Multiphase Flows
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Lecture - 09
Pressure Drop Calculation for Homogeneous Flow

So, welcome back last class what we have done we have derived the formula for Navier stoke equation and we have reduced the Navier stoke equation for the 1 dimensional flow, now what formula we have derived is dP by dx or you can say minus of dP by dx, if you are writing phi minus initial will be equal to m naught dV upon dx, if you are writing in the though in the x direction plus P upon A tau w minus rho into g ok.

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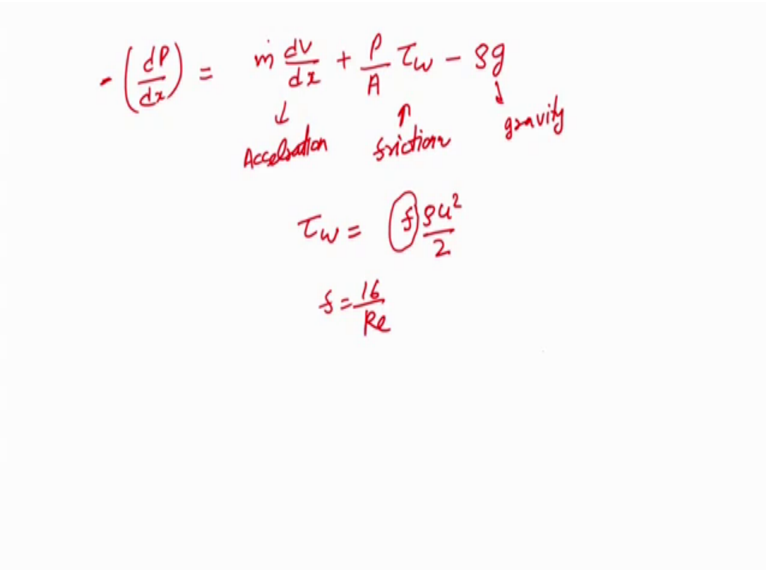

Now, if you are writing in one direction it can be gs, I can write rho g plus and minus is just showing the direction of the gravity. So, in this case this is what we have said that the dP by dx the single phase flow is actually can be reduced in 3 parts, one is the acceleration term delta V due to acceleration, another is your frictional term or viscosity term custom friction and then gravity term.

So, what we have said that this is what the delta P equation is if you want to find dP by dx in a pipeline for the single phase flow this is what the delta P will be the contribution will be from acceleration fluid acceleration from fluid friction and because of the gravity field. So, all these 3 fields will play their role and you will see the combined effect that will be in terms of the overall pressure gradient across the pipeline or across any column for the single phase flow.

We have discussed that; what is the problem the problem comes with the tau w the frictional term and tau w we have written as f rho u square by 2. So, we said that the problem is with the friction factor how to find it for laminar flow f equal to 16 by Re and for turbulent flow we said that you have in the Colebrook equation, Blasius equation all you have to go to moody chart depending upon e upon d value or K upon d value which is K is nothing, but the roughness factor you will get the different value of the f. So, that was the challenge in the single phase flow.

Now, what we are going to do now is to start writing the equation for the multiphase flow and I will start with the gas liquid flow and we will see that how to write the equation or how to change this equation for the different flow regime. Now, to start with the first flow regime which we have studied is the homogeneous flow regime homogeneous flow.

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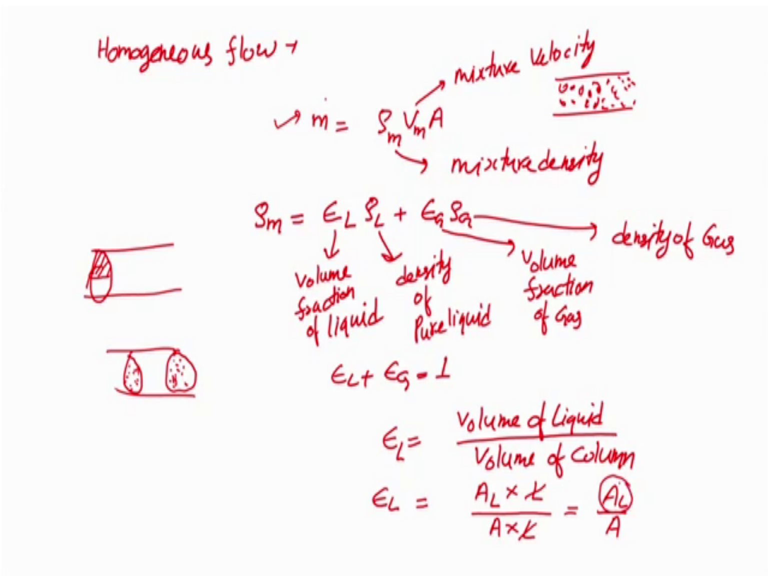

