Chemical Engineering Thermodynamics
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Lecture – 04
The First Law of Thermodynamics for Open Systems

Hello and welcome back. In the last few lectures in Chemical Engineering Thermodynamics, we have talked about the first law, the development of the first law for closed system, followed by the development of the first law for open system. Then we define enthalpy, and the specific heat capacities at constant pressure and constant volume. We solved a few problems based on these concepts.

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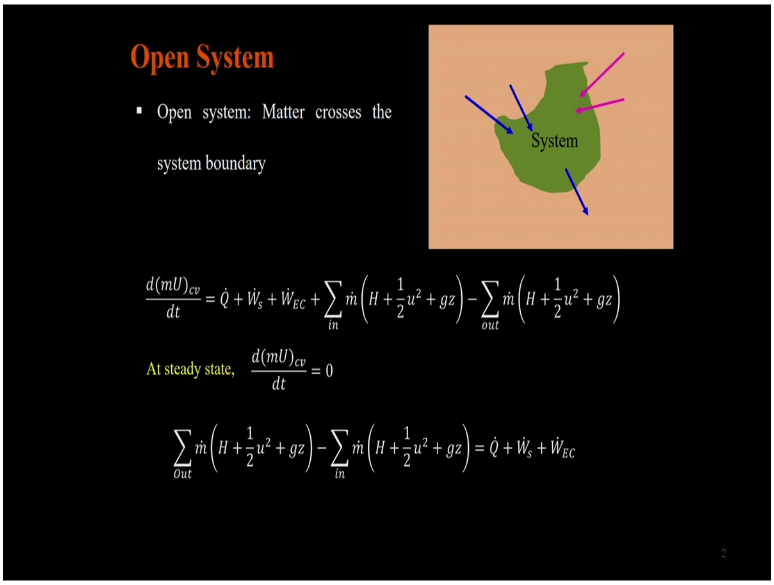

Open System

- Open system: Matter crosses the system boundary

$$\frac{d(mU)_{cv}}{dt} = \dot{Q} + \dot{W}_s + \dot{W}_{EC} + \sum_{in} \dot{m}\left(H + \frac{1}{2}u^2 + gz\right) - \sum_{out} \dot{m}\left(H + \frac{1}{2}u^2 + gz\right)$$

At steady state, $\frac{d(mU)_{cv}}{dt} = 0$

$$\sum_{Out} \dot{m}\left(H + \frac{1}{2}u^2 + gz\right) - \sum_{in} \dot{m}\left(H + \frac{1}{2}u^2 + gz\right) = \dot{Q} + \dot{W}_s + \dot{W}_{EC}$$

Let us quickly summarize the first law for an open system that we looked at in the last class. Turns out open system is one where the matter can cross the system boundary. And the blue ones, we see here are the streams carrying the mass in and out of the system. The two pink lines are for transfer of energy Q dot, and W dot work and heat transferred into the system.

Now, we derived the expression for the first law, and it reads that the rate of accumulation in the system m dot times U in the control volume over d t is the rate of heat added to the system, the work added to the system both the shaft work, and the expansion contraction work when we have a system that whose control volume changes.

And then the enthalpy of the streams going in and out of the system; the kinetic energy of the streams going in and out of the system; and finally, the potential energy of the streams going in and out of the system.

Remember that we get the enthalpy term H instead of the internal energy term for a closed system, because we have an additional work which we call as flow work. And once you add the internal energy and the flow work together this H term appears instead of the u term. So, this is the final expression, we had for an open system energy balance or the first law for an open system. Then turns out that at steady state the rate of change, I am sorry this is d of m U by dt since we already have time in the denominator.

The rate of change with respect to time will be 0 at steady state right. And once that happens, it turns out that the change in the enthalpy plus kinetic and potential energies should equal the amount of the work done and heat added to the system. Now, with this in mind what we will do today is try to solve a few problems based on application of the first law for both open and closed systems, very simple problems, but then we will try to illustrate the concepts based on these energy balance equations.

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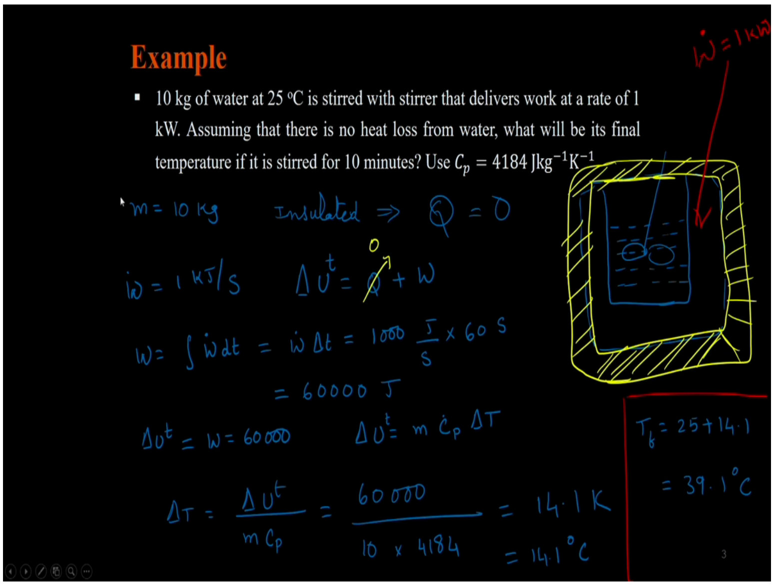

