Multiphase Microfluidics
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Lecture – 14
Taylor Flow: Heat transfer 2

So, in the last class we looked at the basics of heat transfer in single phase flow, some basic definitions and then some typical characteristics of heat transfer in the Taylor flow regime. Based on those, based on the understanding that we developed in the last class, let us try to develop a general correlation for the heat transfer in the Taylor flow regime. Of course, we would need to make some assumptions to make this a closed system of equations.

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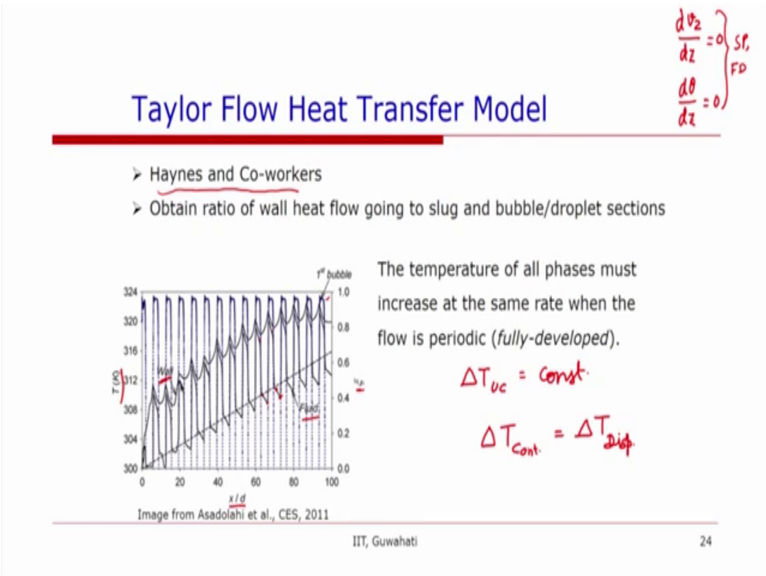

So, that correlation which can be used or a system of equations which can be used to predict the overall heat transfer in a unit cell for fully developed asset number in a channel.

So, up before we go on, let me try to say few words or sensitized you a bit about the fully developed Taylor flow, because our general definition of fully developed is that for velocity. We say that del or dvz over dz is equal to zero and d theta over dz is equal to zero, my apology, because I keep exchanging this bit, either x or an z. Probably you will understand that these are, whenever I use this, most of them, most of the time it is referring to x axial coordinates sometimes I call it x and sometimes I call it z. So, this is for true, for single phase fully developed flow, but when we talk about Taylor flow, it is not possible to achieve this, because the velocity at the middle of the bubble and the velocity in the liquid slug or even in the liquid slug at two different location, is going to be different

So, what we mean by fully developed Taylor flow or what somebody else mean by fully developed Taylor flow is that, the flow has become fully periodic. See if you look at two periodic locations then the flow has achieved period of periodicity. So, if you compare the velocity at two periodic locations, then this should be equal. Then it has become, flow has become fully periodic hydrodynamically. So, if the model that we present here, is based on professor Brian Haynes and his students and collaborators professor David Fletcher

So, what we are looking at in this plot, is along the distance, the temperature and there are two different temperatures have been plotted; the temperature at the wall of the channel and the temperature of the fluid at the at the bulk temperature of the fluid and this has been done for a number of bubbles and this is a snapshot in time. So, one can look at the different bubbles from the inlet and the wall temperature is changing. So, it has not become steady, yet it is almost. So, what we look at that, the temperature is increasing continuously and these are the, this is the non dimensional radial location

So, this is from zero to 1. So, that is basically smaller by capital R and this is the wall of the channel and the blue symbols represent the shape of the bubble. So, one can look at the temperature of the liquid and the temperature of the gas, both of them decreases, but there is a jump here. So, if we look at the corresponding location. For example, this point and this point, there is a corresponding increase in delta t.

And similarly there is corresponding increase in delta here, and that the same like. So, there is a increase in temperature in both. So, when the flow becomes fully developed, then delta T in a unit cell is a constant and it will also require that the temperature of the two phases. So, the temperature of the continuous phase and the temperature of the dispersed phase arise in the temperature of the continuous phase, and the temperature of the this dispersed phase are same.

