Multiphase Flows
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Lecture – 11
Lagrangian Tracking of Single Particle Under Different Forces

So, welcome back.

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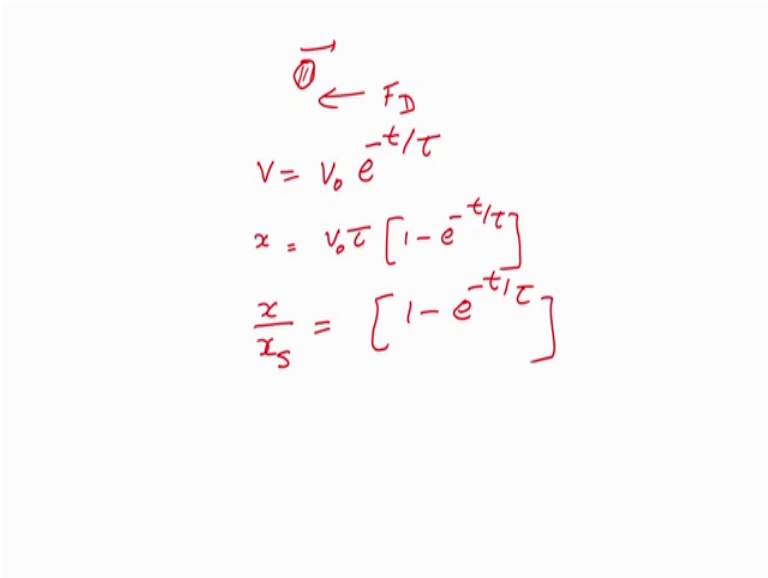

Last class what we have done, we have found that if suppose a particle which is travelling horizontally and only drag is opposing it is motion. So, how, so, this is the particle which was travelling and suppose only drag if F D is only opposing the motion we have derived the formula for the stokes regime and found that V, the how the velocity of this particle will be changing if the initial velocity is V naught.

So, V naught into e raised to the power minus t upon tau where tau is nothing, but the response time and similarly, we have defined that how the position of the particle will change and that will be nothing, but it will be changing as per this formula that x will be equal to V naught tau where V naught is nothing, but the initial velocity of the particle to 1 minus e raised to the power t upon tau and V naught can be written as also the initial position of the particle. So, it can be x upon x s the starting position will be equal to 1 minus e raised to the power t upon tau.

So, this says that how the particle position and velocity is going to change and this has numerous application I have already discussed to track the motion of the particle. Now, what we have assumed here we have said that only drag is acting. So, we assume that the particle is moving horizontally. If it is moving horizontally, gravity component will be 0, if there is any buoyancy force that is also not going to act because we are measuring the horizontal motion.

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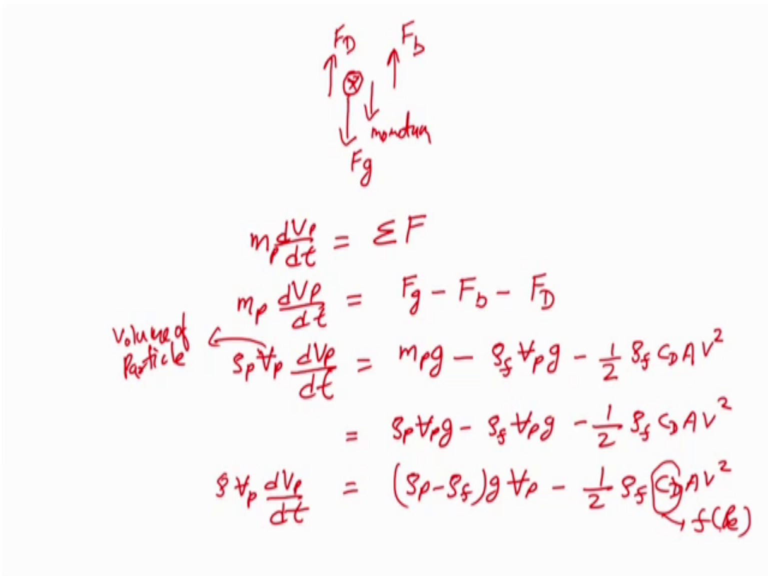

So, now, what we are going to do? We are going to complicate the problem little bit and now, we will say that the same problem. But, suppose particle is moving in downward direction, ok. So, now, if the particle is moving on the downward direction or I can say upward direction only the directions if I will change the direction what will happen only the direction of the forces will change. So, suppose if it is moving downward direction the gravity is going to play F g that will be acting on the down side this is the momentum if I assume that initial velocity was there as a V naught. So, this is momentum, drag is going to oppose the relative motion if the air is stationary or the fluid in which the particle is settling or moving down is stationary then definitely drag is going to move at upward F d and if there is some buoyancy that is also going to act upward direction buoyancy will always be upward.

So, that will be the overall force balanced. So, if I write the rate of change of equation the rate of change of momentum ok, that is nothing, but m dV upon dt is equal to say V particle I will write it as a particle will be equal to summation of the forces. Now, what would be the forces which will be acting. So, this will be mp dV p upon dt and the summation of forces will be this will be equal to F g, I am taking downward as a positive that upward as a negative minus it will be F b minus F d. So, this will be the force which will be acting on the body and this is the initial momentum that will be equal to the m dV upon dt.

Now, what we can do? We can do exactly same thing which we have done previously and what we our target is to derive the formula for the velocity and the change in the position. So, now, what we can do, in this case we can write it m p is nothing, but mass of the particle that we can write it as a rho p density of the particle into volume of the particle. So, this is volume. I am writing it as a V cross so that this velocity and volume does not get one should not get confused. So, m dV p upon dt.

Now, what is F g? F g is what? Is the gravitational force. So, this will be again m of particle into g. So, that is gravitational force. What will be the buoyancy force? Buoyancy force will be rho of fluid into volume of particle into g. So, that will be the buoyancy force and the drag force is nothing, but we have already discussed half rho of fluid into C D into area into V square. So, this will be what that is the drag force, ok.

So, now if we just try to solve this equation we want to simplify this equation, what we can do? We can write it as a rho p and m p again, here can be written as rho p into V of particle into g minus rho f, V of particle into g minus half rho f, C D A into V square now here what I can do it will be rho p minus rho f, I can take g and volume of particle outside minus half rho f C D A V square, ok.

Now, what is the we have already this rho p into V p into dV p upon dt, ok. Now, what we want? we want to write the terms in terms of the Reynold number because we know that the C D is a function of Reynold number. So, we know this C D is a function of Reynold number. So, I will convert the whole equation in terms of the Reynold number and to convert that the Reynold number will be defined for the particle Reynold number.

