_r} = \dot{S}_G$$

So, let us try to write that this is similar to an energy balance for an open system except we will have an additional term. So, entropy balance for an open system, let us consider an open system meaning we have a flow stream. So, I have an open system have a stream coming in, I have a stream going out. There can be multiple streams more than one stream coming in, more than one stream going out. There is heat exchange and work done with the surroundings. So, I have Q, I have W S and again there can be more than one stream which exchanges heat and more than one stream which exchanges work with the surroundings.

So, flow rates are \dot{m} in and \dot{m} out, the heat exchange and work exchange are \dot{Q} and \dot{W} . Now, for this scenario if I take my control volume and write the energy balance equation first; we have already derived this. The first law is it reads something like this. The derivative of the total internal energy of the control volume plus the change in the enthalpy plus the kinetic energy plus the potential energy gravitational potential energy. This change should be equal to \dot{Q} plus \dot{W} or \dot{W}_s that is the shaft now, the flow work is already be included in the h term. Now, this is the energy balance or the first law for an open system.

Now, we can similarly write an entropy balance equation except that. In this case unlike the energy there can be generation of entropy. So, entropy out minus entropy coming in this is change in entropy due to the flow streams plus entropy change of the delta entropy

change for the surroundings plus delta entropy change for control volume. All of these put together would be equal to entropy generated right. Entropy of the flow streams coming or going out minus coming in that is the entropy change, due to the flow streams the entropy change of the surroundings plus the entropy change of control volume all of these put together would equally entropy be generated or in mathematical terms, I can write this as $\Delta m \dot{S}$ for the flow streams, this is out minus in for the flow streams plus dS total for the surroundings over dt . This is the rate of entropy change for the surroundings plus entropy change for the control volume which would be mass of the control volume times the entropy specific entropy for the control volume or the rate of change of that quantity will be equal to $S \dot{G}$ the rate of entropy generated and according to the second law this $S \dot{G}$ and rate of entropy generation should always be greater than or equal to 0; equals 0 for irreversible process greater than 0 otherwise right.

On top of that we can replace the quantity for the surroundings with $Q \dot{}$ the rate of heat exchange with the surroundings over $T \sigma$ which is the temperature of the surroundings right. So, then if $Q \dot{}$ is the heat exchange with the surroundings then the heat added to the surroundings is negative of that quantity. So, in that case I can rewrite this equation as follows: $\Delta m \dot{S}$ for the flow stream plus d of the change in entropy for the control volume over $d t$ minus the rate of heat exchange with the surroundings over $T \sigma$; is going to be equal to $S \dot{G}$.

If we have more than one heat exchange with the surroundings at different temperatures then we simply add additional terms. So, instead of the rate of entropy change of the surroundings we simply replace it with the heat exchange with the surroundings over the temperature of the surroundings. So, this equation then will give us what is known as an entropy balance. All we have is entropy of the flow streams, entropy of the control volume and the entropy generated in there. Now, for a reversible process $S \dot{G}$ or the rate of entropy generation is 0 like we said right.

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$$\Delta(\dot{m}S)_{fs} - \frac{\dot{Q}}{T_\sigma} + \frac{d(mS)_{cv}}{dt} = \dot{S}_G$$

For S.S. condition

$$\Delta(\dot{m}S)_{fs} - \frac{\dot{Q}}{T_\sigma} = \dot{S}_G$$

For S.S. and reversible

$$\Delta(\dot{m}S)_{fs} - \frac{\dot{Q}}{T_\sigma} = 0 \Rightarrow \dot{Q} = T_\sigma \Delta(\dot{m}S)_{fs}$$

Energy balance (reversible at S.S.)

$$\dot{W}_{rev} + \dot{Q}_{rev} = \Delta \left[\left(h + \frac{1}{2}u^2 + gz \right) \dot{m} \right]$$

$$\Rightarrow \dot{W}_{rev} = \Delta \left[\left(h + \frac{1}{2}u^2 + gz \right) \dot{m} \right] - T_\sigma \Delta(\dot{m}S)_{fs}$$

$$\dot{W}_{rev} = \Delta \dot{m} (h - T_\sigma S)_{fs}$$

So, let us write that last equation one more time over here delta of m dot times S for the flow streams minus Q dot over T sigma or simply considering one stream here plus d of m times S for the control volume over dt is going to be equal to S G dot right. For steady state conditions the change in entropy of the control volume will go to 0 and we simply have right. So, what we get is then delta m dot S for the flow streams minus Q dot over T sigma is going to be equal to S G dot.