Example

- 10 kg of water at 25 °C is stirred with stirrer that delivers work at a rate of 1 kW. Assuming that there is no heat loss from water, what will be its final temperature if it is stirred for 10 minutes? Use $C_p = 4184 \text{ Jkg}^{-1}\text{K}^{-1}$

$m = 10 \text{ kg}$ Insulated $\Rightarrow Q = 0$

$\dot{W} = 1 \text{ kJ/s}$ $\Delta U^t = \overset{0}{\cancel{Q}} + W$

$W = \int \dot{W}\,dt = \dot{W}\,\Delta t = 1000 \frac{J}{s} \times 60 \text{ s}$

$= 60000 \text{ J}$

$\Delta U^t = W = 60000$ $\Delta U^t = m\,C_p\,\Delta T$

$\Delta T = \frac{\Delta U^t}{m\,C_p} = \frac{60000}{10 \times 4184} = 14.1 \text{ K}$

$= 14.1 \text{ °C}$

$T_f = 25 + 14.1$

$= 39.1 \text{ °C}$

Let us look at problem number-1 for today. We have some amount of water at 25 degrees centigrade, which is stirred, and the stirrer delivers work at a rate of 1 kilowatt. Assuming that there is no heat loss from the water, what will be the final temperature if it is stirred for 10 minutes? And the C P for water is given to us.

So, let us see, if we can solve this problem. Let us draw a simple system here, we have water mass of water in this vessel is 10 kilogram. This vessel is well insulated from the surroundings right, let us put an insulation there. Let us pick a different color. So, we have insulated the system from the surroundings. So, once the system is insulated, the mass of the system also is constant. So, we have insulated the system from the surroundings, and then the mass of the system is constant; m is given to us, it is 10 kilograms and because the system is insulated, this implies the heat exchange with the surroundings Q is 0.

And we are doing with stirring, which essentially boils down to let us pick a different color there. We were stirring which means we are adding work to the system, and the rate of work that is being added is W dot, it is 1 kilowatt right. If we look at this problem, we have a closed system. So, we can get away writing a closed system energy balance equation, which by now we know that delta U is Q plus W delta U total is Q plus W. But, what we have and then of course Q for this particular problem is 0. So, what we have is delta U t is W. What we are given is W dot the rate of work that is being done on the system, we have to change it to W.

So, W is going to be it is a constant, it is stirred with a constant power. So, W is going to be integral of W dot times d t. And then since W dot is constant, it is essentially W dot times delta t, the time for which it is stirred. So, it is going to be 1000 kilo joules per second into 10 minutes, which is 60 seconds. So, the work done W within this time of 10 minutes is going to be 6 followed by let us say four 0's about 60,000 joules I am sorry this is 1000 joules not kilojoules 1 kilojoules, so that is 1000 joules per second times 60, so that 60000 joules is the work that is done on the system.

Once, I have this much amount of work done, the next thing we need to do is calculate the total internal energy change, and because it is a closed system and Q is 0 delta U t would be W, which is 60,000. Now, however what we are interested in is the final temperature not delta U t, to get the final temperature for liquid water whether you call it as C P or C v the, because it is liquid C P and C v are almost same.

So, delta U is going to be C P or rather delta U t is going to be m times C P delta T for liquid right. If C P is constant and it will be m times C P times delta T, and what I am interested in is delta T. So, delta T is going to be delta U t over m C P, so that will be

60000 over 10 kilograms times 4184 joules per kg per Kelvin. And that number turns out to be 14.1 Kelvin, this is delta T.

So, the final temperature, then is going to be the initial temperature plus delta T, so final temperature is going to be the initial temperature, which is 25 degrees centigrade plus 14.1 degrees Kelvin or 14.1 degree centigrade, so that is 39.1. So, if we stir this liquid water, where the 1 kilowatt power for about 10 for 10 minutes, then the final temperature is going to be 39.1 degree centigrade. Assuming that there is that that there is no heat loss to the surroundings, so that is application of the first law for a very simple scenario.

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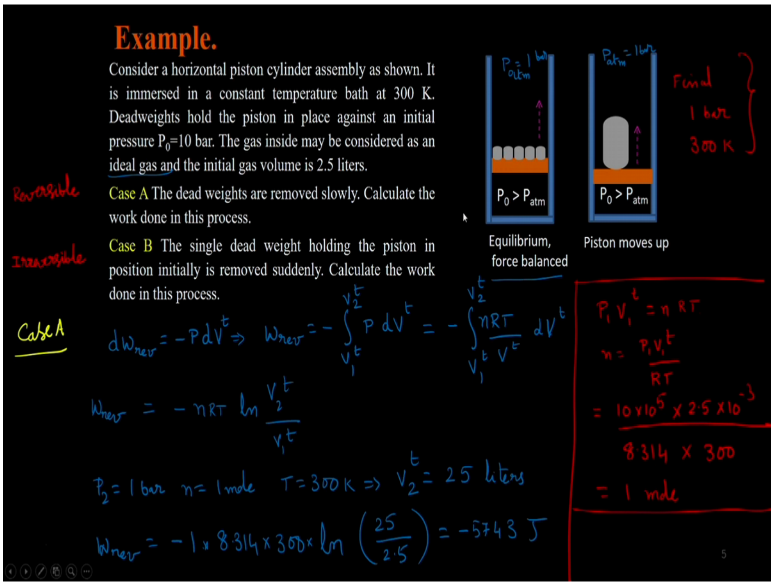

Example.