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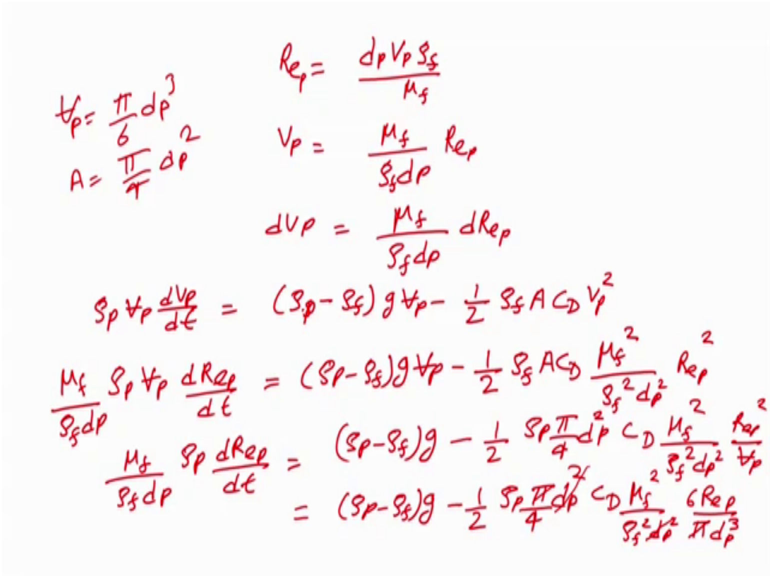

And, which is being defined R e will be equal to what it will be d P it will be V P it will be rho of fluid upon mu of fluid this will be what particle Reynold number, ok. So, particle Reynold number is defined it in this way we have already discussed it earlier. So, this will be the particle Reynold number. So, V P we can write it as mu f upon rho f into d P into R e P ok. So, dV p will be equal to what? Mu f upon rho f d P into d R e f particle.

So, we can replace now this above equation and we can write this dV P in terms of the Reynold number. So, this will be rho or you just write it. So, that the step of simplicity rho P V P dV P upon dt will be equal to we have derived it rho f rho P minus rho f into g into V P minus half rho of fluid A into C D into V square of d P square. Now, we will replace the V P with this and if you do it, it will be just rho P into V P this will be d re upon d R e P upon dt and then you have to multiply it by mu f upon rho f into d P this will be equal to rho P minus rho f into g V P minus half rho f A C D and V P can be written as mu f square upon rho f square into d P square into R e P.

Now, we can simplify it we can replace the d P; d P is equal to what? This is the particle, is spherical. It will be pi by 6 d P cube area if the particle is spherical it is pi by 4, d P square will replace A and V P here. So, we will get mu f upon rho f into d P into rho P and d R e P upon dt, this V P again this will be rho P minus rho f into g this will be equal to minus half rho f, A is pi by 4, d P square this will be C D, mu f square upon rho f square into d P square into R e P square upon V P and V P we can write it as this term rho P minus rho f into g this will be minus half rho P into pi by 4, d P square into C D this will be mu f square rho f square into d p square into R e P and this will be pi by 6, d P cube, the V P.

Now, we will just try to simplify it. So, this just 4 pi-pi will be cancelled out, this d P square this d P 5. So, d P square d P square here will be cancelled out we get the d P cube.

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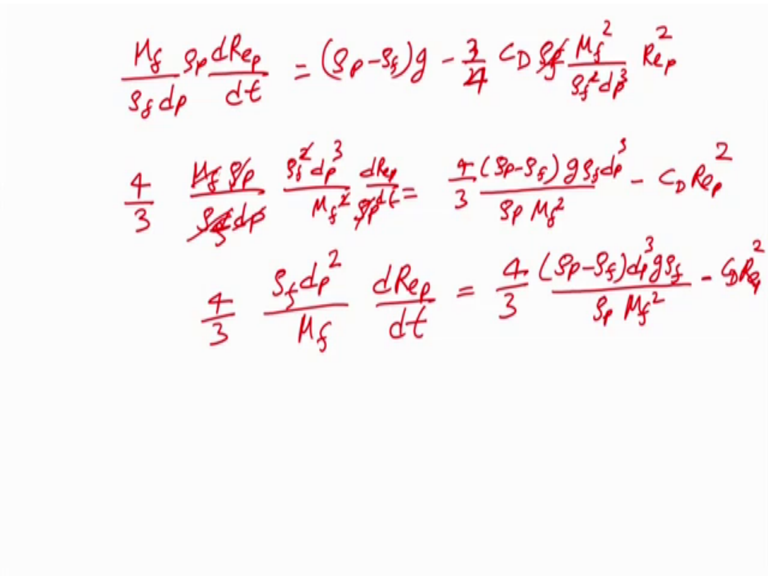

And, if you just simplify this equation what we are going to get we are going to get it as if we just simplify it will be mu f into upon rho f, d P into d R e P upon dt this will be equal to here rho P is also there rho P minus rho f into g and this will be equal to minus half actually it will be 8 and just 3. So, 6, so, this will be 3 by 4. I can write it 3 by 4 because this will be 8 and 6. So, this will be cancelled out. So, it will be 3 by 4 into C D into rho P into mu f square upon rho f square into d P cube into R e P square. So, this also square, so, R e P square.

So, I hope I have written this correct, rho P and you will get this now we will further simplify it we will write it as a d R e P upon dt term. So, what we are going to do we are just writing it in terms of the C D into R e square. So, this if you try to simplify what we are going to do we are just going to divide by this term over all. So, this will be 4 by 3. Now, if you divide by the rho P; rho p – rho P here will be cancelled out. So, this I will write it as it is mu f rho P upon rho f d P, ok. Now, I am dividing by this terms 4 by 3, it will be rho f square d P cube upon mu f square, upon rho P. This will be equal to again here 4 by 3 rho P minus rho f into g upon this will be rho P into mu f square into rho f into d P cube, ok. So, this will be minus C D into R e P square. So, that is what we are going to get.

Now, what we are going to do we are just going to simplify this and this will be sorry here dR e p upon dt is missing. Now, we are going to simplify it. If we will simplify it you will see that this will be what this is rho of fluid ok. So, if you simplify it you see rho of fluid will be cancelled with here, mu f square will be cancelled from this place rho P rho P will be cancelled out, what you are going to get you are going to get it 4 upon 3 into rho of particle sorry rho of fluid into d P square into d P square just d P and one this will be cancelled out.