And, for steady state and reversible when it is reversible there is no entropy generation term; all we have is delta m dot as for the flow streams minus Q dot by T sigma need to be equal to 0. And, again like I said if there is more than one heat exchange with the surroundings then we simply add all those terms as we get additional terms at the end. So, these are different scenarios for entropy balance right. Hopefully when we do a few more problems things will get clear, but one last thing before we start working on a couple of problems. In case of a reversible process, the energy balance equation remember the energy balance equation also right.

So, this means Q dot is going to be T sigma multiplied with delta m dot S for the flow streams right. This is Q for a reversible process. But for a reversible process the energy balance and at steady state would yield W for the reversible process is going to be W plus Q which is this quantity here, Q for the reversible process is going to be simply delta H plus half u square plus this quantity multiplied with m dot. This is the energy

balance equation for a reversible process, at steady state which means the work done in case of a reversible process in terms of entropy will be H plus half u square plus the potential energy term minus Q reversible which is in terms of entropy $T \sigma \Delta m \dot{S}$ for the flow streams.

So, what we have done here is expressed the work done for a reversible process in terms of entropy of the flow streams and the enthalpy term. This becomes even simpler if we neglect the kinetic and potential energy changes right, then this equation simply becomes W reversible is going to be $\Delta H - T \sigma S$ multiplied with $m \dot{}$, let us bring that $m \dot{}$ here and call it as this for the flow streams. That would be the W reversible in case of a steady state condition, in terms of entropy and enthalpy.

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$$\eta_{\text{(work required)}} = \frac{\dot{w}_{\text{ideal}}}{\dot{w}_{\text{real}}}$$

$$\eta_{\text{(work produced)}} = \frac{\dot{w}_{\text{real}}}{\dot{w}_{\text{ideal}}}$$

$$\dot{w}_{\text{lost}} = \dot{w}_{\text{real}} - \dot{w}_{\text{ideal}} \quad \dot{w}_{\text{lost}} \geq 0$$

$$\dot{w}_{\text{lost}} = T_G \dot{S}_G$$

$$\dot{S}_G \geq 0$$

Now, remember that we define the thermodynamic efficiency in terms of work required. If work is required for the process then the efficiency was W for the ideal process over W for the let us call it as real process. And, efficiency work extracted or work produced if work is produced then efficiency is defined as W for the real process over the W for the ideal process. These were the definitions for thermodynamic efficiency right. So, the lost work is defined as the difference between the two scenarios. Either case it does not matter required or produced, in either case the lost work is going to be the work for the real process minus the work for the ideal process.

And because, of the definitions of the efficiency in both cases W is going to be always greater than or equal to 0, it is going to be 0 for an ideal scenario or a reversible process, it is going to be greater than 0 for an irreversible process. This is the lost work and finally, turns out that this lost work will also equal the entropy generation term. It will be related to the entropy generated term were this equation the rate of entropy generation multiplied with the temperature of the surroundings would equal lost work. And, because lost work is greater than 0 entropy generated again is going to be greater than or equal to 0 depending on the type of the process; if it is reversible process it is going to be equal to 0. So, what do we have done here then is derived a general expression for entropy generation for flow processes right.

So, for a flow process this is the entropy generation or the entropy balance equation and for specific cases of steady state or steady state and reversible processes we can derive those expressions. And, we can also write reversible work done in terms of the entropy because we can replace the heat exchange for a reversible process with this term. So, use the first law and we can write relate the reversible work done to the entropy of the streams. And finally, we can relate the lost work which is the difference between the real work and the ideal work is what we call as the lost work that will be related to the entropy generated term. Now, this lost work essentially signifies the amount of work lost because of the irreversibilities whether its work required or work produced it does not matter.

If it is work required we need to do additional work more than the ideal or reversible scenario because there are irreversibilities we need to do more work. So, the difference is the lost work. If it is work produced where we end up extracting less amount of work than what it should be in a reversible process. So, that difference again is equal to the lost work. So, this is lost work due to the irreversibilities and the entropy generated is going to be a measure of the lost work. It will be related to the lost work via the temperature of the surroundings through this equation. So, in that sense entropy generation is an important concept, it will give us a measure of the lost work or the irreversibilities in the process. So, let us look at these concepts more closely by solving a few problems.

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Problem

- An ideal gas at 25 bar is throttled to 1 bar at a rate of 100 mol s⁻¹. If the temperature of the surroundings is 300 K, calculate the rate of entropy generation and lost work.


