Multiphase Flows
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Lecture - 15
Algebraic Slip method and Euler Euler Method

Welcome back. So, in the last class, we were discussing about the different modeling approach and I have said that what is the advantages and disadvantages of each modeling approach.

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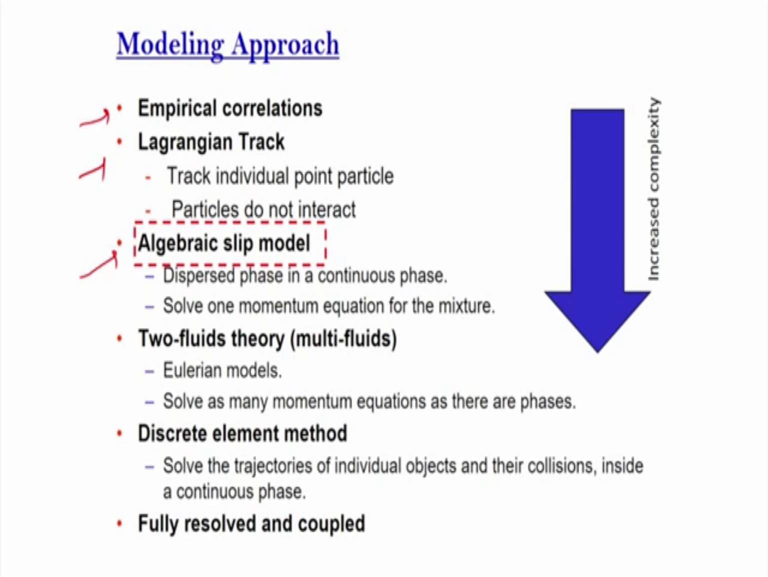

Modeling Approach

- Empirical correlations
- Lagrangian Track
  - Track individual point particle
  - Particles do not interact
- Algebraic slip model
  - Dispersed phase in a continuous phase.
  - Solve one momentum equation for the mixture.
- Two-fluids theory (multi-fluids)
  - Eulerian models.
  - Solve as many momentum equations as there are phases.
- Discrete element method
  - Solve the trajectories of individual objects and their collisions, inside a continuous phase.
- Fully resolved and coupled

And we have already covered about empirical correlation and Lagrangian track in the previous classes. In Lagrangian track is the same whatever we were solving the Newton's second law of motion by taking the different forces into the account. Whether, it is a buoyancy, it is a drag it is a gravity all those forces we are balancing the demo metafilter force or the momentum integral. So, that is what we are doing it.

Now, what we are going to discuss now is the other methods and one of the most kind of one of the other method are kind of very initial methods which is being used for the simulation or for the modeling of the multi-phase flow is the algebraic slip method. Now, why it has been used? Because, the major advantage of this model is that it is computationally very less expensive as we are solving only one equation for both the phases.

So, there is a continuous or dispersed phase both the phases we solve only one equation and we assume that the phase should be interpenetrating continuum. So, they are kind of inter penetrating each other and we solve the equation based on the mixture properties. So, that is the algebraic slip model. And it is initially developed and it is computationally very, very kind of simple or you can say the less rigorous and computational power requirement is less, compared to the other model whatever we are going to discuss.

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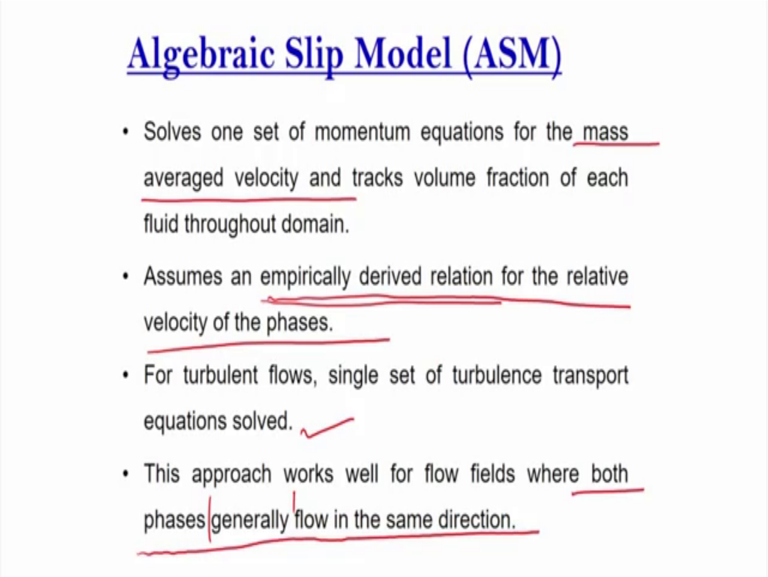

Algebraic Slip Model (ASM)

- Solves one set of momentum equations for the mass averaged velocity and tracks volume fraction of each fluid throughout domain.
- Assumes an empirically derived relation for the relative velocity of the phases.
- For turbulent flows, single set of turbulence transport equations solved.
- This approach works well for flow fields where both phases generally flow in the same direction.

So, let us begin with the algebraic slip model. So, what is algebraic slip model? As I said that it actually depend on the slip velocity and we solve the algebraic equations. That is why it is called algebraic slip model because for slip velocity we solve the algebraic equation. What we do we solve one set of momentum equation for the mass average velocity and we track the volume fraction of each fluid throughout the domain.

So, what we do? We solve once momentum equation one continuity equation which is based on the mass average velocity of the mixture. And what we do? We solve the volume fraction of each fluid throughout the domain. So, we are solving only one phase 1 momentum one continuity equation based on the mass average velocity, volume fraction throughout the domain. So, what we can do we can get the velocity we can get the your mass fraction or mass distribution and then with the volume fraction we will get that where the phases are actually present. So, that is the whole idea of algebraic slip model.

And because you are using the mass average velocity, you are just you just need to solve only one equation for both the phases and the mass average velocity will depend on the both the phases. We will see the equation then it would be more clear. So, what we do here the major assumption is an empirically derived relation for the relative velocity of the phases and that is what I said that and this equation is the algebraic equation which is empirically derived and that is the major assumption.

Because we are going to solve the velocity based on that what is the relative velocity and slip velocity in between the 2 phases and we are going to take the mass average velocity for the mature equation. And this relative velocity formula this relative velocity equation is actually empirically developed equation. So, your accuracy or your prediction will depend on the accuracy of this empirically derived relation and that is the major assumption of this kind of this modeling method. And that is why the accuracy of this modeling method is not very high particularly, when the dispersed phase concentration is very high or the slip velocity is very, very high and we will see that.

So, for turbulent flows again a single set of turbulent transport equations are solved. If you have the fluid which is turbulent you can solve the momentum equation, you can solve the continuity equation, you can solve the volume fraction equation and you can solve the turbulent equation along with it. So, what will happen you have now we will increase, number of equation will increase, the computational time will increase, but still as well or the model way it gives you the luxury to solve the turbulent flow also.

This approach is actually very good for the flow fields where both the phases or (Refer Time: 04:22) generally flow in the same direction. Please mind this word generally it is not like you cannot do the counter current, but generally it works well when both the phases are flowing in the co current mode. It means they are following in the same direction. Why? Because in that way the slip velocity is actually becomes lower.

So, in that this is the way over all kind of you can say that the overall approach which is being used in the algebraic slip model one of the initially developed model which actually starts solving the multi-phase flow equation from the one dimensional to the 3 dimensional domain, but because the computational power was not very high the equation was developed it in this way that you can still solve this problem of the multi-phase typical multi-phase problems and get some solution and we will discuss that what kind of a solution you can get from it.

Overall, what you do you solve one momentum equation one continuity equation for based on the mass average velocity. And then you solve one equation for the volume fraction and you solve one equation for turbulent flow if the flow is turbulent, if the flow is not turbulent you just solve the 3 equations and this is the reason that why your computational time is requirement is relatively lower compared to other models which we will discuss later in this course. That is the best part of this and if you just see the equation I am writing the equation here, this equation is for the continuity equation.

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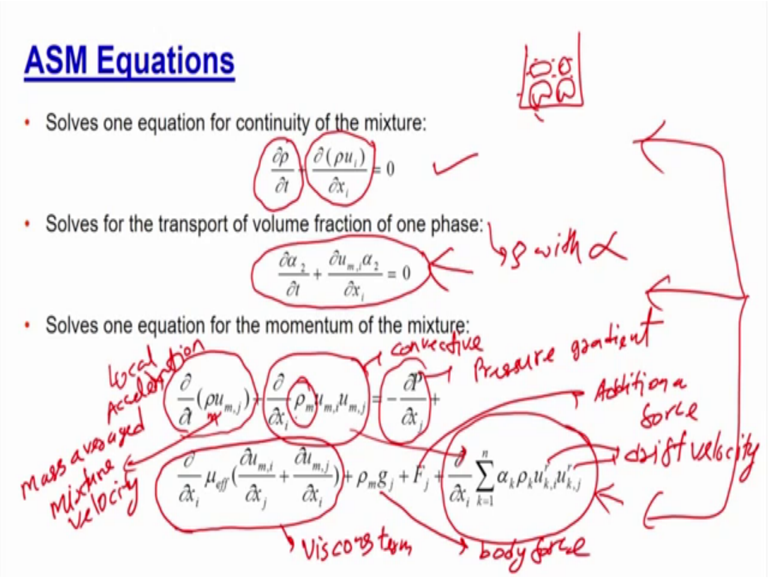

ASM Equations

- Solves one equation for continuity of the mixture:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0$$

- Solves for the transport of volume fraction of one phase:

$$\frac{\partial \alpha_2}{\partial t} + \frac{\partial u_{m,i}\alpha_2}{\partial x_i} = 0$$

- Solves one equation for the momentum of the mixture:

$$\frac{\partial}{\partial t}(\rho u_{m,j}) + \frac{\partial}{\partial x_i}\rho_m u_{m,i} u_{m,j} = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_i}\mu_{eff}\left(\frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i}\right) + \rho_m g_j + F_j + \frac{\partial}{\partial x_i}\sum_{k=1}^{n}\alpha_k \rho_k u^r_{k,i} u^r_{k,j}$$

If you see that this del rho upon del t term is there, this is the unsteady state mass transfer term and this is the convective mass transfer term. So, this is dou by dou x of rho u will be equal to 0, you are getting it from the ui and x i, it means this will be in all the 3 dimension, it will be kind of dou by dou x del u by del x plus del v by del y plus del v by del z or you can say that del rho u n upon del x 1 plus del rho u 2 upon del x 2 plus del rho u 3 upon del x 3. So, you can write it in this form and you can solve this equation the continuity equation. You solve the volume fraction transport equation which was I was take talking about that how the volume fraction is being transported.

Because you are solving only one equation and at the end I want that where is my gas? Where is my liquid? Suppose, if I am talking about a bubble column, again the same bubble column liquid is filled. Bubbles are being passed here, air is being kind of you can say it is passed and bubble is moving. At the end, I also want that where is my bubble? Where is my liquid? What is the fraction of the bubble? What is the fraction of air fraction? Or what is the water fraction available at any domain?

So, to do that, if you do not have that information then the solution will have very limited application. So, to get that information we solve a additional equation here that is the volume fraction tracking equation and that volume fraction tracking equation actually differentiate between the phases that which phase is there; so because we are solving only one cut momentum equation and one continuity equation. So, this is sure actually equation a is gives you the benefit or give you the luxury to separate the 2 phases and find it out that where the fractions are there.

And if you see that volume fraction tracking equation is very similar to the mass conservation equation. Instead of the rho we have replaced the rho with alpha in the volume fraction equation if you will see here. So, if you see that equation this is del alpha by del t and this a kind of unsteady state for a term for the volume fraction that how the volume fraction is changing with the time. And then this is the convective volume fraction transport term. So, how the volume fraction is being transported from one location to other location or you say the convective motion because of that how the kind of fraction of the phases is being transported.