So, rho of fluid d P square upon mu f into d R e P upon dt this will be equal to 4 upon 3 rho P minus rho f into d P cube into g into rho of fluid upon rho P into mu f square minus C D R e square R e P square, now this rho P should be cancelled out actually. So, rho f rho f and rho of particle this is rho of particle failure there is something somewhere something is wrong this is also rho of fluid actually, not rho of particles this is rho of fluid.

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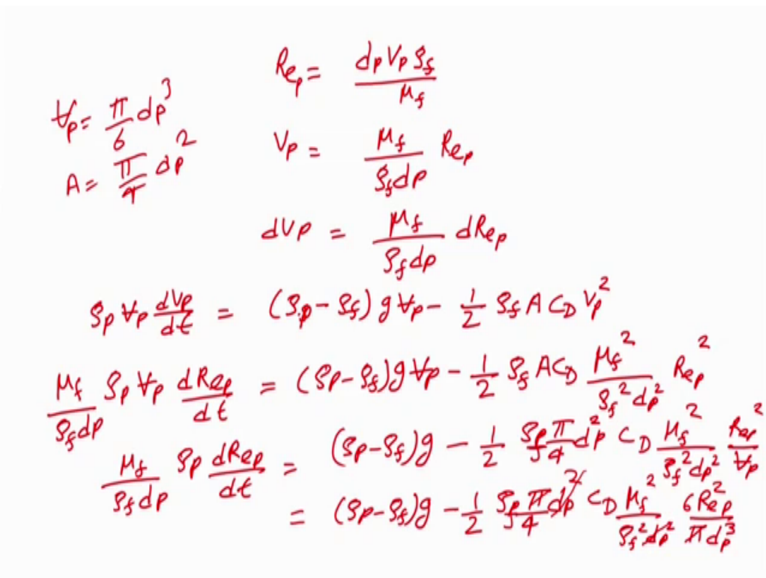

So, that is why it is making strong here rho of fluid will be there. So, if you do the rho of fluid this will be if I cancelled it rho of fluid rho of fluid will be cancelled out. So, rho P will not be cancelled out here. So, pi by 6 rho of fluid. So, this will be C D ok, here rho of fluid will be there instead of rho P.

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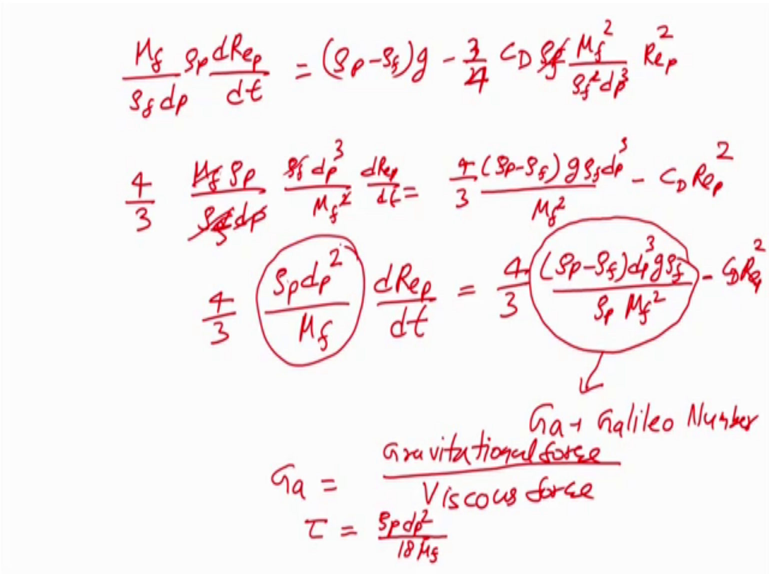

C D 3 by 4 rho of fluid C D rho f square rho f square d P cube upon R e square R e P square f. So, this is rho of fluid. Now, if it will be rho of fluid it will be just divided by the here rho of fluid. So, this will be rho of fluid again. So, rho of fluid will come here one rho of fluid because rho of fluid rho of fluid this will be cancelled out. So, only one rho of fluid will be there that will be here this rho P will go. There will be no rho P here, sorry, please check it, just do it and here again it will be once you will divide it it will be rho of fluid square it will not be the square it will be only the rho of fluid. So, it will be rho of fluid into d P cube mu f square and there will be no rho P here.

So, in that case if you do that this will be what rho f rho f will be cancelled out and rho will be as remain as it is. So, this will be not rho f it will be rho P. So, this will be rho P d P square upon mu f d R e P upon dt into this. Now, this number is called this number is called the Galileo number, G a; represented with G a and G a is nothing, but Galileo number. And, the Galileo number G a is nothing, but it is the ratio of gravitational force to the viscous force. So, ratio of gravitational force to the viscous force; it means what, where the fluid is settling what is the importance if the fluid is getting settling. So, how much gravity is pulling it up and how much the viscosity of the fluid is actually resisting the motion of the particle. So, that is what is the, Galileo number. It is very important once the settling is going on are anything any object is moving under the gravity.

So, it shows that how much is the gravity is playing role to pull the particle or the particle and how much is the viscosity of the system or viscosity of the fluid is resisting the motion of the particle. So, that is the Galileo number. Now, we already know that tau which is response time is nothing, but rho P into d P square upon 18 mu f. So, what we can do? This is what we can write it in terms of the tau this quantity we can write it in terms of the tau.

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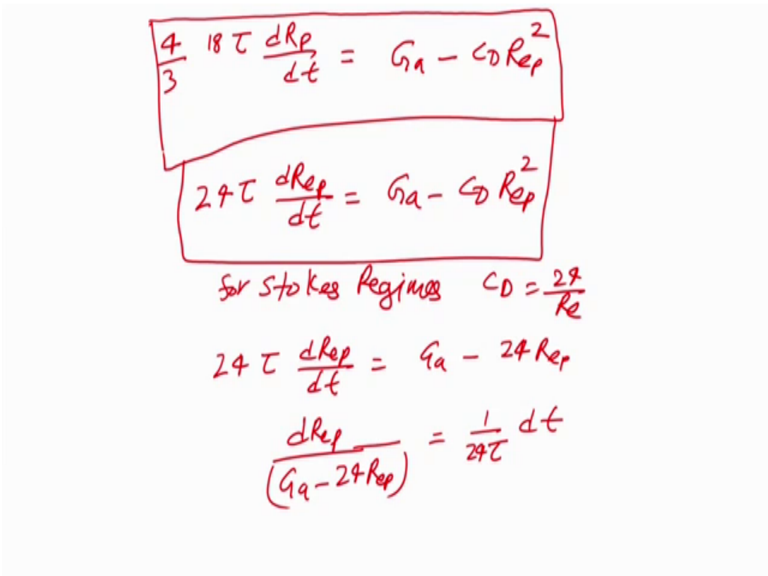

Now, if we do the replacement, this will be 4 by 3 into 18 tau into d R e P upon dt this will be equal to Galileo number minus C D into R e square R e of particle square. So, this will be the equation you will get to see that how the particle motion is changing with the time. Now, if you just simplify it this will be nothing, but 24 tau into d R e P upon dt will be equal to G a minus C D into R e P square.

