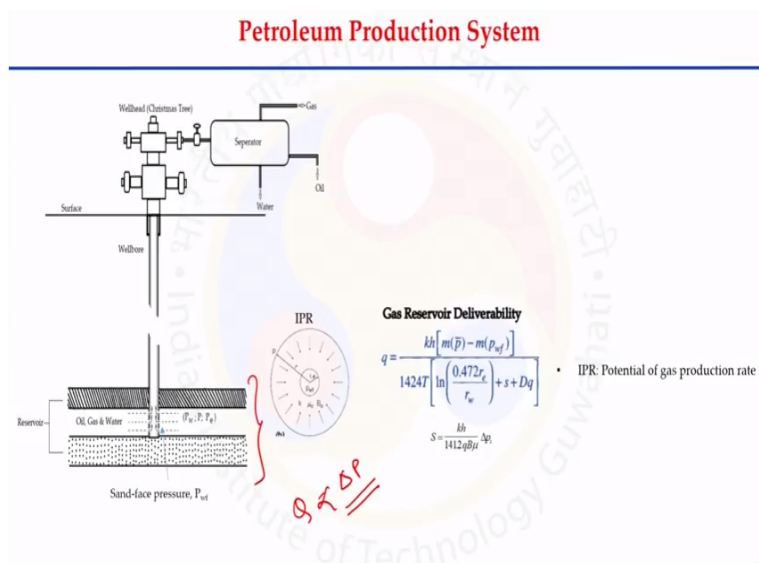


Natural Gas Engineering
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Module No # 02
Lecture No # 08
Inflow Performance Relationship (IPR) – II

Hello and welcome again today's lecture is a continuation of previous lecture where we discuss about the inflow performance relationship so I name this lecture as IPR 2 before we move further let us revise what we did in the last lecture.

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This is a petroleum production system we are working on this IPR relationship has been developed for flow through pores media to wellbore. So in last class we were developing the relationship using the mathematical relationship as material balance and we could establish the relationship how the flow rate is a function of pressure drop down that is the pressure of reservoir minus the pressure of wellbore.

The well bore is also known as the sand phase and with this relationship we could understand how the fluid properties and the reservoir properties are accounted in the mathematical expression so the big expression looks like this.

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Gas reservoir deliverability

Pseudo Steady State: Analytical expression

$$q = \frac{kh [m(\bar{p}) - m(p_{wf})]}{1424T \left[\ln \left(\frac{0.472r_e}{r_w} \right) + s + Dq \right]}$$

Assumptions:

- Single phase flow in the reservoir
- Compressible isothermal fluid flow
- Homogeneous and isotropic reservoir system
- Constant permeability
 - Fully radial flow only
 - Laminar (Viscous flow)
- Constant Pay-zone

- q is the gas production rate in Mscf/d,
- k is the effective permeability to gas in md,
- h is the thickness of pay zone in ft,
- $m(p)$ is the real gas pseudopressure in psi^2/cp at the reservoir pressure p in psi,
- $m(p_{wf})$ is the real gas pseudopressure in psi^2/cp at the flowing bottom hole pressure p_{wf} ,
- T is the reservoir temperature in R,
- r_e is the radius of drainage area in ft,
- r_w is wellbore radius in ft,
- s is skin factor, and
- D is the non-Darcy coefficient in d/Mscf .



So if we see the flow through this reservoir depend on the flow regime that is steady state under the steady state of the transient we discuss all this things in the last class the expression written here for the pseudo steady state condition that is simply says how q the production rate under the standard is related to permeability at the pay zone on thickness h and this is in the form of pseudo real gas pressure.

So in the last class we understand the different form the mathematical expression can be written that is the p approach, p square approach and mp approach, This approach has been to make that non-linear equation that is because of the material balance appear the we can linearize this using the approximation.

We discuss all these things and these expression for this steady state and pseudo steady state in using all three approximation all based on certain assumptions we discuss in the last class we said the flow is single gas flow in the reservoir the gas is compressible in the nature the reservoir is homogenous in terms of porosity and pay zone thickness as well as isotropic in terms of permeability.

The fully radial flow has been assumed and the laminar means discuss flow regime was considered so the Darcy law as applied to stimulate the or to replace the expression in the mathematical balance equation. It is important in what unit each term which is appearing in this

mathematical equation is defined if we change the unit of any of this parameter the numerical coefficient that is in this equation is 1 upon 1424 will take the different numerical value.

So you should be very careful and what unit system is adopted it is US field unit system or the SI unit system as well as in a particular unit system the definition of each parameter should be considered with unit system accordingly the numerical coefficient should be adjusted. For example the area may be given in Inch square or in ft square depends on which unit it is given and how it is going to affect our mathematical equation should be corrected with the conversion factor.

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IPR Curve: Analytical Expression: Diffusivity Equation		
Different form of IPR	Steady State	Pseudo Steady State
General relationship between P & R	$p - p_{wf} = \frac{q\mu}{2\pi kh} \ln\left(\frac{r}{r_w}\right)$	$p - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln\frac{r_e}{r_w} - \frac{r^2}{2r_e^2} \right)$
Inflow equation expressed in term of $p = p_e$ at $r = r_e$	$p_e - p_{wf} = \frac{q\mu}{2\pi kh} \ln\left(\frac{r_e}{r_w}\right)$	$p_e - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln\frac{r_e}{r_w} + \frac{1}{2} \right)$
Inflow equation expressed in terms of average pressure	$\bar{p} - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln\frac{r_e}{r_w} + \frac{1}{2} \right)$	$\bar{p} - p_{wf} = \frac{q\mu}{2\pi kh} \left(\ln\frac{r_e}{r_w} + \frac{3}{4} \right)$

- Similar expression can be written for P² approach and m(p) approach
- The expressions can be formulated to known parameters, like formation volume factor, viscosity, etc

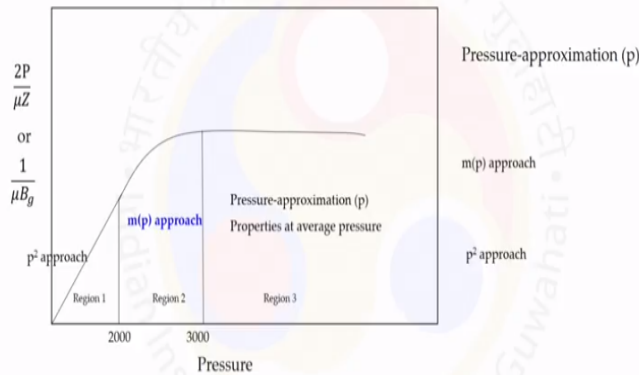
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So the general expression as given for the steady state and pseudo steady state in this table. You will see there is a small difference in a steady state or pseudo state case and that is in the ln something and that is appear in the boundary condition applied. If we choose the average reservoir pressure takes different form here we get 1 / 2 because $r_e = r_e$ if we choose the reservoir pressure at the r_e location and we use as a average pressure is volumetric average pressure.

