

**Multiphase Microfluidics**  
**Dr. Raghvendra Gupta**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Guwahati**

**Lecture – 21**  
**Inertial Microfluidics**

Hello, the topic for today is inertial micro fluidics. So, micro fluidics is often associated with small dimensions as the name suggests that the channel size is small. And the velocities that are present in the micro channels will also be small as a result the Reynolds number will be small in micro channels.

So, the flow is often laminar the laminar flow suggests that the flow will happen in laminar's, with very little or no interaction between the lines or no mixing as it generally obtain in the turbulent flows or the add is that are present in the turbulent flows.

So, this gives us a lot of control over the flow and because no heuristic models are required in laminar flow, heuristic means empirical or everything can be derived from the fundamental principles at least for single phase fluid micro channels.

So, the predictive power becomes very good and one can use this predictive power to control the flow of cells, bubbles, droplet, us particle for a number of applications.

So, in this lecture we are going to look at or examine the fact, that is inertial effect are important. So, when we say that the Reynolds number is low in micro channels, then immediately or the next thing is done that the Reynolds number is we generally try to neglect the Reynolds number or inertial effects.

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## Introduction

- The flows in microfluidics are often laminar
- Inertia is often neglected

*Re < 2300  
Laminar*

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v v) = -\nabla p + \mu(\nabla^2 v + \nabla^2 v^T)$$

*Unsteady term      Convective acceleration      Pressure grad      Viscous term*

- Stokes regime ( $Re \rightarrow 0$ )
- Convective acceleration term neglected
- Linear and time-reversible Stokes equations of motion

IIT, Guwahati 2

So, in the Navier Stokes equation, where we have different terms this is unsteady term, convective acceleration term, pressure gradient and viscous term we are not written any other body forces in this.

So, in this case when the Reynolds number is low we often tend to neglect the convective acceleration term and the unsteady term can be neglected if the flow is steady. So, this suggests the flow to be in this stokes regime and why we tend to do this, because this gives us a linear differential equation which can we solve analytically.

Now this may not be always the case, because we know that in a channel often the flow is laminar below Reynolds number 2300 or so, flow is laminar. But so, for example, if we have a Reynolds number 100, then the inertia is not negligible we cannot neglect the inertia term, but the flow will still be laminar.

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### Mirror Symmetry Time Reversal Theorem

(Bretherton 1962)

"When moving in a steady unidirectional shear flow at small Reynolds number under the action of viscous forces alone, to every orbit of a given finite rigid body there corresponds one of the body of opposite mirror-symmetry. The corresponding orbits are 'mirror images' obtained by reflexion in a plane perpendicular to the streamlines, but are traversed in opposite senses."

- Due to the linearity of the Stokes equation
- For a rigid spherical particle, no lift force acts on a symmetric particle for a simple channel flow.

IIT, Guwahati 3

So, in stokes regime what we have there are some beautiful results that one can see in the stokes regime. For example, in the Mirror-Symmetry Time Reversal Theorem: Bretherton suggested that when, moving in a steady unidirectional shear flow. So, the shear flow can be represented where we have a gradient in the velocity.

So, in a unidirectional shear flow at small Reynolds number under the action of viscous forces alone; that means, at small Reynolds number only viscous forces are important. So, the inertial forces has been neglected the flow is stores; to every orbit of a given finite rigid body there corresponds one of the body of opposite mirror-symmetry. The corresponding orbits are mirror images obtained by the reflection in a plane perpendicular to the streamline, but are traversed in opposite senses.

So, this suggests that if a rigid spherical particle or a sufficiently symmetric particle, which has which is symmetric about it is rotational axis no lift force will act on a symmetric particle for a simple channel flow. So, the particles will not experience any lift force and by lift force here, I mean the force on the particle in the transverse direction the direction normal to the streamlines ok.

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The slide is titled "Non-linear Effects" in blue text. Below the title is a red horizontal line. A bulleted list follows, with each item preceded by a right-pointing arrow. The footer of the slide contains the text "IIT, Guwahati" and the number "4".

## Non-linear Effects

- Non-linear effects because of:
  - Non-Newtonian fluid
  - Asymmetric channel structure
  - Asymmetric particle shape
  - Deformable particles
  - Non-negligible inertial effect

IIT, Guwahati 4

So, the flow is symmetric in stokes flow; however, in micro channels also we can experience non-linear effects due to a number of facts. For example, if the flow is non-Newtonian it may be following a power law model or it can be say a viscoelastic fluid or and so on.

So, the non-linear effects can come into picture when the fluid is non-linear effect can come into picture when the fluid is non-Newtonian, or when the channel in which the flow is happening the structure of the channel is asymmetric that can also give rise to non-linear effects in the flow. And if the shape of the particle as we have just seen that if the shape of the particle is symmetric about it is rotational axis, then it does not explains any transverse lift, but if the shape of the particle is asymmetric then we can see non-linear effects. If the particles are deformable then deformability of the particle may also induce a non-linear effect and say a lift on the particles. And finally, what we are concerned with in this lecture is if the inertial effects are non-negligible.

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### Inertial Effects

- Inertial effects can be important in microfluidics
  - Flow still laminar but non-negligible Re, say  $Re \sim 100$ 
    - Deterministic nature of flow
    - Fluids and particles can still be controlled and manipulated
    - Nonlinear and irreversible motion of fluid particles
  - Velocity difference across the particle or obstacle length scale are important in inertial effects
    - Large velocity gradients in microchannels

$$\frac{\partial u}{\partial x} \sim \frac{u}{D}$$

IIT, Guwahati 5

So, inertial effects can be important in microfluidics for as we said just the few slides back that the flow, in the micro channels is still laminar, but non-negligible say at about Reynolds number 100. And this have the properties of the laminar flow in the sense that the flow is deterministic in nature, you can still solve the equations in the sense that you do not need heuristic models, you may not be able to solve the equations always analytically, but you can get a numerical solution without the need for any empirical models or heuristic models for the turbulent flow.