So, this is the motion under the gravity this will be the equation which will tell you that under the gravity how this R e P and this will be changed. Now, you can further integrate it you can solve this problem again just by replacing suppose for the stokes law you can say this is nothing, but 24 upon R e P you can just cancelled it out it will be 1 minus x 1 upon x, ln problem and you can solve it you will get the equation which will be similar to whatever we have got earlier. So, you will get the equation for the velocity, you will get the equation for the position. So, you can calculate everything whatever the way you want, ok.

Now, based on that you can find it out that how the particle velocity will change or particle Reynold number will change with the time, how the particle position will change with the time. So, that all calculation we can do again just suppose if I say that in stokes regime for stoke regime C D is equal to what 24 upon R e, now this will be 24 tau d R e P upon dt this will be equal to G a minus 24 upon R e if I do I will say 24 R e P.

So, what I will get I will get this value or we can say that d R e P upon G a minus 24 upon R e P is equal to dt into 1 upon 24 tau ok. So, we can integrate it in terms of the d A and we can see that how the time is changing suppose we can say that initial is R e naught if it is has certain velocity finally, say R e P then we can integrate it and we can find it out that how the Reynold number will change with the time. The Reynold number can be converted in terms of the velocity by using the same R equal to V rho d upon mu. We can calculate in terms of the velocity, we can find it out how velocity is changing with the time, velocity can be converted in terms of dx upon dt, we can find it out how the position is changing with the time.

So, under this influence again you can track the particle trajectory you can track the particle velocity.

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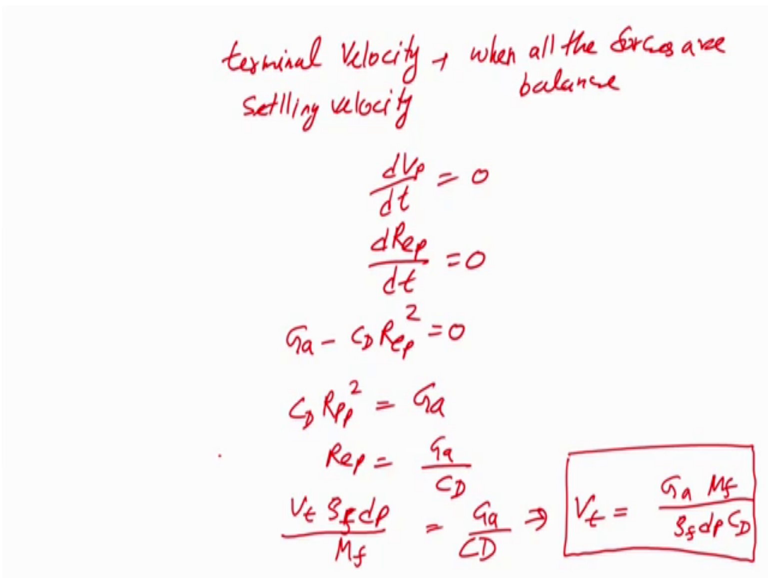

But, what we are more interested is in is suppose the term which we called as a terminal velocity or settling velocity. Now, what is terminal velocity or if the particle is moving down or settling down I will say it as the same as a settling velocity. So, how it has been defined terminal velocity? Terminal velocity is being defined when all the forces are balanced. It means what that dV upon dt term is going to be 0.

So, it means if I neglect that the initial transition period where the particle velocity is changing with the time and before it achieve a constant velocity this equation if I neglect that part if I put this dV by dt, the same equation what will happen db by dt is equal to 0, it means if dV by dt is equal to 0 then d R e P upon dt is also going to be 0. So, particle velocity then this equation will be modified and this equation will be G a minus C D R e square R e P square will be equal to 0 or you can say that R e P square C D is equal to genetic algorithm.

So, you can find it out the correlation here directly and for stokes regime again if I do that; so, before going to the stokes regime what I will do I will write just normal equation. So, this we can say that R e P is G a upon C D and V t if you do this R e P in terms of the velocity of the particle you can write it as a V of terminal into rho of particle into sorry rho of fluid into d of particle upon mu of fluid, so, will be equal to G a upon C D. So, you can write it as V t is nothing, but G a into mu f upon rho f into d P upon C D. So, that will be the terminal velocity of the particle.

In general case, you can write it in this way; the only thing is you have to find that the C D verses R e correlation and C D versus R e correlation you have to find it out with the terms that how the C D is changing with the Reynold number. So, you have to find the Reynold number, it will be iterative solution. So, you have to find the Reynold number or you have to guess a Reynold number for that Reynold number you will calculate the value of C D. Once you will calculate the value of C D you will get the value of V t.

Now, calculate again with the new Reynold number you V t you calculate the Reynold number and see that if the assumed Reynold number and this calculate Reynold number has certain difference. If they are same it means your assumption is correct, your Reynold number assumption is correct. If they are not same, you choose the new V t Reynold number the Reynold number calculated by this V t and again keep on doing that iteration till you are not getting the number of value or Reynold number within a certain percentage and that percentage error generally we take as a less than 5 percent. So, with the iterative solution you can get the value of V t, ok.

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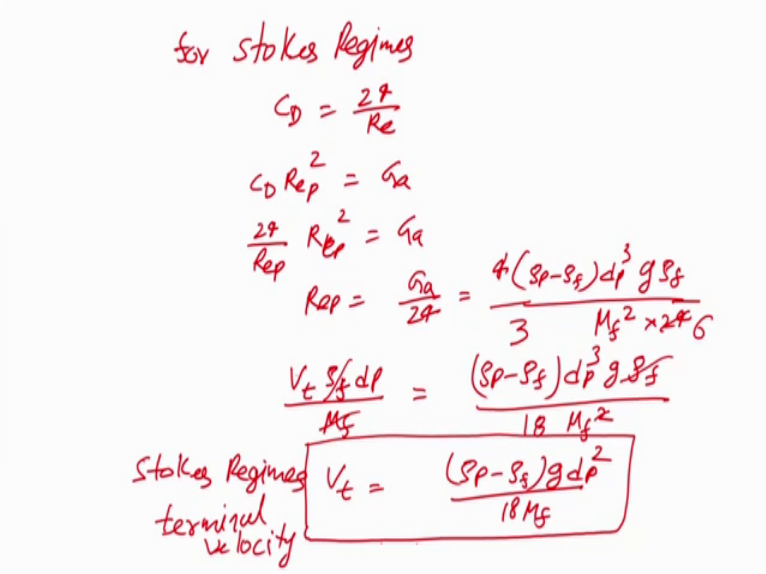

For stokes regime you do not need to do anything you can further we can simplify and that will be what the stoke regime C D will be equal to 24 upon R e. So, now, what we will do C D R e P square was equal to G a. So, we will say 24 upon R e P into R e P square will be equal to G a. So, this will be R e P will be equal to what, this will be G a upon 24. Now, you can open the G a; G a was nothing, but rho P minus rho f into d P cube into g upon mu f square, and it was this, ok. So, this will be this R e or this was 4 by 3. I think this 4 by 3 term we have included here. So, this will be 4 by 3, this was what the Galileo number was.