Now, what does homogeneous flow? Homogeneous flow is where both are homogeneously mixed or you can also say it is a mixed flow ok. So, it means both the phases gas and liquid are homogeneously mixed inside and they are flowing together. So, how the whole equation will be modified in case of homogeneous slope?

Now, if you think about the homogeneous flow then what is how we will write the mass balance? So, the mass balance m naught will be written as rho into V into A, now we have this is for the single phase flow. Now we have 2 phases. So, how the rho will be there and if both the phases are homogeneously mixed, it means suppose this is the pipe line both gas and liquid are homogeneously mixed and they are flowing together so, because they are comb now behaving as a mixture. So, I can reduce it in terms of the mixture density. So, I will write it rho m where rho m is nothing, but the mixture density ok.

So, it means what instead of writing the equation 2 equations for both the phases why can write the equation in terms of the mixture density, mixture velocity or it because both the phases are homogeneously mixed, instead of considering them as a 2 different phases I can consider them as a single phase and I can take the property which will be the property of the mixture. So, the overall mass balance equation or continuity equation we can write it as a rho m into V into A and V is what? V will be of mixture velocity. So, V so, this will be the mixture velocity.

Now, the problem is how to find the mixture velocity? How to find the mixture density? If we know that then we can find it out we can write the continuity equation at least. So, for mixture density; how we write? We write that rho m of it means the mixture density is nothing, but epsilon of L into rho L plus epsilon of G into rho G ok.

So, it means volume fraction of liquid phase inside the pipeline multiplied by the density of this is the fraction or volume fraction, so fraction of liquid this is density of pure liquid. Similarly, this is volume fraction of gas and this is density of gas it means if I know the volume fraction of the liquid, if I know the density of the liquid, then I and if I know the volume fraction of gas and density of gas I can find it out the rho m and we know that the formula one more and that is epsilon L plus epsilon G this is equal to 1.

So, it means if I know either epsilon L or epsilon G, I can find it out that what will be the and if I know the density of pure species or pure liquid and gas then I can find it out the rho m.

Now, the problem is that how to calculate the rho m? How to calculate the epsilon L? So, we know that if volume fraction is being defined say liquid volume traction is defined as a volume of liquid upon volume of reactor or I will say column, say a pipeline ok. So, this is the way we have already introduced the volume fraction epsilon L and that is nothing, but the volume of that phase divided by the total volume. So, that is the way we can find it out that epsilon L.

Now, volume of liquid is what suppose, the cross section this is a pipeline and a particular fraction or this is suppose the area of the pipeline and particular fraction in the area has been occupied with the liquid. So, in that case I will say that to simplify it I can say this is A L, it is the area of liquid which has been occupied multiplied by the length of the pipeline divided by the total area total volume. Total volume will be what area cross sectional area into L, it means because they are homogeneously mixed I am assuming that a particular fraction in the pipeline or particular area fraction has been occupied by the liquid and that is remaining same throughout the length of the reactor..

So, this will be A L into L, now this L can change with the length of the column, but that overall value of A L remains same, it means if I am saying that 60 percent of the area is being occupied by the liquid that 60 percent may change. So, that 60 percent can be distributed like this, that 60 percent somewhere can be distributed like this, that 60 percent somewhere can be distributed like this. So, that distribution may change over the length of the reactor, but that A L remains same, the fraction of the area which is being occupied by the liquid is remain same.

So, I can write it epsilon L by this or I can write it as A L upon A if length will be cancelled out you will see A L upon A ok. Similarly you can write epsilon G.