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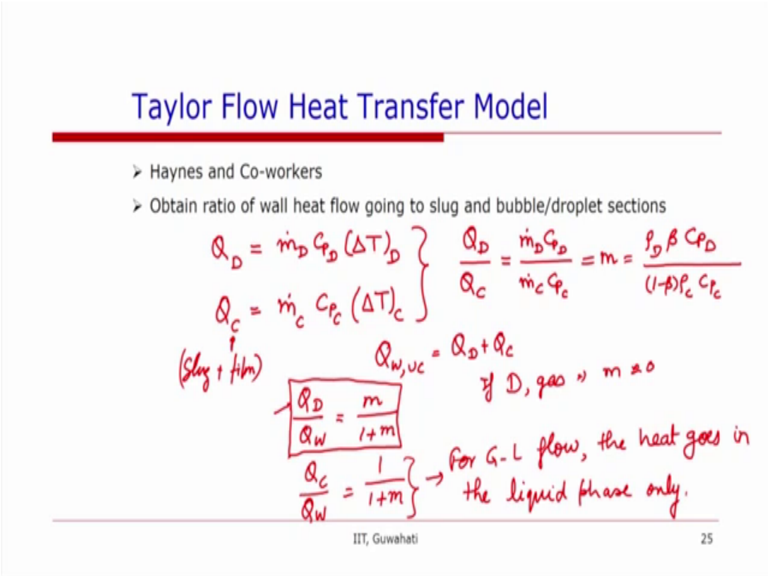

So, based on this we can write the heat flux Q by the dispersed phase is equal to m dot dispersed phase CPD into delta T of dispersed phase. Similarly the Q for slug will be equal to m dot slug CP slug or we can say, because what we are looking at rather than slug we are looking at, here is the continuous phase which is slug plus the film. So, m dot c is CPC delta T continuous phase

Now, based on this we can get, because delta T of dispersed phase and delta T of continuous phase is going to be same. So, we can basically obtain QD over QC is equal to m dot DCPD over m dot c. CPC is equal to m and the total heat that will be coming from the wall. So, Q wall in a unit cell will be equal to QD plus QC. So, one will have QD over Q wall is equal to m over 1 plus m and Q continuous phase divided by Q wall is equal to 1 over 1 plus m. You can also write m in terms of rho d into beta CPD divided by 1 minus beta rho C CPC

We will have in both the cases u here, but then the u will cancel out. So, that is the definition of m. Basically the ratio of thermal mass mass flow rate multiplied by the heat capacity. So, that is important relationship that how much or what fraction of the heat is going to the dispersed phase and what fraction of the heat is going to the continuous phase, if d is gas. So, then we can see that m, because beta and the minus beta they will be equivalent numbers, CPC are also of equivalent and. So, because rho gas is very small, when you compare with that of liquid. So, m is almost negligible; so you can neglect and most of the heat, then this shows that for gas liquid flow the heat goes in the liquid phase only ok.

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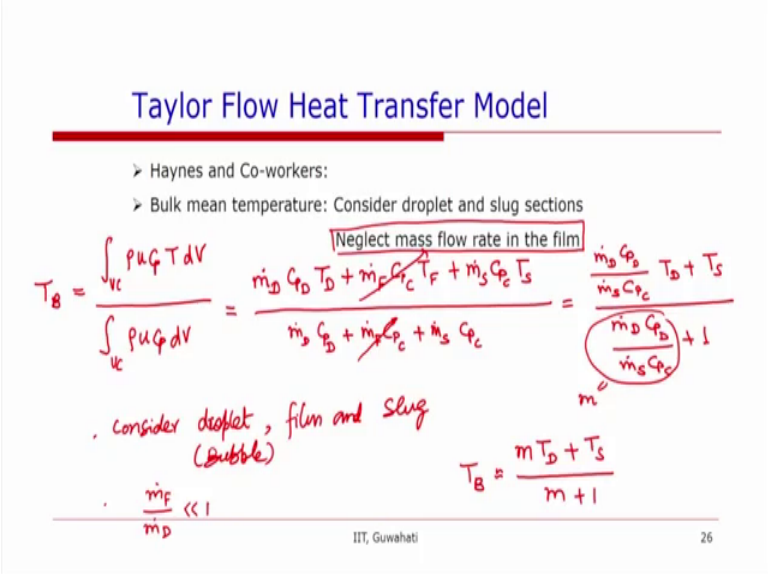