Consider a horizontal piston cylinder assembly as shown. It is immersed in a constant temperature bath at 300 K. Deadweights hold the piston in place against an initial pressure P_0=10 bar. The gas inside may be considered as an ideal gas and the initial gas volume is 2.5 liters.

Reversible

Case A The dead weights are removed slowly. Calculate the work done in this process.

Irreversible

Case B The single dead weight holding the piston in position initially is removed suddenly. Calculate the work done in this process.

Case A

$$dW_{rev} = -P\,dV^t \Rightarrow W_{rev} = -\int_{V_1^t}^{V_2^t} P\,dV^t = -\int_{V_1^t}^{V_2^t} \frac{nRT}{V^t}\,dV^t$$

$$W_{rev} = -nRT \ln \frac{V_2^t}{V_1^t}$$

$P_2 = 1$ bar $n = 1$ mole $T = 300\text{ K} \Rightarrow V_2^t = 25$ liters

$$W_{rev} = -1 \times 8.314 \times 300 \times \ln\left(\frac{25}{2.5}\right) = -5743\text{ J}$$

$$P_1 V_1^t = nRT$$

$$n = \frac{P_1 V_1^t}{RT} = \frac{10 \times 10^5 \times 2.5 \times 10^{-3}}{8.314 \times 300} = 1 \text{ mole}$$

Let us take another example. This time we have a horizontal piston cylinder assembly as shown in the figure. And this whole assembly is immersed in a constant temperature bath at 300 Kelvin. Now, we place dead weights on the piston which hold it against a pressure of the gas inside, and the pressure of the gas initial pressure of the gas inside is 10 bar, and these weights will keep the piston in position right. Outside is atmospheric pressure, which is 1 bar let us say, the atmospheric pressure is 1 bar ok.

Now, the gas inside is ideal gas, and the initial volume is 2.5 liters. What we need to do is calculate the work done in the process, there are two cases for which we need to calculate the work done. The first case if we place very small dead weights, and they are removed slowly, calculate the work done in the process.

When we say we are there the dead weights are removed slowly, and then it is also given for case A. At each removal of the weight everything is in equilibrium and the forces are balanced. So, what we are saying is that we were trying to follow a reversible path in the first scenario. These are small dead weights and when we remove them slowly, and at each step if the system is at equilibrium due to balance of forces, then what we are saying is that the process itself is reversible. So, the first process is a reversible process.

In the second case, we have a large dead weight. It is a single dead weight holding the piston position initially, and we remove it suddenly, then of course the as we have looked at the example earlier, it is going to be an irreversible process. In both cases, we are expanding the gas.

In both cases we will end up at the same final position, because we are expanding it against atmosphere which is at 1 bar, and we it is an isothermal process. So, the final state is going to be 1 bar and 300 Kelvin right, the final conditions are same in both cases. But, in one case we were doing the process reversibly, and in the other case it is an irreversible process. And in both cases we want to calculate the amount of work done.

As we have seen earlier work is a path function. So, we are following different paths one is a reversible path, the other one is an irreversible path. So, we will have different work that is being done on the system in both the cases, and we were interested in calculating that.

So, let us take case A first. For case A, it is a reversible process, right for a reversible process we know that the work done in a reversible process is negative P d v d W is negative P d v right. So, if I integrate this, this will be W is going to be negative integral of V 1 to V 2 you go from initial state to the final state P times d v right. And then this is an ideal gas it is given to us. So, we know that P is RT over V, so it will be negative integral of V 1 to V 2 R T over so V d v.

Now, the total volume is 2.5 liters, we do not know how many moles are inside. So, let us just say that all of these actually are the total volumes right, so that way we do not have to calculate the number of moles inside. And now if I integrate this equation is n R T by V right P is n R T. So, this will be n R T over V. Now, if I integrate this equation right, it will be negative n R T is an isothermal process l n V 2 t over V 1 t, it is going to be negative n R T l n V 2 t over V 1 t. So, this is the work done in a reversible path. So, I know the temperature, I know the initial volume, what I do not know is the number of moles and the final volume, so we still have to calculate the number of moles.

Let us apply the ideal gas law, and see what we can do P 1 V 1 t is n 1 R T or simply n R T. T is constant that it is a closed system, so n is constant. So, we will drop the subscripts, it simply n R T right. So, n is going to be P 1 V 1 t over R T. So, if we substitute these numbers 10 bar 10 into 10 power 5 are 2.5 liters 2.5 let us write it a little better here, so this is 10 bar 10 into 10 power 5 Pascal's times 2.5 10 power minus 3 meter cube that is 2.5 liters, R is 8.1314 joule per mole per Kelvin. And the temperature given to us is 3 Kelvin. If you simplify this, it turns out to be actually 1 mole. So, what we have initially is 1 mole of gas. And obviously, the final number of moles also is 1.

Then to calculate V 2 I could simply use the number of moles I have, and the final pressure P 2 is going to be 1 bar, because it balances with the atmospheric pressure right, n is 1 mole, I do not know V 2, but the temperature is still the same 300 Kelvin. So, if we use this and then whichever way you look at it, you use ideal gas law or you use that the fact that P V is a constant for an ideal gas. Either way what we end up with is a volume, because the pressure has decreased by tenfold the volume final volume is going to increase by tenfold. And we will have 25 liters.

So, I have V 2 t which is 25 liters, V 1 t is given to me which is 2.5 liters right. So, then W reversible is going to be negative 1 mole times 8.314 is the value for R times 300 is the temperature l n of 25 liters over 2.5 liters. And this number turns out to be negative 5743 joules. Now, notice that the value is negative, because the system is expanding, system is doing work on the surroundings. So, it is a negative number. And its value turns out to be negative 5743 joules, when it expands tenfold this is case A.

