Multiphase Flows

Dr. Rajesh Kumar Upadhyay
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture – 13
Multiphase Interactions: Multi-particle Drag, Virtual Mass Force, Basset Force and Lift Force

In last class, what we were discussing is about the drag and we have discussed the different drag closure, and in the different during the discussion of the different drag closure we have always come to know that how the drag has been developed and in that discussion I said that, what we were discussing is actually for a single particle drag. Now most of the practical application we have multiple particles. Now if you have a multiple particle then what will happen? Your drag will get modified.

So, what we have seen that most of the correlation is actually based on either the Schiller Naumann or on the Ergun equation. So, the last equation which we have discussed particularly for the multiple solids that was either Gidaspow Wen and Yu. So, Wen and Yu Gidaspow anyone if you discuss is actually or Ergun was actually based on the two equation; first one on the Schiller Naumann correlation for the certain Reynolds number certain void fraction range and other is for the Ergun equation.

Now, briefly I would explain that how the Ergun equation can be written in terms of the drag force which I have not done in the yesterday class. So, now, I will do that first, and then we will see that if you have a multiple solids say you have a packed bed or you have a fluidized bed then how the equations should be modified the drag equation should be modified.

So, we have written the Ergun equation and Ergun equation we have written it in form just to.

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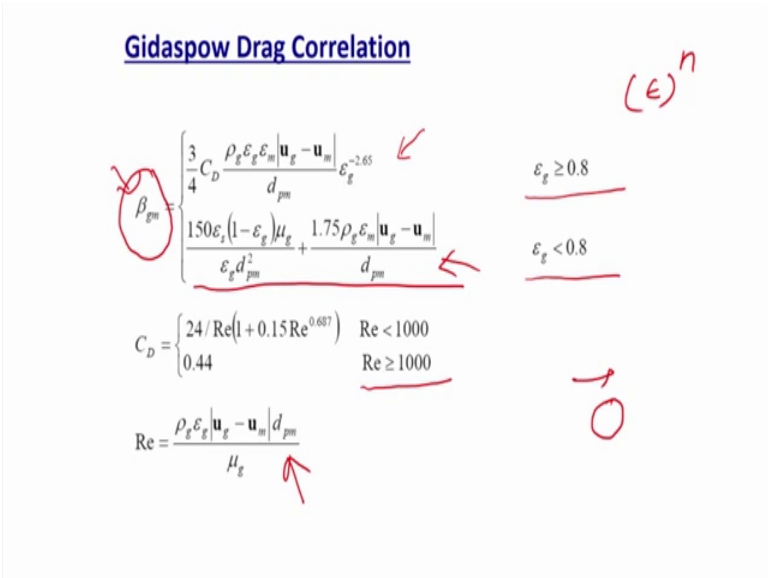

Revise as 150 into epsilon s into this into one point epsilon, mu g upon epsilon g dp square plus 1.75 rho g epsilon of solid and mu g minus upon dp m. So, I will just write it in this form.

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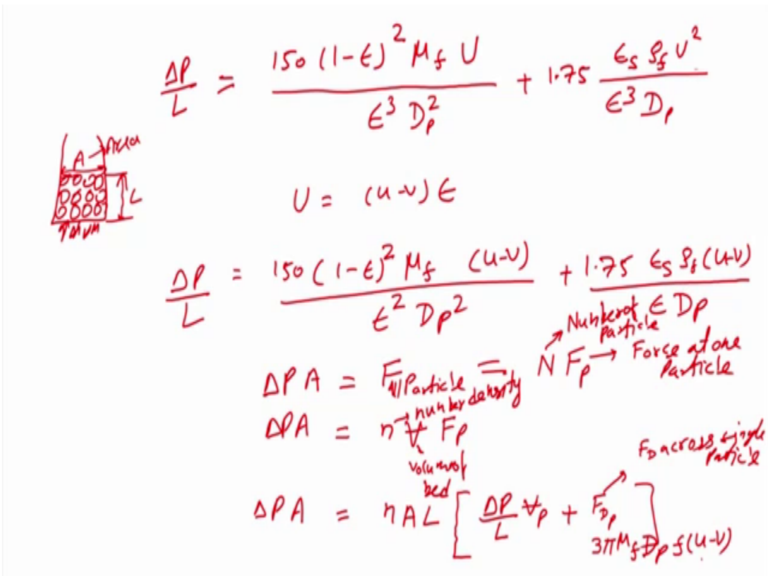

So, we have written delta P upon L 150 into 1 minus epsilon square into mu of fluid into U velocity. Now, I am writing in terms of the superficial velocity right now. So, this will be in terms of epsilon cube plus 1.75 into epsilon s into rho of fluid into u square again I am writing it in terms of this. So, it will be epsilon cube into d this will be D square now this is D of particle

Now, if you write it in terms of the slip velocity as I said, what you will be the slip velocity? The U will be written as u minus v into epsilon ok. So, if you do it in this way, then you can write it in form of 150 into 1 minus epsilon, 1 minus epsilon square it will be mu of f, now you can write it as in terms of epsilon u minus v upon epsilon cube into Dp square plus 1.75 into epsilon s into rho f into you can write u minus v upon this will be epsilon into Dp. And this will be we can also write it in terms of we can remove this epsilon and we can make it instead of 3 we can make it 2. So, this is what the Ergun equation they have written in form of the slip velocity.

Now, if suppose a bed is there which is completely filled with the solid and Ergun equation as I discussed is mainly for the packed bed. So, suppose if I have a bed, which is completely filled with the solids and this they are in the packed bed condition and some fluid is flowing.

So, what will happen? The delta P across the bed will be equal to the forces acting on the particles on all the particles or within that volume. So, this volume; so, if I assume that length is L and the area is say A is area we are taking it. So, then what we can do? We can write it out that the overall force acting on the body acting on the particle will be actually equal to the delta P upon L. So, I can write simply the delta P upon L or delta P and if I multiply it with the area, that will be the total force acting on this bed. So, across the delta P that will be equal to the total force acting on the particle. So, total force on the particle.

Now, what will be the total force on the particle? So, the total force on the particle will be nothing, but the drag and the pressure drop across the each particle. So, how to write that? We can write the total force across the particle, now the total force across the particle can be written as in terms of the single particle force, now single because the drag we have defined in terms of the single particle drag. So, I want to write it in terms of the single particle force.

If I want to do that, I will first do what? I will multiply it with the number density where n is the number density. Now how the number density has been defined? So, number density has been defined is number of particles per unit volume. So, now, I have to do what? Because I want the total number of particle the force on one particle across the one particle. So, what I have to do? The F across all the particle I will say all the particle will be equal to what? Number of particle see I will write the number of particle as N multiply by the force across one particles Fp ok. So, this is force at one particle and N is number of particle.

