**Multiphase Flows**
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**Lecture – 03**
**Fundamental definitions and terminology used in Multiphase-II**

So, last time we were discussing about the bulk density of this first phase and as we said that bulk density is nothing but the mass of dispersed phase per unit volume of the mixture. And based on that we have defined some formula and that was rho d bar which is the bulk density is nothing, but the del dot M d upon dou V, where V is the volume of the mixture. So, complete volume.

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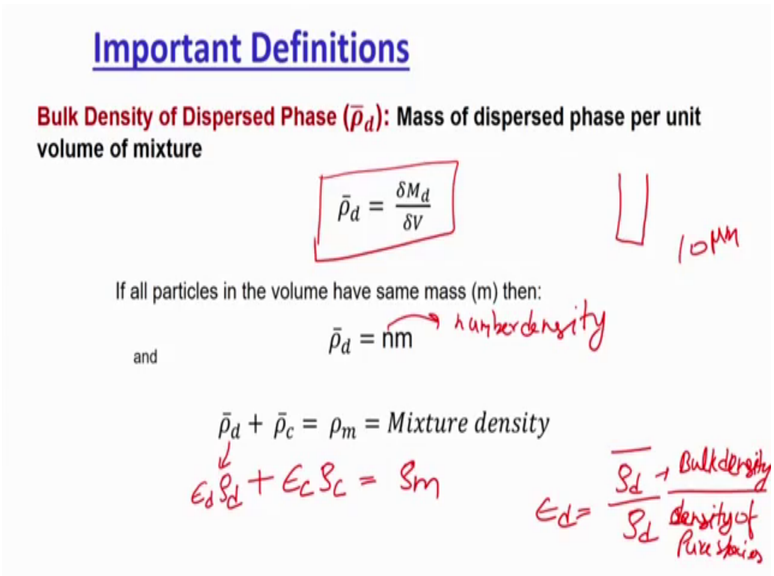


And it can also be written as rho d bar is equal to n where n is the number density multiplied by the mass of a single particle. So, in that way you can also find the number density sorry, bulk density of the particle.

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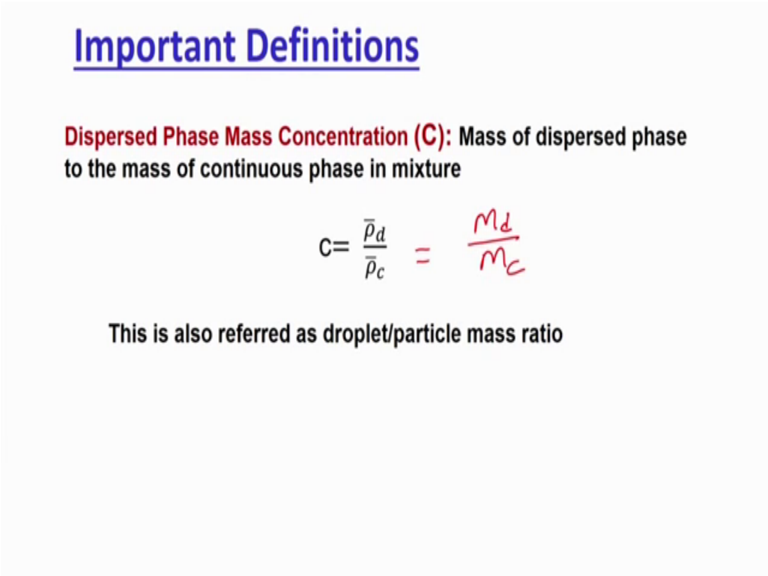


Now, moving to the next the another important definition which we actually found or we were about to discuss and we stopped the last class is about the inertial effect.

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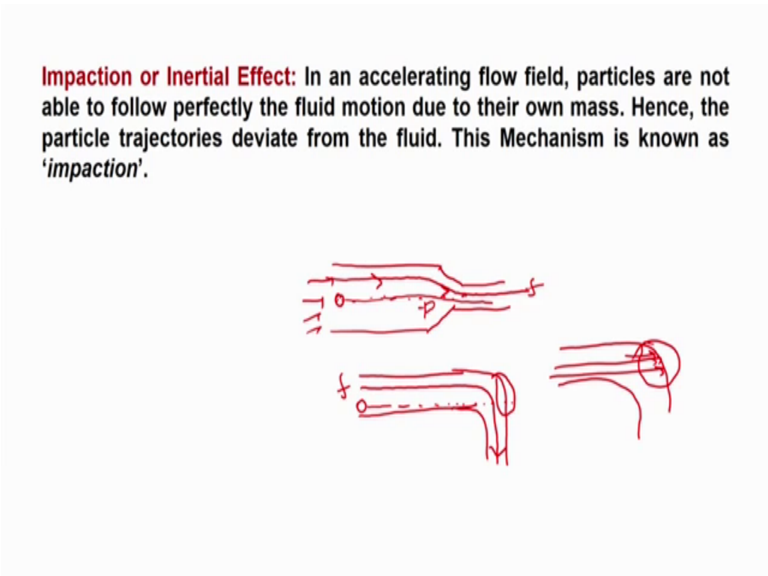


Now, what we mean by inertial effect? And the inertial effect is the effect as the name suggests that there is an inertia. So, now, we are discussing about the two phase flow or multi phase flow in which more than one phases are available. And if suppose in the pipeline a fluid is flowing and suppose air is flowing at a particular velocity and there is some particle which is suspended ok. So, suppose there is a one particle which is suspended. Now, if I change the velocity of the air by any means say suppose if I put a divergence here ok. So, if I put a divergence what will happen the velocity of air will change and with that ideally the velocity of particles would also change, but the problem is sometimes because of the inertial effect the particle does not follow the path of the fluid exactly.

So, suppose what will happen though is suppose there is a stream line flow and then what will happen the fluid will pass something like this, but it is not needed that the particles would also follow exactly the same path this is say fluid path though is not needed the particles should also follow the same path. It may be possible that because of the inertia the particle may go straight it does not move at all it does not change the path and that is called the inertial effect ok.

So, inertial effect is being defined is that basically because of the acceleration in the flow field and the particles are not able to follow the fluid motion. And why they are not able to follow the fluid motion? Because of their own mass they have certain mass and because of that because they are moving with a certain velocity they have certain momentum. So, they actually go with their momentum and they may not follow the path of the fluid exactly.

Now, how much it deviates from the path of the fluid that is what is called the impaction or inertial effect. So, we have to find it out the two things the first is whether they will follow the path of the fluid or not and if they will not follow how much they will kind of deviate from their path. So, impaction effect or inertial effect needs to be found. Now, why this effect is important? Because these effect is very critical particularly for the separation application.

Suppose, if I want to separate there is some dust which is being suspended in the air or you are doing some operation say in fluidized bed and gases are passed and those gases are actually carrying some of the fine particles, or in a boiler we are passing the gas for the oxidation reaction of a combustion reaction and with the gas or with the flue gas some of the ash particles or soot particles are being carried away. So, what you need? You have to actually separate those particles and those particles actually can be separate by using the mechanism of impaction.

Now, how it is possible to separate? What I have to do suppose if there is a flow in which the particles are flowing or the fluidant particles both are flowing I need to just define in such a way I have to give the particle such a momentum that it should have some inertial effect. And then it should say if there is a path here and I can if suppose I put some filter or some kind of a catch here where the particle we go and get stuck. Then what will happen? The fluid actually will move this in elbow kind of a shape this is the fluid, but if suppose there is a particle which is moving here because of the impaction it can go and hit on this wall. So, the there can be a separation.

Now, this can be desirable and this can actually worsley effect also it can be undesirable phenomena. So, what can possible? Possibly if you are not designing the system properly and you are having the flow of a gas and solid or liquid and solid then in the same bed what will happen the particle will go all the time and hit this bed. Now, if they will hit it multiple time the erosion will occur and your bend make damaged after some time. So, that is why it is very important to understand that how the particle will follow the path of the fluid.