And the fluids and particles can still be controlled and manipulated like we can do in the stokes regime, plus the additional effects that come into picture that the flow becomes non-linear and the motion of the fluid is irreversible unlike in the stokes flow. And these effects can be quite important. Now these effects are often, because of say for example, on a particle there is velocity difference across the particle or across a obstacle length. So, these velocity differences or the velocity gradients are responsible for the inertial effects.

Now, in the micro channel these velocity gradients, because the velocity gradient is  $\frac{\partial u}{\partial x}$  or they can be typically scaled by  $\frac{u}{D}$  over typical dimension of the channel. So, these gradients become large in the micro channels and they can affect or the non-linear forces or the non-linear effects can become important, when you compare with those effects in the conventional channel.

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### Inertial Effects

	Microchannel	Large channel
Diameter	100 microns	1 cm = 10000 $\mu\text{m}$
Flow rate	150 microliter/min	
Max Velocity	0.375 m/s	37.5 m/s
Velocity gradient	0.375/50 (m/s /micron)	37.5/ 5000 (m/s /micron)
Re	25	250000 (Turbulent flow)

Chaotic flow

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$$U_{avg} = \frac{Q}{\frac{\pi}{4} D^2}$$

$$U_m = 2 U_{avg}$$

So, let us look some numbers here if you compare a channel of diameter 100 micron and a large channel say of diameter one centimeter. And the flow rate in the micro channel is about 150 micro liters per minute. So, the velocity in this case will be the  $u$  average will be equal to  $Q$  divided by  $\pi$  by 4  $B$  square. So, and the maximum velocity will be 2  $u$  average. So, from that we can calculate the maximum velocity. So, this maximum velocity comes out to be about 0.38 meters per second and the velocity gradient the velocity gradient across the channel will be the maximum velocity divided by the radius of the channel. So, this velocity gradient is about 0.37 meters per second divided by 50 micron.

Now, if we want to achieve the same velocity gradient in a large channel. So, in the large channel where the channel size is one centimeter, which is about 10 mm or 10 000 microns; so the radius will be 5 000 micron. So, across a 5 000 micron radius one will need to have 37.5 meter per second of velocity to achieve the same velocity gradient. So, that is why we have this maximum velocity of 37.5 meters per second.

Now if we want to have the 37.5 meters per second velocity then the Reynolds number in this case become 2.5 into 10 raised to the power 5 or 2.5 lakh, which means the flow is turbulent; whereas for the same velocity gradient the Reynolds number is only 25 in the microns (Refer Time: 12:33) channel. So, the fluid is still laminar we can achieve such flows probably in a large channel, but the flow will be turbulent and highly chaotic. So,

we will not be able to manipulate the particle or we cannot control the flow of the particle in such a case.


So, that is why this effect becomes quite important in micro channels and it has been utilized or it has been used to good effect to develop various microfluidic devices for a number of applications especially in biological applications in (Refer Time: 13:15) and so forth.

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### Inertial Microfluidics

Inertial microfluidics:  
"Unconventional use of fluid inertia in microfluidic systems"

- To focus, concentrate, order, separate, transfer and mix particles and fluids
- **Inertial focusing**: Lateral migration and control of particles
- Structured channels: secondary flows



IIT, Guwahati 7

So, this inertial micro fluidics the term refers to use of inertia of the fluid in microfluidic systems to do different things in channels especially to manipulate the particles to focus particles to concentrate them or to bring an order to the flow of particles, separate the particle transfer and mixed particles and fluids. And this requires a clever use of the particle properties, fluid properties, fluid flow rates, Reynolds number, channel structure, or the general orientation and so on.

So, inertial focusing is one of the related topic of inertial micro fluidics or one of the effects that has been utilized in recent years for microfluidic applications very extensively. The other effect which is combined with inertial focusing is the secondary flow. So, secondary flow refers to the flow in a direction normal to the main stream line flow. So, if any channel the flow is happening in the streamline direction then at a cross section normal to this the flow velocities normal to the streamline are generally termed

as secondary flows, which is in addition to the primary flow which is in the stream line direction.

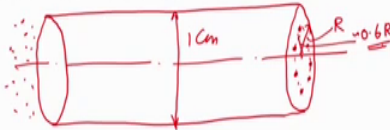
So, this secondary flow can also often be combined with the inertial focusing and this. Secondary flow can be achieved by having some changes in the geometry of the channel for example, in the curved channel one can have secondary flow, or if one can have grooves from the channel, or obstacles in the channel, or some pillars used as obstacle in the channel. So, these structured channels can alone or coupled with the inertial focusing can have different effects in the microfluidic applications.

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### Tubular Pinch Effect

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Segre and Silberberg (1962) effect:  
"Randomly distributed millimetre-sized particles migrated laterally to focus on an annulus of radius 0.6 times the channel radius in a circular channel of one cm diameter."



- A force in the transverse direction (lift force) is required
- Equilibrium position: At least two types of opposing forces

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IIT, Guwahati

8

So, coming to inertial focusing in of 1962 Segre and Silberberg they observe they did an experiment that in a circular channel of one centimeter diameter, they introduced some particles randomly and different locations. So, no preference for any location and what they observed that after some distance the particles focused at a particular position. And this position was about 0.6 times of this if this is are then the radius of this annular location was about 0.6 or about 0.63 times of the channel radius.

And this effect they termed as tubular pinch effect, because this happens at the time when this they observe this there was no explanation for this phenomena. And after that there was lot of effort to explain the phenomenon.