So, that will be the force around the total number of particles ok. So, delta P by a I can now because we do not know how much number of particle is there, but we can always find the number density. So, I can write it in n multiply by the volume, this is volume of bed into F of p. So, that will be my total delta P into area; it means the total force acting across the bed will be the total force acting around one particle multiply by the total number of the particle ok. So, that will be the what will be the pressure balanced.

Now, if you want to solve this what we can do? We can write it delta P into A is number density into volume, and volume we can write if the area of the bed is A length is L, I can write it in terms of the A into L and Fp. Now total force across the particle is what? It is going to be the drag plus the drop of the pressure across the one particle. So, we can write it as delta P upon L into volume of particle volume of the particle one particle volume of single particle plus we can write FD across the single particle FDp. So, FD single particle.

Now, we can do that FD across the single particle is what? It is nothing, but the way we have defined we can write it as a 3 pi mu of fluid D of particle f into u minus v the way we have described in the last class.

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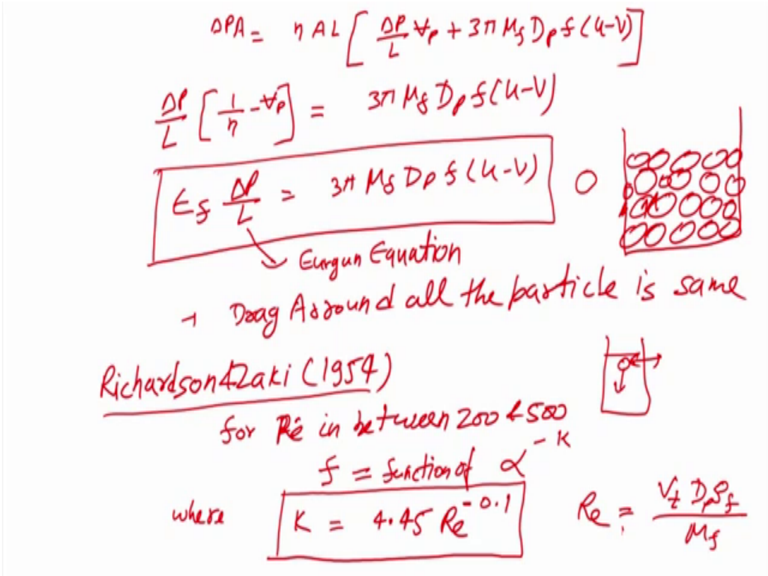

So, I can write it delta P upon A is equal to nothing, but number density into area into L into delta P upon L, volume of a single particle plus 3 pi mu of fluid, D of particle f into u minus v this way we can write it.

Now, what I will do? I will put the delta P together. So, that we can write the delta P in terms of the drag force and that can be replaced in the Ergun equation to use the as a drag closure. So, what we can do? We can write it in terms sorry this is not delta P upon A this is delta P into A this is delta P into a. So, what I will do? I will just bring it together. So, A A will be cancelled out, it will be delta P upon L and delta P upon L will be common. So, it will be 1 upon n minus v of particle. We can write it in this way, this will be equal to 3 pi mu of f D of particle f into u minus v.

Now, this delta P upon L this is what is the number density of the particle. So, it will be what? It will be number of particle per unit volume, per unit volume of the reactor. So, if you do that per unit volume of reactor minus if you do that this will be n will come here number of particle into total volume will be what? Volume of one particle will be total volume of the particle. So, volume of reactor minus total volume of the particle will be equal to what? Volume of gas divided by the total. So, I can write it as epsilon f.

I hope you understand this I am multiplying by the number density here. So, number density is nothing, but number divided by volume, number will be multiplied by here. So, it will be one particle volume multiply by the total number. So, it will be what? It will be the total volume of the particle. So, that will be the total volume of the reactor divided by the volume, that will come as a epsilon f ok. So, this will be equal to 3 pi mu f Dp f into u minus v ok. So, from there you can find the drag force correlation and this delta P upon L you can replace it in the Ergun equation. So, you will get the Ergun equation in the form of the drag closure ok. So, that is the way the people has developed the Ergun drag closure and this delta P by L can be replaced with Ergun equation.

So, you will get the drag force across a packed bed. Now the problem is what we have assumed the major assumption which have made that, drag around all the particle is same. So, what we did? We neglected the particle cloud effect. So, what is happening? There is one particle and we are assuming that this one particle drag whatever is acting the similar drag will act across all the particles.

But the problem is now there is a particle this is not a single particle which is suspended, in a packed bed suppose if I assume this is a packed bed there are multiple particle is there multiple particle is there. So, the problem is assuming that one particle drag when it is suspended alone in air is going to be the same on all the particle is a big assumption and it is never going to be satisfied.

So, what is being assumed always is a particle cloud model and it says that most of the time it is being assumed in most of the drag closure whatever we have done, that drag across one particle is same that is the way the drag organ has defined the drag closure and that has been followed. So, what was needed is that, we should modify this drag. So, that we should get that once the multiple particles are there then the drag closure need to be modified because we understood the drag in this case and drag when the one particle is suspended is going to be completely different because the ad exposure area is also different.

So, to do that Richardson Zaki in 1957 had actually done some work, and I hope you have heard this name this and in 1954 has done some work for the settling experiments where what he did? He performed the settling experiment and tried to correlate he did some settling experiments what he did? He take a liquid, he settled one particle inside and he said that how the settling velocity of what measured the settling velocity of one particle, and then he has used the same experiment with the multiple particle and what he did? He tried to find the effect of the settling velocity of one single particle in comparison with when there is multiple particle is there, and when this wall is also playing a role. So, in the settling normal settling experiments, we assume that the wall is not playing any role the wall is far from the particle. So, they are not going to play any role in the settling of the single particle and that is the way the settling formula which we have used in your undergraduate studies, we have also derived here has been used. But if the wall also play a role then the settling velocity will be different, in presence of the multiple particle again the settling velocity will be different.

So, what he did? He did the experiment to find that how the settling velocity will be affected in the presence of the other particles. So, for that, he did several experiments and he found that for Reynold number in between 200 in between 200 and 500 he found that a correlation the f actually the this drag force f can be defined in the new way, and that f will be actually the function of it will be the function of alpha raise to the power minus k.

And this k value is actually is a function of Reynold number and that varies with a empirical equation again because it is a fitted equation Re raise to the power minus 0.1. this is the way it vary and Re has been defined based on the settling velocity. So, v of settling of v of terminal into D into rho of fluid upon mu of fluid.