Now, if you do that this is rho f now you do this upon 24 you will get that R e P is nothing, but V if you want to find the terminal velocity it will be this into 24 here will be coming. Now, V P R e P is nothing, but V of terminal into rho of particle sorry rho of fluid into d of particle upon mu of fluid this will be equal to 4 upon 3 into this will be actually 18, I will write just this is 4, 6. So, I will write it as a 1 upon 18 or I will write rho P minus rho f into d P cube into g into rho f upon 18 mu f square.

Now, if you do that if I have to find the V t what I will do I will cut this mu and mu this rho of fluid and rho of fluid will be cancelled out and this d P will make a d P square. So, V t will be equal to what it will be rho P minus rho f into g into d P square upon 18 mu f.

So, this will be the terminal velocity in case of stokes regime terminal velocity. So, this you have already done, but how this terminal velocity has been derived that is what I have just shown you that this is nothing, but a stokes regime how this terminal velocity will be defined or derived. So, this is rho P minus rho f g d P square upon 18 mu f.

So, if you have a turn you have to find the particle terminal velocity in stokes regime you can use this formula directly. Everything is known if you know the particle diameter if you know the fluid under which it is being settled down. If you know the particle density you can calculate the terminal velocity very easily, but what will happen in case of non this stokes regime you need to do the iteration.

Now, again I would like to explain the procedure of this.

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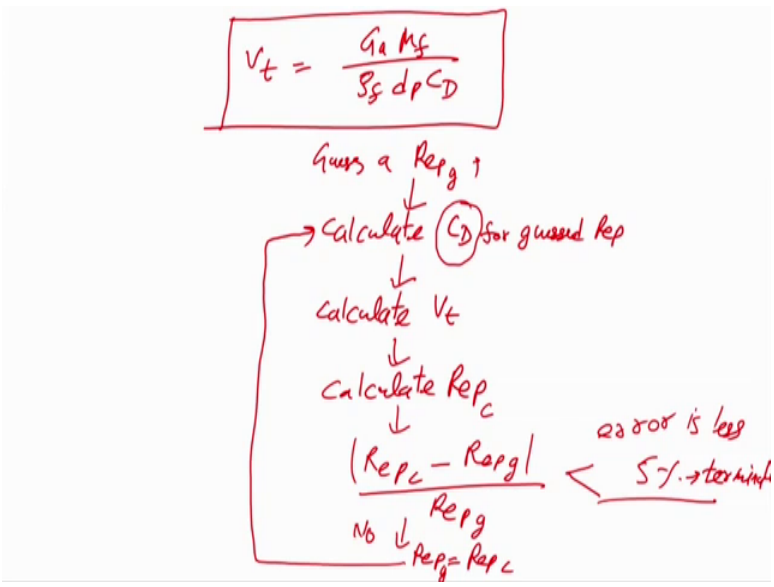

So, this what we have done we have defined that V t is nothing, but this is equal to the non-stokes regime this is G a mu f G a mu f upon rho f d P C D mu f upon rho f into d P into C D this is in the non-stokes regime or any other regime. So, what we need to do we need to guess a Reynold number R e P, for that R e P calculate C D value from the chart or from the correlation which you are using for the C D value there are different correlation has been given in the literature and we will discuss that correlation for the different drag for different C D values correlation is given. Calculate the C D value, calculate V t. I will write here more elaborately calculate the C D value for guessed R e P.

Now, calculate the V t value calculate R e R e P now this I will say calculated R e P C which is calculated this is R e P g which is the guessed one. So, this is C, now, calculate that R e P C minus R e g R e P g upon say R e P g this you take as the mod.

If this error is less than 5 percent ok, error is less than 5 percent then you assume that whatever the C D you have done is calculated is fine. If it is more than 5 percent then now you replace if less than 5 percent say I will write it here then terminate, terminate your program suppose if you are writing the code and you can write try to write a code for this that then it will be terminate. If this is no, if this is not correct then R e P will be equal to you say R e P g will be equal to R e P C and then you again go back and calculate this . So, you have to keep on repeating this loop till your V t your R e P which was guessed or an R e P which is being calculated does not come within the 5 percent range.

So, that will be the iterative, you can write a program and for any particle you can calculate that what will be the terminal velocity. The only thing is the determination of the C D and how to find that C D we will discuss it later to the different classes that how the C D correlations are given by the many papers and depending on the correlation you can take a C D correlation and you can keep on doing the same iteration again and again till you are not conversed to this value, once you terminal through this value your program will be terminated.

So, you can write your code you can we can discuss it in the assignment or I will give a assignment within which you can write a code if you want we will see that we will discuss it further to understand that how to do it. If you do not want to write a code you can still do it numerically by your own hand, but it will take time because every time you have to do the computation.

So, you can still do it without writing a code, you can write a code both are both up to you how you want to do it the overall idea is that how to calculate the C D value.