We get additional term here in the steady state -1 / 2 and -3 / 4 and the pseudo steady state conditions. This we had done in the last class similar expression can be written for P square approach and MP approach.

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IPR Solutions: Approximation approaches



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So the approximation we understood how the approximation should be chosen which we approach p square approach or mp approach should be chosen depends on this curve roughly because the properties are competition of properties like p upon Mu z or 1 upon Mu Bg, Bg is volume formation factor how this particular term that is appearing in our material balance equation is a function of pressure.

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IPR Curve: Analytical Expression- Pseudo steady state

$$q = \frac{kh}{1422T \left(\ln \frac{r_e}{r_w} - 0.75 \right)} \int_{p_{wf}}^{\bar{p}_{res}} \left(\frac{2}{\mu_g} \frac{p}{TZ} \right) dp$$

$$q = \frac{7.08(10^{-6})kh}{1422T \left(\ln \frac{r_e}{r_w} - 0.75 \right)} \int_{p_{wf}}^{\bar{p}_{res}} \left(\frac{1}{\mu_g B_g} \right) dp$$

$$q = \frac{kh \left(m(\bar{p}_{res}) - m(p_{wf}) \right)}{1422T \left(\ln \frac{r_e}{r_w} - 0.75 + s \right)}$$

$$B_g = B = 0.00504 \frac{ZT}{p}$$

$$m(p_{wf}) = 2 \int_0^{p_{wf}} \frac{p}{\mu Z} dp$$

$$m(\bar{p}_{res}) = 2 \int_0^{\bar{p}_{res}} \frac{p}{\mu Z} dp$$

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And another thing which I mention just we can change from one unit system to the other one parameter side to the other parameter set for example here the formation volume factor if ZT and P are not known if the (()) (05:27) formation factor value is given volume factor value is given to


us we can always use this relationship to replace some of the parameters and those has been done here for a pseudo steady state case.

Similar can be done for steady state case or any other form and as well as here I had mentioned in terms of the pseudo real gas pressure that can be done for pressure approach and P square approach.

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Steady State: IPR

✓ Pressure approximation approach	$q = \frac{kh[\bar{p} - p_{wf}]}{141.2 \times 10^4 B_g \mu \left[\ln \left(\frac{r_e}{r_w} \right) \right]}$	$m(p) = \int_{p_b}^p \frac{2p}{\mu z} dp \approx \frac{p^2 - p_b^2}{\mu \bar{z}}$
✓ Pseudo pressure approach	$q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1424T \left[\ln \left(\frac{r_e}{r_w} \right) \right]}$	<ul style="list-style-type: none"> • q is the gas production rate in Mscf/d, • k is the effective permeability to gas in md, • h is the thickness of pay zone in ft, • $m(p)$ is the real gas pseudopressure in psi²/cp at the reservoir pressure p in psi, • $m(p_{wf})$ is the real gas pseudopressure in psi²/cp at the flowing bottom hole pressure p_{wf} • T is the reservoir temperature in R, • r_e is the radius of drainage area in ft, • r_w is wellbore radius in ft, • s is skin factor, and • D is the non-Darcy coefficient in d/Mscf.
✓ Pressure square approach	$q = \frac{kh[\bar{p}^2 - p_{wf}^2]}{1424 \mu \bar{z} \left[\ln \left(\frac{r_e}{r_w} \right) \right] T}$	

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So if we summarize steady state condition how IPR looks like in a different form you will see always everything is same except how we are changing from pressure term to pseudo reduce pressure term and from pseudo reduce pressure term to P square approach and that is given here how $m(p)$ and p square can be approximated if the pressure is less than 2000psi. So it depend on the approximation that we discuss the particular form can be taken as well as if you see here in the first expression the B_g volume formation factor is considered in state of the temperature here.

(()) (06:30) properties are known to us we can choose appropriate form of the IPR equation as well as the accuracy needed or the computation facilities is available we can choose particular approach like p , p square or $m(p)$ approach. Then again the unit system should be defined for each expression otherwise the numerical coefficient in the expression will be changed.

In pseudo steady state case if you compare the steady state cases pseudo steady state case we only think is here that is 0.472 is appearing in the log form and this is because of if you

remember the table when we compare this steady state or pseudo steady state case we are having addition term – 3 / 4. If we club this with $\ln r_e / r_w$ we are going to this r_e / r_w and this expression is having this addition term in the denominator for the pseudo state otherwise everything is same.

So what we can say like now we understand if a particular problem given to us the properties are given to us for the reservoir formation or the fluid properties or at least the pressure conditions at the desired locations are given to us we can estimate the relationship how my flow rate will change with a particular parameter.

That particular parameter could be the pressure or at the wellbore or could be the change in the permeability could be change in the viscosity of the gas could be change in the wellbore radius could be change in the radial means radius the reservoir or any other parameter that is appearing in the expression. For example q is directly proportional to permeability is increasing our flow rate may get increase.

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Inflow Performance Relationship (IPR)

➤ Example

$\gamma_g = 0.65$

$D_i = 7 \frac{7}{8} \text{ inch}$

$h = 78 \text{ ft}$

$A = 160 \text{ metres}$

$\tau = \frac{2580 \text{ Pa} \cdot \text{m}^3}{4613 \text{ Pa} \cdot \text{m}^3}$

β_g

P_2	\uparrow	P_{wf}
P		
$m(P)$		

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So for example if a gas of specific gravity of 0.65 is being produced from a reservoir and the diameter of wellbore is given to us. Let us say X and the Z pay zone thickness are given to us for example is given in this example is 78 feet. So from 78 feet long pay zone thickness radially fluid is flowing where diameter of D_i that is also I can write in a numerical term that is $7 \frac{7}{8}$ inch.

And the area from the drainage area other temperature and pressure is given to us in what we can do temperature at the reservoir or the average temperature is given to us and the condition at the wellbore what is the pressure at the wellbore or we have given like what is the average pressure of the reservoir given to us. Now if we know everything or we can calculate what is not given to us like if we are using the formation volume factor B_g we can calculate that thing.

Idea is that if we do this thing of course the question becomes which method should be chosen so what we can do we can check the pressure value the approach we can choose depends on the pressure value in what range this pressure or the pressure through which the gas will flow gas will be flowing like reservoir pressure to wellbore pressure in what range these two values are if they are below 2000psi we can go with p square approach if higher than 3000 psi we can go with p approach.