$P_1 = 25 \text{ bar}$ $P_2 = 1 \text{ bar}$ $\dot{m} = 100 \text{ mol/s}$
 $T_\sigma = 300 \text{ K}$

$\frac{d(m\dot{u})_{cf}}{dt} + \Delta \left[\left(u + \frac{1}{2}u^2 + gz \right) \dot{m} \right] = \dot{Q} + \dot{W}_S \Rightarrow \Delta H = 0 \Rightarrow \Delta T = 0$

$\Delta S_{fs} = -R \ln \frac{P_2}{P_1} = -8.314 \ln \left(\frac{1}{25} \right) = 26.76 \text{ J mol}^{-1} \text{ K}^{-1}$

$\Delta(\dot{m}S)_{fs} + \frac{d(m\dot{S})_{cf}}{dt} - \frac{\dot{Q}}{T_\sigma} = \dot{S}_G$

$\dot{S}_G = \Delta(\dot{m}S)_{fs} = \dot{m} \Delta S_{fs} = 100 \frac{\text{mol}}{\text{s}} \times 26.76 \frac{\text{J}}{\text{mol K}} = 2.676 \frac{\text{kJ}}{\text{s K}}$



The first one, we have an ideal gas at 25 bar is throttle to 1 bar at a rate of 100 moles per second. If the temperature of the surroundings is 300 Kelvin calculate the rate of entropy generation and lost work. So, this is a pretty straightforward scenario, we have ideal gas at high pressure. So, let us say it is flowing at a particular and it is throttle at a particular flow rate. So, let us say I have a pipeline like that where, I have flow of an ideal gas and to be able to throttle all we need is a small pinhole. So, we are going to throttle it at the end there right. So, the ideal gases throttle.

So, the pressure inside P 1 is 25 bar and the final pressure P 2 is 1 bar, the flow rate m dot is 100 moles per second. The surroundings are at 300 Kelvin so, T sigma is 300 Kelvin. We want to calculate the rate of entropy generation and lost work. To be able to do that we first need to calculate or write an entropy balance equation. But, before we do that and to write the entropy balance equation, I need to calculate the initial and final conditions; all I know is the pressures I do not know about the temperatures. So, the first thing to do is see what the change in temperature is going to be. To be able to do that we are going to write the first law equation. The first law for a flow process will be plus delta of H plus half u square plus the potential energy times m dot will equal Q dot plus W S dot.

We are not doing any work here in fact, in that sense everything is lost work; we are not doing work at all in this particular process it is just throttled. So, the shaft work is 0,

there is no heat exchange with the surroundings right ah. So, the steady state process there is a continuous stream of ideal gas coming in and being throttled. Let us also ignore the potential and kinetic energy changes, then all we have left is $\Delta H = 0$. And, this is an ideal gas $\Delta H = 0$ means only one thing for an ideal gas, it means there is no temperature change right. And, there is no temperature change we can readily calculate. So, I know from the energy balance equation, but there is no temperature change and it is an ideal gas.

We have done a similar problem yesterday, when the gases are ideal or and there is no temperature change the entropy difference or entropy change is going to be simply $\ln \frac{P_2}{P_1}$ minus $\ln \frac{P_2}{P_1}$. So, that is going to be negative $8.314 \ln \frac{1}{25}$, P_2 is 1 bar P_1 is 25 bar and this turns out to be 26.76 joule per mole per Kelvin. This is the total change in the entropy. Now, I can also write an entropy balance equation, as we have done earlier. The entropy balance equation for this particular scenario we can derive that, it will be this ΔS of \dot{m} times S for the flow streams plus $\dot{m} S$ for the control volume over dt minus $\frac{Q}{T} \sigma$ or \dot{Q} by $T \sigma$ is going to be equal to the entropy rate of entropy generation.

And, in this particular equation there is no change in entropy of the control volume because, it is a steady state process. There is no heat exchange with the surrounding. So, those two terms are going to disappear. What I will be left with is the rate of entropy generation \dot{S}_G is equal to ΔS of \dot{m} for the flow streams because, it is a steady state process \dot{m} is constant which means $\dot{m} \Delta S$ for the flow stream. Or, the ΔS is in terms of specific quantity because I have the molar values, I can write it as \dot{m} times ΔS if they are the molar values. So, that turns out to be 100 moles per second multiplied with 26.76 joule per mole Kelvin. Remember that I do not have to convert \dot{m} into \dot{m} , I can simply use both of them to be molar quantities instead of the specific quantity and \dot{m} . This value turns out to be 2676 2.676 kilo joules per second Kelvin or kilowatts per Kelvin.

