Multiphase Flows
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Lecture – 10
Pressure Drop Calculation for Separated and Annular Flow Regime

Welcome back. So, last class what we have done, we have seen the how to calculate pressure drop in by using the Lockhart-Martinelli correlation and before that we have written the equation for homogeneous flow model where everything was based on the mixture properties.

Now, what we are going to do now, is to see that how to calculate the pressure drop or how to write the pressure drop equation for a stratified flow and annular flow.

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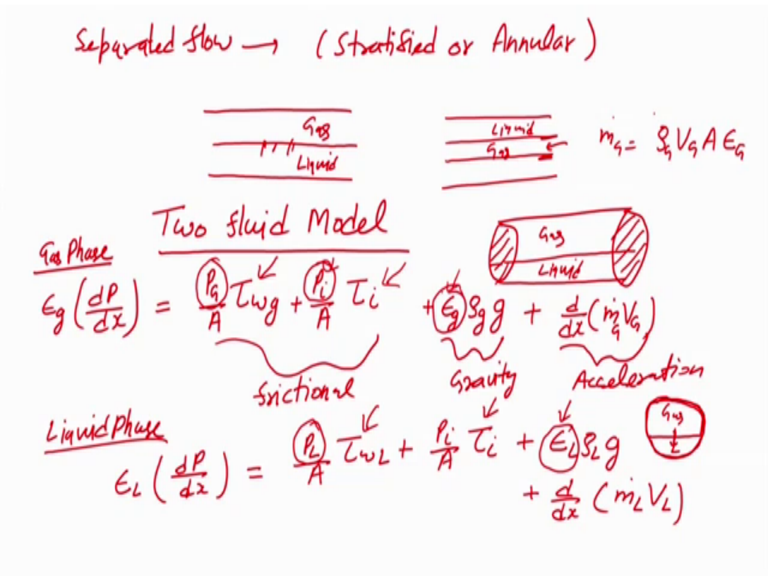

So, we call it as a separated flow. Why separated flow, because the flow are separated, the two phases are separated it can be either stratified or annular. What is stratified flow just to rewind ah, again it means gas and liquid are separated. So, liquid will be at the bottom of the column of pipeline and gas will be on the top and annular means the gas will be flowing inside, ok. So, this will be gas and it will be surrounded by the liquid. So, this is stratified flow or annular flow and both are separated because both the phases are simply separated from each other. So, we can also call it as a separated flow.

So, what we are going to see that how to write the delta P equation for this kind of a flow. Now, for writing the delta P equation for stratified or annular flow generally we use two fluid model ok. So, this two fluid model is mostly mini book if you will go they will say that it is a two fluid model or annular model.

Now, this model is very popular to calculate the delta P for stratified and annular flow. Now, why this two fluid model it means now what we have done in homogeneous model we have written one equation and that one equation was based on the mixture property ok. So, we have taken the v superficial of mixture, we are taking the mixture density, we have taken the mixture viscosity, we have taken the friction factor for the mixture, Reynold number for mixture everything we have calculated for the mixture. Why, because both the phases was homogeneously mixed. So, that is why we have used the mixture model.

Now, in two phase model as the name suggests that now, we will write the equation for both the fluid individually and we will simultaneously solve both the equations to get the delta P. So, to write the delta P equation again I will do the same thing which we have derived from the RTT, the basic equation of the dP, which says that dP by dx is nothing, but it actually you see in any pipeline or any system because of the three facts factors the first factor is gravitational, second one is frictional or due to viscosity and third is due to the acceleration.

So, again we are going to do the same thing. So, dP by dx now in two fluid model if I will write that will be equal to again for the three phases, three things one is for acceleration frictional and gravity. Now, because there is a two phase flow suppose there is a this is a pipeline there is a pipeline here and if I am writing for the separated flow, the suppose liquid layer is there this is the liquid layer and on the top gas is flowing. So, what will happen the dP by dx whatever you will see that we will see at the pressure you will see at direct cross section.

So, some fraction of the pressure loss will be because of the liquid some fraction of this pressure will be because of the gas. Now, we are writing both the equation individually. So, what we need to do this dP by dx, if I am writing for the liquid phase I have to multiply with epsilon L, if I am writing for the gas phase I have to multiply with the volume fraction of the gas.

So, suppose if I am writing for gas phase first. So, I will write it is a epsilon g, it means the volume fraction the contribution of dP by dx because of the gas and that will be equal to what we will write it as P G upon area into tau w g. So, that is the contribution because of frictional contribution. So, P G will be the parameter which is occupied by the gas is the cross sectional area and tau w g ok. Then, what you will see? You will see I am leaving one term here for certain things; why I am leaving, I will we will discuss it later, but I am reading space for one term.

Then, we will write it for the gravitational now what is gravitational gravitation will be rho g into g ok. So, it will be rho g into g; now, again what you have to do because this rho g into g will be there, but you have to multiply with the overall volume and this volume will have a certain fraction only will be occupied because of the gas. So, that is why you have to multiply it with the epsilon g ok. So, that you will see that rho g is there this is the complete volume because which this will come as a unit of volume, this volume will also come into the pressure per unit volume. So, you have to multiply with the epsilon g to find it out that what fraction is occupied by the gas inside, ok.

So, this will be this plus it will be the acceleration part. Acceleration part will be what? We can write it d by dx of m naught into V and V of gas m naught of gas. Now, m naught of gas is what if I write m naught of the gas, it will be rho into V g rho g into A multiply by epsilon of G, clear. So, that how much fraction of the area is being filled with the gas, so, that the way we will calculate the m naught G. So, this will be the two phase flow equation. Now, why I have left a term because there will be another frictional term and that will be the frictional term which will be acting at the interface.

So, suppose this is a front view which you are seeing here, so, this is liquid this is gas. So, what will happen there will be stress which will be acting on this wall because of the liquid the friction which will be acting on this portion because of the gas ok, again I am saying this because of the gas and there will be certain friction which will be acting at the interface because of the gas and liquid both. So, what you need to do you have to define the inter facial or interface stress component. Now, interface stress component will be what it will be written as P i which is parameter of the interface divided by area divided by tau and interface, tau i. I will write it as a tau i not tau w i, because there may or may not be a wall at the interface. So, interface is between the gas and liquid. So, I will write it as a tau i.

So, this is what is the frictional term, this is frictional, this is gravitational because of the gravity and this is due to the acceleration ok. So, again what we have done we have written that dP by dx is going to be in the same three forms. There will be a frictional component, the one new term will be added and that will be the frictional component because of the interface which has been created between the gas and liquid. So, whether is the separated flow or annular flow you are going to see the interface ok. So, whether is the flow is like this or the flow is like this here you will see again the interface, here in this case also you will see the interface.

So, interface will always be there and because there is a interface it is a cylindrical geometry. So, do not assume that there will be two interface here. So, it is the same interface which is being forming. So, there will be interface in this case annular also interface in case of a stratified also and that is why the equation remains same only the some values will change we will see with the once we will discuss in detail about this. So, that is the way you can write the gas phase equation.