So, that is the way Re has been defined and he says that f is going to be the function of this and how to write it.

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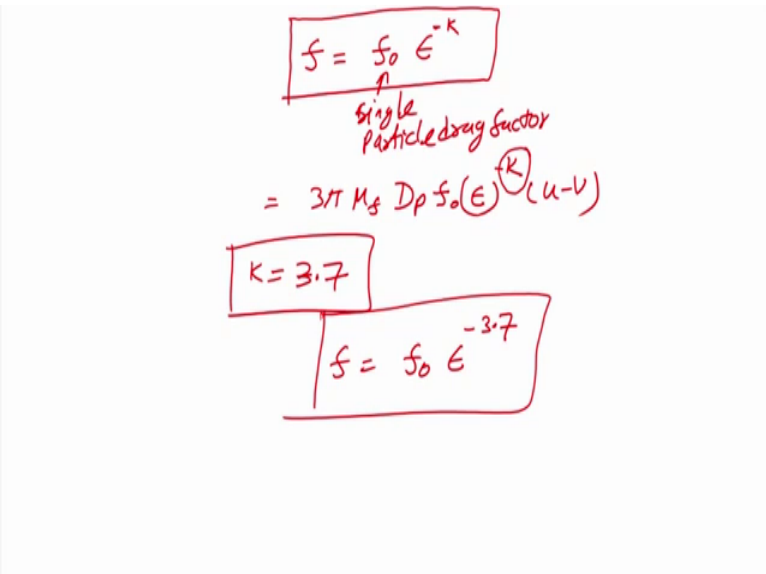

So, he has done that the f is nothing, but actually f naught which is the single particle drag factor particle drag factor into epsilon this alpha is nothing, but actually epsilon you can write it in because we are using the volume fraction as epsilon will write it in terms of the epsilon raise to the power this. So, this is epsilon raise to the power minus k. So, it means what? My whole drag equation will be modified and I will write the whole drag equation in form of 3 pi mu f, D of particle into f naught for f naught into epsilon raise to the power minus k into u minus v.

If I do it in this way, Richardson Zaki found that this fitted actually well for the particle cloud method. The only thing is the value of the k how it is should be found and then he found that if you use this f naught as a Schiller Naumann drag closure, this equation gives perfectly fine. The only problem is the value of the k that how to find this value of the k. So, the value of the k has been found by several researcher they have done a lot of experiments, and they have found that the value of the k in most of the cases is equal to 3.7 and that is the reason most of the drag closure we have written it was epsilon raise to the power minus 3.7. So, I can write f as f naught into epsilon raise to the power minus 3.7.

Now, several researcher have found different values some says minus 2.65 some says minus 3.7, but it has been well established that this correlation f is the particle cloud method, f is going to be the valid is equal to epsilon raise to the power minus k and k will be the function of Reynold number and it can keep on changing.

So, if you see remember these equations which we have discussed earlier there is always a epsilon g, and that is what I said that this why this epsilon g comes, I will explain later this is the reason the epsilon g comes here. In this case Gidaspow has says that epsilon g the k value will be actually equal to minus 2.65.

Now, there are several correlation you will find for the drag in literature and this k this value the exponent value will be keep on changing. So, do not get confused, but you will always see the epsilon function in the f naught. So, the main particle cloud drag will be equal to the single particle drag multiply by volume fraction of the fluid raise to the power n value or k value which is the k constant, which will be keep on changing and different researcher for different experiment had found the different value of the k.

So, if I do that ideally speaking as per the experimental data says, you can predict the drag on a particle cloud or a group of the particle which are moving. So, this solves the problem of the particle cloud. So, what we have discussed till now, if I just try to quickly summarize, we have discussed the single particle drag different drag closures which are available for the single particle, and then thereafter we have tried to discuss how that if you have a multiple particle how the single particle drag should be modified in terms of the multiple particle. We have seen that how the Ergun equation should be written in form of the drag f value. So, that there will be no confusion that how the Gidaspow has given the f values there.

So, all these things have been discussed now, what we are going to do, as I said till now whatever the forces we have considered we have said that one particle is moving say vertically moving downward, we have considered the force say F D will be moving this, FD will be there and Fg will be there and this is the initial momentum.

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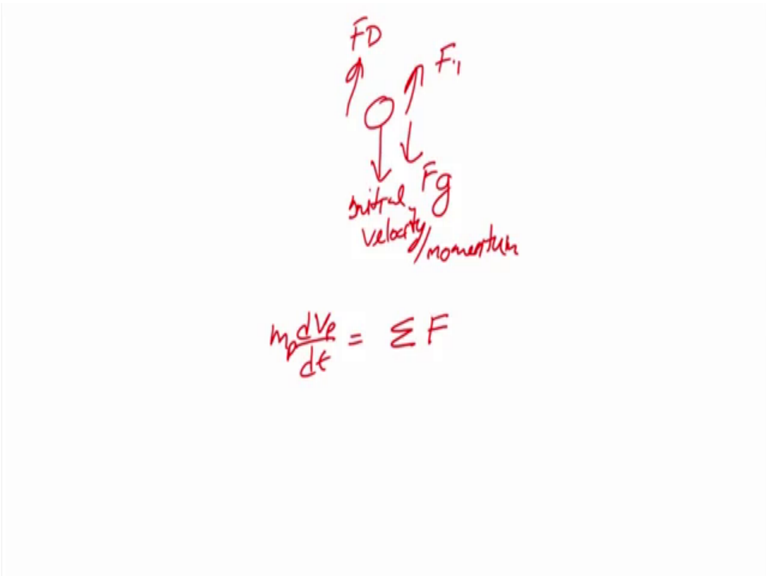

But different other forces also act on the particle, some forces are important for the gas liquid application, some forces are important for the gas solid applications, but there are other forces also which affect the motion of the discrete phase. Now again I am saying discrete phase because it can be droplet, it can be bubble, it can be particle. So, other forces also act and we need to define now those other forces and those other forces are nothing, but actually virtual mass force, lift force, Basset history force and there are others also, but mostly these three forces are very critical.

So, now what we are going to do, we are going to first discuss about these forces how to write these forces what is the significance of these forces and how to mathematically write the forces. Now once you know that matrix formulation, what you can do? We can do the same equation mdV upon dt of the particle is equal to summation of the forces. Now you can add those forces depending upon their direction in this equation, you can solve the particle tracking equation the way we have done in the previous classes which we have seen that, the similar way you can use to solve those.