So, this is exactly similar to the continuity equation the only thing is now, instead of the mass conservation or mass conservation you are solving the volume conservation equation or you can say that you are solving the volume transport equation instead of the mass transport equation. So, that is the way it has been written then we solve one momentum equation and that momentum equation is actually based on the mixture property mass average mixture velocities and we also introduced a drift velocity inside.

So, that we can understand that what is the slip between these 2 velocity of 2 phases. So, what we do? We solve the momentum equation if you see this is a clear cut or Navier stokes equation which is I have written in the 2-dimensional domain. It is an Navier stokes equation and then you add a extra term which shows that how the drift taking place between the 2 phases.

So, let us discuss this term this is the unsteady state acceleration term, this is how the particle is moving. So, del u by del t local acceleration or any steady state velocity term that it momentum term you can say a local acceleration I will say local acceleration ok. Then this I is again based on that the mass average mixture velocity and then you solve the convective acceleration term, this is convective term yeah you solve the convective term. And then the pressure gradient you solved that what will be the pressure gradient this is pressure gradient.

Now, this is viscous term I have written in the 2 dimension that is why you are seeing the 2 thing. This is viscous force or viscous term, this is the gravity of body force, this is any extra force any additional force which is acting on that. Please see that we have not taken the drag here because we are solving only one equation. So, we are solving maintain the equation based on the mixture property, that is why we are not including the drag in the momentum equation term.

Now, this is the term which has been extra internal included. So, that we can get the individual information of both the phases and that is very close to whatever the convective term is there. If you see these 2 terms which are very close there the convective term the only difference is that you are now multiplying it with the alpha I. So, what you are going to do you are going to solve all these 3 equations simultaneously. We are going to solve all these 3 equations simultaneously to get that how the individual drift is acting on each phases? That is why there is a summation time.

If you have 2 phase, you will solve this will be the 2 term one will be the 4 come say phase 1 and another term will be the same thing will be phase 2 and you will calculate this drift velocity. So, this u r is nothing but is a drift velocity and how to calculate. So, what is there days more unknowns are now you have introduced 2 more unknowns one is which is the mass average mixture velocity. This is mass average mixture velocity mixture velocity.

So, we need to define that what is this and we need to define your; which is nothing but the drift velocity. So, we have just modified the momentum equation we have just added one extra term, rest of the momentum equation if you will see it is very close it is similar to the single-phase flow. Only thing which is being added is this drift velocity term which is nothing but the convective acceleration of individual phases.

How the individual regions are moving and that is why the alpha is being introduced here. So, that we can have the information of the phase which we are the phase of the interest or for which we are trying to solve the problem and then we solve the alpha overall alpha volume and volume fraction equations so that we can sign that how the volume fraction transport taking place.

The accuracy or the unknown terms here is now is still the and ur that how they should be defined, how the ur should be defined, and as I said that the accuracy of these models depend on the definition of this and ur which are being defined mostly through the empirical correlations. How this rho is being defined? Because, if you see here also rho is everywhere the rho is the mixture rho this is also rho m this is the mixture rho.

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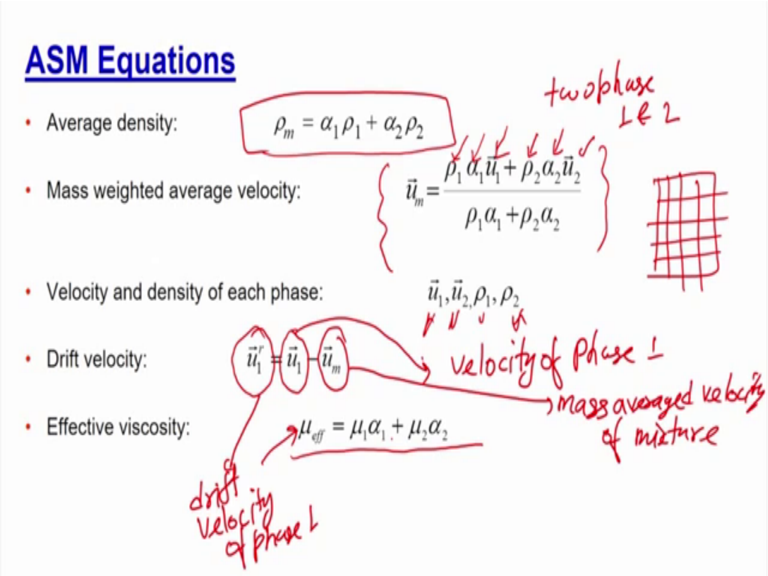

ASM Equations

- Average density: $\boxed{\rho_m = \alpha_1 \rho_1 + \alpha_2 \rho_2}$
- Mass weighted average velocity: $\vec{u}_m = \dfrac{\rho_1 \alpha_1 \vec{u}_1 + \rho_2 \alpha_2 \vec{u}_2}{\rho_1 \alpha_1 + \rho_2 \alpha_2}$
- Velocity and density of each phase: $\vec{u}_1, \vec{u}_2, \rho_1, \rho_2$
- Drift velocity: $\vec{u}_1^r = \vec{u}_1 - \vec{u}_m$
- Effective viscosity: $\mu_{eff} = \mu_1 \alpha_1 + \mu_2 \alpha_2$

So, mixture rho is being defined as we have already discussed during the introduction phase that the mixture rho is nothing but volume fraction of the phase 1, plus volume fraction of the phase 2. So, that is the way this mixture model this rho is being defined, a logistic model through mixture has been defined, then we need to define the mass average velocity.

Now, how to define the mass average velocity; so, mass average velocity has been defined by this empirical correlation which says that it is nothing but rho alpha one into un. If suppose it is a 2 phase, if it is a 2 phase 1 and 2 then it will be the density of the first phase multiplied by the fraction of that phase which is present at that location for where you are solving this equation.

If you are solving it numerically you will what you do you will discretize the whole domain, if suppose you are doing a numerical simulation we are going to discretize the whole domain and you will solve this equation for each cell for each node point. Depending upon what approach you are solving you are solving the cell weight wise all your solving the node wise you will solve it and then for each cell you fine to calculate this. So, what will happen? That alpha rho 1 is going to be remain same that is the density of that phase 1 alpha one will be changed and it will be updated through the volume transport equation for each cell and for each location and for each time. U n will be the velocity of that phase at that location which will be calculated rho 2 is nothing.

But that what is the density of that face alpha 2 again what is the holdup of volume fraction of that phase at u 2 and then it will be averaged with the mass average we are calculating the velocity. So, we have to divide by the total mass now what will be the total mass per unit volume if you take then it will be rho 1 into alpha 1 plus rho 2 into alpha 2 that will be the mass per unit volume. So, that is the way the mass average velocity has been calculate it means calculated in this cases and then you can use this here un u 2 rho 1 and rho 2 or the velocity and density of each phases respectively.

Then we calculate the drift velocity. So, how to calculate the if you remember that the drift velocity term which we have introduced, which actually multiplied by the alpha which shows that the phase information actually comes from that place the individual phase information. So, the drift velocity is being defined by ur drift velocity is the velocity of that phase minus the velocity of mass average velocity of the mixture.

So, this is mixture velocity of mass average mixture velocity and this is the velocity of that phase that will be the drift velocity of that phase. So, u and r it means the velocity drift velocity this is drift velocity of phase 1 of phase 1 will be equal to u n is the velocity of that phase velocity of phase 1 minus mass average velocity of mixture .

So, it what information it gives? It gives that how much fast or how much slow this velocity phase is moving compared to the mass average velocity of the mixture. So, you get the individual information of that particle and this ur you calculate and then from ur you can get the un. So, you can get the velocity of each phase there and then mu is being solved with mu 1 alpha 1 plus mu 2 alpha 2 as we have already discussed that the accuracy of these equation or mu this way to calculate the effective viscosity of the mixture is always questionable.

But this is the one of the best possible way to calculate the mu or you can develop your mu rating or your mutual relation based on the experimental observation by using some viscometer, you can calculate that. The best way of calculating it is this it may not be the correct way or 100 percent correct way, but this is the one of the best way to calculate the viscosity and you can do that with the mu 1 alpha 1 plus mu 2 alpha 2.

So, what you will do? Now, if you see this equation you are almost able to know everything. Now you can solve this and you can get the value of velocity for individual phase.

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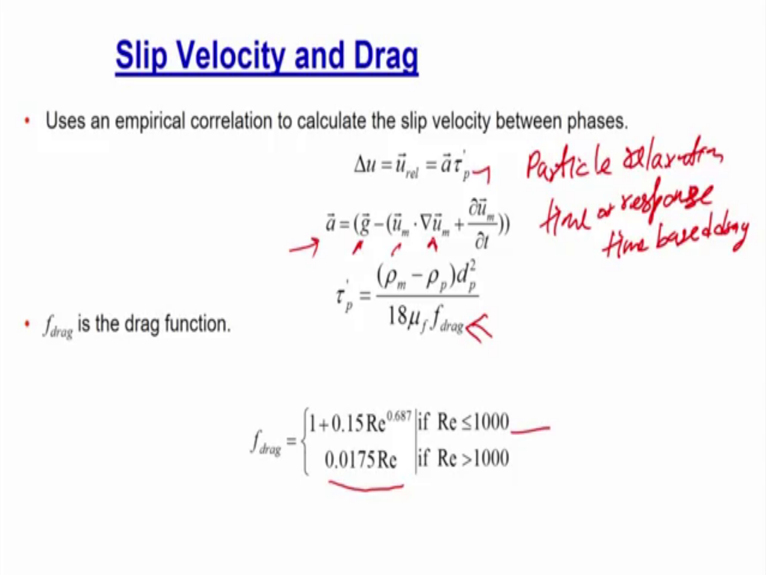

Slip Velocity and Drag

- Uses an empirical correlation to calculate the slip velocity between phases.

$$\Delta u = \vec{u}_{rel} = \vec{a}\tau_p'$$

Particle relaxation time or response time based drag

$$\vec{a} = (\vec{g} - (\vec{u}_m \cdot \nabla\vec{u}_m + \frac{\partial \vec{u}_m}{\partial t}))$$

$$\tau_p' = \frac{(\rho_m - \rho_p)d_p^2}{18\mu_f f_{drag}}$$

- f_{drag} is the drag function.

$$f_{drag} = \begin{cases} 1 + 0.15\,\mathrm{Re}^{0.687} & \text{if } \mathrm{Re} \le 1000 \\ 0.0175\,\mathrm{Re} & \text{if } \mathrm{Re} > 1000 \end{cases}$$

So, that is the way it has been solved and then what you do you use some empirical correlation because what we have not included till now is the drag we have not included somewhere the drag and we know that if the 2 phases are flowing together, one phase is in discrete, then what is going to add the most important force is going to be the drag.

So, what we need to do we need to include the drag in our equation then only being able to see the relative motion between the 2 features or the slip motion between the 2 phases. So, the drag is being defined that what we do is define the relative velocity that what will be the relative velocity or the slip velocity and the slip velocity or relative velocity has been defined with the a bar of tau p prime. So, this is not the real particle relaxation time this is very close to particle relaxation time that is why we have defined it as a tau p, but I put a prime. So, that it is looks little bit different from whatever the particle relaxation time which we have discussed earlier.

So, please do not get confused this is the particle relaxation time, but it is not purely the particle relaxation time. It is the particle relaxation time based on the drag, that is the way we define inside that this the particle relaxation time or response time based on drag or if I have been defined there is a 2 term have been introduced one is a and these all are empirical equation developed equation. This is a times tau p prime and it is being defined as a overall acceleration which the particle is seeing.