Now, because the particles which are randomly introduced in the channel they settle at a particular equilibrium position. So, there is radial motion of the particles of course, this radial motion will be driven by a force on the particles. So, that means, they are had to be a lateral force or what we term as lift force on the particle, which drives the particles in the radial or the transverse direction.

Now, because there is an equilibrium position; so there has to be or there have to be 2 opposing forces, because if the force acts away from the center of the channel then the particle should go and stick to the wall or if the force is directed towards the center of the channel, then all the particles should be in the middle of the channel.

However, this equilibrium position was observed near the or it was observed between the center and the wall of the channel. It must be pointed out that the concentration of the particles was small. So, they are not particles are not very highly concentrated in the experiments.

So, there has to be at least 2 forces which are acting in the opposite the opposite direction one force that acts towards the center and another force that acts towards the wall. And by the compa by the competition or the equilibrium of these forces the equilibrium positions will be attained. So, there has been a number of analytical studies and computational studies to understand these forces.

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### Lift Forces on the Particles

Two main forces account for the tubular pinch effect

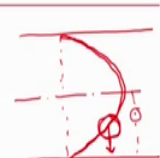
- **Shear gradient lift force**

$$F_{SG} = \frac{C_{SG} \rho U_{max}^2 a^3}{D_h}$$

$D_h \rightarrow$  Hydraulic diameter

$C_{SG}$  is a function of Reynolds number and position

- force directed towards the side with a higher relative velocity



Parabolic (Quadratic)  
 $U = 2U_{avg} \left[ 1 - \frac{r^2}{R^2} \right]$   
 $H_s$  and  $Lead$   
 $Len$

IIT, Guwahati

9

And now, it is established that there are 2 forces on the particle that are responsible for this tubular pinch effect. So, one force is caused by the shear gradient lift force. So, the shear flow is the force, because of the gradient in the velocity or the shear flow where we have gradients in the velocity. And the shear gradient lift force is caused by the gradients in the shear so; that means, the velocity, gradient, are not uniform, but they are varying along the channel.

So, this example for example, in a parabolic profile which is quadratic you can see this the velocity profile is  $2 U_{\text{average}} \left( 1 - \frac{r^2}{R^2} \right)$  in a channel.

So, this force in a channel and the force on the particle acts away from the wall this was seen by Ho and Leal and Cox separately. So, they observed that the particle will have when it is in a shear gradient force then it will have 2 different velocities. And these velocities will not be equal and to compensate for that the fluid will the flow field will change and a net force will act on the particle. And this net force will be in general be directed towards the side with a higher relative velocity and it. So, happens that in a channel the higher relative velocity will be always towards the wall. So, the force will be directed towards the wall.

The scaling for this force has been given as a lift coefficient which is called  $C_{SG}$  or constant for  $cl \text{ gradient } \rho u_{\text{max}}^2 a^3 / d_h$  where  $d_h$  is the hydraulic diameter of the channel and this  $C_{SG}$  is not necessarily constant as a function of the Reynolds number of the channel and the position of the particle; so the distance of the particle from the center of the channel.


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### Lift Forces on the Particles

Two main forces account for the tubular pinch effect

➤ Wall lubrication force  $F_{WI} = \frac{C_{WI} \rho U_{max}^2 a^6}{D_h^4}$

$C_{WI}$  is a function of Reynolds number and position



Particle lags behind the fluid.  
Pressure builds up in the constriction between particle and wall.

IIT, Guwahati 10

Another force that is required that should be acting away from the wall and that comes from the wall lubrication force that when particle moves near the wall, then 2 things happen; one because of the wall effect the particle lags behind, the particle lags behind the fluid. And the streamlines the bulk of the fluid moves away and a pressure builds up in the constriction in the constriction between particle and wall.

So, when the fluid that moves bulk of the fluid moves away from the wall the stream lines become dense at the pressure is high compared to the other side of the particle of course, there will be a force directed away from the wall. And this force is scaled as a constant, which is constant for lift coefficient of wall lubrication into  $\rho U_{max}^2$  into  $a^6$  divided by  $D_h^4$ ; so there while this velocity scaling it same, but the dependence of the 2 forces on the size of the particle and the size of the channel is different.

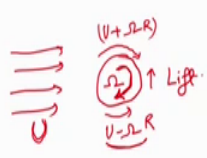
And of course, the  $C_{WI}$  is also a function of the Reynolds number and the position of the wall. So, the dependence of the velocity is not necessarily, what we see this equation because the universe number also have the velocity.

So, by these 2 opposing forces the effect of the or the equilibrium position of the particles due to the inertial focusing was explain or this tubular pinch effect was explained.

(Refer Slide Time: 25:21)

### Other Lift Forces

➤ Rubino and Keller (1961): Lift force on a rigid sphere translating in a quiescent liquid with a rotation perpendicular to the translation

$$\underline{F_{RK}} = \pi a^3 \rho \underline{\Omega} \times \underline{U}$$


IIT, Guwahati 11

Now, one might also think of some more lift forces for example, a lift force acts on a particle, which is rotating in a uniform flow that there might be a relative velocity between the particle and the fluid.

So, if the particle is rotating with a velocity  $\omega$  and there is a relative velocity between the particle and the fluid. So, the fluid which on the side where it supports the rotation of the particle supports, the velocity then one will have a higher velocity. Let us say  $U + \omega R$  on the other side the velocity will be reduced  $U - \omega R$  not necessarily the same magnitude, but.

So, that will create velocity difference and by considering the Bernoulli theorem we know that when the velocity difference is so, at the lower side we will have high pressure. So, the force will be directed from this side to the other side. So, there is a lift force. And this lift has been defined here the magnitude of this lift can be given by this formula has given by Rubino and Keller, but the magnitude of this lift force on a rotating particle was significantly low, when you compare the shear gradient lift force or the wall lubrication forces on the body ok.