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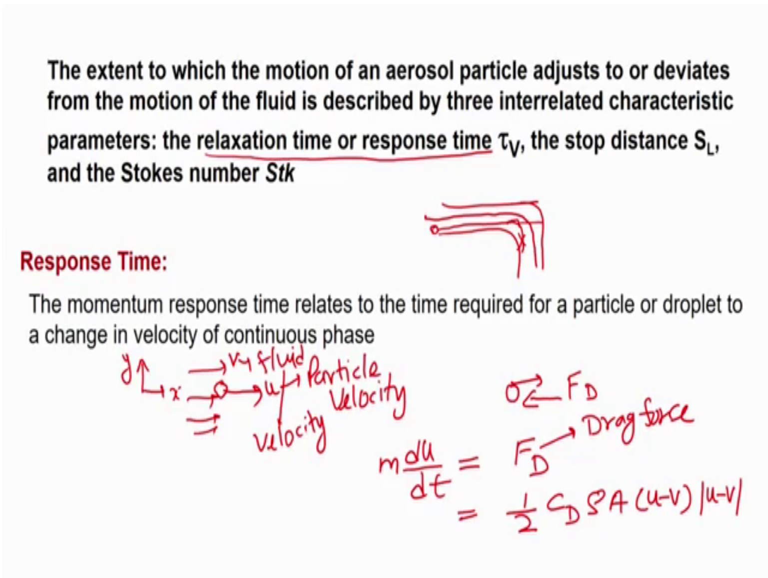


Now, again I am telling that it can be used for the separation mechanism to separate the particle clearly. So, that is the way the impaction has been studied and actually the impaction can be studied by the 3 quantities and that 3 quantities are relaxation time, or we say it a response time. We can also find the stop distance, the stop distance means where the particle will go and stop and third is the stokes number. Now, we will define all these 3 separately and we will see the important of each phases, but the impaction generally is being defined with these 3 numbers or these 3 values.

Now, how these 3 values are defined? So, first is response time or relaxation time as it name suggests that what is the response time response time is the time which particle will take to respond to the change in the fluid velocity or the continuous phase velocity as the name suggests clearly.

So, it is the momentum response because we are talking about the momentum we are talking about the velocity it is a momentum response time that relates to the time required for a particle or a droplet to change of the in the velocity of the continuous loop. It means it is the time required a particle for a particle to respond to change in any particle velocity. So, before that it will not respond it means what it will continue its path.

So, suppose if this is a bend again I am taking bend is a very simple example. If the pursued is moving it in this way and the particle response time is say very high then what will happen particle will not move in this side in this direction instead of going in this direction it will continue its motion in the same direction. So, what will happen? It will go and hit the bends.

So, that will depends on the particle response time that how fast it can response to the change in the fluid velocity. So, that is called particle response time. So, that is the one phenomena which actually describe that how the relaxation factor or whether the impaction will play a role or not if it will play how much important role it will play. So, that is the definition of the response time.

Now, how to calculate the response time? So, for that we can do a derivation. Now, how to do the derivation is a very simple thing suppose if a particle for the same case is moving and it is moving horizontally. So, I am neglecting the effect of the gravity and buoyancy will be very small. So, what will happen? If suppose there is a particle which is moving in horizontal direction in say direction of x, if suppose this is x this is y it is moving in x direction along with the fluid.

So, the fluid is also moving and particle is also moving. So, what will happen say the fluid velocity is v and the particle velocity is u. So, these are the particle velocity u is particle velocity and this is fluid velocity.

So, a fluid is moving in which some particle is suspended and if I am assuming that the particle velocity is u meter per second fluid velocity is v meter per second if they are moving then what will happen what are the forces which are going to act. So, if you see the forces which are acting because the motion is in horizontal there is not gravity is not going to play any role if I neglect the buoyancy and I am just considering the motion in the horizontal direction or in the x direction then the buoyancy will also not play a role. So, there will be two forces which will be acting, one is the particle momentum or particle inertia and that will be opposed with the drag force.

So, I can write a equation which can say that m d u by dt. So, rate of change in the momentum is nothing, but that will be equal to the F D which is nothing, but a drag force. So, you can say this F D is nothing, but drag ok. Now, how the drag force are defined? Well revisit it again, but I hope in your undergraduate or in your basic chemical engineering courses or basic engineering courses this might have been introduced that drag is nothing, but it is half into C D it is rho A u minus v square.

So, because this both are moving we are defining it based on the slip velocity. So, I am saying that it is u minus v and mod of u minus v. So, some book says that is u minus v square I am writing it in the directions u minus v into mod of u minus v so that you can get the direction of the drag force. So, finally, these are the forces which is acting on the particle if it is moving horizontally.

So now, we can what we can do we can try to see that how to calculate the response time. So, we will try to define or derive the formula for the response time. Now, to do that we did the force balance the only problem is the C D value as we know that the C D is a function of Reynolds number and if you keep on changing the Reynolds number the value of C D will change and that the value will be different for the laminar and turbulent flow or you can say there is stokes regime and Taylor regime or turbulent regime the values will be different.

So, let us assume that the velocity is less and we are still in the stokes regime then the C D will be actually defined with the 24 by Re.

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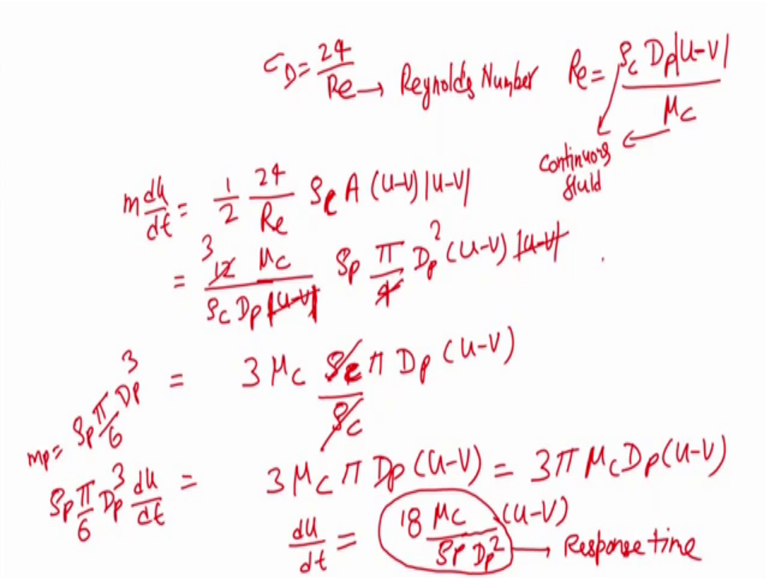

Now, this Reynolds number Re is nothing, but the Reynolds number ok. And this is Reynolds number is defined based on the slip velocity. So, what will be the Reynolds number? Is this nothing, but rho of continuous fluid d of particle u of particles because the particle velocity is v u, I am writing it as a u and this will be based on the slip velocity. So, you will say that u minus v upon mu of continuous.

So, that is the Reynolds number the weight has been defined, it has been defined based on the slip velocity. So, this rho is for the continuous fluid, these two is for continuous fluid rho and mu, and this is for the discrete phase or discrete phase and the relative motion. So, that is the way the Reynolds number has been defined.

So, we know that if this is following in the stokes regime or it means the Reynolds number is very very less or is less than one then C D can be written as 24 by Re and the whole equation can be further simplified as m d u by dt is nothing but half C D is 24 by Re and Re I am going to this is a Re and rho of particle into area into u minus v to mod of u minus v ok. So, that is the way it is going to be defined.

Now, what is this area? Area is the area which is on the projected area. So, it means this area for a spherical particle will be nothing, but pi by 4 4 square. So, I will just open this Reynolds number and area and this will be equal to 12 upon Reynolds number is rho of continuous fluid into d of particle into u minus v and then this is mu of c continuous because 0 d upon mu it will go up it will be rho p it will be pi by 4 D p square this is u minus v mod of u minus v.

So, that is the way one can define this. Now, Reynolds number is the based on the mod. So, I will write it here the mod. So, that it values would be positive. Now, if you solve this what will happen? This two will be cancelled out and we can write it in the most simplified form and this will be cancelled out 4 and this. So, this will be 3. So, you can write it 3 mu c it will be rho p into pi this D p 1 D p will be cancelled out it will be D p and u minus v and this will be rho p upon rho c. So, you will get a value which will be of this kind.