So, that is being the gravity if the particle is moving or discrete phase is moving anywhere, the gravity will be acting what is the convective acceleration which is going to take a role. So, this is dot del so, that is the convective acceleration term and then what is the local acceleration term del upon del t and is being calculated is nothing but the mass average velocity. So, that is the way we calculate the a value if you know the you can calculate the a value very easily then this is the tau p prime n.

Now, again I said that we have not included the drag earlier. So, we have include the drag in the slip velocity or relative velocity and that we are using again question this is being empirically developed. That is the tau p prime and tau p prime is being actually defined as rho m minus rho p, which is the mixture velocity mixture density minus the rho of that phase rho a particle or discrete phase into multiplied by dp square which is nothing but the dp square is nothing but the that phase diameter or discrete phase diameter upon 18 mu f into f drag.

So, we multiply it with the drag factor and this drag factor is being calculated based on the shear Norman correlation for Reynold number less than thousand and for the Reynold number greater than thousand it is being used with 0.0175 into re. Now, you can always use your own drag you can write a you do for you can write your own program to use your own drag whatever you have kind of you want to use or you feel that it will be best suited for your system, but this is the way it has been introduced.

So, drag is also being introduced. So, if you see that very smart way of modeling the thing in a simplistic solution where you can solve very few equation, most of the equations are these equations are algebraic equations. So, it does not numerically does not require much time to solve and you have only 3 equations which are coupled together and you solve it. So, all these equations we solve together to get the velocity field to get the volume fraction field and we have also included the drag so that we can have the idea about the slip velocity and relative velocity also.

This is the way we solve the algebraic slip modern equation, these are the equations which we solve we solve is simultaneously and we get the data. The only problem or restriction is that as I said that, we are what we are doing we are solving only one equation instead of the 2 individual equations for individual phases now we are solving only one equation and we are assuming that the slip velocity is not very high. You have to see that there will be several restriction of this flow you cannot use it the accuracy is limited.

Because we have oversimplified our model accuracy is going to be limited, but it predicts well under certain conditions and there are certain condition it is now you cannot use it. So, the major restrictions which are there for this model this is not only these restriction, these are the few restrictions which are very, very critical and one need to be understandable need to be careful before using this kind of approach is first after this the restriction is the particle relaxation time.

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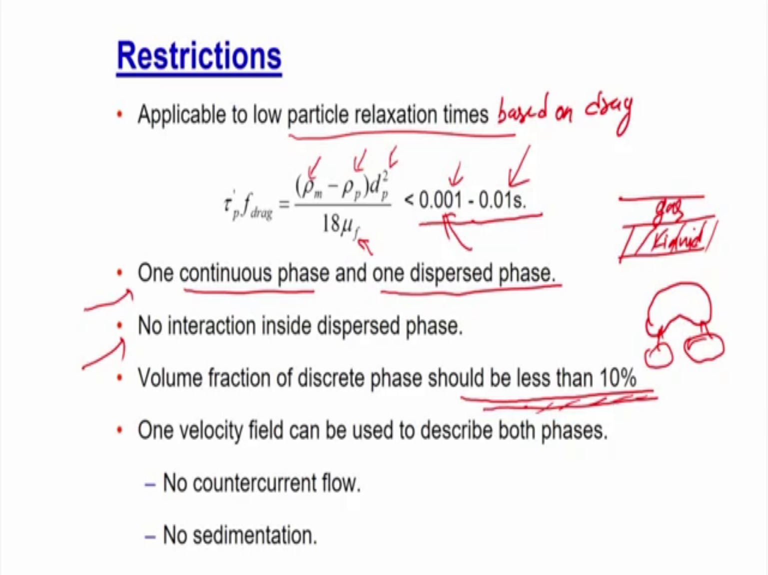

Restrictions

- Applicable to low particle relaxation times based on drag

$$\tau'_p f_{drag} = \frac{(\rho_m - \rho_p) d_p^2}{18\mu_f} < 0.001 - 0.01s.$$

- One continuous phase and one dispersed phase.
- No interaction inside dispersed phase.
- Volume fraction of discrete phase should be less than 10%
- One velocity field can be used to describe both phases.
  - No countercurrent flow.
  - No sedimentation.

The way we define the particle relaxation time, I will say that particle in relaxation time based on drag. So, that there should be no confusion with the initial particle relaxation time response time the way we have defined. So, tau p prime into f drag which is being nothing but the particle reactions in time rho m minus rho p into dp square upon 18 mu f and should be less than 0.001 to 0.01 second, it should be in this region or it should be less than that.

Why I have given 2 range because some book says this some book follows this. So, that is why I have given both the range it should be definitely less than this. This value lesser the accuracy of the model will be much higher, but some books also say it is odds group also says that it should be even less than 0.01, it will be working fine that is the way and if you see that if I know about my system, I do not need to do the simulation and that is why we define it in this way that it is tau p prime into f dragged because f drag.

If you want to calculate you have to calculate the non-number for that you need the sitting inside everything to need. So, that is why tau prime into f dragged that way which is nothing but rho m minus rho p dp square upon 18 mu f, you can calculate all these things rho m can be calculated.

If you know that what is your lined off density of individual phases, what is the volume fraction of the individual phases? If you know the density of the discrete phase, if you know that roughly that what should be the size of your discrete phase particle or discrete phase bubble. It can be bubble it can be droplet, it can be particle what which will be you can calculate the mu f if you have again some idea about the volume fraction if it is less than 0.001 you can use this model or less than 0.01 you can use this model.

But above than that is this model applicability is really low and accuracy will be very, very low then again there is one limitation again the other limitation is that one phase should be continuous and one phase should be dispersed. If 2 continuous phases are moving together then it is difficult to model. So, it means if you want to model a separated flow it is suppose there is a pipe, where gas and liquid are flowing this is liquid this is gas ok. So, this is liquid this is gas then you cannot use the algebraic slip model.

So, 1 phase need to be in discrete one or dispersed 1 phase need to be continuous then only this model can be use then, if suppose the phase is disposed you cannot see the interaction inside the dispersed phase. So, suppose if there is a bubble and that bubble is rotating or is changing the shape of it. Say this and it is happened with the Danckwert's theory or surface renewal theory, if you will see that this is that the bubble actually keep on changing the shape is the surface stretch theory. Those kind of interactions if you want to model which is inside the dispersed space you cannot model it ok.

Even we are not modeling anything about the bubble coalescence. So, suppose if the 2 bubbles are there they are coming and close to each other and they are being form a bigger bubble. We are not doing we are not modeling all those things. So, the interaction inside the dispersed phase we are not modeling. So, that is again a limitation because most of the time in the bus a gas liquid system particularly bubble column, we have seen that bubble shape bubble size keep on changing ok. So, that that the interactions you cannot actually mind this cannot solve or cannot model by using the algebraic slip model. So, that is the one of the limitation.

Another limitation is that the discrete phase fractions would be less than 10 percent and this severely limit the application of this. Now, again there is a debate here some people say 10 percent, some people says 5 percent. I am just writing all the numbers I am giving you the bigger number here say 10 percent definitely above than 10 percent you cannot do that because discrete phase interactions will be very, very high one way coupling will cannot be used you have to have 2 way, 3 way, coupling at least or 4 way coupling and that is why here whatever we are doing we are solving just one way coupling we are solving the equation of both the phases it is only troubled with the drag only ok.

And that drag is also not the part of the main equation drag, we are actually taking into the account in the slip velocity ok. So, we kind of that is the major limitation of this that if your discrete phase fraction is more than 10 percent PSM model algebra slip model cannot be used. And in most of the industrial problem industrially relevant thing kind of reactors or element vessels the volume fraction of discrete phase is generally more than 10 percent almost all the time ok.

Other than the few specialized cases and that is why the accuracy or kind of just the model prediction is very, very limited ok. The applicability of this model is very, very limited then again as I said that it is desirable that both the phases would flow in the same direction, then the accuracy of the model is higher. So, in this model ideally cannot be used for the counter current flow and for sedimentation slope it means where the particles are being settled out and the fluid is moving up.

In this kind of a flow, you cannot use this model and even in case of the counter current flow say gas is coming from the top, liquid is going from the bottom or vice versa gas is going from the bottom to top and liquid is coming from the top to the bottom you cannot use this model the accuracy of this model will be limited. So, that is the major kind of you can say restriction. So, I have shown here the example of a bubble column which has been solved by using the algebraic slip model.

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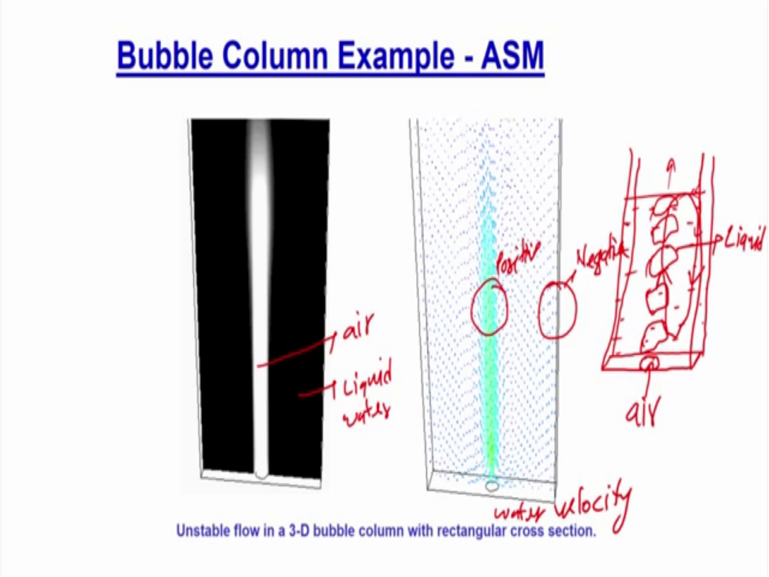

Bubble Column Example - ASM

Unstable flow in a 3-D bubble column with rectangular cross section.

And the geometry is there is a column is a 2-dimensional column and what we have done is a small bit here to show that the column can be used or the model can be used for even the 3-dimensional domain this is the geometry and at the centre we have made a hole of certain dimensions. I think this dimension is more around one centimeter and we have passed the air from this. So, this whole column is filled with the liquid this whole column is filled with the liquid and air is passed at the center of the column by air is this passed through the center of the column.

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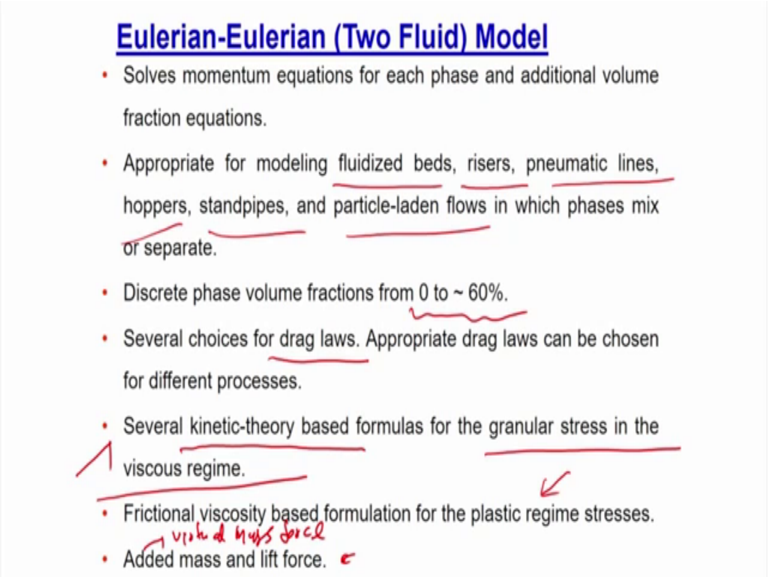

Eulerian-Eulerian (Two Fluid) Model

- Solves momentum equations for each phase and additional volume fraction equations.
- Appropriate for modeling fluidized beds, risers, pneumatic lines, hoppers, standpipes, and particle-laden flows in which phases mix or separate.
- Discrete phase volume fractions from 0 to ~ 60%.
- Several choices for drag laws. Appropriate drag laws can be chosen for different processes.
- Several kinetic-theory based formulas for the granular stress in the viscous regime.
- Frictional viscosity based formulation for the plastic regime stresses.
- Added mass and lift force.

virtual mass force

So, this is liquid and this is the air is passed there. So, what will happen? The air will lift at the center in the downward directions well the bubble will lift here at this location, the velocity is very low. So, the bubble fraction is very low bubble will lift it in this way liquid will move upward at the center of the column which you can see here. So, most of the bubble fraction actually will be at the center of the column. You can see here the air fraction which is actually at the center of the column it is showing here the white color is actually shows air this is air and this is liquid or water in this case we have taken water.

