Multiphase Flows
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Lecture - 12
Multiphase Interactions: Drag Force

Welcome back. So, what we are going to do today is what we have done first, I will discuss that briefly. So, we have seen the force balance for a particle tracking and we have seen that how the kind of different forces like electric forces or magnetic forces or any forces affect the particle motion, we have seen how in presence of the drag, buoyancy and gravity the particle motion is being changed.

So, what we have covered till now is the particle motion or the pressure drop calculation for the gas-liquid flow or liquid-liquid flow in 2 phase flow and gas-solid flow. So, 2 phase flows, we have completely covered whether it is a gas-liquid or liquid-liquid or gas-solid ok. So, that is what we have covered.

Now, in everything what we have seen that the drag force play a very important role and how to calculate the drag force? So, the drag force below Reynold number 1 which is called stokes regime. We have already defined the formula and we said that below the Reynold number 1, it is C D upon C D equal to 24 upon R e ok. So, that is what we have already done that it is below Reynold number 1.

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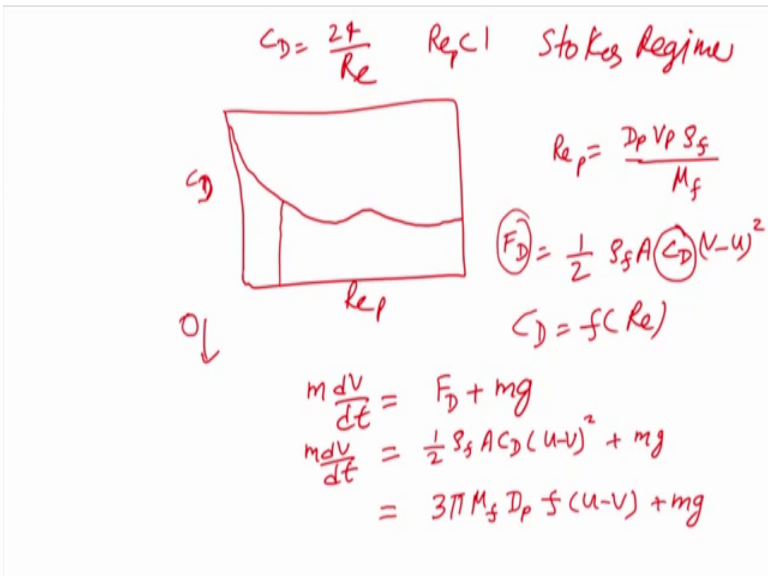

C D will be equal to 24 upon R e. Now, whatever we have done whether it is for gas-liquid, whether it is for the gas-solid ok. The gas-liquid can be also extended to the liquid-liquid. Everywhere the drag plays a very critical role. And we need to understand that how this drag force will be calculated. And during the class, I always keep on telling that different correlations are available for the drag force. For below Reynold number 1, this correlation is fixed this is called stokes regime. And in the stokes regime, we have already known that drag is 24 upon R e, but what happened below Reynold number more than 1, Reynold number of particle more than 1 ok?

Then, we do not know actually that how the Drag Force will be defined. So, for that, there is a graph available which plot the C D versus Re and I think I have already discussed that graph. That that graph is available which is along the C D versus Re; this is C D, and this is R e. So, there is one way that you can always go to the graph for your Reynold number ok. And Reynold number of particles once we say whether it is a particle or based on the discreet phase, say it is a bubble, it is a droplet anything.

You can define the Reynold number and Reynold number will be what? Reynold number of particle will be D of particle into V of particle into rho of fluid upon mu of fluid. So, that is the way Reynold number of particle has been defined. The particle can be bubble, particle can be droplet, particle can be solid.

So, this is one way that the people have plotted the C D versus R e. One can go to C D versus R e graph, see that for respectability Reynold number what is the value of the C D? And based on that you can calculate the drag force and drag force is being defined as half rho of fluid into area into C D into V square or it is the slip velocity square ok. So, let me write it in terms of the slip velocity. So, I am writing it as a v minus u. So, you can write it in in that way. So, you can calculate that. What we need is only C D.

So, the one way is the graphical method, but the problem is if I am writing a numerical solution; if I want to write a code, I do not want the code should be a my program should always go and look for the graph, because it will make the program very slow or the system very slow; even your calculation very slow.

So, what we want? We want the people have done several experiments. And different people have done experiments for the different conditions and proposed the different drag loss. This I have already told that there are several drag laws is available for

Reynold number greater than 1. What we are going to do now? We are going to discuss some of those drag laws which are popularly used or widely used in the multiphase flow field. And we will try to understand those drag law.

And then those drag laws actually do what? For a particular experimental condition they give the correlation between C D which is a function of R e. So, they are going to give the correlation. And we are going to discuss those drag laws. They are very very critical because in all your programming, in all your calculation, you use the drag law instead of using the graph. And based on that what you have to do you have to calculate the C D value, that C D value will be used to calculate the F D value and this F D value will be used in your equation ok.

So, in that way the whole equation solution you can get and you can find that the drag correlation. And as I said that drag is a very important because it is acting between the mean motion mean velocity of the particle to the mean velocity of the fluid ok. So, this is a very critical force.

So, now, let us again go back and I am writing the same equation which we have discussed; a particle which is moving in vertically, I am neglecting the buoyancy then what we will be writing that suppose the particle which is moving down, then I will write m d V upon d t is equal to F D plus mg this we already done. So, if I write it for that it will be what I can write? I can write the F D in terms of half rho f A; C D, I can write, u minus v whole square plus mg ok. This is the way we have solved it all the problem our.

Now, what we can do in several places to define the C D, people have defined another value which is called F and that is called as a drag factor. And the whole drag equation F D can be written as in terms of the stokes drag. Now what is strokes drag? It will be 3 pi mu of fluid into D, D of particle into I will write as a F and I will u minus v plus mg. We can also write this equation in terms of the stokes drag and we can define a function which is called drag factor F and drag factor F.