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$$\begin{aligned}\dot{w}_{ideal} &= \dot{w}_{rev} = (\Delta H - T_\sigma \Delta S) \dot{m} = (\cancel{\Delta H} - T_\sigma \Delta S) \dot{m} \\ &= -\dot{m} T_\sigma \Delta S_{molar} = -100 \times 300 \times 26.76 = -802.8 \frac{kJ}{s} \\ \dot{w}_{real} &= 0 \\ \dot{w}_{lost} &= \dot{w}_{real} - \dot{w}_{ideal} = 802.8 \frac{kJ}{s} = T_\sigma \dot{S}_G\end{aligned}$$

Now, this is the rate of entropy generation, the ideal work done as we have seen earlier is for the reversible process and for the reversible process this is ΔH dot minus T times ΔS or rather ΔH minus $T \Delta S$ times \dot{m} . This is the reversible work done, again we can replace everything with molar quantities and multiply with \dot{n} instead of \dot{m} . So, this will be ΔH minus $T \Delta S$ times \dot{n} , let us molar values right.

So, for this particular process ΔH we have already identified that it is 0. So, all I have left is minus $\dot{n} T \Delta S$ molar value. So, that is minus a 100 times $T \Delta S$ or the temperature of the surroundings is 300 Kelvin. And, the molar value is 26.76 joule per mole per Kelvin. If I multiply this what I get is a negative 802.8 kilo joules per second right. So, this is the work that I should be producing if the process were reversible, but in real scenario the work I am producing is 0 right. There is no work produced due to the throttling process. So, the lost work is then W for the real process minus W for the ideal process which is I am losing all of that work 802 kilo joules per second; I lost this much amount of work.

Notice that this quantity will also be equal to $T \Delta S$ times the rate of entropy generation. So, what we have calculated in this problem then is the entropy generated via the entropy balance equation and the lost work could have been calculated using the entropy generated or, we can calculate the reversible work and then subtract it from the real

world that has been done in the process. Either way the lost work is 800 kilojoules per second and because, we are throttling we are not extracting any work all of it is lost.

So, let us extend this idea and try to solve another problem, I have a refrigeration unit and it is used to cool water. Flow rate of water is 100 kilograms per second and we are cooling water from 45 degrees centigrade to 5 degrees centigrade. The refrigeration unit itself rejects heat to the atmosphere at 30 degrees centigrade; it operates with some efficiency 30 percent efficiency. We want to calculate the ideal work, the lost work and the entropy generated, the specific heat capacity for water is constant over this particular range the value is given. So, let us list out what is given to us so, far in the problem.

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Problem

- A refrigeration unit is used to cool water (flow rate 100 kg/s) from 45 °C to 5 °C. The unit rejects heat to atmosphere at 30 °C. If it operates with an efficiency of 30 % calculate the ideal work, lost work and entropy generation. Assume specific heat capacity for water over this range is constant at 4.186 kJ kg⁻¹ K⁻¹

$$T_0 = T_{\text{sur}} = 303.15 \text{ K} \quad T_1 = 318.15 \text{ K} \quad T_2 = 278.15 \text{ K} \quad C_p = 4.186 \frac{\text{J}}{\text{kg K}}$$

$$\eta = 0.3 \quad \dot{m} = 100 \frac{\text{kg}}{\text{s}}$$

$$\Delta H = \int_{T_1}^{T_2} C_p dT = C_p \Delta T = 4.186 (278.15 - 318.15) = -167.4 \text{ kJ kg}^{-1}$$

$$\Delta S_{\text{water}} = \int_{T_1}^{T_2} \frac{C_p}{T} dT = C_p \ln \frac{T_2}{T_1} = 4.186 \times \ln \left(\frac{278.15}{318.15} \right) = -0.5624 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

$$\dot{w}_{\text{ideal}} = (\Delta H - T_0 \Delta S) \dot{m} = 100 [-167.4 - 303.15 (-0.5624)] = 305.2 \frac{\text{kJ}}{\text{s}}$$

$$\dot{w}_{\text{real}} = \frac{\dot{w}_{\text{ideal}}}{\eta} = \frac{305.2}{0.3} = 1017.2 \text{ kW}$$

T sigma or the temperature of the surroundings to which heat is rejected is 30 degrees centigrade or 303.15 Kelvin. The initial temperature T 1 is 45 degrees centigrade or 318.15 Kelvin or cooling water from this T 1 to a final temperature of T 2 5 degrees centigrade or 278.15 Kelvin. The specific heat capacity for water is given 4.186 joule per kilogram Kelvin. We are also given a couple of other things. The first one is efficiency which happens to be 30 percent or efficiency eta is 0.3 and mass flow rate for the stream that is being cool in this case water is 100 kilograms per second. These are the quantities that are given to us. We want to calculate ideal work, the lost work and entropy generator or the rates of these quantities probably in this scenario.