Similarly, you can write for liquid phase. So, what we will do we will write it epsilon L dP by dx because some portion will be because of liquid. So, epsilon L now, what we have to do we have to write the frictional component because of the liquid. So, I will write it P L upon A into tau w L plus I will write again the interface. So, it will be P i upon A into tau i plus this will be epsilon L rho L into g plus it will be d by dx of m naught liquid into V liquid ok. So, that will be the overall component which you will see that the frictional pressure drop for the gas phase and liquid phase in case of the gas and in case of the liquid and you can calculate it.

Now, what is the problem? The problem is to calculate the epsilon g that how to calculate the epsilon g and epsilon L value you do not know you do not have idea about that value because there is nothing like a no slip condition. So, epsilon here is not equal to the alpha holdup. So, you have to find the alpha epsilon g and epsilon L then you have to find the value of tau w g tau w L. Now, additional term has been added that is tau i and this is tau i that how to calculate the tau i, how to calculate the P i. So, this is the additional term which has been added, rest everything is as this complicated as it is a single phase flow, but these are the extra terms which comes and because of that again you need some correlations to solve this.

So, now what we will see we will see how to calculate tau w g, tau w L and tau y.

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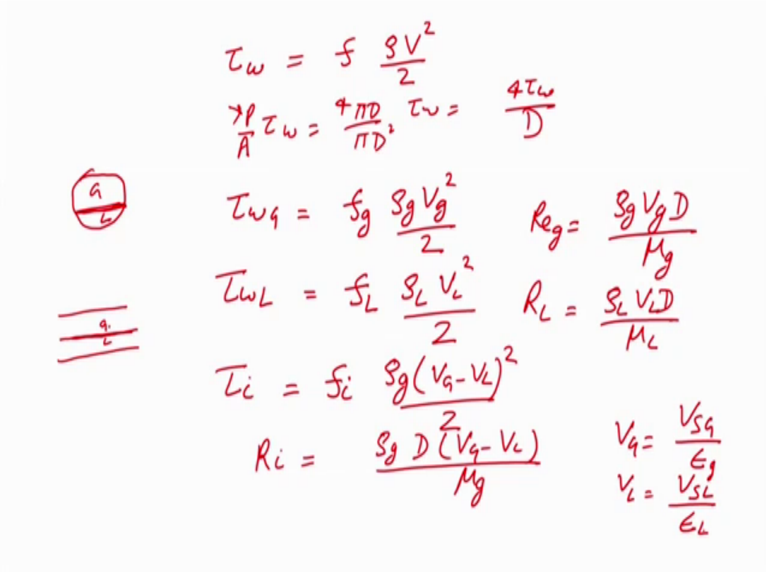

Now, tau w g is very simple the way we have defined in single phase flow. So, tau w tau is nothing, but the way we have defined so, it will be what that will be friction factor f into rho into V square upon 2 and for a cylindrical pipe we have written already that P upon A tau w is nothing, but it is pi D upon pi D square by 4 into tau w. So, we have written, this will be cancelled out; it will be 4 tau w upon D. But, now because the flow is separated or annular we cannot use P as a pi D, it is not occupied in the full full pipeline. So, what will happen your P will also be modified? So, we will see how the P will modify, but first we will see that how the write the tau w g and tau w L. So, tau w G will be written as f g, so, friction factor based on the gas velocity ok, it will be rho g it will be V g square divided by 2.

Now, friction factor based on the gas velocity means the Reynold number of the gas will be calculated based on rho g into V g into D upon mu g, that is the way the Reynold number will be calculated, ok. Similarly, tau w L that will be based on the liquid. So, friction factor based on the liquid velocity or liquid Reynold number it will be rho L V L square divided by 2. So, again Reynold number of the liquid will be depend on will be rho L V L D upon mu L.

Now, the problem comes with the interface, because interface what you are going to see if you again I am writing the drawing the same thing this is liquid, this is gas and this is interface. So, if you will see the interface, say in a front view this interface will be moving with a relative velocity of the gas and liquid. So, what we are going to do, we are going to define tau i as f i rho of g and I will take the relative velocity and relative velocity is V g minus V L square divided by 2 ok. So, that is the way we define the interface tau i. Now, one can ask that why rho g why not rho L, so, we have defined it in that way so that because the rho L value is less the interface tensor the interfaces stress values will come lower if you will put low rho L then the interface stress value will be very very high and actually in the case of this the gas phase which have lower viscosity is going to actually dominate the overall interface stress.

So, that is why we write it it in this term and we define the R i also based on the interface R i. So, f i will be calculated based on the R i and R i for the interface is being defined based on the interface velocity which will be again rho g D and V g minus V L mu g. So, this is the way the interface thing has been defined the Reynold number is defined interface. You can calculate the interface Reynold number, you can calculate the values and V g and V L you can calculate based on that the superficial velocity and superficial velocity you can know that that this is equal to V g is equal to V of superficial gas divided by epsilon, V L is equal to V of superficial liquid divided by epsilon; so, epsilon g, epsilon L.

So, if you know the epsilon you can calculate V g and V L you can calculate the inside velocity, you can calculate all this parameter. Now, what we need to do we have to find that how to calculate the perimeter at the interface and what will be the perimeter of the gas, what will be the perimeter of the liquid; so, it means these quantities, perimeter of the gas, perimeter of the interface, perimeter of the liquid and also the value of epsilon L and epsilon g.

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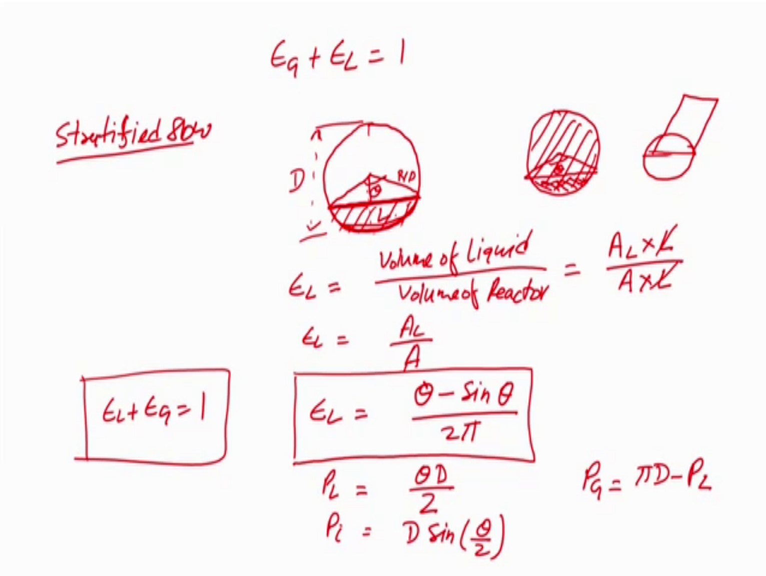

Now, we know that the correlation. That epsilon g plus epsilon L is equal to 1. So, it means the summation of both will be equal to 1. So, if I calculate epsilon g or epsilon L, I can calculate that what will be the value of the next volume fraction. So, how to do that? So, first what we need to do, we have to calculate the value of D L and that will be calculated based on first thing that suppose this is my pipeline and I am showing you the front view, this is the liquid level at this is the centre of the column and it in forming angle theta from this place, ok.