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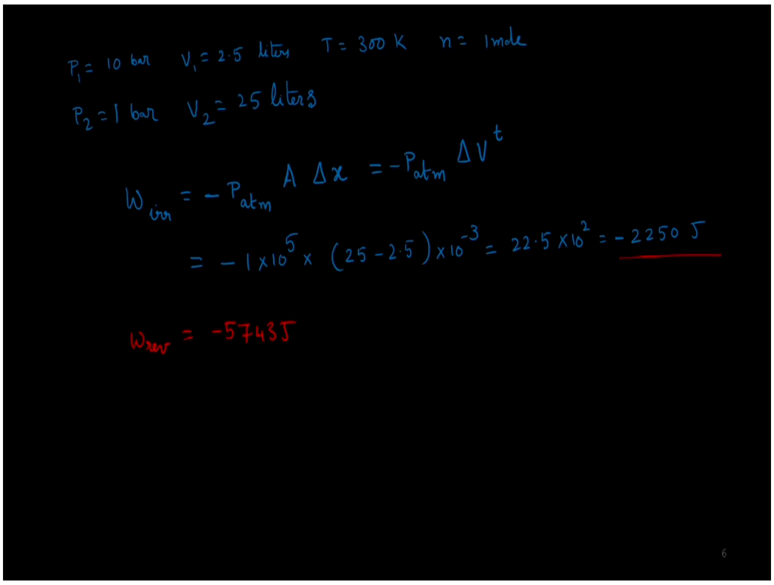

$P_1 = 10$ bar $\quad V_1 = 2.5$ liters $\quad T = 300$ K $\quad n = 1$ mole

$P_2 = 1$ bar $\quad V_2 = 25$ liters

$$W_{irr} = -P_{atm}\, A\, \Delta x = -P_{atm}\, \Delta V^t$$

$$= -1 \times 10^5 \times (25 - 2.5) \times 10^{-3} = 22.5 \times 10^2 = \underline{-2250\ J}$$

$$W_{rev} = -5743\ J$$

Now, for case B we have a different scenario alright. We are taking the weight away in one shot. The final equilibrium conditions are still the same. So, for case B, P 1 is 10 bar, V 1 is still 2.5 liters, T is 300 Kelvin, n is 1 mole, P 2 is still 1 bar, and because ideal gas law still holds we do is still going to be 25 liters. What is different is the way, we moved from state-1 to state-2.

The first one was that you reversible step, this is going to be an irreversible step, because we were going to take a weight suddenly this is 1 single large weight, which is balancing the force we take it away suddenly, and the gas is going to expand in a irreversible path. So, the intermediate steps are not at equilibrium there right. So, you cannot integrate or the work done is non-going to be negative P d v, we cannot do this integral, but we can employ another trick.

If you look at it the work done by the system, which is the gas inside on the surroundings, it is doing the work against a pressure which is equivalent to which is equal to the atmospheric pressure, and the displacement is delta V going from the initial state to the final state. So, in such a scenario W for this sudden change in an irreversible step is going to be negative P atmosphere, the area of cross section multiplied with delta x, which is exactly equal to negative of the atmospheric pressure times delta volume total right. And this delta volume total is essentially the difference between 25 and 2.5.

So, then this will be the atmospheric pressure, which is 1 bar or 10 power 5 Pascal's multiplied with V 2 minus V 1 25 minus 2.5, so that is 22.5 10 power 5, this is 25 and 2.5. So, this is actually meters, so in terms of meter cube, it will be 10 power minus 3 meter cube. So, this will be 22.5 into 10 power 2 or 2250 joules negative 2252 joules.

Now, notice that this number is way lower than the work that was done by the system during a reversible process. In that scenario the work done was negative 53, let us write that down. In a reversible process, it was negative 5743 joules in an irreversible process it is way lower. So, the work done in a irreversible process is always going to be lower than the work done in a reversible process, if the system is doing work on the surroundings that is what the negative sign indicates here. So, this is how we solve the problem for the two cases of a reversible process, where we integrate the P d v term. And in case of an irreversible process, where we simply calculate the work done by the system on the surroundings against the pressure of the surroundings.

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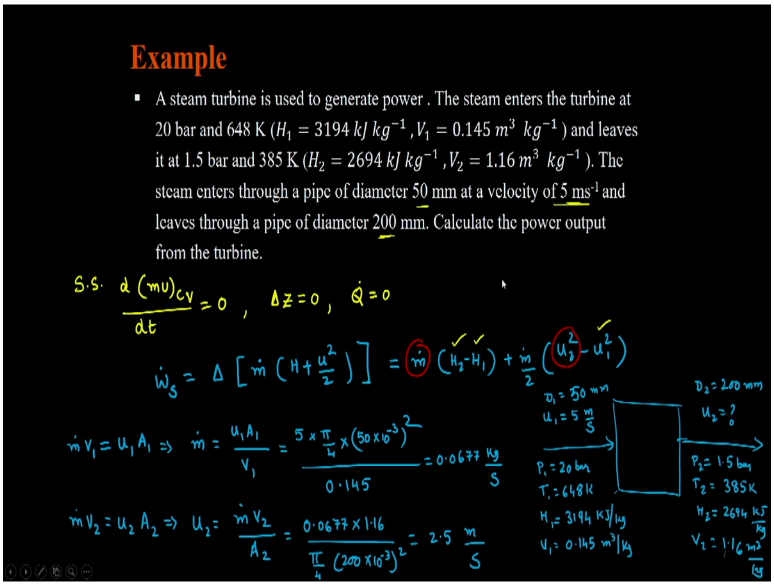