So, you can see the air is flowing at the center there is no very small amount of the air or almost no air is there near the wall and most of the liquid is actually near the wall. This is the air this is the water you are able to discriminate the 2 phases very clearly and that is you are able to do because you are solving the volume transport equation. So, though we are solving only one equation, we are able to discriminate between the phases that which phase is present at got location. Entirely, see that there is more air fraction at the center there is all no air fraction or very low air fraction near the wall.

We are able to differentiate between the phases and we are able to do it why because we are solving the volume transport equation, we are able to also solve the drift equation drift velocity and we are coupling it with the slip velocities too. So, you can solve that the relative motion then we have also calculated the velocity of the water why we have not plotted the velocity of air because the velocity of the air will be only at the center end and it will be moving upward.

So, we have also tried to show the velocity of the water this is the velocity of water this is water velocity and this simulations are being done in and it is fluent. And so, what we have this the air is being passed because the air is moving up at the center of the column water will also move up at the center of the column and you can see that if I just zoom it, you can see that the velocity are positive at the center of the column.

If you will see this the velocity is positive look at these location you can see that the velocity is positive ok. So, you can see this and near the wall the velocity is negative because, what will happen? If you see this location again that near the wall the velocity is negative and why it is happening because if you see positive here, it is negative velocity negative I do not know whether, you are able to see it earlier positive and zoom it you can see this velocities that is here the particle is moving up and in this location the part this water is actually going down.

So, here liquid or water is moving up here, it is going down why it is moving up here because air is moving up a bubble is moving up that side it is pushing some of the liquid with it. And that is why the liquid is also moving up then at a certain height at this height what happened that air actually disposed to the atmosphere and move out of the column and goes to the atmosphere while liquid have not that much momentum it has no more buoyancy force, it will fall down again and then it will start coming down from this side.

So, you are seeing a simple circulation cell and we are able to predict that circulation. So, the ASM model ok. So, these are the simulations this simulation has already been shown in the literature we have just repeated the simulation to see that ok. You are able to get the same profile this is a standard way to solve the bubble column. So, if your bubble is the discrete phase volume fraction is very low say less than 5 percent or 10 percent you can use this column to get the information and we have able to get that those information in a very, very in a very quick way or relatively faster way compared to the models which we will discuss here after. So, that is the advantage.

But the major problem is as I said that the particle relaxation time multiplied by f drag should be less than 0.01 or 0.01 depending upon which group you follow I would recommend 0.001 here and second thing that the flow should not be counter current and the discrete phase fraction should be less than 10 percent for sure. So, lower the discrete phase fraction better accuracy that is the way it is kind of the model has been developed is relatively simpler as we seen the equations and you can solve that approach. So, that is all about the algebraic slip model which is being used some people also called it as a mixture model or algebraic slip model which is being used here I have shown here that how-to kind of calculate a model in the this 2 phase flow.

In some of the similar kind of commercial software's though if you can want to write you can write your own course, but if you are planning to use some commercial software be aware some of the commercial software does not allow to use more than 2 phases for ASM model. It does not mean that ASM model cannot solve 3 phase problem it can still solve the 3-phase problem you just need to modify the equations we just need to modify the mixture velocity, the mixture density mixture viscosity the way the mass average velocity has been defined you have to modify those equation and can easily modified the third component will be added or third phase will be added.

But most of the commercial software's actually restrict the ASM application for 2 phase flow only. So, kind of do not worry do not be confused that it can be used only for the 2 phase it can also be used for the 3 phase definitely the volume fraction if it will be 3 phase will be further higher interactions of the phases will be further higher. So, accuracy of the productions will be lower.

But mathematically there is no restriction you can use it theoretically there is no restriction you can use it ok. So, that is the overall about the algebraic slip model now we will move to the next model which is one of the most commonly used and very popular model to model the multi-phase flow reactor and that model name is Eulerian Eulerian model as I said earlier also in discussion that Eulerian Eulerian model. What we do we assume all phases to be interpenetrating continuum, ok.

And that is the one of the major assumption which you do I have already discussed about that a junction that what interpenetrating continuum means it means suppose, there is a solid phase there is a gas phase they are moving together. You will allow the solid to go inside the gas you will allow the gas to go inside the solid physically in the real world it is not possible, but in the Euler models you. So, that all you kind of describe it it in that way.

So, what happens for any small cell whatever you are solving you will not get that where is the particle exactly where is the solid gas exactly, you will get the fraction of the solid fraction of the gas in that cell and that is the way the kind of the phases has been defined with the volume fraction here again and we will multiply the equations with the volume fraction for each word term. So, that we can get that for each velocity for each cell whatever the velocity or whatever the stress we are calculating what is the contribution of the each region.

You will see the equations, but that is the major assumption that you assume that the particle should be interpolated continuum there is no discrete nature of the particle and that is the major limitation also, because you do that several interaction forces you cannot solve it or you have to solve by using the empirically developed equation or you have to model it by using some equations, but directly you cannot solve this and that is limits that kind of use or you can say the accuracy of this approach.

But still this is the only approach till date which can be used at all the scale you can solve the laboratory scale problem you can solve the pilot plant a scale problem you can solve the industrial scale problem, but only the completion time will be higher, but it is has a potential to do that and that is why this model is widely used this model is also being known as a 2 fluid model.

Particularly if you are solving it for both the fluid phases and you see is called as a 2 fluid model and why 2 fluid model is being solved called because both the phases whether it is a solid or it said any other phase it is a solid also gas also liquid whether it is a dispersed stage both the phases you assumed to be fluid and that is why it is also called 2 fluid model commonly.

So, what we do in this we solve the momentum equation and continuity equation for each phases individually. So, what we were doing earlier we are solving only one momentum one continuity here you will solve n number of momentum if you have n number of phase you will solve n number of momentum equation and number of continuity equation. So, it means if I have a 2 phase I will solve 2 continuity 2 momentum for both the in this kind of approach.

So, what is going to happen your computational power requirement is going to increase. Because earlier, you were solving only 3. Now, you are going to solve 4 if you have a turbulent model you will solve actually 5 you will also include the turbulence if you are including the turbulence in both the phases you are going to solve 6. So, your number of equations are increased and that is why the computational time means higher compared to the algebraic slip model.

But the accuracy is also better compared to the algebraic slip model because now you are solving the individual equation for individual phases has been widely used for the modeling of fluidized bed risers pneumatic lines hoppers standby is particle laden flow bubble columns slurry bubble column all kind of a reactor this approach has been widely used to model other than the pure granular flow where only granules are flowing ok. So, that is the place where it is kind of it is being not model or it is decorations will be hampered.

But other than that, it can be used anywhere even for the flow which is being with the both the phases are separated then also this model can be used if they are dispersed then also this model can be used. So, the applicability of this model is very wide you can use for almost all the cases in this model and that is why the acceptability of this model is also relatively higher. Because, we are solving the individual phases equations and we are coupling those equations are those phase with the drag and the other forces which will be required to solve the a higher order coupling.

So, we discuss about the equation, but that is the way this whole thing is there and it has been approved used for modeling all kind of a reactor the discrete phase volume fraction can be anywhere between 0 to 60 percent. So, it gives you almost the luxury to solve all kind of a reactor. So, even if you solve the pad bed the generally the impact, but the maximum volume fraction goes in the range of 60 percent. So, you can solve till the back bed then you can solve the pad bed to find that how the velocity distribution is taking place in the trickle bed also people have used that this is gives a lot of flexibility.

Now, if you see the restriction of all the ASM restriction is now being removed actually and then you can use the drag of whatever the drag you want for your system. You can write your UDF or use your own code to use the drag. So, it does not restrict on your drag model you can use the kinetic theory of granular flow or you can based on that to solve the granule stress in the viscous regime and it means what it is going to solve the equation between if you remember, the closure equation you will solve the drag if you want to do the find that the interaction between the mean motion of the solid mean motion of the gas or I will say mean motion of the discrete phase and mean motion of the continuous phase.

You can take the turbulence equation you can include the turbulence equation, if you want to solve the mean motion of the gas range or the fluid phase to the fluctuating motion of the fluid phase. You can use the kinetic theory of gases if you want to solve the interactions between the mean motion of the solid phase and subjugating motion of the solid phase all these things. You can include, you can solve, the viscous regime flow you can solve the granularity stress by using the KTgl.

So, that flexibility is also there you can use that the frictional velocity formula in the plastic regime can also be used. It means it can be used not only for the viscous regime it can also be used for the plastic regime you can add all the forces. Whatever we have discussed whether, it was virtual mass force or added mass force it is also called virtual mass force lift force all these equations you can include and you can solve this and that is why this is the applicability of this model is enormous.

Because, it gives you the flexibility to do whatever you want to do the only problem is that accuracy of all this equations this model depends on the closure equations or the closure models which you used to solve these equations and because these equations are now the number of variables are more number of equations are less. You have to use the closure equation to close that and your accuracy of the solution, now is going to depend that how good are your closure equations.

So, how good are your drag force equations we have discussed, several drag forces for the same thing for either it is a gas liquid flow liquid. You can flow a gas solid flow we have discussed several type of drags. Now, depending upon how accurately your drag is able to predict the motion it will the accuracy of this Euler model will depend on that.

How accurately the kinetic theory of granular flow models are being used to predict the fluctuations; that means, the way it will able to predict the motion or how accurately your turbulence model. Whatever, you are using whether it is a k epsilon k omega or any RNG model whatever the accuracy; however, it is able to predict the fluctuation motion.

The accuracy of the island model is going to depend on that. So, that is the way it has been solved and if you use for the granular flow regime before I go and discuss the equation.

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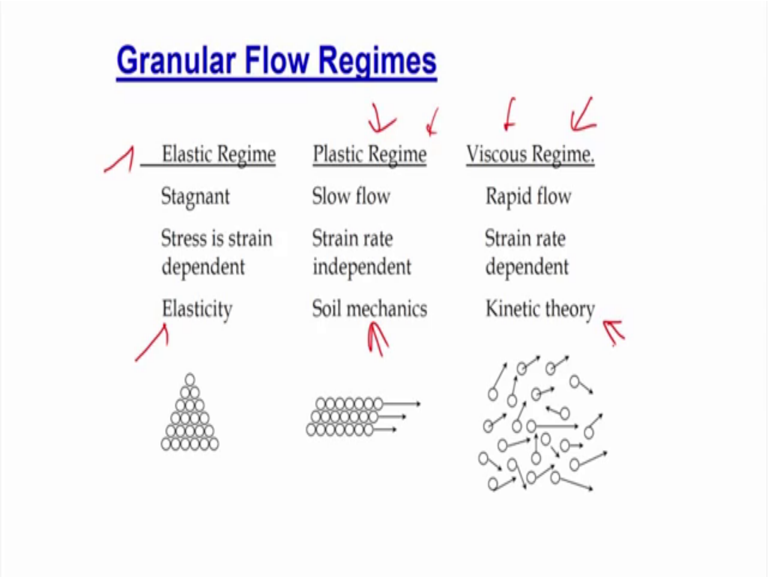

Granular Flow Regimes

Elastic Regime	Plastic Regime	Viscous Regime.
Stagnant	Slow flow	Rapid flow
Stress is strain dependent	Strain rate independent	Strain rate dependent
Elasticity	Soil mechanics	Kinetic theory

So, I said that it can be used for plastic regime it can be used for the viscous regime for the granular flows. So, generally for the granular flow 3 regimes are there one is a elastic regime plastic regime and viscous regime this things you might have been done in your solid mechanics course.

