Introduction to Polymer Physics

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Lecture – 19
Viscoelasticity: Mechanical Models

Hello everyone, in today's lecture we will be talking about the Viscoelasticity of polymeric materials. In the previous lecture we discussed mechanical properties of Polymers. So, today what we will do is look at characteristic property of poly materials which is velocity and where we will see that polymeric materials respond to a deformation in a way which resembles both in elastic solid and a viscous liquid.

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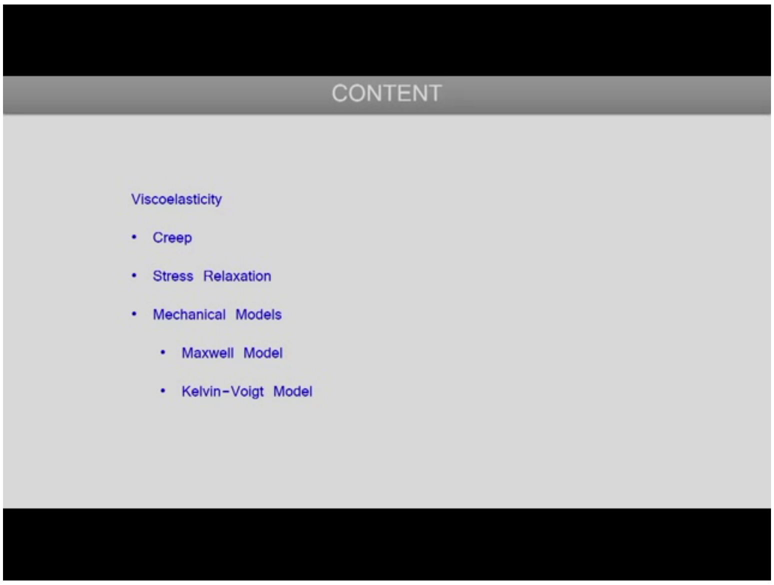

In today's lecture the content will be after an initial introduction to viscoelasticity we will talk a bit about a couple of experimental techniques which are used to study that time dependent mechanical response of such viscoelastic polymeric materials. And, then we will also look at a couple of mechanical models that are simple, but somewhat effective representation of viscoelastic behavior of polymeric materials. It must be said at outset that viscosity viscoelasticity of polymers is complex phenomenon, and of course it is a subject where many advanced text and articles are available.

So, in our discussion today we will the limit ourselves very introductory coverage of this viscoelastic properties of polymers.

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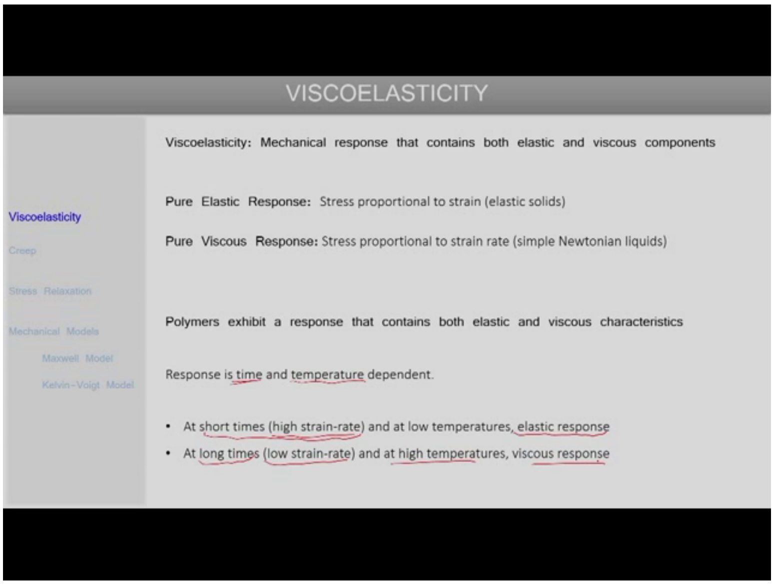

When we say viscoelasticity it refers to a mechanical behavior or mechanical response to load or a deformation where the response actually is representative of both an elastic solid and a viscous liquid. So, the mechanical response of such viscoelastic materials would have characteristics of the response of an elastic solid material as well as that of a viscous liquid material. And apart from that it usually is not just a linear combination of these two responses, but can be more complicated by the fact that the elastic and viscous responses themselves can be coupled in some cases.

So, viscoelasticity is observed in many polymeric materials and most of the polymeric materials and it is. Another way to look at it is a time dependent mechanical response to a deformation. So, if we contrast this kind of a response or behavior to that of a purely elastic behavior or a purely viscous behavior then we will be able to better appreciate the this concept.

So, if we consider pure elastic response, so in a pure elastic response if first stress is applied then the corresponding strain produced in the material is directly proportional to the stress applied or in other words its materially strained the stress is proportional to the strain produced. So, that is what the behavior of a purely or perfectly elastic solid would be.

In contrast, if we have a purely viscous liquid. So in that case the response to any mechanical deformation is that, stress that is produced in a material is a proportional not to the strain applied, but to the rate of strain. So, the rate at which the strain is changing that is the quantity which is proportional to stress for purely viscous substance like simple liquids. Viscoelastic response actually lies intermediate to these two extreme responses and this viscoelastic behavior in response is what we will study in more detail in today's lecture.