So, what will happen suppose the diameter is d the radius is R by 2 and the diameter is D the whole diameter of the section is say D then what you can do you can use the simple trigonometry you can find it out that what will be the epsilon L; epsilon L is nothing, but the volume of liquid divided by volume of reactor. Now, this we can write as area which is occupied by the liquid into length area of the reactor into the length, now this is going to be the same. So, that will be cancelled out because in case of a stratified flow now we are doing this calculation alpha stratified flow ok. So, this will be cancelled out. So, epsilon L will be equal to A L upon A, because they are being separated with this.

Now, what we need to do, we need to calculate the A L value. Now, how to calculate the A L value, for that what you have to do? You have to find it out that what is this fraction ok, what is this fraction. So, if you calculate that fraction we know this theta value you can divide it you can draw a perpendicular here and you can calculate that what will be the this fraction this length, ok. If you can calculate this length you can just divide it and you can find it out what will be the area. So, if you do the small trigonometry epsilon L you will get it as gamma or theta because we are writing in terms of the theta minus sine theta upon 2 pi, ok. So, what we are doing is nothing, but this is value is nothing, but pi D square what we need to do we need to just find it out that will be the area overall area that will be pi by 4 D square. So, pi by 4 D square will be there.

Now, what we need to do we need to just find that what is the fraction. So, pi by D square is nothing, but the whole area if you do the pi by D square, you will see that this is the whole cross section now you will get. Now, what you need to do? We have to find it out that what is this cross section, what is this? So, what how you can find it out now you can have is this angle is theta you can say that what will be the theta minus sine theta into pi D square that will be the value will come pi D square by 4 and then once you will do that the theta minus sine theta you have to divide it by the pi that will be the half angle you will get that how much of this fraction will be there.

So, if I know that I will get the A L; A will be the same that pi D square by 4 pi D square by 4 will be cancelled out and you will get the epsilon L value. So, this is the small trigonometry, you can try to solve it I have already told you the procedure. So, you will get the epsilon L value that will be nothing, but it will be theta minus sine theta upon 2 pi. So, it means what, if I know the liquid level if I know that what is the level of liquid inside the pipeline I can calculate the value of the theta. Once, I can calculate the value of the theta I can find it out that what is the fraction of epsilon L.

So, we can calculate the epsilon L values. Now what, similarly you can calculate the simple trigonometry again the P L; P L will be nothing, but this, this is the P L. So, the P L will be what, if you calculate the P L that will be theta into D upon 2 ok. So, that will be the P L, because you will get the perimeter, ok.

Now, similarly you can calculate the P i and P i is what? This value that what is the parameter for the interface, now interface is complete. So, the P i is actually this complete value if suppose this is the interface. So, this is the whole P i is the interface value if you do that again these are small trigonometry problem you can try to solve, if you are not able to solve we can discuss it further that will come as D sin theta by 2. So, that will come as a P i. Now, P L, P G, P G will be what? It is nothing, but pi D minus P L. So, you will get the P G value. So, if you know the liquid level inside or you know the gas level inside you can calculate all epsilon L, epsilon g, P L, P i and P G by using very small trigonometry.

So, you have to just solve the trigonometry things and you will get all this parameter value P L value P i value I will recommend that you definitely solve it. If you face any problem please write us back and we will try to solve it further or we will discuss it further in detail.

So, once you have epsilon L you can calculate the epsilon G also by using the same formula which is epsilon L plus epsilon G is equal to 1. So, what now it means you can solve the whole stratified flow problem in a one dimensional domain. Again, this is only one dimensional problem if you go for the 3 dimensional the; you have to use the numerical approach you have to use the advanced CFD approach to solve these things, ok.

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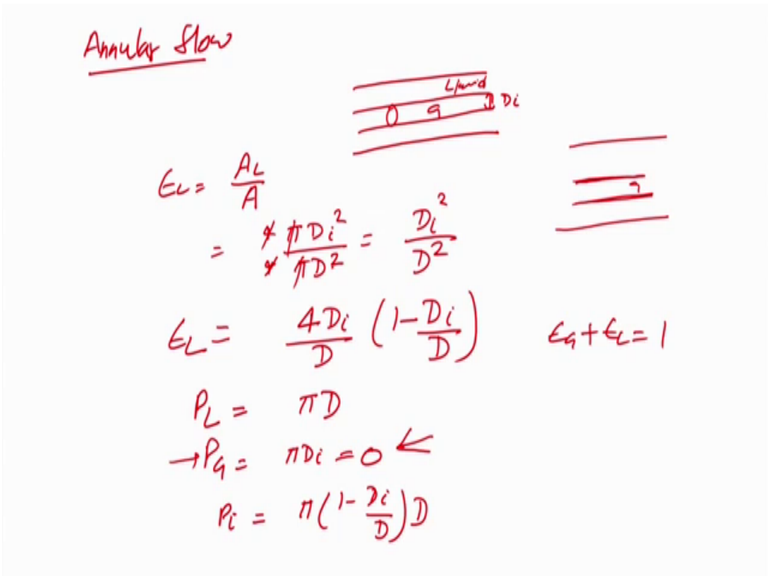

Similarly, for annular flow, what is annular flow again it means this is the pipeline the gas is flowing at the centre and liquid is on the outside again you can do that same trigonometry and if this is D i if the inner diameter is D i where the gas is flowing you can do that all this, you can find it out epsilon L and epsilon L is nothing, but A L upon A, you can get the A L; A L will be what A L will be in terms of simply pi D i square and divided by pi D square, this will be 4 here and 4, here this will be cancelled out, pi-pi will be cancelled out, ideally it will be D i square upon D square.

So, you can find it out or if you do the further trigonometry you can find it out that this is nothing, but in a domain it will be 4, if you write in terms of the hydraulic diameter it will be D i upon D into 1 minus D i upon D. So, this will be the epsilon L value. So, very small trigonometry I am not solving it because this is very very small trigonometry if you do just a small pen and paper work you will able to calculate all this.

Similarly, the P L will be what P L will be the outside perimeter that will be pi D. The P G value, now P G value will be pi D i, some book says that P G value will be 0, because in comparison to if the annular flow the liquid thickness or gas thickness is very very small compared to the liquid thickness we can easily neglect the P G. So, that will be there and P i will be nothing, but it will be pi 1 minus D i upon D. So, that the fraction that how much fraction will be there at the interface.

So, what you will get you will get D i minus D will be nothing, but it will give you the ratio. So, one minus D i by D will show you that what is this fraction and that you will multiply it with the pi D you will get that what will be the parameter of the interface. So, because of that many people because this is the interface P G is being said to be 0, because if suppose this is the annular flow this is the liquid this is the pipeline this is the gas ok, so, this is the parameter of the gas and similar is the parameter of the interface. So, because we are taking the interface with the same thing what we do the P G value we make it as a 0, because they we say that there is nothing parameter which is being occupied with the gas.