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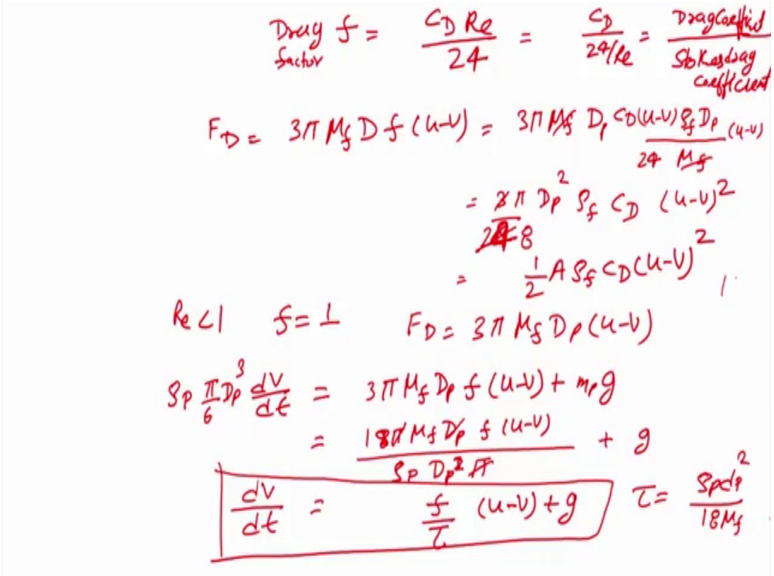

Is defined as C D into R e upon 24, it means what? It is the ratio of drag factor drag value C D, drag coefficient value to the value of the stokes drag ok.

So, I can say that it is actually C D upon 24 by R e. So, drag coefficient divided by stokes drag, stokes drag coefficient. And f is being defined and because I am writing in terms of the stokes drag F D, I have changed I have modified it in terms of the 3 pi mu. So, F D we have written as 3 pi mu of fluid D f into u minus v ok.

Now, f you will write it in this form whatever the way we have written what will be this this will be 3 pi mu of f D into f will be C D into R e; R e I am writing in terms of the v, u minus v because based on the slip velocity u minus z into rho of fluid into D of particle; this is particle v of particle divided by mu of fluid into 24 and u minus v. So, if you see this this equation; what it will be? It will be 3 to pi D p square into mu f mu f will be cancelled out into rho f into C D into u minus v whole square and this divided by 8 ok.

So, if you do that this will be actually 24, sorry this divided by 24. If you do that this will be 8 and we know that pi by 4 D p square is area. So, it will come as a half a rho of fluid C D, u minus v square. So, that is the way it has been defined it has been converted. So, what people have done people have defined a function which is called drag factor ok. And drag factor, this drag factor f has been defined as a ratio of drag coefficient at that condition divided by the stokes drag coefficient.

So, in that way we get the f value and they took this equation can also be defined in terms of the f and if stokes regime flow; if the flow is under the stokes regime, it means if R e is less than 1, if R e is less than 1, then f value will be 1. So, for the stokes regime, f value will be one and you will get that in the stokes drag law it will be nothing but 3 pi mu f D p into u minus v ok. So, in that case F D will be equal to 3 pi mu f D p into u minus v. So, this you will get the f value will be equal to 1 that is nothing but equal to the stokes drag, which you have already done we have already discussed that.

So, in that way we can define. So, what we can do? We can write the equation d v upon d t into mass of the particle mass of the particle. I can write it as rho of particle into the volume of the particle and volume of the particle can be written for this spherical particle it will be pi by 6 D p cube and that will be equal to 3 pi mu of fluid into D of particle into f into u minus v plus M g m into g, M of particle into g.

Now, if I divide it by this the M of particle and M of particle will be cancelled out here. You will see a value which will be 3 pi mu f D p into f u minus v ok, this will be 6 will be multiplied here. So, you will see 18 ok. Now it will be rho of particle D p cube into here; it will be coming pi and this will be equal to g because M of particle will be cancelled out with the m of the particle.

Now, if you see that this D p and this will be cancelled out. So, this will be becomes D p square. So, rho p D p upon 18 mu f will be nothing but tau, this pi pi value will be cancelled out. So, we will get a equation which will be f upon tau into u minus v plus g that will be dv upon dt where we know that tau is nothing but is equal to rho p into D p square upon 18 mu f, mu of fluid. So, that is the way we can reduce it into that place and we can define the drag forces either in terms of the f or in terms of the C D.

Now, why I have said this? Because some books we will see the C D value, we will see the drag value drag coefficient value; how it is going to change with this R e for the different correlation, but some books write the correlation in terms of the f which is drag coefficient. Some book write in terms of the C D which is the drag coefficient. So, one should not get confused or you should not get confused if you see the terms in terms of the f; f is nothing but the drag factor and the whole equation will be modified at it in this way.

So, I hope now you can easily convert if you see some correlation somewhere in terms of the drag factor or f or in terms of the C D, you can interchangeably use those parameters. And this equation we have already developed. Now what we need we need the value of f. We have developed the same equation in terms of the C D, we need the value of the C D or if we need the value of f. Once we have done that, we can track that how the particle position is changing with the time ah, how the particle velocity is changing with the time and how the particle Reynolds number is changing with the time. We can do all those things, we can find the particle trajectory completely.

So for that, now what we need we need the value of f or value of C D and several researcher have done lot of work and they have tried to find it out how to calculate the f value or the C D value ok. They perform the experiments and we briefly also try to discuss the experiments that how the experiments has been performed, what is the advantages and disadvantages of each method and then we will discuss those things.

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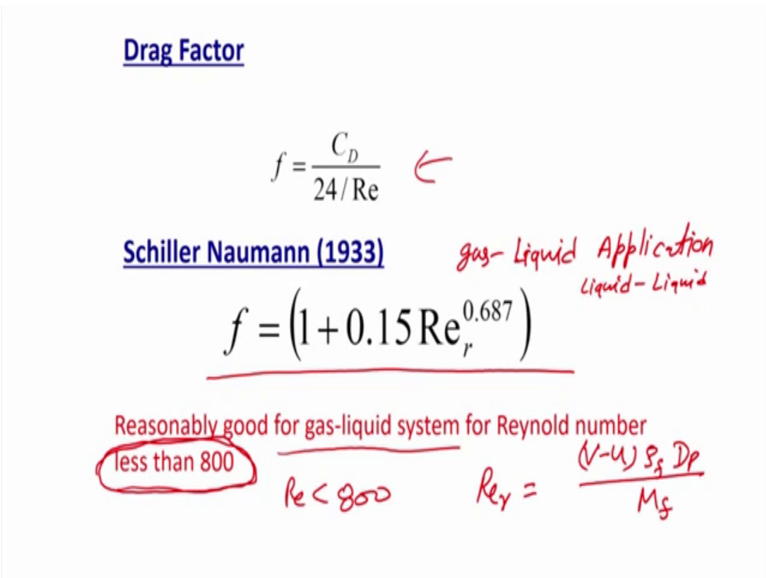

So, the f we have already discussed is nothing but drag factor which is C D upon 24 by R e. So, the first correlation actually, not the first correlation per say, but the most important correlation which is widely used for the gas-liquid application people, also use it for the liquid-liquid but it is more commonly used for the gas-liquid application is the Schiller Naumann correlation, Schiller Naumann drag correlation.