But just for a revision I am doing it that this Euler model can be used for this places the elastic regimes are generally stagnant. They do not move till move and the stress and strain are dependent to each other and may still clean the collision is modeled with the elasticity that, how they will move or the collision is modeled totally through the elasticity ok. So, that is this kind of a flow is called elastic regime.

The plastic regime is where the flow is very, very slow the stress and strain rate is independent that does not depend it on that the motion is not depend on the strain rate.

And generally, the models and when the flow models has been done through the solid mechanics that how the solid mechanics problems we solve we solve in the plastic regime in the viscous regime the flow is very fast it moves very rapidly because they are in the viscous regime.

A strain rate are dependent ok. So, if you change the strain rate your overall mission will also get changed your collision will also get changed and the flow is missed mostly dependent on the kinetic theory..

So, the kinetic theory of granular flow is being used to model the flow as the flows are very much collisional dominating and they are moves very fast and the strain rate is dependent these are the 3 regimes I am not going in detail of this. I am just giving you just trying to revise it you can see it in your solid mechanics courses that what are the details of these regimes, but that is the basic classification of these 3 regimes and Euler Euler can be used for the plastic regime and viscous regime.

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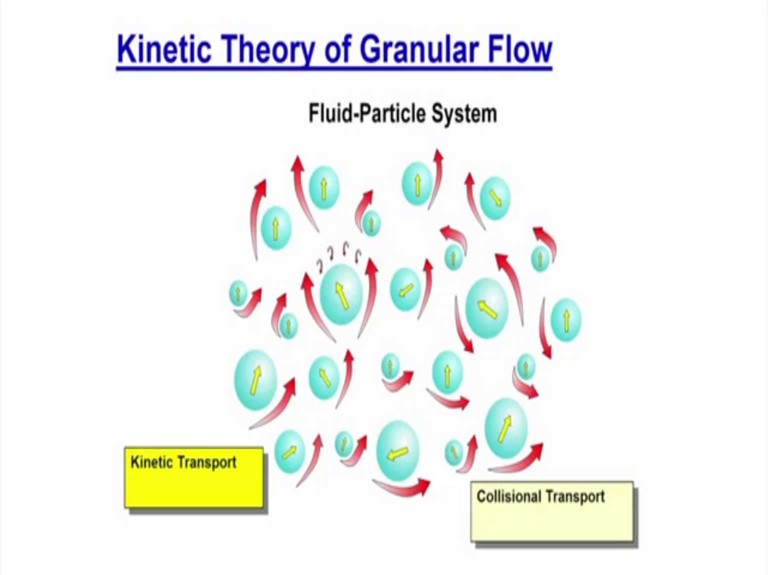
Kinetic Theory of Granular Flow

Fluid-Particle System

Now, what I said that in the viscous regime, but maybe most of the fluid ice bits comes under the viscous regime what we need we need a kinetic theory of granular flow to model the collision to model the flow actually because the flow is depend on the kinetic theory of the flow now kinetic theory of granular flow it is itself is a very vast topic and cannot be covered here.

So, the domain of this course is not to discuss the kinetic theory of granular flow in detail or the derivation of the kinetic theory of granular flow you can follow any book to see that, what is the how the kinetic theory of granular flow has been defined or derived? And you will see that it is actually nothing but the kinetic theory of the granular flow is very similar to the kinetic theory of gases you take that approach instead of the gases in gaseous kinetic theory of gases. We assume the gas molecular to be a point particles the size is infinitely small here we take a finite size of the particles.

Now, the moment you take the finite size of the particle and the particle is being placed in the fluid which is moving and the particles are free to move randomly. Anywhere, they will have the collision and the flow will be mainly depend on the kinetic transport it means the transport which is because of the fluid and again, the another transport which will be dominating is the transport because of the particle collision. So, that is the way the solute particle system is being defined.

And for this kind of a flow kinetic theory of granular flow is being used and being modeled how the kinetic theory of granular flow model has been described this all a brief description again I am telling that it is a brief description it is very difficult to cover it in a small time it requires abstention time to show you that how the character theory of grammar flow has been defined.

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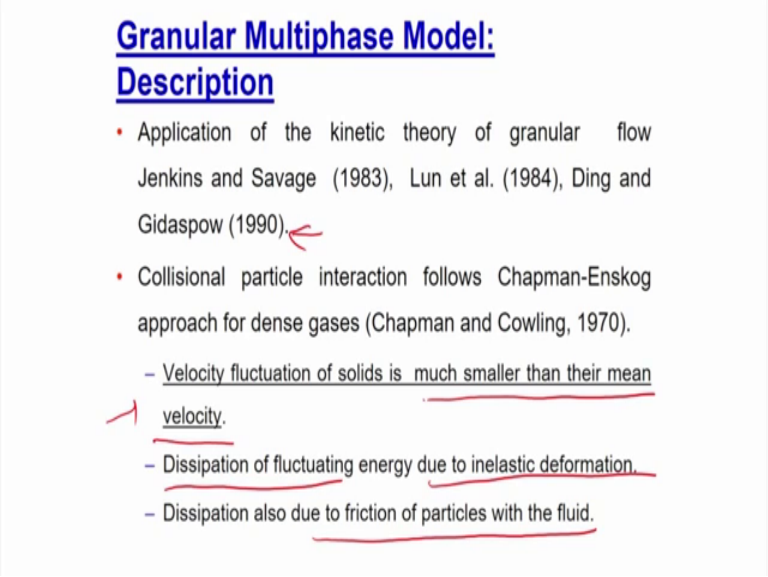

Granular Multiphase Model: Description

- Application of the kinetic theory of granular flow Jenkins and Savage (1983), Lun et al. (1984), Ding and Gidaspow (1990).
- Collisional particle interaction follows Chapman-Enskog approach for dense gases (Chapman and Cowling, 1970).
  - Velocity fluctuation of solids is much smaller than their mean velocity.
  - Dissipation of fluctuating energy due to inelastic deformation.
  - Dissipation also due to friction of particles with the fluid.

So, basically kinetic theory of granular flow is very close to the Chapman Enskog theory, it is not very close to the actually the kinetic theory of the ideal gases or kinetic theory of gases, but it is very close to the Chapman Enskog theory where the Chapman Enskog theory. If you remember your kinetic courses then it has been might have been told that Chapman Enskog theory what we do it is for the real gas.

So, you do not assume that the particle is a point particle you assume the particle has certain size and then you try to calculate that what will be the interaction forces will be acting on that. So, the Chapman Enskog and then you define the attraction potential repulsion potential energy own potential all those quantities you define.

Kinetic theory of granular flow also is very close to the Chapman Enskog theory and we assume that the particle has a finite size and that is particle finite size we see the interactions. So, the basically if you see that the application of 4 4 4 sorry kinetic theory for granular flow has been developed initially by the Jenkins and savage in 1983, then Lun et al has further modified it in 1984 ding and get Gidaspow has done the same in 1990.

So, this is a chronological development of this; what is based on this is based on the collisional particle interactions. So, what we are doing we are seeing that how the particle will you have a collision and because of that collision how the motion of mean motion will be correlated to the fluctuating motion of the particle. So, that is the whole idea and the kinetic theory of the underflow.

So, what we do the velocity fluctuation, what is the assumptions are that the velocity fluctuation of the solid is much smaller than their mean velocity. So, that is the one thing which we should keep in the mind that if you are using the kinetic theory of the granular flow your kinetic, it means the velocity fluctuations in the solid should be much lower compared to the mean velocity of the solid.

If the flow is completely collisional dominating and the mean velocity of the solid is much lower, then the fluctuating velocity of the solid may be this model will not give you the much accurate position, this kind of prediction and why I have said this because some of the bits. And we will discuss it while we discuss the binary fluidized bed and all where you have the distribution of the particle, which you use may be it may possible that the flow is actually collisional dominating.

We will discuss those cases, where the flow you will see that the velocity of fluctuation velocities are much higher compared to the mean velocity. So, in these kind of cases the accuracy of the granular model predictions the kinetic theory of granular gas granular flow will be limited then dissipation of the fluctuation energy due to inelastic deformation. We do that the dissipation of the fluctuation energy due to the inelastic deformation that we take and then dissipation also due to the friction of the particle with the flow.

So, these all dissipation we take into the account and we use the Chapman Enskog theory model to do good you find that, what is the kinetic theory of granular we develop the kinetic theory of the granular flow.

So, what is the major thing this is that the particle fluctuation velocity should be less than the mean velocity. We calculate the dissipation of the fluctuation energy due to the inelastic inferred deformation. So, particle deformation is also being taken into the account the dissipation due to the friction of the particles with the fluid is also being taken into the account. So, all these things is taken into the account and based on that a granule phase kinetic theory of granular flow has been developed and a quantity has been defined that is called granular temperature.

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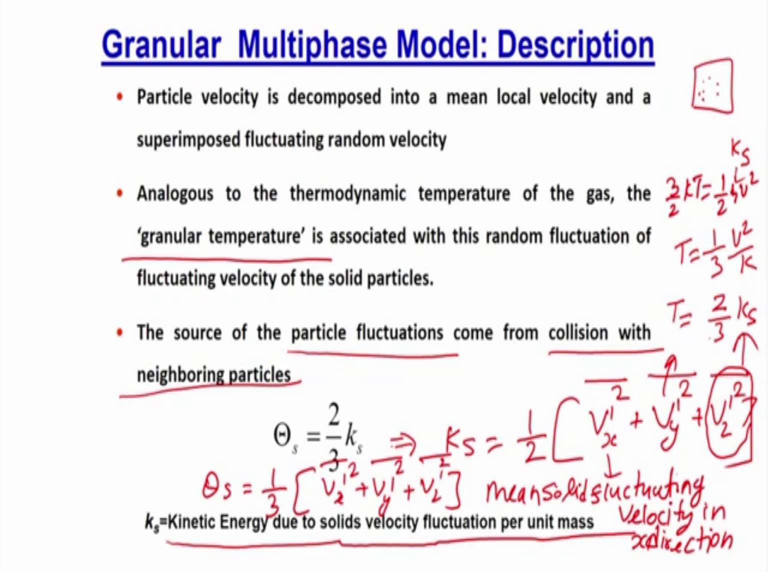

Granular Multiphase Model: Description

- Particle velocity is decomposed into a mean local velocity and a superimposed fluctuating random velocity
- Analogous to the thermodynamic temperature of the gas, the 'granular temperature' is associated with this random fluctuation of fluctuating velocity of the solid particles.
- The source of the particle fluctuations come from collision with neighboring particles

$$\Theta_s = \frac{2}{3}k_s \Rightarrow k_s = \frac{1}{2}\left[\overline{v_x'^2} + \overline{v_y'^2} + \boxed{\overline{v_z'^2}}\right]$$

$$\Theta_s = \frac{1}{3}\left[\overline{v_x'^2} + \overline{v_y'^2} + \overline{v_z'^2}\right]$$

mean solid fluctuating Velocity in x direction

k_s=Kinetic Energy due to solids velocity fluctuation per unit mass

$$\frac{3}{2}kT = \frac{1}{2}v^2 \quad k_s$$

$$T = \frac{1}{3}\frac{v^2}{k}$$

$$T = \frac{2}{3}k_s$$

So, what we do we take the velocity of particle is decomposed in the mean velocity and superimposed it with the fluctuation regime of fluctuation thing random velocity. So, we assume that there is a particle which has certain mean velocity we super imposed that mean velocity with the flux random fluctuating velocity. So, that we do then we define a term which is very analogous to the thermodynamic temperature of the gas, when we use define the kinetic theory of gas we define the thermo dynamic temperature.