Now, if we see that sorry this is not rho p, this is the rho c actually ok. So, in that way this will be actually rho c, and this rho c rho c will be cancelled out ok. So, I think you have written it correctly here it is rho c. So, this is the rho c. Now, if this two will be canceled out what you will get is thrice mu c into pi D p u minus 3 or I will write it to a more familiar term which will be 3 pi into mu c D p into u minus v.

So, if you see that that is the formula which you have used for the drag force in the stokes regime and in terms of that it will be 6 pi mu r into v. Now, because based on the slip velocity it is two phase flow we are defining it based on the slip velocity.

So, you are going to have that is m d u by dt is nothing, but 3 pi mu c D p into u minus c. Now, what I am going to do? I am going to integrate it, but before that I would like to open this mass which mass is nothing, but the mass of the particle. So, this is mass of the particle it can be written as m p can be written as it will be the rho of particle into volume of the particle and volume of the particle for a spherical particle it will be 5 by 6 D p cube. So, I will replace mp with that and if I do that mp if I replace with this I can just write it instead of mp it will be rho p pi by 6 D p cube.

Now, I will just try to simplify it if you will try to simplify I will get the values it is like pi pi will be cancelled out you will get d u by dt will be equal to you just bring everything here it will be 18, this pi will be cancelled out it will be mu c upon rho p ok. Now, D p cube will be cancelled out. So, this will be upon d square D p square and u minus v ok. So, this will be what is being turned out the equation and the value if you see this value this is the unit of this is the reciprocal of the time and that is called the response time. So, this is nothing, but is called response time or relaxation time.

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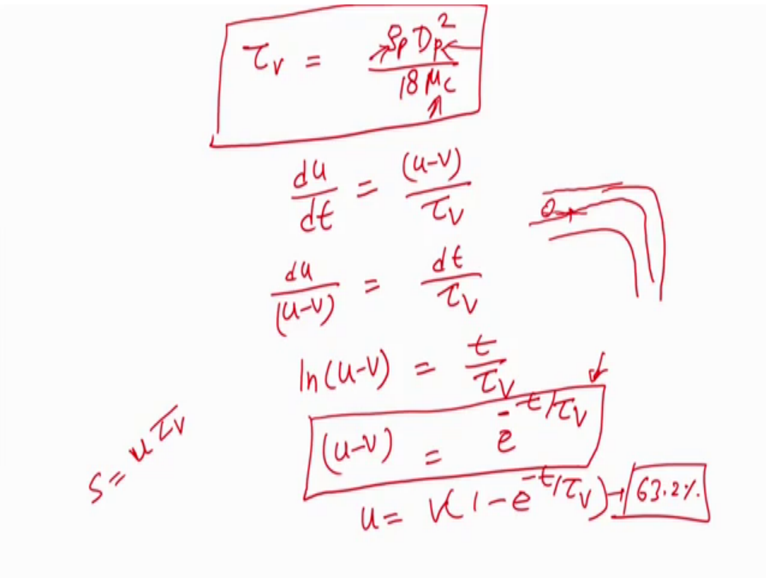

So, the response time or relaxation time is being defined say if I say that response time by tau v if I represent then it is nothing, but rho p into D p square upon 18 mu of continuous.

So, this is called the response time ok. So, that is much time a particle will take if you are putting a particle in the system which is moving a gas which is moving. So, depending upon how much is the density of the particles what is the size of the particle, and what is the viscosity of the gas it will have certain time it will take before it will respond to any change in the fluid motion velocity of fluid velocity.

Now, do you see that it is function of the density it is function of the particle diameter and it is inversely proportional to the part fluid viscosity. It means what? In the fluid viscosity is very high the response time is going to be low, if your particle density is low then the response time is going to be the low, if your particle diameter is low then the response time is going to be low. So, it means what if this times will be low particle will respond to the flow very fast if this time is very high particle will not respond to the fluid or it will respond very lately to any change in the fluid velocity of fluid motion of fluid direction. So, that is the way response time has been derived.

Now, it can be further continued and if I just want to find it out the stop distance I can say that d u by dt was equal to if I just look here it was u minus v upon tau v. So, that is the way we can write it. Now, if you do it in this way what you will get you will get that

1 upon d u upon u minus v is equal to dt upon tau v ok. Now, if you do the integration of the same if I integrate it then what I will get I will get the function which will be in the form of ln. So, this will be ln u minus v will be equal to this will be t upon tau v.

So, now, if you do that t upon tau v value; now, what will happen? I can take the integral and it will be u minus v will be equal to e raised to the power t upon tau v. So, that will be what you will get as the formula. Now, mostly what will happen the u will be smaller v will be higher. So, this value is going to be negative. So, I can take that negative sign here to reduce this value to make this value positive. So, finally, you are going to get that u minus v is nothing, but e raised to the power minus t upon tau v. So, that will be your overall time which it will take before the particle will respond to the motion and you can see that how the tau v is the time and how the u will change with the time.

So, you will find it out that u and v correlation you can see that will change in the v how the u is going to be changed. I hope this is clear that why we have taken negative, we have taken it negative because here the particle mainly is moving because of the gas velocity. So, u is going to be smaller than the v and because of that this value is going to be negative and if that is true I have to take the minus out and e raise to the power minus t by tau it will be there.

So, once we are decaying the velocity we are changing the velocity the particle motion is also going to decay or particle motion is also going to change and we can calculate that how the motion change will take place. So, in that way we are defining this.

Now, if we have defined this we can also find it out suppose if the particle have b started with the initial velocity 0, then what will happen I can include the whole thing from 0 to say a velocity u if I do that what will happen. If the time t is equal to 0 u v will be equal to 0 I can calculate that what will be the value after you and then we can say that at time t equal to t if the velocity is u if you do that then what you will get you will just do this ln. So, you will get that u is nothing, but it will be v into 1 minus e raised to the power t upon tau v. So, what you need to do? You have to just integrate this integral will be from 0 to time t and at 0 you can say the velocity is u is 0 and at time t the velocity is u.

So, if you do that calculation if you solve it you will get the value of this kind and it will show you that how the particle velocity is actually going to change with the time. So, if you change the fluid velocity motion how the particle velocity is going to change. Now, from there you can find it out that whether the particle is responding to the change in the fluid velocity and you can also track the velocity of the particle with the time. So, that is very very important very very critical and if you see that it is actually following if you just try to revise your control it is actually following that when the u will be equal to v once the particle will change or it will be nearly equal to b.

So, if you find that the e raise to the power is this value whatever we are saying that t minus v is equal to 1 then what will happen? The e raised to the power one value will come it will be 1 upon 1 then this whole value will come 1 upon sorry 2.736. So, this whole value will come at 63 percent, you 63.2 percent, but for the sake of simplicity you can find that it is 63 percent.

So, it means what that the momentum response time can also be defined as a time which is required to achieve the 62 percent of the particle velocity or 63 percent of the velocity of the free stream velocity or of the 63 percent of the particle velocity of the free streamed in the particle will as find or will particle we attain that is called the response time.

So, that can also be defined it, it in this way which is coming directly from this. So, what will happen u and p will be then how the response time will be defined. So, you can define the response time it in this way. So, this is the another way to define the response time, but we will go with our own definition which says that it is nothing, but the time required for a particle to follow or to response to any change in the fluid velocity. So, that is the way we have defined this tau v.

Now, once the tau v is defined we have also defined that how the particle velocity will change with the time. So, now, what you can do, you can solve any problem in which I have the initial particle velocity is given if the fluid velocity is given and it is given that the flow is very small you can use this formula to calculate that with the time how the particle velocity will change.

Now, once you know that with the time how particle velocity is changed you can also track the position of the particle, that how much position, how much distance it will travel before it will stop. So, in that way you can find it out that the penetration distance or stop distance. So, you can also calculate the stop distance for these particles that if they are moving at what distance it is going to stop. So, that is called the stop distance.

And it has been defined actually the stop distance is nothing, but is that l this is nothing, but the velocity into the tau v. So, if you have know the as stop distance is nothing, but what is the velocity say is you into tau v if you do that we will get the stop distance that what will be the final distance at which it will stop.