Example

- A steam turbine is used to generate power . The steam enters the turbine at 20 bar and 648 K ($H_1 = 3194\ kJ\ kg^{-1}, V_1 = 0.145\ m^3\ kg^{-1}$) and leaves it at 1.5 bar and 385 K ($H_2 = 2694\ kJ\ kg^{-1}, V_2 = 1.16\ m^3\ kg^{-1}$). The steam enters through a pipe of diameter 50 mm at a velocity of 5 ms^{-1} and leaves through a pipe of diameter 200 mm. Calculate the power output from the turbine.

S.S. $\frac{d(mU)_{cv}}{dt} = 0, \quad \Delta z = 0, \quad \dot{Q} = 0$

$$\dot{W}_s = \Delta\left[\dot{m}\left(H + \frac{u^2}{2}\right)\right] = \dot{m}(H_2 - H_1) + \frac{\dot{m}}{2}(u_2^2 - u_1^2)$$

$$\dot{m} V_1 = u_1 A_1 \Rightarrow \dot{m} = \frac{u_1 A_1}{V_1} = \frac{5 \times \frac{\pi}{4} \times (50 \times 10^{-3})^2}{0.145} = 0.0677\ \frac{kg}{s}$$

$$\dot{m} V_2 = u_2 A_2 \Rightarrow u_2 = \frac{\dot{m} V_2}{A_2} = \frac{0.0677 \times 1.16}{\frac{\pi}{4}(200 \times 10^{-3})^2} = 2.5\ \frac{m}{s}$$

So, those were two examples based on a closed system. Let us take two examples based on open system energy balances, where we apply the first law. In this example, we were go to apply first law to an open system, we were given that a steam turbine is used to generate power.

The inlet and the outlet conditions exit conditions for the turbine are given to us. The specific enthalpies and the specific volumes are also given as well as the velocities at the inlet and the exit of the turbine are given or in fact I should say the velocity at the inlet is given 5 meters per second, whereas the diameter of the exit and the entrance are given to us. And we were asked to calculate the power output from the turbine.

Now, for this example we were going to assume a steady state operation, so that we calculate a power output at steady state conditions. And from the open system energy balance for steady state condition, If I can write d of the control volume or the d of the internal energy of the control volume by d t to be equal to 0. And in addition to that the other terms I will assume to be 0 are the change in the position of the inlet and exit streams or in other words the change in the potential energy of the two streams will ignore that change.

Also we were going to assume that the turbine is going to operate under adiabatic conditions. So, there are very minimal heat loss from the turbine to the surroundings, which in fact makes Q dot to be equal to 0. So, the steady state operation no change in the potential energy and then Q dot is going to be equal to 0 for the adiabatic process.

With these assumptions the open state energy balance equation will reduce to the change in shaft work will be equal to delta of the change in enthalpy of the flow streams m dot times H the change in enthalpy plus if there is a slight change in the kinetic energy will account for that. So, we will add the kinetic energy term over there your square over two. So, W S dot is going to be delta of H plus u square over 2 multiplied with m naught of course that would be the rate in the rate of change in the enthalpy and the kinetic energy that should equal to the shaft work produced by the turbine.

And remember the delta notation, we are using is for exit minus the inlet. So, if I were to write this would be m dot notice that I am not using the subscript for m, because we are operating in steady state conditions. So, m dot is going to be constant multiplied with H 2 minus H 1 plus m dot times u 2 square minus u 1 square over 2 that would be the shaft work produced.

Now, let us see what is given to us, I have a steam turbine, and I have an inlet stream that is coming in at 20 bar the temperature is 648 Kelvin H 1 is 3194 kilo joules per kg, and V 1 is 0.145 meter cube per kg, and at the exit P 2 is 1.5 bar, T 2 is 385 Kelvin, H 2 is 2694 kilo joules per kg. And v 2 also is given to us, it is 1.16 meter cube per kg. Notice that the specific volume has increased, because the pressure has decreased considerably.

And also what we do know are the velocities u 1 is 5 meters per second, and I do not know u 2. But, what I do know is the area of cross section or rather the diameter. So, let me write the inlet diameter is 50 mm, whereas the exit diameter is 200 mm. Now, if you look at the energy balance equation and the given information, let me see what terms I have from the energy balance equation, I do not know the mass flow rate, but I do know H 1, H 2, and u 1. So, what I need to calculate is u 2 as well as m dot. Those are the terms that are missing in this puzzle m dot, and u 2 I have to calculate these two terms, so that I can calculate the shaft were produced by the turbine.

So, to be able to do that let us start with the m dot first, notice that m dot times the volume molar volume of the specific volume at the inlet will be equal to the velocity times the cross sectional area A 1. So, this implies m dot is going to be u 1 A 1 over the specific volume V 1. So, u 1 is 5 meters per second, A 1 is going to be pi by 4 D 1 squared, which is 50 times 10 power minus 3 meters squared over the specific volume at the inlet 0.145. And if you simplify the numerical, it turns out to be 0.0677 are kg per second. This is the mass flow rate, and because we were operating at steady state the mass flow rate at the inlet and the exit of course is saying.