So, that is why the P g value is kept 0. If you just substitute these values in the equation which we have written for the two-fluid model you will get the equation for the annular flow and again you can solve the dP by dx values because now we have everything if you know that what is the area or what is the D i value what is the annular section diameter which the pi this liquid or gas is being covered. You can calculate epsilon L you calculate epsilon G by using this epsilon G plus epsilon L equal to 1 and you can calculate the P L, you can calculate P G, you can calculate P i. So, all those unknowns you can calculate and once you able to calculate that you can easily calculate the delta P, ok.

So, in this way what we can do whatever we have done till now we can calculate the dP by dx value or delta P value in homogeneous flow model, we can use Lockhart-

Martinelli kind of correlation, empirical based correlation to calculate the delta P in pipeline, we can use two fluid model for stratified flow or annular flow to calculate the dP by dx value. So, if you remember we have covered almost everything the way we stratified flow and all the things will remain same only this value of epsilon P G and P L will change and one has to do the small trigonometry to find these values. So, if you see that everywhere all those regimes two-fluid model will be used.

So, if you see what we have done we have calculated the dP by dx for the entire flow regime of gas liquid flow. So, whether it is horizontal, whether it is vertical; if horizontal pipe is there the gravity term will be 0, if it is a vertical pipe gravity term will be there if the flow is fully developed the acceleration term you can neglect. So, for any kind of a flow with small assumptions or with valid assumptions you can calculate the dP by dx for one-dimensional model. So, all these models are one-dimensional which for which you can get the analytical solution very easily. If you go for the 3D model if you then what you need to do, you cannot have a analytical solution, you cannot have the simple equations and then you have to go for the advanced CFD approach or the CFD approach or the numerical approach to find the values of all these P G, P i and P L and to solve a complicated equation.

So, to get a first end idea again the one-dimensional equations are widely used one should know that how to write a one-dimensional equation how to solve those one-dimensional equation and that was the motivation of this part of the course which we have covered. So, with this we have covered the topic number-3, which was the about the flow regimes, flow regime delta P for the separated flow and homogeneous flow.

Now, what we are going to do we are going to see that how to write the same equation for or the momentum equation for gas solid flows. So, till now what we were we were doing is for the gas liquid, now onward what we will do we will do it for the gas solids.

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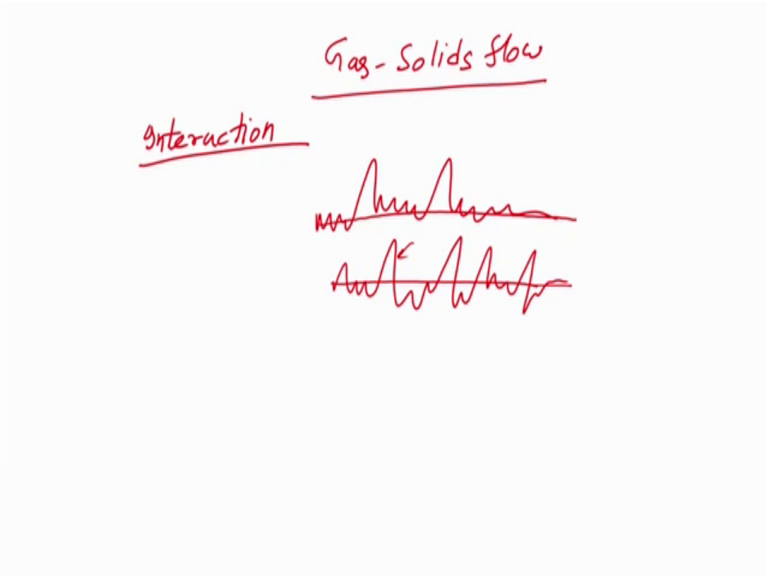

Before we move to the gas solid flow, what I would like to see that to see that what are the interactions possible between the gas and solid, ok? So, there is a closure problem it is called and the interactions are we will discuss it again once we go for the CFD simulation, but just to give a brief idea that what are the interactions which we is there which we see in case of the gas solid. So, gas solid is entirely different then whatever we are seeing in the gas liquid, why because in gas liquid both the phases were in continuum, both the phases was continuous other than the homogeneous flow where there was one full of water in bubbly, but we assume that they are completely mixed and we can use the mixture model and based on the mixture property we have solved the equation.

So, what we can do here for the gas solid flow because the solids are discrete in nature itself they always discrete they cannot be continuous for sake of simplicity sometimes we will use the continuum approach here for the gas solid, we will discussed those things once we will see how to solve the three dimensional equations the rigorous model, but for the one-dimensional problem what we will do we will simplify it and we will try to see that first what are the interactions, what are the forces acting once a gas and solid flows take place.

So, first thing what we are going to see is the interaction now what do we mean by interactions. So, interactions means if suppose we always say that how you are interacting with the others; it means suppose if two phases are flowing together or two people are flowing together or two objects are flowing together how these objects are interacting with each other, how they are communicating between each other.

So, in case of the gas solid the communications are there at the for level. So, what we have if there is any flow and if the flow is turbulent what will happen there will be a mean motion and there is a fluctuating motion. So, what do you mean by that if suppose the flow is turbulent. So, this is what is the turbulence nature is the fluctuation and I can say that this is the mean and over the mean the fluctuations are inbuilt. So, there is a mean motion here which is moving in it in this direction and there is a fluctuations too. So, what is happen all the this will be true for gas, this will be true for the solids or I will say this will be true for the fluid and this will be true for the solid too. So, the interaction will also be at the two level; one will be at mean level and another will be at the fluctuation level.

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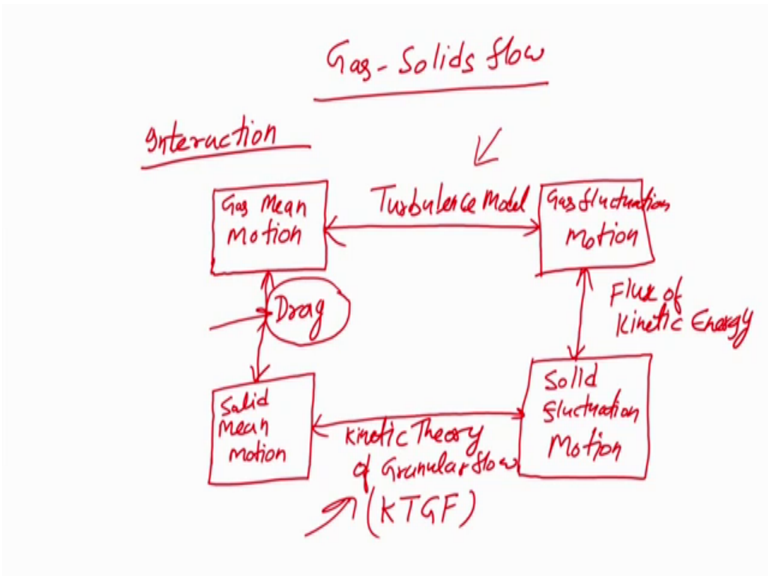

So, what I am going to do, I am going to make four compartment and I am showing this is as a gas mean motion, and then I will say that gas will also have because it is a turbulent flow it will also have a fluctuation motion. So, gas fluctuation motion. Similarly, I will say it for the solid. So, solid mean motion and this will be solid fluctuation motion solid. So, these are the four motions. So, gas have it is own mean velocity it is turbulent fluctuating velocity solid will also have it is mean velocity it is fluctuating velocity.