But if you want to calculate that how the food plot will move or how the particle will move along the fluid path you can use this formula you can convert it to the in terms of the velocity. The u in terms of the you can calculate to the position and you can track that how the particle positions say if there is a pipeline where there is elbow and the fluid is moving particle is also moving we can track that how the particle will start following the path of the fluid.

So, it means if you change the path of the fluid how fast the particle will able to accommodate that change you can calculate that by using tau b. You can calculate the particle path line because calculating the position of the particle you can also calculate the particle velocity how it will decay the velocity and what will be the final velocity it will get. So, all those things can be found with the stop distance also. So, you can also find that when the particle will go and finally, stop at some place. So, these all things can be handled can be completed.

So, with this; now, similar to particle momentum response time there is thermal response time also. So, particle thermal response time is nothing, but the. So, what we have discussed is the momentum response time.

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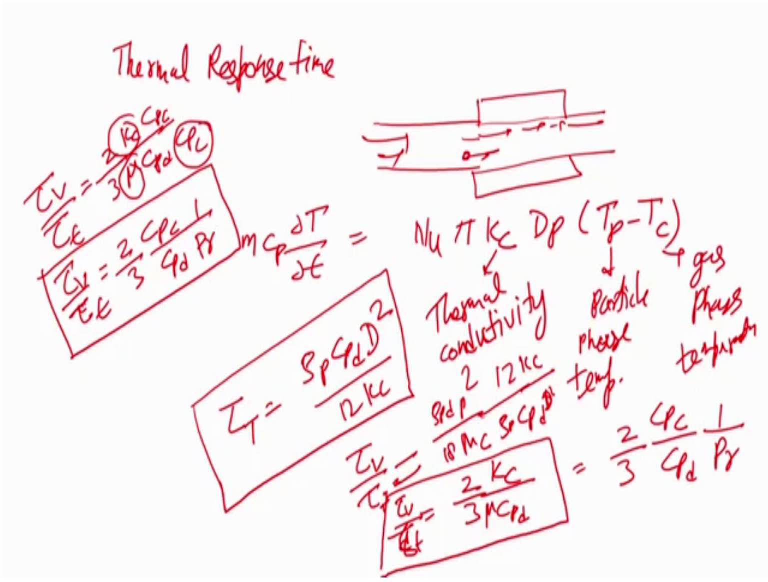


Similarly, in similar line we can discuss the thermal response time, and thermal response time is nothing but the response time needed by the particle to respond to any change in the temperature of the continuous fluid. So, if suppose the fluid is moving and I put certain a heating zone here at certain location and the fluid is moving what will happen the fluid temperature once it will come here we start developing and you will see a proper fluid velocity.

Now, if there is a particle here then the particle will take actually some time before it will respond to any change in the food flow in any change in the temperature of the continuous fluid or of the fluid which is slowing. So, that is called the thermal response time and it can again calculate the wave we have balance the momentum or the force we can balance the energy and based on the energy balance we can calculate that what will be the response time which will be needed for the thermal or thermal what will be the thermal response time.

Now, to just for the example that how to do that how to do the energy balance what will be the total energy contained by the particle it will be nothing, but mc p dou T upon dou t you. So, it means this capital T is the temperature small t is the time. So, that is the total energy any particle is having or the total energy the particle can have.

Now, this will be equal to nothing, but the Nusselt number which is the number and kind of responsible for the heat transfer it should be the pi k c, k c will be the thermal conductivity of the continuous phase and it will be D p into T p minus T c. So, once I said T p minus T c it is nothing, but the temperature of the particle this is the particle temperature and this is the temperature of the continuous phase. So, I will say that this is a gas phase temperature and this is particle phase temperature.

So, you can calculate similar way that how much time it will take for the particle to respond to any change in the temperature. So, you can do all this balance and if you solve this you can find it out that what will be the thermal response time of the particle and just I am leaving it to you as an assignment and maybe we will do this as a assignment.

The tau T is the thermal response time of the particle will be nothing, but it will be rho p into c p d, where the c p d is nothing, but the heat capacity value for discrete phase or the particle phase into d square upon 12 k c. So, this will be the particle thermal response time and it is nothing, but you have to just balance this open the end the way we have done in terms of the rho p into vp, the vp is the volume of the particle. And then ncp dp you have to write this and then you just equate it put the value of Nusselt number in terms of that k and then you solve it you will get that rho t is nothing, but equal to rho p c p d D square upon 12 k c.

So, it means what if I try to find it out the value of tau v upon tau t and just try to find it out. So, tau v is nothing, but it was rho p d p square upon 18 mu c and tau t is nothing, but it will be rho p into c p d d square upon 12 k c. So, this is the way this is I c p d it is this if you solve it out it will be cancelled out many things and you will get that 2 by 3 it will be k c upon c p d.

So, that you will get the value of tau v upon tau t sorry tau v upon tau t that will be the value which will tell you that whether the momentum response time is higher or thermal response time is higher and if you want to correlate it with the this. So, this will be mu c p d there will be one mu which is missing here. So, it will become mu c p d. So, that is the way it will be found. So, if you want to find it out if you want to correlate it you can also correlate this with the Prandtl number and if you want you have to what you need to do you have to just multiply by this cpd up and down and then you can calculate the Prandtl number of continuous flow.

So, you can do that you can write it in terms of the Prandtl number also and if you write in terms of the Prandtl number it will be nothing, but 2 by 3 it will be c p c it means the heat capacity value for continuous phase will be c p d into one upon Prandtl number and. So, this is the way you can you can define. Now, how this is being defined? How what I have done to derive that? I have just multiplied here in this equation. So, if I take that tau v upon tau t thermal response time is nothing, but it comes 2 upon 3 this will be k c upon mu it will be c p d.

Now, if I multiplied with c p c up and down it will be c p c then what will happen? We know that c p mu by k that is nothing, but it is the Prandtl number. So, this will be 2 by 3 it will be c p c upon c p d and this c p, this c p this mu upon k is going to be the Prandtl number. So, we will say one upon Prandtl number.

So, similarly you can find the response time the way we have derived the things remain same you can find the response time you can find that momentum response time, you can find the thermal response time. The idea is that impaction it means if the fluid path is changing of fluid velocity is changing particle may take some time before it will respond to that change and that is called the response time.

Now, once we are talking about the momentum that is called momentum response time when we are talking about the temperature or the heat we will say that it is the thermal response time. So, that is the way the response time has been derived. And one can use that response time to calculate that how the particle will behave and the same calculation can be used to calculate the stop distance the way I have told you earlier. Same calculation can be used to track the particle trajectory of particle motion; same equation can be used to calculate the particle velocity. So, if you know this you can calculate how the particle will move with the time.

Now, moving towards the next; so, we have defined response time, we have defined this stop time. Now, the third one is that stokes number.

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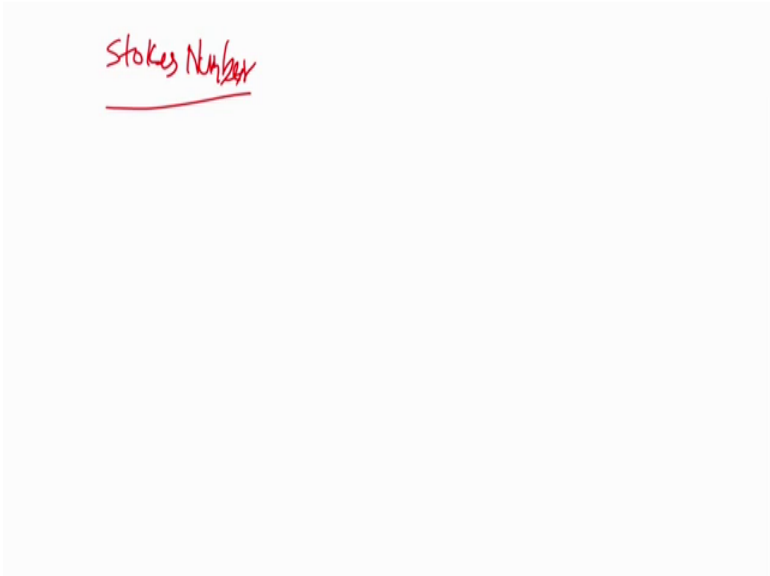


Now, what is a Stokes number and how it has been defined.