And how do we define the thermodynamic temperature? We define it with the collision between the particles or collision, between the gas molecules and we say that the collision between the gas molecules are the kinetic energy developed because of that collision is equal to half rho v square and that is equal to nothing but 3 by 2 KT where k is the Boltzmann constant and t is the thermodynamic temperature.

So, from there you define the kinetic theory of gas this temp analogous to the temperature you say 3 by 2 KT is equal to half mv square ok. So, you say that this is the kinetic theory of this who you define the kinetic energy and you say that this kinetic energy is basically because of the collision and that is why you feel the temperature. So, the temperature you define as 1 upon 3 V square upon K and if I take it the velocity as a of this velocity per unit mass if I will take or I will divide that the temperature in the mass independent quantity if I define in that way it will be coming it in this Y phis.

Similar way, we define the granular temperature which is nothing but it is associated with the random fluctuation of the fluctuating velocity of the solid particle in the kinetic theory of gases just go and revise it. And if there is any confusion you can write me I we can try to talk those problems we assume that the particles or molecules are having a collision with each other and they are randomly moving and because of that random motion their kinetic energy. Because, of this collision and the random motion the kinetic energy has increased and that is because of that only the temperature of the body will increase.

So, the temperature of the body is actually related to the random motion or the kinetic energy. So, what will happen if you start hitting any fluid say gas what will happen it is temperature will increase if it is temperature will increase then kinetic energy will increase number of collision will increase. So, you can say that increase in the temperature is being general seen only because of the increase in the collision.

Similar things we are also doing here in the kinetic theory of granular slope I am not deriving it, but what we do we take that mean velocity of the particle we superimpose it with the fluctuating velocity then with the analogous to the thermodynamic temperature. We define a quantity called granular temperature and that is actually being defined as a quantity which is associated with the random flux creation of the fluctuating velocity of the solid particle.

So, what does it means? That the fluctuation particle whatever the particle is which is there what is it is fluctuating velocity and how the variation in the fluctuating velocity is taking place. That is the way the granular temperature has been defined and the source of the granular temperature or the particle fluctuation is actually comes from the collision of the neighborhood particle. The particle fluctuation source whatever we account it is a counted based on the collision which the particle is having with the neighboring particle.

So, in that way the granular temperature has been defined and it has been defined as the exactly same way 2 upon 3 into Ks. So, suppose if I take it as a Ks which is the kinetic energy then it will be the temperature will be nothing but it will be 2 upon 3 of Ks divided by K I am not including the Boltzmann constant here.

So, exactly same way analogous to that this whatever thermodynamic temperature is being calculated or defined analogous to that the granular temperature has been defined which will be nothing but 2 by 3 Ks and the Ks is nothing but the kinetic energy due to the solid velocity fluctuation per unit mass. So, I did the per unit mass. So, that this m will be removed and it will be nothing but half V square that is the case kinetic energy due to the solid velocity fluctuation per unit mass exactly same way the kinetic energy has been defined here that in the kinetic theory of the gases that is why this granular temperature and that thermodynamic temperature are a very analogous quantity.

And the ks which is the kinetic energy of fluctuation per unit mass it will be defined as the same way the ks will be half it will be V X prime square plus v y prime square plus VZ prime square. So, suppose you have 3-dimensional velocity it will be VX prime is the fluctuating velocity velocity in X direction. So, solid fluctuating velocities I will write it let us make it very solid fluctuating velocity in the X direction this is solid fluctuating velocity in the Y direction this is solid fluctuating velocity in the J direction.

So, what you are saying that how the solid fluctuations are taking place in all the 3 directions you are adding them and then you are dividing it by 1 by 2 you are square with them and then I had actually. So, it will be actually averaged these all are averaged. So, what you do suppose in a small cell? If the particle is there is 10 particles are there you take the fluctuation particle fluctuating velocity of all the particles you square them and then you add them ok.

So, that is the way it has been there. So, it is a mean fluctuating velocity of in the X direction this is the mean solid I will write it here mean also means solid fluctuating velocity in the X direction means solid fluctuating velocity in the y direction mean solid fluctuating velocity in the J direction you take average all together. So, that is the way that Ks has been defined and theta is nothing but 2 by 3 of Ks or I can say theta s is nothing but 1 upon 3 it will be v x prime square v y prime square VZ prime square.

So, that is the way the granular temperature can be defined. So, this quantity has been defined to model the collision or to predict the collision that, what will be the collision forces? And how the fluctuation velocities are being correlated?

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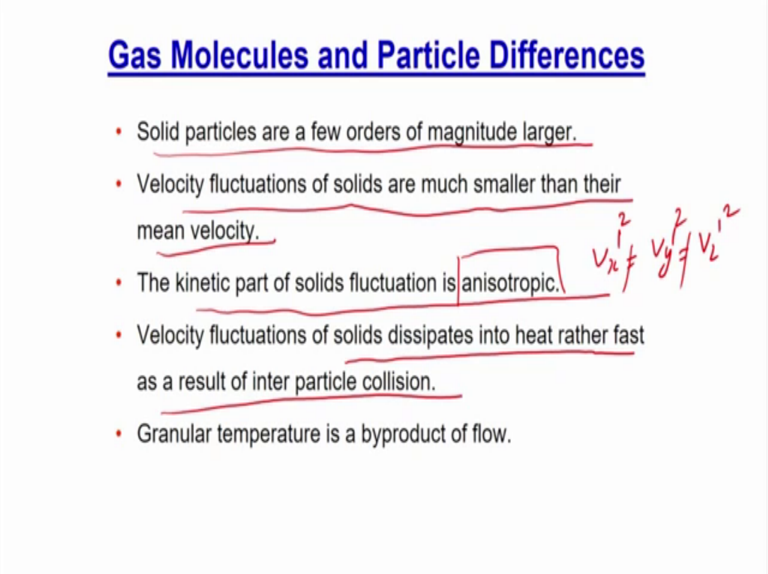

Gas Molecules and Particle Differences

- Solid particles are a few orders of magnitude larger.
- Velocity fluctuations of solids are much smaller than their mean velocity.
- The kinetic part of solids fluctuation is anisotropic. $v_x'^2 \neq v_y'^2 \neq v_z'^2$
- Velocity fluctuations of solids dissipates into heat rather fast as a result of inter particle collision.
- Granular temperature is a byproduct of flow.

So, that is the way the kinetic theory of granular flow has been defined again this is a very brief way I have just tried to cover give the very basics of this.

But what is the difference between the kinetic theory of the particle kind of gases and the gas molecules or kinetic theory of granular flow and kinetic theory of gases the difference is actually whatever the difference is there between the gas molecules and the particle. So, that is the main difference between the kinetic theory of gas and kinetic theory of granular flow that, kinetic theory of the granular flow solid particles are a few order of magnitude larger compared to the gas molecules.

So, the gas molecules even if you check the check the Chapman Enskog theory. We assume to the finite size of the gas molecules, but the particles are generally few order magnitude higher. So, suppose a gas molecules even if you take a finite size generally will be in the order of any strong here you will get in the order of micrometer or nano meter even if you are taking nanometer particles. Then also, it is one order of magnitude higher if you are talking about the micrometer level particle the 4-order magnitude higher.

So, velocity fluctuation of the solids are much smaller than the mean velocity and that is most of the time it is true while, in the case of gases it may not true the fluctuating velocity may be higher compared to the mean velocity the kinetic where a part of the solid fluctuation is anisotropic again this is very critical most of the in the gas kinetic theory of the gases we always assume that the fluctuation are isotropic in nature it means what dx prime is equal to VY prime will be equal to VZ prime or you can say VX prime is square VY prime with square VZ prime is quite extreme anyway you can define.

This is the isotropic fluctuation, but whatever VZ in the kinetic theory of the solids that they are their fluctuation are mostly an isotropic in nature. So, if they are anisotropic it means they are not equal then velocity fluctuation of the solids dissipate into heat rather fast at the result of inter particle collision. So, if you are having a collision the particle actually the energy will dissipate very fast compared to the way it has been dissipated in the kinetic theory of gases or with the gas molecule and granular temperature the way we have defined is actually the byproduct of the flow or the collisional dominating flow or by product of the collisional slope that is the way we define it.

And that is the difference major difference the way gas molecules and particles are different or the way the kinetic theory of gases or the kinetic theory of granular flow is different. So, what we do with this what we have done we have taken we have defined the third order and this kind of integration or third closure and that third closure was between the particle mean motion to the particle, fluctuating motion which has been defined with the kinetic theory of granular flow and has been could defined a term of this granular temperature which will be actually correlating them.

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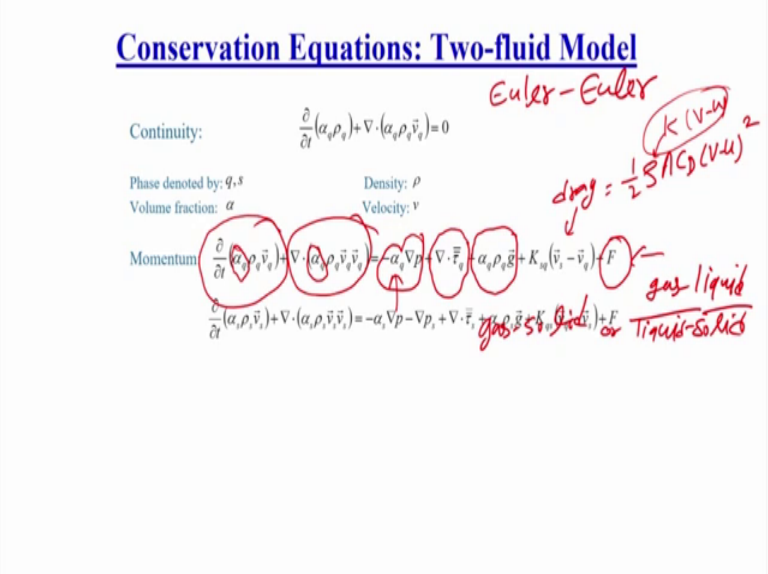

Conservation Equations: Two-fluid Model

Continuity: $\frac{\partial}{\partial t}(\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = 0$

Phase denoted by: q, s Density: ρ

Volume fraction: α Velocity: v

Momentum: $\frac{\partial}{\partial t}(\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = -\alpha_q \nabla p + \nabla \cdot \bar{\bar{\tau}}_q + \alpha_q \rho_q \vec{g} + K_{sq}(\vec{v}_s - \vec{v}_q) + F$

$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{v}_s) + \nabla \cdot (\alpha_s \rho_s \vec{v}_s \vec{v}_s) = -\alpha_s \nabla p - \nabla p_s + \nabla \cdot \bar{\bar{\tau}}_s + \alpha_s \rho_s \vec{g} + K_{qs}(\vec{v}_q - \vec{v}_s) + F$

So, based on that we again back to the 2-fluid model or Euler Euler model ok; so, please do not get confused if the name is same Euler Euler or 2 fluid model and what we do we solve the continuity equation. We solve the momentum equation for the individual features. So, if suppose you have a 2 phase you solve the 2 continuity equations say this is for the phase q the similar thing you will solve for the phase s.