Now, we define the bulk temperature. So, bulk mean temperature can be defined as integral per unit cell rho u Cp Tdv divided by integral for a unit cell rho u Cp dv. So, in an average, we can write this, because rho u is rho udv refers to the mass flow rate. So, we can write in the sense m dot for the dispersed phase. Actually we can consider three phases or consider three zones, consider droplet or bubble film and slug

M dot d CPD and temperature of the dispersed phase plus m dot film Cp of the continuous phase, because film is in the continuous phase only, and this is temperature of the film plus m dot slug cp, continuous phase temperature of the slug divided by m dot d CPD plus m dot FCPC plus m dot s CPC. So, we can neglect the flow rate in the film. So, we are neglecting the mass flow rate in the film and; that means, we say that m dot film is small compared to m dot T, not m dot slug

So, we do that then we have this relationship turning as m dot d CPD divided by m dot s CPC td plus Ts divided by m dot d CPD over m dot s CPC plus 1. Now this is approximately equal to m, why approximately, because we have neglected the mass of the film in both the cases for the continuous phase. So, this Tb is mTD plus Ts over m plus 1. So, one of the approximations that we have made in this case is that we have neglected the mass flow rate in the film ok, which is quite ok. If we are using for gas liquid flow for, but for liquid, liquid flow or for film that surrounds the slug that is not or that may not be the case

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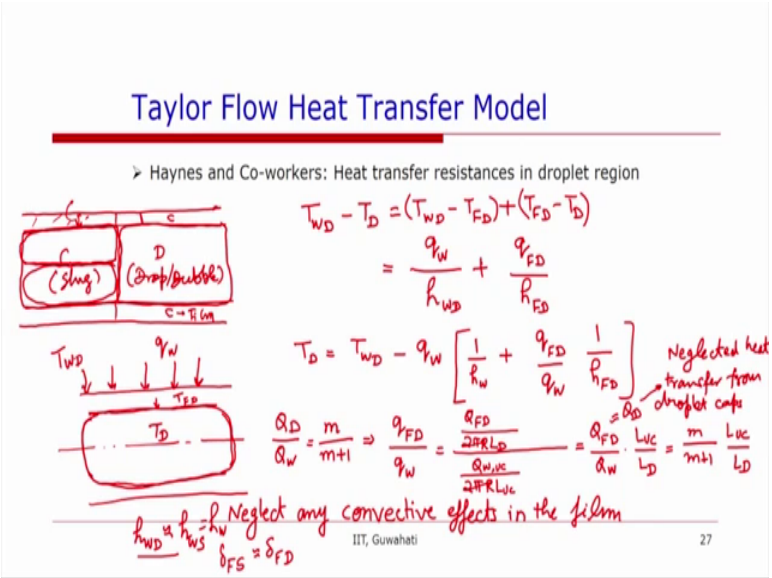

So, now we will be looking at the heat transfer in the droplet regime. So, the picture that we have in this case, is like this. We have a continuous film in which there is a stream of continuous phase, which is slug and dispersed phase which is drop or bubble move. Now we can treat this, the films in the slug and drop.

So, it is a very clear the that there is a film surrounding the droplet or surrounding the bubble, because this is of continuous phase; however, for the slug one needs to think a bit, because the liquid recirculates mainly in the middle region and then beyond this recirculating region, there is a small film and this film, the thickness of this film may or may not be same of the film, surrounding the droplet or bubble, but for our analysis we will assume the two films to be equal to simplify to simplify things

So, what we are going to look at the in this case is, account for the resistances in the droplet regions. So, we will draw it again, if the channel wall, a droplet and the heat entering on the wall, the heat flux being qw and then it comes into the film and from film, it enters in the droplet region and so on. So, the temperature difference T wall in the droplet region. So, the temperature of the wall is TWD minus T droplet which is temperature of the droplet we can also write this as this is the film. So, the film temperature we write as TFD or the temperature at the interface between the film and droplet

So, we can write this as TWD minus DFD plus TFD minus 3D. So, we have just subtracted and added the film temperature. Now we can write this T wall minus TFD is equal to q wall; that is the heat coming in divided by the heat transfer coefficient. So, delta T is equal to q wall over F and that will be a h of wall. In the droplet region we will neglect any difference. So, we will we are going to assume hwd is equal to hw slug regime. So, and that is when we do this, when we write this equation we also neglect convective effect.