Now, Schiller Naumann drag correlation is actually being developed in 1933 and it has been developed for a bubble which is rising in a liquid. So, in that that is why, because it is developed for that, it is being widely used for the gas-liquid correlation. The accuracy is very high for the gas-liquid, but people also use the Schiller Naumann correlation for the gas-liquid, liquid-liquid and even for the gas-solid or for the liquid-solid all those places people use, but the accuracy is really there for the gas-liquid system.

So, Schiller Naumann correlation says that the experiments was performed for the moving drag bubbles, for the different sizes, for different condition and it has been found that the drag factor f will be nothing but is equal to 1 plus 0.15 into R e number R e r Reynold number or relative Reynold number raised to the power 0.687.

Now, what is the R e r? R e r is relative velocity, it means it is v minus u velocity of the fluid minus velocity of the particle into rho of fluid into D of particle divided by mu of f; based on that it has been defined and Schiller Naumann claim that this is valid for all the Reynolds number ok. So, anything which is Reynold number more than 1, it will be valid and you can calculate the f value, that is the claim of the Schiller Naumann made in 1933 ok. And they have said that that it can be used for anything because the basic parameter remains same that one particle is moving in a liquid or in a kind of a fluid.

So, this has been used and this has been widely used even now; this is one of the correlation which is still valid and predicts the equation or predict very well for the gas liquid application; however, it has been found recently that this correlation too reasonably good for the gas liquid system for Reynold number less than 800. So, please see this value; this is very critical that it do the good job for Reynold number less than 800; R e is less than 800. Though the Schiller Naumann claim that it can be worked everywhere but it has been found with the literature many people have used this and they found that if R e is less than 800, this correlation gives a good prediction of f for the gas liquid system; not for the gas-solid; for the gas-liquid system ok.

So, this is one of the correlation has been developed and it is a pure empirical correlation. You can see the value all the constants they are empirically fitted constant. They have the experiment has been performed for the different conditions and based on that the empirical equation has been fitted, the C D versus R e plot has been given or f verses R e plot has been plotted and then empirically fitted equation has been developed.

And because this has been fitted for all the Reynold number, they assume that it will be valid for all the Reynold number for all kind of a flow but later time it has been found that this is only good for the gas-liquid system for Reynold number less than 800. So, that is the way it has been comes and this is called Schiller Naumann correlation widely used for the gas liquid.

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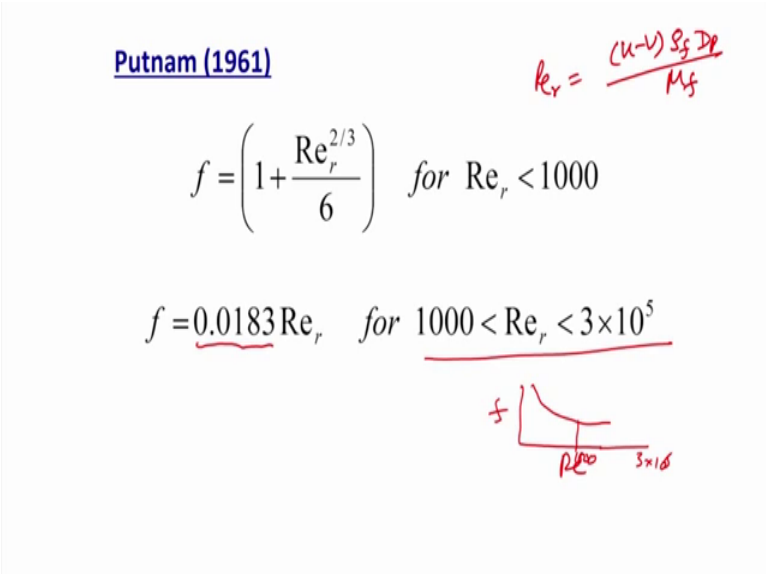

What happened there after Putnam has actually did some more experiment in 1961. I does not mean that between 1933 to 1961, no one has done anything; people has worked on that, but I am just discussing those correlations which are widely used in the literature for either the gas-liquid or liquid-liquid or gas-solid application.

So, I am discussing very few; there are lot of correlation available. You should not get confused. These correlations are most widely used correlations and most of the literature if you will go; see the papers on the gas-liquid liquid-liquid or gas-solid; You will see these correlation frequently used.

So, Putnam in 1961 has given one correlation for the f and that correlation is f is equal to one plus R e r raised to a power 2 by 3 divided by 6. It has been again developed for a solid sphere suspended in a fluid and then again R e r is based on the relative velocity. So, it will be again the same R e r will be nothing but is equal to u minus v into rho of fluid into D of particle divided by mu of fluid.

So, they have done this and they have found that this is for R e r less than 1000; they in fitted empirical correlation that is 1 plus R e r raised to the power 2 by 3 divided by 6. And for R e r value greater than 1000 and less than 3 into 10 to power 5, this is a big Reynold number they found that f is equal to 0.183 into R e r. Please see the correlation these all are empirical fitted correlations. So, do not get confused that why these odd numbers are appearing. These odd numbers are appearing because these all are empirically fitted correlation. Experiment has been performed plot has been drawn between the f versus R e and whatever the the f versus R e things has come they fitted the correlation.

So, what they have done. They have break the graph in 2 part: one is R e less than 1000 and one is R e greater than 1000 into 3 into 10 to the power 5 ok. And then they see that, the f value for the initial part is nothing but 1 plus R e r raise to the power 2 by 3 by 6 and f greater than 1000 and less than 3 into 10 to the power 5 it is 0.183 into R e r. Now what these correlations are doing? I do not need to go on the graph. I can just keep on doing the calculation. I can write my program and I can do the correlation, I can find the correlations. So, that is the Putnam 1961 correlation for the f.

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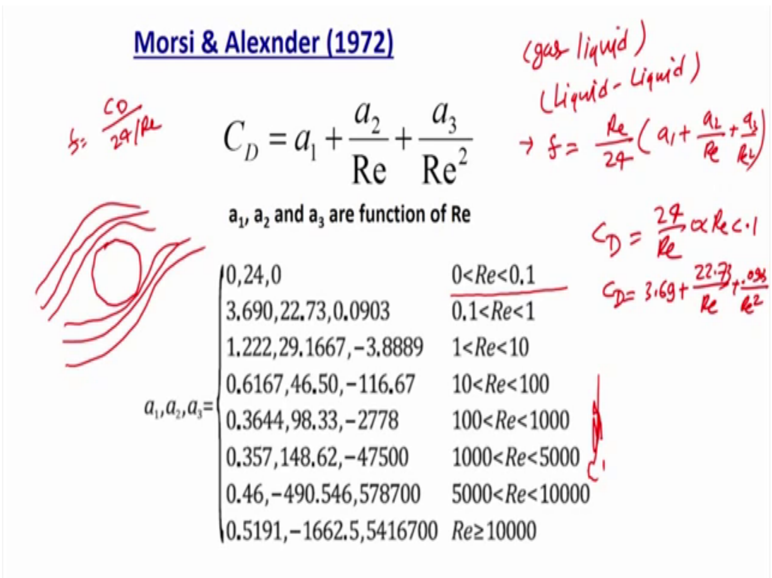

After that Morsi Alexander has found a correlation in 1972 and this is also widely used for the gas-liquid system also for the liquid-liquid system. The Morsi Alexander has developed the correlation in terms of the C D; you can also write it in in terms of the f.