Those are the simplest approach and if it is in between 2000, 3000 we should go with mp approach and what we can do we can calculate the relationship between how value in a different conditions like p square p or mp approach what is the value of flow rate I am going to get and if I change the flowing condition so this is my Q and if change my flowing condition bottom wellbore pressure if I change how this Q is going to change we can record this and we can have a IPR curve construction.

In this example it was given like you are having the pressure at around 4600 reservoir pressure is 4600 135 psia and it was ask now calculate the flow rate when the bottom wall pressure is 3000 psi. So it is very clear from this the pressure range is 3000 to 4000 we can go directly to simplest form of the IPR that is the p approach and we can calculate but let us consider for the comparison purpose.

If we calculate with p square approach and mp approach how much deviation we will get we will discuss these things later on. But we can do that thing and every method will give us the different value and the different value will come because the parameters those are involved in mathematical expression are calculated at different pressure conditions.

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Inflow Performance Relationship (IPR)- II

- Inflow performance
 - Productivity Index
 - Absolute Open Flow
- Analytical Expression
 - Skin Effect
 - Non-Darcy Effect
- Empirical Methods- Field Tests – Single Point or Multiple Points
 - ❑ Simplified treatment approach- Back pressure approach
 - ❑ Laminar-inertial treatment approach- Forchheimer approach
- Modification in IPR for
 - Horizontal Well
 - Multiple Wells



In this class we are going to understand further about this IPR relationship and the inflow performance curve or the relationship can be expressed with two other parameters. So the two other parameters that is commonly used in the gas industry or gas production system or the productivity index and the obsolete often flow condition that we will discuss from these two are..

Analytical expression so far we developed are based on certain assumptions like only radially flow is happening only gas is getting flowed from this reservoir to wellbore only Darcy law is applicable is flow is under the laminar conditions or the permeability is constants means isotropic reservoir condition. So we will extend some more realistic data we are deviating from ideal gas or towards the real gas were not comprehensive real gas but we can consider two more phenomena those happen when the production happens like this skin effect and non-Darcy effect those two we will be discussed.

Further as we had seen in the analytical expression we are we need certain parameters like permeability, pay zone thickness, viscosity of the gas and some other things. So the solutions for ahh the situation when we are not having the values for those parameters is the empirical method. In fact empirical methods are more accurate than the analytical expression and the field test data can be used to have those empirical relationship.

The empirical relationship established in the gas industries we can call them the back pressure approach that is also the called the deliberate approach. We will discuss that in detail that the

simplified treatment approach. Another approach is developed by the Forchheimer and that is account more regress about non laminar behavior means when the laminar inertial turbulent treatment approach it is called and in that case the quadratic form of the flow equation with respect to DP with respect to pressure gradient is proposed by Forchheimer.

Modification and IPR equation should be done when we are having the fracture or the horizontal conditions in the well flow and or multiple wells. So the discussion of IPR 2 will be around this subject productivity index can be defined as the flow rate divided by pressure draw down. That is simply says the J that is productivity index is flow rate divided by the pressure draw down.

Here I am saying in the form of the pseudo real gas pressure m_p similar expression can be written for p square or p approach also. So we re-adjust our equation of the IPR for m_p approach in this case we will get if we take Q on the numerator and pressure gradient or pressure draw down in the denominator.

Whatever remains on the side is kh upon the part that should be here and all these kht are the reservoir properties like permeability and page on thickness will be we can say J depends only on reservoir properties and J value is very important because it can give us the flexibility to find out certain things.

So for example if we can adjust our equation what we are going to get this very simple form $Q = J$ multiply by the pressure difference here the pressure difference in m_p form and this situation when the reservoir pressure and bottom whole pressure. Bottom whole pressure means sand phase pressure sometimes it is called the flowing bottom hole pressure if both becomes equal or what I am trying to say if $P_{wf} = P_e$ in that case whatever the approach we are using either the p , p square or m_p approach the Q will be simply 0 means no production is happening and that will be the situation here.

So when we are not producing anything from the reservoir the pressure at the bottom whole will be equal to reservoir average pressure and that we can say here in this situation point 1 we can have it and another situation could be there where the bottom hole pressure is set as a atmospheric pressure or obsolete atmospheric pressure. Or in other term my p_{wf} is 0 m_{pwf} is

also 0 and in that case where i will be in this chart at point 2 and the flow rate under this condition will be the maximum flow rate.

There is major pressure draw down that is happening when my Q_{wf} is set to 0 and the production under that condition will be the maximum production can be achieved from that particular well and that situation is called the AOF absolute open flow conditions and we can write it is here AOF and so this is a curve between pressure and flow and so we do so if we are having 2.0 more than 2 points we can get the curve and that curve simply allow us.

When this is intersecting at Q we can say this is absolute open flow conditions the maximum flow rate can be obtained when it is intersecting the Y we can say this is reservoir pressure and the slope of this curve will give us $1/J$ that is a reciprocal of productivity index. So productivity index is having the unit that depends on the approach we are using is Pe square approach p approach, p square approach or mp approach accordingly this will get change but the processor is same.

Q is the maximum flow and U_F is the absolute open flow potential what is the advantage of this sometime this is the terminology that is used just to communicate in the reservoir industrial like what is the productivity index of a particular reservoir particular well as well as this can allow us to understand the IPR relationship with just a single data point set. So for example where we are having just because of XYZ region we cannot shut down our well for a longer duration we are having just one set data where our flow rate at a particular bottom hole pressure is still we can develop the IPR relationship that allow us to construct the IPR.

So for a single point analysis when we are having we can get this maximum Q_{max} at a particular condition when the P_{wf} is 0 and using this relationship when we are having like equation 1 and equation 2 what we can do we can just divide Q / Q_{max} and we will get this relationship. So if we know what is Q_{max} or AOF condition for my reservoir I know my reservoir pressure I can develop the relationship how my Q will change at a particular P_{wf} .

So the relationship can be developed just using 1 single point data and that is always available when our reservoir is being operated or the well is being operated at a particular flow rate you

know what is the Pwf of the at the bottom hole condition. And some cases when we need multiple data set or at least two data set will be discussed later on.

So as discussed so far what we did we consider just a laminar flow conditions and very ideal condition when the flow is just radially happening and before it is going to the wellbore it is having perforated region the well bore is perforated so the gas is just defusing in the well bore without having any additional resistance but that the ideal case near the well bore some other phenomena happens that is we call the skin factor.

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Skin Effect

Skin Factor $S = \frac{kh}{141.2qB\mu} \Delta p_s$

- Additional pressure change due to heterogeneities close to the wellbore.
- Deviation from the ideal inflow.
- Analogous to the film coefficient in heat transfer
- Skin factor is a variable used to quantify the magnitude of the skin effect

Handwritten notes:

damage → S +ve
 $K_{skin} < K$
 → S -ve
 $K_{skin} > K$

$(\Delta p)_{\text{net}} = (\Delta p)_{\text{reservoir}} + (\Delta p)_s$
 $\Delta m(p)_s = \text{skin factor}$

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And the skin factor is because of several reason the simplest reason is there is damage happen near the well bore and the damage may happen during the drilling part during the well completion time or because of some other reason like over the time the sand particular deposited in that region and it is restricted it is not allowing the gas to flow as it was flowing in the ideal condition.