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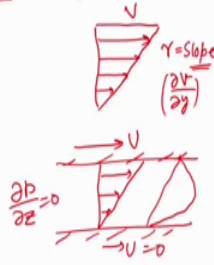
### Other Lift Forces

- Saffman (1965) studied the lift force in a simple unbounded shear flow

$$F_S = 6.46a^2\gamma^{0.5}(\rho\mu)^{0.5}V$$

- Shear rate  $\gamma$
- Relative velocity of particle  $V$
- One order of magnitude larger than force due to rotation  $F_{RK}$

Sphere would migrate towards the side where the fluid velocity is largest



IIT, Guwahati 12

Then the other force is very closely related to the shear gradient lift force. So, in this case there was not a gradient, but a simple quite Couette flow. So, those do not know quite flow Couette flow is the flow in which then the velocity profile is linear. And that is often achieved by flow between 2 parallel plates in which the upper plate moves with a velocity and lower plate is fixed. There is no pressure gradient in the flow, because if there is a pressure gradient and one can superimpose, the parabolic velocity profile and the linear velocity profile and the resulting velocity profile we will look something like this depending on the pressure gradient and the velocity so anyway.

So, the Couette flow is the simple shear flow. So, Saffman studied that lift force in a simple unbounded shear flow can be given by this formula. Where  $V$  is the relative velocity between the particle and the fluid  $\gamma$  is the slope of this; that means, it is the velocity gradient  $\frac{dv}{dy}$  and  $a$  is the radius of the particle  $\rho\mu$  under properties of the fluids. So, he introduced a force which acts on the particle.

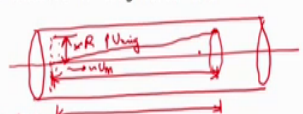
Now, this force was one magnitude one order of magnitude larger than Rubino Keller or F R K, which is because of the rotation of the particle, but this and it will act on the sphere because of this which the escaped migrate towards the side where the fluid velocity is larger, but the magnitude of this force was also not able to explain that it was also significantly small or negligible when you compare with the shear gradient lift force.

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### Channel Length

Can be estimated based on the lateral velocity

➤ Lateral velocity: calculated from the balance of drag force and shear gradient lift force



$U_{\text{migration}}, t_{\text{max}}$   
 Lateral velocity of the particle  
 Time required for the particle to reach equilibrium position

$L_{\text{focusing}} = U_m \cdot t_{\text{max}}$   
 $= U_m \cdot \frac{D_h^2 (3\pi\mu)a}{C_{sg} \rho U_m^2 a^3}$

Balance between shear gradient lift force and drag force  
 $\frac{C_{sg} \rho U_m^2 a^3}{D_h} = 3\pi\mu U_{\text{mig}} a$

$L_{\text{focusing}} = \frac{3\pi\mu D_h^2}{2 C_{sg} \rho U_m a^2}$

13

So, now having understood the forces or the physical effects behind that tubular pinch effect, if you want to develop a device in which the fluid velocity we want to calculate the focusing positions, then it is important to know that; what is the channel length that is required for the focusing of the particles? So, the length let us say the particles are introduced at this position and the length that will be required in which the particles can focus from the random positions to their equilibrium positions.

Now, so, this can be calculated  $L_{\text{focusing}}$  will be equal to  $U_{\text{migration}}$  into time. So, the migration is the velocity lateral velocity of the particle and time is the time required for the fluid or time required for the particle to reach and for design purposes we should consider this  $t_{\text{maximum}}$ . So, which will be  $t_{\text{maximum}}$  will be equal to the let us say the maximum distance that will be slightly less than  $R$ ; if the particle which is near the center and want to reach the equilibrium position. So, the maximum distance that we might need to travel a fraction of so, because you do not know that fraction. So, let us say that this distance is  $R$  and the velocity the lateral velocity is  $u_{\text{migration}}$  and sorry this is naught.

So, these are the 2 factors that will be required for calculating the  $L_{\text{focusing}}$ , but that will not be equal to like this. So, what  $L_{\text{focusing}}$  will be equal to the streamline velocity? So, let us say we take  $U$  or  $U_{\text{maximum}}$  into  $t_{\text{maximum}}$ . So, that will be the focusing

distance the time in which the particle will be traveling in a path like this, if we decompose these 2 paths 1 distance is this and the other distance is the radial distance.

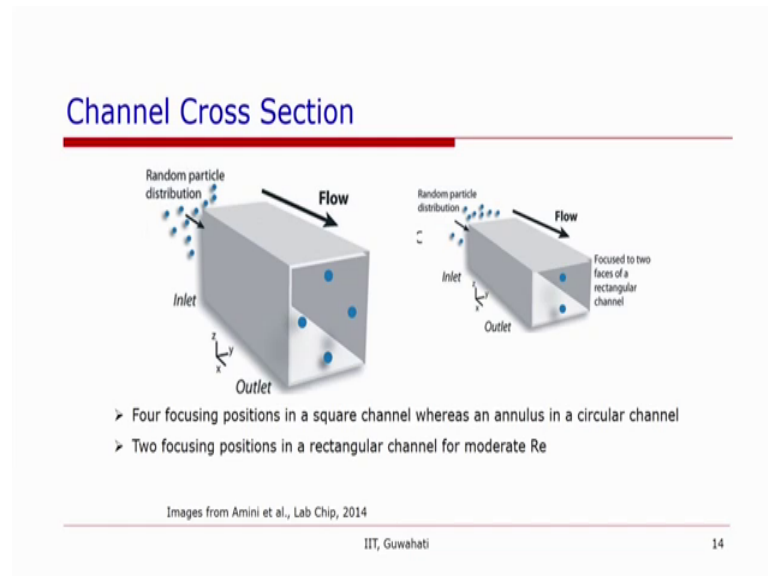
So, what we want to find focusing or distance and this distance we have we have we have approximated as  $R$ . So, the time to reach this position in both to cover both the distances will be equal, but the velocities will be different the velocity in the axial direction we scale it as and velocity in the radial direction is  $U_{migration}$ .