The poly polymeric materials in general exhibit response to mechanical deformation which corresponds to a viscoelastic behavior. And response that we have the response this viscoelastic response is seem to be dependent both on the time of observation as well as the temperature of observation ok. So, typically when we subject the viscoelastic material like a polymeric to some deformation or let us apply some load to it to produce a deformation then it seen that the corresponding response of the material it varies with time and it can be vary temperature as well.

And this is something that one would not see in purely elastic solids. For an elastic solid, if a given the load is applied correspondingly the formation will be produced which will not vary with time as long as the load itself is not varying with time, whereas, for a viscoelastic material even if a constant load is applied the corresponding deformation produced can vary with time. So, in general the response varies with time for viscoelastic materials and it varies with temperature as well. In the limit when the time of observation is a short or in other words when the temperature is low. So, in such cases the typically the elastic kind of response is observed in viscoelastic materials. And in the other extreme when the time of observation is large or when the temperature is high in that case viscous kind of response can be expected.

So, this short time behavior which can correspond to high rate of strain applied this typically produces a kind of elastic response. And if in the long time limit when observation is done for a very long time that is in the strain rate correspondingly is low. And also at high temperatures the viscous response is what is observed. So, we see that the behavior of these viscoelastic polymeric materials is intermediate to that of an elastic solid and a viscous liquid.

So, before we move on to discussing some simple models for this viscoelastic behavior of a polymeric materials let us first go ahead and look at a couple of experimental techniques which are used to study the this time dependent mechanical response of the viscoelastic polymers.

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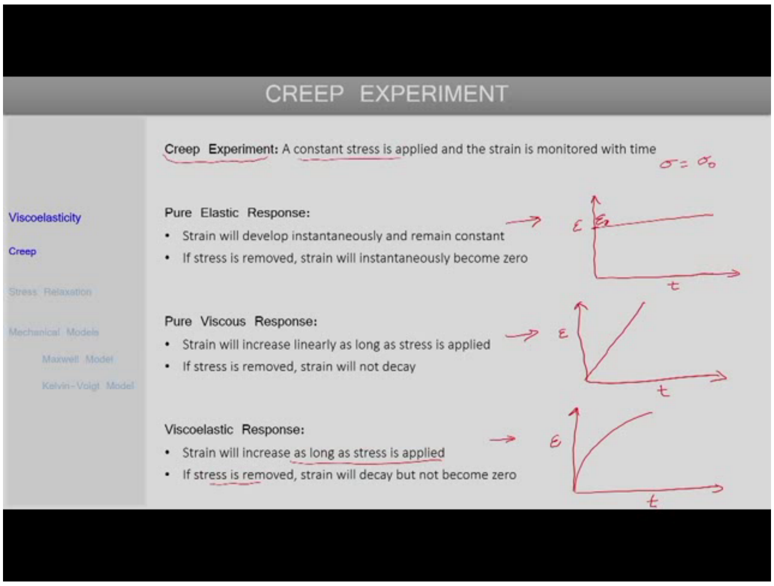

So, the first experimental technique that we will discuss is this creep experiment or creep loading. And here what is done is the sample is subjected to a constant stress and the strain corresponding strain or deformation developed in the sample that is monitored with time. So, for a given constant stress that has been applied how the corresponding strain changes with time. Before, we look at the behavior or response of a viscoelastic material let us first again discuss what how the response of a purely elastic or a purely viscous material will look like.

So, if we have a purely elastic material a perfectly elastic solid, then as soon as this constant stress is applied instantaneously up corresponding strain will be developed in the material and this strain will not change with time. So, since the stress applied is constant the strain developed will also remain constant and will not change with time. So, if we try to plot strain so let is represent our strain by some quantity epsilon symbol epsilon and if we wish to see how it changes with time; then since the applied stress sigma is some constant value. Then, correspondingly the strain produced will also be a constant as at some constant value let us say epsilon naught.

And, let us say if the applied stress is removed then instantaneously the strain in the elastic solid that will also disappear ok. So, there will not be any residual strain or deformation upon removal of the stress applied. So, this creep experiment if let us say if it is done for a perfectly elastic solid, then it will just produce a constant strain because, the stress impose is a constant value. If we on the other hand look at the response of a viscous liquid and how it responds to constant stress that supplied, then in that case the since for a liquid viscous liquid the stress is proportional to strain rate. So, since the stress is a constant in the creep experiment the strain rate will be constant, which means that the strain will increase linearly with time.

So, you in the case of a viscous response so this one is for elastic for the case of a viscous purely viscous response when the applied stress is constant. So, if we monitor strain as a function of time we will see that it increases linearly because, an as long as the stress is a constant stress is applied the strain will keep on increasing linearly. And if let us say the stress is removed or the load is removed at certain point of time, then the strain will not decay to zero; the deformation that has been induced due to the application of that stress that will be permanent. So, if for a viscous material if the constant stress is applied the strain will increase linearly and if the stress has removed at a point of time then whatever strain or deformation that has been induced that will remain it will not decay.