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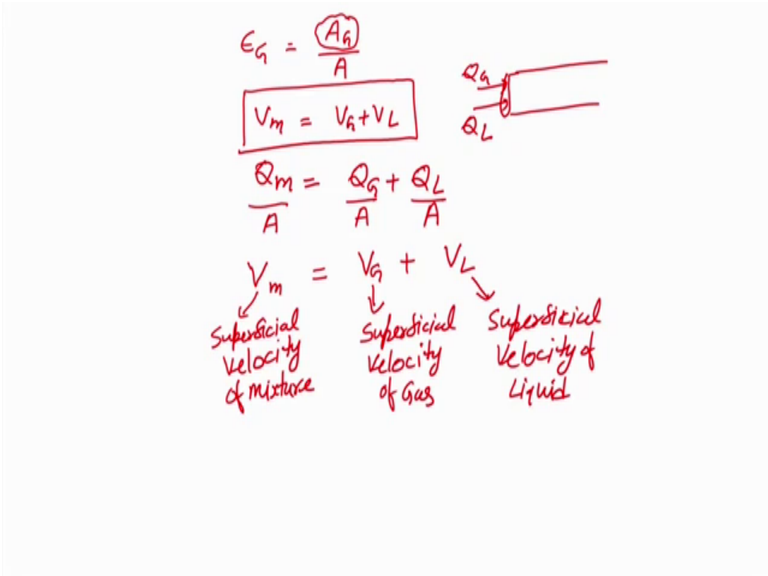

Similar way and epsilon G will be nothing, but the A G upon A. So, if I know the fraction of the area which is being occupied, I can find it out epsilon L and epsilon G, because area we know already that is the pipe area.

Now, the problem comes with the mixture velocity so, Vm. Now, Vm is what Vm will be nothing, but it is the flow rate of or superficial gas velocities of liquid plus superficial gas velocity of the gas. So, why this way we can write because total mixture flow rates say Qm if in the pipeline we are in sending gas and liquid say gas flow rate is Q G, liquid flow rate is Q L, then Qm mature flow rate will be what Q G plus Q L ok.

Now, if I want to find it out the mixture velocity in terms of the superficial velocity because that is only mixture is which is being filled inside the column, what I can do? I can divide it by the area ok. If I divide it by the area what I will get? This will be Qm upon A, it means flow rate mixture flow rate divided by the cross sectional area, it will give me the Um or Vm. Let us denote it with the V, because we are following notation V.

So, this will be Vm, where Vm is what nothing, but the superficial and I hope how you know that how to define the superficial velocity superficial velocity is nothing, but the volumetric flow rate divided by the area of empty column it means you are not concerning any internals, any other thing, only the area of the empty column we divide with the volumetric flow rate we get that superficial velocity that is why this team is called superficial.

Similarly, if you Q G is the flow rate we are knowing that how much of the gas flow is going in gas volumetric flow rate, this will be written as this will be equal to V into G, where V G is nothing, but superficial of gas. It means what? We are assuming only gas is flowing inside the pipeline what will be the velocity of the gas, that is called superficial velocity of the gas. We are neglecting the liquid fraction; it means we are assuming the whole pipeline is only filled with the gas.

Similarly, this will be this term will be equal to V of L, where V L is nothing, but superficial velocity of liquid, it means we are assuming that only liquid is filled inside. So, it means what we have found the correlation for the Vm, Vm is nothing, but V G plus V L and if we know that what is the volumetric what is the volumetric flow rate through which the gas is flowing inside the pipeline, if I know the volumetric flow rate of the liquid, I can find out the mixture velocity. And mixture density I can find it out if we know that A Z upon L, then we will know that mixture velocity.

Now, in that case what will happen we can write the continuity equation and we can solve it. So, at least the continuity equation portion we can solve, the only problem is to find that A L and A G. These are the 2 problems to find that how to find what is the fraction of area which is being occupied by the gas? What is the fraction of the area which is occupied by the liquid? So, if we know that we can at least solve the continuity equation.

Now, if you write the momentum equation or velocity equation for the homogeneous model, what I will do I will just write first.