Now, for the exit condition again I can apply the same equation m dot times V 2 will be equal to u 2 times A 2, and because I already know m dot, now I can calculate u 2 from this expression, it would be m dot times V 2 over A 2. And this time it is 0.0677 times the specific volume at the inlet 1.16 meter cube per kg defined by the area at the exit pi by 4 D 2 square, which is 200 times 10 power minus 3 squared. And if you simplify these numeric curves, it turns out that this u 2 is 2.5 meters per second.

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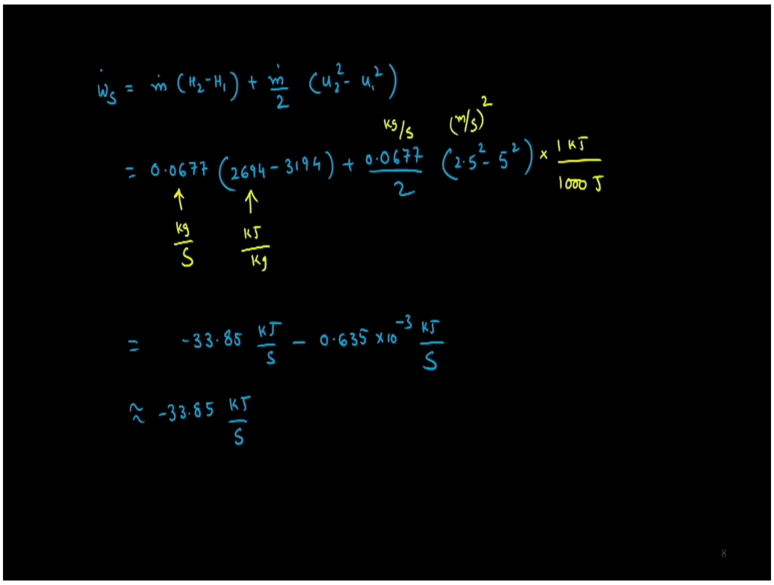

$$\dot{W}_S = \dot{m}\,(H_2 - H_1) + \frac{\dot{m}}{2}\,(u_2^2 - u_1^2)$$

$$= \underset{\uparrow \atop \frac{kg}{S}}{0.0677}\,(\underset{\uparrow \atop \frac{KJ}{Kg}}{2694} - 3194) + \overset{Kg/s}{\frac{0.0677}{2}}\,\overset{(m/s)^2}{(2.5^2 - 5^2)} \times \frac{1\ KJ}{1000\ J}$$

$$= -33.85\ \frac{KJ}{S} - 0.635 \times 10^{-3}\ \frac{KJ}{S}$$

$$\approx -33.85\ \frac{KJ}{S}$$

So, now I have used the given information, and obtained from the velocity at the inlet I obtained the mass flow rate, and from using that mass flow rate at the steady state condition, we obtain the velocity at the exit. Once we have this, we can go back to the energy balance equation.

And calculate the shaft work produced. Shaft work is going to be W S dot that is going to be m dot times H 2 minus H 1 plus m dot over 2 times u 2 squared minus u 1 squared. And this value of m dot is 0.0677, H 2 is 2694 minus 3194 plus m dot over 2 times 2.5 squared minus 5 squared. Now, notice that the units for enthalpy are kilojoules per kg and this is kg per second. So, what I have is kilo joule per second, whereas this is meters per second for velocity so meters per second squared and this is kg per second.

So, we have to convert it into kilojoules. So, what I need to do is multiply this with 1 kilo joule over 1000 joules right. So, there is an additional factor of 1000 that needs to be used in the second part just to make the units consistent. And once we simplify these numerical's, the first term will be minus 33.85 kilo joules per second, and the second term is going to be 0.635 10 power minus 3 kilojoules per second, so that is going to be 33.85 kilo joules per second.

Notice that the contribution of the change in kinetic energy term to this overall shaft work is very very negligible. Majority of this comes from the change in enthalpy that is the first term, so that is the shaft work the turbine is going to produce right is negative, which means the system is doing work on the surroundings, the work is being produced by the turbine. And it is produced iterate of 33.85 kilowatts, so that is how we apply the first law to an open system in a very simple scenario once we have the information on the enthalpies and the velocities of the stream so.

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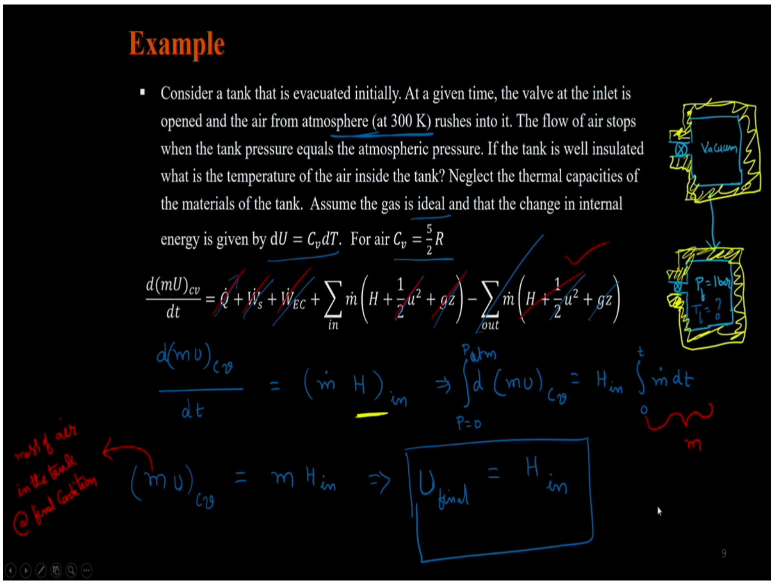