Now, what they are going to do they are going to interact with each other. So, what happened there is a certain level of interaction between the gas mean motion and gas turbulent motion and that is being modelled by using the turbulence model and you might have heard the name of k epsilon, k omega, LES, RNG all those are model to understand that how the fluid mean motion and fluid fluctuating motions are correlated ok.

So, if you develop a correlation if you use a model then if you want to model the gas mean motion as well as gas fluctuation motion you need certain models some additional parameter, some additional model and that models are known as turbulent model, ok. Sometimes we also call it as a turbulence closure. So, you need another extra equation to close the problem and that will be in terms of the turbulence model.

Now, the gas mean motion and gas solid mean motion will also interact with each other and that interaction is actually being modelled by using drag this is a term which we have used earlier also many times if you have heard this term that drag is acting and all, so, what happen drag act between the mean motion of the fluids and solid or the fluid and object, it is not act between the fluctuating motion.

So, once we are talking about the drag we are talking about the mean motion of the object and of the fluid between that whatever the forces are acting or if you want to model the interaction between the mean motion of the gas to the mean motion of the object or the solid you need additional model and that model or that closure is called drag closure, ok.

Similarly, the part solid mean motion and solid fluctuating motion is being correlated and gas is being modelled by using kinetic theory of granular flow, more commonly known as KTGF and we will try to briefly discuss KTGF once we will discuss the CFD part that. So, solid mean motion and solid fluctuating motion as you use turbulence model to model the solid gas mean motion to the gas fluctuating motion that how they are correlated, similarly KTGF is used to model the solid mean motion with the solid fluctuating motion and both the fluctuation the solid fluctuation and the gas fluctuation they also interact with each other and that is being modelled flux of kinetic energy flux of kinetic energy.

Now, this is correlated with mainly basic fluid mechanics. So, I am not going to touch it the basic this is the basic fluid mechanics part what we are going to discuss in this course is about these two and mainly about the drag because this is something which is acting once a discrete body is moving in a continuous loop, ok. So, the drag is the main force which is actually interacting between the mean motion of the gas or of the fluid with the mean motion of the object, suspended object and that is why the drag calculations are very very critical and we will see that how to do the drag correlation calculations and how to model the object movement by using the drag.

So, this is the forces which is acting we are mainly focused on the drag and kinetic theory of granular flow and this all together is called closure problem. So, if you know all this you will able to close the problem and you will able to solve everything. So, you will able to see how the gas mean motion is interacting with the gas fluctuating motion how the gas mean motion is interacting with the solid mean motion how the solid mean motion is interacting with the solid fluctuating motion and how the solid fluctuating motion is interacting with the gas fluctuating motion. Everything you can solve if you have all these four models which are valid which are applicable to your application, ok.

So, what we are going to discuss mainly about is the drag and the kinetic theory of the granular flow. What is drag? Drag is nothing, but it is a interaction force between the solid mean motion to the fluid mean motion ok.

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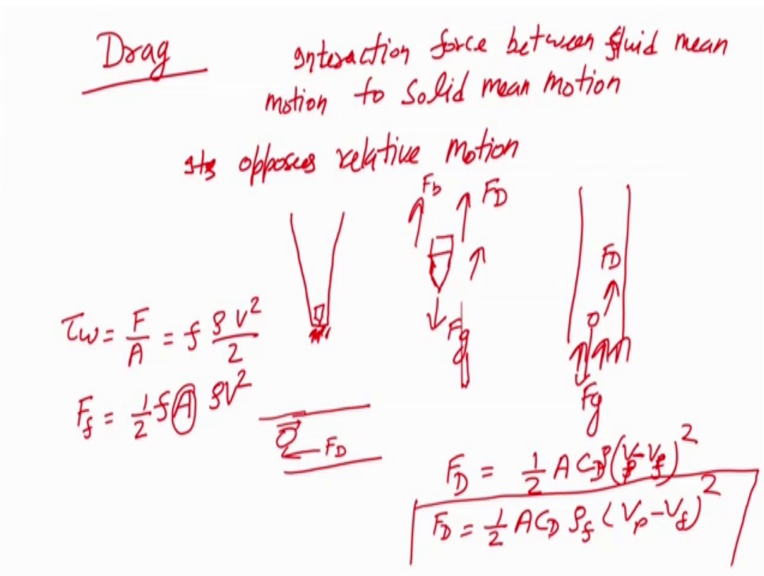

So, how the drag is being defined, because we are going to do that so, drag is nothing, but it is a interaction force , fluid mean motion to solid mean motion and opposes relative motion. So, the direction of the drag always opposes the relative motion. Relative motion means the motion between this is the relative between the gas and the fluid and the object, ok. It does not mean that it opposes the motion of the object that is the wrong motion which many people always say, but drag is a resistive force. It resists the relative motion of the object it does not more resist the motion of the object it is resist the relative motion.

What does it mean? It means that suppose in case of rota meter, you all know the rota meter, I hope that there is this kind of assembly there is a float at the bottom of this which will be sitting idle at the bottom once the fluid will pass, what will happen around this float. So, if the fluid will pass what will happen the fluid momentum will have upward, the buoyancy will act upward, the gravity of this will be acting downward F b, F g and the drag will act on which direction because the float will moving in the upward direction, but still the drag is going to oppose the relative motion and the relative motion because of that the drag force will also be in this direction because the relative motion of the object is towards the downward object wants to move downward, gases or fluid is moving upwards, so, the relative motion is towards the downwards once the fluid is the float is not moving and that is why the drag will act upward and it will favour the motion of the float ok. So, that is drag force.

Similarly, if suppose object is moving in a stationary fluid ok. So, what will happen, the object is moving at this direction, so, the relative motion of the object is in the direction of say x. So, what will happen the drag will be acting to oppose the direction? Now, similarly, if suppose a fluid is moving from bottom to top and the fluid velocity is much higher very very high and the solid is being there initially stationary now, with the fluid velocity it is moving and so, what will happen, the solid will try to catch the velocity of the fluid.

So, in that case the fluid is moving upward the relative motion of the solid is actually the solid is initially was stationary or ah. So, what will happen it will try to resist it is motion and in that case the drag will act F D will act upward the F g will act downward, till the time when the solid velocity is not equal to the gas velocity. The moment it will be there the drag will start opposing the motion of the solids.

So, what we are trying to see or say that the drag is a force which actually act between the mean motion of the solid and mean motion of the fluid and it opposes the relative motion of the object, ok. So, that is what is the drag and what is the difference between the drag and friction? Both are the surface force the friction act on the continuous surface, but once the body is suspended the instead of the friction the force which actually resists the motion of the fluid or motion of the object is called drag. So, it is very analogous to the friction the friction act on the continuous surface, drag act on a suspended surface.