So, if they have a 2 phase you will solve it in this way the alpha is actually the volume fraction do not get confused this alpha is actually whatever we are representing here alpha is actually nothing but epsilon. So, it is the volume fraction inside and then you solve the momentum equation. So, if you have 2 phases you solve this momentum equation and you keep or you solve it twice. So, if the both the phases are liquid you solve this equation twice.

So, if you see this here again local acceleration convective acceleration you solve the pressure gradient term, but now the pressure gradient is being multiplied with the volume fraction of that phase it means now you are solving that what is the pressure difference or what is the pressure gradient which is the contribution because of that phase similarly if you see their local activation also we are solving we are multiplying with the alpha it means the phase fraction then we are multiplying with the alpha here again means what this is the contribution only of that phase.

Then we solve the tau that what is the stress viscous term we solve this then any body force definitely gravity is going to act. So, gravity on that stage that is why we multiply here again with the alpha and then we include a drag. So, drag is here this is the drag we have write in the drag we modified generally in the commercial software in the drag we write it in this way ks into v as minus d and the case will be defined with the rest of the term, whatever the way we have discussed.

So, the drag we defined as half rho a c d into slip velocity say v minus u square . Now, what we do they define this is as a k and v minus u and rest of the term will be defined in terms of the k which will be also correlated with the ceiling. So, we will discuss that some of this terms, but that is the way the drag has been defined which is being acting between the 2 phases and you are modeling the mean motion interaction of the 2 phases which is the part of the equation now which was not there in the algebraic slip model.

In algebraic slip model, you are defining the drag you are including the drag in the slip velocity or in the relative velocity and that you are being defined with the empirically developed correlation. Now, here we are using the model of the drag directly into the equation. So now, if suppose you have the 2 phases both the phases are fluid it means say if you are solving for the gas liquid system fluid does not mean that this would be the same phase.

So, if you are solving for the gas liquid system. So, the same equation once you will solve for the gas phase once we will solve for the liquid phase what will happen you have 2 continuity equation 2 momentum equations. So, that what we saw and then we try to find it out the closure for the drag if you have any other interactions forces like virtual mass force if you want to use the lift force any other force you can also include it here to solve this equation. So, that is what in the 2-fluid model or Euler model we do.

Now, once the flow is solid what we do is in case of the solid flow in the things becomes a little bit different. So, in the gas phase or both the phases which are fluid then the things are little bit simpler you solve exactly this equation is a pure damage to equation or you can say the name is very close to Navier stokes equation where you just include the drag force which is the interaction between the 2 phases you just multiply with the volume fraction of each phases in that place ok. So, the each term will be multiplied by the volume fraction. And so, that if you sum those both the equation you will get the total forces which is acting on the body of it in the wholes kind of domain.

So, that is the way it has been solved continuity equation is also solved individually for one individual phases. So, the mass conservation equation always validated you never see a problem of the mass loss that your mass conservation is not being balanced. So, that is the major advantage, but once the solid comes into the picture suppose now you are want to solve for the gas solid or liquid solid this equation will be modified and some of the term will be this equation will be modified.

How this equation will be modified? We will see it here that how this equation will be modified you know you just enriching this so that this equation will be more visible. So, if you see the left id signed I order term is not being modified because that is nothing but drag federation term local acceleration convective acceleration they are exactly same for the solid phase also and they are just being multiplied with the volume fraction. So, this is being this terms has been used alpha s term.

Now, we include here if you see there is only one pressure term here we include one more pressure and that pressure is the solid pressure term and we keep this term exactly same alpha s into delta p instead of this delta p alpha q or gas phase say volume fraction. Now, you multiply with the solid phase volume fraction and we subtract that solid pressure term we include that solid pressure term ok.

Now, why the solid pressure term has been include how it has been defined we will discuss in the next slide then we use a solid stress term now this is very critical because now, what we are going to model? We are going to model the delta tau s and the tau s is being defined for the fluid with the same stress strain dependent ratio.

So now you are treating the solid as a fluid you are taking it as a viscous fluid and we will see that that is why you need to modify this tau s values also and we will see those things later. So, these 2 terms are very, very critical because you are defining it for the solid ideally the solid has no viscosity or you can say that infinitely high viscosity.

So, if I take the solid if I start rubbing it I will not deform that is not the curve this my fluid nature this is not the nature of the solid so, but what we are doing to do? We are taking that viscous deformation forces here in they are defining it it in that way. So, what we need to do we need to define the mu value of the solid.

Now, mu value of the solid you cannot get. So, you have to define some correlation to find the mu value of this and that is why now you will see that though the equations looks very simple the approach looks very simple you need to define many quantities. And that is why the accuracy of this equation will depend the accuracy of all those models which we are using to define the solid pressure term to solid viscosity term the drag force term all these terms how you are defining your accuracy is going to be limited with that.

And if you are going taking the granular temperature also and what is the terms which we are including to model the granular temperature. We will discuss that there certain terms will be needed this is the solid pressure term.

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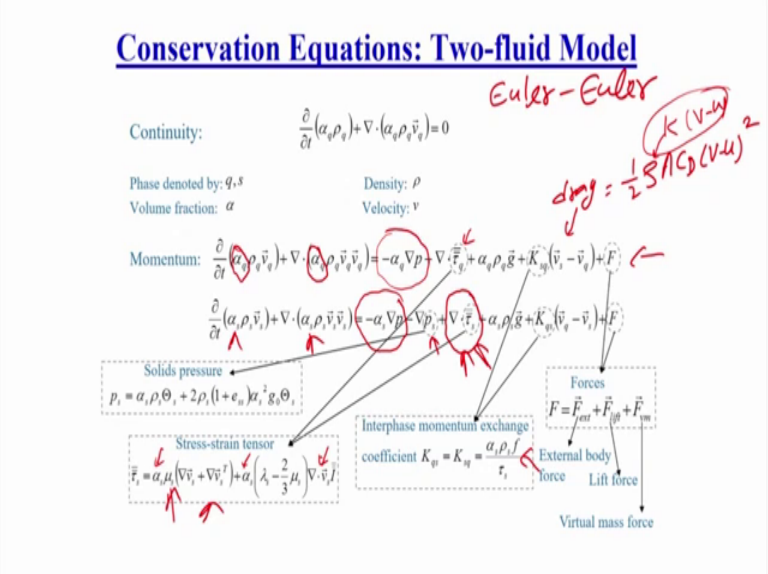

Conservation Equations: Two-fluid Model

Euler-Euler

Continuity: $\frac{\partial}{\partial t}(\alpha_q\rho_q)+\nabla\cdot(\alpha_q\rho_q\vec{v}_q)=0$

Phase denoted by: q, s

Volume fraction: α

Density: ρ

Velocity: v

drag $= \frac{1}{2}\rho A C_D (v-u)^2$

$K(v-u)$

Momentum: $\frac{\partial}{\partial t}(\alpha_q\rho_q\vec{v}_q)+\nabla\cdot(\alpha_q\rho_q\vec{v}_q\vec{v}_q)=-\alpha_q\nabla p+\nabla\cdot\bar{\bar{\tau}}_q+\alpha_q\rho_q\vec{g}+K_{sq}(\vec{v}_s-\vec{v}_q)+F$

$\frac{\partial}{\partial t}(\alpha_s\rho_s\vec{v}_s)+\nabla\cdot(\alpha_s\rho_s\vec{v}_s\vec{v}_s)=-\alpha_s\nabla p-\nabla p_s+\nabla\cdot\bar{\bar{\tau}}_s+\alpha_s\rho_s\vec{g}+K_{qs}(\vec{v}_q-\vec{v}_s)+F$

Solids pressure

$p_s=\alpha_s\rho_s\Theta_s+2\rho_s(1+e_{ss})\alpha_s^2 g_0\Theta_s$

Stress-strain tensor

$\bar{\bar{\tau}}_s=\alpha_s\mu_s(\nabla\vec{v}_s+\nabla\vec{v}_s^T)+\alpha_s\left(\lambda_s-\frac{2}{3}\mu_s\right)\nabla\cdot\vec{v}_s\bar{\bar{I}}$

Interphase momentum exchange coefficient $K_{qs}=K_{sq}=\frac{\alpha_s\rho_s f}{\tau_s}$

Forces

$F=\vec{F}_{ext}+\vec{F}_{lift}+\vec{F}_{vm}$

External body force

Lift force

Virtual mass force

Which is being defined at it in this way we will discuss anyway I do not want to kind of get into it. So, the solid pressure term that is what this would want to focus that if you see that for both the fluid phase or in the solid phase. The equation is same the way it has been defined is same and it is nothing but Newton's law of viscosity.

So, if you see that that is the way it has been defined the only thing has been done it has been multiplied by the alpha. So, that if you are solving for the gas phase it will tell you about the gas phase thing if you are solving for the liquid phase it will tell you the information about the liquid phase. So, this is the way it have been defined this is the if you guys go ahead and open the transport phenomena book you will see that the tau has been defined it in this way, there is a region to define it in this way because the tau is a symmetric tensor. So, that is why the other transpose term has been used. So, that you get the symmetric nature.

The volume change can affect the viscosity that is why this terms has been used and if the fluid is incompressible this del dot v term will be 0 with the continuity equation anyway. So, this is the way it has been defined and that is the major limitation because the solids which cannot have any mu value this mu value. Now, you need to define that mu value and for that you need to depend on some of the equation which is being developed and those equation accuracy will limit the accuracy of all these models.

This is the as I said that this is nothing but a drag and the drag is being defined this is the interface momentum is or drag which has been defined as alpha as rho s as f upon tau s ok. So, what we have done we have already said that as I said that in this force we have actually defined it in this we have put the drag factor we have defined the alpha s rho s upon tau p if you do that you will actually get this value ok.

And these are the forces which can be any additional force as I said it can be any external force a electrical say you can have magnetical magnetic field it can have a lift force it can be virtual mass force it can be message history force any force which you want to include you can include. So, that is the beauty of this model it gives you a lot of luxury, but the major limitation of this model is that for these things your accuracy will depend based on the accuracy of these closure equations which you are using to solve this equation.

In case of the solid, the accuracy will depend on the model equation or the closure the equations which we are using to solve the delta ps or del p s and delta tau s values and also the drag forces.

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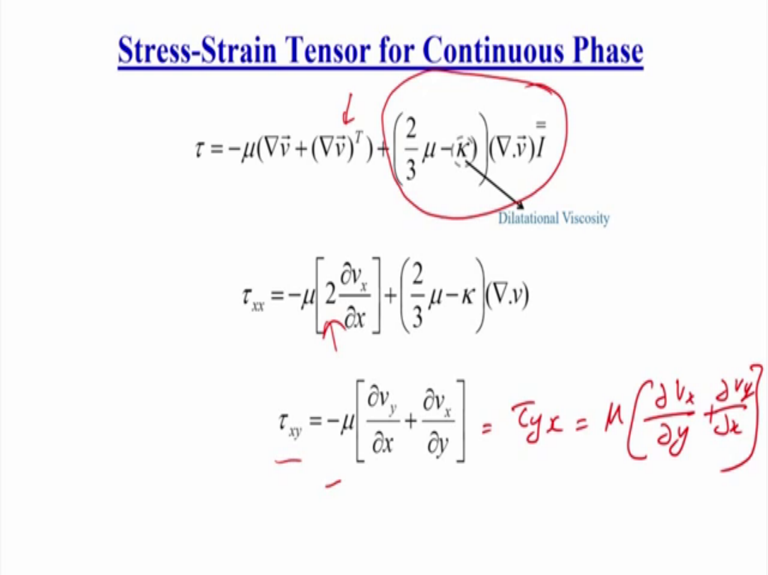

Stress-Strain Tensor for Continuous Phase

$$\tau = -\mu(\nabla\vec{v} + (\nabla\vec{v})^T) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla.\vec{v})\bar{\bar{I}}$$

Dilatational Viscosity

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x}\right] + \left(\frac{2}{3}\mu - \kappa\right)(\nabla.v)$$

$$\tau_{xy} = -\mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] = \tau_{yx} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

We will see that, how this individual terms are being defined. So, shear stress strain tensor term as I said that it is for the continuous phase it has been defined and it has been defined as mu del v plus x transpose 2 upon 3 mu minus k where k is the dilatational viscosity into del dot v i which actually this term take care of when the volume change the dilatational viscosity is there which is caused.