What you need to do you just need to multiply here by 24 by R e. So, if you want to write the same correlation in terms of the f, I just do not need to do anything. I have to just multiply this 24 upon R e, a 1 plus a 2 upon R e plus a 3 upon R e. That will be f correlation because f is nothing is equal to C D upon 24 by R e ok.

So, if I want I have the C D correlation. I want f correlation, what I need to do? I need to sorry the here I need to divide it. So, this is the C D correlation, I need to divide it by 24 by R e, not the multiply by 24 by R e. So, this will be R e upon 24, this will be R e upon 24. I have to divide it by 24 upon R e the C D value, I will get the f. So, this will be R e upon 24 and this will be what you will get that Morsi Alexander factor in terms of the f.

So, you can interchangeably use some books, you will find the correlation in terms of the C D, some places in terms of the f. I have written in terms of the f in the original paper. If the value is given in terms of the f, I have written here in terms of the f in the original work if the value is given in terms of the C D, I have written in terms of the C D, but you can easily interchange C D to f and f to C D ok. So, if the correlation is given in terms of the f, you can easily convert it in terms of the C D. If the correlation is given in terms of the C D, you can easily convert in terms of the f ok.

So, this is the way you can write and they said that this fitted it in terms of the constant and they say that the C D actually works in a polynomial equation form which will be a 1 plus a 2 upon R e plus a 3 upon R e square. It means what the C D is inversely proportional to R e and they said that this equation is valid everywhere and they found that they follow the stokes law only for R e is less than 0.1. So, though we say that R e is less than, R e p is less than 1, the flow is in the stokes regime but Morsi Alexander found that R e is less than 0.1, it is following the stokes regime because if you put this value what will happen, the a 1 value will be 0 ok.

If I put it here, the a 1 value will be 0; a 3 value will be 0. So, what you will get? You will get C D is equal to 24 upon R e and they are saying that this is valid if R e is greater than 0 and less than 0.1, above then that this is not valid. So, stokes regime is not valid at all. And they have given the different correlation and they said that if R e is greater than 0.1 and less than 1, then you are not getting the stokes regime equation, you are getting the value of a 1, you are getting the value of a 2, you are getting the value of a 3. Though the value of 80 is very close to 24, they said that for R e p is less than 1 and greater than this, you are getting the value which is C D will be equal to 3.69 plus 22.73 upon R e plus 0.0903 divided by R e square.

So, it will follow this. It means Morsi Alexander found in their experiments that is stokes regime is valid only for Reynold number which is less than 0.1. And how they have performed the experiment; they have suspended a solid object in a moving fluid and they have performed the experiments for the different flow rate, different velocities, different Reynold number and try to find it out how the drag force is actually changing.

So, this experiment has been performed this is a widely perform they performed the experiment for huge set of data, huge set of conditions and they found that the particle was suspended and the fluid was passing through it, a shear was suspended and fluid was passing through it and they found the value of C D, they have written it in terms of the polynomial equation and the value of coefficient they have found for the different Reynold number.

So, these all are the value of coefficient for the different Reynold number. You can see this table. The only one interesting parameter they have found that R e p or Reynold number less than 0.1 is only falls under the stokes regime ever than the 0.1, it does not fall under the stokes regime ok. So, it is something very close, but it is something different and you will see that the a 1 and a 3 will also have some value ok.

Similarly, if you keep on increasing the value, you will see that if the coefficient value is keep on changing and somewhere you are getting the value in minus; it means now the value of the drag force will be actually keep on reducing. So, they have to perform the experiments and they perform the experiments for lot of Reynold number and their break date and they said that for Reynold number greater than 10000, the value is remains same always.

So, this is also used widely for the gas-liquid and liquid-liquid systems to calculate the C D value or f value and found that the accuracy is quite good, but many times the Schiller Naumann predicts better result compared to the Morsi Alexander. So, if you are writing or you are simulating or you are trying to solve the gas-liquid equation or liquid-liquid equation, the 2 equations which are widely used or commonly used; one is Schiller Naumann and one is Morsi Alexander ok. For Reynold number less than 800, it is found that Schiller Naumann, Morsi Alexander performs approximately good, approximately well or approximately same and for more than 800, the Schiller Naumann accuracy is really poor and Morsi Alexander in that conditions is being used.

So, this is the way it has been developed.

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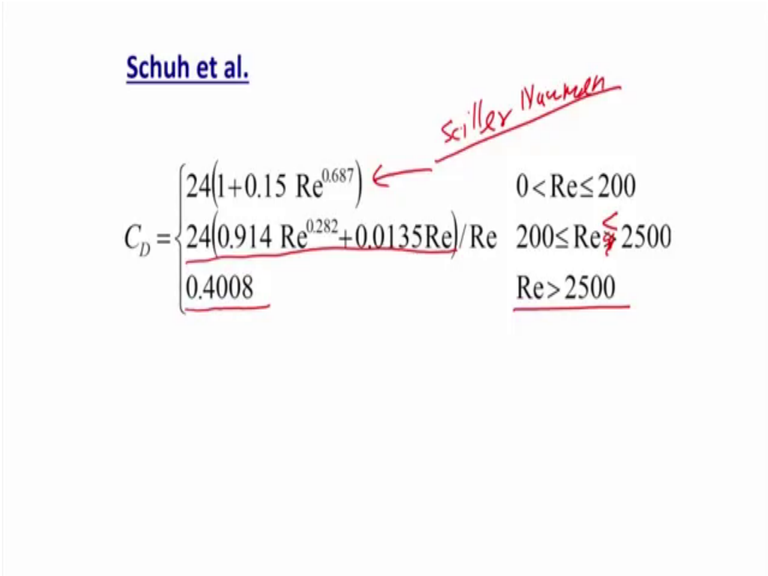

After that Schuh et al in 1978 has actually developed another correlation for the C D and they have said that the C D correlation is nothing but 24 plus 1 plus 0.15 into R e raise to the power 0.687.