Now, to calculate  $U_{migration}$  we can consider the balance between shear gradient lift force and drag forces shear gradient lift force is  $C S G \rho \frac{a^3}{d_h} \rho U_{max}^2$  over hydraulic diameter on the particle.

That will be equal to the drag force on the particle, which will be because the drag forces. So, will be acting in the radial direction and this will be because of the motion in the radial direction. So, the velocity that we considered is  $3 \pi \mu$  and because the velocity magnitude will be small, so the stokes flow we can expect the flow to be stokes. So, that is why we can use by stokes drag here and we can use the definition or the balance of the forces to calculate the migration velocity and from that we can substitute it into  $t_{max}$  and get the length for focusing lens.

So, that is equal to  $U_m$  into  $t_{max}$ . So,  $R$  is  $D_h$  by 2 divided by  $U_{migration}$  which is  $C S G \rho U_m^2 \frac{a^3}{D_h}$  will go there we have also  $3 \pi \mu$ . So, one you have we will cancel out and we will have into  $a$ . So, we also have  $3 \pi$  by 2  $\mu D_h^2$  and we will this will all  $3 \pi \mu$  into  $a$ . So, we also have  $a$  here. So,  $\mu D_h^2$  1 of  $a$  will cancel out and we will have  $\rho U_m^2 a$  as into  $C S G$ . So, by this formula we can make and estimate of the focusing length.

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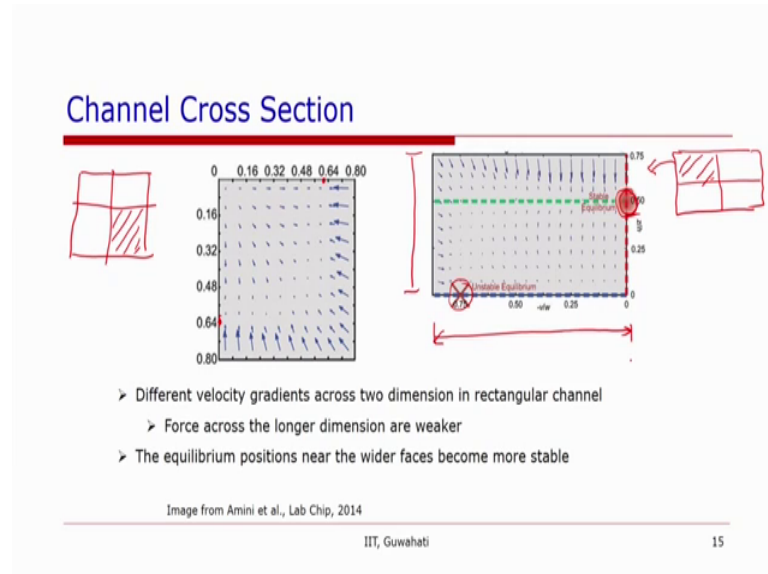


Now, looking at the effect of the cross section of the channel so, in microfluidics it is often convenient to have the channels, which are rectangular in shape or square in shape. So, what we have seen till now that there are focusing positions or there is because the geometry of a cylindrical or circular cylinder is symmetric about the axis. So, the particles will focus on an annular location and, but in a rectangular channel it is not so, obvious.

And so, number of experiments have been done especially by Decarlo and others. So, it has been observed that there are 4 focusing positions in a square channel and this position is about 0.6 at a distance of  $0.6 d_h$  by 2 from the center of the channel.

Now if the particle aspect ratios increased; that means, not the particle aspect ratio, but the channel aspect ratio is increased; that means, the channel becomes rectangle rectangular, then at moderate Reynolds number it has been observed that the focusing positions changed from 4 to 2.

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This is explained by Decarlo and others by the magnitude of the forces that act on the particle. So, in a rectangular channel, what is shown here is the force field in a quarter of a channel. The particle experiences similar forces from both the sides of the wall, because the velocity gradient are same. So, the particle focusing position it is somewhere in one of the there will be one focusing position here and another focusing position somewhere here.

However, when it is rectangular channel the velocity gradient in this direction so, this is a part of a rectangular channel and that is what is shown here. And while the velocity this distance and this distance is different. So, the velocity gradient in this direction, because the distance it is small. So, gradient is large is this gradient is small. So, the force on the particles is larger here then in this direction. So, this position seems to become unstable and the particles focus on these 2 locations. And this has been utilized in a number of applications to change the equilibrium position of the particles to reduce the equilibrium positions of the particles or 2 only 2 positions; so that the particles can be separated easily.

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### Reynolds Number Effect

- At higher  $Re$ , the equilibrium position shifts slightly towards the wall.
- The increase in shear gradient lift force is relatively larger than for wall lubrication force
- In rectangular channels, can lead to increase in number of focusing positions

IIT, Guwahati 16

However, the Reynolds number can also affect true the equilibrium positions though only slightly, by increase it has been observed that the shear gradient lift force is a stronger function of Reynolds number. So, the increase in shear gradient lift force is relatively large, then the wall lubrication force. So, what will happen that the force on the particle the shear gradient lift force will be larger than the wall lubrication force? So, the equilibrium positions are they will shift towards the wall.

In rectangular channels this might sometime also lead to the focusing position increasing. So, the number of focusing position can be come again 4.