So, that is a pure viscous kind of response; if you look at now a typical response of a viscoelastic material. There what happens is that the response again we can see as being intermediate between a elastic and viscous response. So, what we will see is that as the constant stress is applied to a viscoelastic material the strain actually increases with time, but the increase is not linear. So, unlike a viscous purely viscous material for a viscoelastic material the strain increase will be there, but it will be non-linear with the time.

So, the strain will increase with a the applied stress, but the increase is not necessarily linear and if the strain is stress is removed, in that case the strain can decay in this case there will not be complete partner and deformation of the material. So, due to the elastic part of the strain deformation and that will decay if the stress is removed, but the strain need not decay to a zero value. So, the original again the shape of the material need not be the recovered and partly permanent deformation might be there in the material.

So, if we now try to plot this change of strain with time for a constant applied stress that is of in the case of a creep loading. Then for a viscoelastic material typically the change in strain with time will look something like this it. Initially it will increase, the strain will increase at a higher rate and as time progresses the rate of change of strain actually itself decreases. So, the d epsilon dt actually will decrease with time, but the strain itself will grow with time as long as a constant stress has been applied. If the stress is removed then in that case the strain will the decay or start decreasing, but it did not reach zero ok.

So, some permanent deformation might be there even if the stress is removed. This is the kind of viscoelastic response that one gets for simple polymers subjected to creep load. And, this kind of a loading is important studying this kind of a mechanical response for polymers is important because, there are many applications in which the polymeric material might be subjected to, subject to a certain constant load. And it is important to know what how the deformation in the polymer takes place if it is subjected to a certain amount of load for a long period of time. So, that is why these creep measurements are important.

Another class of experiments are that is important again from the point of view of this time dependent mechanical response of polymeric materials, is what is called the stress relaxation experiments. So, in contrast to the creep experiments where the stress is maintained constant in the stress during the relaxation experiments the constant strain is maintained. So, the material is strained by a certain amount and that constant strain is maintained. And the stress required to maintain that constant strain that is monitored. So that is it seen how that says changes with time.

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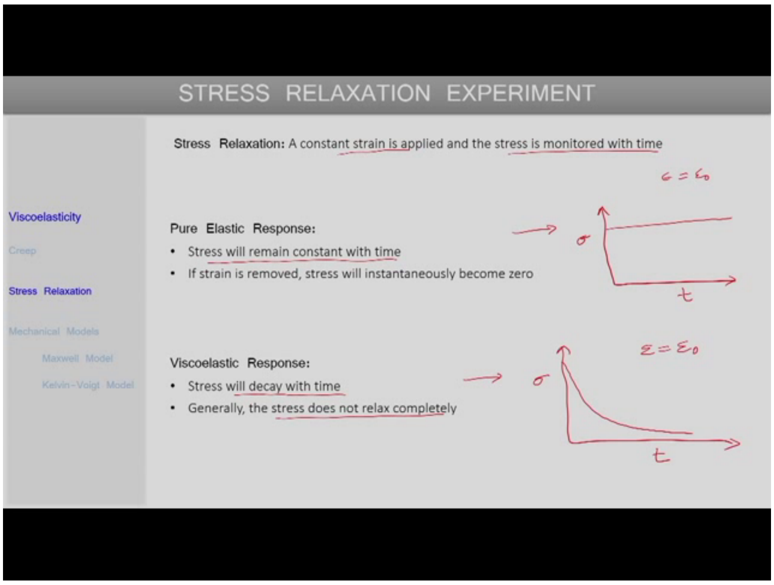

So, for a stress relaxation experiment, a constant strain is applied and the corresponding stress that is monitored with time. Again if we talk about the purely elastic solid so you first let us look at that limit what happens when the stress relaxation kind of condition is imposed on a purely elastic solid. So, for such a solid if we apply a constant strain then, immediately a corresponding constant stress is will develop and as long as this constant strain is applied the same constant stress will be present ok. So, for a constant strain that is applied the stress will remain constant with time and let us say if strain is removed at certain point of time then this stress will again instantaneously become zero. So, that is the response that one will get from a purely elastic solid again.

And we are talking about the response of purely elastic or purely viscous material just to contrast it with that of a viscoelastic material and to emphasize the fact that the viscoelastic response typically is intermediate to these two. But, it is not necessarily a linear combination of these two and it can be more complex coupling of the elastic and viscous response.

So, now if we try to again draw the change in stress with time so, now since the strain has been maintained constant at some value epsilon equal to epsilon naught then for a purely elastic response the corresponding stress will also stay constant with time. It will not change very with time and if at some point of time the strain is removed the stress will also come down to zero again. If we have a purely viscous material then, since the constant strain is applied there is no strain rate there the strain is not changing with time. So, correspondingly there will not be any stress. So, initially as the constant strain is applied at that very instant sharp striking in the stress might be observed, but immediately it will decay to zero because, the strain remains constant after that so strain rate is zero and stress also will be zero. If we now look at the response of a viscoelastic materialso, in that case what happens is a strain is a constant strain is applied, the stress that develops in a material that slowly decays to zero.

And stress actually, but for a typical viscoelastic polymers does not decay all the way to zero it will decay to a small value, but some residual stress might still be present even after long times. So, if we look at the viscoelastic response as we discussed stress decays with time, but the stress need not relax completely. What that means, is that the stress need not go all the way to zero even at long times. So, if we again draw sigma versus time or stress versus time, with if a constant strain is applied as is an inner stress relaxation experiment.