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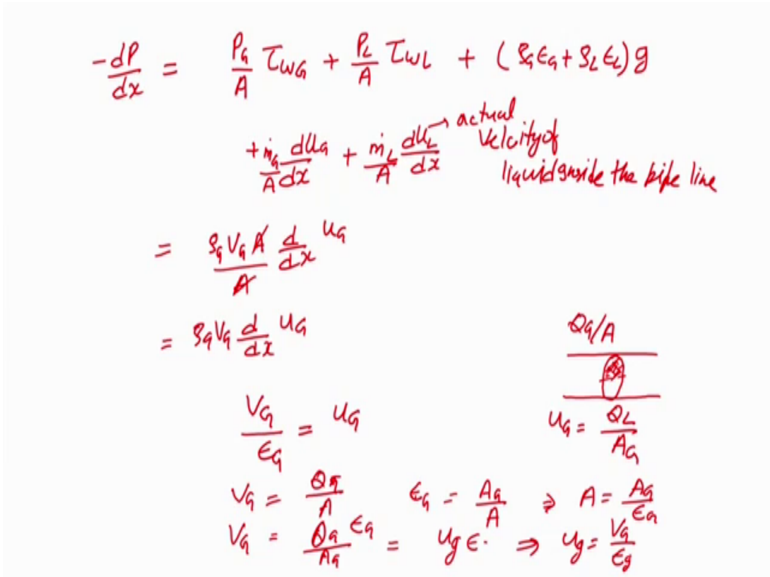

The dP by dx, this will be equal to minus in terms of now I am writing first for individually 2 phases and both are flowing together. So, I will just reduce the term, this will solve the single phase single term we have written, now we will I will write the 2 terms and then we will try to combine them it means what I will write it the tou P G instead of this I will write P G upon A, it means perimeter of the gas which is the parameter which is being occupied by gas upon tau W G, it means wall shear stress because of the gas plus I will write P L upon a tau W L ok. It means wall shear stress because of the liquid plus the gravity term, gravity term we will write it as a rho G epsilon G plus rho L epsilon L into G because we have to write the mixture viscosity mixture density instead of mixture density I am writing these 2 terms ok.

So, this will be the rho G epsilon G plus L and then we have to write the momentum term or acceleration term and acceleration term is nothing, but we will write it as a d by dx we are writing it in terms of the V of gas ok. Ideally it should be what? The liquid velocity inside will write it as a V G, right now if you write it it into the V G you have to write the V into dV upon dx. So, we were writing it in terms of V or m naught of gas into dV G upon dx plus m naught of liquid into dV L upon dx ok.

So, you can write it out what is the mass flow rate of the gas which is coming inside? What is the mass flow rate of liquid which is coming inside in terms of the vl ok. Now, this should be ideally what it should not be the superficial velocity, it should be the inside velocity and because it is inside velocity I am writing going to write it in in terms of not with the V, I am going to denote it in terms of the U and this U L ok. Which is U L and U

G shows the velocity of the gas and liquid inside ok. So, this is the actual velocity liquid inside ok. Similarly, this will be the actual velocity of the gas inside the pipeline.

Now, what we can do? We can work on these terms these 2 terms particularly. So, I will write it as this term and not G, I can write it it in terms of what it will be nothing, but rho G into V G into area the V G will be nothing, but the superficial velocity ok, now d by dx of U G you can write it it in this way.

Now, if I divide it by the area then what will happen? Because we have already divided by the area this is m naught upon A, this is upon A. So, this A and A will be cancelled out so, what you will get is rho G into V G now we can write it in terms of this as a d by dx rho G into V G upon U G.

Now, we have already said that that what is the U G and U correlation V G correlation? So, we can write it out V G upon epsilon is nothing, but U G ok. This we have already discussed it means what the inside velocity will be nothing, but the superficial velocity divided by the volume fraction of the gas inside the pipeline, that will be what the inside velocity of the gas you will see because this is what vg is what superficial velocity.

Now, superficial velocity how we have derived this we have already discussed, but just again one for the revision; so, this is suppose the pipeline what we have assumed while calculating the superficial velocity which is V G, V G is nothing, but Q G upon area and we have assumed that complete area is being filled with the gas.

Now, U G is what? U G is Q G upon area of gas ok, now area of the gas what we know that that this is the cross sectional area some portion is being filled by the liquid say and some portion is being filled by the gas. So, what will be you can write it you can write that A we have already seen that epsilon G is nothing, but we have already seen that epsilon G I have just shown here that epsilon G is nothing, but A G upon A. So, it means what A you can replace it as A is nothing, but A G upon epsilon G ok.