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The slide is titled "Deformability effects" in blue text, underlined with a red line. It contains a bulleted list of points. The footer includes "IIT, Guwahati" and the slide number "17".

- Deformability leads to additional lift forces
  - Lift possible even in Stoke's flow
- Di Carlo et al. (2014) categorised deformable particles in 3 categories
  - Elastic solid particles
  - Deformable drop
  - Deformable capsules
- Equilibrium positions closer to the channel centre for deformable particles than for rigid ones
  - Deformability induced lift acting towards centre

IIT, Guwahati 17

Now, what we have been discussing till now is the motion of rigid spherical particles or the focusing of rigid spherical particles, what if the particles are deformable? So, the deformable particle can be classified into 3 different categories or where they are they can be elastic solid particle, where the particle outer wall behaves elastically and the inside of the particle also behaves as an elastic solid.

The other category is droplet where you have a fluid. So, the behavior at the interface is governed by the interfacial phenomena or the surface tension. So, that elastic effect is coming from surface tension and there is fluid inside the particle, because of the shearing motion at the interface the fluid inside will also have a motion and that will reduce the drag on the droplet.

So, the deformable droplet is another category and the deformable capsules. So, in the capsules the capsule might have been filled with a fluid, but the outer wall is a solid elastic or a solid wall. So, that will have a wall that will act as a solid wall and the fluid inside will have the filling inside the particle will be fluid.

So, broadly it has been observed that there is a lift force even in the Stokes flow and the particles are deforming. So, when you look at the inertial effects on the deformable particle and there is superposition of the or there are 2 different forces, which are interacting the inertial lift force and the lift force induced by the deformability of the particle. And by comparison of the equilibrium positions for deformable and rigid

particles, it has been observed that the equilibrium position shift towards the center of the channel for deformable particles when compared with the rigid particle.

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The slide is titled "Deformability effects" in blue text. Below the title, there are two bullet points: "➤ Tanaka et al. (2012) studied migration of deformable cancer cells along with rigid beads" and "➤ Longer focusing length required for rigid beads". A handwritten red equation  $L_{focus} \propto (U_{mig})^{-1}$  is written on the right side of the slide. At the bottom, there is a footer with "IIT, Guwahati" on the left and "18" on the right.

So, that means, that the deformability induce lift it acts towards the center of the channel, utilizing this fact tanaka et al developed a microfluidic device to separate the cancer cells, which are deformable from the rigid beads they did experiments with other cells also, but what we are discussing or what is concerned here that the cancer cells, which are deformable and their motion was compared with rigid particles and they found that the focusing length that was required for rigid particles was more.

So, as we have just seen that  $L$  focusing will be proportional to  $U$  migration. So, this migration velocity or  $L$  focusing will be inversely proportional to  $U$  migration.

So, the migration velocity higher the migration velocity lower the focusing length required or the lower the time required for the particle to reach there. And this migration velocity comes from the shear gradient lift force. So, higher the shear gradient lift force, higher the migration velocity and smaller the focusing length.

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### Application

- No extra force fields or devices required
- Can be parallelised by branching off many channels from a single inlet
- Application in high-throughput sheathless flow cytometry

Image from Hur et al., Lab Chip, 2010

Can process 1 million cells  $s^{-1}$   
A standard cytometer 10000  $s^{-1}$

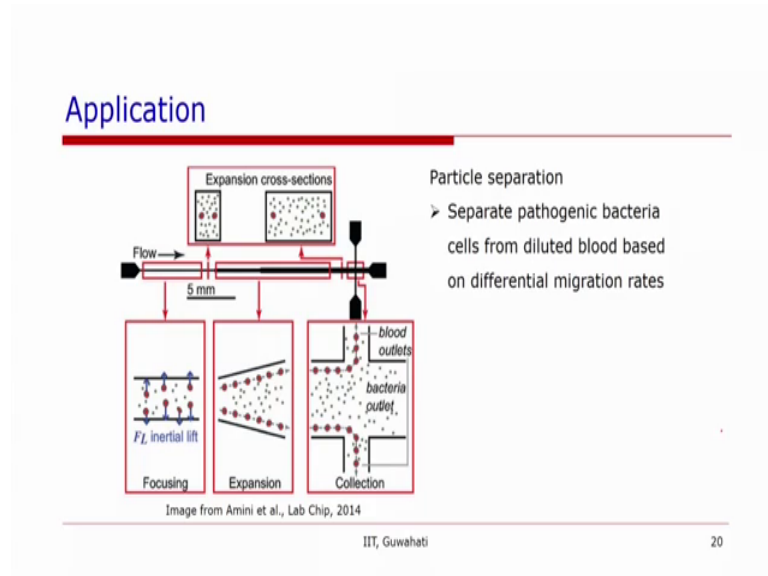
IIT, Guwahati 19

Now, we look at some of the applications of these inertial fluidic effects so, the first application is in C plus flow Cytometry. So, the flow cytometry referred to the focusing of the cells for cells sorting or for cell counting in a number of biological applications.

And this is done by hydrodynamic focusing in a standard cytometer, where the processing rate or the throughput is about 10 000 cells per second. They have shown it has been shown by hur et al that the processing rate can be increased up to 1 million cells per second by using inertial micro fluidics. One of the there are 2 factors that account for the higher processing rate; one is the higher flow velocity and the other is parallelization is possible in the inertial microfluidic channel.

So, in the inertial micro fluidics application or the inertial focusing applications so, number of the had probably 256 channels in parallel. And those particles they achieve the equilibrium position there was no extra fields force fields or device, which was required. So, this method of focusing is Paseo focusing, where the structure of the channel and flow velocity are utilized to focus the particles and then the counting of the particles can be done.