So, the stress will at some initially will be at some value and it will decay with time, but even after long times some residual stress might be present. So, this is the kind of behavior when a constant strain is present. So it is not that the strain has been removed at any point of time; even in the presence of a constant strain throughout the stress in the material actually reduces with time, decreases with time unlike an elastic material where the stress remains constant with time.

So, here the stress reduces with time and, but even at the long time some residual stress might still be present so that is a viscoelastic response. So, these are a couple of important experimental measurements done to characterize the mechanical response of this elastic material especially the time dependent mechanical response. Some other kind of experiments are also typically done where the material is exposed to some kind of an oscillating stress or strain where the stress or strain might be varying sinusoidally with time and the corresponding response of the material is observed.

So, such experiments basically come under the class of dynamic mechanical experiments and in the next lecture we will briefly look at some of the some such experiments and the corresponding mechanical response of polymers under oscillating strain or stress conditions. But today from here what we will do is move on to the discussion of a few simple mechanical models of viscoelasticity. So, we will restrict ourselves to what is called linear viscoelasticity.

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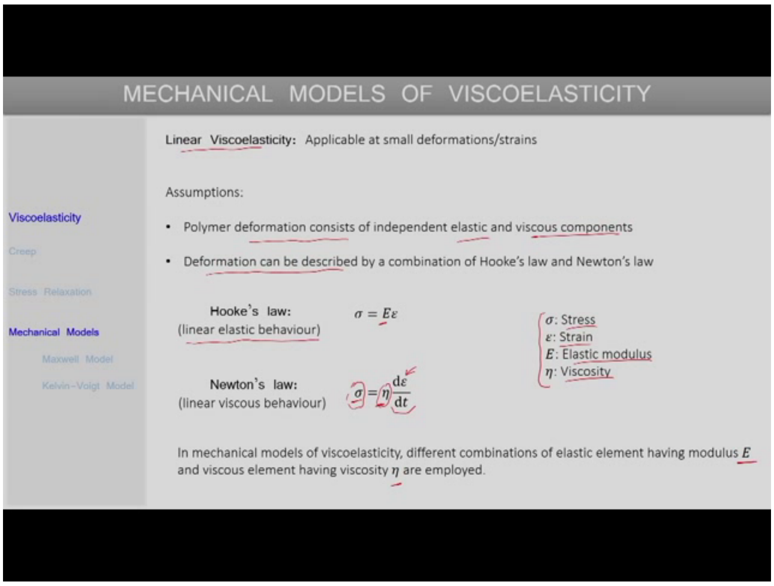

In that whatever elastic and viscous response that we are considering to and combining to give the risk elastic response the individual elastic and viscous responses themselves will be considered linear. And a direct linear kind of combination of these two responses will be assumed to describe the viscoelastic behavior of the material. So, this kind of linear viscoelasticity if normally is valid only when the applied strain or deformation is quite small and as a strain or deformation becomes large this linear kind of this linear assumption will fail. So, since we our discussion is limited to simple cases in simple systems, we will talk only about models describing linear viscoelasticity.

So, the assumptions involved here in this linear viscoelasticity is that or the mechanical models that we will be discussing is that deformation of the polymer consists of an independent contribution from elastic kind of response and a viscous kind of response. So, an elastic and viscous component will in independently contribute to the overall response of the viscoelastic polymer material. And, another simplification that we will make is that all the deformation of this viscoelastics in the material that will be described by a combination of Hooke's law which is valid for linear elastic materials and Newton's law which is valid for linear viscous materials. So, we will combine these two mathematical simple mathematical laws and try to describe the behavior of viscoelastic materials.

So, if we talk about Hooke's law this is something that we discuss in the previous lecture as well; it is a simple law that just relates the stress to the strain through a linear kind of relationship. So, Hooke's law says that the any stress is proportional to strain and this proportionality constant E is refer to the elastic modulus. So, if the stress strain that we are talking about the other tensile stress or strain typically this will be the corresponding Young's modulus.

If we are talking about the shear stress the shear strain then even in that case if we can talk out the kind of Hooke's law that linearly relates the shear stress or shear strain and in that case the elastic modulus will be the corresponding shear modulus. So, here the E that we have it we are not limiting it to any particular specific modulus, but depending on the type of deformation involved it can be the elastic Young's modulus for tensile kind of deformation or it can be the shear modulus for shear kind of deformation.

So, we know that this Hooke's law describes linear elastic behaviors of course, our models will only be only be valid where this linear kind of behavior is expected to be valid. Similarly for a purely viscous kind of a behavior one can use a Newton's law which again describes a kind of linear viscous behavior and here the stress is proportional to the strain rate ok. So, the rate at which the strain epsilon is changing with time, that is what the stress depends on and the proportionality this proportionality constant in this case that is referred to as the viscosity of the material.

So, the corresponding symbols are epsilon sigma stress and strain is elastic modulus eta is the viscosity. The aim of for the any of the simple mechanical models is to try to combine these two simple laws describing linear elasticity and linear viscous behavior. And, to come up with a combination that can give a reasonable description of viscoelastic behavior.