So, you can write it in terms of what now if you reduce this equation it means what you can write V G which is nothing, but Q G upon area now area can be written as Q G into A G into epsilon G. Now, this value is what Q G into A G is equal to U G into epsilon G is equal to what V G. So, you can write it out you G will be nothing, but V G upon epsilon G ok.

So, by using that what if I can do I can change this differential and I can go this.

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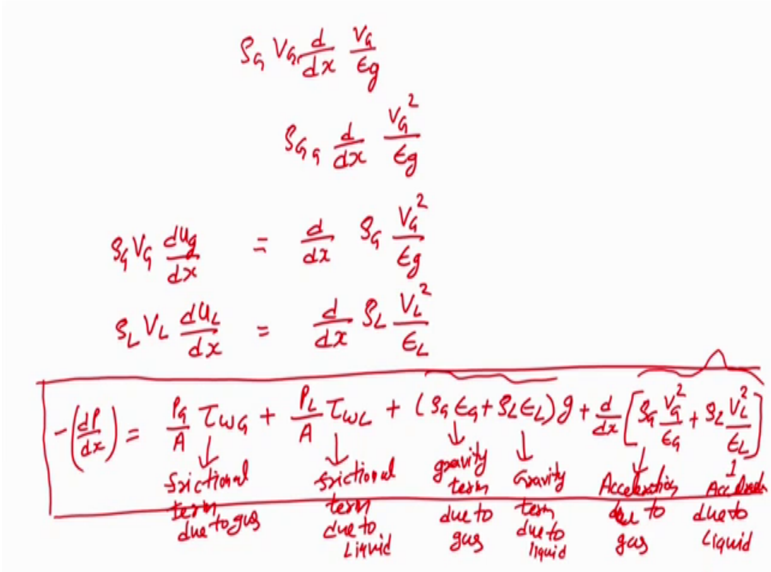

As a rho G into V G d upon dx and instead of U G, I can write it out as a V G upon epsilon g. Now, if I just solve this equation I can write it as rho G into d by dx and I can write it out as a V G square upon epsilon g ok. So, or if I can put it rho G also inside so, I can write it as d by dx rho G into V G square upon epsilon g.

Similarly, for the liquid also I can reduce the same term. So, this term is equal to what this term was equal to rho G into V G into d U g upon dx this I can write it in this form. So, what I can do I can write similarly rho L into V L dU L upon dx can be written as d by dx of rho L into V L square upon epsilon L. So, what I can do again I can write the equation for the dP by dx in this form. So, minus dP by dx will be equal to P G upon area tau W G plus P L upon area tau W L plus rho G into epsilon G plus rho L into epsilon L gravity plus I can write it d by dx and this will be rho G V square G upon epsilon G plus rho L V square L upon epsilon L.

So, this will be my equation for the homogeneously makes two-phase flow ok, now we have not assumed that both the phases are moving with the same velocity or something we have said that this is the general equation of homogeneous flow, where both the phases are flowing together. Now, this will be nothing, but the frictional term term due to gas, this is frictional term due to liquid, this is gravity term due to gas, this is gravity term due to liquid, this is acceleration due to gas and acceleration due to liquid.

So, what we are saying if the flow is homogeneously mixed what you will see you will see the dP by dx. Now, instead of the single phase flow whatever you are seeing that the overall dP by dx is a function or is the summation of the dP by dx occur because of the friction, dP by dx occur because of gravity, dP by dx occurs because of the acceleration.

Now, you will see the effect which will be the combined effect dP by dx which will be frictional term, but the friction will be now caused because of the gas and liquid both. So, these 2 terms has been kind of written, the gravity term which will be because of the gas and liquid both. So, this term has been added and acceleration term again that will be because of the gas and liquid which is because that this term has been added.

So, this is the general equation for the homogeneous flow, now the homogeneous flow generally what we assume is that the whole flow is flowing as the mixture and there is No Slip in between. So, homogeneous flow with No Slip, that is mostly we will write it it it in that way that homogeneous flow with No Slip.