So to account that the restriction offered by this layer we can see there is kind of a layer that is getting deposited so if I draw this here. So this is kind of situation where we are having this well bore and near the well bore something getting deposit here. Assume here is something happening over the time and that could be like pressure that is offer some additional pressure change due to this deposition or damage is happening near the well bore.

The additional pressure change due to heterogeneity close to the well bore occurs the deviation from the ideal inflow happens near the well bore and that is should be accounted when we are developing the more realistic mathematical expression. This deposition can be understood with the concept of film coefficient in heat transfer where there is additional film on this surface of the heat extended devices that is offer the resistance.

The skin factor is very well used to quantify the magnitude of the skin effect that is happening here on the surface. So we can see how this is going to be so in ideal case we were having the reservoir pressure draw drop like ΔP because of that the pressure is happening so we will call this this is happening because of the Darcy law and now the actual pressure drop will be not only because of Darcy flow but because of this skin effect also.

This is skin effect could be positive could be negative in most of the cases it is positive until unless something has been done in the well to reduce the damage to eliminate the damage that happen to the value. That could be done by (()) (22:47) the well cleaning the well other way this is my r and the pressure that was happening like this. In the ideal case I was having like this now the pressure is somewhere happening like this or some other mean like this but we are having this additional pressure drop and this addition pressure drop ΔP_s is going to affect my Q .

Because I know my Q is related to the pressure gradient now I got additional pressure gradient and that additional pressure gradient will reduce by flow rate. So what exactly can be done we can do either in form of the mp or square or p approach we can say this addition pressure in Δp form is because of this skin factor. We can write this skin factor S in the form of like this so if we do this material balance as we did for the just a Darcy case we will get some additional pressure drop.

That additional pressure drop can be written in form of the other parameter like how permeability kh and the formation of volume factor and viscosity the fluid is going to effect the expression can be written in form of the Δp whatever is appearing here we can denote that as S as skin factor S . So for positive skin factor the S value will be added in the numerator of the expression.

There are two cases when the well is getting damaged and well is damaged S is actually positive this skin factor is positive and it means near the well bore the permeability has been lower down so the K is skin in this case when the S is positive K is skin is lesser than K. So what exactly having the fluid was flowing from here at a different at a particular permeability and suddenly it got some reason when the permeability is low.

When the permeability is low it means the pressure drop will happen there and that pressure drop can be accounted and when the S is negative it means near well bore is been improved by any other by cleaning or acidizing or something in that case when the fluid was flowing from this constant permeability because we assume the constant permeability and suddenly it experience of permeability region where the K of this region that we called skin region is greater than K.

So the fluid will flow at a higher rate it will often less resistance and this situation of that type will be like this. So here the pressure drop is small compared to when the skin is there or when the ideal gas is there when the pressure will be lesser than the ideal case and the flow will get improved. There might be situation when there is no skin and that case the K is skin near region will be equal to K.

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Productivity Index (J)

$$J = PI = \frac{q}{m(p_{res}) - m(p_{wf})} = \frac{kh}{1422T \left(\ln \frac{r_e}{r_w} - 0.75 + S \right)}$$

$$q = J \left[m(\bar{p}_{res}) - m(p_{wf}) \right]$$

$$AOF = q_{max} = J \left[m(\bar{p}_{res}) - 0 \right] = J \left[m(\bar{p}_{res}) \right]$$

J = PI = Productivity Index (Mscf/d/ psi²/cp)- Depends only on Reservoir properties
 q_{max} = Maximum gas flow rate (Mscf/d)
AOF = Absolute open-flow potential (Mscf/d)

m(p_{wf}) = m(\bar{p}_{res}) - \frac{1}{J} q

Pseudo pressure m(p) approach

Pressure Square approach

IPR Curve based on Single Point

Single Point

$$\frac{q}{q_{max}} = 1 - \frac{m(p_{wf})}{m(\bar{p}_{res})}$$

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And if we write the expression considering everything as we did for the analytical expression without the skin effect what we are going to get the expression where we are having this S in the denominator of the expression. So my Q if I take this here again the same expression as we are

having the standard equation for the IPR we will see the addition term S appears in the denominator and that is reciprocal to flow rate.

If S is positive if damage happen to near wellbore the Q will get reduced so this is the way it is going to affect we can get these similar productivity index as we did without the skin effect in the previous two slides back we can get the similar analysis how the EOF and Q is going to be affected with the skin effect. And knowing that thing is still we think we are having the single point test that allow us to develop the IPR relationship or by sketching the relationship between the pressure and Q we get the AOF reservoir pressure and inverse of the productivity index by the slope.

So the skin effect is just going to add in the denominator and going to offer the resistance if it is negative then of course the flow rate get improved but what exactly it is doing it is not changing the nature of the equation. The nature of the equation in the sense it was related like Q is related to pressure gradient while if we going to another concept of non-Darcy effect you will see the equation becomes quadrate effect is still in the first order system where Q is related to pressure gradient.

It may be non-linear so it may be non-linear equation because other parameters are involved so the of course the pressure drop down and flow rate is having the non-linear relationship but it becomes second order or more non-linear when we having the second concept of the non- Darcy flow regime.

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Non-Darcy Flow Regime

Non-Darcy flow occurs in the near-wellbore region of high capacity gas and condensate reservoirs: As the flow area is reduced substantially, the velocity increases, inertial effects become important, and the gas flow becomes non-Darcy.

- Near bore-well Turbulent flow exists: Additional pressure drop
- IPR relationship for Non-Darcy condition: Quadratic in term of flow rate, q

Non-Darcy Flow Forchheimer Model

$$\frac{dp}{dx} = \frac{\mu_g}{k_g} v_g + \beta_0 \rho_g v_g^2$$

$$\frac{dp}{dx} = \frac{\mu_g}{A \cdot k_g} q + \beta_0 \beta q^2$$

β_0 is the turbulence parameter to gas and function of permeability and porosity, and tortuosity.

- Simplified treatment approach- Back pressure approach
- Laminar-Inertial-Turbulent (LIT) treatment approach- Forchheimer approach

The skin factor and non-Darcy coefficient (D) can be estimated on the basis of pressure transient analysis

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There is a well which is producing under the Darcy law and under the viscous condition under the isotropic condition but why any mean it happens like the skin gets deposited we could account that thing also in the mathematical expression but is still the assumptions of laminar flow is there while exactly what happens when we are having the radial flow. In radial flow the area far from the well is having significant radius and the flow is going just like this towards the well.