In the mechanical models of viscoelasticity that will talk about the elastic component or elastic element is typically represented by what is called a Hookeian string. So, a kind of a string which has a constant elastic modulus E, the viscous element will have a constant viscosity eta and the viscous element is actually is normally represented by a what is called a dashpot, which is kind of a damper kind of a system containing a liquid having viscosity eta.

So, typically these mechanical models comprise string as elastic element and simple dashpot as a viscous element. We will the talk about a couple of these simple mechanical models and what we will see is that the combination of the viscous element and the elastic element in different ways that can give us these different models and correspondingly that can produce different kinds of responses. And, then we can see how well the responses of these models compare with the response of actual viscoelastic polymers.

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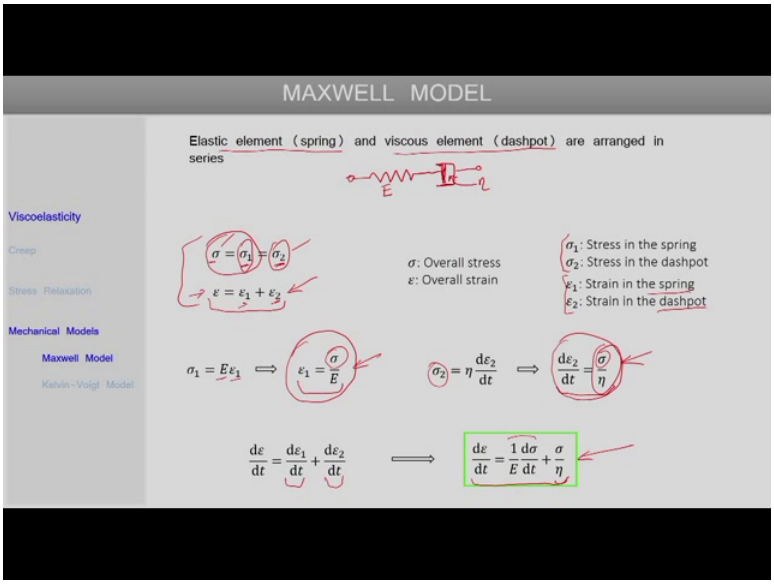

So, if connect our elastic element which is a string and the viscous element which is a dashpot in series. So, that is what gives us the Maxwell model of viscoelasticity or Maxwell model of linear viscoelasticity. So, if we typically the spring will be represented by something like this and then we will have a kind of a dashpot that is represented by this symbol. And, these two combined in series and this spring will have the elastic modulus E, the liquid inside the dashpot will have viscosity eta. So, this combination is what is called the Maxwell model of a viscoelasticity.

So, a elastic element and the viscous element combine in series. So, now, let us say for this model if overall stress sigma is applied and correspondingly overall strain epsilon is produced then what we can say is that the sigma overall stress will be equal to the stress in the elastic element that is the string and that will also be equal to the stress in the viscous element which is a dashpot. So, sigma 1 sigma 2 represents the stress in the string and dashpot the epsilon 1, epsilon 2 represent the strain in the spring and dashpot.

So, epsilon in this case will be additive because the system the two elements are in series. So, the strains produced in the individual elements epsilon 1 and epsilon 2 when they are added they will get that will give the overall strain produced in this entire Maxwell model ok. Whereas, the stress the overall stress that is applied that will be identical to the individual stresses present in the two elements because of the fact that the two are in series or assumed to be in series. So, if we start with these considerations which is due to the fact that we are using a series model, then we can for the sigma which corresponds to elastic element we can describe it by the Hooke's law and sigma 1, that is, and sigma 2 which is the viscous element that can be described by the Newton's law of viscosity.

So, if we do that sigma 1 we can write as capital E times epsilon 1 where E is the elastic modulus and this we can just rewrite as epsilon 1 equal to sigma by E. And here we have removed the subscript 1 from sigma, because we see that sigma and sigma 1 they are the same they are equal. Similarly for the viscous element we can write the Newton's law and again rearrange that, and again instead of sigma 2 we have just written sigma here and removed the subscript, because sigma 2 is also equal to sigma.

Now, we have these two expressions. Using these two expressions and differentiating these expressions with respect to time and by combining all of them, we will develop the equation for the Maxwell model. So, first let us differentiate this expression with respect to time. So, if we do that we get that the rate of change of overall strain is just the sum of the rate of change of strains in the two elements. And this first one d epsilon 1 dt that we can obtain from the Hooke's law expression directly by simple differentiation. And this second term here is directly given by the Newton's law for the viscous element.

So, if we combine the two; then this is the expression that we get and this first term as we discussed comes by just differentiating this expression, and the second term is what we have on the right hand side here. So, this is the equation describing the Maxwell model of linear viscoelasticity. And next let us try and see how this model how good is this model in predicting the mechanical response of actual viscoelastic polymers ok.