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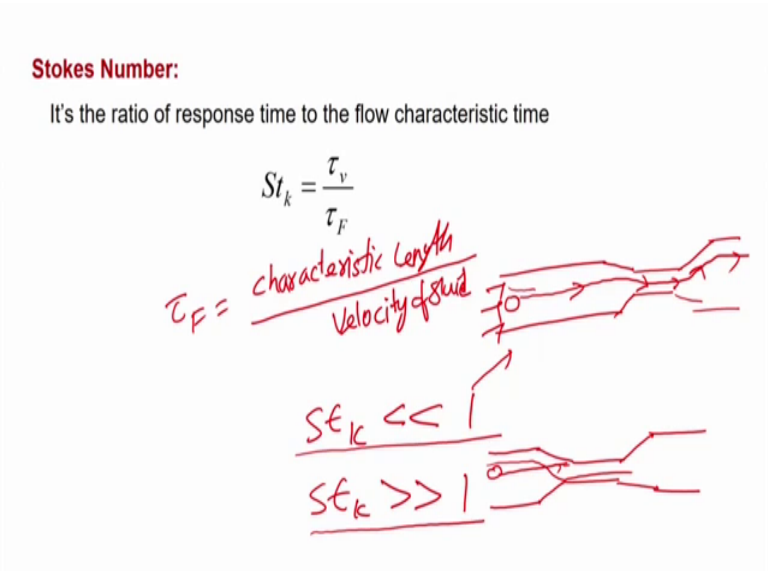


So, a Stokes number is nothing, but it is the ratio of response time to the flow characteristic time. So, the stokes number is represented with St subscript k and it is ratio of tau v upon tau F, where the tau F is the flow characteristic time.

Now, what do you mean by tau F, flow characteristic time? So, suppose there is a fluid which is flowing in a pipe line, it is a fluid which is flowing in a pipe line and I have to calculate that what will be the characteristic flow time I will say that what is say a is the length suppose this is the time line and suppose it is changing the dimension, let us assume that it is changing the dimension and again it is going get it in this way.

So, suppose your fluid is flowing here and the particle is also suspended. Now, because the particles will fluid will move particle will also move along the fluid. Now, they will reach to this divergent section they will go and diverge to a small section of throat and then again they will go to again separate it and they will kind of instead of contraction. Now, they will have expansion and the fluid will move out and particles would also move out.

So, now, if we find it out the tau F, tau F is will be nothing, but the characteristic flow length characteristic length divided by the velocity of the fluid. So, that is the tau F. So, the characteristic length in this type will be what, is the length of the throat or you can say that. So, that will be the tau F. So, how much is the fluid characteristic time will be there that is the time. So, you can calculate that the tau F tau v we already know we can calculate the tau v and we can calculate the stokes number and to find it out that whether the particle is going to follow the path of the fluid or not.

Now, if the stokes number is very very less than 1, if suppose St k is very very less than 1. It means what? It means the particle response time is very low compared to the fluid flow time ok, it means the characteristic time of the flow it means what the particle response time because it is very very low it will response is very low it means the particle has ample time or very long time to respond to the change in the fluid motion because tau v is very very small ok.

So, it means what tau is very small tau F is very high, particle length. So, characteristic time of the fluid is much higher compared to the characteristic time of the particle. So, it means what? Particle will just follow the path of the fluid it will have ample time to respond. So, in that case if this is less than one the particle will actually flow and it will just change is stay here and then go out. So, that will be the path of the particle exactly same will be the path of the fluid also. So, it has ample time to respond to any change.

Now, if the stokes number is say is very very high then 1, then what does it mean that particle response time is very very high compared to the flow characteristic time. It means the flow characteristic time is very very low and particle response time is very high it means what in that case particle will have no time to respond to any change in the fluid velocity ok. So, it will have its own inertia or action is going to work it will move with its own inertia and it will not respond to any change in the fluid velocity of flow direction.

So, what will happen? For the similar case now, if suppose this is the case. Now, what will happen? The particle will go and it will just hit the water. So, that is the way it is not going to respond to any change in the fluid motion. So, the fluid motion may change, but particle is not willing to do anything it is just going and hitting the ball. So, that is called once will happen when the Stokes number is very very more than 1 and its stokes number is less than 1. Then particle will have ample time and it will respond to all the changes you will make with the fluid velocity.

So, that is the way we can find the impaction effect. So, we can calculate the stop distance, we can calculate the response time and we can calculate the stokes number. So, in that way you can find the effect of impaction with the help of stokes number, with the help of response time one can also derive that how the particle will be have how the particle location will change the position can be calculated, one can also calculate that with that time how the particle velocity will change.

So, you can see that whether the particle is accelerating or it is deaccelerating, if you know how it is deaccelerating you can calculate that what will be the stop distance when the particle will deaccelerated this much that the velocity of the particle will go to 0.

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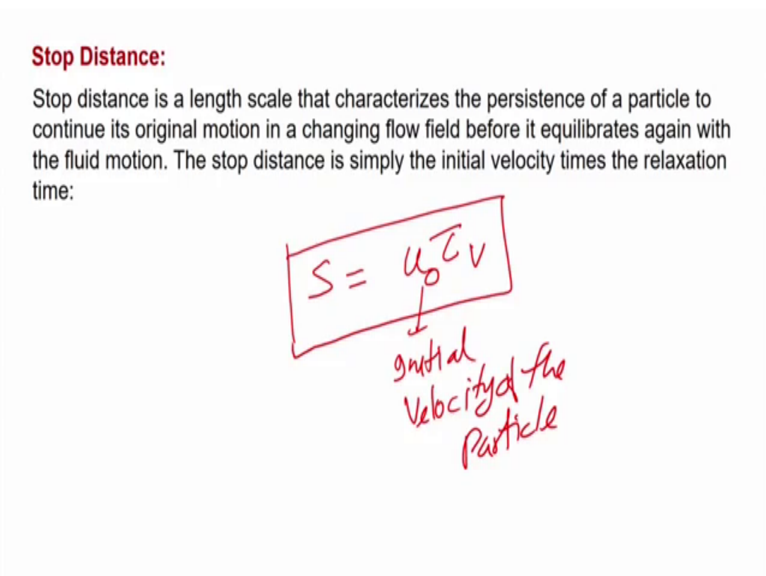

So, all those calculations can be done and the similar way as I said the stop distance has been defined it is nothing, but the initial velocity of the particle into the relaxation time. So, stop distance S is nothing, but what is the initial velocity of the particle. Now, I am saying that particle velocity as a u. So, I will denote it as say u naught into tau v and that is nothing, but the stop distance. So, you can also calculate that if you know the tau v we after how long the particle will actually stop. So, this u naught is the initial velocity of the particle.

So, these 3 quantities together can actually help to track the motion of the particle to find that if the particle want to stop where the particle will stop. So, we can suppose if I ask a question that there is a vein high speed vein in which some particles has been suspended. And let us assume that initial velocity of the particle is say 1 meter per second and can you tell me after how much time that particle will stop, if I give you the density of the particle, if I give you the velocity of the particle, initial velocity of the particle, if you give the velocity of the fluid, if I give you the properties of the fluid, it means the density and viscosity you can actually calculate that after this much length the particle will actually stop it will not move.

So, in that way we can calculate that we can also see that how the deacceleration will take place if the particle velocity is reducing and we can also track the motion of the particle of position of the particle because once you have the velocity you can just do the dx by dt or the dz by dt or dy by dt and integrate it to get the velocity of position of the particle. So, everything can be done by using this 3 ok.