So far from this it is not having the experience of change in the velocity very significantly what when it approaches near the well bore what exactly happens the area that is $2\pi rh$. So h is phase on thickness that is constant but the radial R is getting reduce when R is getting reduce your area is getting reduce and by the definition of velocity $V = q/A$. So when the area is getting reduce our velocity will go up and this velocity increase crosses laminar region to high flow region high velocity region.

And that is always the turbulent region the velocity approaches so the velocity increase and that is happen because the area is getting reduce another region is gas expenses and happens because our gas is a however fluid compressible gas when it goes near to this region velocity increases pressure drop happens addition pressure drop happens and the volume formation expansion or the gas get expanded.

So because of this region primary reason is the velocity increase and the velocity increase says the deviation is happening from the Darcy law and we have to account for this turbulent or the

non- Darcy behavior so Forchhiemer has developed mathematical relationship he said as the flow is happening under the non- Darcy reason and the inertial forces also becoming important and the equation should be modified to account both Darcy flow and non-Darcy flow.

So the pressure gradient in a this is given for the axial position like if you writing a radial coordinate system the minus will go away so the pressure gradient is because of the Darcy flow that is our Darcy equation and this is because of the non-Darcy equation in terms of the velocity will be related to square of velocity some constant and the density. So the expression is given which account for Darcy and non- Darcy flow we can convert this expression using the relation like $V = qA$ in this form.

Where addition g are given for the gas case that is can be avoided simply it is a μ means because our system is just a gas system. So this is the pressure gradient this is the summation of Darcy and Non – Darcy with the help of this equation written like this we can replace the density. We know density = 29 gamma g or some other 29 gamma g is P / zRT we can replace similarly the Q that is the reservoir condition we can replace to standard condition is that we say Q_{sc} what exactly we can do we can say using this non- Darcy coefficient.

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Non-Darcy Flow Regime

Reference	Correlation	Unit for β	Unit for k
Cooke (1973)	$\beta = \frac{b}{k^2}$	atm.v/g	darcy
	<i>a and b experimentally determined constants</i>		
Tharvin & Mahany (1998)	$\beta = \frac{3.1 \times 10^4 r^2}{k}$	1/cm	darcy
Geensma (1974)	$\beta = \frac{0.005}{k^{1.75}}$	1/cm	cm ²
Tek et al. (1982)	$\beta = \frac{5.5 \times 10^6}{k^{1.5} \rho^{0.5}}$	1/ft	md
Liu et al. (1995)	$\beta = \frac{8.91 \times 10^6 r}{k \rho}$	1/ft	md
Engen (1992)	$\beta = \frac{a}{\rho^{0.5} (10^{-4} k^2 \rho)^{0.5}}$	1/cm	darcy
	<i>a = 1.75, b = 150</i>		
Jankov & Katz (1955)	$\beta = \frac{1.82 \times 10^6}{k^{1.5} \rho^{0.5}}$	1/cm	md
Fiscal et al. (1980)	$\beta = \frac{4.8 \times 10^3}{k^{1.5}}$	1/in	md
Jones (1987)	$\beta = \frac{6.35 \times 10^6}{k^{1.5}}$	1/ft	md
Coles & Hartman (1998)	$\beta = \frac{1.07 \times 10^{10} \rho^{0.5}}{k^{1.5}}$	1/ft	md
Coles & Hartman (1998)	$\beta = \frac{2.40 \times 10^9 \rho^{0.5}}{k^{1.5}}$	1/ft	md
Li et al. (2001)	$\beta = \frac{11500}{k \rho}$	1/cm	darcy
Wang et al. (1999)	$\beta = \frac{(10)^{0.5} \rho^{0.5}}{k^{1.5}}$	1/cm	cm ²
Wang (2000)	$\beta = \frac{1}{k^{1.5}}$		
	<i>r is tortuosity</i>		

$(\Delta P)_{act-r} = (\Delta P)_{Darcy} + \frac{(\Delta P)}{s} + (\Delta P)_{Non-Darcy}$

Total pressure gradient happening in the system under the actual condition is now summation of pressure gradient is happening because of the Darcy law or Darcy flow + pressure gradient is happening because of the skin factor similarly pressure gradient is happening because of non-

Darcy flow and as we did the equation adjustment for the simply Darcy flow later on we did with the inclusion of skin factor we can do similar for the non-Darcy flow also.

In the non- Darcy component there is some beta prime that is beta prime is beta parameter of gas and it is a function of porosity and permeability some time it is also function of the (β) (32:54) and several expressions are given literature in the expression based on the experience based on the several filters how this turbulence coefficient or the turbulence equation in this parameter gets effected.

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Pseudo Steady State: IPR

Pressure approximation approach

$$q = \frac{kh[p - p_{wf}]}{141.2 \times 10^3 \bar{\mu} \bar{z} \left[\ln \left(\frac{0.472r}{r_w} \right) + s + Dq \right]}$$

$$S = \frac{kh}{141.2 q B \mu} \Delta p_i$$

Pseudo pressure approach

$$q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{14247 \left[\ln \left(\frac{0.472r}{r_w} \right) + s + Dq \right]}$$

$$m(p) = \int_{p_b}^p \frac{2p}{\mu z} dp \approx \frac{p^2 - p_b^2}{\mu z}$$

Pressure square approach

$$q = \frac{kh[\bar{p}^2 - p_{wf}^2]}{1424 \bar{\mu} \bar{z} \left[\ln \left(\frac{0.472r}{r_w} \right) + s + Dq \right] T}$$

- q is the gas production rate in Mscf/d,
- k is the effective permeability to gas in md,
- h is the thickness of pay zone in ft,
- $m(p)$ is the real gas pseudopressure in psi²/cp at the reservoir pressure p in psi,
- $m(p_{wf})$ is the real gas pseudopressure in psi²/cp at the flowing bottom hole pressure p_{wf} ,
- T is the reservoir temperature in R,
- r_e is the radius of drainage area in ft,
- r_w is wellbore radius in ft,
- s is skin factor, and
- D is the non-Darcy coefficient in d/Mscf.

Production rate as a nonlinear function of pressure drawdown (reservoir pressure minus bottom hole pressure)

The skin factor and non-Darcy coefficient can be estimated on the basis of pressure transient analysis

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With the help of some other non- parameters are converting from 1 unit to the other we can write this beta in the form of F here and in the form of F we can write here sorry from beta we can write in F form and F2 we can write in D form. Here you can see the beta that is appearing in the non- Darcy coefficient is a function of permeability and porosity and that beta is related to additional term we can say related to the fluid properties or the reservoir properties in this form.