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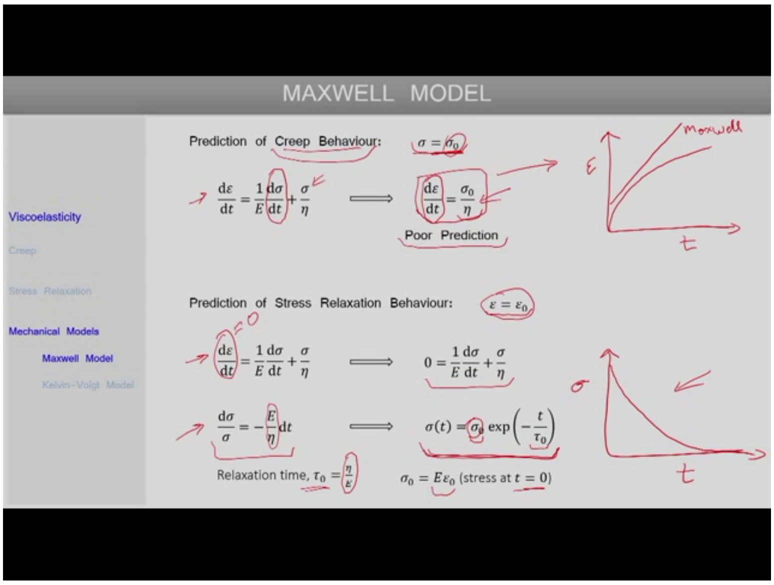

So, for the Maxwell model let us first see under creep kind of condition or creep loading condition, what kind of prediction does a Maxwell model get and whether that is an acceptable kind of prediction or not. So, for creep behavior we know that constant stress sigma naught is imposed on the system. So, if we start with our equation for the Maxwell model that we discussed in the previous slide. Then, since then the stress now is constant at sigma naught this time derivative of stress under creep condition will be 0. So, the maximum model will simply reduce to this equation here and instead of sigma we have written sigma naught because, the stress is kept constant during a creep experiment.

And we see that this rate of change of strain with time that comes out to be a constant for from Maxwell model. So, maximum model predicts that the strain will increase at linear rate with time ok. So, d dt is a positive constant and this kind of prediction is actually poor prediction for the creep behavior of actual polymeric materials. So, if I draw this the kind of prediction that we are getting from this equation epsilon versus strain versus time, then Maxwell model is predicting behavior like this where epsilon is increasing linearly with time. So, this is the Maxwell model prediction.

And the actual creep experiments done on the viscoelastic polymeric materials the response produced in such cases is where this actually epsilon is increasing with time, but the rate of increase of epsilon that actually reduces rate time. So, initially the rate at which epsilon increases is high and later on it goes down. So, this Maxwell model is not a very good model for predicting creep behavior of polymer viscoelastic materials. And next lecture see how will it predicts the stress relaxation behavior. So, we know that in the stress relaxation experiments the constant strain is imposed in the system and the relaxation of that stress or how the stress is changing with time that is monitored. So, in stress relaxation we will say that epsilon is being kept at epsilon naught a constant value. And then again starting with the Maxwell model equation, if we substitute this constant value for strain here then this term actually becomes 0 because, the strain is constant so it will not change.

So, in that case the expression reduces to this and which we can easily rearrange in the form of this where we have separated the terms containing sigma and taken it to one side and the remaining terms are on the other side. So, this equation or can be simply integrated an integration and again and simplification will lead to a final expression that looks something like this. So, that the time dependence stress that actually is given by constant stress value sigma naught which is the initial stress multiplied by an exponential of minus t by tau naught ok. So, we see an exponential kind of decay of this stress with time in the prediction of Maxwell model.

Here there before we actually see how this stress relaxation behavior looks like for a Maxwell model let us talk about this tau naught. So, upon comparison one can say upon comparison with especially this term when what one can say is that this tau naught is nothing, but this ratio of viscosity and the elastic modulus eta by E and for them this is a relaxation time for the Maxwell model. And, what it signifies is that if we are talking about time scales much smaller than a relaxation time then at those time scales Maxwell model will behave like an elastic solid. And, if the time scales involved are much larger than the relaxation time then the viscous kind of behavior will be more dominant for Maxwell model. So, that is the significance of the relaxation time that we have.

The sigma naught that we have here, that is just the initial stress at t equal to 0 and that we can write as just, the elastic modulus times the strain which is constant in the case of stress relaxation experiment. So, if I plot again stress versus time, so in the stress relaxation experiment strain is constant stress is where stress is monitored with time. So, what we will see is that the Maxwell model predicts a kind of exponential decay with time and at long enough times or at almost time tending to infinity the stress will decay to 0 ok.

So, what we see is that this stress relates the relaxation behavior this behavior actually as described by the Maxwell model is a somewhat similar or to a first approximation it can capture the kind of trend and behavior that is shown by actual viscoelastic polymers. So, what we can conclude is that for maximum model the stress relaxation predictions are reasonable. The prediction of creep kind of experiments they are not acceptable ok. So, the creep the predictions are poor whereas the stress relaxation predictions are reasonably ok.

So, in the Maxwell model the two elements are combined in series. So, in the previous case we had combined them in series. Now, the natural alternative would be to see what happens if the two elements are combined in parallel and that is what the second mechanical model that we will be studying talks about.