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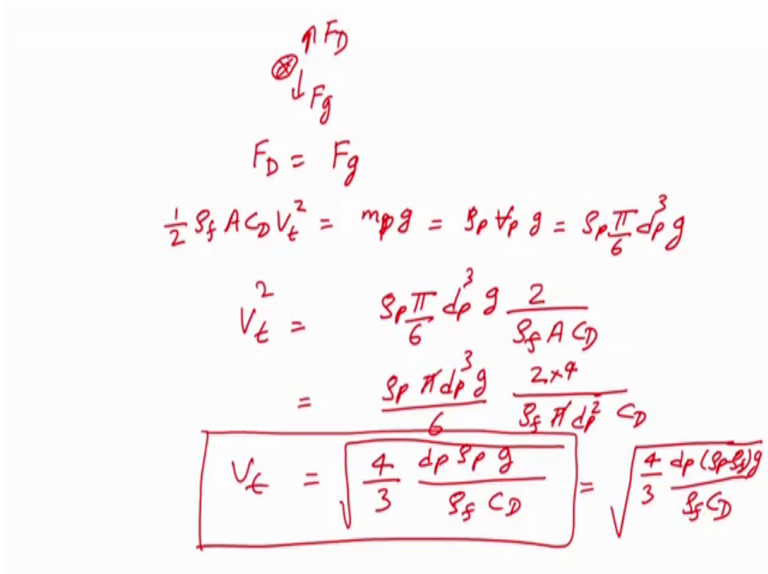

Now, what we are going to do which is the most popular formula which we discussed which we use here when the particle is being only settled in presence of F g and F d. So, if I am neglecting the buoyancy say if the rho f value is very very low and if I am neglecting the buoyancy effect then this F g will be equal to we can derive the formula for that and why I am deriving because this is what you have done in your undergraduate, F d will be equal to F g and in that case half rho F d is what half rho of fluid into area into C D into V square now because this is settling we have neglected the dV upon dt term it means all the forces are balanced. So, this will be the settling velocity or the terminal velocity and this will be m g into g, m of particle sorry, into g.

Now, m of particle can be written as rho of particle into volume of particle into g and this can be written as rho of particle into pi by 6, d P cube into g. Now, we can solve this V t. So, V t will be what? V t will be equal to this values pi by 6 this. So, you can calculate this value this area will be equal to what. So, this will be rho P pi by 6 d P cube into g and this will be 2 times of rho f, sorry this will be 2 times of rho f upon A into C D.

Now, you can further simplify it and you can write it as this will be 3. So, rho P into pi d P cube into g upon 6, 2 upon rho f and area can be written as pi by 4 d P square into C D, ok. Now, if you further simplify it this we will get what, this will be 8 and this so, this will get a 4 upon 3 and pi-pi will be cancelled out d P cube will be d P only. So, this will be d P, ok. Now, this will be rho P this will be g upon rho f into C. So, this will be what you will get V t square value here. So, V t will be equal to what this will be under root of this value.

So, what you will get? You will get that formula for the V t versus C D again you have to do the if you do not know the C D value. So, what you have to do you have to guess the Reynold number again the same look will be there. You have to guess the Reynold number if you have to find that Reynold number guess the Reynold number find the C D value; for that C D value whatever you have guessed you can calculate that V t value. Again, from this V t value you can calculate the Reynold number.

If both the Reynold number is same then it is fine, you can terminate your calculation or your program. If they are not same you can take that newly calculated value of Reynold number as a guessed value. Again, you can find the C D value for this you can again find the V t value and you can keep on running this loop till your error is not coming within a margin and margin generally we take as a 5 percent .

Suppose, if there is a density difference if we have to do that. So, this will be 4 upon 3, it will be d P generally rho P minus rho f into g upon rho f into C D. So, you can use this formula again to calculate the terminal velocity. So, what you can do? What we have done? Now, in this part we have seen that how the value of particle terminal velocity of particle settling velocity will change under the influence of different forces, how it will change for the stokes regime and for the other than the stokes regime, how to calculate the particle terminal velocity in case of any flow and how to calculate the position of the particle, how it is changing with the time, how the velocity of the particle is changing with the time if we do not want to neglect the transient term if we want to calculate we can do that.

So, what we can do, we can calculate the particle position, we can calculate the particle velocity, we can change the find the particle position with the time, we can find the particle velocity with the time. So, you can track the particle in every domain if only drag is acting once we have seen and another case we have seen when the drag as well as the other forces like buoyancy and gravity is also acting.

So, with this calculation whatever we have done till now, any particle which is flowing or the group of the particle is flowing we can track the motion of the particle for the group of the particle you have to solve the equation for individual particles. The only change will be the C D value will be modified and we will discuss all these things that how the C D value will be modified for a particle cloud and based on that you can calculate it.

So, it means what whether it is a you have a particle which you have a hundred particles which are flowing together; you have to solve the hundred Newton second law of motion to track the particle position, ok. So, whether it is a fluidized bed, whether it is a pneumatic conveying, whether it is a general solid is suspended in the air and they are moving, you can track the velocity of those particles you can track the position of those particles. And, if you know the velocity if you know the position you can also if you know the position you can also calculate the fraction of the solids present there. So, you can understand about the system.

So, all the gas solid system where the Lagrangian approach is there you can solve the Navier–Stokes equation to get the terminal velocity to get the settling velocity to find how the position of the particle is changing with the time, how the velocity of the particle is changing with the time, ok.

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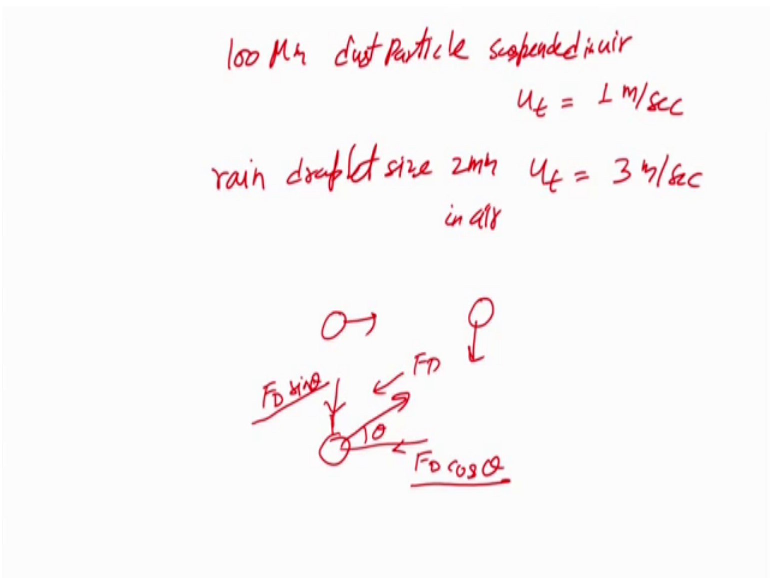

So, now, the typical value I am just seeing that giving you that about the terminal velocity; so, if suppose if I have a 100 micro meter dust particle which is settled in air ok, suspended in air then its terminal velocity suppose, once the everything will be percent the terminal velocity will be around 1 meter per second. I have assumed the density of the dust is equal to the 2900 which is the density of the sand. So, you will get it as a 100 meter 1 meter per second and if suppose the rainfall which we have always seen the drop if suppose I am assuming that the droplet size of the rain is 2 mm which is very typical then the u t in air, in air will be around 3 meter per second, ok.