And we write in the form of D this expression can be adjusted here equation can be adjusted for the IPR which account all these pressure drop down happening. Here I am showing like this different scientist has done the analysis how to calculate the turbulence parameters and it is the function of porosity, permeability (β) (34:02) or it is just a function of porosity or permeability so in different format is given we can choose one of them and the mostly used expression is this.

So what we can see now we got the overall expression for our system IPR curve that says how much Q is a function of Darcy flow as well as how it accounting as the skin factor and how it is accounting the D non-Darcy flow so D represent the non-Darcy coefficient or non- Darcy flow coefficient it can be called and here you can say if we adjust this equation Dq will multiply with Q and we will get the quadratic equation and that is exactly what Forchheimer said the in the case of Non- Darcy flow your Q and pressure will be having a relationship and Q will be having the quadrate form.

But we have adjusted in this equation simply says Q left hand side can be evaluated with this big expression. Now in this expression we are having Q on the left hand side Q on the right hand side thus iteration process is required like this newton Raphson method is a required. Not only because of this region Q is that the both the places some properties like the viscosity calculation need some parameters those depends on the Q or the pressure conditions.

So the iteration is required with some objective function to have the Q value in this approach of P similar can be written for MP approach we call the pseudo approach and P square approach. We already discuss under which condition we should use but again I could emphasize if we are having the capacity to calculate to numerical integrate the pressure range we should always go with the mp approach because this is the most accurate approach.

So similar expression can be written for the pseudo steady state case the IPR curve for pseudo steady state are shown here the difference is here only terms of 0.472 in each form otherwise other expression the numerical expression all are same. Again I am emphasizing the numerical coefficient will depend on the unit chosen to represent a particular parameter in the system this is the US field unit system.

So the production rate from here we can say the production rate as a non-linear function of pressure draw down and so my Q is here if I adjust this become non-linear in fact under certain assumption if we are assume it become the quadratic nature. Now the parameter which we are seen here is s and D how to estimate them either we are having sufficient information of the parameter those appears in there expression like for non-Darcy coefficient D we need to know

the beta and beta depends on a permeability and porosity first and later on it depends on the pay zone thickness and other parameter.

So this can be done the skin factor non- Darcy coefficient can be estimated on the basis of pressure transient analysis. This is type of the analysis that we will discuss in the next lecture when we will be discussing how the gas well testing can be performed what does it means when we are having a reservoir when we are having a well ready to produce we do certain type of the testing or the reservoir to do certain parameters unknown parameters as well as know the features performance of a well we do analysis that is called the gas well analysis and discuss later on but here I would like to emphasis. These values s and D can be estimated using some field test

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Pseudo Steady State: IPR

Pseudo pressure approach

$$q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1424T \left[\ln\left(\frac{0.472r_e}{r_w}\right) + s + Dq \right]}$$

Handwritten annotations:

- $\alpha = \frac{kh}{1424T}$
- $q = \frac{\alpha [m(\bar{p}) - m(p_{wf})]}{\ln\left(\frac{0.472r_e}{r_w}\right) + s + Dq}$
- $\alpha = \frac{\alpha [m(\bar{p}) - m(p_{wf})]}{\ln\left(\frac{0.472r_e}{r_w}\right) + s + Dq}$
- $\alpha \left[\ln\left(\frac{0.472r_e}{r_w}\right) + s \right] + \frac{Dq^2}{2B} = m(\bar{p}) - m(p_{wf})$
- $m(\bar{p}) - m(p_{wf}) = Aq + Bq^2$ (Non Darcy flow)
- $m(\bar{p}) - m(p_{wf}) = \text{Darcy}$

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So let us summarize it again since pseudo pressure approach for the pseudo steady state conditions we got this expression and this expression simply can be adjusted to get other form that I can write it for example if I say my kh all this thing is constant and I can say this is let us say x so what I will get qx mp bar means sometimes it is written mp bar sometime it is written mpe sometimes mpr – mp wf divided by ln 0.472 / re / rw + s + Dq I can write that way correct.

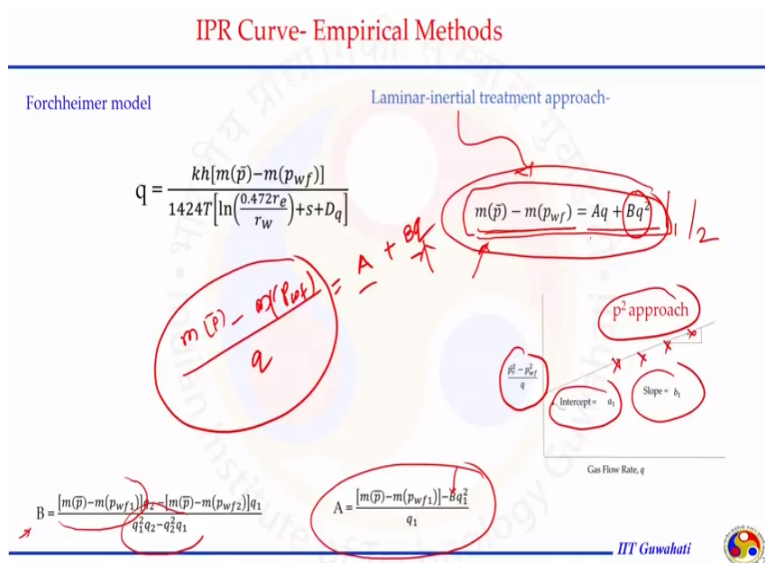
Now I will take all of them here so what I can write my q sorry this ln is just here ln so my q multiply by ln 0.7472 re / rw + s + Dq square divided by X and I will get mp – mpw. So what I got something that could be the constant if my reservoir is having a constant radius re if my

wellbore is constant radius of r_w the skin at a particular time is S_s similarly D is the porosity in permeability or none of the reservoir or a constant we can say all this can be the constant.

So what I will get if I write I can write this expression $m_p - m_{pwf}$ and should write average $m_{pwf} = Aq + Bq^2$ square this entire thing I can represent as A and this as a B . So what I did I can simply transform my relationship of this big expression in this form x is kh upon 14240 so the k permeability h is pay zone thickness t is the temperature. So all these are the part of the system when no Darcy means non- Darcy flow was not there it was case when the laminar flow was happening and the Darcy equation was at adequate enough to represent the flow.

So what we can say the A here is just represent Darcy conditions with the skin effect of course and here B what is in the B that says D that to be the non- Darcy flow coefficient divided by some parameter that KH upon 1424 temperature. So this simply because of the non- Darcy or the pressure draw down it difference is non-linear function of the flow rate or in fact it is quadratic relationship with the flow rate with the coefficient A and P , A represent for Darcy flow P represent for non- Darcy flow.