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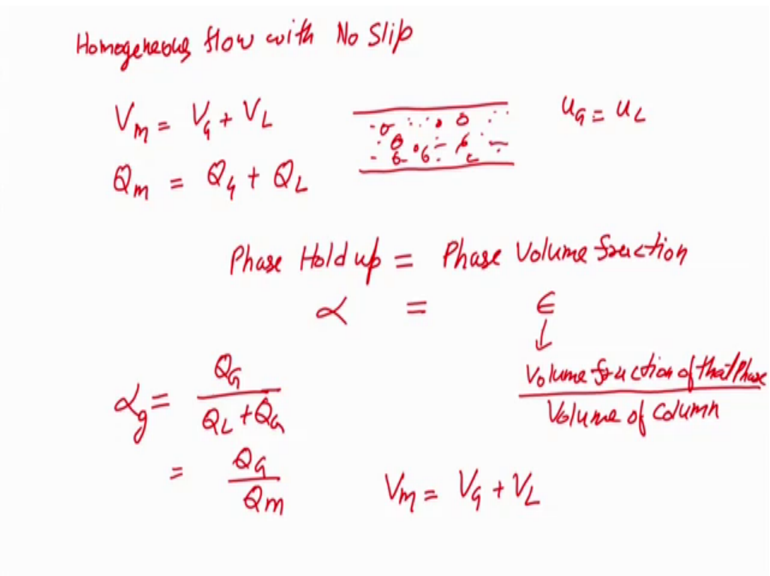

Now what do you mean by No Slip? No Slip means there is No Slip between the 2 phases in sight it means if the both the phases are flowing inside say gas and liquid. So, gas and liquid both are flowing inside ok, I am writing the gas in form of the bubble, there is No Slip in between it means what? U G is equal to U L, it means the velocity of the gas inside the column is equal to the velocity of liquid inside the column, it means there is No Slip in between.

So, what will happen? The velocity remains same. If the velocity remains same, it means what now you can write it out the mixture velocity and everything you can write because both are the velocity is same, you can write everything in terms of the mixture velocity. Now, how the mixture velocity we can write we have already shown that U G or V of G mixture superficial velocity is nothing, but velocity of superficial velocity of the gas plus superficial velocity of the liquid or you can say Qm is nothing, but Q of G plus Q of L you can write it in this frame.

So, what happens with the No Slip? No Slip means it says that phase hold up is equal to phase volume fraction ok that is what it means. Now, there will be confusion bit confusion we have already discussed; what is volume fraction? Volume fraction is being denoted by epsilon G and epsilon G or epsilon and phase hold up is defined by alpha, it means what we are saying that alpha will be equal to epsilon.

Now, how the phase hold up has been defined epsilon we have already seen that how the epsilon has been defined. So, epsilon has been defined as volume fraction of that phase divided by volume of reactor, I will write column volume of column. Now, how alpha is defined alpha phase holdup is actually being defined the way the liquid or the gas or the different phases are going inside, it means what? Alpha is nothing will be different say alpha of g will be defined as Q G upon Q L plus Q G.

So, what we are saying that if there is No Slip and the phases are distributing accordingly according to their flow rate inside then what will happen the alpha g the fraction the phase holed up inside the gas holdup will be inside will be nothing, but what is the volumetric flow rate? Now if you see here the volumetric flow rate that is what we are saying there is No Slip it means both are being occupied same there is no velocity difference inside and they are being kind of occupied the fraction according to their inlet flow rate ok. So, that is called phase holdup and if their fraction occupied inside is equal to the according to their inlet flow rate it means the phase volume fraction which is the volume fraction inside the reactor is equal to the phase holdup which is the holdup or the fraction of the gas which is being fed compared to the total flow rate.

So, this is the Q G, it means this is what is the gas flow rate divided by the total flow rate so, Q this is nothing, but the mixture. So, I can write as Q G upon Qm, it means the holdup is defined as what is the fraction of flow rate of the gas divided by the total flow rate. It means the fraction of the gas flow rate divided by the amongst the total flow rate ok.

So, alpha g if suppose 60 percent it shows. So, it was that the fraction of the gas at the inlet is 60 percent and 40 percent is the liquid, it means the total flow rate if I know I will just multiply by 0.6 I will get the gas flow rate, if I and and if I multiply by 0.4 I will get the liquid flow rate. So, that is what alpha g is there and if there is No Slip inside it means both the phases is being flowing together without having any difference in the velocity.