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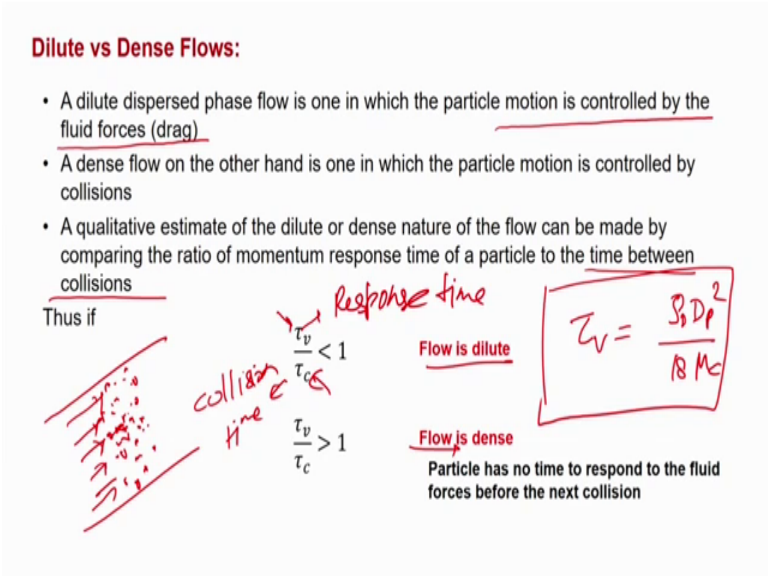


Now, the next thing which is critical in multi phase flow, now we are defining it. So, in the definition the one of the most critical thing is that whether the flow is dilute or dense. So, actually speaking of frankly if you see that the whole multi phase can be divided in 2 parts and the treatment will be entirely different for these 2 parts.

Once the flow is dilute and another once the flow is dense. Now, there is a critical difference between these two. Now, what is the difference? Once as the name suggests dense means the fraction of the discrete phase will be very high that is called a dense flow and when the fraction of discrete phase is very low and that is called the dilute flow. So, that is the broadly definition has been defined the way the dense and dilute flows are being defined in the multi phase flow. And the whole treatment is different.

We will see the treatment once we will discuss the dilute flow and dense flow pertain the gas solid flows we will see that how those differences will occur once the flow is dilute how you will treat the flow or how you will model the flow or how you which technique can be used to see the flow are kind of diagnosis the problem and if the flow is dense how it will be modeled or how it will be investigated. So, we will see that later on. But what is the definition and how you are going to quantify that whether the flow is dilute or dense it is very very critical and rough definition or you can say the layman definition is if the discrete phase fraction is very high the particle is actually dense the flow is actually dense if the discrete phase fraction is very low less than 5 percent or so, the particle or the flow is called as a dilute flow.

That these are the weight definition there are certain concrete definition and terms to find that whether the flow is dilute or dense and the concrete definitions whatever it is is given here and it says that once or dilute the dispersed phase is one in which the particle motion is controlled by the fluid forces. It means what? In the say there is a fluid which is flowing in a pipeline and they have only few particles suspended, so 4 5 particles which are suspended here.

Now, what will happen? Because the number of particles are very very small the fluid is going to dominate the motion of the particles ok. So, I am not considering the particle response time here, I am not considering that the vein and all these things are there if that is not there then what will happen the particle motion will be primarily depend on the fluid and it is actually you can say that it will depend on the drag as we have already seen that if one particle is suspended how you can calculate, how the particle motion will take place.

So, particle motion will finally, depend on the fluid only. So, that is the cause as a dense or dilute phase. So, once the particle motion is mainly governed or controlled by the fluid forces like drag it is called a dilute film.

Now, a dense phase just contrary to each other; suppose if I pack it with lot of particles say if I just put lot of particles here inside. Then what will happen? The particle motion actually will not only depend on that how the fluid motion is taking place, but it also depends on that how these two particles are moving together or how these particles are moving together, are they having a collision, if they are having a collision how they are responding to that collision. So, whether they break whether it is a complete elastic collision, whether is a inelastic complete inelastic collision, whether it is in between the elastic and inelastic visco elastic kind of a collision, whether they are changing the dimensions after the collision, whether they are kind of changing the velocities after they are having collision.

So, the particle motion will not only depend on the fluid forces the particle forces will also have a radar meaning role, like particle collision or particle, particle collision we talk about that then the flow is called as a dense flow. So, that is the major difference between the dilute flow and dense flow.

Now, how to quantify? This all our definition once I am saying that the particle forces are dominating or whether the discrete phase fraction resolved our qualitative the quantitative we do not have the picture till now. So, how to find the quantity that whether it is dilute or dense phase? So, for that we actually see the ratio and we see the ratio of tau v upon tau c, where tau v is nothing but the response time and tau c is nothing but it is the collision time collision time.

Now, how to calculate tau c we will see that. So, qualitatively we can calculate the whether the fluid is dense dilute based on these two ratios, so tau v by tau c. So, if the tau v by tau c is less than 1 the flow is dilute if it is greater than 1 the flow is dense. Now, if you see that value it means greater than one means what the response time is very high compared to the your tau c collision type which is very very low if the collisional time is very very low. It means what? What you understood about that that the particles are densely packed with each other or they densely packed then well if the collision time will be low.

So, how the collision time is being defined? It is the time between the two successive collision of the particles. So, that is called the collision time ok. So, that is the two successive the time between the collisions or two successive collisions of the particle is called the collision time.

Now, if this is greater than 1 collision time is this overall tau e upon tau c ratio is greater than 1, it means what collision time is very very small if the collision time is very small it means the particle will have a very frequent collisions and that is possible only if they are packing fraction of the particle is very high. It means the fraction of the particle is very high, it means it is going to be a kind of dense phase or dense flow. A project way around if we see that tau v upon tau c is very very less than 1 then which means what the tau c value is very high the tau c value will be high if only if the frequency of the collision is less and it will be less only if the particles are very far from each other.

So, what we need? We need to actually find with this we can classify that whether the flow is dilute or dense, but what I need is the value of tau v and tau c. Now, we already know how to calculate the value of tau v. So, if I know my fluid system if. Now, I know my particle if I know the diameter of the particle then I can easily calculate as rho p into D p square upon 18 we can easily calculate the response time the only problem is how to calculate the collision time that is tricky.

Now, once you calculate the collision time you can find it out without doing any experiment or without physically seeing two other things you can say that whether the flow is going to be dilute or the flow is going to be dense. So, to calculate the this collision time we will use the basic collisional approach or theory of the collision of frequency and that theory of collision frequency has been actually derived with the granular temperature theory we discussed that theory later our kinetic theory of granular flow which will discuss later.

And we will try to see that how the collisions has been found by using the very simple approach which is being used also in the kinetic theory of granular flow also in the kinetic theory of the gases both the things, where this collision frequency can be calculated exactly in the similar manner and how to do that we are just going to see that. So, to calculate the collision frequency what we need to do we have to assume a system.

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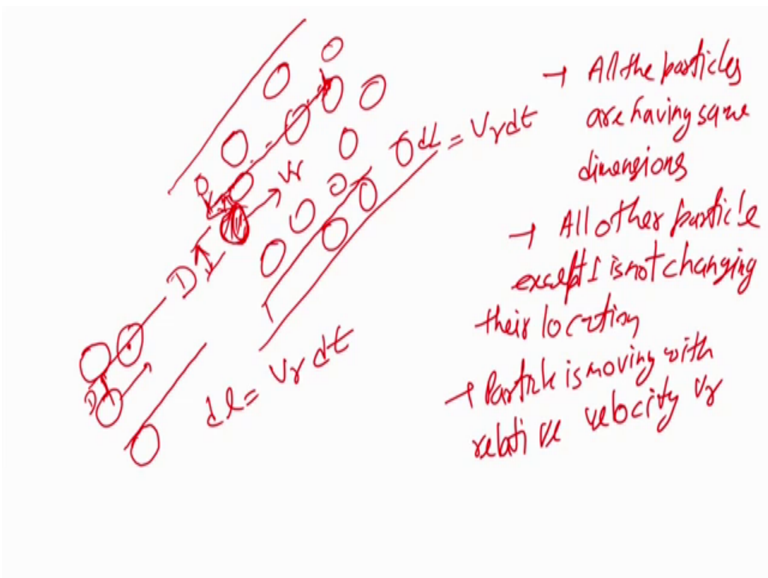


And let us assume that I have a system I am making a particle which is very big and let us assume that this is my one particle which is going to move. So, I am assuming that this only particle is moving ok. And let us assume that it is moving because we are talking about the relative velocity say, it is moving with a velocity v r they have realized the relative velocity and we are assuming right. Now, that only one particle is moving at a time and rest of the particles are constant ok. So, that is the assumption which we use in kind of granular flow also, we also use in the kinetic theory of the gases.