So, when we neglect the convective effect, basically we are writing. So, we will do that later on anyway, but by doing this approximation what we are saying that delta f in the slug region will be equal to the delta f in the droplet region. So, that is qw over sw in the droplet region plus TFD minus T D. So, TFD minus T D will be qFD divided by h FD. now qFD over hFD. So, we can write this expression as td is equal to TWD minus qw into 1 over hw plus q Fd over qw 1 over h FD. What we have done is? we have taken T d on this side and wrote it like this dw minus qw has been brought out

So, 1 over sw we have just approximated is, this sw dropped d subscript here plus qFD over qw, because qw has come out. So, we have a qw in the denominator into 1 over hFD. Now we need to write this q FD. So, because we knew that qd over qw is equal to m over m plus 1. You can remind yourself from here qd over qw is equal to m over m plus 1. So, that gives us q FD over qw is equal to q FD divided by the droplet region. So, that will be 2 pi r length of the droplet

Remember this rh, we are approximating r is equal to r droplet. So, we are neglecting the film thickness in this case divided by qw over 2 pi, this is qw for the entire unit cell. So, that is why we have 2 pi r L u c. So, 2 pi r 2 pi r will be cancelled out and what we have is q FD over qw into Luc over Ld, and we can substitute qfd over qw as approximating that q FD is about same as qd. So, what have we just talked about in this m in a minute m over m plus 1 L u c over ld

Now, why qFD over qd. So, the heat what is qd? qd is the heat that is being entered in the droplet or the droplet can have heat entering from the film region and the two caps. So by doing that we have neglected heat transfer from bubble or because we are using droplets; so, from droplet kept at the front and rear ok.

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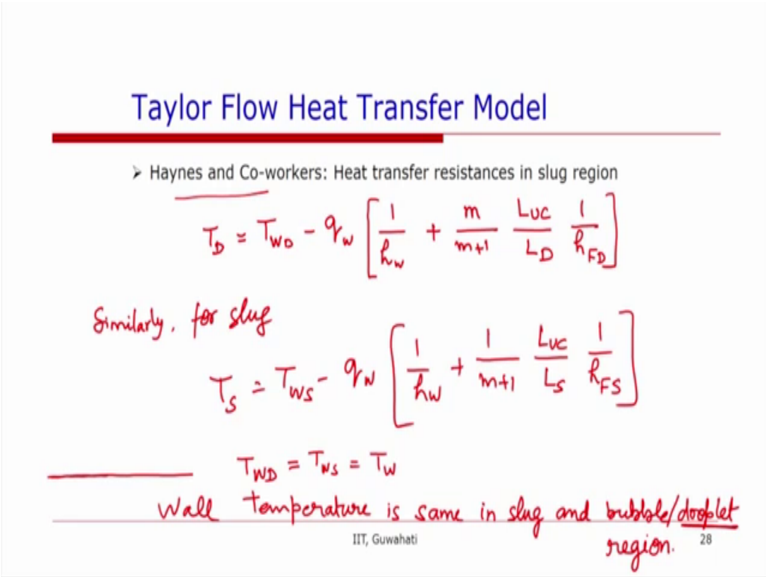

So, when we substitute this, then we will have temperature of droplet is equal to T wall d minus qw 1 over hw plus m over m plus 1 Luc over ld 1 over h F droplet. Similarly for slug one can write T slug is equal to T wall slug minus q w 1 over hw plus 1 over m plus 1. So, we have this as because now it is slug. So, the heat will be 1 over m plus 1 and you see over Ls 1 over h as slug ok. So, we have a relationship between the temperature differences and all this.