So, that will be the settling velocity of the rain droplet if it is size is 2 mm you will see that it is moving at the 3 meter per second velocity. If you have a suspended particle 100 micrometer dust particle which is suspended in the air, you want to find that the terminal velocity you will see that the terminal velocity will be in the range of 1 meter per second.

So, that is the way you can do that if suppose again if suppose if we assume that the particle whatever we have assumed right now, either particle is moving horizontally or particle is moving vertically suppose if the particle is moving at a particular angle. So, what you can do the drag will act it.

It in that angle, you can say angle is theta the drag F D will be here you can say this will be F D cos theta this will be component F D sin theta and you can do the balance you can solve the horizontal component force, you can solve the vertical component force you will get a particle horizontal velocity by using this motion with dV by dt. You can get the water particle vertical velocity by using this F D sin theta force and what you will have you will have the particle x velocity you will have particle y velocity, you can find it out the resultant velocity magnitude and direction of the vector of the velocity and you can find that how the particle will move.

So, using the 2 dimensional domain, we are right now solving the 1 dimensional domain you can easily convert this problem to 2 dimensional domain to find it out, how the particle is moving with the time how the particle position will change with the time. So, in the most realistic cases the domain is actually 3 dimensional, you can further do it in the third dimension you can find you can just resolve it in terms of the vector another angle will come this is of the theta 5 will also come.

You can resolve the F D value in r theta and z coordinate or in xy and z coordinate and you can find it out the velocity in the x direction velocity in the y direction velocity in the z direction and the combined motion of the particle that where the particle is moving how the particle position are changing. So, these are the more realistic cases which I am just giving the examples.

So, with this what you can do you can track the position of the particle, with the time you can track the velocity of the particle with the time in 2 dimensional domain, in 3 dimensional domain, in one dimensional domain. So, what you need to do, you just need to solve this now the only problem is the, it is computational very costly.

Computationally it will be very costly, we will discuss it again later the computational cost why it will be computationally very costly whatever we have done we have done for one particle, but even if I take a small fluidized bed say a diameter of 2 inch a height of 20 centimetre there will be millions of the particle will be present inside if the particle diameter is say around 50 micron or 75 micron I am talking about a group a particle then millions of the particles will be present there.

And, then what you need to do you have to solve these equations for the million of the particle. We have neglected the collision between the particle we have not seen the particle-particle interaction this is the one way interaction. We are just seeing that the drag is actually interacting with the mean motion of the particle the particle interaction the particle fluctuations we have not taken into the account.

So, what you have to do you solve several equations together to get that how the position of each particle is going to change with the time, how the velocity of each particle is going to change with the time if you know that you can calculate how the pressure is going to be modified with the time and you can have the complete statistics you can have a complete hydrodynamics of flow dynamics of the system. We will discuss again about these things later on, right now the idea is the one dimensional motion that how to solve this.

So, I hope now you can solve the particle motion in one-dimensional whatever the forces is acting on it you can solve it. Now, what is left is that external forces. So, external forces can be in many of the application what we see through the for the particle separation particularly, we use either the electric field or we use the magnetic field to separate the particle from the main fluid. So, suppose like a particle is suspended from like typical industries where kind of power plants where the cool particles are burned to get the energy.

Now, the cool particles are burn they form the flue gases the flue gases is being used to heat the water to generate the steam, but at the end of the day some of the soot particles or ash particle is being carried away with the flue gases you cannot discharge your fluid gases to the fluid gases to the environment because they have the suspended particles. You have to remove that suspended particle and to remove that suspended particles what we use we use electrostatic precipitator. First we use the back filter, but after a certain particle size then the back filter is not able to do cyclone separator is not able to separate then we need to use the electric field.

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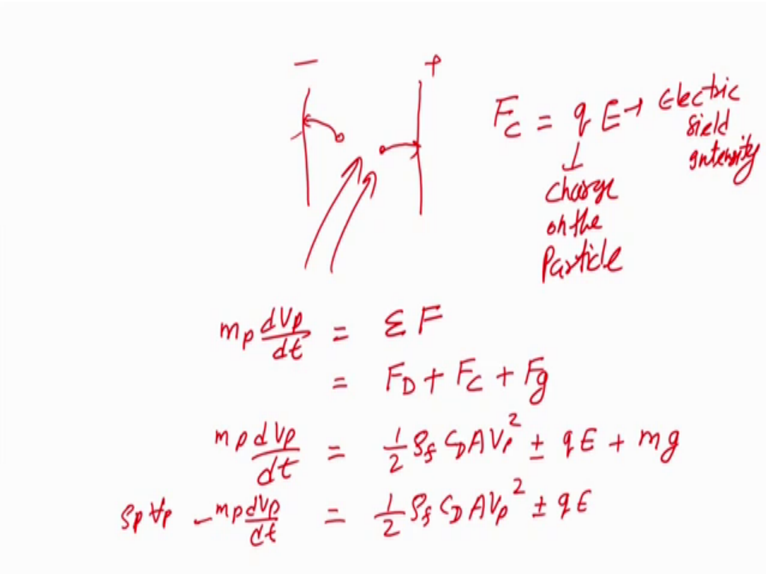

And, what ESP do, it have a plate. It has a plate where the electrical charge is being generated gas flows from there now because of this the particle has a certain charge and it moves either through the anode or the positive or negative depending upon what is the charge generated on the particle it will move on this side.

Now, in this case if external force is there similar thing is true with the magnetic particle magnetic force if the particle has some magnetic field or suppose it is iron or something I can put a magnetic field I can put the magnet and then what will happen the particle will be move towards the magnet. So, the motion of the particle will change. So, what we need to see now what we are going to do now next that in case in presence of the external field in external force field how the particle trajectory will change with the time, how to calculate the particle settling velocity of particle velocity that when the external force field is also present.

So, what we are going to do we are going to case take a typical case of the ESP we will see that how in the ESP the particle is going to be changing its motion. So, now, what will happen again in case of this, this ESP what we are going to do we are going to take the electrostatic charge or the coulomb forces inside and coulomb forces F C is being defined as q into E where q is nothing, but is the coulomb and E is nothing, but volt per meter or you can say the q is nothing, but the charge on the particle on the particle and E is electric field intensity. So, that will be the coulomb forces. If suppose if I put an electric field then this will be the force which will be acting on the particle. So, what we can do we can write the same equation again mp into dV p upon dt is equal to summation of the force.

Now, the summation of the forces can be written as this will be what are the forces which is going to act that is going to be F D that is going to act here plus F C which is the F C the coulomb forces or electric forces plus F g ok, because once the particles will move vertically downward also.

So, if we do that we can write it as md P upon dt m P dV P upon dt will be equal to C D this will be half rho of fluid into C D into A into V of particle square plus q into E, ok. Now, that q into E will be there which will be plus or minus depending on the charge of the system plus mg, ok.