So, what I am assuming that suppose there is a system in which suppose this is a pipeline or system you need some particles are suspended and I am assuming that only one particle is moving rest everyone is stationary and I am giving a velocity of the particle this particle to v r which is the relative velocity it means I am assuming that other velocities other particles are also moving physically they are not changing their position ok. So, in that way we are just defining that how the particle is moving.

Now, what will happen once the particle will move? Suppose the diameter of this particle is d suppose this particle is d diameter then what will happen. Once it will move it will form a collisional cylinder. Now, while it will form a cylinder the region is once it is moving say it is moving in this direction in this circular pipe and the particles are spherical then what will happen it will travel certain distance.

So, let us assume that it is traveling certain distance. Now, how much distance it will travel what will be the distance that is very obvious that if the particle velocity is v r then in a time t or dt it is going to travel a distance l which is nothing, but v r into dt. So, that is the distance it will travel and that is nothing, but this. Now, during the travel it will have some collisions with the other particles.

Now, I have already assumed that the particles other particles are stationary they are not changing their position only one particle changing the position. So, on the hooch which particles this particle will start will having going to have a collision it means what are those particles or whose are those particles the true it is going to have a collision. Now, that can be determined by the collisional cylinder dimensions. So, collisional cylinder length I have already calculated that is v r into dt it means time delta t or dt is very small time it will move a distance say dl and that dl distance is given here.

Now, with whom it is going to have the collision only with the particle if suppose this diameter is d it will have a collision with a particle which is coming in the one r space it means the particle is suppose all the particles are having the same dimensions. Means I am assuming that all the particles are having same dimensions first. Second I am assuming that all other particle except one is not changing their location and particle is moving with relative velocity v r.

So, these are what we have assumed that the particles are having the same dimensions it means all the particles is of have the same dimensions t. Then second that the particle other than one particle or rest other particles are stationary and their positions are not changing it means the velocity of the other particles are 0 and I am allotting a velocity v r which is the relative velocity of the particle to the particle which is moving.

Now, once I allotted that what will happen it will during its path say for a small time delta t it will travel certain distance and that distance will be nothing, but the dl and which we have already discussed that that is not going to be something, but it is going to v r into dt. So, in a very small time it will travel certain distance that distance is d r, but during this distance travel it will have collision with the other particles.

Now, which particle it will have the collision with the particle which is following falling between the one diameter of the particle. It means what? Suppose if I am just writing this as I separately say this is my particle path if any particle which is b like that or it is inside of this is going to have a collision. Now, what is this? This will be nothing, but if the particle say it is not going to because any particle center which is actually lying from a distance at 1 diameter. So, any particle center which will be lying on a distance within the 1 diameter of the particle from the center of the particle it is going to have the collision with it ok.

So, let me again clarify it. So, suppose these are the particles this is say let me raise it and then again kind of draw it to have more clarity on it. Now, suppose I am just assuming this picture here this, this whole portion I am just doing here. So, one particle we have assumed that there is only one particle which is moving and it is moving forwards particular distance, so for a small time. So, say this traveling a distance of dl this is the distance it is traveling.

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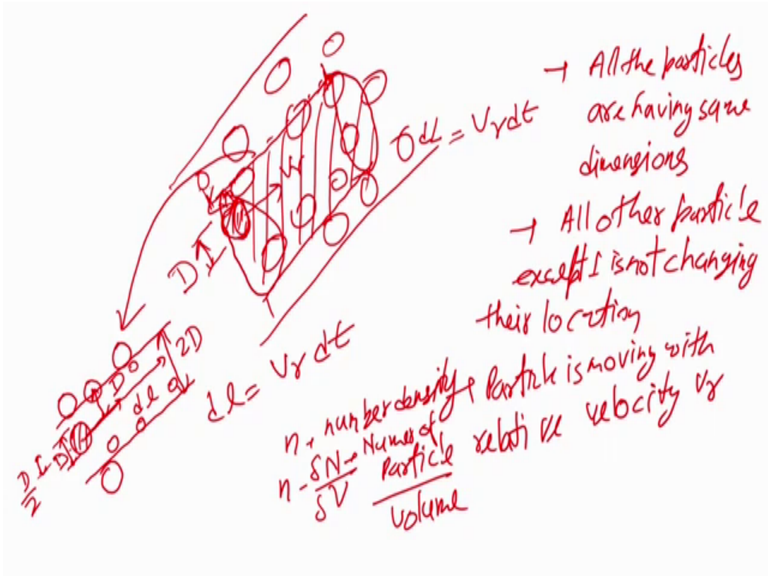


Now, during its path it will have a position with the other particle. Now, each particle it is going to have a collision with the particle suppose this is the diameter which is d ok. So, this will be this distance will be what? It will be d by 2, d by 2. So, this will be the d by 2 distance which is actually nothing, but the radius d by 2 ok. So, this is the centerline radius.

Now, any particle suppose which is following their center. So, which is their center is it in such a way that the center of this particle is at a difference at a distance of d from the center of this particle. So, say this is the as you that as the center of the particle. So, if this distance is also equal to the d. If any particle which is following within one diameter of the particle which is moving and please remember one diameter from the center of the particle not from the wall. So, from the edge of the particle one radius and from the center of the particle 1 diameter, if any one, any particle which is following within that it is going to have a collision with this particle.

So, suppose if I draw a line here and I draw a similar line here. So, any particle which will be fall within this is going to have the collisions with this particle, any particle which is falling outside even like that it is not going to have any collision with the particle because this will not touch each other ok.

So, in that way we have defined the collisional dimension or you can say the collisional diameter and that collisional diameter will be nothing, but the 2D and collisional length will be nothing, but the v r into dt. So, suppose now, if I come back to the same place I will say that a collision cylinder will form and any particle which is following that in this cylinder is going to have a collision with the particle. So, I can find it out that how many number of collision the particle will have.

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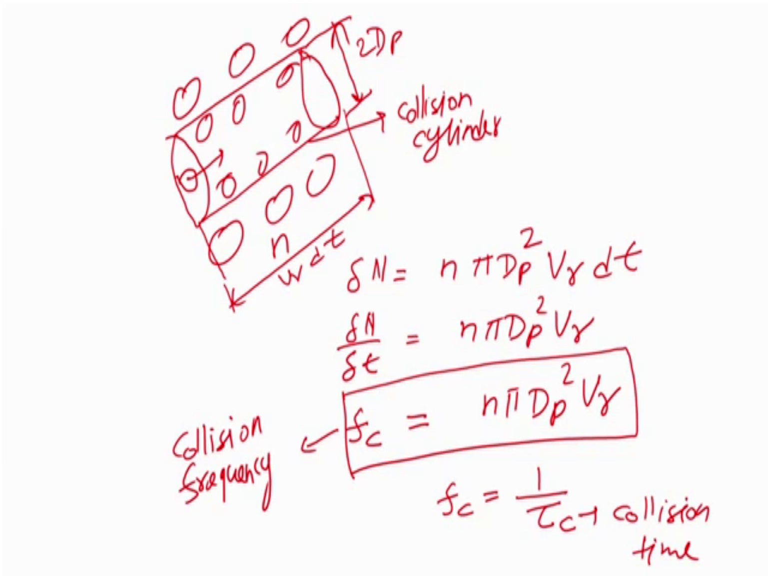


Now, how to find the number of collision? If suppose that the packing density of number density of the particle which is there present in this pipeline or in this system is n. So, n is the number density and we have already defined the n as nothing but number of particle and divided by v or dou n by dou v, where n is this is the number of particles per unit volume.

So, you can calculate that how much is the number density is particle if that is known then what we can do we can find it out that how many number of collision or particle which is moving in that system is going to have. Now, that number of particle which is going to be printed.

So, for that what I am going to first calculate that this is my collisional dimensions ok, this is my collisional cylinder, one particle which is moving they are some other particles is available the some particles are actually out of the system and this is my collision cylinder. So, if I know the number density of the system first what I want to find it out that number of particles which are present within this collisional cylinder.