So, if you will see this, this is equal to the Schiller Naumann correlation. So, this is equal to Schiller Naumann. So, Schuh et al says that the Reynold number greater less than only 200, less than or equal to 200, the Schiller Naumann correlation is valid; after that the Schiller Naumann correlation is not valid. They have given another correlation for Reynold number is greater than 200 and less than 2500 sorry this sign is opposite way around. This will be less than 2500. They said that this Reynold number, this value of the C D will be valid and they said that for Reynold number more than 2500, your C D value is always constant and the value is 0.4008.

So, they have said that now Schuh et al says that what you are getting is that your Reynold number. The Schiller Naumann correlation is only valid for less than 200. Reynold number less than 200. While most of the practical application it has been found that Schiller Naumann do well reasonably well for Reynold number less than 800, but as per their experimental data, the Schiller Naumann correlation is good only for Reynold number less than 200 ok.

After that they have given another correlation. So, that they are several such correlation available; everyone have their own opinion. Everyone have their own equation. I am not discussing that if you want to follow, you can follow the paper, you can follow the journals, you can follow the journals in international journal of multiphase flow, you can follow the chemical engineering science, chemical engineering general, AICHe or any multiphase flow books and each multiphase flow book if you will see, we will see a different correlation has been given or different set of correlation has been given. What I am doing, I am just summarizing something and trying to see those give you those correlation which are widely used.

So, Schuh et al is also used for the gas-liquid and the liquid-liquid system and has found well and many people have used this correlation to find the value of C D; however, most of the correlation whatever we will see other than the Morsi Alexander, people have tried to write the correlation in terms of the Schiller Naumann and they have modified the coefficients like this and they have tried to change if the Schiller Naumann is not valid, they have just tried to fit it empirically and found the different correlation as per whatever the value they have got.

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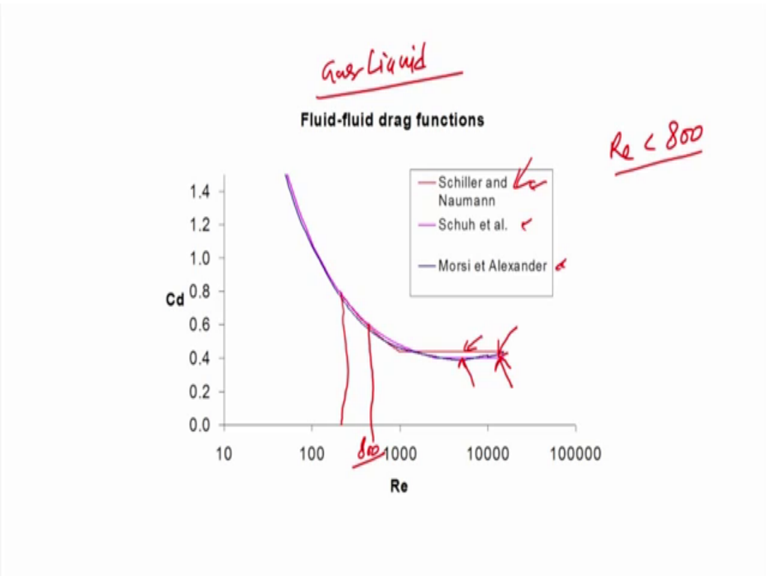

So, now based on that again, another correlation has been developed and this is the graph actually which I have taken from the literature where the experiment has been performed for the fluid and fluid. It is actually gas-liquid simulation and we have done the simulation for the gas-liquid and we have tried to compare the C D versus R e for the different correlations.

So, if you see this correlation and I have used the 3 correlation which are widely used; Schiller Naumann, Naumann, Schuh et al and Morsi Alexander ok. So, Morsi if you see this correlation values, then what you will find that the Schiller Naumann, Morsi Alexander and Schuh et al approximately predict well, approximately till this point, they are predicting equal and that is the point where Reynold number is around 800. So, that is what we said that I keep on telling that if Reynold number is less than 800 , then Schiller Naumann do a good job.

So, Schiller Naumann, Schuh et al or Morsi Alexander all 3 are approximately same. Once your Reynold number goes beyond 800, then what happen; you will find a difference and if you see that above 1000, the Schiller Naumann actually does not change anything. The value of the drag is almost remains same ok. So, they are not doing anything Schuh et al if you will see, above 2500 their value is still same if you see this values, the pink colour graph ok. If I zoom it little bit to show you. So, if you see this, this is constant Schiller Naumann constant, after around 800 or 900 this value is not changing and that is the reason that Schiller Naumann predict good only till 800 because after 900, it values remains always same.

After that if you see this Schuh et al the 20 after 2500, the value is almost same because that says that the after that the value will be around 0.4008. And Morsi Alexander if you will see they are keep on changing because this is based on the polynomial equation they are keep on changing. But above 10000 if you will see the Morsi Alexander, what they are saying? They are saying that your drag value is actually now increasing with the Reynold number, you drag value if you see here is actually increasing if again I zoom it. So, they are now increasing. That is why the Morsi Alexander is valid only up to 10000 of Reynold number after that because it is the equation in the reverse polynomial and the value is in terms of the minus also, it will increase ok.

So, what we found that if you are having a Reynold number which is definitely less than 200, everything is same around 200, everything is exactly same; between 200 to 800, there is a slight difference, but still Schiller Naumann correlation is very close to all other predictions. After that what happen Schiller Naumann correlation gives a flat correlation flat value while the other correlation keep on giving you the different values which you can use and the application and that is what they predict the different drag coefficient.

So, for Reynold number less than 800 that is why it is found in the literature that the value of the Schiller Naumann correlation predicts very good for Reynold number less than 800, after that one should either use the Morsi Alexander or Schuh drag coefficient. If you are not very sure about your Reynold number that how your Reynold number will change, how much it will change then one can use the Morsi Alexander or Schuh model to be independent of the Reynold number if you are not very sure that what is your Reynold number, but because these all are having different correlation. Morsi Alexander is computationally expensive.

Because what you have to do, you have to again go see your Reynold number, check your value of a 1, a 2 form the table and then use that value it takes long time. So, if your Reynold number values are less , this correlation Schiller Naumann is having only one single correlation. So, it is very fast ok. It respond very fast while if you have to Schuh et al then what you have to do you have to put a loop, you have to find a search thing that if your Reynold number is less than this, use this correlation If your Reynold number is less than more than 200 less than 2500, use this correlation. If your Reynold number is more than 2500, use this correlation.