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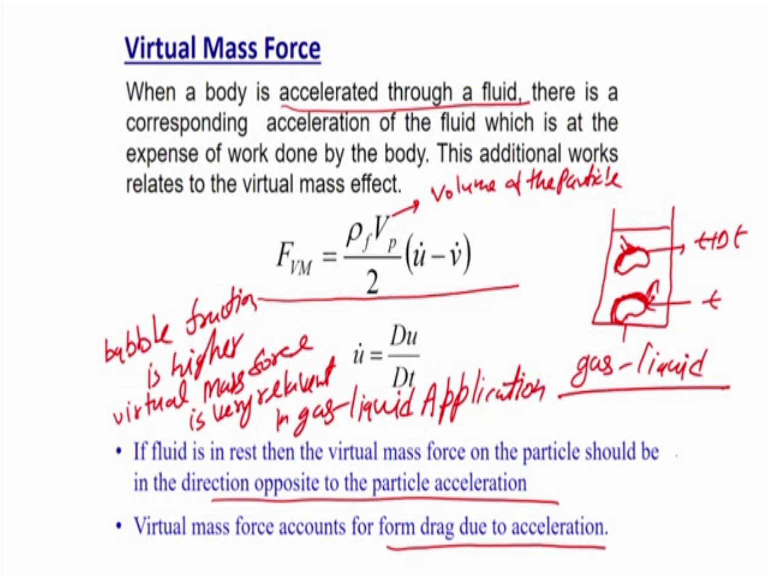

So, first thing the first force which is very critical is virtual mass force. Now what is the virtual mass force? As the name suggests the force is nothing, but it is a virtual force. Virtual force means it is a force which you are actually filling virtually ok. So, what happens suppose I will take a bubble column in this suppose liquid is stationery and a bubble is injected at the bottom, and this bubble is accelerating through this fluid.

So, what will happen? If the bubble is moving, it will transfer some of the liquid from this location to the next location because it is accelerating it is moving. So, if it will move the liquid adjacent to it, which was earlier at this location. So, this is the liquid film I am darking it. So, that this is in form of the film, which was earlier position here say at time t it was here. It if the bubble moves at this location this liquid will at t plus delta t say t plus delta t this liquid actually will move here ok

Now, this is mainly because the fluid has moved because of the acceleration of the body and because of that the additional work has been done and additional mass which has been transferred from this location to this location. So, this location using additional mass and because of that additional mass, one force will act on the body and that force is called the virtual mass force.

That is the way the virtual mass force is being defined that when the body is accelerated through a fluid there is a corresponding acceleration of the fluid which is at the expense of the work done by the body. So, because the body is moving the fluid will also move, some of the mass of the fluid will be transferred from one location to another location and that is actually the virtual mass effect which has been transferred and that will act upon a force, which will be acting on the body ok.

So, that body force is called virtual mass force and because this is the Lagrangian motion of the particle this is the Lagrangian motion of the discrete particle I will say bubble as well as the fluid, we define the virtual mass force exactly in the same way as we have defined the drag instead of the slip velocity, we take the Lagrangian velocity ok.

So, how we define? We define the drag as nothing, but rho f into V p, the Vp is nothing, but the volume of the particle this is the volume of the particle and into this. So, what you are seeing here is actually nothing, but is the acceleration and that is called the virtual mass force ok. So, this is m dv upon dt. So, this will be V p is nothing, but volume of the particle rho f is the density of the fluid. So, if you do that, this is actually effective mass which has been transferred into the slip velocity dv upon dt because this is the substantial derivative. So, this u naught into v naught is nothing, but the substantial derivative and dv by dt is nothing, but the force and that force is being generated with the virtual mass.

Now, why substantial derivative has been taken? Because this is the Lagrangian motion, this is the motion where the particle or bubble has transferred the fluid from one location to the another location ok. So, this is going to be the function of both position as well as time this is both coming into the picture and that is the way the substantial derivative has been defined because the mass has been virtually transferred from one place to another place.

So, this mass is also very important and this mass is very very critical particularly for the gas liquid system, where the liquid gas actually or gas bubble transfers some of the liquid with it. In case of the gas solid the this force is not very critical, because the solid this gas solid density is much higher and the transportation of the solid virtually is very very difficult and most of the solid is being transferred because of the velocity of the gas or. So, that is why they move. So, that is why this motion this force is more critical in case of the gas liquid system and particularly where the gas volume fraction is higher.

So, if your gas wall suppose you are using the churn turbulent flow or something, the virtual mass force is very very important and this force is also a count in a way for the form drag which created due to the acceleration. So, the skin drag and form drag. So, this force is also accounts for the form drag, which causes because of the acceleration of the fluid and this force is actually acting to the direction opposite to the particle acceleration.

Because this is the virtual mass force the particle is being accelerated, you have created the additional force on the upper side. So, this force will actually act in the opposite direction to balance the force on the body. So, this is the way the virtual mass force has been defined that how the virtual mass force act on anybody, and it is very critical for the gas liquid application of higher volume fraction of the discrete phase.

So, if the discrete phase or bubble volume fraction is higher, virtual mass force is very relevant in gas liquid application. So, this is one kind of force which is acting on the body and critical for the gas liquid flow acted on the body once again because of the extra mass which has been transferred from one location to another location, because of the bubble acceleration or discrete phase acceleration ok. So, that is the way the virtual mass force has been defined.

Now, another force which is very critical is called Basset Force and Basset force has been actually defined account for the viscous effects ok. So, what it does?

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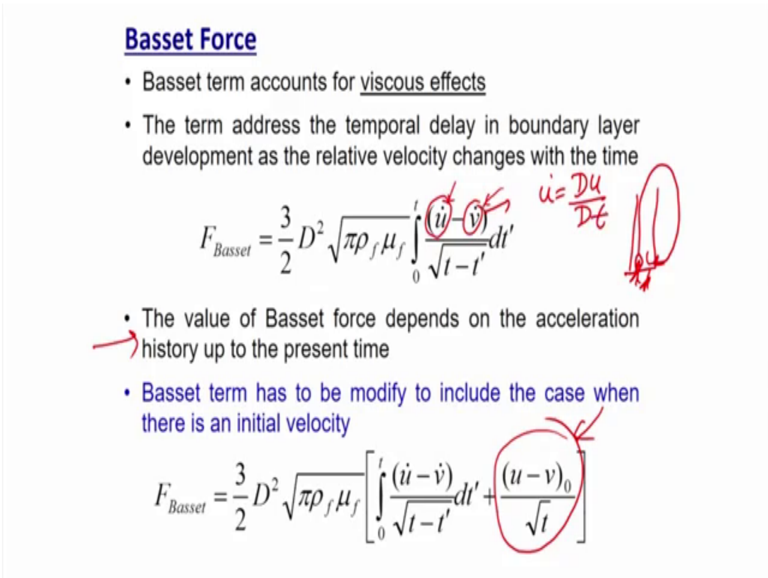

Because of the viscosity of a some force act on the body and that force is called the Basset force ok. This force actually also act the temporal delay in the boundary layer development as the relative velocity changes with the time.