So, what will happen there phase holdup will be equal to the volume fraction it means they will be there inside of the column they will occupy the similar fraction as per their flow rate. So, if you do that this is actually the same if there are there is No Slip then what will happen this is nothing, but the volume of gas and this is the total volume the time time will be cancelled out. So, you will get that epsilon so, if there is No Slip alpha will be equal to epsilon, it means they will be distributed exactly same way as they are coming in ok.

Generally this is a condition a very big assumption which is being there and very small cases percentage of the cases you can see the homogeneous flow with No Slip condition, we will see those cases which are being used for such kind of a flow. So, once I say homogeneous flow with No Slip it means what I am meaning that alpha is equal to epsilon. Now because alpha is equal to epsilon I can write everything in terms of the because I know the now I can calculate the mixture velocity. So, I can write everything in terms of the and that the way we write it.

Now, what we will do we will write everything in terms of the Um and Vm or Vm, Vm we have already said is nothing, but V G plus V L and if I know the flow rates individual flow rates I will know the mixture flow rate or if I know the total flow rate anyway I am knowing the Vm. So, what I can write? I can write the complete equation like a single phase flow equation and I will say.

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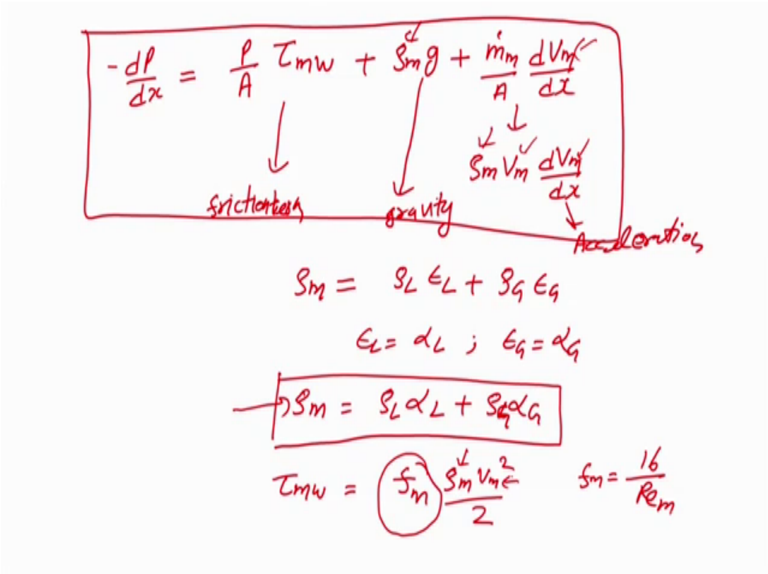

Minus dp by dx is nothing, but will be equal to P upon A ok, now this will be tau m w plus I will write it rho m into g plus it will be rho m into into d by dx of Um square upon epsilon G term will not come it will be upon this.

So, I can write it as in terms of the earlier way the way we have written and that will be you can write it as m naught of mixture divided by area into dVm upon dx or if you write you can also write it this term as instead of this you can write it as a rho m into Vm into dVm upon dx.

So, whole equation will now be reduced it it in this form, you will want to write in terms of the mixture mass flow rate or you want to write it in terms of rho m you can find it out. Now the thing is that you now calculated you reduce everything in terms of the single phase flow and because there is No Slip, you can easily write everything in terms of the mixture flow rates and this tau m is nothing, but the mixture frictional this this is the friction term friction term, this is the gravity term and this is the acceleration term and you are writing every term in terms of the mixture.

Now, the problem here is what is the problem? Again the tau m is going to be the problem; rho m is going to be the problem. So, rho m we can now calculate, now rho m is no more a problem actually for homogeneous flow with No Slip because I can write it out row m as rho L into epsilon L plus rho G into epsilon G. Now because epsilon L is equal to alpha L and epsilon G is equal to alpha G, it means the volume fraction and phase holdup is equal rho m can be written as rho L into alpha L plus rho G into alpha G, sorry rho G into alpha G.

So, now we know the alpha G we know the alpha L, we know the individual density of the both liquid and gas, the rho m we can easily find it out. So, that the problem one problem we have reduced and we have simplified it so, we can find it out this rho m very easily this value Vm, we can anyway find it out. So, this value we can also reduce defined.