Example

- Consider a tank that is evacuated initially. At a given time, the valve at the inlet is opened and the air from atmosphere (at 300 K) rushes into it. The flow of air stops when the tank pressure equals the atmospheric pressure. If the tank is well insulated what is the temperature of the air inside the tank? Neglect the thermal capacities of the materials of the tank. Assume the gas is ideal and that the change in internal energy is given by $dU = C_v dT$. For air $C_v = \frac{5}{2}R$

$$\frac{d(mU)_{cv}}{dt} = \cancel{\dot{Q}} + \cancel{\dot{W}_s} + \cancel{\dot{W}_{EC}} + \sum_{in} \dot{m}\left(H + \cancel{\frac{1}{2}u^2} + \cancel{gz}\right) - \cancel{\sum_{out} \dot{m}\left(H + \frac{1}{2}u^2 + gz\right)}$$

$$\frac{d(mU)_{cv}}{dt} = (\dot{m}H)_{in} \Rightarrow \int_{P=0}^{P_{atm}} d(mU)_{cv} = H_{in}\underbrace{\int_0^t \dot{m}\,dt}_{m}$$

mass of air in the tank @ final Condition

$$(mU)_{cv} = mH_{in} \Rightarrow \boxed{U_{final} = H_{in}}$$

Now, let us look at another example. This is the final example, we were going to discuss today. We have a tank that is evacuated initially that is evacuated initially. So, let us draw a tank I have a tank, and then it is evacuated initially. And let us put a valve in the tank, so that we hold the vacuum in that tank. So, initially there is nothing in this tank right.

So, let us draw the tank here. So, I have some space, so that I have some space right I have a tank initially put it out there, and then the initial pressure is 0 is a vacuum complete vacuum. And then we open the valve, and once you open the valve of the air from outside is going to enter this tank, and then fill the whole tank until the pressure inside is going to be same as the atmospheric pressure right. So, the final pressure is 1 bar let us say, it fills up the whole tank. So, this is our process from vacuum to 1 bar by opening the valve, we fill in the air from the atmosphere. It also says that the tank is well insulated.

So, let us insulate this tank let us insulate this tank, like we have done before. So, this is an installation we are providing for the tank, so that there is no heat loss to the surroundings right, there is no heat loss to the surroundings. So, I have a tank now, and then this is insulated. Initially, the valve is closed and it is a complete vacuum, we open the valve atmospheric air enters inside the tank, and then we close the valve after that, so that we reach the final state.

Question asked is what would be the final temperature of air, the final temperature of air inside the tank what is the value of T f. What we do know is that the air can be assumed to be an ideal gas, and for an ideal gas the change in internal energy is d U is C v dt, and for this particular gas under consideration that is air C v is given as 5 by 2 R. So, these are the three things that are given to us. The gas is ideal d U is C v d t, because it is ideal.

We will come back and discuss why d U is C v dt for an ideal gas, but for now let us just say d U is C v dt, and then C v is 5 by 2 R. This is the process I have we were interested in finding the final temperature. But, if my system is what is inside this tank, then obviously it is an open system, we were allowing some matter from the surroundings to enter my system. So, it is an open system.

So, we will write an open system energy balance in this case, we will write an open system energy balance in this case right and let us see what terms we can we can drop from this particular equation. Obviously, there is no work done or heat exchange with the surroundings, because the tank is insulated so that term drops out. The tank is at the same location, we were just filling air in to this tank.

So, the potential energy term drops out, at the initial and the final conditions although there is flow, but once the flow stops there is no change in the kinetic energy. So, the kinetic energy terms can also be dropped from this particular expression. There is no shaft work, and there is no expansion and contraction work, because the volume of my some of the control volume for my system is fixed right. So, what I have done is this equation, which reads derivative of the internal energy total internal energy inside the control volume with time or change of that with time is the rate of change in the enthalpy.

Now, notice that there is only an inlet to this tank, there is no outlet. So, this is in right there is no outlet, we only have the inlet. So, in fact even this term can be completely dropped right. All I have done is if I then write the equation again d m U C v over d t is going to be there is only one inlet stream m dot H right at the (Refer Time: 37:25) right. So, this is what I have.

So, my energy balance equation boils down to this for the case we are considering can. Now, this is an unsteady state process. So, this is not a steady state process, because we are continuously filling air into the tank. So, we cannot drop the d by d t term right. But, what we can do is because the air atmospheric air can be considered it from atmosphere is at a constant temperature, and it is an ideal gas the is at atmospheric pressure the temperature is constant. So, this term here on the right hand side, on the right hand side the enthalpy term at the inlet is going to be constant H n is constant right.

If that be the case, I can rewrite this expression and integrate it right d of m U over the control volume is going to be H at the inlet integral of m dot d t over the entire time over which we fill it, right. And the total mass that comes into the tank within this time is going to be exactly the same the mass inside the control volume after the filling is complete right, then we have an integral on this side as well, after the filling is complete. This is from the initial condition at which the pressure is 0 to the final condition at the atmospheric pressure.

Now, the left hand side at initial condition when the pressure is 0, there is nothing inside. And in the final condition if we say the amount inside the tank is m, then this will be m times U for the control volume at the final condition. So, then what we were saying is m, this m is the mass of air in the tank at the final condition right.