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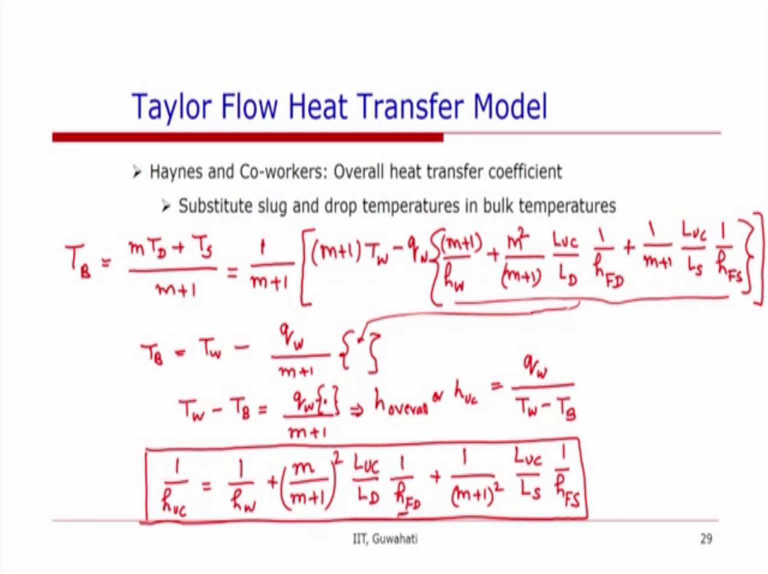

Now, next that the substitute this slug and droplet temperature in the bulk temperature. So, we had this bulk temperature Tb is equal to m TD plus Ts divided by m 1 plus 1. So, we can substitute the two values from here, and we will get m over m plus 1. So, we can write this is 1 over m plus 1 and we will have m plus 1 Tw

Ideally we will have, we had these two terms m TWD plus T ws. So, another approximation we are going to make here, that was what they have made is in their correlation is T wd is equal to T wall plus is equal to T wall. So; that means, saying that the temperature at the wall is same in the slug, wall temperature is same in slug and bubble regions bubble or this approximation is more valid in the droplet region, where the heat capacity of the two phases will be of the same order. So, the temperature will be about same for the two phases.

So, this is m plus 1 Tw minus qw into m over hw or m plus 1 over hw plus m square over m plus 1 luc over ld 1 over h FD plus 1 over m plus 1 Luc over Ls 1 over hFS bracket close. Now we can substitute, we can spend this and we see that Tb is equal to T wall minus qw over m plus 1, and this entire thing in bracket. So, we can just represent this entire expression in the mind metal bracket things out and their expressions in copy here. Now when we substitute this then we will get Tw minus Tb is equal to qw over m plus 1 and we can define an h overall or huc is equal to q wall divided by T wall minus. So, this will be T wall minus q wall into this bracket, entire thing in the brackets where uc is equal to q wall divided by T wall minus T bulk

So, from this relationship we get 1 over huc is equal to 1 over hw. As you can you can see this m plus 1 and m plus 1 will cancel plus m over m plus 1 squared m square and we have one more m plus 1 Luc over Ld 1 over hFD plus 1 over m plus 1 squared Luc over L s 1 over hFS. So, this is a relationship between the overall heat transfer coefficient wall. The heat transfer coefficient at the wall and m which is the ratio of the thermal mass of the two phases slug lengths entire unit cell length and the heat transfer coefficients for the FD and hFS

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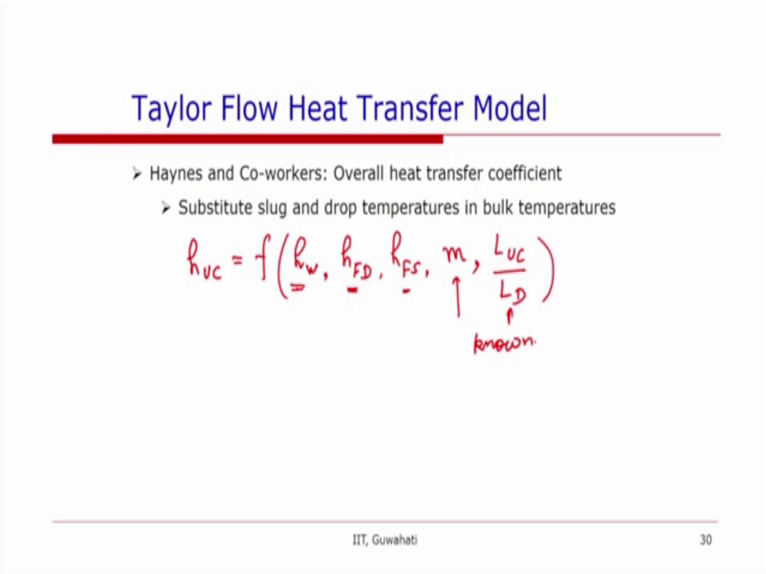