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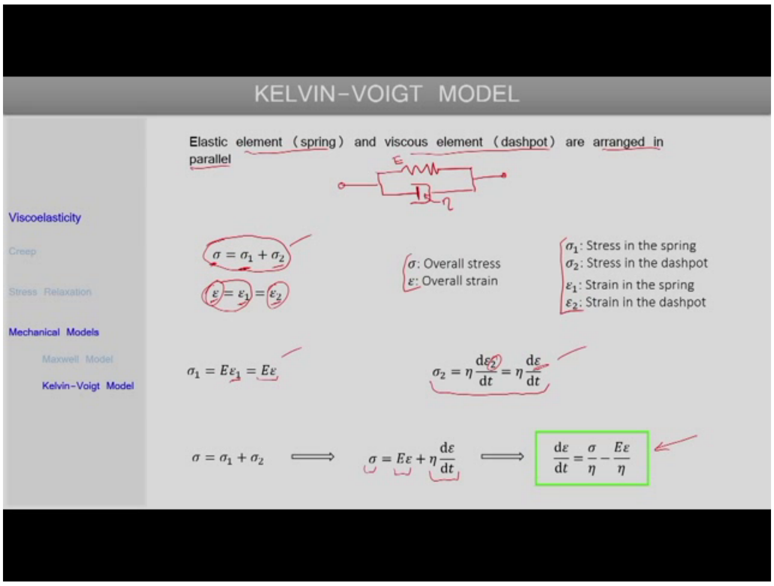

So, in this model which is referred to as the Kelvin-Voigt model or many times it is also referred to simply as a Voigt model of linear viscoelasticity. So, here the elastic element or that is the string and the viscous element that is a dash part they are arranged in parallel. So, in Maxwell they are there they were arranged in series now, here they are arranged in parallel. So, now based on again this arrangement let us see. So, you the arrangement schematically one can represent something like this. So, again the spring will have elastic modulus E En, the dashpot will contain a fluid having viscosity eta. So, this is the kind of arrangement that we have in the Kelvin-Voigt model. And, based on the assignment we can again try to establish or identify the relations between the overall stress and strain and the corresponding stress and strain in the individual elements ok.

So, if we do that in this case since our arrangement in is in parallel, the overall stress actually will be the sum of the stress in the first and the second element whereas, the overall strain in will be equal to the strains in the individual element. So, epsilon will be equal to epsilon 1 and epsilon 2 because, they are in parallel. So, any strain produced overall strain that will be the same as a strain produced in the individual elements. So, the nomenclature is the same as before and again we can apply the Hooke's law for the elastic element and Newton's law for the viscous elements. So, if we do that Hooke's law gives us sigma 1 equal to the elastic modulus times epsilon 1, which you can just write as capital E times epsilon; dropping the subscript because epsilon and epsilon 1 are equal.

Similarly, for the viscous element Newton's law of viscosity can be written. So, sigma 2 we can write as eta d epsilon dt; again the subscript 2 has been dropped here because epsilon 2 is also equal to epsilon ok. So, if we combine these two so now the overall stress we see is the sum of the individual stresses sigma 1 and sigma 2. So, using this equation in combination with the Hooke's and the Newton's law what we can do is write them together. And that gives the overall stress as the stress in the elastic element plus the stress in the viscous element and just rearranging the terms we get the typical equation that is there for this Kelvin-Voigt model.

And here the rate of strain or strain change in strain with time that d epsilon dt that is equal to sigma by eta minus this elastic modulus E times epsilon which is the strain divided by again eta. So, this is the Kelvin-Voigt model equation and again let us see how this model and now predicts the response when under creep conditions and under stress relaxation conditions.

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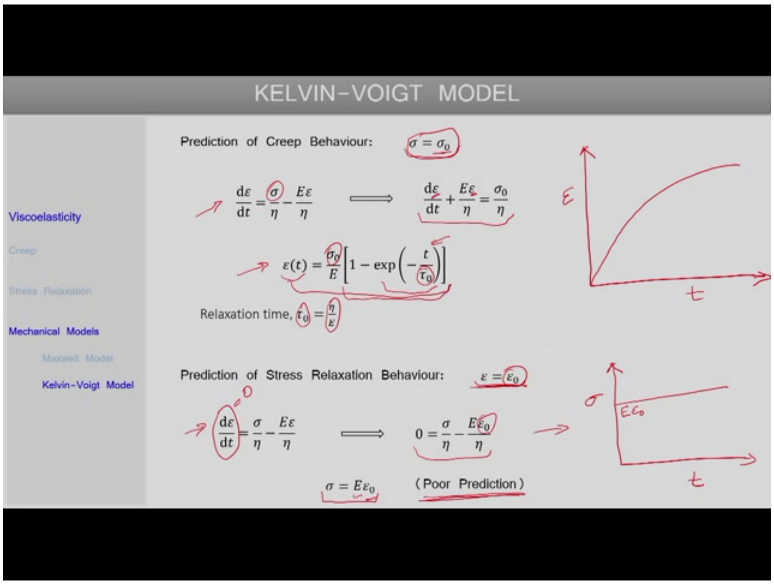

So, if we first look at the creep behavior as tracked by Kelvin-Voigt model; so under creep conditions the stress is maintained constant at sigma naught. And if we use the Kelvin-Voigt model equation here then this stress sigma that will be equal to a constant value the sigma naught in this case and if we do that and rearrange the terms the equation that we get is of this type. And we can see that this is actually an ordinary differential equation of first order in epsilon. These kinds of equations can be easily solved by the integrating factor technique.