So, let us first go ahead and calculate ΔH and ΔS so, that we can calculate the ideal work right. ΔH for this case is integral of $C_p dt$ going from T_1 to T_2 , we are considering the water stream that is being cooled and for this stream ΔH is integral of $C_p dT$ from T_1 to T_2 because, C_p is constant this value turns out to be $C_p \Delta T$ or 4.186 times T_2 minus T_1 which is 278.15 minus 318.15 Kelvin. Or, that quantity there turns out to be negative 167.4 kilo joule per kg; that is ΔH for the water stream. Similarly, the entropy change for water that is being cooled is integral of $\frac{C_p}{T} dT$ from T_1 to T_2 again C_p by T and because C_p is constant this will be $C_p \ln \frac{T_2}{T_1}$ or this is 4.186 times \ln of $\frac{T_2}{T_1}$ which is 278.15 or, T_1 which is 318.15.

So, this quantity there is going to be negative 0.5624 kilo joules per kg per Kelvin because, it is entropy right. So, once I have these 2 quantities I can calculate the ideal scenario. It is simply ΔH minus $T \Delta S$ multiplied with \dot{m} , this is what we derived right. This is an energy balance equation ΔH , steady state ΔH minus \dot{Q} will be dealt W_{ideal} , \dot{Q} can be written in terms of ΔS in this form. So, it will be ΔH minus $T \Delta S$ times \dot{m} , \dot{m} is 100 kilograms per second, ΔH is negative 167.4 minus $T \Delta S$ is 303.15 times negative 0.5624 is of the S . And, if I simply this number it turns out to be 305.2 kilo joules per second kilo joules per second by multiplying kilo joules per kg with kg per second.

So, what I get is kilo joules per second or kilowatts. So, the ideal scenario ideal or reversible work done in this process is 305.2 kilowatts, that is the rate of reversible work done. But, my process itself is irreversible, my process itself is irreversible and I have to do work to be able to cool the water from 45 to 5 right. So, sorry the irreversible work done then is going to be ideal work over the efficiency. In this case it will be 305.2 over 0.3 which will be 1017.2 kilowatts that is the real work done. So, I have calculated 2 quantities: one from the entropy change, the other one using the efficiency given to me. Once, I have these 2 quantities I can actually go ahead and calculate the lost work in the process.

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$$\dot{w}_{\text{lost}} = \dot{w}_{\text{real}} - \dot{w}_{\text{ideal}} = 1017.2 - 305.2 = 712 \text{ kW}$$
$$\dot{S}_G = \frac{\dot{w}_{\text{lost}}}{T_G} = \frac{712 \text{ kJ/s}}{303.15 \text{ K}} \approx 2.35 \text{ kJ K}^{-1} \text{ S}^{-1}$$

The rate of lost work is simply the real work minus the work required in case of an ideal process. This is the work that is being used in an ideal process and this difference is 1017.2 minus 305.2 which turns out to be 712 kilowatts. So, this is the amount of work I am losing due to the inefficiency in the process or irreversibility in the process. So, I have calculated all the 3 quantities I need and we can extend this and calculate the entropy generation rate also; this is the lost work over the temperature of the surroundings. So, this will be 712 kilo joules per second over the temperature of the surroundings which is at 303.15 Kelvin. So, this will be 2.35 kilo joule per Kelvin second. This is the rate of entropy that has been generated right.

So, in this case we have calculated the entropy generation what we are the lost work. And, we calculate the lost work using the efficiency given to the for the process. So, these are various scenarios which will explain the concepts we have learned today. So, that completes our discussion on the second law, the entropy and lost work. I hope you enjoyed the course so far, when we come back we are going to see how we are going to translate these entropy into engineering calculations. We are going to use this entropy for a variety of engineering calculations in the coming lectures.

Thank you. See you in the next lecture.