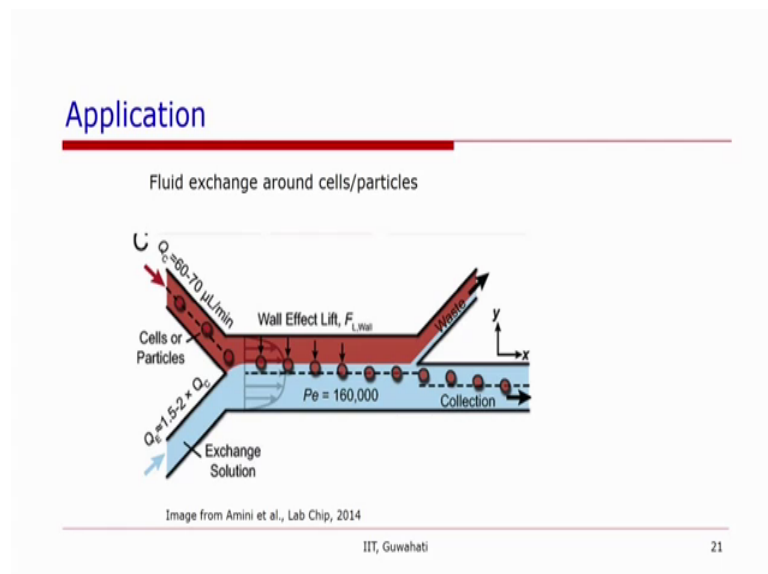
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Another application is in the separation of the particles, where the blood cells they are separated from the bacteria. So, the size of the blood cells are larger than the bacteria and due to the inertial force the particles the blood cells, which are significantly larger in size they focused to the equilibrium positions.

And then by using a expansion channel expansion the focusing position was shifted towards the sidewalls and then these particles were separated collected. So, the particles were separated from the pathogen bacteria in the fluid. So, that is another application.

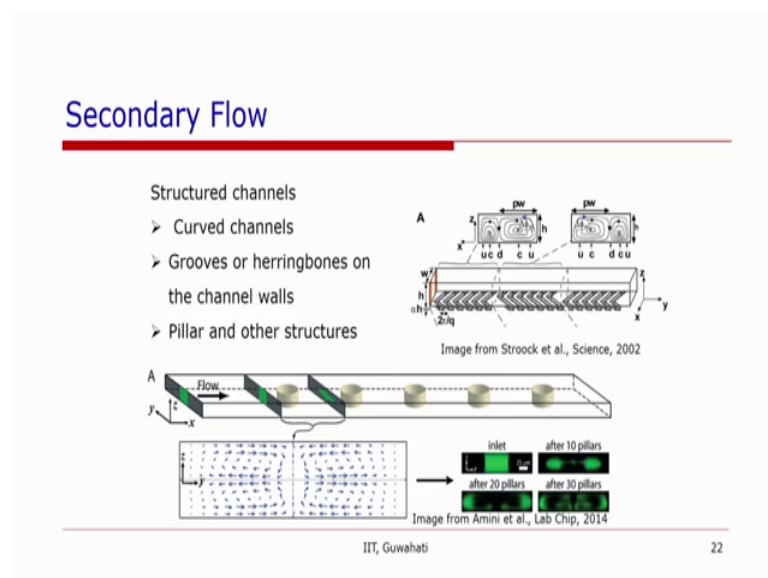
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Then the third application is in fluid exchange around cells and particles. So, here the particles are present in one fluid and the particles are exchanged or migrated towards another fluid by so, in this application the rectangular channel was used. So, we have there are 2 focusing positions and by using the different flow rates of the fluid the focusing positions were manipulated.

So, that the focusing positions are in the second fluid. So, the particles when they arrive from the 2 separate channels into this channel, they migrate to a position where the focusing position is in the second fluid. So, a particle migrate from the first fluid to the second fluid and then these fluids are these particles are separated.

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Now, so, till now we have looked at the flow of particles in a structured channel, in a straight channel without any structured in the flow or the channel without any modification in the channel geometry except the cross section of the channel, where the cross section we have looked at the cross section of the channel to be circular or rectangular or square shaped. Some people have also worked with trapezoidal channel and have got some interesting visuals there.

In the structured channel what we are looking at that by changing the channel path or by creating some structures in the channel some secondary flows can be generated. So, we have listed here 3 different structured channels the channel shape which is which can be curved. So, we will look into it a bit later.

The other structured channel is grooves or herringbones on the channel walls [vocalized-noise]. So, there are grooves created on the one wall of the channel. And what this does that this causes a secondary flow in this plane in this plane is shown here. So, you can say that in the  $x y z$  and this is an  $xyz$  plane. So, there is motion in the secondary direction or the secondary flow in this case.

And this causes good mixing and this because there is secondary flow. So, the focusing position this will also change the focusing position of the particles. So, one can utilize such structures to good effect to change the equilibrium position say from 2 equilibrium position to one equilibrium position or can use different applications using the a structured channel coupled with inertial focusing.

The other structured channel is creating the pillars in the channel. So, if one creates the simple geometry in which the pillars are kept at different locations regular at the regular interval in the space and once is secondary flow in the particle. And then combined with energy and focusing one can again have different equilibrium positions change the number of positions to develop applications.

So, now coming to curved channels, in the curved channels there is an interesting phenomenon.

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### Secondary Flow

Curved channel:

- Centrifugal forces come into play
- Max velocity shifted to concave wall
- Sharp velocity gradient between point of maximum velocity and wall
- Sets up a pressure gradient in the transverse direction
- Leads to recirculation of fluid from the centre towards the wall
- More vortices at high Re

IIT, Guwahati
23

When the flow is the flow happens from say into the page in this direction. So, the inner wall which is the convex wall and the outer wall, which is concave wall there will be a force, which is centrifugal force on the fluid and that force will shift the velocity profile towards the inner wall.