So, if you look at now, what we need is, that overall heat transfer coefficient is a function of three heat transfer coefficient h wall or the film heat transfer coefficient h film to droplet h film to slug and m which is ratio of the thermal mass and Luc over Ld and Luc over Ls what else yeah. So, these are the factors. So, these things we can know from the characteristics of the flow. Now what we need to know are these three coefficient hw, hFS and hFD and if we know, then it becomes a closed system and we can calculate the overall heat transfer coefficient for the Taylor flow region.

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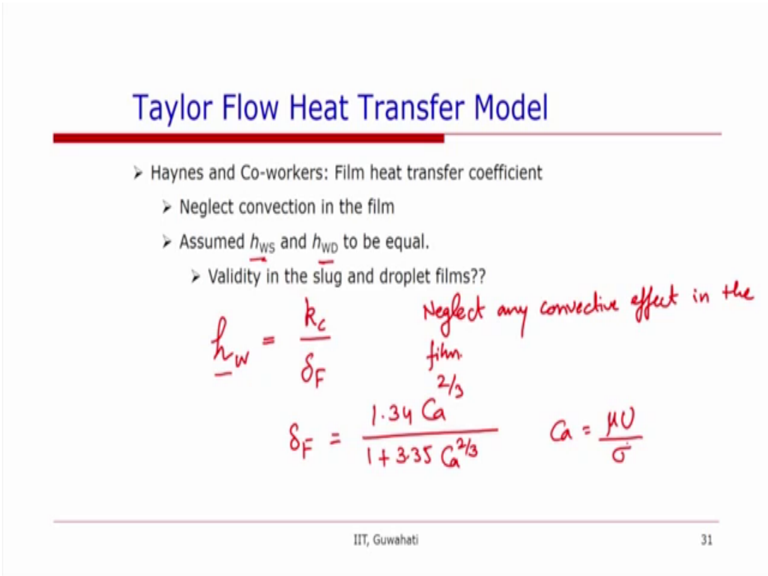

So, as we have already discussed that we have assumed in this case, that the wall heat transfer coefficient for the slug and the droplet regime is assumed to be same, and we can consider only conduction. So, hw is equal to k over delta film, where k is the thermal conductivity of the convective phase. So, we neglect any convective effect in the film. So, the pure conduction will give heat transfer coefficient is thermal conductivity divided by the diffusion length.

So, this is delta F and delta F 1 can obtain from (Refer Time: 34:43) relation of (Refer Time: 34:45) which is 1.34 Ca, which is capillary number raised to the power 2 by 3 into 1 plus 3.35 Ca raised to the power 2 by 3 Ca capillary number defined as mu u over sigma ok

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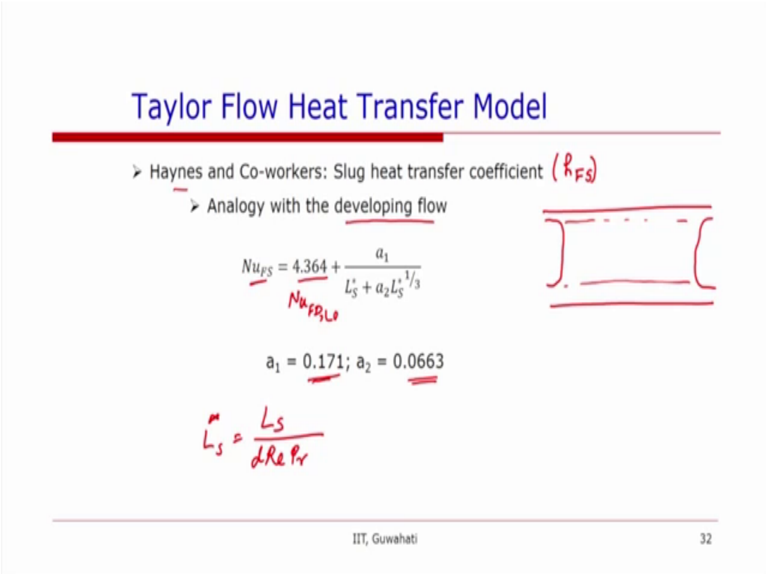