So, if I am writing a program, I am writing thing, I have to put a search loop, I have to use a if command that is going to take time. So, if you are very sure that if your Reynold number is not going anywhere more than say 500 or 600 or less than 800, one should go for the Schiller Naumann correlation. If you are not sure about the Reynold number, one should go for a Schuh or one should go for Morsi Alexander.

Now, they all are computationally expensive. Schuh is Morsi Alexander is more computationally expensive compared to the Schuh ok, but only again if you are sure that your Reynold number is less than 2500. So, based on that your flow condition, you can use any of these Reynolds number and they can provide a good value, a good correlation for the C D value and which can be used to predict the drag force ok. So, predict the C D and from the C D, you can calculate the drag force.

So, that is the way for the gas-liquid and liquid-liquid system has been used. It does not mean that people do not use for the gas-solid. People have used it for the gas solid, but the accuracy in the gas solid is really poor because why, it has been developed either for the liquid-liquid or a solid which is suspended and the fluid is moving like Morsi alexander. So, in that case mostly it is a drag on the fluid and that is why they are actually working well with the fluid-fluid system ok.

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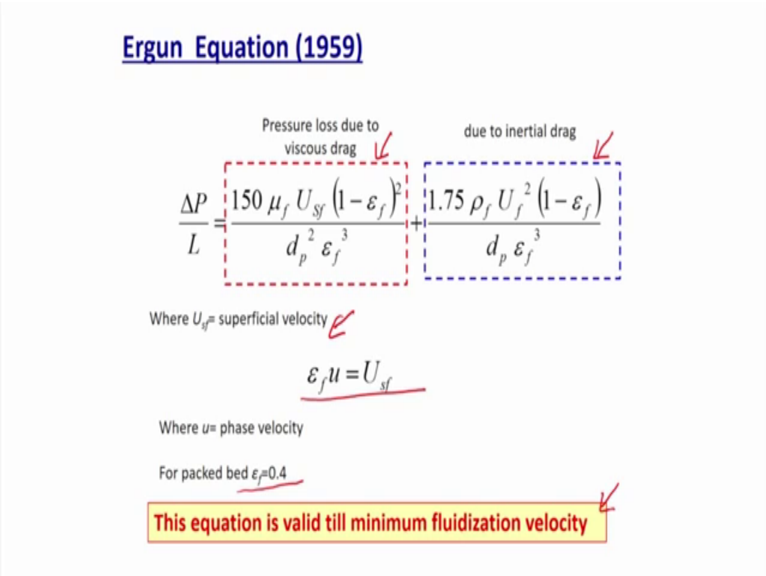

So, the next is the gas solid system and I hope Ergun equation you all been must be knowing from your undergraduate studies from the basic flow through the packed bed you will be knowing the ergun equation which is being developed in 1959 to calculate the delta P in the packed bed condition and that has been used as a drag coefficient ok.

So, this is the Ergun correlation. I hope you have already known this, but I am just introducing it this is the Ergun correlation in terms of the superficial velocity which you have might have seen in your previous courses that U sf is nothing but the superficial velocity and they say that delta P upon l is nothing but is equal to 150 mu f U sf into 1 minus epsilon f where epsilon f is nothing but the packed bed void fraction when the value is f around generally at around 0.4 epsilon f value. Then d p into epsilon f cube plus 1.75 rho f into u f square 1 minus epsilon f upon d p into epsilon f.

So, this has been actually break the organ equation is of 2 part: one is for the pressure loss due to the viscous drag, this is for the viscous drag and this is due to the inertial drag which we have already discussed that the skin drag and frictional drag. So, this is because of the, the first term is called conjuncoe correlation and that is actually the correlation because of the viscous drag.

The other one is called black loma and black loma is mainly because of the inertial drag. So, the whole organ equation is being found it actually represents the pressure loss because of the inertial component and because of the viscous component the way we have done earlier that how the delta P will be there ok. So, there will be inertial component, there will be the viscous component, there is the gravitational component ok.

So, this is the viscous, this one is the inertia. So, that is the way the whole drag has been formulated in the Ergun equation this is written in terms of the superficial velocity. If you want to write it in terms of the phase velocity, then what you need to do we know that the correlation is nothing but epsilon f into u will be equal to u of superficial.

So, what you will get, you will get the gas phase velocity or that phase velocity inside of the system instead of the superficial velocity. So, you can co change the correlation in terms of the phase velocity to get the drag because drag is going to act whatever is the velocity inside; not of the velocity which is on the outside of the system. So, it is going to act on the inside velocity or the phase velocity.

So, the Ergun equation you need to modify in terms of the phase velocity and that what you need to do; you have to just divided it by the U sf divided by the epsilon f, you will get it in terms of the u. So, this whole equation can be changed in terms of the phase velocity and that will be the way we will calculate the drag ok.

And now this has been found that the Ergun equation is valid mostly applicable when the void fraction is very low or the solid fraction is very high and it gives very good result up to the minimum fluidization velocity.

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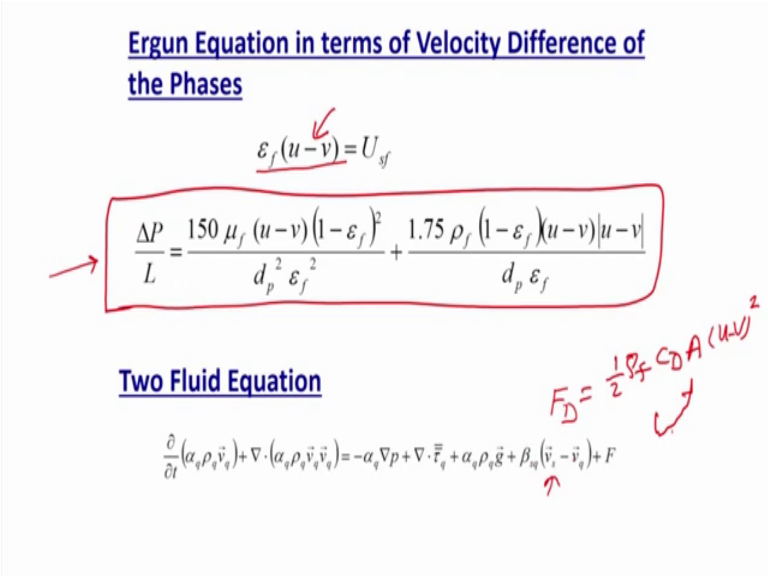

So, what we can do? We can also write in the Ergun equation because we want to write always in terms of the relative motion because both the gas and particle might be moving. So, we can also write in terms of the relative equation, it is in terms of u minus v.