So, the term addressed the temporal. So, Basset force what is does that, once there is a boundary layer development there is a temporal delay in that and this force also account for the relative velocity change which happen because of the temporal delay in the boundary layer development across the particle.

So, that is the Basset history force, it actually has been defined is more important when the particle is recirculating. So, it is being said that actually the particle the Basset history force that is why its called Basset history force, that the particle or discrete phase actually remember their history and why they remembered their history because of the viscous effect, and the temporal delay in the boundary layer development.

So, suppose if there is a flow in which of discrete particle is moving and if this discrete particle is again recirculated back before the boundary layer development take place, then what will happen? It will remember that the path of the previous location that how it has moved and that is called the Basset history force, which actually account the temporal delay in the boundary layer and is because of the viscous effect the damping effect which is remembered. So, it has been defined by 3 by 2 dp square into mu f mu f into rho f into pi and again it is a Lagrangian velocity of Lagrangian slip velocity,which is being there because of the relative velocity change in the boundary layer development and with the time how that development is going to be the account. So, this is going to be the function of this and it is affect the motion, and it says that the previous time step the particle actually remember that the previous time step phenomenon that is why the t minus t prime has been taken into the account.

So, the value of Basset force depends on the acceleration history up to the present time. It means when the particle is recirculating, what was the acceleration history how it is started accelerating and when the particle again come back to the same location, what is the acceleration. So, its remember it is the history of acceleration, how it is becomes accelerated ok. And if suppose there is some initial velocity, then we have assumed that initially the particle is started with the 0 if the initial velocity will be there, then the Basset history force term will be modified and you have to use the initial velocity also in this place.

So, you have to say that what was your initial acceleration before you come into the column and then once you come into the column the boundary layer is start developing, because of that days that some temporal change or temporal delay in the relative velocity development. So, how your acceleration of the particle is being modified because of that change?

So, if the initial velocity is not zero if the particle is coming inside at a rest position at a zero velocity, this component will be 0 because u minus v will be 0 and if it is not then the initial acceleration will actually also get modify and you have to take that account that in the history of the force. So, that is also called Basset history force this these are very weak forces whatever I am talking about, but it is very sometimes critical particularly for a recirculating system. So, this is the way the Basset history force has been defined. Please remember here again u naught and v naught is the substantial derivative. So, u naught is nothing, but its capital D u upon Dt ok. So, that is the way u naught has been defined.

Now, Basset history force as I said that these are weak forces and insignificant for most of the time. So, the Basset history force and virtual mass force becomes insignificant for application, where rho f upon rho d it means rho of fluid upon rho of discrete phase is approximately tens to the power minus 3 ok

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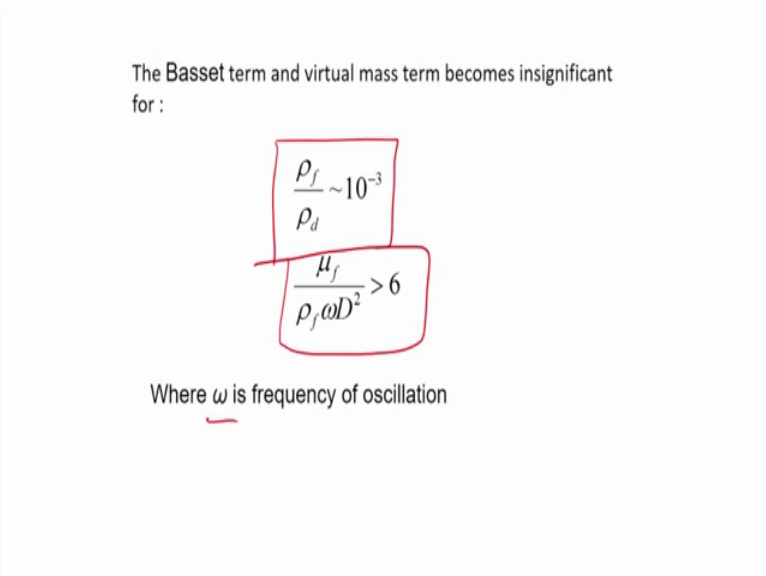

So, if this is the condition, the Basset history force and virtual mass force becomes insignificant. Now if you see that most of the time other than if it is in the order of 10 power minus 3, if you see that other than gas liquid system most of the time you will see this condition to be true other than gas liquid and liquid liquid system.

So, if you have a gas solid system, you will always comes into the range of 10 is to the power minus 3 virtual mass forces will be not be significant in the gas solid system, and Basset history force if this term is greater than 6, then the Basset history force has not been significant and where the omega is the frequency of oscillation, it means it is saying that the frequency when it is coming back again and if it is that is greater than 6 then again the Basset history force will be insignificant.

So, if this values are this then these forces are insignificant, and most of the cases actually say in the circulating fluidized bed or normal fluidized bed these values the circulation rate is much higher. So, this values are actually more than 6 and because of that what happens that, your forces is not very critical ok. So, these are the forces, but in case of the thing if you want a very accurate prediction, you need to account for this particularly the virtual mass force for the gas liquid system or liquid liquid system, if your the volume fraction of the discrete phase is high you need to actually account for that ok.

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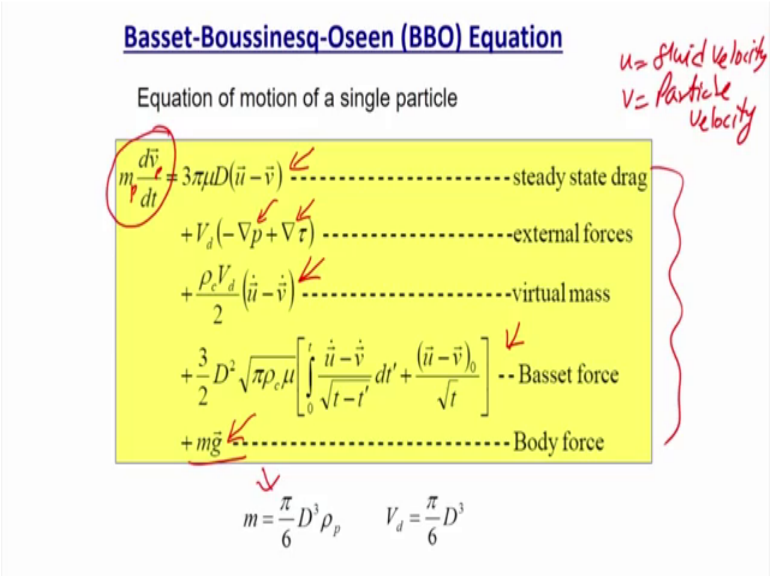

So, that is what is the Basset history force virtual mask force. So, finally, if I write the forces which is acting across one particle which is given by the Basset Boussinesq Oseen equation which is famously known as BBO equation, which we have done earlier also we can write that at steady state for one particle not a steady state sorry, in the motion equation of motion for a single particle should be given in this form.