So, that is the way this force is being defined and F D is being defined very clear or very close to the way friction has been defined or frictional force has been defined. So, friction force tau w or you can see is being defined tau w is what is the frictional force divided by the area that was being defined as rho u V square upon 2 multiply by f. Now, what we do here we define the F D exactly similar way as of the frictional force and what we say this is nothing, but half into A, so, the tau f of friction is nothing, but half f area into rho V square.

Now, this area is what is the surface area at which the friction is acting or the object is moving, here it will be F D will be defined as half a instead of the friction factor we say that the drag coefficient D into V square or relative motion V of fluid minus V of particle or I will say V of particle minus V of fluid whole square does not matter because you are going to square it ok. So, this is the way, this is at and they will be one more rho term.

So, F D will be equal to what it will be half A C D rho of this fluid into V of fluid V of particle minus V of fluid this one, this is what is called drag force is very analogous to the frictional force. Here A is the surface area at which this is acting; this A is the projected area. So, which the drag is acting or the surface at which the drag is acting C D is the drag coefficient and there is lot of correlations we will discuss that over the time.

There are lot of correlation like the f C D is also not known C D for the laminar flow is known very well and it has a direct correlation with the R e and C D is equal to 24 by R e for the laminar flow, for the turbulent flow or the Taylorism flow where Reynolds number values are relatively higher, C D value is not known and again we have a chart which is C D versus R e where you can calculate the C D value or different authors or different researchers have derived the correlation to find the C D values in the different environment. You can use any of these correlation depending upon at which environment you are using. So, we will discuss about all those correlation later on, but this is the way the drag has been formulated which actually opposes the relative motion and act between or the force which act between the mean motion of the solid to the mean motion of the fluid ok.

So, with this what we are going to do we are going to define the way we have done till now, for the gas liquid flow we have written the momentum equation we have tried to find it out the delta P here what we are going to do we are going to first see that how to find that how to track the motion of the particle if suppose a particle is being suspended or object is being suspended in the fluid how the object will move how far object will travel, how to find the object location with the time now, how to find the object position with the time.

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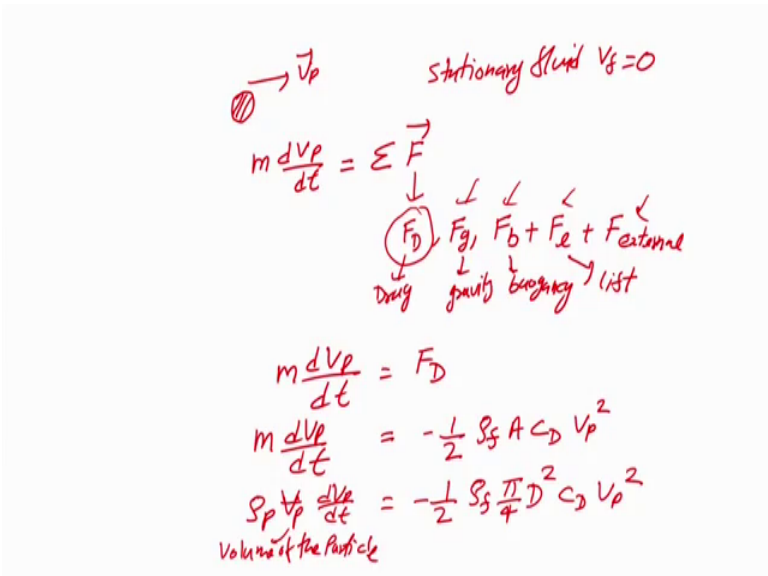

So, first we will do that and then we will do the delta P calculation. So, what we are going to do? We are going to first make the problem very simple and say that suppose, there is a particle which is being thrown with a velocity u P or V P horizontally no vertical motion. So, that I can simplify the equation I can neglect the gravity and this is moving the particle is moving. There is no other particle; there is only one particle which is being thrown in the horizontal direction which is suspended in the fluid.

So, what I am going to do I am going to write that how this particle will move and because this is a motion of a discrete particle which force is going to act it will be acted the Newton will follow the Newton second law of motion and the position of the particle or velocity of the particle can be calculated by using the Newton's second law of motion. Now, what is Newton's second law of motion? The rate of change of momentum is equal to the overall forces acting on the body. So, what will do, we will write the rate of change of momentum for the particle which will be m dV P upon dt that will be equal to the summation of forces which is acting on the body, ok.

Now, what are the forces which will be acting on anybody? It will be the forces can be drag force, gravitational force, buoyancy force, any lift force or any other external force. So, it can be drag, gravity, buoyancy, lift any other external forces, any force can act. Now, we are simplifying the case we are saying that a particle is moving in the air this is being thrown horizontally with a velocity and that what we need to calculate that how the particle will move, how the particle position will change with the time and how far the particle will move, if I have to derive the equation for that.

So, this so the force acting on this body will be F D drag force ok. Now, drag force is going to oppose the motion of the particle. So, this will be minus half this will be rho of fluid into V square I will write area into area into C D, because the gas velocity is 0 we are assuming that gas is stationary the particle is moving in a stationary fluid. So, V of f is equal to 0. So, I can write it as a V P square directly if the V f is not 0, then you have to write V P minus V f square. So, this will be the way we can solve it this is m dV P upon dt.

Now, what we can do we can write it the m is mass of the particle now the mass of the particle I can write it in terms of rho of particle into volume of particle now volume and V is looking same. So, I am writing the volume is it in this way. So, volume of the particle into mass of the particle this is volume of the particle. So, this dV P upon dt now this will be minus half, this will be rho f area of the particle if the particle is a spherical the area will be pi by 4, D square into C D into V P square, ok.

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$$\rho_p \frac{\pi}{6} D_p^3 \frac{dV_p}{dt} = -\frac{1}{2} \rho_f C_D \frac{\pi}{4} D^2 V_p^2$$

$$\frac{dV_p}{dt} = -\frac{3}{4} \frac{\rho_f}{\rho_p} C_D \frac{V_p^2}{D_p}$$

$$dt = -\frac{4}{3} \frac{\rho_p D_p \, dV_p}{\rho_f C_D V_p^2}$$

$$\int_0^t dt = -\frac{4}{3} \frac{\rho_p}{\rho_f} \int_{V_0}^{V} \frac{D_p \, dV_p}{C_D V_p^2}$$

$$Re = \frac{V_p \rho_f D_p}{\mu_f}$$

$$dRe = \frac{\rho_f D_p}{\mu_f} dV_p \Rightarrow dV_p = \frac{\mu_f}{\rho_f D_p} dRe$$

Now, what we can do we can write rho p and volume of the particle will be what pi by 6 dP cube into dV P upon dt this will be equal to minus half it will be rho f into C D into pi by 4 D square into V P square ok. So, this pi-pi will be cancelled out, you can write it here. So, this will be dP square and dP cube this will be cancelled out. So, this will be here. So, dV P upon dt will be written as minus 3 by 4 rho f upon rho P into C D upon into dP with V P square upon D P, ok.