Now, we can write it as we can solve this as bps equation we can solve these equations again further and we can write it in terms of the Reynold number. We can calculate the Reynold number, how it is changing with the time. We can calculate the this values because this q is constant. You can calculate you can solve it the way we have solved it earlier the only thing one more constant value will be added and you can have all those things you can find it out that how the particle position will change with the time, how the particle velocity will change with the time. You can write it in terms of the Reynold number also you can calculate that how the Reynold number is changing with the time.

I am not doing this calculation you can do it very easily we have already done q is a constant force which is acting there. So, that value is just that is going to act as a constant ok. So, now, we can have all this calculation we can find it out. Now, in the more simplified case if suppose I am assume that the particle is moving horizontally the q E force is dominating over the mg it is just moving horizontally then in that case the same equation will be modified as half rho f C D A V P square plus minus q E and this is the more realistic case because in the electrostatic precipitator particle first move towards the wall gravity does not play much role once it is stick to the wall then it will fall down because of the gravity.

So, this is what is the more realistic case and you can again do the calculations you now, integration will be very simple this is constant you can convert everything in terms of the Reynold number or you can write it in terms of the V P and dt and you can find it out m P you can calculate again as a rho P into V P. V P you can write it in terms of the d P. The way we have done I do not want to do it the same thing again and again. So, you can do this exercise take it as a assignment, I will put it as actually as assignment in the or next assignment which will be uploaded and if there is any problem we can discuss it further.

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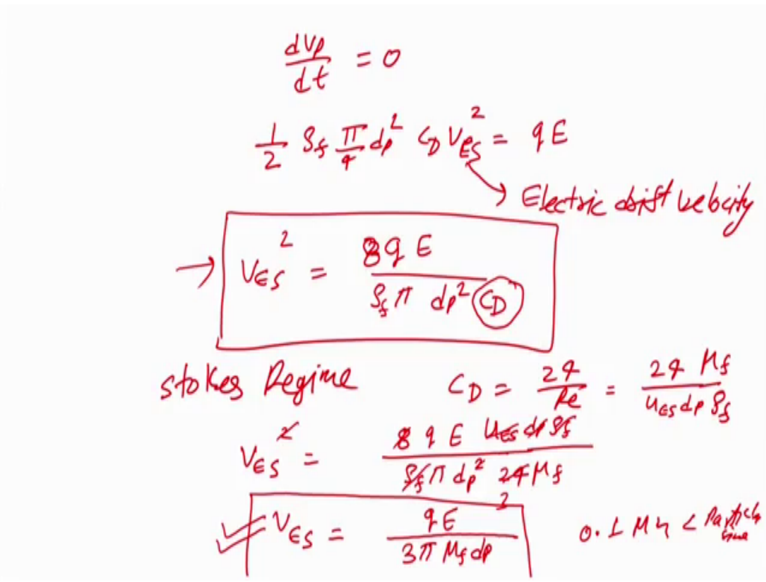

So, what we are more interested is in once the velocity or this kind of the dvb d V P upon dt is equal to 0. It means all the forces are balanced and then the particle is moving with a particular thing in that case you can write it as half rho of fluid area of particle is pi by 4 d P square into C D into V P square will be equal to q into E, ok.

We can find it out or V instead of V P we called it as a electric drift velocity or E S which is called electric drift velocity, because now the particle is moving in influence of the electric field and the drag is opposing the motion of that. So, that is the electric drift velocity V is called. So, this will be half rho. So, V E S square will be equal to what q E upon 2 into rho f actually this will be 8 upon pi d P square into C D.

So, if you want to find it out in the normal regime again, what you need to do? You have to guess a Reynold number, you have to find the C D value, you have to find the V S value, with the V S again you find the Reynold number. If both are same then your guessed value of Reynold number is correct and your calculation of V E S is correct if not the new Reynold number which we have calculate you again use the Reynold number to calculate the C D, keep on running the same loop again and you can calculate that what will be the electric drift velocity of the particle.

So, if you write a same program or you make a excel sheet where the same calculation is being done then you can simply calculate very easily that what will be the V E S. Now, in stokes regime what will happen the C D will be replaced by 24 upon R e and R e is nothing, but 24 upon u E S because we are writing it in terms of this into d of particle into rho of fluid into mu of fluid. So, I will replace it here and if you do it it will be V E S square will be equal to 8 into q into E upon this was rho of fluid, ok. So, this will be rho of fluid into pi into d P square and C D value will be what this will be 24, this will be mu f, this will be u E S, d P into rho f ok. So, now, this rho f-rho f will be cancelled out. This u E S this will be cancelled out, this will be 3 and this d P and d P will be cancelled out. So, you will get V E S will be q E upon 3 pi mu f into d P.

So, you will calculate the ESP value, V E S value, the electric drift velocity values in terms of the stokes regime if the particle Reynold number is less than 1. You can calculate in the general regime in general regime the solution will be iterative in the stokes regime the solution will be straightforward and what you have to do you can just find that how the electrical mobility will take place. So, how the particle velocity will be there, so, that is the way the things can be defined in most of the cases in ESP whatever the way we have discussed the ESP is used only for micron or below particle size ok.

So, the particle size should be less than 1.1 micron then ESP is used and most of the time we are in the stokes regime because particle size is very very small, it pulls the pull Reynold number very low and you can use this value to calculate that what is the V E S. If it is not in the stokes regime you can this use this value you can do the iterative solution and you can find it out that what is the E S value.

Now, the only problem whatever we have discussed till now is the value of C D and I said that the C D value for the stoke regime it is very clear it is 24 by R e; other than the stokes regime it is difficult to calculate either you have to do on the chart which will be changing with the particle diameter which will be changing with the particle size or particle shape. So, many authors have done lot of work to find the C D value and several C D correlation has been developed.

Now, in the next time what we are going to do we are going to discuss those C D correlations that what are those C D correlations, how those C D correlation has been developed, what is the advantage of each correlation and when to use which correlation. So, if you know the C D correlation what you need to do? You can do all these calculations if you have the C D correlation, you can just do the iteration find the C D value from the correlation use the Reynold number guess the Reynold number find the C D value for that Reynold number by using the correlation find the u V E S value or u t value for that and then again keep on doing the iteration.

So, we will see that the different drag loss which is being developed by many researcher for the different applications and how that drag loss can be implemented or can be integrated with this C D values to calculate the velocity of the particle, with the time to calculate the position of the particle with the time or to calculate the settling velocity or terminal velocity so, that we will discuss in the next class.

Thank you.