So, the main velocity profile velocity profile in the main direction will be shift it in the straight channel it is supposed to be parabolic whereas, this maxima will shift towards the concave wall. And this will caused velocity gradients to be sharp here and that will set up a pressure gradient in the transverse direction. As a result the fluid will move from the center of the channel towards the wall and then again come back. So, the recirculation will set up in the secondary direction or in the transverse direction.

So, at low Reynolds number this is that the stoke regime such effect is not observed, but when inertial regime.

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### Secondary Flow

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➤ Dean number:  $De = Re \sqrt{\frac{d}{D}}$

$d$  = channel dia  
 $D$  = Radius of curvature

➤ Dean effect has been utilised in microfluidics for a number of applications

- Enhance mixing
- For 3D hydrodynamic focusing
- For creating an inertially modulated variable focal length lens
- Inertial focusing

$$De = \frac{\sqrt{\text{Centrifugal force} \times \text{inertial force}}}{\text{Viscous force}}$$

IIT, Guwahati
24

Then there are vertices that are observed at very high Reynolds number the number of vertices are observed to increase. And this effect is characterized by dean number, which is Reynolds number into D by capital D and this D number is ratio of viscous forces and centrifugal force and inertial force small D is channel dia and capital D is 2 into radius of curvature of the curve channel. So, this has been utilized this effect has been utilized in a number of applications in microfluidics.

For example the flow is that, because the flow is laminar in micro channels. So, if one had single phase flow one would like to enhance the heat transfer or mass transfer. Then utilizing this effect one can have intense mixing of the fluid. So, this has been used as a means of mixing in channels and it has been observed the heat transfer coefficient or the mass transfer coefficients can increase to 2 to 3 4 when you compare with single phase fully developed flow in a channel. It has also been used for a hydrodynamic focusing or creating an inertially modulated variable focal length lens or it has been used for inertial focusing which we are going to discuss.

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## Secondary Flow

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Curved channel:

- Lateral motion of particles is enhanced by Dean flow
- Equilibrium positions reached faster
  - Shorter distance required for focusing
    - Same focusing length at low Re or De (22 or 4)
    - At higher Re or De (89 or 17), focusing length 5 times shorter
- Modified equilibrium positions
  - Additional drag force on particles due to secondary flow
  - If Dean drag is too strong, can lead to mixing

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IIT, Guwahati
25

So, in the inertial focusing this lateral motion of the particle, because particles are moving in the lateral direction because of the inertial lift, and the secondary flow that further enhances the lateral motion of the particle; so this helps in part of in reaching the equilibrium position faster and that will result in the at distance, that is required for focusing become this is smaller. One can also play around with the number of focusing positions and it has been shown that the focusing length remains unaffected at low Reynolds number or dean number, but at higher dean or Reynolds number the focus length can become significantly shorter.

Equilibrium positions can be modified. So, these equilibrium positions can be modified, because when the particle is moving it will experience more drag and that drag additional drag on the particle, because of the secondary flow. So, it will experience more drag and

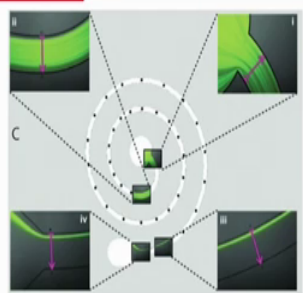
the particle equilibrium position is going to change. However, if the drag caused by the secondary flow by or because of the dean flow is more than this can lead to mixing.

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### Secondary Flow

Spiral channel:

- One direction of curvature
- Steady state Dean flow
- Employed for differential focusing and separation of microparticles



The diagram illustrates a spiral channel with a central spiral path and four inset images (I, II, III, IV) showing the cross-section of the channel at different points along the spiral. The insets show the flow profile and the position of particles, demonstrating how the steady-state Dean flow leads to differential focusing and separation of microparticles.

IIT, Guwahati 26


So, there are 2 different kind of curved channels has been used; one in which the channel it is spiral. So, the curvature is in one direction as you can see here and in this case the dean flow will be steady and this is often employed for differential focusing and separation of particles. So, you can see here that the particles separate and moved to a particular position here. So, this is a spiral channels.

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### Secondary Flow

➤ Asymmetric curving channel:

- Alternating curvature
- Opposing channel segments should be asymmetric
- Initially introduced to reduce number of focused streams
- Enhance inertial focusing: shorter focusing length



The diagram shows an asymmetric curving channel with an inlet labeled 'Inlet (Random)' and an outlet labeled 'Outlet (Ordered)'. The channel has alternating curvature, and the flow profile shows particles being focused into a single stream. The diagram also indicates 'longitudinal ordering' and 'lateral focusing'.

IIT, Guwahati 27

The other one is the curved channel where it is alternating curvature, but notice that this curvature is not symmetric it is not same here and in this. So, one might need to make sure that these opposing channel segment segments they are not symmetric. So, that the curvature effects are not nullified by the 2 symmetric portions, which are alternating. And in this it has been shown that the focusing length can be reduced and number of focused streams can also be reduced in the in the channel. So, this has also been used for a for applications of cell separation or cell focusing.

So, in summary we need to take into account the inertial effects in microfluidics and this has led to a the inertial effects or the non-linear effects, in microfluidics has led to a number of interesting applications, in particular in the inertial focusing. So, in this lecture we have first look at the tubular pinch effect, which suggests that in a channel at high Reynolds number in laminar flow the particles achieve a equilibrium position. And this equilibrium position is a function of channel Reynolds number, channel cross section and the structure of the channel.

And these have led to a number of applications, where microfluidic devices can be developed by playing around with the channel structure, the particle shape, the particle deformability and so on and so forth.