Now, for slug heat transfer coefficient for hfs, because if we look at the flow, the flow in the slug which is moving, it has an analogy with the developing flow. So, is similar correlation that is used for the developing flow in the liquid. only flow case or this liquid flow in the channel, is used, which h for the catch boundary condition 4.364 which is n u for fully developed liquid, only plus a 1 by Ls star plus a 2 Ls star raised to the power 1 by 3. So, where Ls star is inverse regime number or Ls over d Renaults number Prandtl number ok.

So, by fitting their data for the experiments. So, it has been found that the values of a 1 and a 2 are 0.171 and 0.0663, and so that gives that correlation, gives the heat transfer in the liquid slug.

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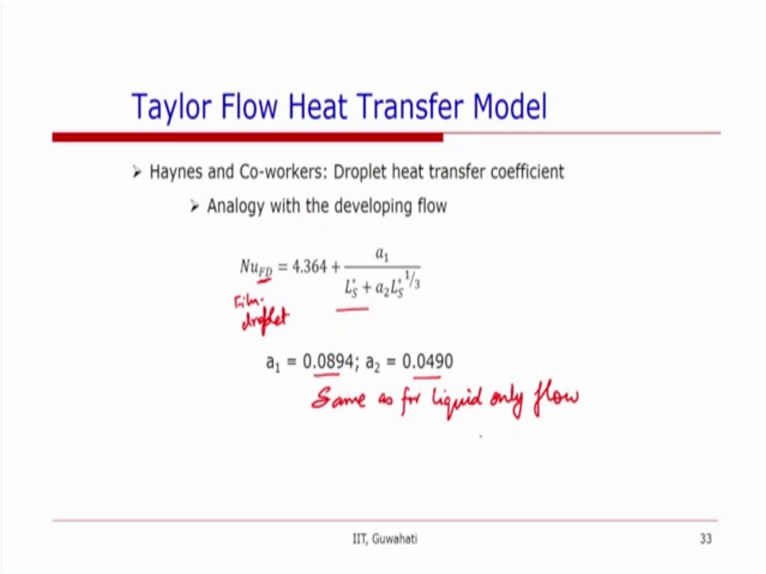

Using similar arguments they have also obtained a relationship for the, this is not fully developed. So, this is film to droplet. So, the Nusselts number is 4.36 for the same correlation, but a 1 and a 2 these are same as for liquid and the flow ok. So, one can obtain Nusselts number for the liquid only from this correlation, and then it is become system of equation becomes closed. So, one can calculate the overall or over all Nusselts number for a in a unit cell in the Taylor flow regime

So, basically in this lecture what we have done is, we have looked that the a simple model based on the fundamental principle for the over all nusselt number in a Taylor flow regime, or in a unit cell which consists of a droplet or a bubble and the liquid slug. The model is based on the assumption that there is a continuous thin film of liquid on the wall and the droplets and slugs. The stream of droplets and slug move, move over it; there are a number of assumptions that we have made along the, because to make a system of equations to be closed one of them, is that the film thicknesses in the around the slug and around the droplet they are same. This may not be the case every time

Another assumption that has been made that the heat transfer to the droplet phase is happening from the film or from to the slug phase, is happening from the film only, and there is no interface in exchange at the interface between the two phases ok. So, once we get to calculate this, we need to calculate the heat transfer at the wall, which can be done by assuming only pure conduction in the channel wall and the correlations for heat transfer coefficients in the droplet. And in the slug flow regime they are calculated based on the developing flow correlations for liquid only developing a correlations for developing flow for liquid only flow in a channel ok.

Thank you.