So, we can write it it in this form. Now, what we can do, we can calculate separate this V P and dt. So, I can write it it as D t is equal to minus 4 upon 3 dt will be equal to minus 4 upon 3 rho of particle minus rho of fluid into dV P upon V P square into D P upon C D ok. So, we can write it it in this way. Now, what we can do we can integrate it we can integrate it from say 0 to time t, dt is equal to say initial velocity was V naught final velocity is V. So, I can write it as it will be minus 4 upon threw rho P and rho f is constant it will come outside ok, it will be integral u naught or V naught minus V of D P into dV upon C D into V P square, ok.

Now, we are not writing the C D outside why because C D is going to be the function of Reynold number as we have already discussed. So, what we have to do we have to write the C D in terms of the Reynold number. So, what I am going to do I am going to change this equation in terms of the Reynold number. Now, how the Reynold number will be defined this is will be V of the particle into rho of fluid into D P upon mu of fluid that is the Reynold number will be defined now we know that V and R e is the variable. So, I can write it dR e is equal to rho f into DP upon mu f into dV p, dV p.

Now, we can replace it is dV p with R e. So, if you do that what you will get you can implies that right here dV p is nothing, but mu f upon rho f into DP upon dR e, ok.

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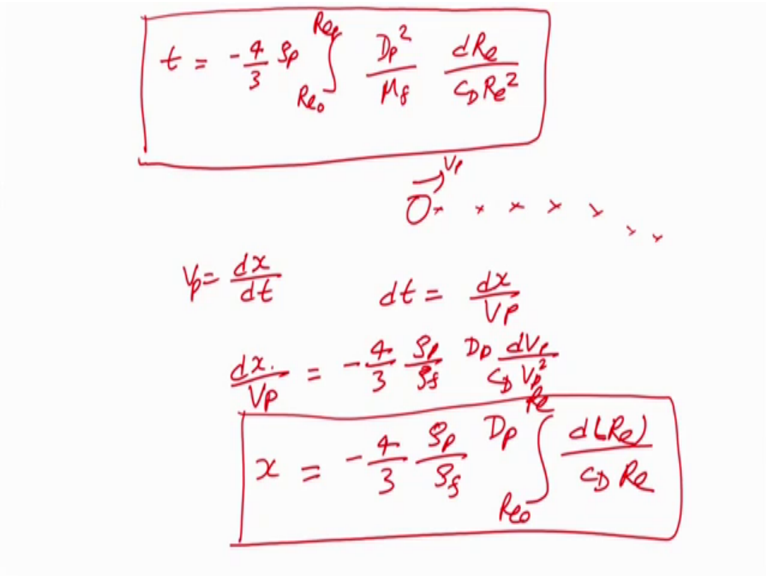

So, if you do that substitution you will get t is equal to minus 4 upon 3 it will be rho P at u naught the value will be R e 0 to R e P and this will be equal to mu dP square. So, if you solve that you will get D P square upon mu f to dR e upon C D into R e square we get this equation.

Now, if you know the R e versus C D correlation you can integrate it and you can find it out for a particular Reynold number values how that with the time the particle will move ok. So, so the particle movement you will get with the time now if you want to find it out the particle position because I want I am interested to track the particle trajectory. So, say particle has been started from this place a particular position it has been thrown with a initial velocity of V P of V naught then what will happen how the particle will move and which time how the particle trajectory will change. So, it means I am interested suppose the particle is moving it it in this way or any direction I am also interested that what is the position of the particle. If I know that I can track the position of the particle or I can track the movement of the particle.

So, what I can do I can write we know that V is nothing, but is equal to dx upon dt V P. So, dt will be equal to dx upon V P. Now, if I do that and I substitute in the initial equation which is in this equation ok. So, what will happen dt will be written as dx upon

V P will be equal to minus 4 upon 3 rho P upon rho f into D P into dV upon C D into V P square dV P.

So, if you do that this is dx, x will be equal to minus 4 upon 3 rho P upon rho f D P integral of d of R e if you calculate in terms of the R e this will be D of R e C D R e from R e 0 to R e. So, if you calculate it just what we have done we have calculated the V P in terms of the Reynold number and you will get exactly same correlation whatever it is. So, what you will see you will get this as the R e correlation here again C D versus R e, you can see the position of the particle is actually the inversely proportional to Reynold number, the time with how it will be change will be inversely proportional to R e square.

So, with this you can calculate what you if you know the C D versus R e correlation you can easily calculate that how the particle trajectory will be there, how the particle will move, which time how the particle velocity will change you can find from this correlation you can find that with the time with the how the particle position will change. So, you can find the particle trajectory and if you know the particle trajectory in any fluid solid motion you can find it out that how long the particle will take to go out of the system and how the particles are moving all those calculations you can do. Once you know the particle position you can calculate the particle velocity. If you know the particle velocity particle this position you can calculate the volume fraction, you can calculate every quantity. So, that is the way to find it out for any gas solid flow this is a very simple problem where we are talking about one particle we can make the problem complicated by using many particle tracking.

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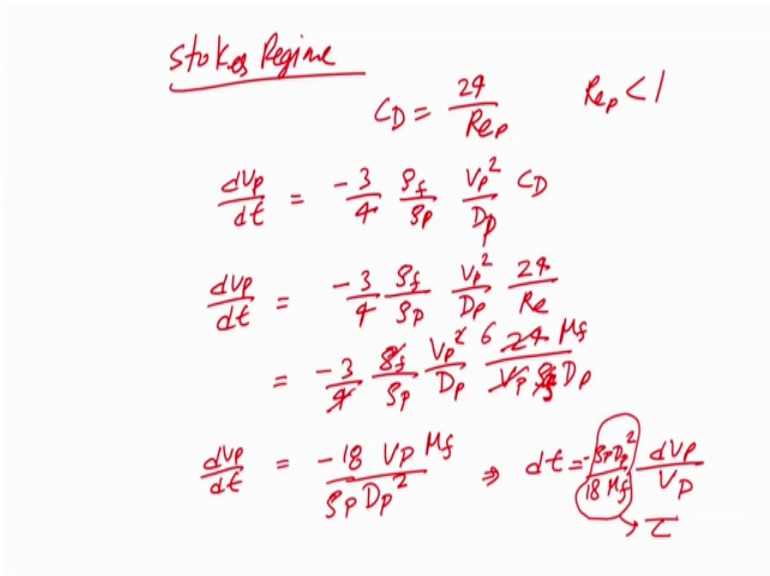

So, now what we are going to do we are going to solve the same equation for the stokes regime and the stokes regime means I hope you have done this in your undergraduate book that is stokes regime means C D is equal to 24 by R e or particle when R e particle is less than 1, then the stokes regime is valid and there the C D is being defined as 24 by R e.

Now, if I use the 24 by R e in the numbers whatever we have defined we can calculate that how the V is changing with the time how this position is changing with the time. So, what we will do, we will just write again the same correlation dV P upon dt. So, we will write dV P upon dt was equal to minus 3 by 4 just the and dV upon D P is equal to f D minus 3 by 4 from there I am starting again rho f upon rho P we have already derived this. So, I am writing this equation directly into V P square upon D P into C D ok. So, this is f D and m dV upon dt was there and I have converted in terms of the rho f into pi by 6 D P cube this we have written in terms of the pi by 4 area we have written in terms of the pi by 4 D square. So, from there we are doing it.