So, what will be the single particle motion equation mdv by dt where m is nothing, but the mass of the particle. So, I will write it as a mp of dv p upon dt will be equal to. So, v here is giving actually the particle phase velocity, u is here the liquid phase velocity or the gas phase velocity of fluid phase velocity. So, u is fluid velocity, v represent the particle velocity. So, if you do that a momentum change force acting on the body m dv upon dt will be equal to what? Summation of all the forces.

Now, what will be the forces acting on the body? That will be the drag force we have taken written in terms of the stokes drag, it can be in terms of any drag. So, it will be actually the FD then your external forces what will be the external forces? External forces will be because of the pressure, because of the stress, which will be acting then it will be the virtual mass force which will be acting on the body, it will be the Basset history force or Basset force which will be also acting on the body and plus the any other external force like gravitational. So, that will be the complete force balance across the particle, and if you want to accurately track the position of a discrete phase position of the particle ideally one should solve the BBO equation which balances all the forces across the particle.

So, what we did? We take a single particle equation m dv by dt will be equal to drag force plus any external force, external force will be nothing, but the pressure drop across the bed plus the stress because of the across the bed the losses due to the stress, then virtual mass force acting on the discrete phase plus Basset history force which is acting on the discrete phase plus any other external force or body force. So, all these forces will be the combine acting on the body and for accurate prediction one should solve this, but for the sake of the simplicity in many of the application, depending upon the application we neglect some of these forces and we simplify the equation and we solve it ok.

Here m is mass of the particle. So, for the single particle mass if the particle is spherical, it will be 5 by 6 dp cube into rho of p, and v d is nothing, but the discrete phase volume fraction which is the volume of the particle. So, pi by 6 into D cube. So, that is the way it has been defined and this is called BBO equation, which actually one should solve across the one particle to find it out the particle trajectory or the particle motion ok.

So, with this what we have done? We have covered most of the forces acting on the body which is acting on the vertical position in the vertical direction. There are certain forces which is acting in the tangential direction or not in the normal direction and that one of the very critical force on that is lift force ok. And that is why because it is not acting in the normal direction, we have not included the lift force in the BBO equation. Because this BBO equation is in the same direction of the motion of the particle, we will see that lift will not act in the same direction it will act in some other direction other than the normal.

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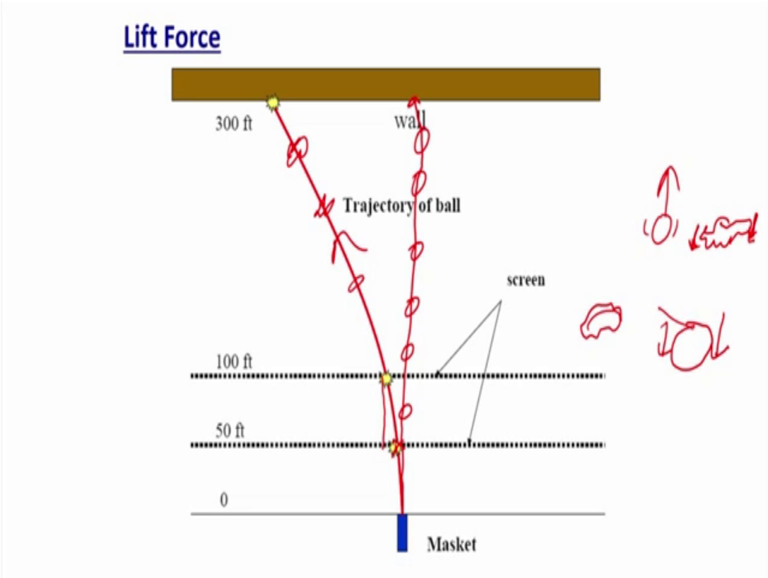

So, what is the lift? The lift is suppose if I take a basketball on this and we try to follow the marking of the ball if I throw the ball at a particular angle, what will happen? The ball will not move straight the ball will move at a certain angle. So, what will happen because of this change what will happen? If I find the two different places the particle location, the location of the particle will be actually different.

And why it is different it is because of the lift. So, why this lift occur? Suppose if I take gadgets suppose where we can throw the particle which is throwing the particle in vertically, and if suppose the particle is not a spherical in nature. Let us assume that the particle is not a spherical in nature particle is of some irregular shape.

If the particle will be of some irregular shape, once you will inject the particle or once you will throw the particle initially the particle will lift upward or it will move upward then what will happen? There will be pressure acting on. So, the particle which is moving up say one particle.

Now, if the particle is spherical both the side forces is same and that is why it is not changing its direction, but suppose if I am using a taking a particle which is of irregular shape something like this what will happen? The force acting on this side and the force acting on this side will be different ok. So, because of that, the particle will tilt it motion and it will try to move. So, 0 will be some friction acting on this side some friction acting on this side. So, there will be loss in the friction because of the pressure wherever the frictional losses will be more, the pressure drop will be higher wherever the frictional losses will be less, pressure drop will be lower.

So, because of this irregular shape the friction will be different of the different sides and because of that there will be a pressure gradient on both the side and because of that pressure gradient the particle will start moving from higher pressure to the lower pressure side, and instead of moving upward it will follow a trajectory it will follow a trajectory and this trajectory is mainly because of the lift force, which is acting on the particle.

Originally the particle actually should travel straight and hit the wall here while it is hitting somewhere at this location. So, what it means that, if I just solve the normal forces and I want to track the trajectory of the particle what will happen? If the lift if the particle shape is irregular then the lift becomes very critical and in that case if you solve the BBO equation to get the particle position even if you take all the forces into the account, you will get the particle trajectory in this line while ideally the particle is moving in this line.