So, if the this equation is solved then the corresponding epsilon as a function of time is obtained as this expression. So, here tau naught is the relaxation time which is again eta by E and sigma naught is the constants stress applied during the creep experiments. So, we see that if we look at this equation and see how it describes the creep response then we see that at time t equal to 0 if time t is 0 then epsilon will come out to be 0 in this case because, at t equal to 0 the exponential term this term will be 1. So, the bracketed term will become 0 and the corresponding strain will also be 0.

So, at t equal to 0 the strain will be 0 and as time increases the strain will increase, but not linearly it will increase in a kind of exponential fashion. So, if we draw again the change of strain with time. So, in a creep experiment how it is straight for a corresponding strain is changing with time. Then the Kelvin-Voigt model actually predicts kind of behavior that looks like this. So, we see that at least the Kelvin-Voigt model is able to predict the fact that the strain is not linearly increasing with time, but it is a the rate of increase of strain with time actually decreases as time progresses.

So, an initial part the strain increases rapidly with time, but as time progresses the rate at which the strain is increasing that also slows down and ultimately tries to reach a constant value. So, what we can comment here is that the Kelvin-Voigt model gives a better prediction of the creep behavior of a viscoelastic polymers than the Maxwell model. If we on the other hand try to see how the stress relaxation behavior is directed by the Kelvin-Voigt model and noting that in the stress relaxation experiments the strain is maintained constant and epsilon naught.

Again, then again if we write down the Kelvin-Voigt model equation and identify this term to be 0, because the strain is now constant. So, in this case the equation simply reduces to this and this epsilon has been changed to epsilon naught due to the constant strain. And, if we rearrange the items ultimately we get sigma equal to the elastic modulus times epsilon naught.

So, Kelvin-Voigt model actually predicts the stress to be a constant which is a purely elastic kind of response. So, we see that under stress relaxation conditions the Kelvin-Voigt does not capture the viscosity response at all, it just predicts the elastic kind of response. So, if I draw the stress versus time here in that case Kelvin-Voigt model that is just predicts a constant value of stress with the value being equal to the elastic modulus times the constant strain imposed.

So, this prediction from Kelvin-Voigt model is actually quite poor because, it does not describe the stress relaxation behavior in actual viscoelastic polymer at all where the stress actually decays with time. So, overall what we can say is that Maxwell model is provides a decent description of the stress relaxation behavior of viscoelastic polymers whereas, it fails to capture the creep behavior properly. On the other end the Kelvin-Voigt model captures the creep behavior of viscoelastic materials to a good approximation. However, it completely is unable to capture the stress relaxation behavior so, both models have their advantages and disadvantages.

So, a natural extension of these mechanical models can be to somehow combine these two models the Maxwell and the Kelvin-Voigt model in such a way that both the creep responses as well as a stress relaxation response are captured to a good approximation for viscoelastic materials. So, one such combination that has introduces is proposes what is called the standard linear model, in that what is done is to a Maxwell element. So, Maxwell element will actually include a elastic string and viscous dash dashpot in series. So, if we consider these two together in series and if we add another string to it in parallel; so, in that case this represents a standard linear model. And, this kind of this addition of this parallel string or a parallel elastic element to the Maxwell model that actually leads to an improvement in the prediction of creep behavior by this model.

So, this standard linear model is a relatively simple model which can capture both the stress relaxation and creep behavior. Of course, if we can combine more elements in different kind of fashions to create more types of mechanical models. And of course, as we increase the number of components in the model the mathematical complexity increases. A general kind of model is what is called the mechanical model, what is called the generalized Maxwell model where many the Maxwell elements are actually are combined in parallel. So, a maximal element which is a elastic and viscous elements in series many of them are combined in parallel to create a generalized Maxwell model ok.

So, many such mechanical models are there and they have utility of up to a certain point in that, they can describe the mechanical response of polymers, but a shortcoming of these models is that since the models have been developed from purely from by mechanical concentrations. So, there is no molecular insight present. So, these models actually cannot give any information about the molecular level rearrangements happening in the polymer. And, what are such rearrangements are relaxations that correspondly correspondingly produced a viscoelastic response.

So and no molecular level insight can be gained from these purely mechanical models so, that is one disadvantage. So, we will conclude this lecture here where we have studied two very simple mechanical models of viscoelasticity or rather linear viscoelasticity. And in the next class what we will do is, in the next lecture what we will do is to talk a little bit about the response of our viscoelastic response to oscillatory stress or strain being imposed on the system.

So, if we have a sinusoidal stress or strain, what can a viscoelastic response it can produce and that comes under the domain of dynamic mechanical experiments or dynamic mechanical analysis.

We look at that in the next lecture and we will also briefly take a look at the fact that the polymeric liquids like solutions and melts, when they are flowing they typically do not show Newtonian behavior and many times the behavior is non Newtonian. So, the stress is not linearly related to the rate of strain. So, we will look at the non Newtonian behavior of this polymeric liquids say, in some detail as well in the next lecture.

Thank you.