Now, C D value I can put as 24 by R e. So, dV P upon dt will be equal to minus 3 by 4 rho f upon rho P into V P square upon D P into C D is equal to 24 upon R e and R e we are going to write it as the way we have done rho f upon rho P this will be this will be V P square upon D P 24, R e is nothing, but V P rho P into D P sorry it will be rho fluid rho fluid into mu fluid because it is Reynold number of the particle it will be rho of fluid and the mu of fluid.

Now, this is the way we can write it; this V P and this V P will be cancelled out, this will be 6. So, we can write it as minus 18, rho f rho f will be cancelled out it will be your minus 18 V P D P square rho P into mu f. So, that is what you will get this will be is equal to dV P upon dt. So, you can write it in terms of the dt in clause that this dt will be equal to your dV P upon V P and this will be equal to rho P into D P square upon 18 mu f ok. So, this will be minus you can do it in this way.

Now, we know that rho P D P square upon 18 mu f is what, is the relaxation time which we have defined with the tau.

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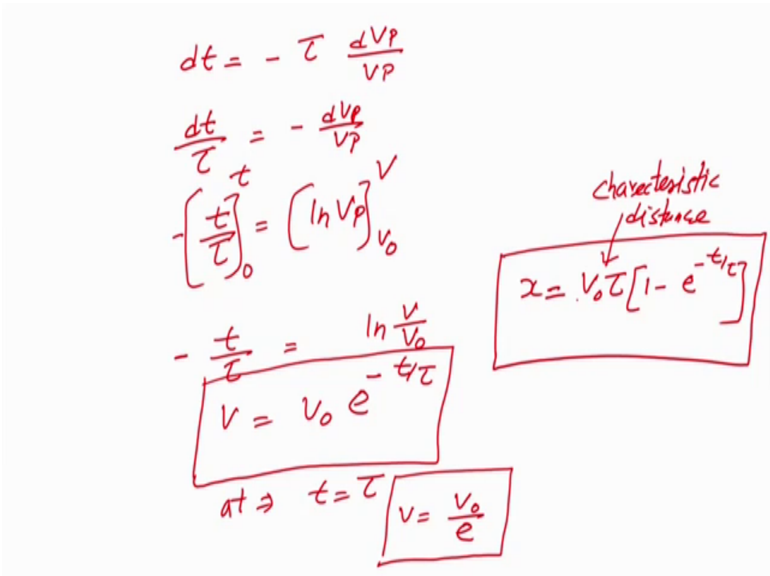

So, I can say that dt is nothing, but will be equal to minus tau dV P upon V P, ok. Now, we can integrate it. So, this will be dt upon tau will be equal to minus dV P upon V P. Now, we can integrate it if you will integrate it you will get that t upon tau minus will be equal to ln V P ok.

Now, initial velocity if I am taking 0 it will be 0 and final velocity is V p. So, this will be the V P value. Now, if you do that the initial velocity say if I assume that at initial velocity at time t equal to 0 the initial velocity was say V naught and at time t equal to t the initial velocity is V, then what you have to do you this will be minus t upon tau this will be equal to ln V P, V upon V naught, ok.

Now, you can write it as V equal to V naught into e raised to the power minus t upon tau. So, you can find it out that for the stokes regime if a particle has been thrown in a stationary fluid how the velocity of the particle will change with the time ok, that will be the function of relaxation number and the initial velocity V naught, ok. So, you can calculate it and add suppose if we do that at t equal to tau this e value will be equal to minus 1, it means V will be equal to V naught upon e. So, one relaxation time you will be reduced by 2.73 times ok. So, V will be equal to V naught upon e. Similarly, to relaxation time it will be V naught upon e square and so on.

So, you can find it out the values that how the velocity will change with the time in case of the stoke's regime. Similarly, you can calculate the position you can write this and dV P, V P as dx upon dt and you can write the dt in terms of the dx and you can calculate the x value and again this is very small thing you can do it again the calculation, it will come as one minus e raised to the power minus t upon tau. And, please remember V naught tau is nothing, but the characteristic distance. So, V naught tau is nothing, but the characteristic distance which it will flow in the fluid.

So, you will find the position of x with the time you can find the velocity with the time. So, it means what, in any gas solid flow or any fluid solid flow if you tell me that what is the initial velocity of the flow what will be the final velocity of the fluid or you want to calculate the final velocity of the fluid with the time I can give you that how the particle will move with the time, how it will this position of the particle will change with the time.

So, in the dust strong mainly we see that sometimes we see that though I have my area have no dust, but still we see lot of suspended solid particles inside ok. Like Delhi used to complain that lot of soot particles are coming from Haryana and all. So, we can do all those calculation whether it is feasible or not. You know the particle size you know the initial velocity you can assume the initial velocity of the wind with the mat department you will have that available. You can see that if the particle initial velocity was 0, if it is being picked up with that initial velocity of the gas and we assume that a velocity of the solid was equal to the velocity of the gas. We can find it out with the time what will be the velocity and what will be the position of the particle, we can track the complete trajectory of the particle and we can find it out that in any storm the dust particle which has been suspended in the storm what is the origin of that.

So, we can do all the calculation in terms of the nature, in terms of the particle flow, if there is a pneumatic conveying or the solid particle which is flowing with the fluid inside you can calculate that what is the initial velocity of the solid which is it has been thrown , how the particle velocity will change with the time how the position of the particle will change with that time.

Now, what we have done, we have done the equation calculation only for the stokes regime because the C D versus R e correlation was very simple at that place you can do it you can use different drag model which we will discuss later on. You can use those drag model you can put it here in the same equation and you can find it out that how the particle motion will take place.

Now, whether it is a fluidized bed whether it is a fast fluidized bed, circulating fluidized bed, whether it is a pneumatic conveying whatever we have discussed whether it is a dune flow you can calculate the particle velocity you can calculate the particle trajectory everything you can do if you know that the velocity of the particle, ok. If the case the fluid is also moving what you need to do? Whatever the values calculation we have done it is only for the velocity of the particle you have to do the relative velocity that fluid velocity will be constant, it will come out of the integral and you can use that relative velocity formula.

So, you can do all this calculation in terms of the relative velocity, you can do the calculation in terms of the drag force which will be the real drag force acting on the particle for that different model has already been given we will discuss all those drag models, ok. Now, we can also do that right now, we have done it by avoiding the gravity what we are going to solve next we will include the gravity in it, we will include the buoyancy in it and we will see that now how the equation will get modified. So, this summation of f right now, we have taken only drag, what we are going to do we will slowly include all the forces and you can calculate that how the particle movement will get affected in presence of different forces which is acting on the body and if you know that you actually know the particle tracking of the in any gas solid flow you can do the particle tracking.

So, with this we will stop here and in next class what we are going to discuss we will include the drag gravity, we will include the buoyancy and we will see that how the equation will get modified.