So, there the irregular pressure difference of friction is there the lift becomes very important, and that is mostly critical either the velocity when the case where the velocity field itself is not same or the particle shape is irregular. Like why I am saying velocity field, suppose I have a motion I have the particle where this side is moving at a higher velocity, this side is moving at a lower velocity again you will see a pressure gradient, then again the particle will be actually start moving in one direction.

So, that is the force the force, which is acting because of pressure difference or that is created because of the velocity or the pressure gradient that is created because of the friction because of the irregular surface a tangential force act on the body which changes the trajectory of the body of the body position and that tangential force is actually the lift force ok. So, it changes the whole trajectory of the particle which I have tried to show here and that is why we make a particle which is irregular shape to explain the fundamentals ok. So, this is what is the lift force.

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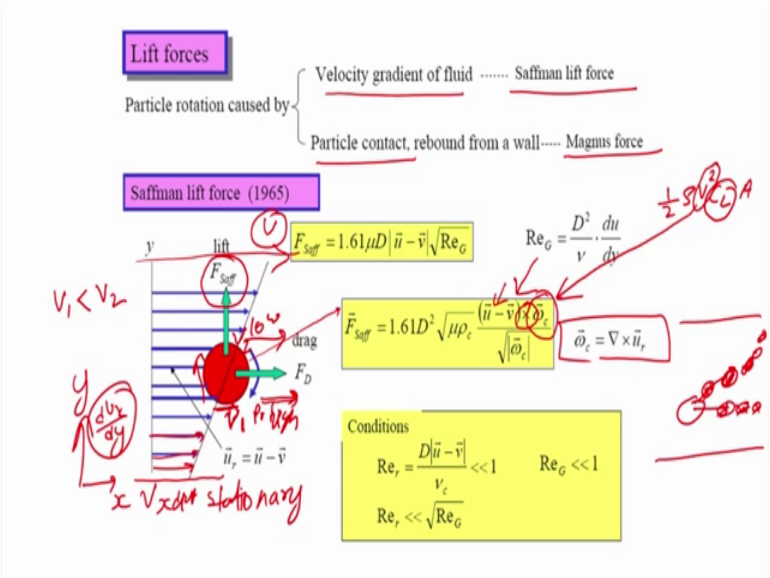

Now, how to account the lift force? So, lift force is actually as I said that it can be created because of the velocity gradient, it can be created because of the wall friction ok. So, based on that velocity gradient or wall friction the lift force has been divided in two part, the first lift force is called velocity gradient which is created because of the velocity gradient is called Saffman lift force the lift force which is created because of the contacting it means because of the wall surface is called Magnus lift force ok.

So, this has been defined it in two ways. So, let us assume that I have a this is very typical problem which you might have solved earlier in your transport phenomena, that two plate motion between the two plate ok. So, suppose there is a one plate here one plate here at this location, and suppose if I start pushing this plate if I start pulling this plate at a velocity v and this plate is stationary.

What will happen because this plate is moving at a velocity v the fluid adjacent to this will start also moving with a velocity v because of the no slip boundary condition. So, you will see higher velocity here, here again mostly boundary condition will prevail. So, the fluid at this surface will be moving at a zero velocity. So, you will see a linear change in the velocity from the 0 to the v this way. So, you will get this kind of a velocity vector you will get this kind of a velocity vector the way we have shown here.

If suppose a particle has been placed here and which is also moving in this direction in this direction with the plate because finally, the fluid motion is it in this direction, but there is a velocity gradient in the y direction. So, suppose this is the x direction, this is the y direction. So, the primarily motion is in the vx direction, but there is a velocity gradient is in the dv x upon d y exists. It means there is a gradient in the y direction of the velocity if a particle is being suspended here which is moving because of the fluid. So, primarily the motion of the particle will be in this direction and drag will also act in the same direction which will oppose the relative motion ok.

So, if the particle velocity is lower than the velocity of the fluid it will initially favour the particle motion. So, let us assume that is the case, that initially the particle is stationary and when they start pulling this, then the particle start moving ok. So, the F D will be favouring here what will happen? The because of the velocity gradient you will see that here the velocity is less here the velocity is less.

So, suppose this is V 1 if I assume this surface the velocity is V 2, V 1 is less than V 2. Now if the V 1 is less than V 2 then as per the Bernoulli what will happen? The pressure here will be higher compared to the pressure at this point ok. So, there it will be we will see the pressure will be high here it will be low and because of that the particle will start moving from high pressure to the low pressure side and you will see a recirculation it in this form. So, the particle will start moving it in this way.

So, instead of just moving this side the particle will be start moving in some other direction also vertical direction also, and it will follow a trajectory something like this ok. So, that is the wave is being there and this recirculation which you are seeing actually the particle start recirculating, it is mainly because of the lift forces and this recirculation will cause the additional force on the particle which is being given by the lift force and because this is the motion which is not in the normal direction, it is somewhere the other direction of the force normal direction to the force which is acting, there is a cross product comes into the picture and it has been defined u minus v cross w c, where wc is nothing, but del cross u r which ur is the relative velocity upon omega c.

So, this is the way the Saffman lift force has been defined, which clearly says that it is nothing, but it is the normal force of these velocity because we are doing it in the cross product. So, there is a recirculation omega c has been created, the recirculation is nothing, but how the circulation has been defined it is del cross v. So, it will be del cross the velocity of the particle or relative velocity will be equal to this will be the direction of the Saffman force will be equal to u into this.

So, this is if you will see has been actually defined similar to the drag, only the thing is that instead of the velocity square where we did u minus v whole square, we did u minus v multiplied by del cross u r which is actually giving me that del cross u r is giving me the recirculation velocity or rotational velocity and the cross product is telling me that the direction of the Saffman force will be the perpendicular to the motion of this. So, this will be the direction of the lift force.

So, you will see that the particle is not moving only horizontally in this way, the particle will now move in this direction somewhere. So, it will start moving upward in this way. So, if I track the position of the particle instead of this position which ideally use if you will solve the BBO you will get. If you solve BBO and lift together you will get this kind of a position of the particle which is the more realistic position ok.

So, this is the way lift force has been defined. It has defined in the similar way do not go with this empirical values,they are empirical values, but finally, it is defined in the similar way it is half rho v square into CL instead of the cd, the CL value is there, I have written the CL value here that is why it is looking at it in this form into area in that way it has been defined the area I have written in terms of the D square I have just simplified it here CL value I have taken the v square actually is being written it in this form so that, we can know that this force is actually in the vertical direction.