So, the phase velocity will be what instead of the phase velocity we can write it in terms of the relative velocity. So, it will be epsilon f u minus v into will be equal to U sf. So, the overall superficial velocity will be equal to what it will epsilon f into u minus v that what is the slip inside.

So, what you can do? You can change your correlation in terms of the Ergun equation, in terms of the relative velocity, you can see this the relative velocity. So, what I have done. I have replaced it the U sf will be equal to u minus v into epsilon f. So, if you do that instead of epsilon f cube here, it will be epsilon f square here; instead of square, it will be epsilon f.

So, we can write it in terms of here delta P upon l will be this, will be the viscous drag term this will be the inertial drag term in terms of the relative velocity and you can use this and this equation can be used in the drag form in the 2 phase flow equation. We will discuss this later, I will not discussing it here, but this beta value you can use here.

So, this is the Ergun drag equation which is also used for the gas-solid flow, but this equation is very accurate up to the minimum fluidization velocity. It means in the packed bed regime, this equation is applicable, this equation predicts very well, but if you move from the packed bed regime to the fluidized bed regime, then it will not work well.

So, what is done. The other correlation has been developed and Wen and Yu has developed a new correlation which is based on that the beta value and I have written it in terms of why I have written it in terms of the beta because here you are writing in terms of the F D, the equation we have developed we are writing it in terms of the F D and F D was nothing but half rho of fluid cd into your area into u minus v square.

So, instead of F D we have defined it in in terms of beta which is a coefficient into u minus v. So, what will be the beta value?

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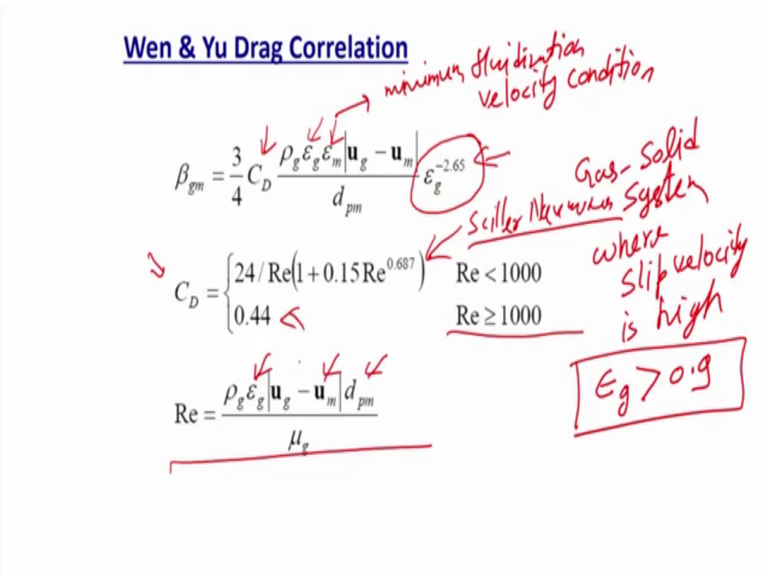

So, beta value will be given as this correlation because now you have a solid fraction inside the beta value is given as 3 by 4 C D rho of gas or you can take it rho of fluid epsilon of fluid or epsilon of gas into epsilon at the minimum fluidization velocity conditions. So, this is minimum velocity condition and u g minus u m upon d pm ok.

So, what is the velocity? So, this is the way it has been defined the beta value and they say that the C D which is again going to be the function the C D will be in terms written, they have written in in terms of the Schiller Naumann correlation for Reynold number less than 1000 and they said that the value will be equal to 0.44 which will be constant for Reynold number is more than 1000. So, Wen and Yu has used this correlation and this Reynold number they have defined it in terms of the slip velocity.

So, this is nothing but rho g and please see that they are multiplied here with the epsilon g because.

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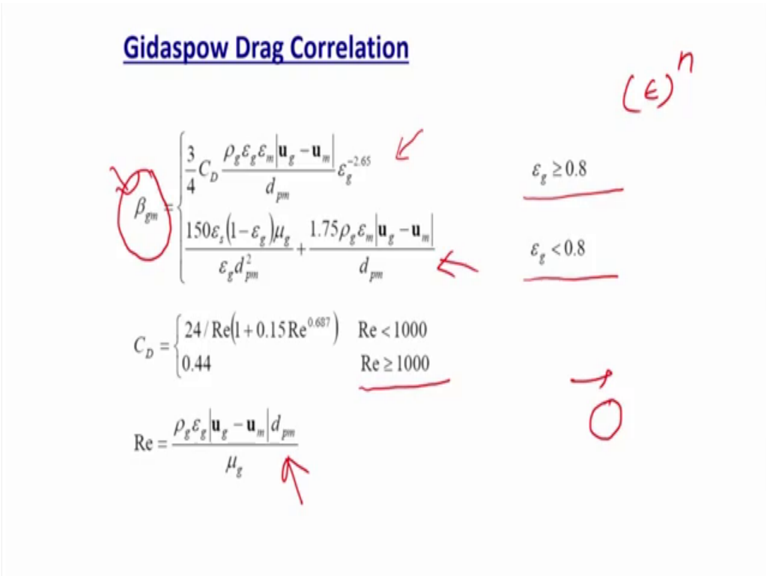

Instead of writing the U sf superficial velocity, now I am writing it in in terms of the phase velocity and phase velocity in terms of the slip velocity. So, that is why epsilon g into u g minus u m that this is u m is nothing but is the velocity of the solid it is a relative slip velocity into d of pm. So, d p of the particle into mu g so, that is the way they have defined the whole correlation this u m and d pm ok.

So, that is the way the Reynold number has been defined. So, instead of the superficial velocity, they have defined the Reynold number in terms of the phase velocity or slip velocity and that is the epsilon g into this. So, Wen and Yu correlation has been developed in 1968, I think I have to check that, but it is in around 1968 and then that is for the gas solid systems. Though they said that it will be valid everywhere, but it has been found that it is very good for gas-solid system where slip velocity is high ok.

So, if the slip velocity is high, they give good prediction and generally they have found that this valid is value is good if you are epsilon, epsilon of void, epsilon of g is greater than 0.9. So, it means for very dilute gas solid flow this correlation has been found very good. They able to predict the C D value, but if the value if the drag for correlation is not good if their C D value is your particle fraction is very high, then the accuracy of this Wen and Yu correlation is not found to be well. So, that is the way it has been discussed and it has been found and they have defined it in in terms of the beta the 2 phase flow equation has been modified. We have already discussed the 2 phase flow equation, but instead of the F D, they have defined it in in terms of the beta into v minus u the slip velocity and the beta has been given it in this term.