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What I just discussed there is explain here that is $m_p - m_{pwf}$ is $Aq + Bq^2$ and this model which is given by the Forchheimer is also called the laminar inertial turbulent treatment model because here we are having this system where other than laminar or deviation from the laminar has been accounted in terms of the Bq^2 in this form. Now the question comes what is this

equation how this equation is going to be useful and how to get the constant A and B those are the big expression that are just represented in the form of A and B.

Either we can get the analytical expression or this can be empirical correlations means the field test data can be collected over the time can put here and we can estimate the value of A and B because we understand if we are having this expression and if this expression further adjusted in $mp_{bar} - mp$, mp_{wf} divided by q we are going to get $A + Bq$. So this quantity if plotted with respect to q we will get a linear relationship and this linear relationship will give us A and B / by intersect and slope.

Similar as been done here in p square approach similar we can do for the mp approach or p approach so the expression is given in the mp approach while the graph is plotted for p square approach just to show yes we have just need to have the left hand side divided by the flow rate q but we will get on the write hand side just $mx + C$ type of expression where we can have the linear fitting for the curve and intersect will give us the A and slope will get B.

A1B1 is written if we are getting the difference form there could be other way this is the way when we are having the multi rate data we can just put those data here and plotted but this method the empirical method allow us to calculate the value of A and B if we are having just 2 data set means well which can be just operated at two difference condition of flow rate we will be having the two different bottom hole pressure and if we get those two conditions we can get the constant value we can get the P and A just by adjusting this equations.

So for example this equation if we put it condition 1 and then condition 2 what we are going to do by adjustment how to get constant B when it is the first condition $mp - mp_{wf}$ one condition multiply by the flow rate of 2 – the pressure drop down and that condition to multiply by flow rate 1 divided by this expression. This can be done if we solve it just algebraically we will get how to get the value of B once we know the value of B here we can put it back and we can get the value of A.

Now this can be done for p square approach as well as p approach similarly as we did for this and by knowing the A and B value we can put any value of P_{wf} and can get the flow rate it means we can operate our reserve our well at any condition P_{wf} to adjust the flow rate q .

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IPR Curve- Empirical Methods

Forchheimer model

$$m(\bar{p}) - m(p_{wf}) = Aq + Bq^2$$

where A and B are empirical constants that can be determined based on test points.

$$B = \frac{[m(\bar{p}) - m(p_{wf1})]q_2 - [m(\bar{p}) - m(p_{wf2})]q_1}{q_1^2 q_2 - q_2^2 q_1}$$

$$A = \frac{[m(\bar{p}) - m(p_{wf1})] - Bq_1^2}{q_1}$$

Laminar-inertial turbulent treatment approach-

Backpressure model


$$q = C [m(\bar{p}) - m(p_{wf})]^n$$

where C and n are empirical constants that can be determined based on test points.

The value of n is usually between 0.5 and 1.

$$n = \frac{\log\left(\frac{q_1}{q_2}\right)}{\log\left(\frac{m(\bar{p}) - m(p_{wf1})}{m(\bar{p}) - m(p_{wf2})}\right)}$$

$$C = \frac{q_1}{[m(\bar{p}) - m(p_{wf1})]^n}$$



So another method back pressure model this is a purely empirical method that is also called the deliberated methods and in this method what is said for example here you are having this expression let us assume we are not having this Dq and that Dq is just because of non- Darcy affect. So we can saw we can write this q = all this is constant I can write let us say I will write C is a constant is mp bar - mpwf.

So this is similar what we got the expression in the productivity index and we can say this C equivalent to what is A there in the Forchheimer model or quadratic model both are equivalent C and A with those are calculated is different however for the case when we are not having the Darcy coefficient C and A are same and this C represent the Darcy flow in the system and when we are having the non- Darcy effect what we can do we can adjust this equation with some exponent N and can say this N is going to represent going to account the changes is happening in our model equation because of the non- Darcy flow.

So for example when n = 1 we are having the fully Darcy flow conditions fully laminar flow conditions is no non-Darcy effect is there but when n is getting changes or for example when it is becoming 0.5 it mean we are having the fully turbulent flow condition. So the n value is easily between 0.5 and 1, 0.5 represent for fully turbulent flow condition and 1 represent for fully laminar condition.

But for having this relationship what we got another empirical relationship that empirical relationship is by any mean if you can get the value of C and n put those in this empirical relation you got the IPR construction curve relationship put different value of Pwf you will get different value of q with the help of that you can construct the IPR curve. How to get the value of C and n? Again if you are having the multi rate test data we can plot by taking the logarithmic of this.

And on log scale if we plot the log q versus log of this expression again here I am showing in terms of p square approach it can be just done in the form of $m_p - m_{pwf}$. If we do that on the log scale what we are going to get this slope will be $1/n$ and with the help of n we can get the value of C from that. So with this expression we can get the C and n with this all test we will be estimate what is AOF we will again go to estimate what is the reservoir pressure and knowing the value of n we can say the flow is still in the Darcy region it is deviation or into non- Darcy reason.

All these kind of the analysis will be done in a separate lecture that is where we will having the gas well testing analysis. If we know the condition of the well at two points for example the well is running we are operating at one point and the second point as we did for the Forchheimer model of the quadratic model we can say similar done for this back pressure model where running at two different condition putting those two condition in this log form of this empirical model.

We can get the value of n just log of flow rate 1/ flow rate to divided by log of pressure draw down one condition divided by pressure drop down is second condition with the help of that we can get n putting n back in this equation we can get for any one condition with the help of one condition we can get the value of C.

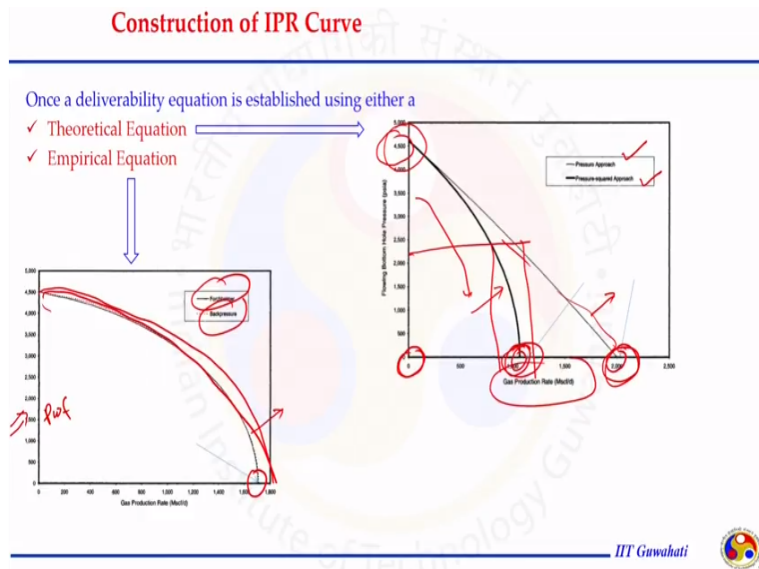
So now C and n are known to me now i got my model equation the model equation is known to me I can get any value of pwf to get both are empirical model and most of the time this empirical models are based on the field experience and until unless there is not major changes is happening in the field for example here is no changes are made in the formation means no permeability has been changed by any meal or there is a more changes are happening in the pay zone thickness there is no changes in the diameter of the pipe is.