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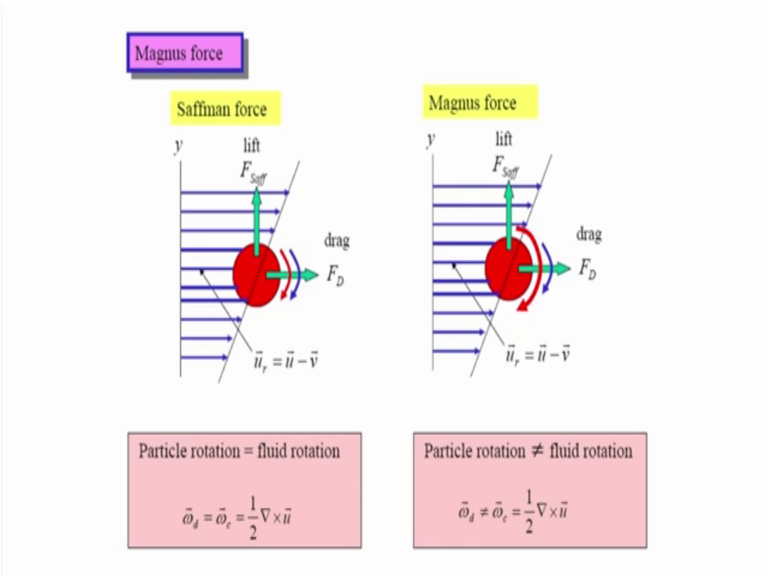

So, similar way Magnus force is also being defined and the Magnus force is being defined because of the particle friction. So, if there is a frictional difference is there between the same particle then the force which will be act is called the Magnus force, the if this the pressure radiant then it is called the lift force.

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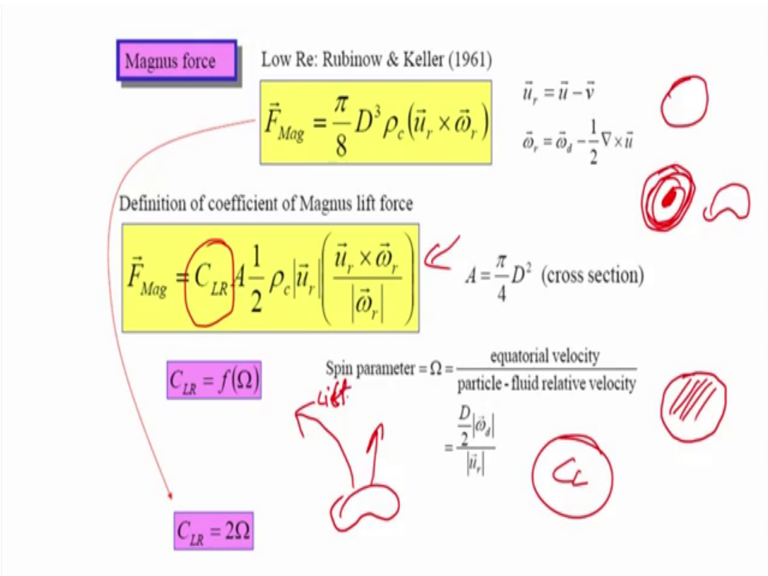

So, now most of us we always talk about the reverse swing in the cricket; most of you might be a fan of reverse swing. So, how the reverse swing is generated is the suppose a ball is there, if I tamper the one side of the ball if I kind of put a friction on the one side of the ball and make the ball very very smooth on one side and another side is rough then what will happen? Suppose there is a ball and this side is pretty much rough another side is very very smooth that is why you see that the fast bowler keep on rubbing the ball on their kin or on their cloth so that, they can make it either very smooth or rough.

Now, if you throw the ball in the air, then what will happen? The friction on the both the surface of the ball will be different one side is very very smooth friction will be low and this side is very very rough or rough the friction will be high. So, what will happen? The pressure drop across this surface which is very smooth will be lower, and the pressure drop across the surface will be where the this is high will be higher.

So, what will happen? The pressure drop is higher it means the pressure will be low on that side pressure will be high on this side and it will move from low pressure to the high pressure. So, the ball will swing in air itself that is what is called Magnus lift force and that is generated mainly because of the roughness on the surface and that has been given exactly same formula which I said that it is nothing, but half rho f into u into a into CL, the u has been defined again in terms of the cross product the wr is nothing, but is the same way del cross u r it has been defined, and you can find it out the lift forces.

The only thing is the value of C LR which you need to find there is different correlation available I am not going to discuss that, but if you see your book different correlation is available to find the CL value some of the correlation some of the value of the CL I have solved and shown the equation in the previous slides, but finally, the C LR value you have to do different literature is available, many people have worked on that tomiyama et al have worked as really significant in the field of the lift force many people work they try to find it out that what will be the value of the CL, they try to find the correlation for the CL similar way as the we have done for the drag force ok.

So, similar kind of a correlation is available several correlation is available if you have a interest you can go and see those things, if you have any problem you can write it to me I can we can discuss it over the assignments and you can see that how the CL has value has been generated. So, we have found that the CL values and in case of particularly the gas liquid system where the bubbles actually shape is irregular; it is not a spherical if you remember always I make a bubble of this shape. The bubble shape is not a spherical bubble shape is of irregular shape, lift becomes very very critical and it has been found that if your velocity of the fluid is very very low then sometimes the lift is even more than the drag force.

So, it means what? If you have a irregular shape bubble, in velocity of the bubble is very very low the bubble will not move up in this direction the bubble will follow some other trajectory like this and that is mainly because of the lift force acting. So, this has been done several paper has been published on this, you can see the papers in chemical engineering science, chemical engineering journals, international journal of multiphase flow will sign several such correlation several paper is still it is a hot area of research, that how with the change in the bubble shape or bubble type or the liquid type or the shear stress the lift forces, how it will be modified.

And people are working on this several correlation is available; again I am saying I am not going to discuss in those correlations. If you have interest you can see that if you have any problem you can come to me.

So, what we have done with this we have completed the forces, which is acting on the particle, you can do all the particle tracking, you can find the trajectory of the particle if lift forces is also acting you have to solve the BBO equation along with the lift forces every point you have to solve that what is the BBO equation position of the particle, then solve for the lift position and see that how much shift in the vertical direction you will get. You will get the new position you will solve the BBO equation again, again go and find the lift force you will again see that how much particle will move and how much change in the direction will take place.

So, in that way you can solve the complete trajectory of the particle ok. So, with this phase our one plot of this is over, now what we are going to discuss in the next time or in the next classes is about the coupling. So, right now whatever we have done we have just found that the fluid the coupling between the mean velocity of the fluid and mean velocity of the particle. Now we will see the other coupling too that how the other coupling will get affected and how to take account of those forces and how to solve the whole equation together ok. So, we will see those things in the next classes.