Now, the problem comes with again tau m w, now tau m w will be written as what again same friction factor which will be based on the mixture into rho m into Vm square upon 2. Now, again rho m we know, Vm we know the only problem is with the fm and now the formula problem becomes little bit more severe compared to whatever we were facing in the single phase flow because let us assume that this is a laminar flow. So, fm will be written as 16 upon Re and this Re will be based on the mixture Reynolds number and mixture Reynolds number Re m will be defined as Vm into rho m into diameter of the pipe divided by mu m.

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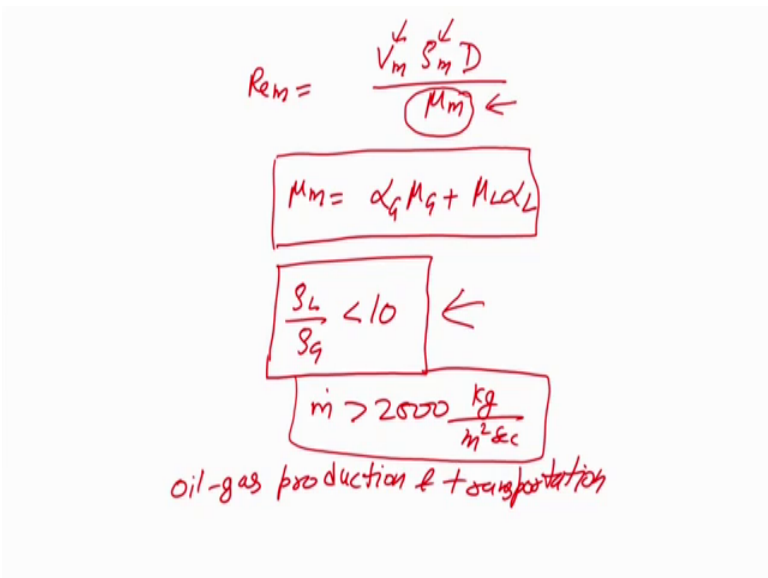

Now, this is the problem in single phase flow, we know the mixture viscosity, we know the sorry we know the single phase flow viscosity, the the fluid viscosity, we know the fluid density Vm and V and d we can we already know here we know the Vm. We can calculate it if we know the total flow rate divided by cross sectional area, we know the

Vm, we know the rho m, because we know the alpha G and alpha L the only problem is mu L that how to calculate the mu m.

So, the mixture viscosity calculation itself is a very big task and it is a whole field of rheology and I am not discussing that, but if you will go and see any petroleum field or any phases the rheology field, you will see that finding the mixture viscosity itself is a big challenge. And it cannot write you cannot write it in a very simple form the way we have written rho m, but for a better approximation or for the first hand approximation the mu m can be written as the way we have written the rho, but this is the very big approximation has been found true for a very few cases, most of the cases this is not true.

And that is why what you need to do you have to depend on the mu m on the experimental data, it means if you are homogeneous model you are using at least one thing you need experimentally and that is the mu m that what is the mixture viscosity you have to use the viscometers, Brookfield viscometer or any other type of viscometer to find it out the mu m or you have to go to the field of rheology, you have to develop the equations for the mu m for different composition.

Now, the problem is that mu m suppose if I am talking about oil transportation pipeline each crude oil have a different property even with the hour or with the time the viscosity the property of the crude oil changes it means the viscosity of the crude oil will change. So, every time you have to go and find it out that what is the value of mu m if you want to use the mixture model homogeneous model or also known as a mixture model?

So, this is the complete formulation of homogeneous model with No Slip condition ok, we have the No Slip condition again I am saying that the phase holdup is equal to the phase volume fraction it means there is No Slip in between the inside. So, this model this kind of a model is being used when the rho L upon rho G is less than 10 and as you can see that rho L upon rho G mostly have a difference of thousand other than the petroleum industries where this is true sometimes the rho L upon rho G is less than 10 or the m naught is greater than 2000 kg per meter square per second ok.

So, only for these conditions this homogeneous model is valid which is true for oil gas transportation. So, in that oil gas production and transportation one can use the homogeneous model, but most of the places because of these 2 limitations particularly this; the homogeneous model with No Slip condition is not valid and one has to go with more rigorous model to understand that how to calculate the dP. So, in homogenous No Slip model I hope you will now able to calculate the dP and how the equation has been simplified and how we have done this job. So, with this we will stop today and we will discuss the other models in the next class.

Thank you.