Because of the volume change of the fluid some of the fluid changes the volume this term has been included to give the symmetric nature to you to you are just out insert tau is the second order tensor which are symmetric to make it symmetric the transpose term has been included and this is the way it has been defined now there is a dilation viscosity I have already discussed that. So, if I try to solve it in the conventional way the way we define the tau for the fluid field it is being defined as minus mu dou v upon dou x into dou v x upon dou y if it is a tau xy ok.

So now if you see that tau xy will be equal to tau y x. So, that does give the symmetric nature I have removed this term because that this term will be there this is normally this term identity matrix or normal this tensor that opponent will be theirs when the normal tensor will be one for the shear stress term this value is anyway going to be 0. So, that is why this value will be defined it in this way. So, this will be tau yx if you write the tau yx it will be also the same mu dou vx upon dou y plus dou v y upon dou x.

So, this gives you the symmetric nature and that is why this way it has been defined for the tau xx term or the normal component of the stress you define it in this way this will be dou v x plus dou v x that is why it becomes twice of dou vx into 2 by 3 mu minus k into del dot v these terms are you already me knowing it it has been very discussed in the transport phenomena. So, I am not touching it much, but that is the way the shear stress tensor in the continuous phase has been defined.

So, the first part if you see here it is not a problem to define the tau which is the continuous stage we can easily define that because for that I can easily find out that what is my new value ok.

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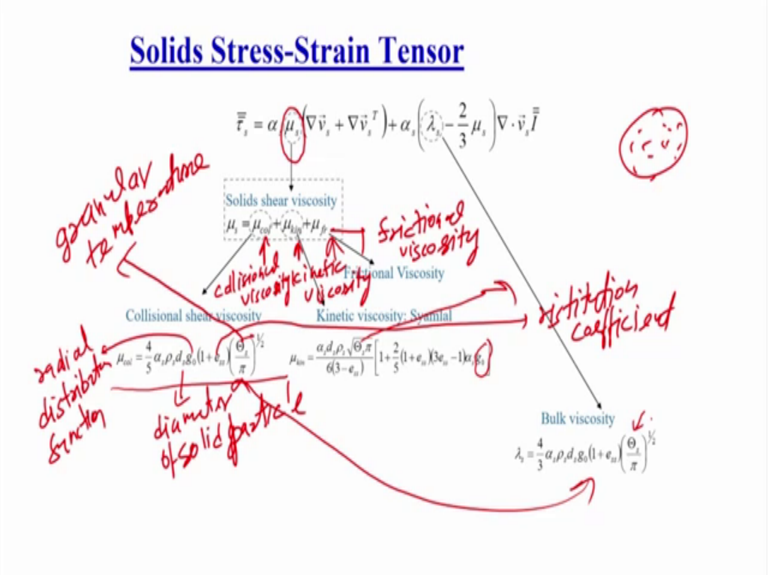

Solids Stress-Strain Tensor

$$\bar{\bar{\tau}}_s = \alpha_s \mu_s \left(\nabla \vec{v}_s + \nabla \vec{v}_s^{\,T}\right) + \alpha_s \left(\lambda_s - \frac{2}{3}\mu_s\right) \nabla \cdot \vec{v}_s \bar{\bar{I}}$$

Solids shear viscosity

$$\mu_s = \mu_{col} + \mu_{kin} + \mu_{fr}$$

Frictional Viscosity

Collisional shear viscosity

$$\mu_{col} = \frac{4}{5}\alpha_s \rho_s d_s g_0 (1 + e_{ss}) \left(\frac{\Theta_s}{\pi}\right)^{1/2}$$

Kinetic viscosity: Syamlal

$$\mu_{kin} = \frac{\alpha_s d_s \rho_s \sqrt{\Theta_s \pi}}{6(3 - e_{ss})}\left[1 + \frac{2}{5}(1 + e_{ss})(3e_{ss} - 1)\alpha_s g_0\right]$$

Bulk viscosity

$$\lambda_s = \frac{4}{3}\alpha_s \rho_s d_s g_0 (1 + e_{ss}) \left(\frac{\Theta_s}{\pi}\right)^{1/2}$$

Now, the problem starts once you go for the solid stress tensor double. So, what happened the solid stress tensor term we define exactly same way as the way we have defined the fluid stress tensor term the only problem in the solid stress tensor term is now you are having a mu s and that is going to create a problem.

Because solid as I said ideally have no viscosity or you can say have a in finite viscosity. So, they do not move they do not deform even for a very high strain rate ok. So, that is you can say in a very infinite viscosity or no viscosity the way you want to tell you can do that, but the major problem comes with the mu s how you could define that dou v s you can still find because the gradient in the velocity will be there in the x direction and the y direction if there is a gradient you will see that gradient.

The only problem is the mu s. So, to define the mu s several people or several researchers have given many correlations, but some of the correlations are very critical or very important I am going to discuss that correlation only. So, the mu s actually which is solid in viscosity has been defined it is a summation of the 3-different viscosity ok. So, viscosity is what it is a resistance to the flow.

So, that approach has been used to define that what will be the solid shear viscosity. So, that we are trying to see that what will be the resistance force will be there if the solids are moving. So, what we are saying that that resistance will be mainly because of the collision it will be because of the kinetic energy it will be because of the friction these 3 will be there. So, the mu cm has been defined as a summation of mu collisional mu kinetic mu frictional ok. So, this is frictional.

So, this is kinetic viscosity term frictional viscosity term and this is collisional viscosity term it has been defined it in this way that mu or total resistance to the flow this comes because in the stress of the solid will come because of the collision of viscosity, kinetic viscosity and the frictional viscosity. In that way, it has been defined for each viscosity several equations has been proposed in the literature.

Again, I am saying only those equations which are most popularly used it does not mean that you have to limit it yourself till this point. You can develop that these equations there are several ways to develop these equations and this several equations are also available. So, we are discussing some of the tentative equations which are widely used. So, collisional viscosity equation has been defined by 4 by 5 alpha as rho s ds into here ds is the solid diameter or diameter of the solid solid for the particle g naught is nothing but the radial distribution function this is radial distribution function g naught.

We will discuss that this radial distribution function and es is the restitution coefficient it is restitution coefficient . So, that is the way it has been defined and if you see that that is also the function of granular temperature. So, this theta s is nothing but it is a granular temperature . So, it says that new collisional is going to be the proportional to under root of granular temperature it is going to be the function of radial distribution function of the solid it means how the solids are radially distributed.

What is the restitution coefficient? Restitution coefficient has been defined that if the particles are having collision if the pollution is completely elastic the value of es s or restitution coefficient will be one if it is completely inelastic the value will be 0, if it is viscoelastic. It means, it is not completely elastic it is not completely in the elastic the value will be between 0 to one ok. So, that is the restitution coefficient it is d prime upon v it means the column velocity of the particle.

After the collision to the velocity of the particle, before the collision if this is a completely elastic both the velocity will be the same. Then it is value will be one if it is complete inelastic then after the collision particle will actually get is stationary. It will value will be 0 if it is viscoelastic, the value will be in between this that is the way the restitution coefficient has been defined and the way it has been defined is that that what is the fraction of the solid present at that place. What is the rho value although it means the total momentum it is going to take place that rho value will depend?

Then what is the particle size then what is the radial distribution of the particle. So, how the particles is being radially distributed? So, suppose this is their how the particle radial distribution is there then what is the restitution coefficient of the particle. It means after the collision whether it will be stopped where there will be elastic. It will be inelastic and then what is the fluctuating velocity component in the particle and that is nothing but the granular temperature it has been included that is the way the collisional shear viscosity term has been defined.

Similarly, kinetic viscosity term has been defined again I am telling again and again that these are not normal equations there are several equations available do not get confused that really these equations are there are other equations are also ever this is the equation which is given by (Refer Time: 74:30) et al and they have also found that the mu kinetic energy is also actually the function of theta s then the temperature.

They are also going to see that; what is the fluctuating velocity of the particle they are also the function of es restitution coefficient and g naught that; what is the radial distribution of the particle. Similarly, I am not detail of that how these equations are discussed then the viscosity is also being defined and it is also in the same way that what is the frictional viscosity you define with the friction and you find it out that what is the value of this.

Lambda s is nothing but the bulk viscosity which has again defined it it in this way and again if you see the bulk viscosity is very close to the wave. We have defined the collisional shear viscosity these 2-viscosity value, if you will see other than some constant the values are exactly same if you see these 2 why it is going to be it means because the bulk viscosity is mainly because of the collision. So, only some parameter is being changed and you get the equation for this also.

Similarly, the solid pressure; so, this is the way the shear strain model is being developed. You use all these collisional equations to model that and you see that how these equations are kind of going to be implemented in this. And the accuracy of this shear stress or kind of all it is stress tensor or solid stress term will depend on how accurately these models are working. So, that is the way and if you see that all this value of g naught we will discuss it, actually are being empirically defined or is being calculated based on some values or some notion like restitution coefficient values calculating for the solid is very different it.

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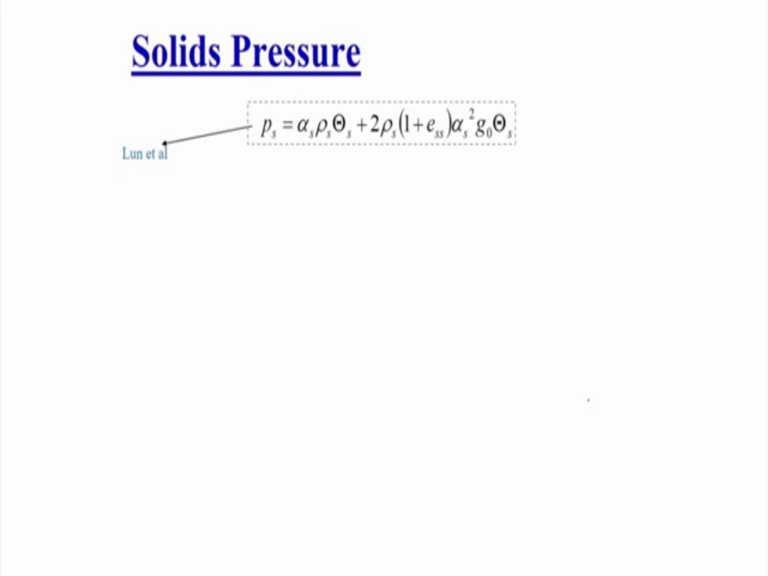

Solids Pressure

Lun et al

$$p_s = \alpha_s \rho_s \Theta_s + 2\rho_s (1+e_{ss}) \alpha_s^2 g_0 \Theta_s$$

So, whether used for the restitution as a bulk parameter or not, the collision should be dumped at a trial situation or not this is all a different question. Altogether, we will discuss translator, but the way it has been defined in the next class. What I am going to do? I am going to discuss about the solid pressure term also and then we will see that how the accuracy of the Euler Euler model will be there? How to use that model? And then after that we will see the discuss the limitation of the Euler Euler model. And then we will use towards the Euler Lagrangian model or which is also called a discrete phase model ok.

Thank you.