Any condition are not changed even the reservoir pressure is not changed significant over the time this equation are pretty much valid if we know the coefficient of equation like A and B for the Forchheimer equation and C and n for the back pressure model. Again I would like to emphasis both the empirical model can be written in p approach, p square approach or mp approach.

And mp approach is more accurate because it accounts all the changes and those happens and those changes happen because the viscosity and compressibility of the function of gas is pressure and the pressure is changing from reservoir pressure to wellbore the pressure is changing not only reservoir pressure to well bore pressure over the time even the reservoir will be changing accordingly pwf will also be changing if we change the flow rate.

So if I summarize the empirical model yes we can use quadratic form or logarithmic form to construct the IPR curve if there coefficient are known to us A and B for the Forchheimer model C and n for the back pressure model.

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And once the equation is known to us in either of the methods we can construct the IPR curve here I have compared theoretical equation and empirical equation so this theoretical equation we are having that is just Q is in a big form where the S and Dq also present in the denominator that

is account for skin effector in the non-Darcy coefficient. So the pressure approach and pressure escort approach are presented here may be mp approach could also be included in this.

But important point is seeing what is the AOF is predicted by both the model there is a last deviation as well as at a particular bottom hole what should be the production rate are represented differently. When they are at early part here it is okay but yes we are going with a higher part flow rate the deviation is more or other way we can say when we are at higher pressure when the changes in the properties are not much we can use the P approach itself there is not a much deviation but as we are going to a lower deviation pressure side the deviation is too much.

The approximation for P and P square approach is completely different and that is where you will end up getting the two different production profile two different AOF condition two different estimation of the well performance and the judgment will be very different so for that empirical correlation should be used. And those empirical correlation give us more accurate data compared to analytical expression because those are based on the field experience.

So for example empirical method for both methods was used to construct the curve knowing the coefficient we can put the different value of the PWF here and can get different flow rate. So for example both the methods are coinciding here and this happening because both are based on the data used from the field test to find out the coefficient that is the where the fully representing the situation.

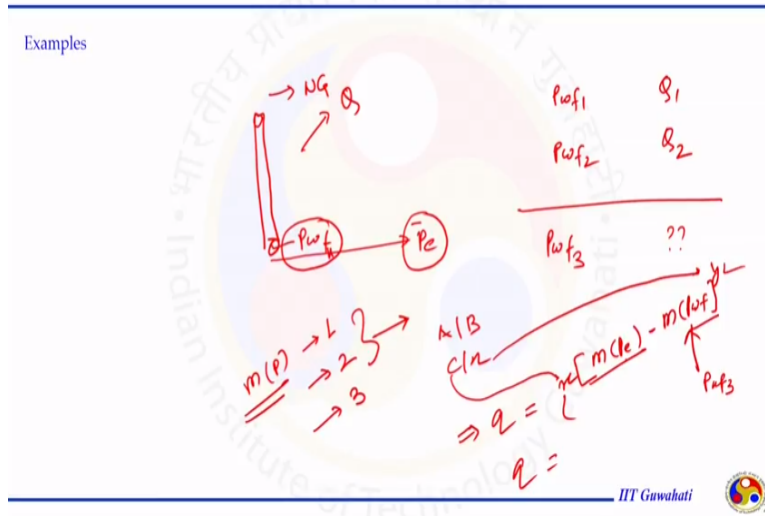
But if you see interesting point here this point is around 1700 and this is around less than 2000 while the other point here is 1000 + with the pressure square approach. But as our reservoir pressure is around 4500 this pressure approach is more close to this. But the nature of curve is the straight line here is the curvature that differentiate that thing we can use the mp approach that is more accurate that we can if we do so we will see the mp approach is more close to or the trend of the change in the flow rate with bottom hole pressure is more close to empirical model.

Those are the Forchheimer model and back pressure model so with the help of this once we can understand the equation we can construct the curve we can predict AOF.

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Pseudo Steady State: IPR- Two Points Test

Examples



So if am having a well which is producing ahhh the natural gas and this is P_{wf} I mean tested in the bottom part only and the reservoir pressure is p_e . So my reservoir pressure is fixed P_e what I did is change this bottom hole pressure to get the Q . So I did this at two condition when I get P_{wf} 1 I got Q_1 when I change I got another condition Q_2 and I was ask when my P_{wf} is 3 what is my Q .

So we can go but P square or mp approach but as I said the mp approach is more accurate we should go with the mp approach and for that we need to calculate mp at point 1 at point 2 or point of interest at this 3. Once we do that thing we can construct that the Forchheimer model or the back pressure model and Forchheimer model we will calculate A and B with the formula given previously and in the with just using these two points and c and n in the back pressure model once the equation is known to us.

Let us see for example in mp form we got this back pressure equation that is simply says I know the value of C let us say non value is X for this C numerical value that is $mpe - mpwf$ and y for the n value numerically i got that is equal to flow rate Q now I can put this third condition $pwf3$ I know this I can calculate the value of q . And i can do with both the models back pressure, Forchheimer and I can check which one is given the most accurate data.

In most of the cases both under giving this similar data again depends on which approach is used mp will give us more accurate data.


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Gas Well Testing

- ❑ A useful tool in to analyze the performance & forecast the productivity of gas wells.
 - static reservoir pressure
 - flow potential of gas reservoirs
 - rate vs pressure depletion
 - to estimate well/formation properties including k, skin factor or damage

- ❑ The results and information gathered during the testing are often used:
 - ❖ by regulatory bodies in setting maximum gas withdrawal rates.
 - ❖ for estimation of gas reserves, and projecting gas well deliveries,
 - ❖ in the preparation of field development program,
 - ❖ in the design of gathering & pipeline facilities, processing plant etc.

- Pressure Transient Test
- Deliverability Test

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So in the next when we will be having the gas well testing lecture we will understand how particular gas well can be put for the test analysis and the test analysis can be done in a pressure transient situation or the (()) (56:06) test situation depend on what is the objective of that well testing it is very important tool because several properties those are not known.

Those are not those cannot be accurately measured like the pay zone thickness the permeability and another parameter can be estimated with the help of this step as well as the production rate what will be future production rate? What will be nature of the curve after certain time can be established with the help of ahh gas well testing. There are several base of performance of testing that we will learn in the next lecture thank you very much.