And that should exactly equal this integral with whatever rate we fill, even if it varies at the end of the day, they told the amount of mass that we feel over the entire time should equal that m once we integrate that m dot dt. So, this will be equal to m times H coming in m times H coming in right. So, we can this implies U for the control volume at the final condition is going to be equal to H coming in. So, this is my energy balance equation for the particular process under consideration right.

So, once we have this, then we should be able to use some of the C v numbers given to us and calculate what it is going to be let us see how we do that. So, to summarize what we were saying is I have an open system right, and I drop the terms that are not applicable, there is no outlet stream right there is no outlet stream. So, this is dropped, there is no expansion contraction work, there is no shaft work, there is no exchange of heat with the surroundings because it is insulated. So, all these terms drop out, there is no kinetic and potential energy terms right.

So, with this or we can ignore them. So, with this what we have is in a differential form the derivative of the total internal energy with time inside the control volume really equal the rate of enthalpy coming in. If we integrate it right if we integrate it, then this will be the total internal energy inside the control volume minus whatever is the initial condition which is essentially 0. So, it is the total internal energy inside the control volume will equal the total enthalpy that has come in through the stream entering, and because both those masses are same.

If we drop them, it turns out that the internal energy smaller or specific internal energy at the final condition should equal the enthalpy of the air at the atmospheric conditions that is my energy balance equation for this particular process.

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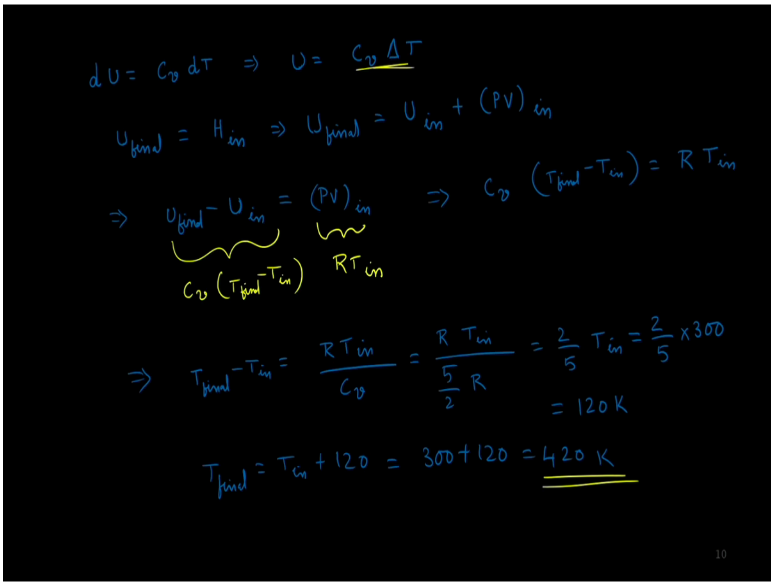

$$dU = C_v\,dT \Rightarrow U = \underline{C_v \Delta T}$$

$$U_{final} = H_{in} \Rightarrow U_{final} = U_{in} + (PV)_{in}$$

$$\Rightarrow \underbrace{U_{final} - U_{in}}_{C_v (T_{final} - T_{in})} = \underbrace{(PV)_{in}}_{RT_{in}} \Rightarrow C_v (T_{final} - T_{in}) = R T_{in}$$

$$\Rightarrow T_{final} - T_{in} = \frac{R T_{in}}{C_v} = \frac{R T_{in}}{\frac{5}{2} R} = \frac{2}{5} T_{in} = \frac{2}{5} \times 300$$

$$= 120\,K$$

$$T_{final} = T_{in} + 120 = 300 + 120 = \underline{\underline{420\,K}}$$

Now, we will take that and then since it is given to us that d U can be d U can be written as C v dt. If we integrate it and take C v to be constant, then U is going to be C v delta T right. So, U is going to be C v delta t. Now, my energy balance equation reads U final is H coming in. I can write H as U plus P V. So, U final is going to be U coming in plus the P V term of the air coming in right.

So, this implies U final minus U coming in is going to be P V coming in, this is an ideal gas. So, P V term is essentially R T, because it is an ideal gas. And this term U final minus U initial, we can use this information. And write it as C v T final minus T for the air coming in. So, then this equation reduces to C v T final minus T in is equal to R T coming in right.

So, then T final minus T in is going to be R T in over C V. And since C v is given to be since C v is given to be 5 over 2 R right, 5 over 2 R, we can substitute that information here as well. So, this is R t at the inlet over 5 over 2 R, so that will simplify to 2 by 5 times of the temperature coming of the air coming in. So, this is delta T. So, and the temperature of the air entering is at 300 Kelvin. So, this will be a 120 Kelvin. So, T final minus T in is 120 or T final is going to be T in plus 120, so that is going to be 300 plus 120 or 420 Kelvin.

So, the final temperature of air inside the tank in this case is 420 Kelvin. Now, that is a large change in the temperature, but then also remember that we have assumed that the thermal capacity of the material of the tank can be neglect or thermal capacity of the material of the tank can be neglected so. Once you heat up the gas inside, if it is in contact with the material of the tank then that material also gets hot, but then we are neglecting that change in the temperature of the material of the tank.

And if that be the case, then we can use the open system energy balance equation to get the number we have a 120 Kelvin for the change in temperature, so that is how we apply the first law in case of a unsteady state process. So, what we have done today is apply the first law in a variety of situations for a closed system, for an open system, and even for a unsteady state process.

That ends the discussion on the first law of thermodynamics and applications of the first law in simple scenarios. When we come back in the next class, we will discuss about I will discuss about how we can extend our understanding of the first law to more practical and engineering applications.

Thank you.