Now, how to calculate that? That number of particles within this we can say that dou n and dou n is nothing, but number density into dou v or the volume of the collisional cylinder and the volume of the collisional cylinder will be nothing, but it will be pi this diameter this whole diameter is actually 2D for 2D p I will write. So, that you do not get confused that D p is nothing, but the diameter of the particle. So, the radius will be D p you can say that pi it will be D p square. Now, we have to multiply with the length. Now, length is nothing, but this length and that length is nothing, but if the particle is moving with a speed v r then it will be v r into dt. So, it will be this d square into v r into dt.

So, this will be the number of particle is going to present inside the collisional cylinder. Now, number of particle present inside the collisional cylinder means that many number of collision it is going to have because whatever the particle is presented in the collision cylinder it is going to have a collision with it.

Now, I can write in terms of the frequency so that I can get the collisional frequency. Now, if I have the number density I can write dou n upon dou t is nothing but n pi D p square into v r ok, and this door upon dou t is nothing, but the number of part collision per unit time it means it is going to be the frequency of the collision I will write it as f c, f c is nothing but is the collision frequency and that is going to be equal to that this is and this is going to be equal to n pi D p square into v r. So, that is going to be the collisional frequency of the particle. So, this many frequency this will be the frequency of the collision ok.

Now, what we are interested is we are interested in the tau c and tau c is what collision time. So, if I have that frequency I can find it out what is collision time and how we can find it out because we know that frequency is inversely proportional to the time. So, this will be nothing, but actually will be equal to 1 upon tau c. So, this is collision time and this is the collision frequency. So, I am going to have both.

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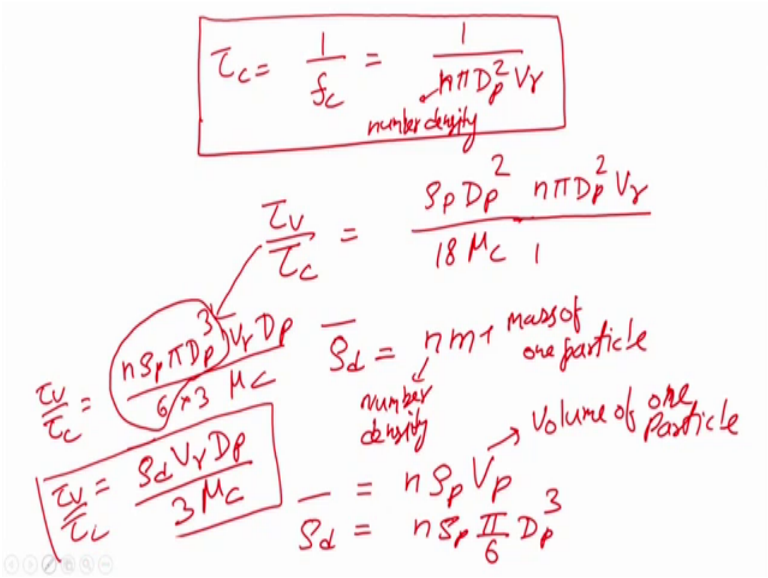


Now, I will just modify it. So, tau c is nothing, but it is going to be one upon f c and that is going to be 1 upon n pi D p square into your v r, that is going to be the your collisional time ok. So, this is your collision time and n please remember is nothing, but this n is nothing, but the number density.

Now, whether the flow is dilute or flow is dense what we need to find we need to find the ratio of tau v upon tau c. Now, tau v upon tau c if it is less than 1 then the flow is going to be damaged, now if we do that, v by tau c what I am going to do is I am just going to calculate the value tau b by tau c. Now, tau c is nothing, but it is rho p into D p square upon 18 mu c and tau c is nothing, but it is and 1 upon n into pi D p square into v r. So, this is going to be the ratio of tau b upon tau c.

Now, what we can do? We can solve it further and we can instead of number density because that is sometimes very difficult to calculate or to find inside the multi phase flow system. But the calculating the bulk density is relatively easier as you already know that if you know the volume fraction you can calculate in the bulk density. I can write this in terms of the and in terms of the bulk density and how to write that we know that the bulk density rho d bar is nothing, but is equal to number density into mass of one particle. So, that is number density we have already introduced that. So, number density into mass of one particle ok.

So, if we do that mass of number density and mass of one particle we can write it. Now, mass of one particle we can again calculate this will be n the mass of one particle is nothing, but rho of particle into V of particle, the V of particle is nothing but the volume of particle and volume of particle if the particle is spherical can be written as n rho p it will be pi by 6 D p cube, that will be equal to rho d bar.

So, we can replace this place in terms of the rho d bar. So, if you want to write that then what we have to do this n. So, I will just try to simplify this equation further here. So, this tau c upon tau c and I will write it in such a way that we can separate this things together I will write these terms together actually. So, I will write n, I will take rho p, I will take pi, I will take D p cube, I will write D p cube into D p and then I will write it as D p v r and D p.

So, I have just braked it and I will write here as 3 this 18, 6 into 3 of mu c. Now, if you see that this is n rho t pi D p cube upon 6 this whole term is nothing, but is equal to the rho d bar or you can say is the bulk density. So, it will be rho d bar q v r into D p upon twice mu of c that is nothing, but tau v upon tau c.

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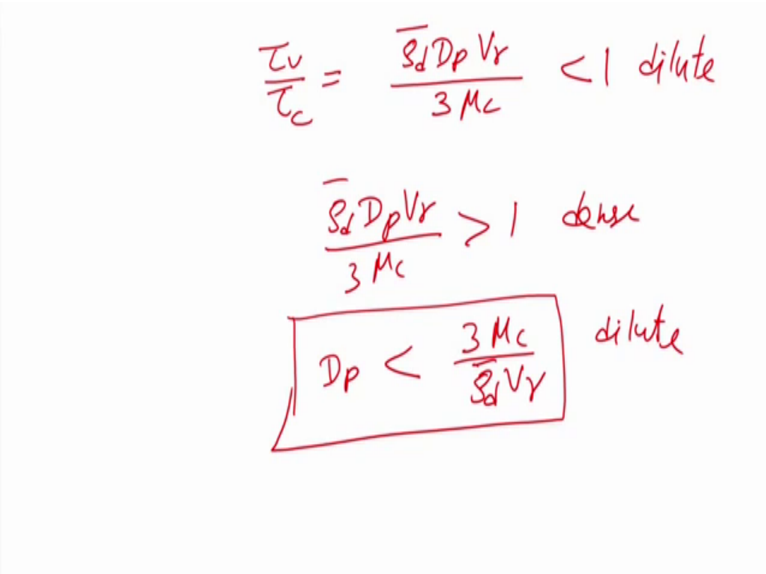


So, now, if we do this again I will write it separately tau v upon tau c and that tau v upon tau c is nothing, but rho d bar into D p into V r upon twice mu c.

Now, if the flow is dilute then what? Then this value should be less than 1, if this value is less than 1 you can say that the flow is dilute if this value rho d bar D p V r upon thrice mu c if this is greater than 1 the slowest dense ok. So, you can. Now, classify the flow is dilute of lowest dense and it can be further seen in terms of the particle diameter. So, suppose if I do it I can say the D p is less than thrice mu c upon rho d bar into here.

If this is correct if the particle diameter is less than this number or this quantity the flow is going to be the dilute if it is more than that the flow is going to be dense. So, these are the definition one can use to find that whether the flow is dilute or the flow is dense. So, that is the whole classification one can do with finding the dilute phase and dense phase system. So, this can be done properly.

So, what we have discussed now, till now, we have discussed that different terms terminology which we are going to use. We have discussed about the impaction, in impaction we have discussed about the stopping time, relaxation time or stop distance relaxation time and then stokes number.

Then we have tried to again divide the flow multi phase in two class major classes dilute flow, and dense flow in the dilute flow we have defined and we have tried to see that how to find that whether the flow is dilute or not and for that we have done the collisional cylinder we have found, we have taken the collision frequency fundamental and based on that we have calculated the collisional time. And we found the correlation which will tell that whether the particle is doing is the diameter is dilute whether the flow is dilute or the fluid states. So, this is the basic definition of the multi phase flow or basic classification of the multi phase flow.

Now, there are certain other things also which we would like to cover. I would like each other recover as a basic example and it is very critical to analyze the multi phase flow reactors that I will do in the next time.