So, these all are empirical correlations that is why you are seeing the different values, epsilon raise to the power minus 2.5 and all. These all are empirically fitted correlations that is why they have defined it in this way there is a region of epsilon raise to the power minus 2.65 and we will discuss it later.

But what is the main important parameter is that C D. The C D has been defined as 24 upon R e into 1 plus 0.15 into R e raise to the power 0.687 which is nothing but Schiller Naumann correlation again. That is why the Schiller Naumann said that my correlation will be valid for all the conditions, but it has been found that it is good do well for the gas-liquid, but even for the gas-solid for Reynold number raise to the 1000 Wen and Yu said that the Schiller Naumann correlation gives good predictions and after the thousand the value becomes 0.44.

But later on it has been found that Wen and Yu correlation is only valid for very dilute flow or the solid concentration is very less and void fraction is more than 0.9. So, that is the way the Wen and Yu correlation has been found to be applicable. Later on much later in the around in nineties Gidaspow has given a drag correlation and again they have written in terms of the beta and beta they have defined exactly same way the way the Wen and Yu have defined and they have found that this beta has been defined it in 2 form: one when the white fraction is more than 0.8 and when the void fraction is less than 0.8.

So, when the void fraction is more than 0.8, they have defined the beta exactly same way as the Wen and Yu correlation has been defined. When the void fraction is less than 0.8, they have defined the beta in terms of Ergun equation in terms of the slip velocity or the phase velocity inside. We have already discussed this correlation.

So, what will happen? The beta value they have said that it will be modified and then they said that the C D value will be the same as of given by the Wen and Yu correlation that if the R e value is less than 1000, the C D value will be 24 upon R e into 1 plus 0.15 or 0.687 and if the C D value is more than 0.44, it will be followed by this correlation.

So, it means what if your void fraction is greater than 0.8 the drag factor will be nothing but it will be Wen and Yu correlation will be used. If your void fraction is less than 0.8, you are going to use Ergun correlations. So, what Gidaspow has done? Gidaspow has merge both the correlation and they said that if your void fraction is very high as I said that Wen and Yu correlation is valid or give good accuracy. If your void fraction is very high, it means your flow is very diluted that is what Gidaspow correlation did that if your void fraction is more than 0.8; one should use the Wen and Yu correlation. If your void fraction is less than 0.8, one should use the Ergun correlation.

So, that is the way they have defined and R e also they have defined in terms of the slip phase velocity or in terms of the slip velocity. So, this equation has also been developed. So, what we have done. There are several other correlations has been developed and if you keep on seeing the paper, lot of correlation has been developed. Recently hkl correlation has been developed for the gas-solid where they said that they have also introduced the bubble property inside ok. In this cases the bubble property has not been introduced. So, we will see what is the bubble property once I will discuss the case study in terms of the fluidized bed.

So, what we have discussed what we have seen till now there are several correlation available a bunch of correlation available and that is the problem in the multiphase simulation or multiphase prediction that your accuracy of the prediction will be limited to the accuracy of the drag closure which you are using ok.

So, if you use different drag closure, you will have a different accuracy altogether ok. So, that is the whole crux and one should know that which drag closure is going to help me and that is why one should understand that drag closure is being developed for what conditions, for which Reynold number it is valid, for which condition it is valid and then only you can have say that my predictions are good. If you are using a wrong drag correlation like if you are using Schiller Naumann correlation for the gas-solid, blindly then what you are going to do, you are having you are going to hamper your accuracy.

Though now you can say that the Schiller Naumann correlation has been used for the Gidaspow also and Wen and Yu also for which is widely used for the gas-solid, but what they have done, they have modified the drag and they have modified the drag in terms of the beta and beta they have defined differently.

So, if I just use F D value, the way we have defined in terms of half rho a C D v square upon 2 the way we have defined earlier and you use the C D value from the Schiller Naumann correlation, your prediction will not be good. So, we are able to get that because we have modified the beta value, we have modified the F D in terms of the beta and slip velocity and beta we have given in terms of we have introduced the void fraction, we have written in terms of the phase phase velocity, we have also multiplied with the solid fraction inside or the discrete phase fraction inside into epsilon is power minus 2.65. Why we have done that, we will discuss it later ok.

So, what we are going to discuss later, we are going to discuss whatever we have done till now is a single particle drag ok. Now in most of the practical application the single particle never flows, it is a bulk of the particle which moves together. Now once the bulk of the particle will move together, what will happen the one particle is going to affect the motion of the another particle ok; like in hindered settling which you have done might be in your initial courses where they say that if one particle I am settling and if their multi multiple particle which is being settled what will happen, the other particle will hindered the settling of the other particle they will hindered the settling of each other and that is why a Richardson Zaki correlation has been developed which says that how the settling velocity is going to change in terms of multiple particle.

So, what happen when the particle cloud is generated, how the drag coefficient will be modified in case of if the multiple particle is there. So, if there is a multiple particle, what we do we write the equation in we modify the drag correlation and we multiply the drag with the Richardson Zaki coefficient and Richardson Zakis coefficient is nothing but epsilon raise to the power n and the value of n depends on the particle fraction.

So, we will see that if you have a particle cloud how the things will change and what we will see, we will see the other forces which will also act. So, right now we have discussed all whatever we have done we have limited our discussion to the drag force, to the buoyancy force and to the gravity force. The other drag also act in the fluid flow system depending upon whether it is a gas-liquid flow, depending upon whether it is a gas-solid flow.

Let me again tell you that the whatever the discrete phase equation I have solved by using the Newton second law of motion, it is not needed that you do it only for the gas-solid even if you can do the same equation you can write it in the same way for the gas-liquid if the gas is in form of the bubble or for the liquid-liquid, if liquid one liquid is in form of the droplet. So, if there is a discrete phase, whether it is a particle, whether it is a bubble, whether it is a droplet, you can do whatever we have done for the discrete phase ok.

So, we can use the Newton second law of motion, we can track the bubble motion, we can track the bubble trajectory, we can track the bubble position ok. Similarly we can track the in the liquid-liquid system, we can track the droplet position, droplet trajectory, droplet velocity how it is changing. So, everything you can do we have done it for the gas-solid, but does not mean that that is limited to the gas-solid, it can be gas-liquid, it can be gas-solid, it can be liquid-liquid.

The only thing is the one phase should be in the discrete form. If it is a separated flow or annular flow of the gas-liquid or liquid-liquid system, definitely you cannot use this Newton second law of motion. It is the only for discrete feature ok. So, next class what we are going to discuss we are going to discuss that how the multiple particle is going to affect the drag force and the other forces which act on the fluid solid system or the fluid-fluid system only the one phase should be in the discrete phase.

Thank you.